A Topological Model for Ecologically Industrial Systems

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Abstract: An ecologically industrial system is such an industrial system in harmony with its environment, especially the natural environment. The main purpose of this paper is to show how to establish a mathematical model for such systems by combinatorics, and find its topological characteristics, which are useful in industrial ecology and the environment protection.

Key Words: Industrial system, ecology, Smarandache multi-system, combinatorial model, input-output analysis, circulating economy.

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§1. Introduction

Usually, the entirely life cycle of a product consists of mining, smelting, production, storage, transporting, use and then finally go to the waste, $\cdots$, etc.. In this process, a lot of waste gas, water or solid waste are produced. Such as those shown in Fig.1 for a producing cell following.
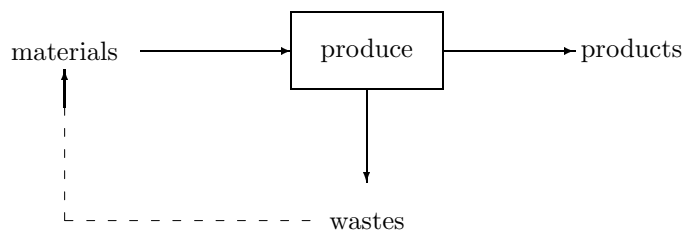

Fig.1

In old times, these wastes produced in industry are directly discarded to the nature without disposal, which brings about an serious problem to human beings, i.e., environment pollution and harmful to our survival. For minimizing the effects of these waste to our survival, the growth of industry should be in coordinated with the nature and the $3R$ *rule*: reduces its amounts, reuses it and furthermore, into recycling, i.e., use these waste into produce again after disposal, or let

them be the materials of other products and then reduce the total amounts of waste to our life environment. An *ecologically industrial system* is such a system consisting of industrial cells in accordance with the $3R$ rule by setting up one or more waste disposal centers. Such a system is opened. Certainly, it can be transferred to a closed one by letting the environment as an additional cell. For example, series produces such as those shown in Fig.2 following.
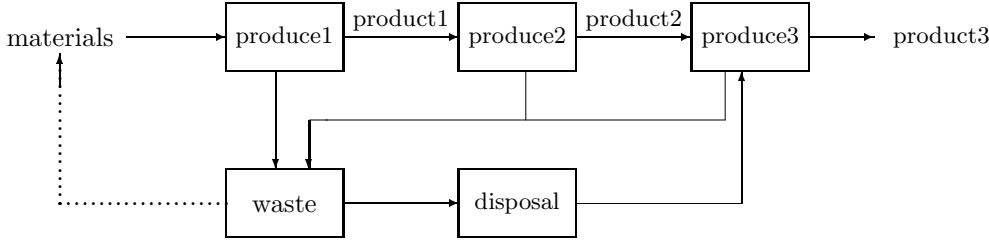

Fig.2

Generally, we can assume that there are $P_1, P_2, \cdots, P_m$ products (including by-products) and $W_1, W_2, \cdots, W_s$ wastes after a produce process. Some of them will be used, and some will be the materials of another produce process. In view of cyclic economy, such an ecologically industrial system is nothing else but a Smarandachely multi-system. Furthermore, it is a combinatorial system defined following.

Definition 1.1([1],[2] and [9]) *A rule in a mathematical system $(\Sigma; \mathcal{R})$ is said to be Smarandachely denied if it behaves in at least two different ways within the same set Σ, i.e., validated and invalided, or only invalided but in multiple distinct ways.*

A Smarandachely system $(\Sigma; \mathcal{R})$ is a mathematical system which has at least one Smarandachely denied rule in $\mathcal{R}$.

Definition 1.2([1],[2] and [9]) *For an integer $m \geq 2$, let $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ be m mathematical systems different two by two. A Smarandache multi-space is a pair $(\widetilde{\Sigma}; \widetilde{\mathcal{R}})$ with*

$$\widetilde{\Sigma} = \bigcup_{i=1}^{m} \Sigma_i, \quad and \quad \widetilde{\mathcal{R}} = \bigcup_{i=1}^{m} \mathcal{R}_i.$$

