Neutrosophic Multi relations and Their Properties

Said Broumi
Faculty of Arts and Humanities, Hay El Baraka Ben M'sikCasablanca
B.P. 7951, Hassan II University Mohammedia-Casablanca , Morocco,
broumisaid78@gmail.com

Irfan Deli
Muallim Rıfat Faculty of Education,
Kilis 7 Aralık University, 79000 Kilis, Turkey,
irfandeli@kilis.edu.tr

Florentin Smarandache
Department of Mathematics, University of New Mexico,
705 Gurley Avenue, Gallup, NM 87301, USA
fsmarandache@gmail.com

June 18, 2014

Abstract

In this paper, the neutrrosophic multi relation (NMR) defined on the neutrosophic multisets [18] is introduced. Various properties like reflexivity,symmetry and transitivity are studied.

Keyword 0.1 *Neutrosophic sets, neutrosophic multisets, neutrosophic multi relations, reflexivity, symmetry, transitivity.*

1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory[40], intuitionistic fuzzy sets[7], rough set theory[25] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. However, All these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, inspired from the sport games (wining/tie/defeating), from votes (yes/ NA/ No), from decision making (making a decision/ hesitating/not making) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F.Smarandache[36] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. This theory and their hybrid structures has proven useful in many different fields such as control theory[1], databases[2, 3], medical diagnosis problem[4], decision making problem [16, 21], physics[26], topology [22], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [5, 6, 10]).

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al.[23] presented the concept of neutrosophic soft set which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache[8, 11] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets set and soft set. Broumi et al. presented the concept of rough neutrosophic set[14] which is based on a combination of the neutrosophic set and rough set models. The works on neutrosophic set combining soft sets, in theories and applications, have been progressing rapidly (e.g. [9, 12, 13, 19]).

The notion of multiset was formulated first in [39] by Yager as generalization of the concept of set theory and then the set developed in [15] by Calude et al. Several authors from time to time made a number of generalization of set theory. For example, Sebastian and Ramakrishnan[34, 33] introduced a new notion is called multi fuzzy set, which is a generalization of multiset. Since then, Several researcher[24, 32, 37, 38] discuussed more properties on multi fuzzy set. [35, 20] made an extension of the concept of Fuzzy multisets by an intuitionstic fuzzy set, which called intuitionstic fuzzy multisets(IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researcher [17, 27, 28, 29, 30, 31]. An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multisets allows the repeated occurrences of membership and non–membership values. The concepts of FMS and IFMS fails to deal with indeterminacy. Therefore Deli et al.[18] give neutrosophic multisets.

The neutrosophic relations are the neutrosophic subsets in a cartesian product of universe. The purpose of this paper is an attempt to extend the neutrosophic relations to neutrosophic multi relations(NMR). This paper is arranged in the following manner. In section 2, we present some definitions and notion about intuitionstic fuzzy set, intuitionstic fuzzy multisets, neutrosophic set and neutrosophic multi set theory which is help us in later section. In section 3, we study the concept of neutrosophic multisets and their operations. In section 4, we present an application of NMR in medical diagnosis. Finally, we conclude the paper.

2 Preliminary

In this section, we mainly recall some notions related to neutrosophic sets[36] relevant to the present work. See especially[2, 3, 4, 5, 6, 10, 16, 21, 22, 26] for further details and background.

Definition 2.1 *[36] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets(N-sets) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0, 1]$. It can be written as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2 *[18] Let E be a universe. A neutrosophic multiset(NMS) A on E can be defined as follows:*

$$\begin{aligned} A \quad &= \{< x, (T_A^1(x), T_A^2(x), ..., T_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), \\ &\qquad (F_A^1(x), F_A^2(x), ..., F_A^P(x)) >: \ x \in E\} \end{aligned}$$

where,

$$T_A^1(x), T_A^2(x), ..., T_A^P(x) : E \to [0, 1],$$
$$I_A^1(x), I_A^2(x), ..., I_A^P(x) : E \to [0, 1],$$

and

$$F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \to [0, 1]$$

such that

$$0 \leq T_A^i(x) + I_A^i(x) + F_A^i(x) \leq 3$$

($i = 1, 2, ..., P$) and

$$T_A^1(x) \leq T_A^2(x) \leq ... \leq T_A^P(x)$$

for any $x \in E$.

$(T_A^1(x), T_A^2(x), ..., T_A^P(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^P(x))$ and $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension(cardinality) of NMS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic multisets on E is denoted by NMS(E).

Definition 2.3 *[18] Let $A, B \in NMS(E)$. Then,*

1. *A is said to be NM subset of B is denoted by* $A\tilde{\subseteq}B$ *if* $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$,$F_A^i(x) \geq F_B^i(x)$, $\forall x \in E$.

