SOFT NEUTROSOPHIC LOOP, SOFT NEUTROSOPHIC BILOOP AND SOFT NEUTROSOPHIC N-LOOP

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic loop,soft neutosophic biloop, soft neutrosophic N-loop with the discuission of some of their characteristics. We also introduced a new type of soft neutrophic loop, the so called soft strong neutrosophic loop which is of pure neutrosophic character. This notion also foound in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

1. Introduction

Florentin Smarandache for the first time intorduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origion, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interal valued fuzzy set. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate, and inconsistentness of date etc. The theoy of neutrosophy is so applicable to every field of agebra. W.B Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings,neutrosophic vectorspaces,neutrosophic groups,neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrsosophic N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali *et al* introduced nuetosophic LA-semigoups.

Molodtsov intorduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [2, 9, 10]. Some properties and algebra may be found in [1] . Feng *et al.* introduced soft semirings in [5]. By means of level soft sets an adjustable approach to fuzy soft set can be seen in [6]. Some other concepts together with fuzzy set and rough set were shown in [7, 8].

Key words and phrases. Neutrosophic loop, neutrosophic biloop, neutrosophic N-loop, soft set, soft neutrosophic loop,soft neutrosophic biloop, soft neutrosophic N-loop.

This paper is about to introduced soft nuetrosophic loop, soft neutrosphic biloop, and soft neutrosophic N-loop and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic loop, soft neutrosophic strong loop, and some of their properties are discuissed. In the next section, soft neutrosophic biloop are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N-loop and their coresponding strong theory have been constructed with some thier properties.

2. Neutrosophic Loop

Definition 1. *A neutrosophic loop is generated by a loop L and I denoted by $\langle L \cup I\rangle$. A neutrosophic loop in general need not be a loop for $I^2 = I$ and I may not have an inverse but every element in a loop has an inverse.*

Definition 2. *Let $\langle L \cup I\rangle$ be a neutrosophic loop. A proper subset $\langle P \cup I\rangle$ of $\langle L \cup I\rangle$ is called the neutrosophic subloop, if $\langle P \cup I\rangle$ is itself a neutrosophic loop under the operations of $\langle L \cup I\rangle$.*

Definition 3. *Let $(\langle L \cup I\rangle, o)$ be a neutrosophic loop of finite order. A proper subset P of $\langle L \cup I\rangle$ is said to be Lagrange neutrosophic subloop, if P is a neutrosophic subloop under the operation 'o' and $o(P)/o\langle L \cup I\rangle$.*

If every neutrosophic subloop of $\langle L \cup I\rangle$ is Lagrange then we call $\langle L \cup I\rangle$ to be a Lagrange neutrosophic loop.

Definition 4. *If $\langle L \cup I\rangle$ has no Lagrange neutrosophic subloop then we call $\langle L \cup I\rangle$ to be a Lagrange free neutrosophic loop.*

Definition 5. *If $\langle L \cup I\rangle$ has atleast one Lagrange neutrosophic subloop then we call $\langle L \cup I\rangle$ a weakly Lagrange neutrosophic loop.*

3. Neutrosophic Biloops

Definition 6. *Let $(\langle B \cup I\rangle, *_1, *_2)$ be a non empty neutrosophic set with two binary operations $*_1, *_2$, $\langle B \cup I\rangle$ is a neutrosophic biloop if the following conditions are satisfied.*

1. $\langle B \cup I\rangle = P_1 \cup P_2$ where P_1 and P_2 are proper subsets of $\langle B \cup I\rangle$.
2. $(P_1, *_1)$ is a neutrosophic loop.
3. $(P_2, *_2)$ is a group or a loop.

Definition 7. *Let $(\langle B \cup I\rangle, *_1, *_2)$ be a neutrosophic biloop. A proper subset P of $\langle B \cup I\rangle$ is said to be a neutrosophic subbiloop of $\langle B \cup I\rangle$ if $(P, *_1, *_2)$ is itself a neutrosophic biloop under the operations of $\langle B \cup I\rangle$.*

Definition 8. *Let $(B = B1 \cup B2, *_1, *_2)$ be a finite neutrosophic biloop. Let $P = (P_1 \cup P_2, *_1, *_2)$ be a neutrosophic biloop. If $o(P)/o(B)$ then we call P a Lagrange neutrosophic subbiloop of B.*

If every neutrosophic subbiloop of B is Lagrange then we call B to be a Lagrange neutrosophic biloop.

Definition 9. *If B has atleast one Lagrange neutrosophic subbiloop then we call B to be a weakly Lagrange neutrosophic biloop.*

Definition 10. *If B has no Lagrange neutrosophic subbiloops then we call B to be a Lagrange free neutrosophic biloop.*

4. Neutrosophic N-loop

Definition 11. *Let $S(B) = \{S(B1) \cup \ldots \cup S(BN), *_1, \ldots, *_N\}$ be a non empty neutrosophic set with N binary operations. $S(B)$ is a neutrosophic N-loop if $S(B) = S(B_1) \cup \ldots \cup S(B_N)$, $S(B_i)$ are proper subsets of $S(B)$ for $1 \leq i \leq N)$ and some of $S(B_i)$ are neutrosophic loops and some of the $S(B_j)$ are groups.*

Definition 12. *Let $S(B) = \{S(B_1) \cup S(B_2) \cup \ldots \cup S(B_N), *_1, \ldots, *_N\}$ be a neutrosophic N-loop. A proper subset $(P, *_1, \ldots, *_N)$ of $S(B)$ is said to be a neutrosophic sub N loop of $S(B)$ if P itself is a neutrosophic N-loop under the operations of $S(B)$.*

Definition 13. *Let $(L = L_1 \cup L_2 \cup \ldots \cup L_N, *_1, *_2, \ldots, *_N\}$ be a neutrosophic N-loop of finite order. Suppose P is a proper subset of L, which is a neutrosophic sub N-loop. If $o(P)/o(L)$ then we call P a Lagrange neutrosophic sub N-loop.*

If every neutrosophic sub N-loop is Lagrange then we call L to be a Lagrange neutrosophic N-loop.