Definition 1.3([1],[2] and [9]) *A combinatorial system $\mathscr{C}_G$ is a union of mathematical systems $(\Sigma_1; \mathcal{R}_1),(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ for an integer m, i.e.,*

$$\mathscr{C}_G = (\bigcup_{i=1}^{m} \Sigma_i; \ \bigcup_{i=1}^{m} \mathcal{R}_i)$$

with an underlying connected graph structure G, where

$$V(G) = \{\Sigma_1, \Sigma_2, \cdots, \Sigma_m\},$$

$$E(G) = \{ \ (\Sigma_i, \Sigma_j) \mid \Sigma_i \bigcap \Sigma_j \neq \emptyset, 1 \leq i, j \leq m\}.$$

The main purpose of this paper is to show how to establish a mathematical model for such systems by combinatorics, and find its topological characteristics with label equations. In fact, such a system of equations is non-solvable. As we discussed in references [3]-[8], such a non-solvable system of equations has significance also for things in our world and its global behavior can be handed by its G-solutions, where G is a topological graph inherited by this non-solvable system.

§2. A Generalization of Input-Output Analysis

The 3R rule on an ecologically industrial system implies that such a system is optimal both in its economical and environmental results.

2.1 An Input-Output Model

The input-output model is a linear model in macro-economic analysis, established by a economist Leontief as follows, who won the Nobel economic prize in 1973.

Assume these are n departments $D_1, D_2, \cdots, D_n$ in a macro-economic system $\mathscr{L}$ satisfy conditions following:

(1) The total output value of department D_i is x_i. Among them, there are x_{ij} output values for the department D_j and d_i for the social demand, such as those shown in Fig.1.

(2) A unit output value of department D_j consumes t_{ij} input values coming from department D_i. Such numbers t_{ij}, $1 \leq i,j \leq n$ are called *consuming coefficients*.
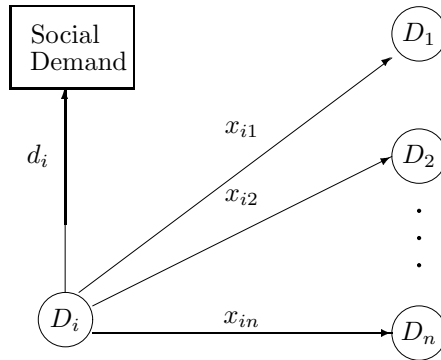

Fig.2

Therefore, such an overall balance macro-economic system $\mathscr{L}$ satisfies n linear equations

$$x_i = \sum_{j=1}^{n} x_{ij} + d_i \tag{1}$$

for integers $1 \leq i \leq n$. Furthermore, substitute $t_{ij} = x_{ij}/x_j$ into equation (10-1), we get that

$$x_i = \sum_{j=1}^{n} t_{ij}x_j + d_i \qquad (2)$$

for any integer i. Let $\mathbf{T} = [t_{ij}]_{n \times n}$, $\mathbf{A} = I_{n \times n} - T$. Then

$$\mathbf{A}\overline{x} = \overline{d}, \qquad (3)$$

from (2), where $\overline{x} = (x_1, x_2, \cdots, x_n)^T$, $\overline{d} = (d_1, d_2, \cdots, d_n)^T$ are the output vector or demand vectors, respectively.

For example, let $\mathscr{L}$ consists of 3 departments D_1, D_2, D_3, where D_1=agriculture, D_2= manufacture industry, D_3=service with an input-output data in Table 1.

Department	D_1	D_2	D_3	Social demand	Total value
D_1	15	20	30	35	100
D_2	30	10	45	115	200
D_3	20	60	/	70	150

Table 1

This table can be turned to a consuming coefficient table by $t_{ij} = x_{ij}/x_j$ following.

Department	D_1	D_2	D_3
D_1	0.15	0.10	0.20
D_2	0.30	0.05	0.30
D_3	0.20	0.30	0.00

Table 2

Thus

$$\mathbf{T} = \begin{bmatrix} 0.15 & 0.10 & 0.20 \\ 0.30 & 0.05 & 0.30 \\ 0.20 & 0.30 & 0.00 \end{bmatrix}, \quad \mathbf{A} = I_{3\times3} - \mathbf{T} = \begin{bmatrix} 0.85 & -0.10 & -0.20 \\ -0.30 & 0.95 & -0.30 \\ -0.20 & -0.30 & 1.00 \end{bmatrix}$$

and the input-output equation system is

$$\begin{cases} 0.85x_1 - 0.10x_2 - 0.20x_3 = d_1 \\ -0.30x_1 + 0.95x_2 - 0.30x_3 = d_2 \\ -0.20x_1 - 0.30x_2 + x - 3 = d_3 \end{cases}$$

Solving this linear system of equations enables one to find the input and output data for economy management.