2. *A is said to be neutrosophic equal of B is denoted by* $A = B$ *if* $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$,$F_A^i(x) = F_B^i(x)$, $\forall x \in E$.

3. *the complement of A denoted by* $A^{\tilde{c}}$ *and is defined by*

$$\begin{aligned} A^{\tilde{c}} &= \{< x, (F_A^1(x), F_A^2(x), ..., F_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)),\\ &\qquad (T_A^1(x), T_A^2(x), ..., T_A^P(x)) >: \ x \in E\} \end{aligned}$$

4. *If* $T_A^i(x) = 0$ *and* $I_A^i(x) = F_A^i(x) = 1$ *for all* $x \in E$ *and* $i = 1, 2, ..., P$ *then A is called null* ns*-set and denoted by* $\tilde{\Phi}$.

5. *If* $T_A^i(x) = 1$ *and* $I_A^i(x) = F_A^i(x) = 0$ *for all* $x \in E$ *and* $i = 1, 2, ..., P$, *then A is called universal* ns*-set and denoted by* $\tilde{E}$.

Definition 2.4 *[18] Let* $A, B \in NMS(E)$. *Then,*

1. *the union of A and B is denoted by* $A\tilde{\cup}B = C_1$ *and is defined by*

$$\begin{aligned} C &= \{< x, (T_C^1(x), T_C^2(x), ..., T_C^P(x)), (I_C^1(x), I_C^2(x), ..., I_C^P(x)),\\ &\qquad (F_C^1(x), F_C^2(x), ..., F_C^P(x)) >: \ x \in E\} \end{aligned}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$,$F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

2. *the intersection of A and B is denoted by* $A\tilde{\cap}B = D$ *and is defined by*

$$\begin{aligned} D &= \{< x, (T_D^1(x), T_D^2(x), ..., T_D^P(x)), (I_D^1(x), I_D^2(x), ..., I_D^P(x)),\\ &\qquad (F_D^1(x), F_D^2(x), ..., F_D^P(x)) >: \ x \in E\} \end{aligned}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$,$F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

3. *the addition of A and B is denoted by* $A\tilde{+}B = E_1$ *and is defined by*

$$\begin{aligned} E_1 &= \{< x, (T_{E_1}^1(x), T_{E_1}^2(x), ..., T_{E_1}^P(x)), (I_{E_1}^1(x), I_{E_1}^2(x), ..., I_{E_1}^P(x)),\\ &\qquad (F_{E_1}^1(x), F_{E_1}^2(x), ..., F_{E_1}^P(x)) >: \ x \in E\} \end{aligned}$$

where $T_{E_1}^i = T_A^i(x) + T_B^i(x) - T_A^i(x).T_B^i(x)$, $I_{E_1}^i = I_A^i(x).I_B^i(x)$,$F_{E_1}^i = F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

4. *the multiplication of A and B is denoted by* $A\tilde{\times}B = E_2$ *and is defined by*

$$\begin{aligned} E_2 &= \{< x, (T_{E_2}^1(x), T_{E_2}^2(x), ..., T_{E_2}^P(x)), (I_{E_2}^1(x), I_{E_2}^2(x), ..., I_{E_2}^P(x)),\\ &\qquad (F_{E_2}^1(x), F_{E_2}^2(x), ..., F_{E_2}^P(x)) >: \ x \in E\} \end{aligned}$$

where $T_{E_2}^i = T_A^i(x).T_B^i(x)$, $I_{E_2}^i = I_A^i(x) + I_B^i(x) - I_A^i(x).I_B^i(x)$,$F_{E_2}^i = F_A^i(x) + F_B^i(x) - F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

3 Relations on Neutrosophic Multisets

In this section, after given the cartesian products of two neutrosophic multisets, we define a relations on neutrosophic multisets and study their desired properties. The relation extend the concept of intuitionistic multirelation [29] to neutrosophic multirelation. Some of it is quoted from [18, 29, 36].

Definition 3.1 *Let $\emptyset \neq A, B \in NMS(E)$ and $j \in \{1, 2, ..., n\}$. Then, cartesian product of A and B is a neutrosophic multiset in $E \times E$, denoted by $A \times B$, defined as*

$$A \times B = \{< (x,y), T^j_{A\times B}(x,y)), I^j_{A\times B}(x,y), F^j_{A\times B}(x,y) >: (x,y) \in E \times E\}$$

where

$$T^j_{A\times B}(x,y), I^j_{A\times B}(x,y), F^j_{A\times B}(x,y) : E \to [0,1]$$

,

$$T^j_{A\times B}(x,y) = \min\left\{T^j_A(x), T^j_B(x)\right\},$$

$$I^j_{A\times B}(x,y) = \max\left\{I^j_A(x), I^j_B(x)\right\}$$

and

$$F^j_{A\times B}(x,y) = \max\left\{F^j_A(x), F^j_B(x)\right\}$$

for all $x, y \in E$.

