

**CONJECTURES ON SOME WELL KNOWN AND LESS
KNOWN TYPES OF PRIMES**

(COLLECTED PAPERS)

INTRODUCTION

This collection of papers brings together several articles regarding primes, submitted by the author to the preprint scientific database Vixra. Apparently heterogeneous, these articles have, objectively speaking, a thing in common: they are all directed toward the same goal – discovery of new ordered patterns in the “undisciplined” set of the prime numbers, using the same means – the old and reliable integer numbers. Subjectively speaking, these papers have, of course, another thing in common: the patterns coming from the very mathematical thoughts and obsessions of the author himself: such “mathematical thoughts and obsessions” are: trying to find correspondents of the patterns found in sequences of Fermat pseudoprimes (a little more “disciplined” class of numbers than the class of primes) in sequences of prime numbers; trying to find chains of consecutive or exclusive primes defined by a recurrent formula; trying to show the importance of classification of primes in eight essential subsets (the sequences of the form $30*k + d$, where d has the values 1, 7, 11, 13, 17, 19, 23, 29) etc.

This collection of articles seeks to expand the knowledge on some well known classes of primes, like for instance Sophie Germain primes, but also to define new interesting classes of primes, like for instance ACPOW chains of primes.

At least in one article, namely “On an iterative operation on positive composite integers which probably always conducts to a prime”, the author rediscovered a notion already known; in the referred case is about the sequence of primes reached by the operation of iterative replacement of a number by the concatenation of its prime factors (see the sequence A037272 in OEIS). We chose to still keep the article in this collection of papers because it treats, beside the primes, some issues about Fermat pseudoprimes.

SUMMARY

1. On a new type of recurrent sequences of primes - ACPOW chains
2. Few recurrent series based on the difference between successive primes
3. A possible unimportant but sure interesting conjecture about primes
4. A formula based on twin primes that generates chains of primes in arithmetic progression
5. On an iterative operation on positive composite integers which probably always conducts to a prime
6. Four conjectures about three subsets of pairs of twin primes
7. Twenty-four conjectures about “the eight essential subsets of primes”
8. Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes
9. Pairs of primes or pseudoprimes that generate an infinity of primes or pseudoprimes via a certain recurrence relation
10. Three conjectures about semiprimes inspired by a recurrent formula involving 2-Poulet numbers

1. On a new type of recurrent sequences of primes - ACPOW chains

Abstract. An interesting type of recurrent sequences of primes which could eventually lead to longer chains of successive primes than known Cunningham chains or CPAP's . Few conjectures including a stronger version of Legendre's conjecture and one regarding the Fermat primes. A classification of the set of primes.

I was studying the numbers of the form $b = 9 + 6 \cdot (10^a + 1)$, where $10^a + 1$ prime, when I noticed that, for many of these numbers b and for many consecutive values of c , the number $b - 2^c$ is a prime.

For instance, for $a = 1$ we get $b = 75$ and $75 - 2 = 73$ (prime), $75 - 2^2 = 71$ (prime), $75 - 2^3 = 67$ (prime), $75 - 2^4 = 59$ (prime), $75 - 2^5 = 43$ (prime), $75 - 2^6 = 11$ (prime), $75 - 2^7 = -53$ (prime in absolute value), $75 - 2^8 = -181$ (prime in absolute value).

This is how I discovered the potential of the recurrent sequences of primes of the type $p_0 + 2^x = p_1$, $p_1 + 2^{(x-1)} = p_2$, ..., $p_{x-1} + 2 = p_x$, where p_0, p_1, \dots, p_x are integers, primes in absolute value.

Descending ACPOW chains

I so define a *descending proper ACPOW chain* of primes the recurrent sequence of primes that I expose it above.

Note: I name these chains of primes ACPOW- k chains, abbreviation from chains of k primes obtained "adding consecutive powers".

Example: the sequence $-181, -53, 11, 43, 59, 67, 71, 73$ is a descending ACPOW-8 chain of primes (in absolute value), because $-181 + 2^7 = -53$, $-53 + 2^6 = 11$, $11 + 2^5 = 43$, $43 + 2^4 = 59$, $59 + 2^3 = 67$, $67 + 2^2 = 71$, $71 + 2 = 73$. It can be seen that the numbers of the terms from the sequence is equal to the power of 2 which we add to the first term plus 1.

I also define a *descending improper ACPOW chain* of primes the recurrent sequence of primes of the type $p_0 + 2^x = p_1$, $p_1 + 2^{(x-1)} = p_2$, ..., $p_{x-y} + 2^y = p_{x-y+1}$, where $0 < y < x$.

Example: the sequence $-1433, -409, 103, 359, 487$ is a descending improper ACPOW-5 chain of primes (in absolute value), because $-1433 + 2^{10} = -409$, $-409 + 2^9 = 103$, $103 + 2^8 = 359$, $359 + 2^7 = 487$.

Note: I name these chains "improper" because the last term is not equal to antepenultimate one plus 2, but plus a power of 2 bigger than 1 (indeed, in the example above, $487 + 2^6$ is no longer prime).

I list below few of these chains that I discovered, with a minimum length 5 and maximum length 8 (because of the special nature of the number 1, I listed also the chains that include it):

Descending proper ACPOW chains:

: 1423, 1439, 1447, 1451, 1453;

: -181, -53, 11, 43, 59, 67, 71, 73.

Descending improper ACPOW chains:

: -769, -257, -1, 127, 191, 223, 239;

: -829, -317, -61, 67, 131, 163, 179;

: -1433, -409, 103, 359, 487.

Note: It can be seen the chains I exposed can't be extended neither before the first term nor further than the last one, so they are "complete" (in the same manner a Cunningham chain is "complete"); a descending proper ACPOW chain is complete "to the right" by definition.

Comment: I generalize these chains to the sequences as the ones I described above but allowing beside the powers of 2 the adding with the powers of any even number, in both proper/improper versions.

We note these chains with the abbreviation $ACPOW(m)-k$, meaning "a chain of k primes obtained adding consecutive powers of m ". With this notation, an ACPOW- k is equivalent to an ACPOW(2)- k .

Example: The sequence 11, 1307, 1523, 1559 would be a descending improper ACPOW(6)-4 chain, cause $11 + 6^4 = 1307$, $1307 + 6^3 = 1523$, $1523 + 6^2 = 1559$ (and $1559 + 6$ is no longer a prime, neither $11 - 6^5$ is not a prime in absolute value).

Ascending ACPOW chains

I define an *ascending proper ACPOW chain* of primes the recurrent sequence of primes of the type $p_0 + 2 = p_1$, $p_1 + 2^2 = p_2$, ..., $p_{x-1} + 2^x = p_x$.

Note: An ascending proper ACPOW chain is complete (in the sence a Cunningham chain is "complete") "to the left" by definition.

It can be seen that an ascending proper ACPOW- k chain with $k > 3$ can start only with a prime having last digit 7 (if would be 3, the second term would be divizable by 5, it would be 9 the third term would be divisible by 5, if would be 1 the forth term would be divisible by 5). If the first term is 7, the last digit of the next terms would be 9, 3, 1 and then again 7, 9, 3, 1 repeatedly.

Example: the sequence 17, 19, 23, 31, 47, 79 is an ascending proper ACPOW-6 chain. It can be seen that the numbers of the terms from the sequence is equal to the power of 2 which we add to the last term plus 1.

I also define an *ascending improper ACPOW chain* of primes the recurrent sequence of primes of the type $p_0 + 2^x = p_1$, $p_1 + 2^{(x+1)} = p_2$, ..., $p_{y-1} + 2^{(x+y-1)} = p_y$.

Note: I named these chains "improper" because the second term is not equal to first one plus 2, but plus a power of 2 bigger than 1. Such a chain is complete to the left if $p_0 - 2^{(x-1)}$ is not a prime.

It can easily be proved that there are just the following possibilities for an ascending improper ACPOW chain to have the length bigger than 3:

: If the first term has the last digit 1, then the power of 2 added to it must be of the form 4^i ; also first term can be only of the form $30^j + 1$ (example of such a chain of length 4: 151, 167, 199, 263);

: If the first term has the last digit 3, then the power of 2 added to it must be of the form $4^i + 3$; also first term can be only of the form $30^j + 23$ (example of such a chain of length 7: 173, 181, 197, 229, 293, 421, 677);

: If the first term has the last digit 7, then the power of 2 added to it must be of the form $4^i + 1$; also first term can be only of the form $30^j + 17$ (example of such a chain of length 5: 617, 1129, 2153, 4201, 8297);

: If the first term has the last digit 9, then the power of 2 added to it must be of the form $4^i + 2$; also first term can be only of the form $30^j + 19$.

Comments:

: I also generalize these chains in the same manner: the sequence 11, 17, 53, 269 is an ascending proper ACPOW(6)-4 chain, because $11 + 6 = 17$, $17 + 6^2 = 53$, $53 + 6^3 = 269$;

I used the terms “proper” and “ascending” to exhaustively define the concept, but it would be better that those two terms to be implied and only the terms “improper” and “descending” to be mentioned.

Another generalization of ACPOW chains

Note: I define further more ACPOW chains only for “proper” and “ascending” senses, the implied ones.

I define an *ACPOW(2,n)-k chain* of primes the recurrent sequence of k primes: $p_0, p_1 = p_0 + 2^n, p_2 = p_1 + 2^{(2^n)}, \dots, p_x = p_{x-1} + 2^{(x^n)}$.

I define an *ACPOW(m,n)-k chain* of primes the recurrent sequence of k primes: $p_0, p_1 = p_0 + m^n, p_2 = p_1 + m^{(2^n)}, \dots, p_x = p_{x-1} + m^{(x^n)}$.

Note: With this notation, an ACPOW-k is equivalent to an ACPOW(2,1)-k.

Examples of ACPOW(2,n)-k chains:

ACPOW(2,3)-4: 29, 37, 101, 613,
where $29 + 2^3 = 37$, $37 + 2^{(3^2)} = 101$, $101 + 2^{(3^3)} = 613$;
ACPOW(2,3)-4: 59, 67, 131, 643.

Quasi-ACPOW chains or QACPOW chains

I define a *quasi-ACPOW chain* of primes or a *QACPOW* a proper or improper, ascending or descending ACPOW chain but with the difference that the terms of the series are not necessary primes but primes or squares of primes.

Examples:

: 107, 109, 113, 11^2 , 137, 13^2 , 233, 19^2 , 617, 1129, 2153, 4201, 8297 is a QACPOW-13
(because: $107 + 2 = 109$; $109 + 2^2 = 113$; $113 + 2^3 = 121$; $121 + 2^4 = 137$; $137 + 2^5 = 169$;
 $169 + 2^6 = 233$; $233 + 2^7 = 361$; $361 + 2^8 = 617$; $617 + 2^9 = 1129$; $1129 + 2^{10} = 2153$;
 $2153 + 2^{11} = 4201$; $4201 + 2^{12} = 8297$);

: 167, 13^2 , 173, 181, 197, 229, 293, 421, 677 is QACPOW-9;
 : 227, 229, 233, 241, 257, 17^2 , 353 is a QACPOW-7.

