CONJECTURES ON PRIMES AND PSEUDOPRIMES BASED ON SMARANDACHE FUNCTION

(COLLECTED PAPERS)

INTRODUCTION

It is always difficult to talk about arithmetic, because those who do not know what is about, nor do they understand in few sentences, no matter how inspired these might be, and those who know what is about, do no need to be told what is about. For those hwo don't know yet, I will appeal to a comparison. Have you seen the movie "We're no angels" with Robert de Niro and Sean Penn? At a turning point, a character from the movie desperately needed help and look through the pockets of clothes for something that he could use. He found nothing. This branch of mathematics, arithmetic, is well known as the least prolific branch of mathematics in the field of material applications, it will not help you going to the moon or invent the atomic bomb. But, on the other side, you don't need any laboratory or suitcases or jacket pockets to possess or wear them after you. Arithmetic is the branch of mathematics that you keep it in you're soul and you're mind not in you're suitcase or laptop. It also will not help you to gain money (unless you will prove Fermat's last theorem without using complex numbers or you will prove Beal's conjecture which is unlikely) but it will give you something more impotant than that: an occupation on the train when you are going to the funeral of an aunt of third degree. No, I was kidding, if you allow me; it will give you an accession to a world equally rich in special symbols and in special people. One of these special people is Florentin Smarandache, who has a large contribution in number theory, including the very important Smarandache function and few hundred sequences, series, constants, theorems and conjectures.

This collection of articles aims to show new applications of Smarandache function in the study of some well known classes of numbers, like prime numbers, Poulet numbers, Carmichael numbers, Sophie Germain primes etc.

Beside the well known notions of number theory, we defined in these papers the following new concepts: "Smarandache-Coman divisors of order k of a composite integer n with m prime factors", "Smarandache-Coman congruence on primes", "Smarandache-Germain primes", "Coman-Smarandache criterion for primality", "Smarandache-Korselt criterion".

SUMMARY

1. The Smarandache-Coman divisors of order k of a composite integer n with m prime factors
2. Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors
3. The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion
4. Sequences of primes that are congruent sco n
5. Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes
6. Two conjectures which generalize the conjecture on the infinity of Sophie Germain primes
7. An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function
8. A recurrent formula inspired by Rowland’s formula and based on Smarandache function which might be a criterion for primality
9. The Smarandache-Korselt criterion, a variant of Korselt’s criterion

1. The Smarandache-Coman divisors of order k of a composite integer n with m prime factors

Abstract. We will define in this paper the Smarandache-Coman divisors of order k of a composite integer n with m prime factors, a notion that seems to have promising applications, at a first glance at least in the study of absolute and relative Fermat pseudoprimes, Carmichael numbers and Poulet numbers.

Definition 1:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way:
$SCD_1(n) = \{S(d_1 – 1), S(d_2 – 1), ..., S(d_m – 1)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is {S(2 – 1), S(3 – 1)} = {S(1), S(2)} = {1, 2}, because 6 = 2*3;
2. $SCD_1(429)$ = {S(3 – 1), S(11 – 1), S(13 – 1)} = {S(2), S(10), S(12)} = {2, 5, 4}, because 429 = 3*11*13.

Definition 2:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way:
$SCD_2(n) = \{S(d_1 – 2), S(d_2 – 2), ..., S(d_m – 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 2 of the number 21 is {S(3 – 2), S(7 – 2)} = {S(1), S(5)} = {1, 5}, because 21 = 3*7;
2. $SCD_2(2429)$ = {S(7 – 2), S(347 – 2)} = {S(5), S(345)} = {5, 23}, because 2429 = 7*347.

Definition 3:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, $n = d_1*d_2*…*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way:
$SCD_k(n) = \{S(d_1 – k), S(d_2 – k), …, S(d_m – k)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 5 of the number 539 is {S(7 – 5), S(11 – 5)} = {S(2), S(6)} = {2, 3}, because 539 = 7^2*11;
2. $SCD_6(221)$ = {S(13 – 6), S(17 – 6)} = {S(7), S(11)} = {7, 11}, because 221 = 13*17.

Comment:

We obviously defined the sets of numbers above because we believe that they can have interesting applications, in fact we believe that they can even make us re-think and re-consider the Smarandache function as an instrument to operate in the world of number theory: while at the beginning its value was considered to consist essentially in that to be a criterion for primality, afterwards the Smarandache function crossed a normal process of substantiation, so it was constrained to evolve in a relatively closed (even large) circle of equalities, inequalities, conjectures and theorems concerning, most of them, more or less related concepts. We strongly believe that some of the most important applications of the Smarandache function are still undiscovered. We were inspired in defining the Smarandache-Coman divisors by the passion for Fermat pseudoprimes, especially for Carmichael numbers and Poulet numbers, by the Korselt's criterion, one of the very few (and the most important from them) instruments that allow us to comprehend Carmichael numbers, and by the encouraging results we easily obtained, even from the first attempts to relate these two types of numbers, Fermat pseudoprimes and Smarandache numbers.

Smarandache-Coman divisors of order 1 of the 2-Poulet numbers:

(See the sequence A214305 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(341)$ = {S(11 – 1), S(31 – 1)} = {S(10), S(30)} = {5, 5};
$SCD_1(1387)$ = {S(19 – 1), S(73 – 1)} = {S(18), S(72)} = {6, 6};
$SCD_1(2047)$ = {S(23 – 1), S(89 – 1)} = {S(22), S(88)} ={11, 11};
$SCD_1(2701)$ = {S(37 – 1), S(73 – 1)} = {S(36), S(72)} = {6, 6};
$SCD_1(3277)$ = {S(29 – 1), S(113 – 1)} = {S(28), S(112)} = {7, 7};
$SCD_1(4033)$ = {S(37 – 1), S(109 – 1)} = {S(36), S(108)} = {6, 9};
$SCD_1(4369)$ = {S(17 – 1), S(257 – 1)} = {S(16), S(256)} = {6, 10};
$SCD_1(4681)$ = {S(31 – 1), S(151 – 1)} = {S(30), S(150)} = {5, 10};
$SCD_1(5461)$ = {S(43 – 1), S(127 – 1)} = {S(42), S(126)} = {7, 7};
$SCD_1(7957)$ = {S(73 – 1), S(109 – 1)} = {S(72), S(108)} = {6, 9};
$SCD_1(8321)$ = {S(53 – 1), S(157 – 1)} = {S(52), S(156)} = {13, 13}.

Comment:

It is notable how easily are obtained interesting results: from the first 11 terms of the 2-Poulet numbers sequence checked there are already foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 1 are equal, as for the seven from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 1 is equal to {6, 6}, the case of Poulet numbers 1387 and 2701, or with {6, 9}, the case of Poulet numbers 4033 and 7957?

