On the Gravity

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Abstract

We outline the three main models of the gravitation of Newton, Fatio-Le Sage and Einstein. All of them explain, more or less, the force of the gravity, but the last one gives at once the geometry of the space. The Fatio-Le Sage model might be a possible solution.

Key words: Fatio-Le Sage idea, cosmic microwave background radiation.

1. Introduction.

There are at least three main theories on the gravity. Namely, the gravitations of Newton, Fatio-Le Sage and Einstein, respectively.

The Newton's gravity is based on his formula of the gravitation:

$$F = -G\frac{m_1 m_2}{r^2} \tag{1.1}$$

F being the force, G the Newton's gravitational constant, m_1 and m_2 the masses of the bodies 1 and 2, respectively, and r the distance between them. F is an attractive force. This formula functions because is based on the third law of Kepler. However, it does not explain how the attraction is done.

In the gravitation of Fatio-Le Sage the attraction is done by the so-called ultramundane corpuscles that push both bodies toward each other. These corpuscles may be substituted, for example, by gravitons [1] or by thermal radiation [2].

In the Einstein's gravity, the rest mass m or the energy E, since $E = \gamma mc^2$, where c is the speed of the light in the vacuum and $\gamma = (1 - v^2/c^2)^{-1/2}$, v being the speed of the body; curves the space. However, it does not explain how the attraction is done.

We focus on this last theory because it gives the geometry of the space.

2. The Einstein static universe.

The Einstein's gravity, called the general relativity (GR), has the advantage, regarding the other two theories, of explaining at once the geometry of the space; that is, the form of the universe. The matter and the radiation are subjected to the geometry of the space.

There are three possible forms of universe: open ($k < 0$), flat ($k = 0$) and closed ($k > 0$), where [3]

$$k = -\frac{2E}{ml^2} \tag{2.1}$$

is the curvature constant of the space, E being the total energy of a test particle in the surface of the universe, m its rest mass and l a constant length. The radius of the universe would be

$$r(t) = a(t)l \tag{2.2}$$

where $a(t)$ is the scale factor, and t the time.

The most interesting universe model is the closed, static, finite and spherical one, because in it the matter and the radiation can go round and round indefinitely transforming into each other in an endless cycle. This is the case of the Einstein static universe. But in this model, it is needed the so-called cosmological constant, Λ, and it has a value of [3]

$$\Lambda = \frac{k}{a_0^2} = const. \tag{2.3}$$

a_0 being a fixed value of the scale factor a. And the radius of the universe would be now

$$r = a_0 l = const. < \infty \tag{2.4}$$

Λ produces the required repulsion to impede the gravitational collapse, giving a static universe.

However, there are some questions to solve with this model. Namely, how the attraction is done, which affects also to the other two remaining models, open and flat. The cosmological redshift. The nature of Λ. And why the universe exists and which are its dimensions.

3. How the attraction is done.

The Schwarzschild's solution of the Einstein's GR equations for the vacuum gives the square space-time differential interval [4] (p. 398):

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2 dt^2 - r^2\left(\sin^2\theta d\phi^2 + d\theta^2\right) - \frac{dr^2}{1 - \frac{r_g}{r}} \tag{3.1}$$

where

$$r_g = \frac{2GM}{c^2} \tag{3.2}$$

is the gravitational (or Schwarzschild) radius, M being the rest mass of the body that produces the gravitational field and r, θ and ϕ the spherical coordinates.

Following [5], note that $r > r_g$, since $r = r_g$ and $r = 0$ would yield $ds^2 = -\infty$ and $r < r_g$ would produce a change of sign in the time and in the space. Note also that we may put

$$r = \frac{2GM}{v^2} \tag{3.3}$$

v being like a gravitational escape speed: $E = K.E. + P.E. = (1/2)mv^2 - GMm/r$, where E, $K.E.$ and $P.E.$ are, respectively, the total, kinetic and gravitational potential energies of a test particle of rest mass m, and from $E = 0$, the escape velocity would be

$$v = \left(\frac{2GM}{r}\right)^{\frac{1}{2}} \tag{3.4}$$

and as $r_g = 2GM/c^2$, it would correspond to a gravitational escape speed of c. As $r > r_g$, $v < c$, and there is not black holes.