Definition 14. *If L has atleast one Lagrange neutrosophic sub N-loop then we call L to be a weakly Lagrange neutrosophic N-loop.*

Definition 15. *If L has no Lagrange neutrosophic sub N-loop then we call L to be a Lagrange free neutrosophic N-loop.*

5. Soft Set

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [10] defined the soft set in the following manner:

Definition 16. *A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \longrightarrow P(U)$.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Example 1. *Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or senctence. Let $E = \{$high rent,normal rent,in good condition,in bad condition$\}$. Let us consider a soft set (F, A) which describes the "attractiveness of shops" that Mr.Z is taking on rent. Suppose that there are five houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ under consideration, and that $A = \{e_1, e_2, e_3\}$ be the set of parameters where*

a_1 stands for the parameter 'high rent,
a_2 stands for the parameter 'normal rent,
a_3 stands for the parameter 'in good condition.
Suppose that
$F(a_1) = \{h_1, h_4\}$,
$F(a_2) = \{h_2, h_5\}$,
$F(a_3) = \{h_3\}$.

The soft set (F,A) is an approximated family $\{F(a_i), i=1,2,3\}$ of subsets of the

set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F,A) as a collection of approximations as below:

$(F,A) = \{$high rent $= \{h_1, h_4\}$,normal rent $= \{h_2, h_5\}$,in good condition $= \{h_3\}\}$.

Definition 17. *For two soft sets (F,A) and (H,B) over U, (F,A) is called a soft subset of (H,B) if*

(1) $A \subseteq B$ and
(2) $F(a) \subseteq G(a)$ for all $a \in A$.

This relationship is denoted by $(F,A) \overset{\sim}{\subset} (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \overset{\sim}{\supset} (H,B)$.

Definition 18. *Two soft sets (F,A) and (H,B) over U are called soft equal if (F,A) is a soft subset of (H,B) and (H,B) is a soft subset of (F,A).*

Definition 19. *(F,A) over U is called an absolute soft set if $F(a) = U$ for all $a \in A$ and we denote it by $\mathcal{F}_U$.*

Definition 20. *Let (F,A) and (G,B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by$(F,A) \cap_R (G,B) = (H,C)$ where (H,C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B$.*

Definition 21. *The extended intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as*

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cap_\varepsilon (G,B) = (H,C)$.

Definition 22. *The resticted union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(e)$ is defined as the soft set $(H,C) = (F,A) \cup_R (G,B)$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.*

Definition 23. *The extended union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as*

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cup_\varepsilon (G,B) = (H,C)$.

6. Soft Neutrosophic Loop

Definition 24. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L \cup I \rangle$. Then (F,A) is called soft neutrosophic loop if and only if $F(a)$ is neutrosophic subloop of $\langle L \cup I \rangle$, for all $a \in A$.*

Example 2. *Let $\langle L\cup I\rangle = \langle L_7(4)\cup I\rangle$ be a neutrosophic loop where $L_7(4)$ is a loop. $\langle e, eI, 2, 2I\rangle, \langle e, 3\rangle$ and $\langle e, eI\rangle$ are neutrosophic subloops of $L_7(4)$. Then (F, A) is a soft neutrosophic loop over $\langle L \cup I\rangle$, where*

$$\begin{aligned} F(a_1) &= \{\langle e, eI, 2, 2I\rangle\}, F(a_2) = \{\langle e, 3\rangle\}, \\ F(a_3) &= \{\langle e, eI\rangle\}. \end{aligned}$$

Theorem 1. *Every soft neutrosophic loop over $\langle L\cup I\rangle$ contains a soft loop over L.*

Proof. The proof is straight forward. □

Theorem 2. *Let (F, A) and (H, A) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then their intersection $(F, A)\cap(H, A)$ is again a soft neutrosophic loop over $\langle L\cup I\rangle$.*

Proof. The proof is staight forward. □

Theorem 3. *Let (F, A) and (H, B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic loop over $\langle L \cup I\rangle$.*

Theorem 4. *Let (F, A) and (H, A) be two soft neutrosophic loops over $\langle L \cup I\rangle$. If $F(a) \subseteq H(a)$ for all $a \in A$, then (F, A) is a soft neutrosophic subloop of (H, A).*

Theorem 5. *Let (F, A) and (K, B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their extended union $(F, A)\cup_\varepsilon (K, B)$ over $\langle L\cup I\rangle$ is not soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their extended intersection $(F, A)\cap_\varepsilon (K, B)$ over $\langle L\cup I\rangle$ is soft neutrosophic loop over $\langle L \cup I\rangle$.
(3) Their restricted union $(F, A)\cup_R (K, B)$ over $\langle L\cup I\rangle$ is not soft neutrosophic loop over $\langle L \cup I\rangle$.
(4) Their restricted intersection $(F, A)\cap_\varepsilon (K, B)$ over $\langle L\cup I\rangle$ is soft neutrosophic soft loop over $\langle L \cup I\rangle$.

Theorem 6. *Let (F, A) and (H, B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their AND operation $(F, A)\wedge(H, B)$ is soft neutrosophic loop over $\langle L\cup I\rangle$.
(2) Their OR operation $(F, A)\vee(H, B)$ is not soft neutrosophic loop over $\langle L\cup I\rangle$.

Definition 25. *Let $\langle L_n(m) \cup I\rangle = \{e, 1, 2, \ldots, n, e.I, 1I, \ldots, nI\}$ be a new class of neutrosophic loop and (F, A) be a soft neutrosophic loop over $\langle L_n(m) \cup I\rangle$. Then (F, A) is called soft new class neutrosophic loop if $F(a)$ is neutrosophic subloop of $\langle L_n(m) \cup I\rangle$, for all $a \in A$.*

Example 3. *Let $\langle L_5(3)\cup I\rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ be a new class of neutrosophic loop and $\{e, eI, 1, 1I\}, \{e, eI, 2, 2I\}, \{e, eI, 3, 3I\}, \{e, eI, 4, 4I\}, \{e, eI, 5, 5I\}$ are neutrosophic subloops of $L_5(3)$. Then (F, A) is soft new class neutrosophic loop over $L_5(3)$, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 1, 1I\}, F(a_2) = \{e, eI, 2, 2I\}, \\ F(a_3) &= \{e, eI, 3, 3I\}, F(a_4) = \{e, eI, 4, 4I\}, \\ F(a_5) &= \{e, eI, 5, 5I\}. \end{aligned}$$

Theorem 7. *Every soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$ is a soft neutrosophic loop over $\langle L_n(m) \cup I\rangle$ but the converse is not true.*

Theorem 8. *Let (F,A) and (K,B) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I\rangle$. Then*

(1) Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over $\langle L_n(m) \cup I\rangle$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$.
(2) Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $\langle L_n(m) \cup I\rangle$ is soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$.
(3) Their restricted union $(F,A) \cup_{R} (K,B)$ over $\langle L_n(m) \cup I\rangle$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$.
(4) Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $\langle L_n(m) \cup I\rangle$ is soft new class neutrosophic soft loop over $\langle L_n(m) \cup I\rangle$.