Remark 3.2 *A cartesian product on A is a neutrosophic multiset in $E \times E$, denoted by $A \times A$, defined as*

$$A \times A = \{< (x,y), T^j_{A\times A}(x,y)), I^j_{A\times A}(x,y), F^j_{A\times A}(x,y) >: (x,y) \in E \times E\}$$

where $j = 1, 2, ..., n$ and $T^j_{A\times A}, I^j_{A\times A}, F^j_{A\times A} : E \times E \to [0,1]$.

Definition 3.3 *Let $\emptyset \neq A, B \in NMS(E)$ and $j \in \{1, 2, ..., n\}$. Then, a neutrosophic multi relation from A to B is a neutrosophic multi subset of $A \times B$. In other words, a neutrosophic multi relation from A to B is of the form (R, C), $(C \subseteq E \times E)$ where $R(x,y) \subseteq A \times B$ $\forall (x,y) \in C$.*

Definition 3.4 *Let $A, B \in NMS(E)$ and, R and S be two neutrosophic multirelation from A to B. Then, the operations $R\tilde{\cup}S$, $R\tilde{\cap}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are defined as follows;*

1.

$$\begin{aligned} R\tilde{\cup}S = \ & \{< (x,y), (T^1_{R\tilde{\cup}S}(x,y), T^2_{R\tilde{\cup}S}(x,y), ..., T^n_{R\tilde{\cup}S}(x,y)), \\ & (I^1_{R\tilde{\cup}S}(x,y), I^2_{R\tilde{\cup}S}(x,y), ..., I^n_{R\tilde{\cup}S}(x,y)), \\ & (F^1_{R\tilde{\cup}S}(x,y), F^2_{R\tilde{\cup}S}(x,y), ..., F^n_{R\tilde{\cup}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cup}S}(x,y) = T^i_R(x) \vee T^i_S(y),$$
$$I^i_{R\tilde{\cup}S}(x,y) = I^i_R(x) \wedge I^i_S(y),$$
$$F^i_{R\tilde{\cup}S}(x,y) = F^i_R(x) \wedge F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

2.
$$\begin{aligned} R\tilde{\cap}S \; &= \{< (x,y), (T^1_{R\tilde{\cap}S}(x,y), T^2_{R\tilde{\cap}S}(x,y), ..., T^n_{R\tilde{\cap}S}(x,y)), \\ &(I^1_{R\tilde{\cap}S}(x,y), I^2_{R\tilde{\cap}S}(x,y), ..., I^n_{R\tilde{\cap}S}(x,y)), \\ &(F^1_{R\tilde{\cap}S}(x,y), F^2_{R\tilde{\cap}S}(x,y), ..., F^n_{R\tilde{\cap}S}(x,y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cap}S}(x,y) = T^i_R(x) \wedge T^i_S(y),$$
$$I^i_{R\tilde{\cap}S}(x,y) = I^i_R(x) \vee I^i_S(y),$$
$$F^i_{R\tilde{\cap}S}(x,y) = F^i_R(x) \vee F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

3.
$$\begin{aligned} R\tilde{+}S = \; &\{< (x,y), (T^1_{R\tilde{+}S}(x,y), T^2_{R\tilde{+}S}(x,y), ..., T^n_{R\tilde{+}S}(x,y)), \\ &(I^1_{R\tilde{+}S}(x,y), I^2_{R\tilde{+}S}(x,y), ..., I^n_{R\tilde{+}S}(x,y)), \\ &(F^1_{R\tilde{+}S}(x,y), F^2_{R\tilde{+}S}(x,y), ..., F^n_{R\tilde{+}S}(x,y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{+}S}(x,y) = T^i_R(x) + T^i_S(y) - T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{+}S}(x,y) = I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{+}S}(x,y) = F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

4.
$$\begin{aligned} R\tilde{\times}S = \; &\{< (x,y), (T^1_{R\tilde{\times}S}(x,y), T^2_{R\tilde{\times}S}(x,y), ..., T^n_{R\tilde{\times}S}(x,y)), \\ &(I^1_{R\tilde{\times}S}(x,y), I^2_{R\tilde{\times}S}(x,y), ..., I^n_{R\tilde{\times}S}(x,y)), \\ &(F^1_{R\tilde{\times}S}(x,y), F^2_{R\tilde{\times}S}(x,y), ..., F^n_{R\tilde{\times}S}(x,y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\times}S}(x,y) = T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{\times}S}(x,y) = I^i_R(x) + I^i_S(y) - I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{\times}S}(x,y) = F^i_R(x) + F^i_S(y) - F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

Here $\vee$, $\wedge$, $+$, $.$, $-$ denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.