Substituting (3.2) and (3.3) in (3.1), we would have that:

$$ds^2 = \left(1 - \frac{v^2}{c^2}\right)c^2dt^2 - r^2\left(\sin^2\theta d\phi^2 + d\theta^2\right) - \frac{dr^2}{1 - \frac{v^2}{c^2}} \tag{3.5}$$

and for given values of θ and ϕ ($\theta = constant$, $\phi = constant$, $d\theta = 0$ and $d\phi = 0$), it would be:

$$ds^2 = \left(1 - \frac{v^2}{c^2}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{v^2}{c^2}} = c^2dt'^2 - dr'^2 = ds'^2 \tag{3.6}$$

which is a generalization of the special relativity ($ds' = ds$) for a radial motion in a gravitational field. As the system S' moves at the escape speed of the field, v, it is pseudo-Euclidean and in it the space and the time have proper values. But in the rest system S, which is within the field, we have the improper values:

$$dr^2 = \left(1 - \frac{v^2}{c^2}\right)dr'^2 \tag{3.7}$$

$$dt^2 = \frac{dt'^2}{1 - \frac{v^2}{c^2}} \tag{3.8}$$

(3.7) is a Lorentz-Fitzgerald contraction. As this length contraction is a radial contraction, this explains how the gravitational attraction is done. (3.8) is a time dilation (or a time dilatation).

4. The cosmological redshift.

In the open and flat models, the cosmological redshift is explained by Doppler effect, because in them the space is expanding. It is the so-called Big Bang (BB, or great explosion) theory.

But the light of the galaxies is red shifted because the light scatters the microwaves of the cosmic microwave background radiation (CMBR) losing energy exponentially with the distance [6-8]. And where the CMBR is the result of a thermal equilibrium in the universe at an absolute temperature of *2.7 K*, which is the temperature of the intergalactic space (IGS) [9].

In effect, it is assumed that all body with a temperature greater than *0 K* emits electromagnetic radiation in the form of thermal radiation [10] (p. 338); consequently, we may suppose that all body emits this type of radiation [11] (p. 261). As in addition, in a thermal equilibrium, all body emits the same quantity of thermal radiation than absorbs, and vice versa, all body absorbs the same quantity of thermal radiation than emits, [10] (p. 345) [11] (p. 261) we conclude that there will always be thermal radiation in the IGS. From the works of Eddington, Regener and Nernst on the temperature of the IGS, that use the law of Stefan-Boltzmann, [9] we deduce that these processes of emission and absorption of thermal radiation produce a thermal equilibrium at a temperature of *2.7 K*, which corresponds to the temperature of the CMBR. Therefore, we conclude finally that the thermal radiation inside of the IGS is the CMBR. In addition, the CMBR is isotropic and homogeneous.

Now, we suppose that the light emitted by the cosmic light sources (stars, quasars, galaxies) when travels in the IGS interacts with the CMBR losing energy. Specifically, the light opens a linear path (without any change of direction) through the microwaves of the CMBR scattering them. We postulate that the light loses energy in this scattering process following an exponential law, that is, following a so-called "tired light" mechanism. Zwicky coined the concept, later called tired light, of that the light would lose energy (by some type of mechanical interaction) in its journey [12].

In effect, our mechanism would be similar, from a quantum mechanical point of view, to the radiation loss by fast electrons, where the mean energy $\langle E \rangle$ of an electron, with initial energy E_0, after having traversed a length x of the medium, is [13] (p. 74) [14] (p. 39)

$$\langle E \rangle = E_0 e^{-x/X_0} \tag{4.1}$$

X_0 being the so-called radiation length, which is inversely proportional to the density of atoms of the medium. The equation (4.1) is obtained from the framework of reference of the electron and considering that this one scatters electromagnetic fields. The fast electron sees the electromagnetic fields of the atoms of the medium like virtual photons because its supposed relative speed is $v \cong c$. The electron loses energy when scatters a virtual photon because suffers an inverse Compton effect.