Theorem 9. *Let (F,A) and (H,B) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I\rangle$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$.

Definition 26. *Let (F,A) be a soft neutrosophic loop over $\langle L \cup I\rangle$, then (F,A) is called the identity soft neutrosophic loop over $\langle L \cup I\rangle$ if $F(a) = \{e\}$, for all $a \in A$, where e is the identity element of L.*

Definition 27. *Let (F,A) be a soft neutrosophic loop over $\langle L \cup I\rangle$, then (F,A) is called Full-soft neutrosophic loop over $\langle L \cup I\rangle$ if $F(a) = \langle L \cup I\rangle$, for all $a \in A$.*

Definition 28. *Let (F,A) and (H,B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then (H,B) is soft neutrosophic subloop of (F,A), if*

(1) $B \subset A$.
(2) $H(a)$ is neutrosophic subloop of $F(a)$, for all $a \in A$.

Example 4. *Consider the neutrosophic loop $\langle L_{15}(2) \cup I\rangle = \{e, 1, 2, 3, 4, \ldots, 15, eI, 1I, 2I, \ldots, 14I, 15I\}$ of order 32. It is easily verified $P = \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}$, $Q = \{e, 2, 5, 8, 11, 14\}$ and $T = \{e, 3, eI, 3I\}$ are neutrosophic subloops of $\langle L_{15}(2) \cup I\rangle$. Then (F,A) is a soft neutrosophic loop over $\langle L_{15}(2) \cup I\rangle$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}, \\ F(a_2) &= \{e, 2, 5, 8, 11, 14\}, \\ F(a_3) &= \{e, 3, eI, 3I\}. \end{aligned}$$

Hence (G,B) is a soft neutrosophic subloop of (F,A) over $\langle L_{15}(2) \cup I\rangle$, where

$$\begin{aligned} G(a_1) &= \{e, eI, 2I, 5I, 8I, 11I, 14I\}, \\ G(e_3) &= \{e, 3\}. \end{aligned}$$

Theorem 10. *Every soft loop over L is a soft neutrosophic subloop over $\langle L \cup I\rangle$.*

Theorem 11. *Every absolute soft loop over L is a soft neutrosophic subloop of Full-soft neutrosophic loop over $\langle L \cup I\rangle$.*

Definition 29. *Let $\langle L \cup I\rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L \cup I\rangle$. Then (F,A) is called normal soft neutrosophic loop if and only if $F(a)$ is normal neutrosophic subloop of $\langle L \cup I\rangle$, for all $a \in A$.*

Example 5. *Let* $\langle L_5(3)\cup I\rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ *be a neutrosophic loop and* $\{e, eI, 1, 1I\}$, $\{e, eI, 2, 2I\}$, $\{e, eI, 3, 3I\}$ *are normal neutrosophic subloops of* $\langle L_5(3)\cup I\rangle$. *Then Clearly* (F, A) *is normal soft neutrosophic loop over* $\langle L_5(3)\cup I\rangle$, *where*

$$\begin{aligned} F(a_1) &= \{e, eI, 1, 1I\}, F(a_2) = \{e, eI, 2, 2I\}, \\ F(a_3) &= \{e, eI, 3, 3I\}. \end{aligned}$$

Theorem 12. *Every normal soft neutrosophic loop over* $\langle L \cup I\rangle$ *is a soft neutrosophic loop over* $\langle L \cup I\rangle$ *but the converse is not true.*

Theorem 13. *Let* (F, A) *and* (K, B) *be two normal soft neutrosophic loops over* $\langle L \cup I\rangle$. *Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not normal soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is normal soft neutrosophic loop over $\langle L \cup I\rangle$.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over $\langle L \cup I\rangle$ is not normal soft neutrosophic loop over $\langle L \cup I\rangle$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is normal soft neutrosophic soft loop over $\langle L \cup I\rangle$.

Theorem 14. *Let* (F, A) *and* (H, B) *be two normal soft neutrosophic loops over* $\langle L \cup I\rangle$. *Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is normal soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not normal soft neutrosophic loop over $\langle L \cup I\rangle$.

Definition 30. *Let* $\langle L \cup I\rangle$ *be a neutrosophic loop and* (F, A) *be a soft neutrosophic loop over* $\langle L \cup I\rangle$. *Then* (F, A) *is called Lagrange soft neutrosophic loop if each* $F(a)$ *is lagrange neutrosophic subloop of* $\langle L \cup I\rangle$, *for all* $a \in A$.

Example 6. *In* (*example* 1), (F, A) *is lagrange soft neutrosophic loop over* $\langle L \cup I\rangle$.

Theorem 15. *Every lagrange soft neutrosophic loop over* $\langle L \cup I\rangle$ *is a soft neutrosophic loop over* $\langle L \cup I\rangle$ *but the converse is not true.*

Theorem 16. *If* $\langle L \cup I\rangle$ *is lagrange neutrosophic loop, then* (F, A) *over* $\langle L \cup I\rangle$ *is lagrange soft neutrosophic loop but the converse is not true.*

Theorem 17. *Every soft new class neutrosophic loop over* $\langle L_n(m) \cup I\rangle$ *is lagrange soft neutrosophic loop over* $\langle L_n(m) \cup I\rangle$ *but the converse is not true.*

Theorem 18. *If* $\langle L \cup I\rangle$ *is a new class neutrosophic loop, then* (F, A) *over* $\langle L \cup I\rangle$ *is lagrange soft neutrosophic loop.*

Theorem 19. *Let* (F, A) *and* (K, B) *be two lagrange soft neutrosophic loops over* $\langle L \cup I\rangle$. *Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not lagrange soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not lagrange soft neutrosophic loop over $\langle L \cup I\rangle$.