Smarandache-Coman divisors of order 2 of the 2-Poulet numbers:

$SCD_2(341)$ = {S(11 – 2), S(31 – 2)} = {S(9), S(29)} = {6, 29};
$SCD_2(1387)$ = {S(19 – 2), S(73 – 2)} = {S(17), S(71)} ={17, 71};
$SCD_2(2047)$ = {S(23 – 2), S(89 – 2)} = {S(21), S(87)} = {7, 29};
$SCD_2(2701)$ = {S(37 – 2), S(73 – 2)} = {S(35), S(71)} = {7, 71};
$SCD_2(3277)$ = {S(29 – 2), S(113 – 2)} = {S(27), S(111)} = {9, 37};
$SCD_2(4033)$ = {S(37 – 2), S(109 – 2)} = {S(35), S(107)} = {7, 107};
$SCD_2(4369)$ = {S(17 – 2), S(257 – 2)} = {S(15), S(255)} = {5, 17};
$SCD_2(4681)$ = {S(31 – 2), S(151 – 2)} ={S(29), S(149)} = {29, 149};

$SCD_2(5461)$ = {S(43 – 2), S(127 – 2)} = {S(41), S(125)} = {41, 15};
$SCD_2(7957)$ = {S(73 – 2), S(109 – 2)} ={S(71), S(107)} = {71, 107};
$SCD_2(8321)$ = {S(53 – 2), S(157 – 2)} = {S(52), S(156)} = {17, 31}.

Comment:
In the case of SCD of order 2 of the 2-Poulet numbers there are too foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 2 are both primes, as for the eight from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 2 is equal to {p, p + 20*k}, where p prime and k positive integer, the case of Poulet numbers 4033 and 4681?

Smarandache-Coman divisors of order 1 of the 3-Poulet numbers:

(See the sequence A215672 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(561)$ = $SCD_1(3*11*17)$ = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = $SCD_1(3*5*43)$ = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = $SCD_1(5*13*17)$ = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = $SCD_1(7*13*19)$ = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = $SCD_1(3*5*127)$ = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = $SCD_1(5*17*29)$ = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = $SCD_1(7*13*31)$ = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = $SCD_1(3*31*47)$ = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = $SCD_1(7*23*41)$ = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = $SCD_1(3*11*257)$ = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = $SCD_1(7*19*67)$ = {S(6), S(18), S(66)} = {3, 19, 67}.

Open problems:

1. Is there an infinity of 3-Poulet numbers for which the set of SCD of order 1 is equal to {2, 4, 7}, the case of Poulet numbers 645 and 1905?
2. Is there an infinity of 3-Poulet numbers for which the sum of SCD of order 1 is equal to 13, the case of Poulet numbers 561 (2 + 5 + 6 = 13), 645 (2 + 4 + 7 = 13), 1729 (3 + 4 + 6 = 13), 1905 (2 + 4 + 7 = 13) or is equal to 17, the case of Poulet numbers 2465 (4 + 6 + 7 = 17) and 8481 (2 + 5 + 10 = 17)?
3. Is there an infinity of Poulet numbers for which the sum of SCD of order 1 is prime, which is the case of the eight from the eleven numbers checked above? What about the sum of SCD of order 1 plus 1, the case of Poulet numbers 2821 (3 + 4 + 5 + 1 = 13) and 4371 (2 + 5 + 23 + 1 = 31) or the sum of SCD of order 1 minus 1, the case of Poulet numbers 1105 (4 + 4 + 6 – 1 = 13), 2821 (3 + 4 + 5 – 1 = 11) and 4371 (2 + 5 + 23 – 1 = 29)?

2. Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors

Abstract. In a previous article I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I sketched some possible applications of this concept in the study of Fermat pseudoprimes. In this paper I make few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors.

Conjecture 1:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 1 equal to {p, p}, where p is prime.

The sequence of this 2-Poulet numbers is: 341, 2047, 3277, 5461, 8321, 13747, 14491, 19951, 31417, ... (see the lists below).

Conjecture 2:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {p, p + 20*k}, where p is prime and k is non-null integer.

The sequence of this 2-Poulet numbers is: 4033, 4681, 10261, 15709, 23377, 31609, ... (see the lists below).

Conjecture 3:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b + 1 is prime.

The sequence of this 2-Poulet numbers is: 1387, 2047, 2701, 3277, 4369, 4681, 8321, 13747, 14491, 18721, 31417, 31609, ... (see the lists below).
Note: This is the case of twelve from the first twenty 2-Poulet numbers.

Conjecture 4:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 is prime.

The sequence of this 2-Poulet numbers is: 4033, 8321, 10261, 13747, 14491, 15709, 19951, 23377, 31417, ... (see the lists below).

Conjecture 5:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 and a + b + 1 are twin primes.

The sequence of this 2-Poulet numbers is: 13747, 14491, 23377, 31417, ... (see the lists below).

Conjecture 6:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b = c + d and a, b, c, d are primes.

Such pair of 2-Poulet numbers is: (4681, 7957), because 29 + 149 = 71 + 107 = 178.

Conjecture 7:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b + 1 = c + d – 1.

Such pairs of 2-Poulet numbers are:
(3277, 8321), because 9 + 37 + 1 = 17 + 31 - 1 = 47;
(19951, 5461), because 23 + 31 + 1 = 41 + 15 – 1 = 55.

Conjecture 8:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where abs{p – q} = 6*k, where p and q are primes and k is non-null positive integer.

The sequence of this 2-Poulet numbers is:
1387, 2047, 2701, 3277, 4033, 4369, 7957, 13747, 14491, 15709, 23377, 31417, 31609, ... (see the lists below).

Note: This is the case of thirteen from the first twenty 2-Poulet numbers.

Conjecture 9:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {a, b}, where abs{a – b} = p and p is prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 10261, ... (see the lists below).

Conjecture 10:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where one from the numbers p and q is prime and the other one is twice a prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 5461, 10261, ... (see the lists below).

Conjecture 11:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c is prime and a, b, c are primes.

The sequence of this 2-Poulet numbers is: 561, 645, 1729, 1905, 2465, 6601, 8481, 8911, 10585, 12801, 13741, ... (see the lists below).

Note: This is the case of eleven from the first twenty 2-Poulet numbers.

Conjecture 12:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c - 1 and a + b + c + 1 are twin primes.

The sequence of this 3-Poulet numbers is: 2821, 4371, 16705, 25761, 30121, ... (see the lists below)

Conjecture 13:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {n, n, n}.

Such 3-Poulet number is 13981.

Conjecture 14:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 2 equal to {5, p, q}, where p and q are primes and q = p + 6*k, where k is non-null positive integer.

Such 3-Poulet numbers are:
1729, because $SCD_2(1729)$ = {5, 11, 17} and 17 = 11 + 6*1;
2821, because $SCD_2(2821)$ = {5, 11, 29} and 29 = 11 + 6*3;
6601, because $SCD_2(6601)$ = {5, 7, 13} and 13 = 7 + 6*1;
13741, because $SCD_2(13741)$ = {5, 11, 149} and 149 = 11 + 6*23;
15841, because $SCD_2(15841)$ = {5, 29, 71} and 71 = 29 + 6*7;
30121, because $SCD_2(30121)$ = {5, 11, 329} and 329 = 11 + 6*53.

Conjecture 15:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 7, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 18705, 55245, 72855, 215265, 831405, 1246785, ... (see the lists below)

Conjecture 16:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 23, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 62745, 451905, ... (see the lists below)

Conjecture 17:

There is an infinity of Poulet numbers which are multiples of any Poulet number divisible by 15 which has the set of SC divisors of order 1 equal to {2, 4, n_1, ..., n_i}, where $n_1 = n_2 =...= n_i = 7$ and i > 0.