Now, by analogy, we can deduce that (in place of the electron) our visible light photon (acting like a particle of "effective mass" hf/c^2, where h is the Planck's constant and f the frequency) scatters (in place of streams of virtual photons) microwaves of the CMBR losing energy following an exponential law similar to (4.1) [7-8]:

$$E(d) = E(0)e^{-d/\delta} \tag{4.2}$$

where

$$d = ct \tag{4.3}$$

is the distance traversed inside of the IGS, and

$$\delta = \frac{k_\delta}{u} \tag{4.4}$$

is a radiation length, k_δ being a constant of proportionality to set and

$$u = nhf_{cm} \tag{4.5}$$

the energy density of the CMBR, where $n = N/V$ is the number of microwaves per unit of volume and f_{cm} their frequency.

u is related with the absolute temperature by the formula [15]:

$$u = \frac{4\sigma}{c}T^4 \tag{4.6}$$

where σ is the constant of Stefan-Boltzmann.

From (4.2), as $E = hf$, then

$$f(d) = f(0)e^{-d/\delta} \tag{4.7}$$

and

$$z = \frac{f(0) - f(d)}{f(d)} = e^{d/\delta} - 1 \tag{4.8}$$

z being the redshift parameter.

And the cosmological redshift increases exponentially with the distance.

For $d/\delta << 1$ ($e^{d/\delta} = 1 + (d/\delta)$)

$$z = \frac{d}{\delta} \tag{4.9}$$

and $z << 1$. And comparing with the redshift of Hubble [4] (p. 486)

$$z = \frac{v_r}{c} = \frac{Hd}{c} \tag{4.10}$$

where $v_r = Hd$ and H are, respectively, the law and the constant of Hubble, and v_r is the speed of recession; we have that

$$\delta = \frac{c}{H} \tag{4.11}$$

Note that from (4.4) and (4.11)

$$H = \frac{c}{k_\delta} u \tag{4.12}$$

Note also that substituting (4.11) into (4.8), and since $f(0) = f_e$ and $f(d) = f_o$, where f_e and f_o are the light frequencies emitted and observed, respectively; we obtain the typical redshift expression of the tired light mechanism

$$z = \frac{f_e - f_o}{f_o} = e^{(H/c)d} - 1 \tag{4.13}$$

All this is in favor of a static universe and rules out the explanation of the cosmological redshift by Doppler effect. That is, it rules out the open and flat models.

Note also that it is not needed any dark energy, for a supposed accelerated expansion of the universe, to explain an exponential redshift.

And note, finally, that with (4.8), it is also explained the so-called intrinsic redshift, which is the excess of redshift of the radio sources (quasars and radio galaxies) in which the light scatters radio waves, and where now d would be the distance traversed by the light inside of the radio source and u, in (4.4), the energy density of the radio waves inside of the radio source [6-8].

The intrinsic redshift cannot be explained in an expanding universe, that is, with the BB model. It only can be explained in a non expanding universe, that is, with a static model.

5. The nature of Λ.

There are authors that relate Λ with a vacuum energy, because this produce repulsion (see the appendix A). Well then, we postulate that it would be

$$\Lambda = \frac{8\pi G}{c^2} u_{vac} \tag{5.1}$$

where u_{vac} is the energy density of the vacuum.

We define the energy of the vacuum as (see the appendix B, (B.3)):

$$U_{vac} = E_0 = \frac{1}{2} h f_{osc} \tag{5.2}$$

E_0 being the energy offset of the quantum harmonic oscillator (QHO). And, also, $U_{vac} = u_{vac}V_{vac}$, where V_{vac} is a volume in the vacuum.

Note also that the factor $1/c^2$ in (5.1) appears considering that

$$U = Mc^2 = \rho c^2 V \tag{5.3}$$

U, M, V and $\rho = M/V$ being, respectively, the relativistic energy, the rest mass, the volume and the mass density of the universe. And from (5.3)

$$\rho = \frac{1}{c^2}\frac{U}{V} \tag{5.4}$$

U/V being the energy density of the universe.

From the velocity equation [3]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{5.5}$$

we see the correspondence between the terms $8\pi G\rho$ and $\Lambda = 8\pi G u_{vac}/c^2$.