(3) Their restricted union $(F,A) \cup_R (K,B)$ over $\langle L \cup I \rangle$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not lagrange soft neutrosophic soft loop over $\langle L \cup I \rangle$.

Theorem 20. *Let (F,A) and (H,B) be two lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.

Definition 31. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called weak Lagrange soft neutrosophic loop if atleast one $F(a)$ is lagrange neutrosophic subloop of $\langle L \cup I \rangle$, for some $a \in A$.*

Example 7. *Consider the neutrosophic loop $\langle L_{15}(2) \cup I \rangle = \{e,1,2,3,4,\ldots,15,eI,1I,2I,\ldots,14I,15I\}$ of order 32. It is easily verified $P = \{e,2,5,8,11,14,eI,2I,5I,8I,11I,14I\}$, $Q = \{e,2,5,8,11,14\}$ and $T = \{e,3,eI,3I\}$ are neutrosophic subloops of $\langle L_{15}(2) \cup I \rangle$. Then (F,A) is a weak lagrange soft neutrosophic loop over $\langle L_{15}(2) \cup I \rangle$, where*

$$\begin{aligned} F(a_1) &= \{e,2,5,8,11,14,eI,2I,5I,8I,11I,14I\}, \\ F(a_2) &= \{e,2,5,8,11,14\}, \\ F(a_3) &= \{e,3,eI,3I\}. \end{aligned}$$

Theorem 21. *Every weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.*

Theorem 22. *If $\langle L \cup I \rangle$ is weak lagrange neutrosophic loop, then (F,A) over $\langle L \cup I \rangle$ is also weak lagrange soft neutrosophic loop but the converse is not true.*

Theorem 23. *Let (F,A) and (K,B) be two weak lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their extended union $(F,A) \cup_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their extended intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(3) Their restricted union $(F,A) \cup_R (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic soft loop over $\langle L \cup I \rangle$.

Theorem 24. *Let (F,A) and (H,B) be two weak lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.

Definition 32. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called Lagrange free soft neutrosophic loop if $F(a)$ is not lagrange neutrosophic subloop of $\langle L \cup I \rangle$, for all $a \in A$.*

Theorem 25. *Every lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.*

Theorem 26. *If $\langle L \cup I \rangle$ is lagrange free neutrosophic loop, then (F, A) over $\langle L \cup I \rangle$ is also lagrange free soft neutrosophic loop but the converse is not true.*

Theorem 27. *Let (F, A) and (K, B) be two lagrange free soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $\langle L \cup I \rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I \rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over $\langle L \cup I \rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I \rangle$ is not lagrange free soft neutrosophic soft loop over $\langle L \cup I \rangle$.

Theorem 28. *Let (F, A) and (H, B) be two lagrange free soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not lagrange free soft neutrosophic loop over $\langle L \cup I \rangle$.

7. Soft Neutrosophic Biloop

Definition 33. *Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop and (F, A) be a soft set over $(\langle B \cup I \rangle, *_1, *_2)$. Then (F, A) is called soft neutrosophic biloop if and only if $F(a)$ is neutrosophic subbiloop of $(\langle B \cup I \rangle, *_1, *_2)$, for all $a \in A$.*

Example 8. *Let $(\langle B \cup I \rangle, *_1, *_2) = (\{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\} \cup \{g \mid g^6 = e\}, *_1, *_2)$ be a neutrosophic biloop and $\{e, 2, eI, 2I\} \cup \left\{g^2, g^4, e\right\}$, $\{e, 3, eI, 3I\} \cup \left\{g^3, e\right\}$ are two neutrosophic subbiloops of $(\langle B \cup I \rangle, *_1, *_2)$. Then (F, A) is clearly soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, eI, 2I\} \cup \left\{g^2, g^4, e\right\}, \\ F(a_2) &= \{e, 3, eI, 3I\} \cup \left\{g^3, e\right\}. \end{aligned}$$

Theorem 29. *Let (F, A) and (H, A) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.*

Proof. Straight forward. □

Theorem 30. *Let (F, A) and (H, B) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$ such that $A \cap B = \phi$, then their union is soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.*

Proof. Straight forward. □

Theorem 31. *Let (F, A) and (K, B) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then*

(1) Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over $(\langle B \cup I \rangle, *_1, *_2)$ is not soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
(2) Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $(\langle B \cup I \rangle, *_1, *_2)$ is soft neutrosophic biloop over

$(\langle B \cup I \rangle, *_1, *_2)$.

(1) Their restricted union $(F,A) \cup_{R} (K,B)$ over $(\langle B \cup I \rangle, *_1, *_2)$ is not soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
(2) Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $(\langle B \cup I \rangle, *_1, *_2)$ is soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Theorem 32. *Let (F,A) and (H,B) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Definition 34. *Let $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop and (F,A) be a soft set over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then (F,A) is called soft new class neutrosophic subbiloop if and only if $F(a)$ is neutrosophic subbiloop of $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$, for all $a \in A$.*

Example 9. *Let $B = (\langle B_1 \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop $B_1 = (\langle L_5(3) \cup I \rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ be a new class of neutrosophic loop and $B_2 = \left\{g : g^{12} = 1\right\}$ is a group. $\{e, eI, 1, 1I\} \cup \left\{1, g^6\right\}$, $\{e, eI, 2, 2I\} \cup \left\{1, g^2, g^4, g^6, g^8, g^{10}\right\}$, $\{e, eI, 3, 3I\} \cup \left\{1, g^3, g^6, g^9\right\}$, $\{e, eI, 4, 4I\} \cup \{1, g^4, g^8\}$ are neutrosophic subloops of B. Then (F,A) is soft new class neutrosophic biloop over B, where*

$$
\begin{aligned}
F(a_1) &= \{e, eI, 1, 1I\} \cup \left\{1, g^6\right\}, \\
F(a_2) &= \{e, eI, 2, 2I\} \cup \left\{1, g^2, g^4, g^6, g^8, g^{10}\right\}, \\
F(a_3) &= \{e, eI, 3, 3I\} \cup \left\{1, g^3, g^6, g^9\right\}, \\
F(a_4) &= \{e, eI, 4, 4I\} \cup \{1, g^4, g^8\}.
\end{aligned}
$$

Theorem 33. *Every soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is a soft neutrosophic biloop over but the converse is not true.*

Theorem 34. *Let (F,A) and (K,B) be two soft new class neutrosophic biloops over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then*

(1) Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over $\langle L_n(m) \cup I \rangle$ is not soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
(2) Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
(3) Their restricted union $(F,A) \cup_{R} (K,B)$ over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is not soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
(4) Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is soft new class neutrosophic soft biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.