Examples:

The Poulet number 645 = 3*5*43, having $SCD_1(645) = \{2, 4, 7\}$, has the multiples the Poulet numbers 18705, 72885, which have $SCD_1 = \{2, 4, 7, 7\}$.

The Poulet number 1905 = 3*5*127, having $SCD_1(1905) = \{2, 4, 7\}$, has the multiples 55245, 215265 which have $SCD_1 = \{2, 4, 7, 7\}$.

(see the sequence A215150 in OEIS for a list of Poulet numbers divisible by smaller Poulet numbers)

List of SC divisors of order 1 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_1(341) = \{S(11 – 1), S(31 – 1)\} = \{S(10), S(30)\} = \{5, 5\}$;
$SCD_1(1387) = \{S(19 – 1), S(73 – 1)\} = \{S(18), S(72)\} = \{6, 6\}$;
$SCD_1(2047) = \{S(23 – 1), S(89 – 1)\} = \{S(22), S(88)\} = \{11, 11\}$;
$SCD_1(2701) = \{S(37 – 1), S(73 – 1)\} = \{S(36), S(72)\} = \{6, 6\}$;
$SCD_1(3277) = \{S(29 – 1), S(113 – 1)\} = \{S(28), S(112)\} = \{7, 7\}$;
$SCD_1(4033) = \{S(37 – 1), S(109 – 1)\} = \{S(36), S(108)\} = \{6, 9\}$;
$SCD_1(4369) = \{S(17 – 1), S(257 – 1)\} = \{S(16), S(256)\} = \{6, 10\}$;
$SCD_1(4681) = \{S(31 – 1), S(151 – 1)\} = \{S(30), S(150)\} = \{5, 10\}$;
$SCD_1(5461) = \{S(43 – 1), S(127 – 1)\} = \{S(42), S(126)\} = \{7, 7\}$;
$SCD_1(7957) = \{S(73 – 1), S(109 – 1)\} = \{S(72), S(108)\} = \{6, 9\}$;
$SCD_1(8321) = \{S(53 – 1), S(157 – 1)\} = \{S(52), S(156)\} = \{13, 13\}$;
$SCD_1(10261) = \{S(31 – 1), S(331 – 1)\} = \{S(30), S(330)\} = \{5, 11\}$;
$SCD_1(13747) = \{S(59 – 1), S(233 – 1)\} = \{S(58), S(232)\} = \{29, 29\}$;
$SCD_1(14491) = \{S(43 – 1), S(337 – 1)\} = \{S(42), S(336)\} = \{7, 7\}$;
$SCD_1(15709) = \{S(23 – 1), S(683 – 1)\} = \{S(22), S(682)\} = \{11, 31\}$;
$SCD_1(18721) = \{S(97 – 1), S(193 – 1)\} = \{S(96), S(192)\} = \{8, 8\}$;
$SCD_1(19951) = \{S(71 – 1), S(281 – 1)\} = \{S(70), S(280)\} = \{7, 7\}$;
$SCD_1(23377) = \{S(97 – 1), S(241 – 1)\} = \{S(96), S(240)\} = \{8, 6\}$;
$SCD_1(31417) = \{S(89 – 1), S(353 – 1)\} = \{S(88), S(352)\} = \{11, 11\}$;
$SCD_1(31609) = \{S(73 – 1), S(433 – 1)\} = \{S(72), S(432)\} = \{6, 9\}$.

List of SC divisors of order 2 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_2(341) = \{S(11 – 2), S(31 – 2)\} = \{S(9), S(29)\} = \{6, 29\}$;
$SCD_2(1387) = \{S(19 – 2), S(73 – 2)\} = \{S(17), S(71)\} = \{17, 71\}$;
$SCD_2(2047) = \{S(23 – 2), S(89 – 2)\} = \{S(21), S(87)\} = \{7, 29\}$;
$SCD_2(2701) = \{S(37 – 2), S(73 – 2)\} = \{S(35), S(71)\} = \{7, 71\}$;
$SCD_2(3277) = \{S(29 – 2), S(113 – 2)\} = \{S(27), S(111)\} = \{9, 37\}$;
$SCD_2(4033) = \{S(37 – 2), S(109 – 2)\} = \{S(35), S(107)\} = \{7, 107\}$;
$SCD_2(4369) = \{S(17 – 2), S(257 – 2)\} = \{S(15), S(255)\} = \{5, 17\}$;
$SCD_2(4681) = \{S(31 – 2), S(151 – 2)\} = \{S(29), S(149)\} = \{29, 149\}$;
$SCD_2(5461) = \{S(43 – 2), S(127 – 2)\} = \{S(41), S(125)\} = \{41, 15\}$;
$SCD_2(7957) = \{S(73 – 2), S(109 – 2)\} = \{S(71), S(107)\} = \{71, 107\}$;
$SCD_2(8321) = \{S(53 – 2), S(157 – 2)\} = \{S(51), S(155)\} = \{17, 31\}$;
$SCD_2(10261) = \{S(31 – 2), S(331 – 2)\} = \{S(29), S(329)\} = \{29, 47\}$;

$SCD_2(13747)$ = {S(59 – 2), S(233 – 2)} = {S(57), S(231)} = {19, 11};
$SCD_2(14491)$ = {S(43 – 2), S(337 – 2)} = {S(41), S(335)} = {41, 67};
$SCD_2(15709)$ = {S(23 – 2), S(683 – 2)} = {S(21), S(681)} = {7, 227};
$SCD_2(18721)$ = {S(97 – 2), S(193 – 2)} = {S(95), S(191)} = {19, 191};
$SCD_2(19951)$ = {S(71 – 2), S(281 – 2)} = {S(69), S(279)} = {23, 31};
$SCD_2(23377)$ = {S(97 – 2), S(241 – 2)} = {S(95), S(239)} = {19, 239};
$SCD_2(31417)$ = {S(89 – 2), S(353 – 2)} = {S(87), S(351)} = {29, 13};
$SCD_2(31609)$ = {S(73 – 2), S(433 – 2)} = {S(71), S(431)} = {71, 431}.