Observations about QACPOW chains and the difference between squares of consecutive primes:

: 5^2 , 41, 7^2
 is a descending QACPOW because $25 + 2^4 = 41$, $41 + 2^3 = 49$
 (incomplete chain, because -199, -71, -7, 5^2 , 41, 7^2 , 53 is the complete improper chain);

: 7^2 , 131, 11^2
 is a descending QACPOW(2,3) because $49 + 2^{(3*2)} = 113$, $113 + 2^3 = 121$
 (incomplete chain, because -463, 7^2 , 131, 11^2 is the complete improper chain);

: 11^2 , 41, 13^2
 is an ascending QACPOW because $121 + 2^4 = 137$, $137 + 2^5 = 169$
 (incomplete chain, because 107, 109, 113, 11^2 , 137, 13^2 , 233, 19^2 , 617, 1129, 2153, 4201, 8297 is the complete proper chain).

Conjecture 1: Between any two squares of consecutive odd primes p_1^2 and p_2^2 there are at least $p_2 - p_1$ prime numbers that can be written as $p_1^2 + \sum 2^x$, where x from i to j , $j \geq i \geq 1$, where x, i, j positive integers.

Check for few first primes:

$3^2 + 2 = 11$ prime;
 $3^2 + 2 + 2^2 + 2^3 = 23$ prime;
 $3^2 + 2^2 = 13$ prime;
 $3^2 + 2^3 = 17$ prime.

Note: between 3^2 and 5^2 there are 4 such primes.

$5^2 + 2 + 2^2 = 31$ prime;
 $5^2 + 2^2 = 29$ prime;
 $5^2 + 2^2 + 2^3 = 37$ prime;
 $5^2 + 2^4 = 41$ prime.

Note: between 5^2 and 7^2 there are 4 such primes.

$7^2 + 2 + 2^2 + 2^3 + 2^4 = 79$ prime;
 $7^2 + 2^2 = 53$ prime;
 $7^2 + 2^2 + 2^3 = 61$ prime;
 $7^2 + 2^2 + 2^3 + 2^4 + 2^5 = 109$ prime;
 $7^2 + 2^3 + 2^4 = 73$ prime;
 $7^2 + 2^4 + 2^5 = 97$ prime;
 $7^2 + 2^6 = 113$ prime.

Note: between 7^2 and 11^2 there are 7 such primes.

$11^2 + 2 + 2^2 = 127$ prime;
 $11^2 + 2 + 2^2 + 2^3 + 2^4 = 151$ prime;

$11^2 + 2^2 + 2^3 + 2^4 = 149$ prime;
 $11^2 + 2^4 = 137$ prime.

Note: between 11^2 and 13^2 there are 4 such primes.

$23^2 + 2^2 + 2^3 = 541$ prime;
 $23^2 + 2^2 + 2^3 + 2^4 = 557$ prime;
 $23^2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 653$ prime;
 $23^2 + 2^4 + 2^5 = 577$ prime;
 $23^2 + 2^4 + 2^5 + 2^6 = 641$ prime;
 $23^2 + 2^6 = 593$ prime.

Note: between 23^2 and 29^2 there are 6 such primes.

Comment: it's obviously that between two squares of consecutive primes, p_1^2 and p_2^2 , there is not necessarily a prime of the form $p_1^2 + 2 + 2^2 + \dots + 2^h$. Example: none of the numbers $y = 23^2 + \sum 2^x$, where $y < 29^2$ (i.e. 531, 535, 543, 559, 591, 655, 783) is prime.

Conjecture 2: Between any two squares of odd primes p^2 and q^2 , where $p < q$, there are at least $(q - p)/2 + 4$ prime numbers that can be written as $p^2 + \sum 2^x$, where x from i to j , $j \geq i \geq 1$, where x, i, j positive integers.

Notes:

: between 3^2 and 5^2 there are 4 such primes (i.e. 11, 13, 17, 23);
: between 3^2 and 7^2 there are 6 such primes (i.e. 11, 13, 17, 23, 37, 41);
: between 3^2 and 11^2 there are 8 such primes (i.e. 11, 13, 17, 23, 37, 41, 71, 73);
: between 3^2 and 13^2 there are 9 such primes (i.e. 11, 13, 17, 23, 37, 41, 71, 73, 137);
: between 3^2 and 17^2 there are 12 such primes (i.e. 11, 13, 17, 23, 37, 41, 71, 73, 137, 233, 257, 263);
: between 3^2 and 19^2 there are 12 such primes (i.e. 11, 13, 17, 23, 37, 41, 71, 73, 137, 233, 257, 263).

Conjecture 3: Between any two squares of consecutive numbers a^2 and $(a + 1)^2$ there are at least 2 prime numbers that can be written as $a^2 + \sum 2^x$, where x from i to j , $j \geq i \geq 1$, where x, i, j positive integers, if a is odd or can be written as $(a + 1)^2 - \sum 2^x$, where x from i to j , $j \geq i \geq 1$, where x, i, j positive integers, if a is even.

Comment: this is a stronger version of Legendre's conjecture.

Check for few first consecutive numbers and few randomly chosen ones:

$3^2 - 2 = 7$ prime;
 $3^2 - 2^2 = 5$ prime.

Note: between 2^2 and 3^2 there are 2 such primes.

$3^2 + 2 = 11$ prime;
 $3^2 + 2^2 = 13$ prime.

Note: between 3^2 and 4^2 there are 2 such primes.

$$5^2 - 2 = 23 \text{ prime};$$

$$5^2 - (2 + 2^2) = 19 \text{ prime.}$$

Note: between 4^2 and 5^2 there are 2 such primes.

$$5^2 + 2 + 2^2 = 31 \text{ prime};$$

$$5^2 + 2^2 = 29 \text{ prime.}$$

Note: between 5^2 and 6^2 there are 2 such primes.

$$7^2 - 2 = 47 \text{ prime};$$

$$7^2 - (2 + 2^2) = 43 \text{ prime.}$$

Note: between 6^2 and 7^2 there are 2 such primes.

$$23^2 + 2^2 + 2^3 = 541 \text{ prime};$$

$$23^2 + 2^2 + 2^3 + 2^4 = 557 \text{ prime.}$$

Note: between 23^2 and 24^2 there are 2 such primes.

$$561^2 + 2 = 314723 \text{ prime};$$

$$561^2 + 2 + 2^2 + \dots + 2^9 = 315743 \text{ prime.}$$

$$561^2 + 2^3 + 2^4 + 2^5 = 314777 \text{ prime.}$$

$$561^2 + 2^7 + 2^8 + 2^9 = 315617 \text{ prime.}$$

Note: between 561^2 and 562^2 there are 4 such primes.

Comment: we could introduce the notion “general”, and note the new series with $GACPOW(m, f(x))$, for the series such that were already defined but where the exponents of 2 (or of a number m) are not the consecutive numbers 1, 2, 3, (...) neither the consecutive products $1*n$, $2*n$, $3*n$, (...), but consecutive values of a bijective function $f(x)$, where x and $f(x)$ are positive integers.

ACPOW pairs of primes

I name an ACPOW pair of primes two primes of the form:

$$[p + 2^m - 2, p + 2^n - 2], \text{ where } p \text{ prime and } n > m > 0.$$

Observations:

: for $p = 3$ the pair becomes $[2^m + 1, 2^n + 1]$ so the only terms of the pair can be only Fermat primes; as the only known Fermat primes are 3, 5, 17, 257, 65537, we have the only known ACPOW pairs of primes having the first term 3: [3,5], [3,17], [3,257], [3,65537]; of course, [5,17], [5,257], [5,65537], [17,257], [17,65537], [257,65537] are also ACPOW pairs of primes;

: for $p = 5$ the pair becomes $[2^m + 3, 2^n + 3]$ and the first few ACPOW pairs of primes having the first term 5, generated for $p = 5$, are: [5,7], [5,11], [5,19], [5,67], [5,131], [5,4099];

: for $p = 7$ the pair becomes $[2^m + 5, 2^n + 5]$ and the first few ACPOW pairs of primes having the first term 7, generated for $p = 7$, are: [7,13], [7,37], [7,2053], [7,140737488355333], [7,9007199254740997];

: for $p = 11$ the pair becomes $[2^m + 9, 2^n + 9]$ and the first few ACPOW pairs of primes having the first term 11, generated for $p = 11$, are: [11,13], [11,17], [11,41], [11,73], [11,137], [11,521], [11,1033], [11,262153], [11,8388617];

: for $p = 13$ the pair becomes $[2^m + 11, 2^n + 11]$ and the first few ACPOW pairs of primes having the first term 13, generated for $p = 13$, are: [13,19], [13,43], [13,139], [13,523], [13,32779], [13,8388619].

: for $p = 17$ the pair becomes $[2^m + 15, 2^n + 15]$ and the first few ACPOW pairs of primes having the first term 17, generated for $p = 17$, are: [17,19], [17,23], [17,31], [17,47], [17,79], [17,271], [17,1039], [17,2063], [17,4111].

Properties of ACPOW pairs of primes:

(1) First we notice that these pairs seems to be relatively rare for a given p as the value of n is growing (especially for $p = 7$, from the cases that I considered).

(2) The series of twin primes and the series of cousin primes are subsets of the series of ACPOW pairs of primes.

(3) At the numbers of the form $N = 2^k + 9$ we noticed something interesting: for k of the form $k = 24 \cdot h - 1$, N has frequently only few prime factors:

$N = 2^{191} + 9$ (where $191 = 8 \cdot 24 - 1$) has just 3 prime factors;

$N = 2^{215} + 9$ (where $215 = 9 \cdot 24 - 1$) has just 3 prime factors;

$N = 2^{239} + 9$ (where $239 = 10 \cdot 24 - 1$) has just 3 prime factors;

$N = 2^{263} + 9$ (where $263 = 11 \cdot 24 - 1$) is a prime number with 80 digits!

(4) If $[q,r]$ and $[q + 2,r + 2]$ are both ACPOW pairs of primes, then we have some interesting values for the numbers $r - q - 2$, but we didn't gather enough data to jump with conclusions.

(5) If $[q,r]$ is an ACPOW pair of primes, and both q and r have the last digit 7, the numbers $q \cdot r - q - 1$ and $q \cdot r - r - 1$ are often primes or products of few primes; we name these numbers the pair affiliated to an ACPOW pair of such primes (having the last digit 7).

: for [17,257] the affiliated pair is [4111,19*229];

: for [17,65537] is [19*229*241,1114111];

: for [257,65537] is [3079*5449,16842751];

: for [7,67] is [401,461];

: for [7,37] is [13*17,251];

: for [7,9007199254740997] is

[11*13*17*22230849662051,23*2741321512312477];

: for [17,137] is [7*313,2311];

: for [17, 8388617] is [659*203669,7*61*333973].

Conjecture 4: I conjecture (against the heuristic arguments of Hardy and Wright) that there are infinite many Fermat primes, that they are all, besides the first two terms, 3 and 5, of the form $10^k + 7$, and that all terms F_n , beginning with the term $F_4 = 257$, satisfy the relations:

- (1) one of the numbers $17 \cdot F_n - 18$ and $17 \cdot F_n - F_n - 1$ is a prime number having the last four digits 4111;
- (2) the other one of the numbers $17 \cdot F_n - 18$ and $17 \cdot F_n - F_n - 1$ is a product of primes as it follows: $19 \cdot 229$ for $n = 4$, $19 \cdot 229 \cdot 241$ for $n = 5$, $19 \cdot 229 \cdot 241 \cdot p$, where p prime, for $n = 6$ and so on (for F_n corresponds a number of $n - 2$ primes).