From the first principle of the thermodynamics:

$$dU = dQ - dW \tag{5.6}$$

Q being the heat and W the work. As $dQ = 0$, because the universe is an isolated system, and $dW = Fdr = pAdr = pdV$, where $F = pA$ is the force, p the pressure, A the area and r the radial distance, and $Adr = dV$; then

$$dU = -pdV \tag{5.7}$$

Differentiating (5.3) with respect to the time and comparing with (5.7), we have that

$$\dot{U} = \dot{\rho} c^2 V + \rho c^2 \dot{V} = -p\dot{V} \tag{5.8}$$

As $V \propto r^3$, and from (2.2), it is

$$\dot{V}/V = 3\dot{r}/r = 3\dot{a}/a \tag{5.9}$$

Substituting (5.9) in (5.8) we have that

$$\dot{\rho}c^2 = -3(\rho c^2 + p)\frac{\dot{a}}{a} \tag{5.10}$$

As

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} \tag{5.11}$$

it is [3]

$$\dot{\rho} = \frac{d\rho}{dr}\dot{r} = -3\rho\frac{\dot{r}}{r} = -3\rho\frac{\dot{a}}{a} \tag{5.12}$$

And comparing (5.12) with (5.10), we have that

$$p = 0 \tag{5.13}$$

which is the state equation for the matter.

As for the radiation in a cavity of volume V, its energy would be [3]

$$U \propto V^{1/3} \tag{5.14}$$

And from (5.7) and (5.14), $p = -dU/dV \propto -(-1/3)V^{(1/3)-1} = (1/3)V^{1/3}V^{1} \propto (1/3)(U/V)$, that is

$$p = \frac{1}{3}\frac{U}{V} \tag{5.15}$$

And from (5.4), $p = (1/3)(U/V) = (1/3)\rho c^2$, that is

$$p = \frac{1}{3}\rho c^2 \tag{5.16}$$

which is the state equation for the radiation.

Differentiating (5.5), and substituting (5.10), we have the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \tag{5.17}$$

From this, we see that Λ opposes to $4\pi G(\rho + 3p/c^2)$.

Or, in other words, substituting (5.1) in (5.17), it is

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\frac{\rho}{2} + \frac{3p}{2c^2}\right) + \frac{8\pi G u_{vac}}{3c^2} \tag{5.18}$$

where $\rho/2$ represents an effective matter density, $3p/2c^2$ an "effective mass density" of the radiation and $u_{vac}/c^2 = \Lambda/8\pi G$ an "effective mass density" of the vacuum that opposes to the other two.

From (5.17) and (5.5), for a static universe: $\ddot{a} = 0$ and $\dot{a} = 0$, it is obtained (2.3) [3].

6. Why the universe exists and which are its dimensions.

If the universe is static, we can assume that it has always existed (which is in accordance with the energy conservation law that establishes that the energy, by principle, cannot be created or destroyed, only transformed). Then, there would be no beginning and no end.

But, then, why the universe exists if it is much easier that nothing existed. A possible answer would be that all the physical magnitudes are zero (or close to zero).

For any particle on the surface of the universe, according to the classical mechanics of Newton, we have that [3]

$$E = K.E. + P.E. = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - G\frac{Mm}{r} \tag{6.1}$$

where E, $K.E.$ and $P.E.$ are, respectively, the total, kinetic and gravitational potential energies of a test particle, m its rest mass and M and r the rest mass and the radius of the universe, respectively.

For a homogeneous and isotropic universe

$$M = \rho\frac{4}{3}\pi r^3 \tag{6.2}$$

ρ being its mass density.

Substituting (6.2) into (6.1), we obtain that

$$\left(\frac{dr}{rdt}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k'}{r^2} \tag{6.3}$$

where

$$k' = -\frac{2E}{m} \tag{6.4}$$

is a constant of curvature. And from (2.1)

$$k = \frac{k'}{l^2} \tag{6.5}$$

For a flat universe, $k' = 0$ and $k = 0$, and from (6.3)

$$\left(\frac{dr}{rdt}\right)^2 = \frac{8\pi G\rho_c}{3} \tag{6.6}$$

where $\rho_c = 10^{-29}\ g/cm^3$ [4] (p. 487) is the critical mass density, with $dr/rdt = H$.