List of SC divisors of order 6 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_6(341)$ = {S(11 – 6), S(31 – 6)} = {S(5), S(25)} = {5, 10};
$SCD_6(1387)$ = {S(19 – 6), S(73 – 6)} = {S(13), S(67)} = {13, 67};
$SCD_6(2047)$ = {S(23 – 6), S(89 – 6)} = {S(17), S(83)} = {17, 83};
$SCD_6(2701)$ = {S(37 – 6), S(73 – 6)} = {S(31), S(67)} = {31, 67};
$SCD_6(3277)$ = {S(29 – 6), S(113 – 6)} = {S(23), S(107)} = {23, 107};
$SCD_6(4033)$ = {S(37 – 6), S(109 – 6)} = {S(31), S(103)} = {31, 103};
$SCD_6(4369)$ = {S(17 – 6), S(257 – 6)} = {S(11), S(251)} = {11, 251};
$SCD_6(4681)$ = {S(31 – 6), S(151 – 6)} = {S(25), S(145)} = {10, 29};
$SCD_6(5461)$ = {S(43 – 6), S(127 – 6)} = {S(37), S(121)} = {37, 22};
$SCD_6(7957)$ = {S(73 – 6), S(109 – 6)} = {S(67), S(103)} = {67, 103};
$SCD_6(8321)$ = {S(53 – 6), S(157 – 6)} = {S(47), S(151)} = {47, 151};
$SCD_6(10261)$ = {S(31 – 6), S(331 – 6)} = {S(25), S(325)} = {10, 13};
$SCD_6(13747)$ = {S(59 – 6), S(233 – 6)} = {S(53), S(227)} = {53, 227};
$SCD_6(14491)$ = {S(43 – 6), S(337 – 6)} = {S(37), S(331)} = {37, 331};
$SCD_6(15709)$ = {S(23 – 6), S(683 – 6)} = {S(17), S(677)} = {17, 677};
$SCD_6(18721)$ = {S(97 – 6), S(193 – 6)} = {S(91), S(187)} = {13, 17};
$SCD_6(19951)$ = {S(71 – 6), S(281 – 6)} = {S(65), S(275)} = {13, 11};
$SCD_6(23377)$ = {S(97 – 6), S(241 – 6)} = {S(91), S(235)} = {13, 47};
$SCD_6(31417)$ = {S(89 – 6), S(353 – 6)} = {S(83), S(347)} = {83, 347};
$SCD_6(31609)$ = {S(73 – 6), S(433 – 6)} = {S(67), S(427)} = {67, 61}.

List of SC divisors of order 1 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

$SCD_1(561)$ = $SCD_1(3*11*17)$ = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = $SCD_1(3*5*43)$ = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = $SCD_1(5*13*17)$ = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = $SCD_1(7*13*19)$ = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = $SCD_1(3*5*127)$ = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = $SCD_1(5*17*29)$ = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = $SCD_1(7*13*31)$ = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = $SCD_1(3*31*47)$ = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = $SCD_1(7*23*41)$ = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = $SCD_1(3*11*257)$ = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = $SCD_1(7*19*67)$ = {S(6), S(18), S(66)} = {3, 19, 67};
$SCD_1(10585)$ = $SCD_1(5*29*73)$ = {S(4), S(28), S(72)} = {4, 7, 6};

SCD_1(12801) = SCD_1(3*17*251) = {S(2), S(16), S(250)} = {2, 6, 15};
SCD_1(13741) = SCD_1(7*13*151) = {S(6), S(12), S(150)} = {3, 4, 10};
SCD_1(13981) = SCD_1(11*31*41) = {S(10), S(30), S(40)} = {5, 5, 5};
SCD_1(15841) = SCD_1(7*31*73) = {S(6), S(30), S(72)} = {3, 5, 6};
SCD_1(16705) = SCD_1(5*13*257) = {S(4), S(12), S(256)} = {4, 4, 10};
SCD_1(25761) = SCD_1(3*31*277) = {S(2), S(30), S(276)} = {2, 5, 23};
SCD_1(29341) = SCD_1(13*37*61) = {S(12), S(36), S(60)} = {4, 6, 5};
SCD_1(30121) = SCD_1(7*13*331) = {S(6), S(12), S(330)} = {3, 4, 11}.

List of SC divisors of order 2 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

SCD_2(561) = SCD_1(3*11*17) = {S(1), S(9), S(15)} = {1, 6, 5};
SCD_2(645) = SCD_1(3*5*43) = {S(1), S(3), S(41)} = {1, 3, 41};
SCD_2(1105) = SCD_1(5*13*17) = {S(3), S(11), S(15)} = {3, 11, 5};
SCD_2(1729) = SCD_1(7*13*19) = {S(5), S(11), S(17)} = {5, 11, 17};
SCD_2(1905) = SCD_1(3*5*127) = {S(1), S(3), S(125)} = {1, 3, 15};
SCD_2(2465) = SCD_1(5*17*29) = {S(3), S(15), S(27)} = {3, 5, 9};
SCD_2(2821) = SCD_1(7*13*31) = {S(5), S(11), S(29)} = {5, 11, 29};
SCD_2(4371) = SCD_1(3*31*47) = {S(1), S(29), S(45)} = {1, 29, 6};
SCD_2(6601) = SCD_1(7*23*41) = {S(5), S(21), S(29)} = {5, 7, 13};
SCD_2(8481) = SCD_1(3*11*257) = {S(1), S(9), S(255)} = {1, 6, 17};
SCD_2(8911) = SCD_1(7*19*67) = {S(5), S(17), S(65)} = {5, 17, 13};
SCD_2(10585) = SCD_1(5*29*73) = {S(3), S(27), S(71)} = {3, 9, 71};
SCD_2(12801) = SCD_1(3*17*251) = {S(1), S(15), S(249)} = {1, 5, 83};
SCD_2(13741) = SCD_1(7*13*151) = {S(5), S(11), S(149)} = {5, 11, 149};
SCD_2(13981) = SCD_1(11*31*41) = {S(9), S(29), S(39)} = {6, 29, 13};
SCD_2(15841) = SCD_1(7*31*73) = {S(5), S(29), S(71)} = {5, 29, 71};
SCD_2(16705) = SCD_1(5*13*257) = {S(3), S(111), S(255)} = {3, 11, 17};
SCD_2(25761) = SCD_1(3*31*277) = {S(1), S(29), S(275)} = {1, 29, 11};
SCD_2(29341) = SCD_1(13*37*61) = {S(11), S(35), S(59)} = {11, 7, 59};
SCD_2(30121) = SCD_1(7*13*331) = {S(5), S(11), S(329)} = {5, 11, 329}.

List of SC divisors of order 1 of the first ten Poulet numbers divisible by 3 and 5:
(see the sequence A216364 that I submitted to OEIS for a list of Poulet numbers divisible by 15)

SCD_1(645) = SCD_1(3*5*43) = {2, 4, 7};
SCD_1(1905) = SCD_1(3*5*127) = {2, 4, 7};
SCD_1(18705) = SCD_1(3*5*29*43) = {2, 4, 7, 7};
SCD_1(55245) = SCD_1(3*5*29*127) = {2, 4, 7, 7};
SCD_1(62745) = SCD_1(3*5*47*89) = {2, 4, 23, 11};
SCD_1(72855) = SCD_1(3*5*43*113) = {2, 4, 7, 7};
SCD_1(215265) = SCD_1(3*5*113*127) = {2, 4, 7, 7};
SCD_1(451905) = SCD_1(3*5*47*641) = {2, 4, 23, 8};
SCD_1(831405) = SCD_1(3*5*43*1289) = {2, 4, 7, 23};
SCD_1(1246785) = SCD_1(3*5*43*1933) = {2, 4, 7, 23}.

3. The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion

Abstract. In two previous articles I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I made few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors. In this paper I define a very related notion, the Smarandache-Coman congruence on primes, and I also make five conjectures regarding Poulet numbers based on this new notion.

Definition 1:

We define in the following way *the Smarandache-Coman congruence on primes*: we say that *two primes p and q are congruent sco n* and we note p ≡ q(sco n) if S(p – n) = S(q – n) = k, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to p sco n respectively k is also equal to q sco n and note k = p sco n = q sco n.