ACPOW chains of the second kind

I define *ACPOW chains of the second kind* the recurrent sequences of primes of the type $p_n = p_{n-1} + 2^k$, where k has the smaller value for that $p_{n-1} + 2^k$ is a prime.

Examples:

: 3, 5, 7, 11, 13, 17, 19, 23, 31, 47, 79, 83, 211 (...);

: 29, 37, 41, 43, 59, 67, 71, 73, 89, 97, 101, 103 (...);

: 53, 61, 317 (...);

: 127, 131, 139, 1163 (...);

: 137, 139, 1163 (...).

We classify the set of prime numbers in the following manner:

: a prime is of class ACPOW I if can be written as $p_n = p_{n-1} + 2^k$, where $p_n = 3$;

: a prime is of class ACPOW II if can be written as $p_n = p_{n-1} + 2^k$, where p_n is the first prime that doesn't belong to the first class;

: a prime is of class ACPOW III if can be written as $p_n = p_{n-1} + 2^k$, where p_n is the first prime that doesn't belong to the first and the second class, and so on.

Comment: the classification is not exclusive: a prime can belong to more than one class.

Note: to the end of the article an addenda will cover the division into classes of the first few primes from the first few classes.

We name the *ACPOW problems* the following two questions:

: is there or not an infinite number of such classes?

: is there such a class with a finite number of terms?

Conclusion: we end here for now this analysis, hoping that we highlighted some of the possible applications of ACPOW chains of primes and ACPOW pairs of primes.

ADDENDA

Primes from class ACPOW I

3, 5, 7, 11, 13, 17, 19, 23, 31, 47, 79, 83, 211, 227, 229, 233, 241, 257, 769, 773 (...).

Note: we didn't find a prime of the form $773 + 2^n$ for n from 1 to 100 ($773 + 2^{100}$ is a number with 31 digits).

Primes from class ACPOW II

29, 37, 41, 43, 59, 67, 71, 73, 89, 97, 101, 103, 107, 109, 113, 241, 257 (...).

Note: we can see that from the number 241 class ACPOW II has the same terms with class ACPOW I.

Primes from class ACPOW III

53, 61, 317, 349, 353, 134218081 (...).

Primes from class ACPOW IV

127, 131, 139, 1163, 1171, 1187, 1699, 263843 (...).

Primes from class ACPOW V

137, 139, 1163 (...).

Note: we can see that from the number 139 class ACPOW V has the same terms with class ACPOW IV.

Primes from class ACPOW VI

149, 151, 167, 199, 263, 271, 1048847 (...).

Primes from class ACPOW VII

157, 173, 181, 197, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW VII has the same terms with class ACPOW VI.

Primes from class ACPOW VIII

163, 167, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW VIII has the same terms with class ACPOW VI.

Primes from class ACPOW IX

179, 181, 197 (...).

Note: we can see that from the number 181 class ACPOW IX has the same terms with class ACPOW VII.

Primes from class ACPOW X

191, 193, 197, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW X has the same terms with class ACPOW VI.

Primes from class ACPOW XI

223, 227, 229 (...).

Note: we can see that from the number 227 class ACPOW XI has the same terms with class ACPOW I.

Primes from class ACPOW XII

239, 241, 257 (...).

Note: we can see that from the number 241 class ACPOW XII has the same terms with class ACPOW I.

Primes from class ACPOW XIII

251, 283, 347, 349, 353, 134218081 (...).

Note: we can see that from the number 353 class ACPOW XIII has the same terms with class ACPOW III.

Primes from class ACPOW XIV

269, 271, 1048847 (...).

Note: we can see that from the number 271 class ACPOW XIV has the same terms with class ACPOW VI.

Primes from class ACPOW XV

277, 281, 283, 347 (...).

Note: we can see that from the number 283 class ACPOW XV has the same terms with class ACPOW XIII.

Primes from class ACPOW XVI

293, 421, 677, 709, 773 (...).

Note: we can see that from the number 773 class ACPOW XVI has the same terms with class ACPOW I.

Primes from class ACPOW XVII

307, 311, 313, 317, 349, 353, 134218081 (...).

Note: we can see that from the number 353 class ACPOW XVII has the same terms with class ACPOW III.

Primes from class ACPOW XVIII

331, 347, 349 (...).

Note: we can see that from the number 349 class ACPOW XVIII has the same terms with class ACPOW XIII.

Primes from class ACPOW XIX

337, 353, 134218081 (...).

Note: we can see that from the number 349 class ACPOW XVIII has the same terms with class ACPOW XIII.

Observations:

: the classes I, II, XI, XII, XVI are converging to the same terms, also the classes III, XIII, XV, XVII and XIX, the classes IV and V, the classes VI, VII, VIII, IX, X and XIV;

: some primes seems to highlight as ***convergence primes for classes ACPOW***: five classes converge to the prime 353, four to the prime 199, three to the prime 241, two to the primes 139, 181, 227, 271, 283, 349, 773.

2. Few recurrent series based on the difference between successive primes

Abstract. Despite the development of computer systems, the chains of successive primes obtained through an iterative formula yet have short lengths; for instance, the largest known chain of primes in arithmetic progression is an AP-26. We present here few formulas that might lead to interesting chains of primes.

I.

I.1.

I was studying twin primes when I noticed that the greater prime (q) from the pair can be obtained from the lesser one (p) through the formula $q = p \cdot (d/2) + d/2 + 1$, where d is the difference between the two primes, namely 2 (the formula is obviously equivalent to $q = p \cdot 1 + 1 + 1 = p + 2$). I applied this formula to the following recurrence relation and I observed that it produces interesting results (*i.e.* many successive primes):

Let be $A(n+1) = A(n) \cdot (d/2) + d/2 + 1$, where $A(0)$ is any chosen prime and d is the difference between $A(m)$ and the next consecutive prime, if the process of iteration stops to the first composite term $A(m)$ obtained. If the iteration is supposed to continue, the right definition is that d is the difference between $A(m)$ and the smallest prime which is greater than $A(m)$.

I list below few such series and I stop the iteration at the first composite term obtained.

We have for $A(0) = 3$:

$$\begin{aligned}A(1) &= 3 \cdot 1 + 1 + 1 = 5 \text{ (because } d = (5 - 3)/2 = 1); \\A(2) &= 5 \cdot 1 + 1 + 1 = 7 \text{ (because } d = (7 - 5)/2 = 1); \\A(3) &= 7 \cdot 2 + 2 + 1 = 17 \text{ (because } d = (11 - 7)/2 = 2); \\A(4) &= 17 \cdot 1 + 1 + 1 = 19 \text{ (because } d = (19 - 17)/2 = 1); \\A(5) &= 19 \cdot 2 + 2 + 1 = 41 \text{ (because } d = (23 - 19)/2 = 2); \\A(6) &= 41 \cdot 1 + 1 + 1 = 43 \text{ (because } d = (43 - 41)/2 = 1); \\A(7) &= 43 \cdot 2 + 2 + 1 = 89 \text{ (because } d = (47 - 43)/2 = 2); \\A(8) &= 97 \cdot 4 + 4 + 1 = 19^2 \text{ (because } d = (97 - 89)/2 = 4).\end{aligned}$$

We choose another prime as a starting term, of course not one resulted from the above iterative process (5, 7, 17, 19, 41, 43, 89), that would conduct to same result because the sequence depends only by the last term obtained.

We have for $A(0) = 11$:

$$\begin{aligned}A(1) &= 11 \cdot 1 + 1 + 1 = 13; \\A(2) &= 13 \cdot 2 + 2 + 1 = 29; \\A(3) &= 29 \cdot 1 + 1 + 1 = 31; \\A(4) &= 31 \cdot 3 + 3 + 1 = 97; \\A(5) &= 97 \cdot 2 + 2 + 1 = 197; \\A(6) &= 197 \cdot 1 + 1 + 1 = 199; \\A(7) &= 199 \cdot 6 + 6 + 1 = 1201; \\A(8) &= 1201 \cdot 6 + 6 + 1 = 7213; \\A(9) &= 7213 \cdot 3 + 3 + 1 = 23 \cdot 941.\end{aligned}$$

We have for $A(0) = 23$:

$$A(1) = 23 \cdot 3 + 3 + 1 = 73;$$

$$A(2) = 73 \cdot 3 + 3 + 1 = 223;$$

$$A(3) = 223 \cdot 2 + 2 + 1 = 449;$$

$$A(4) = 449 \cdot 4 + 4 + 1 = 1801;$$

$$A(5) = 1801 \cdot 5 + 5 + 1 = 9011;$$

$$A(6) = 9011 \cdot 1 + 1 + 1 = 9013;$$

$$A(7) = 9013 \cdot 8 + 8 + 1 = 37 \cdot 1949.$$

Note: We didn't further obtained notable results (long chains of successive primes) for starting terms 37, 47, 53, 59, 61 so it's clear that the formula doesn't conduct to such results if we choose randomly the starting prime.

I.2.

The starting term of the sequence is allowed to be a prime in absolute value; also the terms of the sequence.

We have for $A(0) = -13$:

$$A(1) = (-13) \cdot 1 + 1 + 1 = -11;$$

$$A(2) = (-11) \cdot 2 + 2 + 1 = -19;$$

$$A(3) = (-19) \cdot 1 + 1 + 1 = -17;$$

$$A(4) = (-17) \cdot 2 + 2 + 1 = -31;$$

$$A(5) = (-31) \cdot 1 + 1 + 1 = -29;$$

$$A(6) = (-29) \cdot 3 + 3 + 1 = -83;$$

$$A(7) = (-83) \cdot 2 + 2 + 1 = -163;$$

$$A(8) = (-163) \cdot 2 + 2 + 1 = -5 \cdot 97.$$

We have for $A(0) = -23$:

$$A(1) = (-23) \cdot 2 + 2 + 1 = -43;$$

$$A(2) = (-43) \cdot 1 + 1 + 1 = -41;$$

$$A(3) = (-41) \cdot 2 + 2 + 1 = -79;$$

$$A(4) = (-79) \cdot 3 + 3 + 1 = -233;$$

$$A(5) = (-233) \cdot 2 + 2 + 1 = -463;$$

$$A(6) = (-463) \cdot 1 + 1 + 1 = -461;$$

$$A(7) = (-461) \cdot 2 + 2 + 1 = -919;$$

$$A(8) = (-919) \cdot 4 + 4 + 1 = -3671;$$

$$A(9) = (-3671) \cdot 2 + 2 + 1 = -97 \cdot 227.$$

I.3.

The starting term of the sequence is a prime of the form $2 \cdot k \cdot 11 + 1$.

We have for $A(0) = 67$:

$$A(1) = 67 \cdot 2 + 2 + 1 = 137;$$

$$A(2) = 137 \cdot 1 + 1 + 1 = 139;$$

$$A(3) = 139 \cdot 5 + 5 + 1 = 701;$$

$$A(4) = 701 \cdot 4 + 4 + 1 = 53^2.$$

$$\text{We have for } A(0) = 89: \quad A(1) = 89 \cdot 4 + 4 + 1 = 19^2.$$

$$\text{We have for } A(0) = 199: \quad A(1) = 199 \cdot 6 + 6 + 1 = 1201.$$

We have for $A(0) = 331$: $A(1) = 331*3 + 3 + 1 = 997$.

We have for $A(0) = 353$: $A(1) = 353*3 + 3 + 1 = 1063$.