On the other hand, if $K.E. = -P.E.$, where $K.E. > 0$ and $P.E. < 0$, then $E = K.E. + P.E. = 0$, $k' = 0$ and $k = 0$, and

$$\rho = \frac{3}{8\pi G}\left(\frac{dr}{rdt}\right)^2 \tag{6.7}$$

If $r = \infty$ (flat surface), $\rho = (3/8\pi G)(dr/\infty dt)^2 = 0$. Note that from (6.2), $M = \infty$ and $r = \infty$ implies also $\rho = \infty/\infty^3 = 1/\infty^2 = 0$. In addition, for a static universe, $r = \infty$, would imply, from (2.4), that $l = \infty$, that is, that l is close to infinite. And $k = 0$ would implies the same, from (2.1).

Hence, for a static universe, it would be $k \approx 0$, $l \approx \infty$ and $\rho \approx 0$ but $\rho > \rho_c$.

Also, if U is the energy of the universe, then $U = \sum E = \sum 0 = 0$. As $\sum K.E. + \sum P.E. = \sum (K.E. + P.E.) = \sum E = 0$, then $\sum K.E. = -\sum P.E.$, where $\sum K.E. > 0$ and $\sum P.E. < 0$, or generalizing, the positive energy creates an equal negative gravitational energy. Also, if $\vec{P}$ and $\vec{L}$ are the linear and angular momenta of the universe and $\vec{p}$ and $\vec{\ell}$ the linear and angular momenta of the particle, then $\vec{P} = \sum \vec{p} = 0$, since all way has two opposite directions, and $\vec{L} = \sum \vec{\ell} = 0$, since all rotation is clockwise or its contrary.

Also, as it can be supposed that $\sum K.E. = +\infty$ (or almost) and $\sum P.E. = -\infty$ (or almost), then $U = \sum K.E. + \sum P.E. = \infty - \infty = \Delta E$, and analogously, $P = \left|\vec{P}\right| = \Delta p$, $L = \left|\vec{L}\right| = \Delta \ell$, with $\Delta E \Delta t \geq \hbar/2$ (where $\hbar = h/2\pi$), $\Delta p \Delta x \geq \hbar/2$ (where x is a space coordinate) and $\Delta \ell \geq \hbar/2$, applying the Heisenberg's principle.

Hence, for a static universe, $U \approx 0$, $P \approx 0$ and $L \approx 0$.

In addition, as the quotient U/k_B, where k_B is the constant of Boltzmann, has the dimension of a temperature, it can be used to predict the absolute temperature of the universe: $U/k_B = \Delta E/k_B = \Delta T$, which implies a very low value (in fact, it is $2.7\ K$).

Therefore, and in general, all the physical magnitudes of the static universe are zero or close to zero, except its temperature which is *2.7 K* and its radius which would be close to infinite.

On the other hand, the static universe would be formed, in equal parts, by gamma radiation and by matter and antimatter [16], since: 1) $\gamma = \overline{\gamma}$. 2) $\gamma + \gamma \rightarrow e^+ + e^-$, and vice versa, $e^+ + e^- \rightarrow \gamma + \gamma$. 3) $e^+ + e^- \rightarrow p + \overline{p}$, and vice versa, $p + \overline{p} \rightarrow e^+ + e^-$ [14] (p. 209). 4) p and e^- form a hydrogen atom and $\overline{p}$ and e^+ an antihydrogen antiatom. 5) $p + e^- \rightarrow n + \nu$ and $\overline{p} + e^+ \rightarrow \overline{n} + \overline{\nu}$. 6) $n \rightarrow p + e^- + \overline{\nu}$ and $\overline{n} \rightarrow \overline{p} + e^+ + \nu$. 7) p and n form nuclei of atoms and $\overline{p}$ and $\overline{n}$ form antinuclei of antiatoms. And the different reactions (or events) are governed by the corresponding laws. The particles mentioned: γ, $\overline{\gamma}$, e^-, e^+, p, $\overline{p}$, n, $\overline{n}$, ν and $\overline{\nu}$, correspond, respectively, to the gamma photon, antigamma photon, electron, positron (or antielectron), proton, antiproton, neutron, antineutron, neutrino and antineutrino.

Therefore, the radiation and the matter would be transformed into each other in an endless cycle.

7. Discussion.

As the vacuum is by definition empty and its absolute temperature would be zero, and as the so-called zero (absolute temperature) point energy is an anomaly of the quantum theory (QT), an offset, it have to be taken away, then $U_{vac} = 0$, $u_{vac} = U_{vac}/V_{vac} = 0$, and, from (5.1), $\Lambda = 0$. Hence, the Einstein static universe would not be possible. This rules out in addition the open universe, because it needs also of a vacuum energy for the accelerated expansion. This last model together with the flat one were both already ruled out because they needed of the expansion of the space to explain the cosmological redshift by Doppler effect.