Note:

The notion of *Smarandache-Coman congruence* is very related with the notion of *Smarandache-Coman divisors*, which we defined in previous papers in the following way (Definitions 2-4):

Definition 2:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, n = d_1*d_2*...*d_m, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way: SCD_1(n) = {S(d_1 – 1), S(d_2 – 1), ..., S(d_m – 1)}, where S is the Smarandache function.

Definition 3:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, n = d_1*d_2*...*d_m, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way: SCD_2(n) = {S(d_1 – 2), S(d_2 – 2), ..., S(d_m – 2)}, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is SCD_1(6) = {S(2 – 1), S(3 – 1)} = {S(1), S(2)} = {1, 2};
2. The set of SC divisors of order 2 of the number 21 is SCD_2(21) = {S(3 – 2), S(7 – 2)} = {S(1), S(5)} = {1, 5}.

Definition 4:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, n = d_1*d_2*…*d_m, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way: SCD_k(n) = {S(d_1 – k), S(d_2 – k), …, S(d_m – k)}, where S is the Smarandache function.

Note:

As I said above, in two previous articles I applied the notion of *Smarandache-Coman divisors* in the study of Fermat pseudoprimes; now I will apply the notion of *Smarandache-Coman congruence* in the study of the same class of numbers.

Conjecture 1:

There is at least one non-null positive integer n such that the prime factors of a Poulet number P, where P is not divisible by 3 or 5 and also P is not a Carmichael number, are, all of them, congruent sco n.

Verifying the conjecture:

(for the first five Poulet numbers not divisible by 3 or 5; see the sequence A001567 in OEIS for a a list of these numbers; see also the sequence A002034 for the values of Smarandache function)

: For P = 341 = 11*31, we have S(11 – 1) = S(31 – 1) = 5, so the prime factors 11 and 31 are congruent sco 1, which is written 11 ≡ 31(sco 1), or, in other words, 11 sco 1 = 31 sco 1 = 5; we also have S(11 – 7) = S(31 – 7) = 4, so 11 ≡ 31(sco 7);

: For P = 1387 = 19*73, we have S(19 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1, or, in other words, 6 is equal to 19 sco 1 and also with 73 sco 1;

: For P = 2047 = 23*89, we have S(23 – 1) = S(89 – 1) = 11, so the prime factors 19 and 73 are congruent sco 1;

: For P = 2701 = 37*73, we have S(37 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1;

: For P = 3277 = 29*113, we have S(29 – 1) = S(113 – 1) = 7, so the prime factors 29 and 113 are congruent sco 1.

Note:

If the conjecture doesn't hold in this form might be considered only the 2-Poulet numbers not divisible by 3 or 5.

Conjecture 2:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 3 of a k-Poulet number P divisible by 3 and not divisible by 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that q_1 = d_1 sco n, q_2 = d_2 sco n, ..., q_{k-1} = d_{k-1} sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 3 and not divisible by 5)

: For P = 561 = 3*11*17, we have 7 = 11 sco 4 and 13 = 17 sco 4;
: For P = 4371 = 3*31*47, we have 31 = 7 sco 3 and 47 = 11 sco 3;
: For P = 8481 = 3*11*257, we have 11 = 7 sco 4 and 257 = 23 sco 4;
: For P = 12801 = 3*17*251, we have 17 = 5 sco 2 and 251 = 83 sco 2.

Conjecture 3:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 5 of a k-Poulet number P divisible by 5 and not divisible by 3 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_{k-1} = d_{k-1}$ sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 5 and not divisible by 3)

: For P = 1105 = 5*13*17, we have 13 = 11 sco 2 and 17 = 5 sco 2;

: For P = 10585 = 5*29*73, we have 29 = 13 sco 3 and 73 = 7 sco 3;

: For P = 11305 = 5*7*17*19, we have 7 = 5 sco 2, 17 = 5 sco 2 and 19 = 17 sco 2;

: For P = 41665 = 5*13*641, we have 13 = 11 sco 2 and 641 = 71 sco 2.

Conjecture 4:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_k) of a k-Poulet number P not divisible by 3 or 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_k = d_k$ sco n.

Note:

In other words, because we defined the Smarandache-Coman congruence only on primes, we can say that for any set of divisors d_1, d_2, ..., d_k of a k-Poulet number P not divisible by 3 or 5 there exist a non-null positive integer n such that for any d_i (where i from 1 to k) can be defined a Smarandache-Coman congruence $d_i \equiv q_i$(sco n).

References:

1. Coman, Marius, *The math encyclopedia of Smarandache type notions*, Educational publishing, 2013;
2. Coman, Marius, *Two hundred conjectures and one hundred and fifty open problems about Fermat pseudoprimes*, Educational publishing, 2013.

4. Sequences of primes that are congruent sco n

Abstract. In a previous article I defined the Smarandache-Coman congruence on primes. In this paper I present few sequences of primes that are congruent sco n.

Note:

I will first present again the notion of *Smarandache-Coman congruence*, which is very related with the notion of *Smarandache-Coman divisors*, which I also defined in a previous paper.

Definition:

We define in the following way *the Smarandache-Coman congruence on primes*: we say that *two primes p and q are congruent sco n* and we note p ≡ q(sco n) if S(p – n) = S(q – n) = k, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to p sco n respectively k is also equal to q sco n and note k = p sco n = q sco n.

Note:

Because, of course, S(3 – 1) = 2 and S(3 – 2) = 1, there is no other prime that are congruent sco n to 3. Also there is no other prime to be congruent sco n to 5 so we start the sequences with the prime 7.

Note:

I will consider only the primes 7, 11, 13, 17 and 19 and the primes congruent sco n to them less than 1000 and, because I didn't yet study deeply all the implications of this new notion, I shall restrain myself from any comments or conjectures.

The sequence of primes congruent to 7 sco 2 (= 5):

(n = 2 is obviously the only possible n for such a congruence)

: 17.

The sequence of primes congruent to 11 sco 4 (= 7):

: 23, 37, 107, 317.

The sequence of primes congruent to 13 sco 2 (= 11):

: 79, 101, 167, 233, 277, 827.

The sequence of primes congruent to 13 sco 6 (= 7):

: 41.

The sequence of primes congruent to 13 sco 8 (= 5):

: 11, 23.

The sequence of primes congruent to 17 sco 4 (= 13):

: 43, 199, 277, 397, 421, 433, 659, 719, 823, 977.

The sequence of primes congruent to 17 sco 6 (= 11):

: 61, 83, 281, 797.

The sequence of primes congruent to 17 sco 10 (= 7):
: 31, 73.

The sequence of primes congruent to 19 sco 2 (= 17):
: 53, 181, 223, 257, 359, 461, 521, 563, 937.

The sequence of primes congruent to 19 sco 6 (= 13):
: 71, 97, 137, 149, 331, 461.

The sequence of primes congruent to 19 sco 8 (= 11):
: 41, 173, 239, 283, 347, 503, 701.

The sequence of primes congruent to 19 sco 12 (= 7):
: 47.

The sequence of primes congruent to 19 sco 14 (= 5):
: 29.

References:

1. Coman, Marius, *The Smarandache-Coman divisors of order k of a composite integer n with m prime factors*, Vixra;
2. Coman, Marius, *Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors*, Vixra.
3. Coman, Marius, *The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion*, Vixra.

5. Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes

Abstract. In this paper I define a new type of pairs of primes, id est the Smarandache-Germain pairs of primes, notion related to Sophie Germain primes and also to Smarandache function, and I conjecture that for all pairs of Sophie Germain primes but a definable set of them there exist corespondent pairs of Smarandache-Germain primes. I also make a conjecture that attributes to the set of Sophie Germain primes but a definable subset of them a corespondent set of smaller primes, id est Coman-Germain primes.

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 – 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 – 1), S(p_2 – 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 – 1)$ and $q_2 = S(p_2 – 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Note:

For a list of Sophie Germain primes see the sequence A005384 in OEIS. For the values of Smarandache function see the sequence A002034 in OEIS.

Verifying the Conjecture 1:
(for the first 26 pairs of Sophie Germain primes)

: For [2, 5] we have S(2 – 1) = 1, not prime;
: For [3, 7] we have S(3 – 1) = 2, not odd prime;
: For [5, 11] we have S(5 – 1) = 4, not prime;
: For [11, 23] we have [S(10), S(22)] = [5, 11]
and 5*2 + 1 = 11;
: For [23, 47] we have [S(22), S(46)] = [11, 23]
and 11*2 + 1 = 23;
: For [29, 59] we have [S(28), S(58)] = [7, 29]
and 7*4 + 1 = 29;
: For [41, 83] we have [S(40), S(82)] = [5, 41]
and 5*8 + 1 = 11;
: For [53, 107] we have [S(52), S(106)] = [13, 53]
and 13*4 + 1 = 53;
: For [83, 167] we have [S(82), S(166)] = [41, 83]
and 41*2 + 1 = 83;
: For [89, 179] we have [S(88), S(178)] = [11, 89]
and 11*8 + 1 = 89;
: For [113, 227] we have [S(112), S(226)] = [7, 113]
and 7*16 + 1 = 113;
: For [131, 263] we have [S(130), S(262)] = [13, 131]
and 13*10 + 1 = 131;
: For [173, 347] we have [S(172), S(346)] = [43, 173]
and 43*4 + 1 = 173;
: For [179, 359] we have [S(178), S(358)] = [89, 179]

and 89*2 + 1 = 179;
: For [191, 383] we have [S(190), S(382)] = [19, 191]
and 19*10 + 1 = 191;
: For [233, 467] we have [S(232), S(466)] = [29, 233]
and 29*8 + 1 = 233;
: For [239, 479] we have [S(238), S(478)] = [17, 239]
and 17*14 + 1 = 239;
: For [251, 503] we have S(250 – 1) = 15, not prime;
: For [281, 563] we have [S(280), S(562)] = [7, 281]
and 7*40 + 1 = 281;
: For [293, 587] we have [S(292), S(586)] = [73, 293]
and 73*4 + 1 = 293;
: For [359, 719] we have [S(358), S(718)] = [179, 359]
and 179*2 + 1 = 359;
: For [419, 839] we have [S(418), S(838)] = [19, 419]
and 19*22 + 1 = 419;
: For [431, 863] we have [S(430), S(862)] = [43, 431]
and 43*10 + 1 = 431;
: For [443, 887] we have [S(442), S(886)] = [17, 443]
and 17*26 + 1 = 443;
: For [491, 983] we have S(491 – 1) = 14, not prime;
: For [509, 1019] we have [S(508), S(1018)] = [127, 509]
and 127*4 + 1 = 509.

Conjecture 2:

There exist an infinity of Smarandache-Germain pairs of primes.

Note:

It can be seen that $q_2 = S(p_2 - 1) = p_1$ and also n is often a power of the number 2, so I make a new conjecture:

Conjecture 3:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

1. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, q ≥ 5;
2. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, q ≥ 5.

Note: we call the primes q from the first statement Coman-Germain primes of the first degree; we call the primes q from the second statement Coman-Germain primes of the second degree.

Verifying the Conjecture 3:
(for the first 21 Sophie Germain primes with the property showed)

The first statement:

: For p = 11, 23, 83, 179 we have m = 1
and q = 5, 11, 41, 89;
: For p = 29, 53, 173, 293, 509 we have m = 2
and q = 7, 13, 43, 73, 127;

: For p = 41, 89, 233 we have m = 3
and q = 5, 11, 29;
: For p = 113 we have m = 4
and q = 7.

The second statement:
: For p = 131, 191, 431 we have (m, n) = (1, 5)
and q = 13, 19, 43;
: For p = 239 we have (m, n) = (1, 7)
and q = 17;
: For p = 281 we have (m, n) = (3, 5)
and q = 7;
: For p = 419 we have (m, n) = (1, 11)
and q = 19;
: For p = 443 we have (m, n) = (1, 13)
and q = 17.

Conjecture 4:

There exist an infinity of Coman-Germain primes of the first degree.

Conjecture 5:

There exist an infinity of Coman-Germain primes of the second degree.

Notes:

We have the following sequence of Smarandache-Germain pairs of primes:
[5, 11], [11, 23], [7, 29], [5, 41], [13, 53], [41, 83], [11, 89], [7, 113], [13, 131], [43, 173], [89, 179], [19, 191], [29, 233], [17, 239], [7, 281], [73, 293], [179, 359], [19, 419], [43, 431], [17, 443], [127, 509] (...).

We have the following sequence of Coman-Germain primes of the first degree:
5, 11, 7, 5, 13, 41, 11, 7, 13, 43, 89, 29, 73, 179, 127 (...).

We have the following sequence of Coman-Germain primes of the second degree:
13, 19, 17, 7, 19, 43, 17 (...).

6. Two conjectures which generalize the conjecture on the infinity of Sophie Germain primes

Abstract. In a previous paper ("Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes") I defined two notions: the Smarandache-Germain pairs of primes and the Coman-Germain primes of the first and second degree. The few conjectures that I made on these particular types of primes inspired me to make two other conjectures regarding two sets of primes that are generalizations of the set of Sophie Germain primes. And, based on the observation of the first few primes from these two possible infinite sets of primes, I also made a conjecture regarding the primes q of the form q = p*2^n + 31 = r*2^m + 3, where p, r are primes an m, n are non-null positive integers.

Conjecture 1:

There exist an infinity of primes q of the form q = p_1*p_2*...*p_m*2^n + 1, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the first kind*.

Note:

For [n, m] = [1, 1] the conjecture is the same with the conjecture on the infinity of Sophie Germain primes, *i.e.* the primes of the form q = 2*p + 1.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [2, 1] the primes q are of the form 4*p + 1;
   the sequence of these primes is: 13, 29, 43, ...;
2. For [n, m] = [3, 1] the primes q are of the form 8*p + 1;
   the sequence of these primes is: 41, 89, 137, ...;
3. For [n, m] = [1, 2] the primes q are of the form 2*p_1*p_2 + 1;
   the sequence of these primes is: 31, 43, 67, ...;
4. For [n, m] = [2, 2] the primes q are of the form 4*p_1*p_2 + 1;
   the sequence of these primes is: 61, 157, 229, ...;
5. For [n, m] = [3, 2] the primes q are of the form 8*p_1*p_2 + 1;
   the sequence of these primes is: 281, 409, 457, ...;
6. For [n, m] = [1, 3] the primes q are of the form 2*p_1*p_2*p_3 + 1;
   the sequence of these primes is: 211, 331, 571 (...).