We have for $A(0) = 397$: $A(1) = 397*2 + 2 + 1 = 797$.

We have for $A(0) = 419$: $A(1) = 419*2 + 1 + 1 = 421$.

We have for $A(0) = 463$: $A(1) = 463*2 + 2 + 1 = 929$.

We have for $A(0) = 617$: $A(1) = 617*1 + 1 + 1 = 619$.

Note: This choice of starting term seems to conduct to interesting but not outstanding results.

I.4.

The starting term of the sequence is allowed to be a square of a prime. The starting value of d is now the difference between the starting term and the smaller prime bigger than this (*i.e.* $d = 29 - 25 = 4$ for starting term 25) and after that d is the difference between $A(m)$ and the next consecutive prime.

We have for $A(0) = 3^2 = 9$:

$A(1) = 9*1 + 1 + 1 = 11$ which is prime.

We have for $A(0) = 5^2 = 25$:

$A(1) = 25*2 + 2 + 1 = 53$ which is prime.

We have for $A(0) = 7^2 = 49$:

$A(1) = 49*2 + 2 + 1 = 101$ which is prime.

We have for $A(0) = 11^2 = 121$:

$A(1) = 121*3 + 3 + 1 = 367$ which is prime;

$A(2) = 367*3 + 3 + 1 = 1105$ which is Fermat pseudoprime to base 2.

We have for $A(0) = 13^2 = 169$:

$A(1) = 169*2 + 2 + 1 = 341$ which is Fermat pseudoprime to base 2.

Note: We didn't observed further a notable pattern.

I.5.

The starting term of the series is a prime but the value of d is now the difference between $A(m)$ and the second consecutive prime (*e.g.* $d = 7 - 3 = 4$; $d = 11 - 5 = 6$; $d = 13 - 7 = 6$ and so on).

We have for $A(0) = 3$:

$A(1) = 3*2 + 2 + 1 = 9 = 3^2$.

We have for $A(0) = 5$:

$A(1) = 5*3 + 3 + 1 = 19$;

$A(2) = 19*5 + 5 + 1 = 121 = 11^2$.

We have for $A(0) = 7$:

$$A(1) = 7*3 + 3 + 1 = 25 = 5^2.$$

We have for $A(0) = 11$:

$$A(1) = 11*3 + 3 + 1 = 37.$$

We have for $A(0) = 13$:

$$A(1) = 13*3 + 3 + 1 = 43.$$

Note: We didn't observed further a notable pattern.

I.6.

The starting term of the series is a prime but the value of d is now the difference between $A(m)$ and the third consecutive prime (*e.g.* $d = 11 - 3 = 8$; $d = 13 - 5 = 8$; $d = 17 - 7 = 10$ and so on).

$$\text{We have for } A(0) = 3: A(1) = 3*4 + 4 + 1 = 17.$$

$$\text{We have for } A(0) = 5: A(1) = 5*4 + 4 + 1 = 25.$$

$$\text{We have for } A(0) = 7: A(1) = 7*5 + 5 + 1 = 41.$$

$$\text{We have for } A(0) = 11: \quad A(1) = 11*4 + 4 + 1 = 49.$$

$$\text{We have for } A(0) = 13: \quad A(1) = 13*5 + 5 + 1 = 71.$$

$$\text{We have for } A(0) = 17: \quad A(1) = 17*6 + 6 + 1 = 109.$$

$$\text{We have for } A(0) = 19: \quad A(1) = 19*6 + 6 + 1 = 119.$$

$$\text{We have for } A(0) = 23: \quad A(1) = 23*7 + 7 + 1 = 163.$$

$$\text{We have for } A(0) = 29: \quad A(1) = 29*6 + 6 + 1 = 181.$$

$$\text{We have for } A(0) = 31: \quad A(1) = 31*6 + 6 + 1 = 193.$$

$$\text{We have for } A(0) = 37: \quad A(1) = 37*5 + 5 + 1 = 191.$$

$$\text{We have for } A(0) = 41: \quad A(1) = 41*6 + 6 + 1 = 253.$$

$$\text{We have for } A(0) = 43: \quad A(1) = 43*8 + 8 + 1 = 353.$$

$$\text{We have for } A(0) = 47: \quad A(1) = 43*7 + 7 + 1 = 337.$$

$$\text{We have for } A(0) = 53: \quad A(1) = 53*7 + 7 + 1 = 379.$$

$$\text{We have for } A(0) = 59: \quad A(1) = 59*6 + 6 + 1 = 359.$$

$$\text{We have for } A(0) = 61: \quad A(1) = 61*6 + 6 + 1 = 373.$$

$$\text{We have for } A(0) = 67: \quad A(1) = 67*6 + 6 + 1 = 409.$$

$$\text{We have for } A(0) = 71: \quad A(1) = 71*6 + 6 + 1 = 433.$$

We have for $A(0) = 73$:

$$A(1) = 73*7 + 7 + 1 = 593;$$

$$A(2) = 593*7 + 7 + 1 = 4159;$$

$$A(3) = 4159*26 + 27 + 1 = 108161;$$

$$A(4) = 108161*15 + 15 + 1 = 1622431;$$

$$A(5) = 1622431*20 + 20 + 1 = 32448641;$$

$$A(6) = 32448641*19 + 19 + 1 = 21247*29017.$$

Note: This formula seems also to conduct to interesting results.

II.

We define now another resembling series, based again on the differences between consecutive primes:

Let be $A(n+1) = (A(n) + 1)*(d_1/2) + d_2/2) + 1$, where $A(0)$ is any chosen prime, d_1 is the difference between $A(m)$ and the next consecutive prime and d_2 is the difference between $A(m)$ and the second consecutive prime (the definition is made under the presumption that process of iteration stops to the first composite term obtained).

I list below few such series:

We have for $A(0) = 3$:

$$A(1) = 4*(1 + 2) + 1 = 13;$$

$$A(2) = 14*(2 + 3) + 1 = 71;$$

$$A(3) = 72*(1 + 4) + 1 = 19^2.$$

We have for $A(0) = 5$:

$$A(1) = 6*(1 + 3) + 1 = 5^2.$$

We have for $A(0) = 7$:

$$A(1) = 8*(2 + 3) + 1 = 41;$$

$$A(2) = 42*(1 + 3) + 1 = 13^2.$$

We have for $A(0) = 11$:

$$A(1) = 12*(1 + 3) + 1 = 7^2.$$

We have for $A(0) = 23$:

$$A(1) = 24*(3 + 4) + 1 = 13^2.$$

Note: This formula could also lead to interesting results, but these results still seem to depend on the choice of the starting term.

Conclusion: I believe that especially the first formula is appealing because it is such easy to compute, though the longest chains of primes that I obtained so far using it are just 9 primes long: 11, 13, 29, 31, 97, 197, 199, 1201, 7213 (for starting term 11) and -23, -43, -41, -79, -233, -463, -461, -919, -3671 (for starting term -23). But, on the other side, neither Cunningham chains or AP chains were much more longer before being largely computed.

3. A possible unimportant but sure interesting conjecture about primes

Abstract. Studying, related to Fermat pseudoprimes, my main object of study, the concatenation and the primes of the form $n^*p - n + 1$, where p is also prime, I found incidentally an interesting possible fact about primes. Because the proof or disproof of the conjecture, and also the consequences in the case that is true, are beyond me, I shall limit myself to enunciate the conjecture and give few examples.

Conjecture:

Every prime p that ends in a group of digits that form a prime can be written at least in one way as $p = n^*q - (n - 1)^*r$, where n is positive integer, $n > 1$, and q, r another primes that ends in the same group of digits.

Examples:

The numbers 11, 211, 311, 811, 911, 1511, 1811, 2011, 2111, 2311 are the first ten primes that end in the digits 11, that form a prime.

These primes can be written as:

$$\begin{aligned} &: 11 = 3^*211 - 2^*311; \\ &: 211 = 7^*811 - 6^*911; \\ &: 311 = 6^*811 - 5^*911; \\ &: 811 = 4^*211 - 3^*11; \\ &: 911 = 3^*311 - 2^*11; \\ &: 1511 = 5^*311 - 4^*11; \\ &: 1811 = 9^*211 - 8^*11 = 6^*311 - 5^*11 = 2^*911 - 1^*11; \\ &: 2011 = 10^*211 - 9^*11 = 3^*811 - 2^*211; \\ &: 2111 = 7^*311 - 6^*11 = 2^*1511 - 1^*911 = 2^*1811 - 1^*1511; \\ &: 2311 = 3^*911 - 2^*211. \end{aligned}$$

The numbers 29, 229, 829, 929, 1129, 1229, 1429, 2029 are the first eight primes that end in the digits 29, that form a prime.

These primes can be written as:

$$\begin{aligned} &: 29 = 9^*829 - 8^*929 = 3^*829 - 2^*1229 = 4^*929 - 3^*1229; \\ &: 229 = 7^*829 - 6^*929 = 6^*1229 - 5^*1429 = 3^*1429 - 2^*2029; \\ &: 829 = 4^*229 - 3^*29 = 3^*1229 - 4^*1429 = 2^*1429 - 1^*2029; \\ &: 929 = 3^*1129 - 2^*1229; \\ &: 1129 = 3^*929 - 2^*829; \\ &: 1229 = 6^*229 - 5^*29; \\ &: 1429 = 7^*229 - 6^*29; \\ &: 2029 = 10^*229 - 9^*29 = 4^*1429 - 3^*1229 = 3^*1429 - 2^*1129. \end{aligned}$$

Observation:

Seems that the conjecture above can be extended for primes that end in a group of digits that form not a prime but a square of prime.

Example:

The numbers 3529, 6529, 10529, 21529, 27529, 30529, 33529, 36529 are eight from the first ten primes that end in the digits 529, that form a square of a prime.

These primes can be written as:

$$\begin{aligned}
 : & \quad 3529 = 4*21529 - 3*27529 = 5*27529 - 4*33529; \\
 : & \quad 6529 = 2*3529 - 1*529 = 5*30529 - 4*36529; \\
 : & \quad 10529 = 4*7529 - 3*6529; \\
 : & \quad 21529 = 7*3529 - 6*529 = 6*6529 - 5*3529; \\
 : & \quad 27529 = 9*3529 - 8*529 = 8*6529 - 7*3529; \\
 : & \quad 30529 = 10*3529 - 9*529 = 5*6529 - 4*529; \\
 : & \quad 33529 = 10*6529 - 9*3529 = 2*27529 - 1*21529; \\
 : & \quad 36529 = 6*6529 - 5*529 = 11*6529 - 10*3529.
 \end{aligned}$$

Note: Obviously, in this case is admitted for r from the expression $p = n*q - (n - 1)*r$ to be not just a prime but the ending group of digits itself, that form a square of prime.

Observation:

The conjecture above can also be extended for Fermat pseudoprimes that end in a group of digits that form a prime. For instance, the number 1729 is the first absolute pseudoprime that ends in the digits 29, and can be written as $1729 = 2*1429 - 1*1129$, where 1429 and 1129 are primes.

4. A formula based on twin primes that generates chains of primes in arithmetic progression

Abstract. I was studying recurrences of the form $P(n) = P(n - 1) + 2^k - 2$, when incidentally I found a chain of 5 primes in arithmetic progression that satisfy this recurrence (8329, 8839, 9349, 9859, 10369). But, interesting, instead of find easily other chains of primes based on this recurrence, I obtained easily such chains (up to AP-6) defining in other way, based on twin primes, the relation between those 5 primes.