Assume that $\emptyset \neq A, B, C \in NMS(E)$. Two neutrosophic multirelations under a suitable composition, could too yield a new neutrosophic multirelation with a useful significance. Composition of relations is important for applications, because of the reason that if a relation on A and B is known and if a relation on B and C is known then the relation on A and C could be computed and defined as follows;

Definition 3.5 *Let R(A→ B) and S (B→ C) be two neutrosophic multirelations. The composition S ∘R is a neutrosophic multirelation from A to C, defined by*

$$S \circ R = \{< (x,z), (T^1_{S\circ R}(x,z), T^2_{S\circ R}(x,z), ..., T^n_{S\circ R}(x,z)), (I^1_{S\circ R}(x,z), I^2_{S\circ R}(x,z), ..., I^n_{S\circ R}(x,z)), (F^1_{S\circ R}(x,z), F^2_{S\circ R}(x,z), ..., F^n_{S\circ R}(x,z)) >: \ x, z \in E\}$$

where

$$T^j_{S\circ R}(x,z) = \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\}$$

$$I^j_{S\circ R}(x,z) = \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\}$$

and

$$F^j_{S\circ R}(x,z) = \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\}$$

for every (x, z) $E \times E$, for every $y \in E$ and $j = 1, 2, ..., n$.

Definition 3.6 *A neutrosophic multirelation R on A is said to be;*

1. *reflexive if $T^j_R(x,x) = 1$, $I^j_R(x,x) = 0$ and $F^j_R(x,x) = 0$ for all $x \in E$*
2. *symmetric if $T^j_R(x,y) = T^j_R(y,x)$, $I^j_R(x,y) = I^j_R(y,x)$ and $F^j_R(x,y) = F^j_R(y,x)$ for all $x, y \in E$*
3. *transitive if $R \circ R \subseteq R$.*
4. *neutrosophic multi equivalence relation if the relation R satisfies reflexive, symmetric and transitive.*

Definition 3.7 *The transitive closure of a neutrosophic multirelation R on $E \times E$ is $\hat{R} = R\tilde{\cup}R^2\tilde{\cup}R^3\tilde{\cup}...$*

Definition 3.8 *If R is a neutrosophic multirelation from A to B then R^{-1} is the inverse neutrosophic multirelation R from B to A, defined as follows:*

$$R^{-1} = \left\{ \left\langle (y,x), T^j_{R^{-1}}(x,y)), I^j_{R^{-1}}(x,y), F^j_{R^{-1}}(x,y) \right\rangle : (x,y) \in E \times E \right\}$$

where
$T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$, $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ *and* $j = 1, 2, ..., n$.

Proposition 3.9 *If R and S are two neutrosophic multirelation from A to B and B to C, respectively. Then,*

1. $(R^{-1})^{-1} = R$
2. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof

1. Since R^{-1} is a neutrosophic multirelation from B to A, we have $T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$ and $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ Then,

$$T^j_{(R^{-1})^{-1}}(x,y) = T^j_{R^{-1}}(y,x) = T^j_R(x,y),$$
$$I^j_{(R^{-1})^{-1}}(x,y) = I^j_{R^{-1}}(y,x) = I^j_R(x,y)$$

and

$$F^j_{(R^{-1})^{-1}}(x,y) = F^j_{R^{-1}}(y,x) = F^j_R(x,y)$$

therefore $(R^{-1})^{-1} = R$.

2. If the composition $S \circ R$ is a neutrosophic multirelation from A to C, then the compostion $R^{-1} \circ S^{-1}$ is a neutrosophic multirelation from C to A. Then,

$$\begin{aligned} T^j_{(S\circ R)^{-1}}(z,x) &= T^j_{(S\circ R)}(x,z) \\ &= \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\} \\ &= \bigvee_y \left\{ T^j_{R^{-1}}(y,x) \wedge T^j_{S^{-1}}(z,y) \right\} \quad , \\ &= \bigvee_y \left\{ T^j_{S^{-1}}(z,y) \wedge T^j_{R^{-1}}(y,x) \right\} \\ &= T^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(S\circ R)^{-1}}(z,x) &= I^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ I^j_{R^{-1}}(y,x) \vee I^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ I^j_{S^{-1}}(z,y) \vee I^j_{R^{-1}}(y,x) \right\} \\ &= I^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{(S\circ R)^{-1}}(z,x) &= F^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ F^j_{R^{-1}}(y,x) \vee F^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ F^j_{S^{-1}}(z,y) \vee F^j_{R^{-1}}(y,x) \right\} \\ &= F^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

Finally; proof is valid.