Conjecture 2:

There exist an infinity of primes r of the form r = 2*(p_1*p_2*...*p_m*2^n + 1) + 1, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the second kind*.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] the primes q are of the form 2*(2*p + 1) + 1 = 4*p + 3;
   the sequence of these primes is: 23, 31, 47, ...;
2. For [n, m] = [2, 1] the primes q are of the form 2*(4*p + 1) + 1 = 8*p + 3;

the sequence of these primes is: 43, 59, 107, ...;

3. For [n, m] = [3, 1] the primes q are of the form 2*(8*p + 1) + 1 = 16*p + 3; the sequence of these primes is: 83, 179, 211, ...;
4. For [n, m] = [1, 2] the primes q are of the form $2*(2*p_1*p_2 + 1) + 1 = 4*p_1*p_2 + 3$; the sequence of these primes is: 223, 263, 311, ...;
5. For [n, m] = [2, 2] the primes q are of the form $2*(4*p_1*p_2 + 1) + 1 = 8*p_1*p_2 + 3$; the sequence of these primes is: 283, 443, 523 (...).

Conjecture 3:

There exist an infinity of primes q of the form q = p*2^n + 31 that can be also written as q = r*2^m + 3, where n, m are non-null positive integers and p, r odd primes.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] we have q = 2*p + 31 = 2*r + 3:
   : q = 37 = 2*3 + 31 = 2*17 + 3, so [p, r] = [3, 17];
   : q = 41 = 2*5 + 31 = 2*19 + 3, so [p, r] = [5, 19];
   : q = 89 = 2*29 + 31 = 2*43 + 3, so [p, r] = [29, 43].
2. For [n, m] = [2, 3] we have q = 4*p + 31 = 8*r + 3:
   : q = 43 = 4*3 + 31 = 8*5 + 3, so [p, r] = [3, 5];
   : q = 59 = 4*7 + 31 = 8*7 + 3, so [p, r] = [7, 7];
   : q = 107 = 4*19 + 31 = 8*13 + 3, so [p, r] = [19, 13].
3. For [n, m] = [2, 4] we have q = 4*p + 31 = 16*r + 3:
   : q = 83 = 4*13 + 31 = 16*5 + 3, so [p, r] = [13, 5];
   : q = 179 = 4*37 + 31 = 16*11 + 3, so [p, r] = [37, 11];
   : q = 467 = 4*109 + 31 = 16*29 + 3, so [p, r] = [109, 29].

Annex

The two conjectures from my previous paper mentioned in Abstract where I defined the *Smarandache-Germain pairs of primes* and the *Coman-Germain primes of the first and second degree*:

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 – 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 – 1), S(p_2 – 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 – 1)$ and $q_2 = S(p_2 – 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Conjecture 2:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

3. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, q ≥ 5;
4. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, q ≥ 5.

Note: we call the primes q from the first statement *Coman-Germain primes of the first degree*; we call the primes q from the second statement *Coman-Germain primes of the second degree*.

7. An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function

Abstract. Combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered a very interesting pattern: seems like the recurrent formula f(n) = S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function and f(1), f(2) are any given different non-null positive integers, leads every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely.

Conjecture:

The recurrent formula f(n) = S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function, leads every time to the set of seven consecutive values {11, 17, 28, 24, 11, 15, 16}, set which is then repeating infinitely, for any given different non-null positive integers f(1), f(2).

Verifying the conjecture for few pairs [f(1), f(2)]

For [f(1), f(2)] = [1, 2]:

: f(3) = S(1) + S(2) = 3; f(4) = S(2) + S(3) = 5;
: f(5) = S(3) + S(5) = 8; f(6) = S(5) + S(8) = 9;
: f(7) = S(8) + S(9) = 10; f(8) = S(9) + S(10) = 11;
: f(9) = S(10) + S(11) = 16; f(10) = S(11) + S(10) = 17;
: f(11) = S(16) + S(17) = 23; f(12) = S(17) + S(23) = 40;
: f(13) = S(23) + S(40) = 28; f(14) = S(40) + S(28) = 12;
: f(15) = S(28) + S(12) = 11; f(16) = S(12) + S(11) = 15;
: f(17) = S(11) + S(15) = 16; f(18) = S(15) + S(16) = 11;
: f(19) = S(16) + S(11) = 17; f(20) = S(11) + S(17) = 28;
: f(21) = S(17) + S(28) = 24; f(22) = S(28) + S(24) = 11;
: f(23) = S(24) + S(11) = 15; f(24) = S(11) + S(15) = 16
(...)

For [f(1), f(2)] = [7, 13]:

: f(3) = S(7) + S(13) = 20; f(4) = S(13) + S(20) = 18;
: f(5) = S(20) + S(18) = 11; f(6) = S(18) + S(11) = 17
(...)

For [f(1), f(2)] = [531, 44]:

: f(3) = S(531) + S(44) = 70; f(4) = S(44) + S(70) = 18;
: f(5) = S(70) + S(18) = 13; f(6) = S(18) + S(13) = 19;
: f(7) = S(13) + S(19) = 32; f(8) = S(19) + S(32) = 27;
: f(9) = S(32) + S(27) = 17; f(10) = S(27) + S(17) = 26;
: f(11) = S(17) + S(26) = 30; f(12) = S(26) + S(30) = 18;
: f(13) = S(30) + S(18) = 11; f(14) = S(18) + S(19) = 17
(...)

For [f(1), f(2)] = [5, 11]:

: f(3) = S(5) + S(11) = 16; f(4) = S(11) + S(16) = 17;
(...)
: f(12) = 11; f(13) = 17
(...)

For [f(1), f(2)] = [341, 561]:

: f(3) = S(341) + S(561) = 48; f(4) = S(561) + S(48) = 23;
: f(5) = S(48) + S(23) = 29; f(6) = S(23) + S(29) = 52;
: f(7) = S(29) + S(52) = 42; f(8) = S(52) + S(42) = 20;
: f(9) = S(42) + S(20) = 12; f(10) = S(20) + S(12) = 9;
: f(11) = S(12) + S(9) = 10; f(12) = S(9) + S(10) = 11;
(...)
: f(22) = 11; f(23) = 17
(...)

For [f(1), f(2)] = [49, 121]:

: f(3) = S(49) + S(121) = 35; f(4) = S(121) + S(35) = 29;
: f(5) = S(35) + S(29) = 36; f(6) = S(29) + S(36) = 35;
: f(7) = S(36) + S(35) = 13; f(8) = S(35) + S(13) = 20;
: f(9) = S(13) + S(20) = 18; f(10) = S(20) + S(18) = 11;
: f(11) = S(18) + S(11) = 17 (...)

Open problems

I. Is there any exception to this apparent rule?
II. Is there a finite or infinite set of exceptions?
III. Is there a superior limit for n such that eventually f(n) = 11 and f(n + 1) = 17?
IV. Is the obtaining of a constant repeating set of values a characteristic of other recurrent formulas based similarly on the Smarandache function, having three or more terms?