The formula is: $p + 2 + 30*m + p*30*n$, where m and n are integers, m non-negative and n positive, and p is the smaller prime from a pair of twin primes; giving a constant value to m and consecutive values to n we obtain for the following pairs of twin primes:

For (11, 13): we have $p = 11$.

We take $m = 1$. The formula becomes $43 + 330*n$. We obtain for n from 14 to 17 a chain of four primes in arithmetic progression (4663, 4993, 5323, 5653) and for n from 35 to 38 another such chain (11593, 11923, 12253, 12583).

We take $m = 2$. The formula becomes $73 + 330*n$. We obtain for n from 5 to 8 a chain of four primes in arithmetic progression (1723, 2053, 2383, 2713).

We take $m = 3$. The formula becomes $103 + 330*n$. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1093, 1423, 1753, 2083).

We take $m = 4$. The formula becomes $133 + 330*n$. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1123, 1453, 1783, 2113) and for n from 43 to 48 a chain of six primes in arithmetic progression (14323, 14653, 14983, 15313, 15643, 15973).

We take $m = 5$. The formula becomes $163 + 330*n$. We obtain for n from 36 to 39 a chain of four primes in arithmetic progression (12043, 12373, 12703, 13033).

For (17, 19): we have $p = 17$.

We take $m = 0$. The formula becomes $19 + 510*n$. We obtain for n from 14 to 18 a chain of five primes in arithmetic progression (7159, 7669, 8179, 8689, 9199).

We take $m = 1$. The formula becomes $49 + 510*n$. We obtain for n from 23 to 26 a chain of four primes in arithmetic progression (11779, 12289, 12799, 13309) and for n from 30 to 34 a chain of five primes in arithmetic progression (15349, 15859, 16369, 16879, 17389).

We take $m = 2$. The formula becomes $79 + 510*n$. We obtain for n from 10 to 14 a chain of five primes in arithmetic progression (5179, 5689, 6199, 6709, 7219).

We take $m = 3$. The formula becomes $109 + 510*n$. We obtain for n from 40 to 45 a chain of six primes in arithmetic progression (20509, 21019, 21529, 22039, 22549, 23059).

We take $m = 5$. The formula becomes $169 + 510*n$. We obtain for n from 16 to 20 a chain of five primes in arithmetic progression (8329, 8839, 9349, 9859, 10369).

For (29, 31): we have $p = 29$.

We take $m = 1$. The formula becomes $61 + 870*n$. We obtain for n from 30 to 34 a chain of five primes in arithmetic progression (26161, 27031, 27901, 28771, 29641).

We take $m = 2$. The formula becomes $91 + 870*n$. We obtain for n from 43 to 46 a chain of four primes in arithmetic progression (37501, 38371, 39241, 40111).

We take $m = 4$. The formula becomes $151 + 870*n$. We obtain for n from 42 to 45 a chain of four primes in arithmetic progression (36691, 37561, 38431, 39301).

We take $m = 5$. The formula becomes $181 + 870*n$. We obtain for n from 40 to 44 a chain of five primes in arithmetic progression (34981, 35851, 36721, 37591, 38461).

For (41, 43): we have $p = 41$.

We take $m = 1$. The formula becomes $73 + 1230*n$. We obtain for n from 13 to 17 a chain of five primes in arithmetic progression (16063, 17293, 18523, 19753, 20983).

We take $m = 5$. The formula becomes $193 + 1230*n$. We obtain for n from 4 to 7 a chain of four primes in arithmetic progression (5113, 6343, 7573, 8803).

For (59, 61): we have $p = 59$.

We take $m = 0$. The formula becomes $61 + 1770*n$. We obtain for n from 7 to 11 a chain of five primes in arithmetic progression (12451, 14221, 15991, 17761, 19531).

We take $m = 2$. The formula becomes $121 + 1770*n$. We obtain for n from 17 to 20 a chain of four primes in arithmetic progression (30211, 31981, 33751, 35521) and for n from 25 to 28 another such chain (44371, 46141, 47911, 49681).

We take $m = 4$. The formula becomes $181 + 1770*n$. We obtain for n from 28 to 32 a chain of five primes in arithmetic progression (49741, 51511, 53281, 55051, 56821).

Conclusion: we obtained two AP-6, eight AP-5, twelve AP-4 and many AP-3, considering just first five pairs of twin primes, beside of course the pairs (3,5) and (5,7), values for m up to 5 and values for n up to 50.

5. On an iterative operation on positive composite integers which probably always conducts to a prime

Abstract. By playing with one of my favorite class of numbers, Poulet numbers, and one of my favorite operation, concatenation, I raised to myself few questions that seem interesting, worthy to share. I also conjectured that, reiterating a certain operation which will be defined, eventually for every Poulet number it will be find a corresponding prime. Then I extrapolated the conjecture for all composite positive integers.

Conjecture 1:

For any Poulet number P is defined the following operation which always, eventually, leads to a prime number:

Let P be a Poulet number, $P = p_1 * p_2 * \dots * p_n$, where $p_1 \leq p_2 \leq p_3 \leq \dots \leq p_n$ are the prime factors of P (not distinct, it can be seen from definition: for instance for the only two Poulet numbers non-squarefree known, the squares of Wieferich primes, we have $P = p_1 * p_2$, where $p_1 = p_2$; the reason for writing the number P in this way instead writing $P = p_1^2$ it will be seen further).

Then we consider the number $Q_1 = p_1 p_2 \dots p_n$, obtained by concatenation of the numbers that form the ordered set $\{p_1, p_2, p_3, \dots, p_n\}$.

(Example: $P = 561 = 3 * 11 * 17$; then $Q_1 = 31117$)

Then we have the following possibilities: Q is a prime or a composite number; if it is composite (it doesn't matter if is squarefree or not) we reiterate the operation until is obtained a prime number.

(Example: $Q_1 = 31117 = 29 * 29 * 37$; then $Q_2 = 292937 = 457 * 641$; $Q_3 = 457641 = 3 * 3 * 50849$; $Q_4 = 3350849 = 131 * 25579$; $Q_5 = 13125579 = 3 * 4375193$; finally, the number $Q_6 = 34375193$ is prime)

Note:

Our conjecture is that, reiterating the operation above, from every Poulet number is obtained, eventually, a prime.

Verifying the conjecture for the first few Poulet numbers (beside 561 which was given above as an example):

$P = 341 = 11 * 31$;
:
: $1131 = 3 * 13 * 29$;
:
: $31329 = 3 * 3 * 59 * 59$;
:
: $335959 = 13 * 43 * 601$;
:
: $1343601 = 3 * 3 * 3 * 7 * 7109$;
:
: $33377109 = 3 * 607 * 18329$;
:
: $360718329 = 3 * 120239443$;
:
: $3120239443 = 523 * 5966041$;

: $5235966041 = 419 \cdot 12496339$;
 : $41912496339 = 3 \cdot 7 \cdot 1995833159$;
 : $371995833159 = 3 \cdot 3 \cdot 19 \cdot 1459 \cdot 1491031$;
 : $331914591491031 = 3 \cdot 3 \cdot 709 \cdot 145949 \cdot 356399$;
 : $33709145949356399 = 2011 \cdot 16762379885309$;
 : $201116762379885309 = 3 \cdot 17 \cdot 1357927 \cdot 2904033817$;
 : $31713579272904033817 = 13 \cdot 337 \cdot 7238890498266157$;
 : $133377238890498266157 = 3 \cdot 13 \cdot 53 \cdot 64526966081518271$;
 : $3135364526966081518271 = 3037 \cdot 1032388714839012683$;
 : $30371032388714839012683 = 3 \cdot 3 \cdot 2879 \cdot 16260571 \cdot 72084117943$;
 : $3328791626057172084117943 = 15765766319 \cdot 211140490015097$;
 : $15765766319211140490015097 = 575063 \cdot 27415720224064390319$;
 : $57506327415720224064390319 = 32869 \cdot 1749561210128699506051$;
 : $328691749561210128699506051 = 23 \cdot 61 \cdot 1283 \cdot 1597 \cdot 30391 \cdot 3762309931337$;
 : $236112831597303913762309931337 =$
 : $3 \cdot 509 \cdot 13381 \cdot 11555586205509014201651$;
 : $35091338111555586205509014201651$ is a prime number.

$P = 645 = 3 \cdot 5 \cdot 43$;
 : $3543 = 3 \cdot 1181$;
 : 31181 is a prime number.

$P = 1105 = 5 \cdot 13 \cdot 17$;
 : $51317 = 7 \cdot 7331$;
 : $77331 = 3 \cdot 149 \cdot 173$;
 : 3149173 is a prime number.

$P = 1387 = 19 \cdot 73$;
 : 1973 is a prime number.

$P = 1729 = 7 \cdot 13 \cdot 19$;
 : $71319 = 3 \cdot 23773$;
 : $323773 = 199 \cdot 1627$;
 : $1991627 = 11 \cdot 331 \cdot 547$;
 : $11331547 = 29 \cdot 390743$;
 : 29390743 is a prime number.

$P = 1905 = 3 \cdot 5 \cdot 127$;
 : $35127 = 3 \cdot 3 \cdot 3 \cdot 1301$;
 : 3331301 is a prime number.

$P = 2047 = 23 \cdot 89$;
 : 2389 is a prime number.

Verifying the conjecture for the two squares of Wieferich primes (because they represent a special case):

$P = 1194649 = 1093 \cdot 1093$;
 : $10931093 = 73 \cdot 137 \cdot 1093$;
 : $731371093 = 17 \cdot 223 \cdot 192923$;

: $17223192923 = 2089 * 8244707$;
 : $20898244707 = 3 * 11483606643$;
 : $311483606643 = 3 * 3 * 11 * 11 * 286027187$;
 : $3311286027187 = 3 * 110370428675729$;
 : $3110370428675729 = 21977 * 141528435577$;
 : $21977141528435577 = 3 * 11 * 17351 * 38382455519$;
 : $3111735138382455519 = 3 * 11 * 11 * 113899 * 164113 * 458599$;
 : $31111113899164113458599 = 359 * 86660484398785831361$;
 : $35986660484398785831361 = 162523 * 221425032053301907$;
 : 162523221425032053301907 is a prime number.

$P = 1194649 = 3511 * 3511$;
 : $35113511 = 73 * 137 * 3511$;
 : $731373511 = 11 * 66488501$;
 : $1166488501 = 53 * 2687 * 8191$;
 : $5326878191 = 653 * 8157547$;
 : $6538157547 = 3 * 67 * 32528147$;
 : $36732528147 = 3 * 7 * 37 * 47274811$;
 : $373747274811 = 3 * 3 * 41527474979$;
 : 3341527474979 is a prime number.

Note:

The numbers $P = 1387 = 19 * 73$ and $P = 2047 = 23 * 89$ conducted to a prime from the first step: 1973 and 2389 are both primes. These two 2-Poulet numbers have in common the fact that, in both cases, $p_2 = 4 * p_1 - 3$; indeed, $73 = 19 * 4 - 3$ and $89 = 4 * 23 - 3$. Another such 2-Poulet number is $P = 13747 = 59 * 233$; 59233 is also a prime number.