Proposition 3.10 *If R is symmetric ,then $R^{-1}$is also symmetric.*

Proof: Assume that R is Symmetric then we have

$$T_R^j(x,y) = T_R^j(y,x),$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

Also if R^{-1} is an inverse relation, then we have

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x)$$

for all $x, y \in E$

To prove R^{-1} is symmetric, it is enough to prove

$$T_{R^{-1}}^j(x,y) = T_{R^{-1}}^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_{R^{-1}}^j(y,x)$$

for all $x, y \in E$

Therefore;

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x) = T_R^j(x,y) = T_{R^{-1}}^j(y,x);$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x) = I_R^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x) = F_R^j(x,y) = F_{R^{-1}}^j(y,x)$$

Finally; proof is valid.

Proposition 3.11 *If R is symmetric ,if and only if $R = R^{-1}$.*

Proof: Let R be symmetric , then

$$T_R^j(x,y) = T_R^j(y,x);$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

and

R^{-1} is an inverse relation, then

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

Therefore; $T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y)$.

Similarly

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y)$$

for all $x, y \in E$.

Hence $R = R^{-1}$

Conversely, assume that $R = R^{-1}$ then, we have

$$T^j_R(x,y) = T^j_{R^{-1}}(x,y) = T^j_R(y,x).$$

Similarly

$$I^j_R(x,y) = I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_{R^{-1}}(x,y) = F^j_R(y,x).$$

Hence R is symmetric.

Proposition 3.12 *If R and S are symmetric neutrosophic multirelations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{\cap}S$,
3. $R\tilde{+}S$
4. $R\tilde{\times}S$

are also symmetric.

Proof: R is symmetric, then we have;

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

similarly S is symmetric, then we have

$$T_S^j(x,y) = T_S^j(y,x),$$

$$I_S^j(x,y) = I_S^j(y,x)$$

and

$$F_S^j(x,y) = F_S^j(y,x)$$

Therefore,

1.
$$\begin{aligned} T^j_{R\widetilde{\cup}S}(x,y) &= \max\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \max\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cup}S}(y,x) \end{aligned},$$

$$\begin{aligned} I^j_{R\widetilde{\cup}S}(x,y) &= min\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \min\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cup}S}(y,x), \end{aligned}$$

and
$$\begin{aligned} F^j_{R\widetilde{\cup}S}(x,y) &= \min\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \min\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cup}S}(y,x) \end{aligned}$$

therefore, $R\widetilde{\cup}S$ is symmetric.

2.
$$\begin{aligned} T^j_{R\widetilde{\cap}S}(x,y) &= \min\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \min\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

$$\begin{aligned} I^j_{R\widetilde{\cap}S}(x,y) &= max\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \max\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

and
$$\begin{aligned} F^j_{R\widetilde{\cap}S}(x,y) &= \max\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \max\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cap}S}(y,x) \end{aligned}$$

therefore; $R\widetilde{\cap}S$ is symmetric.

3.

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y)+T^j_S(x,y)-T^j_R(x,y)T^j_S(x,y)\\ &= T^j_R(y,x)+T^j_S(y,x)-T^j_R(y,x)T^j_S(y,x)\\ &= T^j_{R\tilde{+}S}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y)\\ &= I^j_R(y,x)I^j_S(y,x)\\ &= I^j_{R\tilde{+}S}(y,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y)\\ &= F^j_R(y,x)F^j_S(y,x)\\ &= F^j_{R\tilde{+}S}(y,x) \end{aligned}$$

therefore, $R\tilde{+}S$ is also symmetric

4.

$$\begin{aligned} T^j_{R\tilde{\times}S}(x,y) &= T^j_R(x,y)T^j_S(x,y)\\ &= T^j_R(y,x)T^j_S(y,x)\\ &= T^j_{R\tilde{\times}tS}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\times}S}(x,y) &= I^j_R(x,y)+I^j_S(x,y)-I^j_R(x,y)I^j_S(x,y)\\ &= I^j_R(y,x)+I^j_S(y,x)-I^j_R(y,x)I^j_S(y,x)\\ &= I^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

$$\begin{aligned} F^j_{R\tilde{\times}S}(x,y) &= F^j_R(x,y)+F^j_S(x,y)-F^j_R(x,y)F^j_S(x,y)\\ &= F^j_R(y,x)+F^j_S(y,x)-F^j_R(y,x)F^j_S(y,x)\\ &= F^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

hence, $R\tilde{\times}S$ is also symmetric.