Theorem 35. *Let (F,A) and (H,B) be two soft new class neutrosophic biloops over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$. Then*

1. Their AND operation $(F,A)\wedge(H,B)$ is soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their OR operation $(F,A)\vee(H,B)$ is not soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.

Definition 35. *Let (F,A) be a soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, then (F,A) is called the identity soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ if $F(a)=\{e_1,e_2\}$, for all $a\in A$, where e_1,e_2 are the identities element of $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ respectively.*

Definition 36. *Let (F,A) be a soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, then (F,A) is called Full-soft neutrosophic biloop over $B=(B_1\cup B_2,*_1,*_2)$ if $F(a)=B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, for all $a\in A$.*

Definition 37. *Let (F,A) and (H,B) be two soft neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then (H,B) is soft neutrosophic subbiloop of (F,A), if*

1. $B\subset A$.
2. $H(a)$ is neutrosophic subbiloop of $F(a)$, for all $a\in A$.

Example 10. *Let $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ be a neutrosophic biloop where $B_1=(\langle L_5(3)\cup I\rangle=\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a neutrosophic loop and $B_2=\left\{g:g^{12}=1\right\}$ is a group. $\{e,eI,1,1I\}\cup\left\{1,g^6\right\}$, $\{e,eI,2,2I\}\cup\left\{1,g^2,g^4,g^6,g^8,g^{10}\right\}$, $\{e,eI,3,3I\}\cup\left\{1,g^3,g^6,g^9\right\}$, $\{e,eI,4,4I\}\cup\left\{1,g^4,g^8\right\}$ are neutrosophic subbiloops of B. Then (F,A) is soft neutrosophic biloop over B, where*

$$\begin{aligned}
F(a_1) &= \{e,eI,1,1I\}\cup\left\{1,g^6\right\},\\
F(a_2) &= \{e,eI,2,2I\}\cup\left\{1,g^2,g^4,g^6,g^8,g^{10}\right\},\\
F(a_3) &= \{e,eI,3,3I\}\cup\left\{1,g^3,g^6,g^9\right\},\\
F(a_4) &= \{e,eI,4,4I\}\cup\{1,g^4,g^8\}.
\end{aligned}$$

(H,B) is soft neutrosophic subbiloop of (F,A), where

$$\begin{aligned}
H(a_2) &= \{e,2,\}\cup\left\{1,g^6\right\},\\
H(a_3) &= \{e,eI,3I\}\cup\left\{1,g^6\right\}.
\end{aligned}$$

Definition 38. *Let $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then (F,A) is called soft neutrosophic Moufang biloop if and only if $F(a)=(P_1\cup P_2,*_1,*_2$, where P_1 is a proper neutrosophic Moufang loop of $B_1)$ is neutrosophic subbiloop of $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, for all $a\in A$.*

Example 11. *Let $B=(\langle B_1\cup I\rangle\cup B_2),*_1,*_2)$ be a neutrosophic biloop where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=S_3$. Let $P=\{e,2,eI,2I\}\cup\{e,(12)\}$ and $Q=\{e,3,eI,3I\}\cup\{e,(123),(132)\}$ are neutrosophic subbiloops of B in which $\{e,2,eI,2I\}$ and $\{e,3,eI,3I\}$ are proper neutrosophic Moufang loops. Then clearly (F,A) is soft neutrosophic Moufang biloop over B, where*

$$\begin{aligned}
F(a_1) &= \{e,2,eI,2I\}\cup\{e,(12)\},\\
F(a_2) &= \{e,3,eI,3I\}\cup\{e,(123),(132)\}.
\end{aligned}$$

Theorem 36. *Every soft neutrosophic Moufang biloop over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 37. *Let* (F, A) *and* (K, B) *be two soft neutrosophic Moufang biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over B is not soft neutrosophic Moufang biloop over B.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is soft neutrosophic Moufang biloop over B.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over B is not soft neutrosophic Moufang biloop over B.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is soft neutrosophic Moufang biloop over B.

Theorem 38. *Let* (F, A) *and* (H, B) *be two soft neutrosophic Moufang biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic Moufang biloop over B.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic Moufang biloop over B.

Definition 39. *Let* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *be a neutrosophic biloop and* (F, A) *be a soft set over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. *Then* (F, A) *is called soft neutrosophic Bol biloop if and only if* $F(a) = (P_1 \cup P_2, *_1, *_2$, *where* P_1 *is a proper neutrosophic Bol loop of* $B_1)$ *is neutrosophic subbiloop of* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$, *for all* $a \in A$.

Example 12. *Let* $B = (\langle B_1 \cup I\rangle \cup B_2), *_1, *_2)$ *be a neutrosophic biloop where* $B_1 = \langle L_5(3) \cup I\rangle$ *and* $B_2 = S_3$. *Let* $P = \{e, 3, eI, 3I\} \cup \{e, (12)\}$ *and* $Q = \{e, 2, eI, 2I\} \cup \{e, (123), (132)\}$ *are neutrosophic subbiloops of* B *in which* $\{e, 3, eI, 3I\}$ *and* $\{e, 2, eI, 2I\}$ *are proper neutrosophic Bol loops. Then clearly* (F, A) *is soft neutrosophic Bol biloop over* B, *where*

$$\begin{aligned} F(a_1) &= \{e, 3, eI, 3I\} \cup \{e, (12)\}, \\ F(a_2) &= \{e, 2, eI, 2I\} \cup \{e, (123), (132)\}. \end{aligned}$$

Theorem 39. *Every soft neutrosophic Bol biloop over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 40. *Let* (F, A) *and* (K, B) *be two soft neutrosophic Bol biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over B is not soft neutrosophic Bol biloop over B.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is soft neutrosophic Bol biloop over B.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over B is not soft neutrosophic Bol biloop over B.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is soft neutrosophic Bol biloop over B.

Theorem 41. *Let* (F, A) *and* (H, B) *be two soft neutrosophic Bol biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft neutrosophic Bol biloop over B.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft neutrosophic Bol biloop over B.