2.2 A Generalized Input-Output Model

Notice that our WORLD is not linear in general, i.e., the assumption $t_{ij} = x_{ij}/x_j$ does not hold in general. A non-linear input-output model is shown in Fig.3, where $\overline{x} = (x_{1i}, x_{2i}, \cdots, x_{ni})$, $D_1, D_2, \cdots, D_n$ are n departments, SD=social demand. Usually, the function $F(\overline{x})$ is called the *producing function*.
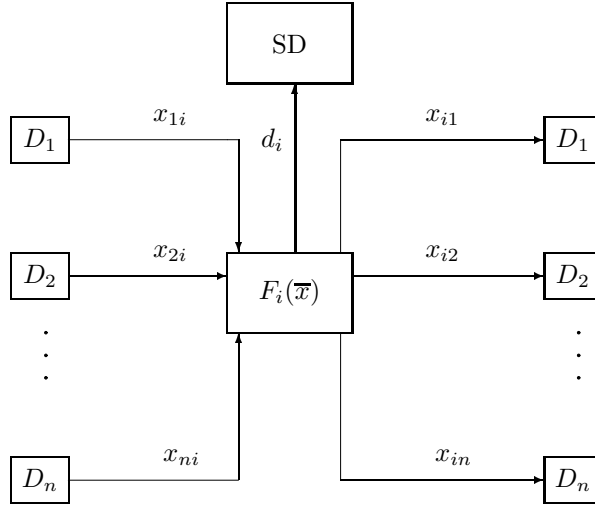
Fig.3

In this case, an overall balance input-output model is characterized by equations

$$F_i(\overline{x}) = \sum_{j=1}^{n} x_{ij} + d_i \tag{4}$$

for integers $1 \le i \le n$, where $F_i(\overline{x})$ may be linear or non-linear and determined by a system of equations such as those of ordinary differential equations

$$1 \le i \le n \begin{cases} F_i^{(n)} + a_1 F_i^{(n-1)} + \cdots + a_{n-1} F_i + a_n = 0 \\ F_i|_{t=0} = \varphi_0, \; F_i^{(1)}\Big|_{t=0} = \varphi_1, \cdots, F_i^{(n-1)}\Big|_{t=0} = \varphi_{n-1} \end{cases} \quad (OES^n)$$

or

$$1 \le i \le n \begin{cases} \dfrac{\partial F_i}{\partial t} = H_1(t, x_1, \cdots, x_{n-1}, p_1, \cdots, p_{n-1}) \\ F_i|_{t=t_0} = \varphi_0(x_1, x_2, \cdots, x_{n-1}) \end{cases} , \quad (PES^1)$$

which can be solved by classical mathematics. However, the input-output model with its generalized only consider the consuming and producing, neglected the waste and its affection to our environment. So it can be not immediately applied to ecologically industrial systems. However, we can generalize such a system for this objective by introducing environment factors, which are discussed in the next section.

$3. **A Topological Model for Ecologically Industrial Systems**

The essence of input-output model is in the output is equal to the input, i.e., a simple case of the *law of conservation of mass*: *a matter can be changed from one form into another, mixtures can be separated or made, and pure substances can be decomposed, but the total amount of mass remains constant.* Applying this law, it needs the environment as an additional cell for ecologically industrial systems and replaces the departments D_i, $1 \leq i \leq n$ by input materials M_i, $1 \leq i \leq n$ or products P_k, $1 \leq k \leq m$, and SD by W_i, $1 \leq i \leq s =$ wastes, such as those shown in Fig.4 following.
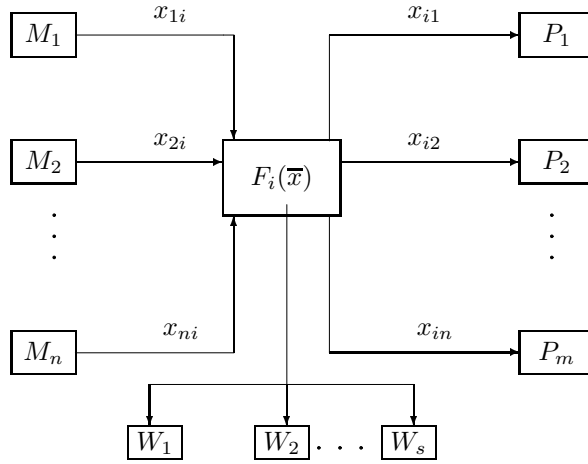

Fig.4

In this case, the balance input-output model is characterized by equations

$$F_i(\overline{x}) = \sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{s} W_i \qquad (5)$$

for integers $1 \leq i \leq n$. We construct a topological graphs following.