Conjecture 2:

For any composite positive integer, the operation defined above, always, eventually, leads to a prime number; so, we have the function f defined on the set of composite positive integers with values in the set of prime numbers; the first five values of f are:

: $f(4) = 211$;
 : $f(6) = 23$;
 : $f(8) = 3331113965338635107$;
 : $f(9) = 311$;
 : $f(10) = 773$.

6. Four conjectures about three subsets of pairs of twin primes

Abstract. In this paper are stated four conjectures about three subsets of pairs of twin primes, i.e. the pairs of the form $(p^2 + q - 1, p^2 + q + 1)$, where p and q are primes (not necessarily distinct), the pairs of the form $(p + q - 1, p + q + 1)$, where p, q and $q + 2$ are all three primes and the pairs of the form $(p^2 + q - 1, p^2 + q + 1)$, where p, q and $q + 2$ are all three primes.

Conjecture 1:

Any pair of twin primes (a, b) greater than $(29, 31)$ can be written as $(a = p^2 + q - 1, b = p^2 + q + 1)$, where p and q are primes (not necessarily distinct).

Verifying the conjecture:

(for the first 5 such pairs of twin primes)

For $(a, b) = (29, 31)$, we have:

$$a = 5^2 + 5 - 1 = 29; \quad b = 5^2 + 5 + 1 = 31.$$

For $(a, b) = (41, 43)$, we have:

$$a = 5^2 + 17 - 1 = 41; \quad b = 5^2 + 17 + 1 = 43.$$

For $(a, b) = (59, 61)$, we have:

$$a = 7^2 + 11 - 1 = 59; \quad b = 7^2 + 11 + 1 = 61.$$

For $(a, b) = (71, 73)$, we have:

$$a = 5^2 + 47 - 1 = 71; \quad b = 2^2 + 47 + 1 = 73;$$

$$a = 7^2 + 23 - 1 = 71; \quad b = 7^2 + 23 + 1 = 73.$$

For $(a, b) = (101, 103)$, we have:

$$a = 7^2 + 53 - 1 = 101; \quad b = 7^2 + 53 + 1 = 103.$$

Note:

The conjecture can also be formulated in the following way: For any pair of twin primes $(t, t + 2)$, where t is greater than or equal to 29, there exist two primes a, b such that $(t + 1) - a^2 = b$.

Verifying the conjecture:

(for the following 5 pairs of twin primes)

$$: \quad 108 - 25 = 83; \quad 108 - 49 = 59;$$

$$: \quad 138 - 25 = 113; \quad 138 - 49 = 89; \quad 138 - 121 = 17;$$

$$: \quad 150 - 49 = 101; \quad 150 - 121 = 29;$$

$$: \quad 180 - 49 = 131; \quad 180 - 121 = 59; \quad 180 - 169 = 11;$$

$$: \quad 192 - 25 = 167; \quad 192 - 121 = 71; \quad 192 - 169 = 23.$$

Note:

The conjecture is also checked for the next twenty pairs of twin primes, with the lesser term equal to: 197, 227, 239, 269, 281, 311, 347, 419, 431, 462, 521, 569, 599, 617, 641, 659, 809, 821, 827, 857.

Conjecture 2:

There exist an infinity of pairs of primes of the form $(p^2 + q - 1, p^2 + q + 1)$, where p and q are primes (not necessarily distinct).

Note:

This conjecture is equivalent with the Conjecture about the infinity of twin primes if the Conjecture 1 from above is true, in such case being representative for the entire set of the pairs of twin primes or may state something different (but still implying the infinity of the pairs of twin primes) if the Conjecture 1 is not true, in such case being representative for an infinite subset of the set of the pairs of twin primes. The two conjectures below also implies the infinity of the pairs of twin primes.

Conjecture 3:

There exist an infinity of pairs of primes (p, q) , where $p + 2$ and $q + 2$ are also primes, such that $p - q + 1 = t$, where t is also prime.

Examples:

$$: 227 - 197 + 1 = 31, \text{ so } (p, q, t) = (227, 197, 31);$$

$$: 239 - 227 + 1 = 13, \text{ so } (p, q, t) = (239, 227, 13).$$

Note:

The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes $(p, p + 2)$, such that $p = t + q - 1$, where t is prime and the numbers q and $q + 2$ are also primes.

Conjecture 4:

There exist an infinity of pairs of primes (p, q) , where $p + 2$ and $q + 2$ are also primes, such that $p - q + 1 = t^2$, where t is also prime.

Examples:

$$: 569 - 521 + 1 = 7^2, \text{ so } (p, q, t) = (569, 521, 7);$$

$$: 851 - 857 + 1 = 5^2, \text{ so } (p, q, t) = (851, 857, 5).$$

Note:

The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes $(p, p + 2)$, such that $p = t^2 + q - 1$, where t is prime and the numbers q and $q + 2$ are also primes.

7. Twenty-four conjectures about “the eight essential subsets of primes”

Abstract. In this paper are made twenty-four conjectures about eight subsets of prime numbers, i.e. the primes of the form $30*k + 1$, $30*k + 7$, $30*k + 11$, $30*k + 13$, $30*k + 17$, $30*k + 19$, $30*k + 23$ respectively $30*k + 29$. Because we strongly believe that this classification of primes can have many applications, we referred in the title of this paper to these subsets of primes as to “the eight essential subsets of primes”. The conjectures state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms.

I.

Conjecture 1:

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = 31$, $a(2) = 61$, $a(3) = 151$ [the first three terms of the sequence are the smallest three primes of the form $30*k + 1$];

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 1$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 2:

Any prime of the form $30*k + 1$ is a term of the sequence $a(n)$ as it is defined by Conjecture 1 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 1$].

Verifying the conjecture 2:

(for the primes of the form $30*k + 1$ up to 421)

: $a(4) = a(1) + a(3) - 1 = 181$;

: $a(5) = a(2) + a(3) - 1 = 211$;

: $a(6) = a(1) + a(5) - 1 = a(2) + a(4) - 1 = 241$;

: $a(7) = a(1) + a(6) - 1 = a(2) + a(5) - 1 = 271$;

: $a(8) = a(2) + a(7) - 1 = a(3) + a(4) - 1 = 331$;

: $a(9) = a(3) + a(7) - 1 = a(5) + a(5) - 1 = 421$.

Conjecture 3:

If the Conjecture 2 doesn't hold, than is true at least that any prime of the form $30*k + 1$ is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1)$, $a(2)$, $a(3)$ are three distinct primes of the form $30*k + 1$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 1$, where $1 \leq i \leq j < n$.

II.

Conjecture 4:

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

$$: a(1) = 37, a(2) = 67, a(3) = 97;$$

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 7$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 5:

Any prime of the form $30*k + 7$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 4 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 7$, for $k > 0$].

Verifying the conjecture 5:

(for the primes of the form $30*k + 7$ up to 367)

$$: a(4) = a(1) + a(3) - 7 = a(2) + a(2) - 7 = 127;$$

$$: a(5) = a(1) + a(4) - 7 = a(2) + a(3) - 7 = 157;$$

$$: a(6) = a(4) + a(5) - 7 = 277;$$

$$: a(7) = a(1) + a(6) - 7 = a(5) + a(5) - 7 = 307;$$

$$: a(8) = a(1) + a(7) - 7 = a(2) + a(6) - 7 = 337;$$

$$: a(9) = a(1) + a(8) - 7 = a(2) + a(7) - 7 = 367.$$

Conjecture 6:

If the Conjecture 5 doesn't hold, than is true at least that any prime of the form $30*k + 7$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 7$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 7$, where $1 \leq i \leq j < n$.

III.

Conjecture 7:

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

$$: a(1) = 41, a(2) = 71, a(3) = 101;$$

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 11$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 8:

Any prime of the form $30*k + 11$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 7 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 11$, for $k > 0$].

Verifying the conjecture 8:

(for the primes of the form $30*k + 11$ up to 367)

$: a(4) = a(1) + a(3) - 11 = a(2) + a(2) - 11 = 131;$
 $: a(5) = a(2) + a(4) - 11 = a(3) + a(3) - 11 = 191;$
 $: a(6) = a(2) + a(5) - 11 = a(4) + a(4) - 11 = 251;$
 $: a(7) = a(1) + a(6) - 11 = a(3) + a(5) - 11 = 281;$
 $: a(8) = a(1) + a(7) - 11 = a(2) + a(6) - 11 = 311;$
 $: a(9) = a(3) + a(8) - 11 = a(4) + a(7) - 11 = 401.$

Conjecture 9:

If the Conjecture 8 doesn't hold, then it is true at least that any prime of the form $30*k + 11$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:
 $: a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 11$;
 $: a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 11$, where $1 \leq i \leq j < n$.

IV.

Conjecture 10:

The sequence $a(n)$, as it will be defined below, has an infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

$: a(1) = 43, a(2) = 73, a(3) = 103;$
 $: a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 13$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 11:

Any prime of the form $30*k + 13$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 10 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 13$, for $k > 0$].

Conjecture 12:

If the Conjecture 11 doesn't hold, then it is true at least that any prime of the form $30*k + 13$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:
 $: a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 13$;
 $: a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 13$, where $1 \leq i \leq j < n$.

V.

Conjecture 13:

The sequence $a(n)$, as it will be defined below, has an infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

$: a(1) = 47, a(2) = 107, a(3) = 137;$
 $: a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 17$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 14:

Any prime of the form $30*k + 17$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 13 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 17$, for $k > 0$].

Conjecture 15:

If the Conjecture 14 doesn't hold, than is true at least that any prime of the form $30*k + 17$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 17$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 17$, where $1 \leq i \leq j < n$.

VI.**Conjecture 16:**

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = 79, a(2) = 109, a(3) = 139$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 19$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 17:

Any prime of the form $30*k + 19$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 16 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 19$, for $k > 0$].

Conjecture 18:

If the Conjecture 17 doesn't hold, than is true at least that any prime of the form $30*k + 19$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 19$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 19$, where $1 \leq i \leq j < n$.

VII.**Conjecture 19:**

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = 53, a(2) = 83, a(3) = 113$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 23$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 20:

Any prime of the form $30*k + 23$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 19 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 23$, for $k > 0$].

Conjecture 21:

If the Conjecture 20 doesn't hold, than is true at least that any prime of the form $30*k + 23$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1), a(2), a(3)$ are three distinct primes of the form $30*k + 23$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 23$, where $1 \leq i \leq j < n$.

VIII.

Conjecture 22:

The sequence $a(n)$, as it will be defined below, has in infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = 59$, $a(2) = 89$, $a(3) = 149$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 29$, where $1 \leq i \leq j < n$ (if such prime exists for any n , as the conjecture states).

Conjecture 23:

Any prime of the form $30*k + 29$, where $k > 0$, is a term of the sequence $a(n)$ as it is defined by Conjecture 22 [in other words, the sequence $a(n)$ is the same with the sequence of the primes of the form $30*k + 29$, for $k > 0$].

Conjecture 24:

If the Conjecture 23 doesn't hold, than is true at least that any prime of the form $30*k + 29$, where $k > 0$, is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1)$, $a(2)$, $a(3)$ are three distinct primes of the form $30*k + 29$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - 29$, where $1 \leq i \leq j < n$.

8. Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes

Abstract. In my previous paper «Twenty-four conjectures about “the eight essential subsets of primes”» are made three conjectures about each one from the following eight subsets: the primes of the form $30*k + 1$, $30*k + 7$, $30*k + 11$, $30*k + 13$, $30*k + 17$, $30*k + 19$, $30*k + 23$ respectively $30*k + 29$. The conjectures from that paper state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms. In this paper are generalized the three conjectures for an infinity of subsets, each having possibly an infinity of terms which are primes or squares of primes, subsets of integers of the form $2*p(1)*p(2)*...*p(m)*k + d$, where $p(1)$, $p(2)$, ..., $p(m)$ are the first m odd primes, k is a non-null positive integer and d an odd positive integer satisfying certain conditions.

Conjecture 1:

The sequence $a(n)$, as it will be defined below, has an infinity of terms that are primes or squares of primes.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = q_1$, $a(2) = q_2$, $a(3) = q_3$, where q_1 , q_2 and q_3 are the first three integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where p_1 , p_2 , ..., p_m are the first m odd primes, k is a non-null positive integer and d is equal to 1 or is equal to any odd positive integer which satisfies the following two conditions: d is co-prime to any of the primes p_1 , p_2 , ..., p_m and $d < 2*p_1*p_2*...*p_m$.

: $a(n)$ is the smallest prime or square of prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - d$, where $1 \leq i \leq j < n$ (if such prime or square of prime exists for any n , as the conjecture states).

Conjecture 2:

Any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of the sequence $a(n)$ as it is defined by Conjecture 1 [in other words, the sequence $a(n)$ is the same with the sequence of the integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where $k > 0$].

Conjecture 3:

If the Conjecture 2 doesn't hold, than is true at least that any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of a sequence $a(n)$ that can be defined as follows:

: $a(1)$, $a(2)$, $a(3)$ are three distinct integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where $k > 0$;

: $a(n)$ is the smallest integer which is prime or square of prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - d$, where $1 \leq i \leq j < n$.

Verifying the Conjecture 2 for m = 1:
(for first few terms)

For $m = 1$ we have $p_1 = p_m = 3$ and only two possible values for d (because d must be odd, smaller than $2 \cdot 3 = 6$ and also co-prime with 3), *i.e.* $d = 1$ and $d = 5$.

I.

For $d = 1$ the first three integers which are primes or squares of primes of the form $6 \cdot k + 1$, where $k > 0$, are 7, 13 and 19:

: Conjecture 1 states that there exist an infinity of integers of the form $6 \cdot k + 1$ which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form $6 \cdot k + 1$ can be defined starting from the primes 7, 13 and 19 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are $5^2 = 1 + 19 - 1 = 13 + 13 - 1$; $31 = 13 + 19 - 1$; $37 = 7 + 31 - 1$; $43 = 7 + 37 - 1$, $7^2 = 7 + 43 - 1 = 13 + 37 - 1 = 19 + 31 - 1$; $61 = 19 + 43 - 1 = 31 + 31 - 1$.

II.

For $d = 5$ the first three integers which are primes or squares of primes of the form $6 \cdot k + 5$, where $k > 0$, are 11, 17 and 23:

: Conjecture 1 states that there exist an infinity of integers of the form $6 \cdot k + 5$ which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form $6 \cdot k + 5$ can be defined starting from the primes 11, 17 and 23 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are $29 = 17 + 17 - 5$; $41 = 23 + 23 - 5$; $47 = 11 + 41 - 5 = 23 + 29 - 5$; $53 = 11 + 47 - 5 = 17 + 41 - 5 = 29 + 29 - 5$; $59 = 11 + 53 - 5 = 17 + 47 - 5 = 23 + 41 - 5$; $71 = 17 + 59 - 5 = 23 + 53 - 5 = 29 + 47 - 5$.

Verifying the Conjecture 2 for m = 2:

For $m = 2$ we have $p_1 = 3$ and $p_2 = p_m = 5$ and the subsets of integers of the form $30 \cdot k + 1$, $30 \cdot k + 7$, $30 \cdot k + 11$, $30 \cdot k + 13$, $30 \cdot k + 17$, $30 \cdot k + 19$, $30 \cdot k + 23$; for these eight subsets of integers we treated a more strict form of the three conjectures from above in the previous paper «Twenty-four conjectures about “the eight essential subsets of primes”».

Verifying the Conjecture 2 for m = 3:

(for $d = 1$, $d = 11$ and the first few terms)

For $m = 3$ we have $p_1 = 3$, $p_2 = 5$ and $p_3 = p_m = 7$ and the subsets of primes of the form $210 \cdot k + 1$, $210 \cdot k + 11$, $210 \cdot k + 13$, $210 \cdot k + 17$, $210 \cdot k + 19$, $210 \cdot k + 23$, $210 \cdot k + 29$, $210 \cdot k + 31$, $210 \cdot k + 37$, $210 \cdot k + 41$, $210 \cdot k + 43$, $210 \cdot k + 47$, $210 \cdot k + 53$, $210 \cdot k + 59$, $210 \cdot k + 61$, $210 \cdot k + 67$, $210 \cdot k + 71$, $210 \cdot k + 73$, $210 \cdot k + 79$, $210 \cdot k + 83$, $210 \cdot k + 89$,

$210*k + 97, 210*k + 101, 210*k + 103, 210*k + 107, 210*k + 109, 210*k + 13, 210*k + 121, 210*k + 127, 210*k + 131, 210*k + 137, 210*k + 139, 210*k + 143, 210*k + 149, 210*k + 151, 210*k + 157, 210*k + 163, 210*k + 167, 210*k + 169, 210*k + 173, 210*k + 179, 210*k + 181, 210*k + 187, 210*k + 191, 210*k + 193, 210*k + 197, 210*k + 199, 210*k + 209.$

: Conjecture 1 states that there exist an infinity of integers of each of these forms which are primes or squares of primes.

: Conjecture 2 states that any prime or square of prime of this form can be defined with the formula from Conjecture 1 starting from the three integers which are primes or squares of primes of the respective form, considering $k > 0$.

: The first three integers which are primes or squares of primes of the form $210*k + 1$, where $k > 0$, are 211, 421 and 631; the following next six integers which are primes or squares of primes of this form are $29^2 = 21 + 631 - 1 = 421 + 421 - 1$; $1051 = 421 + 631 - 1 = 211 + 29^2 - 1$; $1471 = 421 + 1051 - 1$; $41^2 = 211 + 1471 - 1 = 631 + 1051 - 1$; $2311 = 631 + 41^2 - 1$; $2521 = 211 + 2311 - 1 = 1051 + 1471 - 1$.

: The first three integers which are primes or squares of primes of the form $210*k + 11$, where $k > 0$, are 431, 641 and 1061; the following next six integers which are primes or squares of primes of this form are $1481 = 431 + 1061 - 11$; $1901 = 431 + 1481 - 11$; $2111 = 641 + 1481 - 11$; $2531 = 431 + 2111 - 11$; $2741 = 641 + 2111 - 11$; $3371 = 641 + 2741 - 11$.

9. Pairs of primes or pseudoprimes that generate an infinity of primes or pseudoprimes via a certain recurrence relation

Abstract. In this paper are made five conjectures about a type of pairs of primes respectively Fermat pseudoprimes which have the property to generate an infinity of primes respectively Fermat pseudoprimes via a recurrence formula that will be defined in this paper; we name the pairs with this property Coman pairs of primes respectively Coman pairs of pseudoprimes. Because it is easy to show that two given primes respectively pseudoprimes do not form such a pair and it is very difficult to prove that they form such a pair, the correct expression about two odd primes (or pseudoprimes) p , q , where $p = 30*k + d$ and $q = 30*h + d$, where k , h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, is that the pair (p,q) is not a Coman pair respectively that the pair (p,q) is a possible Coman pair of primes (or pseudoprimes).

Definition 1:

We call the pair of odd primes (p, q) , where $p = 30*k + d$ and $q = 30*h + d$, where k , h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of primes* if the sequence $a(n)$, as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = p$, $a(2) = q$;

: $a(3)$ is the smallest number, different from p and q , which is prime from the following three ones: $p + q - d$, $2*p - d$ and $2*q - d$;

: $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - d$, where $1 \leq i \leq j < n$.

Note: the definition implies that the one from the numbers $p + q - d$, $2*p - d$ and $2*q - d$ is prime and that for any n , $n \geq 4$, there exist i, j , where $1 \leq i \leq j < n$, such that $a(n) = a(i) + a(j) - d$, where $a(n)$, $a(i)$ and $a(j)$ are all three primes.

Examples:

The pair of primes $(37, 67)$ is a possible *Coman pair of primes* because:

: $a(3) = 37 + 67 - 7 = 97$ is prime;

: $a(4) = 37 + 97 - 7 = 67 + 67 = 127$ is prime;

: $a(5) = 37 + 127 - 7 = 67 + 97 - 7 = 157$ is prime;

: $a(6) = 127 + 157 - 7 = 277$ is prime.

: $a(7) = 37 + 277 - 7 = 157 + 157 - 7 = 307$ is prime.

(...)

The pair of primes $(97, 127)$ is not a possible *Coman pair of primes* because $2*97 - 7 = 187$ is not prime, $2*127 - 7 = 247$ is not prime and also $97 + 127 - 7 = 217$ is not prime.

Definition 2:

We call a *Coman pair of primes* a *Coman strict pair of primes* if $a(3) = p + q - d$ and $i \neq j$, in other words $a(i) \neq a(j)$.

Examples:

The pair of primes (37, 67) is a possible *Coman strict pair of primes* because:

- : $a(3) = 37 + 67 - 7 = 97$ is prime;
 - : $a(4) = 37 + 97 - 7 = 127$ is prime;
 - : $a(5) = 37 + 127 - 7 = 67 + 97 - 7 = 157$ is prime;
 - : $a(6) = 127 + 157 - 7 = 277$ is prime;
 - : $a(7) = 37 + 277 - 7$ is prime.
- (...)

The pair of primes (37, 157) is not a possible *Coman strict pair of primes* because $37 + 157 - 7 = 187$ is not prime, but is a possible *Coman pair of primes*, because $a(3) = 37 + 37 - 7 = 67$ is prime, $a(4) = 67 + 67 - 7 = 127$ is prime (...).

Definition 3:

We call the pair of odd primes (p, q) a *[possible] generalized Coman pair of primes to base b*, where b is a non-null integer, if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

- : $a(1) = p$, $a(2) = q$;
- : $a(3)$ is the smallest number, different from p and q, which is prime from the following three ones: $p + q - b$, $2*p - b$ and $2*q - b$;
- : $a(n)$ is the smallest prime greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - b$, where $1 \leq i \leq j < n$.

Note: the definition implies that the one from the numbers $p + q - b$, $2*p - b$ and $2*q - b$ is prime and that for any n, $n \geq 4$, there exist i, j, where $1 \leq i \leq j < n$, such that $a(n) = a(i) + a(j) - b$, where a(n), a(i) and a(j) are all three primes.

Examples:

The pair of primes (7, 13) is a possible *generalized Coman pair of primes to base 1* because:

- : $a(3) = 7 + 13 - 1 = 19$ is prime;
 - : $a(4) = 13 + 19 - 1 = 31$ is prime;
 - : $a(5) = 7 + 31 - 1 = 19 + 19 - 1 = 37$ is prime;
 - : $a(6) = 7 + 37 - 1 = 13 + 31 - 1 = 43$ is prime;
 - : $a(7) = 19 + 43 - 1 = 31 + 31 - 1 = 61$ is prime.
- (...)