Definition 40. *Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic biloop if and only if $F(a)$ is Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$, for all $a\in A$.*

Example 13. *Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\{\langle L_5(3)\cup I\rangle,*_1\}$ and $B_2=\{g|\ g^8=1\}$. Let $(P=P_1\cup P_2,*_1,*_2)$ where $P_1=\{e,eI,2,2I\}\subset B_1$ and $P_2=\{1\}\subset B2$ and $(Q=Q_1\cup Q_2,*_1,*_2)$ where $Q_1=\{e,eI,3,3I\}\subset B_1$ and $Q_2=\{1\}\subset B_2$ are Lagrange neutrosophic subbiloops of B. Then clearly (F,A) is a soft Lagrange neutrosophic biloop over B, where*

$$\begin{aligned} F(a_1) &= \{e,eI,2,2I\}\cup\{1\},\\ F(a_2) &= \{e,eI,3,3I\}\cup\{1\}. \end{aligned}$$

Theorem 42. *Every soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 43. *Let (F,A) and (K,B) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(3) Their restricted union $(F,A)\cup_{R}(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.

Theorem 44. *Let (F,A) and (H,B) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is not soft Lagrange neutrosophic biloop over B.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft Lagrange neutrosophic biloop over B.

Definition 41. *Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$, for some $a\in A$.*

Example 14. *Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\{\langle L_5(3)\cup I\rangle,*_1\}$ and $B_2=\{g|\ g^8=1\}$. Let $(P=P_1\cup P_2,*_1,*_2)$ where $P_1=\{e,eI,2,2I\}\subset B_1$ and $P_2=\{1\}\subset B2$ is a Lagrange neutrosophic subbiloop of B and $(Q=Q_1\cup Q_2,*_1,*_2)$ where $Q_1=\{e,eI,3,3I\}\subset B_1$ and $Q_2=\{1,g^4\}\subset B_2$ is not Lagrange neutrosophic subbiloop of B. Then clearly (F,A) is a soft weakly*

Lagrange neutrosophic biloop over B, where

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I\} \cup \{1\}, \\ F(a_2) &= \{e, eI, 3, 3I\} \cup \{1, g^4\}. \end{aligned}$$

Theorem 45. *Every soft weakly Lagrange neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 46. *If $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ is a weakly Lagrange neutrosophic biloop, then (F, A) over B is also soft weakly Lagrange neutrosophic biloop but the converse is not holds.*

Theorem 47. *Let (F, A) and (K, B) be two soft weakly Lagrange neutrosophic biloops over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
(3) Their restricted union $(F, A) \cup_{R} (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.

Theorem 48. *Let (F, A) and (H, B) be two soft weakly Lagrange neutrosophic biloops over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft not weakly Lagrange neutrosophic biloop over B.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft weakly Lagrange neutrosophic biloop over B.

Definition 42. *Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop and (F, A) be a soft set over $(\langle B \cup I \rangle, *_1, *_2)$. Then (F, A) is called soft Lagrange free neutrosophic biloop if and only if $F(a)$ is not Lagrange neutrosophic subbiloop of $(\langle B \cup I \rangle, *_1, *_2)$, for all $a \in A$.*

Example 15. *Let $B = (B_1 \cup B_2, *_1, *_2)$ be a neutrosophic biloop of order 20, where $B_1 = \{\langle L_5(3) \cup I \rangle, *_1\}$ and $B_2 = \{g |\ g^8 = 1\}$. Let $(P = P_1 \cup P_2, *_1, *_2)$ where $P_1 = \{e, eI, 2, 2I\} \subset B_1$ and $P_2 = \{1, g^2, g^4, g^6\} \subset B2$ and $(Q = Q_1 \cup Q_2, *_1, *_2)$ where $Q_1 = \{e, eI, 3, 3I\} \subset B_1$ and $Q_2 = \{1, g^4\} \subset B_2$ are not Lagrange neutrosophic subbiloop of B. Then clearly (F, A) is a soft Lagrange free neutrosophic biloop over B, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I\} \cup \{1, g^2, g^4, g^6\}, \\ F(a_2) &= \{e, eI, 3, 3I\} \cup \{1, g^4\}. \end{aligned}$$

Theorem 49. *Every soft Lagrange free neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 50. *If $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ is a Lagrange free neutrosophic biloop, then (F, A) over B is also soft Lagrange free neutrosophic biloop but the converse is not holds.*

Theorem 51. *Let (F, A) and (K, B) be two soft Lagrange free neutrosophic biloops over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(3) Their restricted union $(F,A)\cup_{R}(K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.

Theorem 52. *Let (F,A) and (H,B) be two soft Lagrange free neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is not soft Lagrange free neutrosophic biloop over B.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft Lagrange free neutrosophic biloop over B.

Definition 43. *Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop where B_1 is a neutrosopphic biloop and B_2 is a neutrosophic group and (F,A) be soft set over B. Then (F,A) over B is called soft strong neutrosophic biloop if and only if $F(a)$ is a neutrosopchic subbiloop of B, for all $a\in A$.*

Example 16. *Let $(B=B_1\cup B_2,*_1,*_2)$ where $B_1=\langle L_5(2)\cup I\rangle$ is a neutrosophic loop and $B_2=\{1,2,3,4,I,2I,3I,4I\}$ under multiplication modulo 5 is a neutrosophic group. Let $P=\{e,2,eI,2I\}\cup\{1,I,4I\}$ and $Q=\{e,3,eI,3I\}\cup\{1,I\}$ are neutrosophic subbiloops of B. Then (F,A) is soft strong neutrosophic biloop of B, where*

$$\begin{aligned}F(a_1)&=\{e,2,eI,2I\}\cup\{1,I,4I\},\\F(a_2)&=\{e,3,eI,3I\}\cup\{1,I\}.\end{aligned}$$

Theorem 53. *Every soft strong neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 54. *If $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a strong neutrosophic biloop, then (F,A) over B is also soft strong neutrosophic biloop but the converse is not holds.*

Theorem 55. *Let (F,A) and (K,B) be two soft soft neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over B is not soft strong neutrosophic biloop over B.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is soft strong neutrosophic biloop over B.
(3) Their restricted union $(F,A)\cup_{R}(K,B)$ over B is not soft strong neutrosophic biloop over B.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over B is soft strong neutrosophic biloop over B.