Construction 3.1 *Let $\mathscr{J}(t)$ be an ecologically industrial system consisting of cells $C_1(t), C_2(t), \cdots, C_l(t)$, R the environment of $\mathscr{J}$. Define a topological graph $G[\mathscr{J}]$ of $\mathscr{J}$ following:*

$$
\begin{aligned}
V(G[\mathscr{J}]) \ &= \ \{C_1(t), C_2(t), \cdots, C_l(t), R\}; \\
E(G[\mathscr{J}]) \ &= \ \{(C_i(t), C_j(t)) \ if \ there \ is \ an \ input \ from \ C_i(t) \ to \ C_j(t), \ 1 \leq i, j \leq l\} \\
&\quad \bigcup \{(C_i(t), R) \ if \ there \ are \ wastes \ from \ C_i(t) \ to \ R, \ 1 \leq i \leq l\}.
\end{aligned}
$$

Clearly, $G[\mathscr{J}]$ is an inherited graph for an ecologically industrial system $\mathscr{J}$. By the 3R rule, any producing process X_{i_1} of an ecologically industrial system is on a directed cycle $\overrightarrow{C} = (X_{i_1}, X_{i_2}, \cdots, X_{i_k})$, where $X_{i_j} \in \{C_i, \ 1 \leq j \leq l; R\}$, such as those shown in Fig.5.

$$X_{i_1}(t)$$

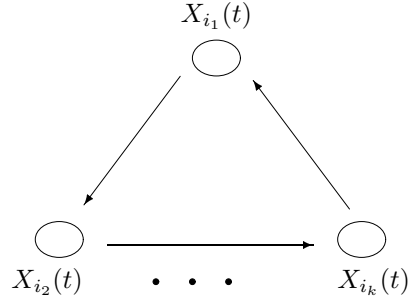$$X_{i_2}(t) \quad \bullet \quad \bullet \quad \bullet \quad X_{i_k}(t)$$

Fig.5

Such structure of cycles naturally determined the topological structure of an ecologically industrial system following.

Theorem 3.2   *Let $\mathscr{J}(t)$ be an ecologically industrial system consisting of producing cells $C_1(t), C_2(t), \cdots, C_l(t)$ underlying a graph $G\,[\,\mathscr{J}(t)]$. Then there is a cycle-decomposition*

$$G\,[\,\mathscr{J}(t)] = \bigcup_{i=1}^{t} \overrightarrow{C}_{k_i}$$

for the directed graph $G\,[\,\mathscr{J}(t)]$ such that each producing process $C_i(t)$, $1 \leq i \leq l$ is on a directed circuit $\overrightarrow{C}_{k_i}$ for an integer $1 \leq i \leq t$. Particularly, $G\,[\,\mathscr{J}(t)]$ is 2-edge connectness.

Proof By definition, each producing process $C_i(t)$ is on a directed cycle, which enables us to get a cycle-decomposition

$$GG\,[\,\mathscr{J}(t)] = \bigcup_{i=1}^{t} \overrightarrow{C}_{k_i}. \qquad \qquad \square$$

Thus, any ecologically industrial system underlying a topological 2-edge connect graph with vertices consisting of these producing process. Whence, we can always call *G-system* for an ecologically industrial system. Clearly, the global effects of G_1-system and G_2-system are different if $G_1 \not\approx G_2$ by definition. Certainly, we can also characterize these *G*-systems with graphs by equations (5) following.

Theorem 3.3   *Let consisting of producing cells $C_1(t), C_2(t), \cdots, C_l(t)$ underlying a graph $G\,[\,\mathscr{J}(t)]$. Then*

$$F_v\big(x_{uv}, u \in N^-_{G[\mathscr{J}(t)]}(v)\big) = \sum_{w \in N^+_{G[\mathscr{J}(t)]}(v)} (-1)^{\delta(v,w)} x_{vw}$$

with $\delta(v, w) = 1$ if x_{vw}=product, and -1 if x_{vw}=waste, where $N^-_{G[\mathscr{J}(t)]}, N^+_{G[\mathscr{J}(t)]}$ are the in or our-neighborhoods of vertex v in $G\,[\,\mathscr{J}(t)]$.