Remark 3.13 *$R\circ S$ in general is not symmetric, as*

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y)\wedge T^j_R(y,z)\right\}\\ &= \bigvee_y \left\{T^j_S(y,x)\wedge T^j_R(z,y)\right\}\\ &\neq T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y)\vee I^j_R(y,z)\right\}\\ &= \bigwedge_y \left\{I^j_S(y,x)\vee I^j_R(z,y)\right\}\\ &\neq I^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned}F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y)\vee F^j_R(y,z)\right\}\\ &= \bigwedge_y \left\{F^j_S(y,x)\vee F^j_R(z,y)\right\}\\ &\neq F^j_{(R\circ S)}(z,x)\end{aligned}$$

but $R\circ S$ is symmetric ,if $R\circ S = S\circ R$, for R and S are symmetric relations.

$$\begin{aligned}T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y)\wedge T^j_R(y,z)\right\}\\ &= \bigvee_y \left\{T^j_S(y,x)\wedge T^j_R(z,y)\right\}\\ &= \bigvee_y \left\{T^j_R(y,x)\wedge T^j_R(z,y)\right\}\\ &T^j_{(R\circ S)}(z,x)\end{aligned}$$

$$\begin{aligned}I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y)\vee I^j_R(y,z)\right\}\\ &= \bigwedge_y \left\{I^j_S(y,x)\vee I^j_R(z,y)\right\}\\ &= \bigwedge_y \left\{I^j_R(y,x)\vee I^j_R(z,y)\right\}\\ &I^j_{(R\circ S)}(z,x)\end{aligned}$$

and

$$\begin{aligned}F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y)\vee F^j_R(y,z)\right\}\\ &= \bigwedge_y \left\{F^j_S(y,x)\vee F^j_R(z,y)\right\}\\ &= \bigwedge_y \left\{F^j_R(y,x)\vee F^j_R(z,y)\right\}\\ &F^j_{(R\circ S)}(z,x)\end{aligned}$$

for every $(x,z)\in E\times E$ and for $y\in E$.

Proposition 3.14 *If R is transitive relation, then R^{-1} is also transitive.*

Proof : R is transitive relation, if $R\circ R\subseteq R$, hence if $R^{-1}\circ R^{-1}\subseteq R^{-1}$, then R^{-1} is transitive.

Consider;

$$\begin{aligned}T^j_{R^{-1}}(x,y) &= T^j_R(y,x)\geq T^j_{R\circ R}(y,x)\\ &= \bigvee_z \left\{T^j_R(y,z)\wedge T^j_R(z,x)\right\}\\ &= \bigvee_z \left\{T^j_{R^{-1}}(x,z)\wedge T^j_{R^{-1}}(z,y)\right\}\\ &= T^j_{R^{-1}\circ R^{-1}}(x,y)\end{aligned}$$

$$\begin{aligned}I^j_{R^{-1}}(x,y) &= I^j_R(y,x)\leq I^j_{R\circ R}(y,x)\\ &= \bigwedge_z \left\{I^j_R(y,z)\vee I^j_R(z,x)\right\}\\ &= \bigwedge_z \left\{I^j_{R^{-1}}(x,z)\vee I^j_{R^{-1}}(z,y)\right\}\\ &= I^j_{R^{-1}\circ R^{-1}}(x,y)\end{aligned}$$

and

$$\begin{aligned} F^j_{R^{-1}}(x,y) &= F^j_R(y,x) \le F^j_{R\circ R}(y,x) \\ &= \bigwedge_z \left\{ F^j_R(y,z) \vee F^j_R(z,x) \right\} \\ &= \bigwedge_z \left\{ F^j_{R^{-1}}(x,z) \vee F^j_{R^{-1}}(z,y) \right\} \\ &= F^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

hence, proof is valid.

Proposition 3.15 *If R is transitive relation, then $R \cap S$ is also transitive*

Proof: As R and S are transitive relations, $R \circ R \subseteq R$ and $S \circ S \subseteq S$. also

$$\begin{aligned} T^j_{R\tilde{\cap}S}(x,y) \ge T^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ I^j_{R\tilde{\cap}S}(x,y) \le I^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ F^j_{R\tilde{\cap}S}(x,y) \le F^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \end{aligned}$$

implies $R\tilde{\cap}S) \circ (R\tilde{\cap}S) \subseteq R \cap S$, hence $R \cap S$ is transitive.

Proposition 3.16 *If R and S are transitive relations, then*

1. *$R\tilde{\cup}S$,*
2. *$R\tilde{+}S$*
3. *$R\tilde{\times}S$*

are not transitive.