Theorem 56. *Let (F,A) and (H,B) be two soft strong neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft strong neutrosophic biloop over B.

(2) Their OR operation $(F, A) \vee (H, B)$ is not soft strong neutrosophic biloop over B.

Definition 44. *Let $B = (B1 \cup B2, *1, *2)$ be a neutrosophic biloop of type II and (F, A) be a soft set over B. Then (F, A) over B is called soft neutrosophic biloop of type II if and only if $F(a)$ is a neutrosopchic subbiloop of B, for all $a \in A$.*

Example 17. *Let $B = (B1 \cup B2, *1, *2)$ where $B1 = \langle L7(3) \cup I \rangle$ and $B2 = L5(2)$, then B is a neutrosophic biloop of type II. Hence (F, A) over B is a soft neutrosophic biloop of type II.*

All the properties defined for soft neutrosophic biloop can easily be extend to soft neutrosophic biloop of type II.

8. Soft Neutrosophic N-loop

Definition 45. *Let $S(B) = \{S(B_1) \cup S(B_2) \cup \ldots \cup S(B_n), *_1, \ldots, *_N\}$ be a neutrosophic N-loop and (F, A) be a soft set over $S(B)$. Then (F, A) over $S(B)$ is called soft neutrosophic N-loop if and only if $F(a)$ is a neutrosopchic sub N-loop of $S(B)$, for all $a \in A$.*

Example 18. *Let $S(B) = \{S(B_1 \cup S(B_2) \cup S(B_3), *_1, *_2, *_3\}$ where $S(B_1) = \{\langle L_5(3) \cup I \rangle\}$, $S(B_2) = \langle g | g^{12} = 1 \rangle$ and $S(B_3) = S_3$, is a neutrosophic 3-loop. Let $P = \{e, eI, 2, 2I, 1, g^6, e, (12)\}$ and $\{e, eI, 3, 3I, 1, g^4, g^8, e, (13)\}$ are neutrosophic sub N-loops of $S(B)$. Then (F, A) is sof neutrosophic N-loop over $S(B)$, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I, 1, g^6, e, (12)\}, \\ F(a_2) &= \{e, eI, 3, 3I, 1, g^4, g^8, e, (13)\}. \end{aligned}$$

Theorem 57. *Let (F, A) and (H, A) be two soft neutrosophic N-loops over $S(B)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic biloop over $S(B)$.*

Proof. Straight forward. □

Theorem 58. *Let (F, A) and (H, C) be two soft neutrosophic N-loops over $S(B)$ such that $A \cap C = \phi$, then their union is soft neutrosophic biloop over $S(B)$.*

Proof. Straight forward. □

Theorem 59. *Let (F, A) and (K, C) be two soft neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, C)$ over $S(B)$ is not soft neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, C)$ over $S(B)$ is soft neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $S(B)$ is not soft neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, C)$ over $S(B)$ is soft neutrosophic N-loop over $S(B)$.

Theorem 60. *Let (F, A) and (H, C) be two soft neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft neutrosophic N-loop over $S(B)$.

Definition 46. *Let $S(L)=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,\ldots,*_N\}$ be a neutrosophic N-loop of level II and (F,A) be a soft set over $S(L)$. Then (F,A) over $S(L)$ is called soft neutrosophic N-loop of level II if and only if $F(a)$ is a neutrosopchic sub N-loop of $S(L)$, for all $a\in A$.*

Example 19. *Let $S(L)=\{L_1\cup L_2\cup L_3\cup L_4,*_1,*_2,*_3,*_4\}$ be a neutrosophic 4-loop of level II where $L_1=\{\langle L_5(3)\cup I\rangle\}$, $L_2=\{e,1,2,3\}$, $L_3=S_3$ and $L_4=N(Z_3)$, under multiplication modulo 3. Let $P=\{e,eI,2,2I\}\cup\{e,1\}\cup\{e,(12)\}\cup\{1,I\}$ and $\{e,eI,3,3I\}\cup\{e,2\}\cup\{e,(13)\}\cup\{1,2\}$ are neutrosophic sub N-loops of $S(L)$. Then (F,A) is sof neutrosophic N-loop of level II over $S(L)$, where*

$$\begin{aligned}
F(a_1) &= \{e,eI,2,2I\}\cup\{e,1\}\cup\{e,(12)\}\cup\{1,I\},\\
F(a_2) &= \{e,eI,3,3I\}\cup\{e,2\}\cup\{e,(13)\}\cup\{1,2\}.
\end{aligned}$$

Theorem 61. *Every soft neutrosophic N-loop of level II over $S(L)=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,\ldots,*_N\}$ is a soft neutrosophic N-loop but the converse is not true.*

Theorem 62. *Let (F,A) and (K,C) be two soft neutrosophic N-loops of level II over $S(L)=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,\ldots,*_N\}$. Then*

(1) Their extended union $(F,A)\cup_\varepsilon(K,C)$ over $S(L)$ is not soft neutrosophic N-loop of level II over $S(L)$.
(2) Their extended intersection $(F,A)\cap_\varepsilon(K,C)$ over $S(L)$ is soft neutrosophic N-loop of level II over $S(L)$.
(3) Their restricted union $(F,A)\cup_R(K,C)$ over $S(L)$ is not soft neutrosophic N-loop of level II over $S(L)$.
(4) Their restricted intersection $(F,A)\cap_\varepsilon(K,C)$ over $S(L)$ is soft neutrosophic N-loop of level II over $S(L)$.

Theorem 63. *Let (F,A) and (H,C) be two soft neutrosophic N-loops of level II over $S(L)=\{L_1\cup L_2\cup\ldots\cup L_N,*_1,\ldots,*_N\}$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft neutrosophic N-loop of level II over $S(L)$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft neutrosophic N-loop of level 11 over $S(L)$.

Now what all we define for neutrosophic N-loops will be carried out to neutrosophic N-loops of level II with appropriate modifications.