8. A recurrent formula inspired by Rowland's formula and based on Smarandache function which might be a criterion for primality

Abstract. Studying the two well known recurrent relations with the exceptional property that they generate only values which are equal to 1 or are odd primes, id est the formula which belongs to Eric Rowland and the one that belongs to Benoit Cloitre, I managed to discover a formula based on Smarandache function, from the same family of recurrent relations, which, instead to give a prime value for any input, seems to give the same value, 2, if and only if the value of the input is an odd prime; also, for any value of input different from 1 and different from an odd prime, the value of output is equal to n + 1. I name this relation the Coman-Smarandache criterion for primality and the exceptions from this rule, if they exist, Coman-Smarandache pseudoprimes.

Introduction

The Rowland's formula was first noticed in 2003 summer camp NKS (New Kind of Science) organized by Wolfram Science and was subsequently proved to be true (transformed in theorem) by one of the participants in this camp, Eric Rowlands, who also conjectured that all odd primes can be generated by this formula. This formula (theorem) is:
: Let be the following recurrence relation: f(1) = 7, and, for n ≥ 2, f(n) = f(n – 1) + gcd[n, f(n – 1)]; then, the formula g(n) = f(n) - f(n – 1) has the exceptional property that it's result can be only a value which is equal to 1 or to an odd prime. The first values of g(n) are (see the sequence A132199 in OEIS): 1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 (...). The first primes resulting from this formula are (see the sequence A137613 in OEIS): 5, 3, 11, 3, 23, 3, 47, 3, 5, 3, 101, 3, 7, 11, 3, 13, 233, 3, 467, 3, 5, 3, 941, 3, 7, 1889 (...).

French mathematician Benoit Cloitre further found a similar formula:
: Let f(1) = 1, and, for n ≥ 2, f(n) = f(n – 1) + lcm[n, f(n – 1)]; then, the formula g(n) = f(n)/f(n – 1) – 1 has also, as result, only a value which is equal to 1 or to an odd prime.

Conjecture 1

Let f(1) = 1 and f(n) = S(f(n – 1)) + lcm[n, S(f(n – 1))], where S is the Smarandache function and lcm the least common multiple. Then the value of the function g(n) = f(n)/S(f(n – 1)) is equal to 2 if and only if n is an odd prime.

Conjecture 2

The value of the function g(n), defined in Conjecture 1, is g(n) = n + 1 for n different from 1 and n different from odd primes.

Verifying the conjectures
(up to n = 17)

: f(2) = 1 + lcm[2, 1] = 3; then g(2) = 3/1 = 3;
: **f(3)** = 3 + lcm[3, 3] = 6; then g(3) = 6/3 = **2**;
: f(4) = 3 + lcm[4, 3] = 15; then g(4) = 15/3 = 5;
: **f(5)** = 5 + lcm[5, 5] = 10; then g(5) = 10/5 = **2**;

: f(6) = 5 + lcm[6, 5] = 35; then g(6) = 35/5 = 7;
: **f(7)** = 7 + lcm[7, 7] = 14; then g(7) = 14/7 = **2**;
: f(8) = 7 + lcm[8, 7] = 63; then g(8) = 63/7 = 9;
: f(9) = 7 + lcm[9, 7] = 70; then g(9) = 70/7 = 10;
: f(10) = 7 + lcm[10, 7] = 77; then g(10) = 77/7 = 11;
: **f(11)** = 11 + lcm[11, 11] = 22; then g(11) = 22/11 = **2**;
: f(12) = 11 + lcm[12, 11] = 143; then g(12) = 143/11 = 13;
: **f(13)** = 13 + lcm[13, 13] = 26; then g(13) = 26/13 = **2**;
: f(14) = 13 + lcm[14, 13] = 195; then g(14) = 195/13 = 15;
: f(15) = 13 + lcm[15, 13] = 208; then g(15) = 208/13 = 16;
: f(16) = 13 + lcm[16, 13] = 221; then g(16) = 221/13 = 17;
: **f(17)** = 17 + lcm[17, 17] = 17; then g(17) = 34/17 = **2.**

Note

The function g(n) = f(n)/S(f(n – 1)) – 1, where f(n) = f(n – 1) + lcm[n, f(n – 1)] might also be interesting to study as a prime generating formula, as it gives prime values (i.e. 5, 17, 23, 191, 383) for the following consecutive values of n: 4, 5, 6, 7, 8; however, for n = 9 the value obtained is a semiprime and for n = 10 is not even obtained an integer value, because m is not always divisible by S(m) so f(n), which is always divisible by f(n – 1), is not always divisible by S(f(n – 1)).

References:

4. Rowland, Eric, *A simple prime-generating recurrence*;
5. Peterson, Ivars, *A new formula for generating primes*;
6. Shevelev, Vladimir, *Generalizations of the Rowland Theorem*.

9. The Smarandache-Korselt criterion, a variant of Korselt's criterion

Abstract. Combining two of my favourite objects of study, the Fermat pseudoprimes and the Smarandache function, I was able to formulate a criterion, inspired by Korselt's criterion for Carmichael numbers and by Smarandache function, which seems to be necessary (though not sufficient as the Korselt's criterion for absolute Fermat pseudoprimes) for a composite number (without a set of probably definable exceptions) to be a Fermat pseudoprime to base two.

Conjecture:

Any Poulet number, without a set of definable exceptions, respects either the Korselt's criterion (case in which it is a Carmichael number also) either *the Smarandache-Korselt criterion*.

Definition:

A composite odd integer $n = d_1*d_2*...*d_n$, where d_1, d_2, ..., d_n are its prime factors, is said that respects *the Smarandache-Korselt criterion* if n – 1 is divisible by $S(d_i – 1)$, where S is the Smarandache function and $1 \leq i \leq n$.

Note:
A Carmichael number not always respects *the Smarandache-Korselt criterion*: for instance, in the case of the number 561 = 3*11*17, 560 it is divisible by S(3 – 1) = 2 and by S(11 – 1) = 5 but is not divisible by S(17 – 1) = 6; in the case of the number 1729 = 7*13*19, 1728 it is divisible by S(6) = 3, S(12) = 4 and S(18) = 6.

Verifying the conjecture:
(for the first five Poulet numbers and for two bigger consecutive numbers which are not Carmichael numbers also):

: For P = 341 = 11*31, P – 1 = 340 is divisible by S(10) = 5 and S(30) = 5;
: For P = 645 = 3*5*43, P – 1 = 644 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;
: For P = 1387 = 19*73, P – 1 = 1386 is divisible by S(18) = 6 and S(72) = 6;
For P = 1905 = 3*5*127, P – 1 = 1904 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;
: For P = 2047 = 23*89, P – 1 = 2046 is divisible by S(22) = 11 and S(88) = 11;
: For P = 2701 = 37*73, P – 1 = 2700 is divisible by S(36) = 6 and S(72) = 6;
(...)

: For 999855751441 = 774541*1290901, P – 1 is divisible by S(774540) = 331 and S(1290900) = 331;
: For P = 999857310721 = 2833*11329*31153, P – 1 is divisible by S(2832) = 59 and S(11328) = 59 and S(31152 = 59).

Comment:
One exception that we met (which probably is part of a set of definable exceptions) is the Poulet number P = 999828475651 = 191*4751*1101811; indeed, P – 1 is not divisible by S(1101810) = 1933, and P is not a Carmichael number.