Proof:

1. As

$$\begin{aligned} T^j_{R\tilde{\cup}S}(x,y) &= \max\left\{ T^j_R(x,y), T^j_S(x,y) \right\} \\ I^j_{R\tilde{\cup}S}(x,y) &= \min\left\{ I^j_R(x,y), I^j_S(x,y) \right\} \\ F^j_{R\tilde{\cup}S}(x,y) &= \min\left\{ F^j_R(x,y), F^j_S(x,y) \right\} \end{aligned}$$

and

$$\begin{aligned} T^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \ge T^j_{R\tilde{\cup}S}(x,y) \\ I^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le I^j_{R\tilde{\cup}S}(x,y) \\ F^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le F^j_{R\tilde{\cup}S}(x,y) \end{aligned}$$

2. As

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) = T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ I^j_{R\tilde{+}S}(x,y) = I^j_R(x,y)I^j_S(x,y) \\ F^j_{R\tilde{+}S}(x,y) = F^j_R(x,y)F^j_S(x,y) \end{aligned}$$

and

$$T^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \geq T^j_{R\tilde{+}S}(x,y)$$
$$I^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq I^j_{R\tilde{+}S}(x,y)$$
$$F^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq F^j_{R\tilde{+}S}(x,y)$$

3. As

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \geq T^j_{R\tilde{\times}S}(x,y)$$
$$I^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq I^j_{R\tilde{\times}S}(x,y)$$
$$F^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq F^j_{R\tilde{\times}S}(x,y)$$

Hence $R\tilde{\cup}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are not transitive.

Proposition 3.17 *If R is transitive relation, then R^2 is also transitive*

Proof: R is transitive relation, if $R \circ R \subseteq R$, therefore if $R^2 \circ R^{-2} \subseteq R^2$, then R^2 is transitive.

$$T^j_{R\circ R}(y,x) = \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \geq \bigvee_z \left\{T^j_{R\circ R}(y,z) \wedge T^j_{R\circ R}(z,x)\right\} = T^j_{R^2\circ R^2}(y,x),$$

$$I^j_{R\circ R}(y,x) = \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee I^j_{R\circ R}(z,x)\right\} = I^j_{R^2\circ R^2}(y,x)$$

and

$$F^j_{R\circ R}(y,x) = \bigwedge_z \left\{F(y,z) \vee F^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee F^j_{R\circ R}(z,x)\right\} = F^j_{R^2\circ R^2}(y,x)$$

Finally, the proof is valid.

4 Acknowledgements

The authors would like to thank the anonymous reviewer for their careful reading of this research paper and for their helpful comments.

5 Conclusion

In this paper, we have firstly defined the neutrosophic multirelations(NMR). The NMR are the extension of neutrosophic relation (NR) and intuitionistic multirelation[29]. The notions of inverse, symmerty, reflexivity and transitivity on neutrosophic multirelations are studied.The future work will cover the application of the NMR in decision making, pattern recogntion and in medical diagnosis.

References

[1] S. Aggarwal, R. Biswas and A. Q. Ansari, Neutrosophic Modeling and Control, Computer and Communication Technology (2010) 718–723.

[2] M. Arora and R. Biswas, Deployment of Neutrosophic Technology to Retrieve Answers for Queries Posed in Natural Language, 3. Int. Conf. on Comp. Sci. and Inform. Tech. (2010) 435–439.

[3] M. Arora, R. Biswas and U.S. Pandy, Neutrosophic Relational Database Decomposition, International Journal of Advanced Computer Science and Applications, 2(8) (2011) 121–125.

[4] A.Q. Ansari, R. Biswas and S. Aggarwal, Proposal for Applicability of Neutrosophic Set Theory in Medical AI, International Journal of Computer Applications, 27/5 (2011) 5–11.

[5] A.Q. Ansari, R. Biswas and S. Aggarwal, Neutrosophic classifier:An extension of fuzzy classifer, Applied Soft Computing 13 (2013) 563–573.

[6] C. Ashbacher, Introduction to Neutrosophic Logic, American Research Press Rehoboth 2002.

[7] K. T. Atanassov, Intuitionistic Fuzzy Set. Fuzzy Sets and Systems, 20(1), 87–86, 1986.

[8] S. Broumi and F. Smarandache, Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8(2) (2013) 130–140.

[9] S. Broumi, Generalized Neutrosophic Soft Set, International Journal of Computer Science, Engineering and Information Technology 3(2) (2013) 17–30.

[10] S. Broumi, F. Smarandache, Several Similarity Measures of Neutrosophic Sets, Neutrosophic Sets and Systems, 1 (2013) 54–62.

[11] S. Broumi, F. Smarandache, More on Intuitionistic Neutrosophic Soft Sets, Computer Science and Information Tech-nology 1(4) (2013) 257-268.