Definition 47. *Let (F,A) be a soft neutrosophic N-loop over $S(B)=(S(B_1)\cup S(B_2)\cup,...,\cup S(B_N),*_1,...,*_N)$, then (F,A) is called the identity soft neutrosophic N-loop over $S(B)$ if $F(a)=\{e_1,e_2,...,e_N\}$, for all $a\in A$, where e_1,e_2,...,e_N are the identities element of $S(B_1),S(B_2),...,S(B_N)$ respectively.*

Definition 48. *Let (F,A) be a soft neutrosophic N-loop over $S(B)=(S(B_1)\cup S(B_2)\cup,...,\cup S(B_N),*_1,...,*_N)$, then (F,A) is called Full-soft neutrosophic N-loop over $S(B)$ if $F(a)=S(B)$, for all $a\in A$.*

Definition 49. *Let (F,A) and (H,C) be two soft neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then (H,C) is soft neutrosophic sub N-loop of (F,A), if*

(1) $B \subset A$.
(2) $H(a)$ is neutrosophic sub N-loop of $F(a)$, for all $a \in A$.

Definition 50. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft Lagrange neutrosophic N-loop if and only if $F(a)$ is Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 64. *All soft Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 65. *Let (F,A) and (K,C) be two soft Lagrange neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F,A) \cup_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F,A) \cup_R (K,C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.

Theorem 66. *Let (F,A) and (H,C) be two soft Lagrange neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not soft Lagrane neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.

Definition 51. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft weakly Lagrange neutrosophic N-loop if atleast one $F(a)$ is not Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 67. *All soft weakly Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 68. *Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F,A) \cup_\varepsilon (K,C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F,A) \cup_R (K,C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.

Theorem 69. *Let (F,A) and (H,C) be two soft weakly Lagrange neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not soft weakly Lagrane neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.

Definition 52. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft Lagrange free neutrosophic N-loop if and only if $F(a)$ is not Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 70. *All soft Lagrange free neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 71. *Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F,A) \cup_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F,A) \cup_R (K,C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.

Theorem 72. *Let (F,A) and (H,C) be two soft Lagrange free neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not soft Lagrane free neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.

Definition 53. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, \ldots, *_N\}$ be a neutrosophic N-loop and (F,A) be a soft set over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then (F,A) over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is called soft strong neutrosophic N-loop if and only if $F(a)$ is strong neutrosopchic sub N-loop of $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$, for all $a \in A$.*

Example 20. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, *_2, *_3\}$ where $L_1 = \langle L_5(3) \cup I \rangle, L_2 = \langle L_7(3) \cup I \rangle$ and $L_2 = \{1, 2, I, 2I\}$. $\{\langle L \cup I \rangle\}$ is a strong neutrosophic 3-loop. Then (F,A) is a soft strong neutrosophic N-loop over $\langle L \cup I \rangle$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, eI, 2I\} \cup \{e, 2, eI, 2I\} \cup \{1, I\}, \\ F(a_2) &= \{e, 3, eI, 3I\} \cup \{e, 3, eI, 3I\} \cup \{1, 2, 2I\}. \end{aligned}$$

Theorem 73. *All soft strong neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 74. *Let (F,A) and (K,C) be two soft strong neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft Lagrange free neutrosophic N-loop.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is soft Lagrange free neutrosophic N-loop.
(3) Their restricted union $(F,A)\cup_{R}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft Lagrange free neutrosophic N-loop over.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is soft Lagrange free neutrosophic N-loop over.

Theorem 75. *Let (F,A) and (H,C) be two soft strong neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft strong neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft strong neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.

Definition 54. *Let (F,A) and (H,C) be two soft strong neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then (H,C) is soft strong neutrosophic sub N-loop of (F,A), if*

(1) $B\subset A$.
(2) $H(a)$ is neutrosophic sub N-loop of $F(a)$, for all $a\in A$.

Definition 55. *Let $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F,A) be a soft set over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then (F,A) is called soft strong Lagrange neutrosophic N-loop if and only if $F(a)$ is strong Lagrange neutrosophic sub N-loop of $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$, for all $a\in A$.*

Theorem 76. *All soft strong Lagrange neutrosophic N-loops are soft Lagrange neutrosophic N-loops but the converse is not true.*

Theorem 77. *All soft strong Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 78. *Let (F,A) and (K,C) be two soft strong Lagrange neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop.
(3) Their restricted union $(F,A)\cup_{R}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop over.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop over.

Theorem 79. *Let (F,A) and (H,C) be two soft strong Lagrange neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is not soft strong Lagrange neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft strong Lagrange neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.

Definition 56. *Let $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F, A) be a soft set over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then (F, A) is called soft strong weakly Lagrange neutrosophic N-loop if atleast one $F(a)$ is not strong Lagrange neutrosophic sub N-loop of $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$, for some $a \in A$.*

Theorem 80. *All soft strong weakly Lagrange neutrosophic N-loops are soft weakly Lagrange neutrosophic N-loops but the converse is not true.*

Theorem 81. *All soft strong weakly Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 82. *Let (F, A) and (K, C) be two soft strong weakly Lagrange neutrosophic N-loops over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F, A)\cup_\varepsilon(K, C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(3) Their restricted union $(F, A)\cup_R(K, C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.

Theorem 83. *Let (F, A) and (H, C) be two soft strong weakly Lagrange neutrosophic N-loops over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft strong weakly Lagrange neutrosophic N-loop over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft strong weakly Lagrange neutrosophic N-loop over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.

Definition 57. *Let $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F, A) be a soft set over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then (F, A) is called soft strong Lagrange free neutrosophic N-loop if and only if $F(a)$ is not strong Lagrange neutrosophic sub N-loop of $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$, for all $a \in A$.*

Theorem 84. *All soft strong Lagrange free neutrosophic N-loops are soft Lagrange free neutrosophic N-loops but the converse is not true.*

Theorem 85. *All soft strong Lagrange free neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 86. *Let (F, A) and (K, C) be two soft strong Lagrange free neutrosophic N-loops over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F, A)\cup_\varepsilon(K, C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(3) Their restricted union $(F, A)\cup_R(K, C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I\rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.

Theorem 87. *Let (F, A) and (H, C) be two soft strong Lagrange free neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft strong Lagrange free neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft strong Lagrange free neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.

Conclusion 1. *This paper is an extension of neutrosphic loop to soft neutrosophic loop. We also extend neutrosophic biloop, neutrosophic N-loop to soft neutrosophic biloop, and soft neutrosophic N-loop. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic loop.*

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