Notice that the system of equations in Theorem 3.3 is non-solvable in $\mathbb{R}^{\Delta+1}$ with Δ the maximum valency of vertices in $G\,[\,\mathscr{J}(t)]$. However, we can also find its $G\,[\,\mathscr{J}(t)]$-solution in

$\mathbb{R}^{\Delta+1}$ (See [4]-[6] for details), which can be also applied for holding the global behavior of such G-systems. Such a $G[\mathscr{J}(t)]$-solution is not constant for $\forall e \in E(G[\mathscr{J}(t)])$. For example, let a G-system with G=circuit be shown in Fig.4.
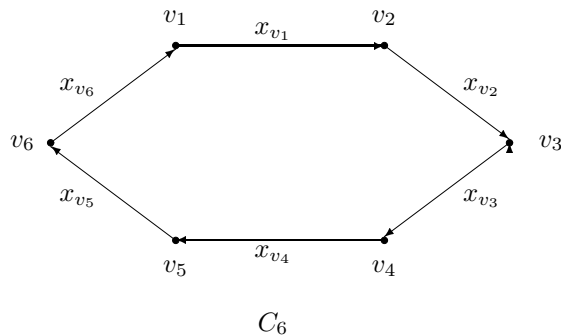

$$C_6$$

Fig.5

Then there are no wastes to environment with equations

$$F_v(x_{v_i}) = x_{v_{i+1}}, \quad 1 \le i \le 6, \quad \text{where } i \bmod 6, \text{ i.e.,}$$
$$F_{v_i} F_{v_{i+1}} \cdots F_{v_{i+6}} = 1 \text{ for any integer } 1 \le i \le 6.$$

If F_{v_i} is given, then solutions x_{v_i}, $1 \le i \le 6$ dependent on an initial value, for example, $x_{v_1}|_{t=0}$, i.e., one needs the choice criterions for determining the initial values $x_{v_i}|_{t=0}$. Notice that an industrial system should harmonizes with its environment. The only criterion for its choice must be

optimal in economy with minimum affection to the environment, or approximately, maximum output with minimum input.

According to this criterion, there are 2 types of G-systems approximating to an ecologically industrial system:

(1) Optimal in economy with all inputs (wastes) $W_{r_1}, W_{r_2}, \cdots, W_{r_s}$ licenced to R;

(2) Minimal wastes to the environment, i.e., minimal used materials but supporting the survival of human beings.

For a G-system, let

$$c_v^- = \sum_{u_1 N_{G[\mathscr{J}(t)]}^-(v)} c(x_{uv}) \text{ and } c_v^+ = \sum_{w \in N_{G[\mathscr{J}(t)]}^+(v)} (-1)^{\delta(v,w)} c(x_{vw})$$

be respectively the producing costs and product income at vertex $v \in V(G)$. Then the optimal function is

$$\Lambda(G) \;\; = \;\; \sum_{v \in V(G)} \left(c_v^+ - c_v^- \right)$$

$$= \;\; \sum_{v \in V(G)} \left(\sum_{w \in N_{G[\mathscr{I}(t)]}^+(v)} (-1)^{\delta(v,w)} c(x_{vw}) - \sum_{u_1 N_{G[\mathscr{I}(t)]}^-(v)} c(x_{uv}) \right).$$

Then, a G-system of Types 1 is a mathematical programming

$$\max \sum_{v \in V(G)} \Lambda(G) \;\; \text{but} \;\; \sum_{v \in V(G)} W_{ri} \leq W_{ri}^U,$$

where W_{ri}^U is the permitted value for waste W_{ri} to the nature for integers $1 \leq i \leq s$. Similarly, a G-system of Types 2 is a mathematical programming

$$\min \sum_{v \in V(G)} W_{ri} \;\; \text{but} \;\; \text{all} \;\; \text{prodcuts} \;\; \text{P} \geq \text{P}^{\text{L}},$$

where P^L is the minimum needs of product P in an area or a country. Particularly, if $W_{ri}^U = 0$, i.e., an ecologically industrial system, such a system can be also characterized by a non-solvable system of equations

$$F_v(x_{uv}, u \in N_{G[\mathscr{I}(t)]}^-(v)) = \sum_{w \in N_{G[\mathscr{I}(t)]}^+(v)} x_{vw} \;\; \text{for} \;\; \forall v \in V(G).$$

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