The pair of primes (11, 17) is a possible *generalized Coman pair of primes to base -1* because:

- : $a(3) = 11 + 11 + 1 = 23$ is prime;
 - : $a(4) = 11 + 17 + 1 = 29$ is prime;
 - : $a(5) = 11 + 29 + 1 = 41$ is prime;
 - : $a(6) = 17 + 29 + 1 = 47$ is prime;
 - : $a(7) = 11 + 41 + 1 = 53$ is prime;
 - : $a(8) = 11 + 47 + 1 = 59$ is prime.
- (...)

Definition 4:

We call a *generalized Coman pair of primes* a *generalized Coman strict pair of primes* if $a(3) = p + q - b$ and $i \neq j$, in other words $a(i) \neq a(j)$.

Note that the term $a(n) = 23$ is not a term of the *generalized Coman strict pair of primes* (11, 17), but is a term of the *generalized Coman pair of primes* (11, 17).

Definition 5:

We call the pair (p, q) of odd Fermat pseudoprimes to the same base b , where $p = 30 \cdot k + d$ and $q = 30 \cdot h + d$, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of pseudoprimes* if the sequence $a(n)$, as it will be defined below, has [possibly] an infinity of terms that are pseudoprimes to the same base b .

The sequence $a(n)$, where n non-null positive integer, is defined in the following way:

: $a(1) = p, a(2) = q$;

: $a(3)$ is the smallest number, different from p and q , which is prime or Fermat pseudoprime to base b from the following three ones: $p + q - d, 2 \cdot p - d$ and $2 \cdot q - d$;

: $a(n)$ is the smallest number which is prime or pseudoprime to base b greater than $a(n - 1)$ that can be written as $a(n) = a(i) + a(j) - d$, where $1 \leq i \leq j < n$.

Note: the definition implies that one from the numbers $p + q - d, 2 \cdot p - d$ and $2 \cdot q - d$ is a prime or a pseudoprime to base b and that for any $n, n \geq 4$, there exist i, j , where $1 \leq i \leq j < n$, such that $a(n) = a(i) + a(j) - d$, where $a(n), a(i)$ and $a(j)$ are primes or pseudoprimes to base b , and, finally, that there is an infinite subset of the set $a(n)$, namely $a(m)$, whose terms are all of them pseudoprimes to base b .

Examples:

The pair of Fermat pseudoprimes to base 2 (2701, 2821), where $2701 = 30 \cdot 90 + 1$ and $2821 = 30 \cdot 94 + 1$, is a possible *Coman pair of pseudoprimes* because:

: $a(3) = 2701 + 2821 - 1 = 5521$ is prime;

: $a(4) = 2821 + 2821 - 1 = 5641$ is prime;

: $a(5) = 2701 + 5521 - 1 = 8221$ is prime;

: $a(6) = 5521 + 8221 - 1 = 13741$ is Fermat pseudoprime to base 2.

(...)

Conjecture 1:

There is an infinity of Coman pairs of primes.

Conjecture 2:

There is an infinity of Coman strict pairs of primes.

Conjecture 3:

There is an infinity of generalized Coman pairs of primes to any base b , where b is non-null integer.

Conjecture 4:

There is an infinity of generalized Coman strict pairs of primes to any base b , where b is non-null integer.

Conjecture 5:

There exist pairs of Poulet numbers which are Coman pairs of pseudoprimes.

References:

1. Coman, Marius, *Twenty-four conjectures about “the eight essential subsets of primes”*, Vixra;
2. Coman, Marius, *Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes*, Vixra.

10. Three conjectures about semiprimes inspired by a recurrent formula involving 2-Poulet numbers

Abstract. Studying the relation between the two prime factors of a 2-Poulet number I found an interesting recurrent formula involving these numbers that seems to lead often to a value which is semiprime; based on this observation I made three conjectures about semiprimes.

Observation 1

We take a pair of 2-Poulet numbers which have a common prime factor, as for instance the pair $[P_1 = 341 = 11 \cdot 31, P_2 = 4681 = 31 \cdot 151]$ or the pair $[P_1 = 1387 = 19 \cdot 73, P_2 = 2701 = 37 \cdot 73]$ and apply on it the recurrent formula $P_n = P_{n-2} + \gcd((P_{n-2} - 1), (P_{n-1} - 1))$.

In the case $[P_1, P_2] = [341, 4681]$ we have:

$$\begin{aligned} &: P_3 = 341 + \gcd(340, 4680) = 361; \\ &: P_4 = 4681 + \gcd(4680, 360) = 5041; \\ &: P_5 = 361 + \gcd(360, 5040) = 721; \\ &: P_6 = 5041 + \gcd(5040, 720) = 5761; \\ &: P_7 = 721 + \gcd(720, 5760) = 1441; \\ &: P_8 = 5761 + \gcd(5760, 1440) = 7201; \\ &: P_9 = 1441 + \gcd(1440, 7200) = 2881; \\ &: P_{10} = 7201 + \gcd(7200, 2880) = 8641; \\ &: P_{11} = 2881 + \gcd(2880, 8640) = 5761; \\ &: P_{12} = 8641 + \gcd(8640, 5760) = 11521; \\ &: P_{13} = 5760 + \gcd(5760, 11520) = 11521. \end{aligned}$$

Starting from P_{14} , we will have $P_{14} = P_{15} = 2 \cdot (P_{12} - 1) + 1$, $P_{16} = P_{17} = 2 \cdot (P_{14} - 1) + 1$ and so on.

In the case $[P_1, P_2] = [1387, 2701]$ we have:

$$\begin{aligned} &: P_3 = 1387 + \gcd(1386, 2700) = 1405; \\ &: P_4 = 2701 + \gcd(2700, 1400) = 2809; \\ &: P_5 = 1405 + \gcd(1404, 2808) = 2809; \end{aligned}$$

Starting from P_6 , we will have $P_6 = P_7 = 2 \cdot (P_4 - 1) + 1 = 5617$, $P_8 = P_9 = 2 \cdot (P_{12} - 1) + 1 = 11233$, $P_{10} = P_{11} = 2 \cdot (P_8 - 1) + 1 = 22465$, $P_{12} = P_{13} = 2 \cdot (P_{10} - 1) + 1 = 44929$, $P_{14} = P_{15} = 2 \cdot (P_{12} - 1) + 1 = 89857$, $P_{16} = P_{17} = 2 \cdot (P_{14} - 1) + 1 = 179713$ and so on.

It can be seen that many of the values of the terms P_i are semiprimes: $361 = 19 \cdot 19$, $5041 = 71 \cdot 71$, $721 = 7 \cdot 103$, $5761 = 7 \cdot 823$, $1441 = 11 \cdot 131$, $7201 = 19 \cdot 379$, $2881 = 43 \cdot 67$, $11521 = 41 \cdot 281$, $1405 = 5 \cdot 281$, $2809 = 53 \cdot 53$, $5617 = 41 \cdot 137$, $11233 = 47 \cdot 239$, $22465 = 5 \cdot 4493$, $44929 = 179 \cdot 251$, $89857 = 59 \cdot 1523$, $179713 = 29 \cdot 6197$.

More than that, between the two distinct prime factors p and q from many of the semiprimes obtained above there exist the relation $q - p + 1 = n$, where n is a prime or a square of a prime:

: $103 - 7 + 1 = 97$;
 : $131 - 11 + 1 = 121 = 11^2$;
 : $379 - 19 + 1 = 361 = 19^2$;
 : $67 - 43 + 1 = 25 = 5^2$;
 : $281 - 41 + 1 = 41$;
 : $281 - 5 + 1 = 277$;
 : $67 - 43 + 1 = 25 = 5^2$;
 : $137 - 41 + 1 = 97$;
 : $239 - 47 + 1 = 193$;
 : $4493 - 5 + 1 = 4489 = 67^2$
 : $251 - 179 + 1 = 73$.

A very interesting thing it happens even if between p and q there is not the relation from the preceding paragraph; in many of these cases $q - p + 1 = n$, where n is a semiprime whose two prime factors admit themselves the relation showed:

: $823 - 7 + 1 = 817 = 19 \cdot 43$ and $43 - 19 + 1 = 25 = 5^2$;
 : $1523 - 59 + 1 = 1465 = 5 \cdot 293$ and $293 - 5 + 1 = 17^2$;
 : $6197 - 29 + 1 = 6169 = 31 \cdot 199$ and $199 - 31 + 1 = 13^2$.

Observation 2

We also observed that the iterative formula $a_{n+1} = 2 \cdot (a_n - 1) + 1$, where a_1 is a square of a prime minus nine, seems likewise to often conduct to primes, power of primes or semiprimes with the characteristics of those from Observation 1.

For $a_1 = 7^2 - 9 = 40$ we obtain the following sequence:

: 79, 157, 313, 625, 1249, 2497, 4993, 9985, 19969, 39937, 79873, 159745, 319489,
 638977, 1277953 (...), where:
 : 79, 157, 313, 1249, 4993, 39937, 79873, 319489, 638977 are primes;
 : $625 = 5^4$ is a power of prime;
 : $2497 = 11 \cdot 227$; $227 - 11 + 1 = 217 = 7 \cdot 31$; $31 - 7 + 1 = 25 = 5^2$;
 : $9985 = 5 \cdot 1997$; $1997 - 5 + 1 = 1993$ prime;
 : $19969 = 19 \cdot 1051$; $1051 - 19 + 1 = 1033$ prime.
 : $1277953 = 101 \cdot 12653$; $12653 - 101 + 1 = 12553$ prime.

Conjecture 1

For any odd prime n there exist an infinity of pairs of odd primes $[p, q]$ such that $q - p + 1 = n$.

Conjecture 2

For any semiprime $p_1 \cdot q_1$, where p_1 and q_1 are odd distinct primes, there exist an infinity of pairs of odd primes $[p_2, q_2]$ such that $q_2 - p_2 + 1 = p_1 \cdot q_1$.

Conjecture 3

For any odd prime n there exist an infinity of pairs of odd primes $[p_i, q_i]$, for any i from 1 to infinite, such that:

$: q_1 - p_1 + 1 = n;$
 $: q_2 - p_2 + 1 = p_1 * q_1;$
 $: q_3 - p_3 + 1 = p_2 * q_2;$
 (...)

$: q_i - p_i + 1 = p_{i-1} * q_{i-1}.$

Note:

This is an interesting way to construct (possible) infinite sequences of semiprimes $p_i * q_i$, starting from a given prime and considering, for instance, the smallest p_i for which the relations from Conjecture 3 are verified. For instance, in the conditions mentioned, we take $n = 13$. We have:

- $: p_1 = 5$ because is the smallest prime such that $n - 1 + p_1 = q_1$ is prime, so $q_1 = 13 - 1 + 5 = 17$;
- $: p_2 = 5$ because is the smallest prime such that $p_1 * q_1 - 1 + p_2 = q_2$ is prime, so $q_2 = 5 * 17 - 1 + 5 = 89$;
- $: p_3 = 5$ because is the smallest prime such that $p_2 * q_2 - 1 + p_3 = q_3$ is prime, so $q_3 = 5 * 89 - 1 + 5 = 449$;
- $: p_4 = 7$ because is the smallest prime such that $p_3 * q_3 - 1 + p_4 = q_4$ is prime, so $q_4 = 5 * 449 - 1 + 7 = 2251$ (...).

We obtained the following sequence of semiprimes $p_i * q_i$: 85, 445, 2245, 15757 (...).