[12] S. Broumi, I. Deli and F. Smarandache, Relations on Interval Valued Neutrosophic Soft Sets, Journal of New Results in Science, 5 (2014) 1–20

[13] S. Broumi, I. Deli, F. Smarandache Neutrosophic Parametrized Soft Set theory and its decision making problem, italian journal of pure and applied mathematics 32 (2014) 1-12.

[14] S. Broumi, M. Dhar, F.Smarandache, Rough neutrosophic sets (2014) submitted.

[15] C. S. Calude, G. Paun, G. Rozenberg, A. Salomaa, Lecture notes in computer science: Multiset Processing Mathematical, Computer Science, and Molecular Computing Points of View, 2235, (2001) Springer-New York.

[16] P. Chi and L. Peide, An Extended TOPSIS Method for the Multiple Attribute Decision Making Problems Based on Interval Neutrosophic, Neutrosophic Sets and Systems 1 (2013) 63-70.

[17] S. Das, M. B. Kar and S. Kar, Group multi-criteria decision making using intuitionistic multi-fuzzy sets, Journal of Uncertainty Analysis and Applications 10(1) (2013) 1-16.

[18] I. Deli and S.Broumi, Neutrosophic multisets and its application in medical diagnosis (2014) (submmited)

[19] I. Deli, Interval-valued neutrosophic soft sets and its decision making http://arxiv.org/abs/1402.3130

[20] P. A. Ejegwa, J. A. Awolola, Intuitionistic Fuzzy Multiset (IFMS) In Binomial Distributions, International Journal Of Scientific and Technology Research 3(4) (2014) 335-337.

[21] A. Kharal, A Neutrosophic Multicriteria Decision Making Method, New Mathematics and Natural Computation, Creighton University, USA, 2013.

[22] F. G. Lupiáñez, On neutrosophic topology, Kybernetes, 37(6) (2008) 797–800.

[23] P.K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, 5/ 1 (2013) 157-168.

[24] R. Muthuraj and S. Balamurugan, Multi-Fuzzy Group and its Level Subgroups, Gen. Math. Notes 17(1) (2013) 74-81.

[25] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci., 11 (1982) 341–356.

[26] D. Rabounski F. Smarandache L. Borissova Neutrosophic Methods in General Relativity, Hexis, no:10 (2005).

[27] P. Rajarajeswari and N. Uma, On Distance and Similarity Measures of Intuitionistic Fuzzy Multi Set, IOSR Journal of Mathematics 5(4) (2013) 19–23.

[28] P. Rajarajeswari and N. Uma, A Study of Normalized Geometric and Normalized Hamming Distance Measures in Intuitionistic Fuzzy Multi Sets, International Journal of Science and Research, Engineering and Technology 2(11) (2013) 76–80.

[29] P. Rajarajeswari, N. Uma, Intuitionistic Fuzzy Multi Relations, International Journal of Mathematical Archives 4(10) (2013) 244-249.

[30] P. Rajarajeswari and N. Uma, Zhang and Fu's Similarity Measure on Intuitionistic Fuzzy Multi Sets, International Journal of Innovative Research in Science, Engineering and Technology 3(5) (2014) 12309–12317.

[31] P. Rajarajeswari, N. Uma, Correlation Measure For Intuitionistic Fuzzy Multi Sets, International Journal of Research in Engineering and Technology 3(1) (2014) 611-617.

[32] S. Sebastian and T. V. Ramakrishnan, Multi-fuzzy Subgroups, Int. J. Contemp. Math. Sciences 6(8) (2011) 365–372.

[33] S. Sebastian and T. V. Ramakrishnan, Multi-fuzzy extension of crisp functions using bridge functions, Annals of Fuzzy Mathematics and Informatics 2(1) (2011) 1–8.

[34] S. Sebastian and T. V. Ramakrishnan, Multi-Fuzzy Sets, International Mathematical Forum 5(50) (2010) 2471–2476.

[35] T. K. Shinoj and S. J. John, Intuitionistic fuzzy multisets and its application in medical diagnosis, World Academy of Science, Engineering and Technology 6 (2012) 01–28.

[36] F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press (1998).

[37] A. Syropoulos, On generalized fuzzy multisets and their use in computation, Iranian Journal Of Fuzzy Systems 9(2) (2012) 113–125.

[38] A. S. Thomas and S. J. John, Multi-fuzzy rough sets and relations, Annals of Fuzzy Mathematics and Informatics x/x (2013) xx-xxx.

[39] R. R. Yager, On the theory of bags (Multi sets), Int. Joun. Of General System, 13 (1986) 23–37.

[40] L. A. Zadeh, Fuzzy Sets, Inform. and Control, 8 (1965) 338-353.