We then might be tempted to choose the closed universe with a BB and a Big Crunch (BC, or great gravitational collapse), but the cyclical version of it would be ruled out because of the second principle of the thermodynamics: the entropy of an isolated (that is, closed) system increases continuously:

$$\frac{dS}{dt} \geq 0 \tag{7.1}$$

S being the entropy. This, in addition, rules out the static universe which is also closed. Note that the closed model is cyclical because it would be a sequence of: BB, BC, BB, BC, etc. It would have also the problem of the cosmological redshift, only explainable with the CMBR [6-8] and considering that now the universe would be in a static phase halfway between a BB an a BC.

Therefore, the only possible universe is a flat, infinite, and hence static one, in which, it would be: $k = 0$, $r = \infty$, $\rho = \rho_c$, $U \approx 0$, $P \approx 0$, $L \approx 0$ and $T = 2.7\ K$, in thermal equilibrium at that absolute temperature which is the temperature of the CMBR, where the gravity [2] and the cosmological redshift [6-8], (4.8) and (4.13), are due to the

CMBR, and where the radiation and the matter would be transformed into each other in an endless cycle.

8. Conclusion.

We have outlined the three main models of the gravitation of Newton, Fatio-Le Sage and Einstein. All of them explain, more or less, the force of the gravity, but the last one gives at once the geometry of the space. The Fatio-Le Sage model might be a possible solution.

Appendix A

From the first principle of the thermodynamics

$$dU = dQ - dW \tag{A.1}$$

where U is the energy, Q the heat and W the work, with $U = uV$, $W = Fr = pAr = pV$, u being the energy density, $V = Ar$ the volume, $F = pA$ the force, r the distance, p the pressure and A the area; for the vacuum, an empty space where $dQ = 0$, $du = 0$ and $dp = 0$, we would have that: $dU = udV$, $dW = pdV$ and $dU = -pdV$, then $udV = -pdV$, and

$$u = -p \tag{A.2}$$

If $u > 0$ (which implies that $U > 0$ because $U = uV$), then $p < 0$, and any particle that enters the vacuum would go to the center. As this would also happen in the adjacent vacuum, then the particles would recede, that is, there would be repulsion.

If $u < 0$ ($U < 0$), then $p > 0$, and any particle that enters the vacuum would go to the periphery. As this would also happen in the adjacent vacuum, then the particles would approach, that is, there would be attraction.

But if $u = 0$ ($U = 0$), then $p = 0$, and any particle that enters the vacuum would not undergo any pressure or force. This would be the normal case because the vacuum is by definition empty and its absolute temperature would be zero. But from the quantum theory (QT) exists an offset, the so-called zero (absolute temperature) point energy (see appendix B).

Appendix B

The energy of the quantum harmonic oscillator (QHO) is:

$$E_n = \left(\frac{1}{2} + n\right) hf_{osc} \tag{B.1}$$

where n is the number of particles and f_{osc} the frequency of oscillation.

Thermodynamically, it would be:

$$\langle E\rangle_T = \frac{1}{2}hf_{osc} + \frac{hf_{osc}}{e^{\frac{hf_{osc}}{k_B T}} - 1} \tag{B.2}$$

k_B being the Boltzmann's constant.

In the vacuum, $n = 0$, which implies $T = 0$, it would be:

$$E_0 = \frac{1}{2}hf_{osc} \tag{B.3}$$

and

$$\langle E\rangle_0 = \frac{1}{2}hf_{osc} \tag{B.4}$$

respectively, then the vacuum energy, or the zero (absolute temperature) point energy, would not be zero. This anomaly is an offset that can be solved redefining the energy of the QHO, respectively, as:

$$E_n - \frac{1}{2}hf_{osc} = nhf_{osc} \tag{B.5}$$

and

$$\langle E\rangle_T - \frac{1}{2}hf_{osc} = \frac{hf_{osc}}{e^{\frac{hf_{osc}}{k_B T}} - 1} \tag{B.6}$$

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