On the Nature of the Newton Gravitational Constant[1]

Casper Spanggaard[2]

[2]Research & Development, ithought, Århus, Denmark. E-mail: cs@ithought.dk

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A definition of G is derived using the product of two Planck point masses and a definition of $\hbar$ based on the speed of light in vacuum and geometry. The theoretical value of G is found to be $6.74981057667161 \times 10^{-11}$ m^3 kg^{-1} s^{-2} yielding a relative accuracy error of the CODATA 2010 G-value of -1.1255%. One experiment resulted in a value with a smaller relative accuracy error than the CODATA 2010 G-value of -0.5098%. Both rest and relativistic mass product equations are derived. These equations relate the relative spacetime spin frequency ω_s, the relative orbital frequency ω_o and (relativistic equation only) the Lorentz factor γ describing relative linear speed of two bodies to the mass product. The Planck mass is a special case mass with $\omega_s \omega_o = \omega^2_{planck} = 1$ s^{-2}. The theoretical value of the Planck mass was found to be $2.16039211144077 \times 10^{-8}$ kg. The relative accuracy error of the CODATA 2010 Planck mass value is 0.7461%. This error is attributed to use of the different definition of $\hbar$. When derived from both $\hbar$ and G constants as well as the rest mass product equation, three kilogram unit definition candidates are all inconsistent. The candidate derived from the rest mass product equation is the only candidate that has equal second and meter exponents suggesting a kind of symmetry. This definition is considered the nominal kilogram unit definition. The other two candidates are considered to be artifacts of the $\hbar$ and G constants.
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1 Introduction

The Newton gravitational constant is of fundamental importance in Newtonian mechanics and general theory of relativity. The value of the constant has been determined by experiments and at present no theoretical definition of its value based on more fundamental constants such as the speed of light in vacuum is widely known. In this paper I present such a definition of the constant.

2 Derivation of the Newton gravitational constant

2.1 Constant derivation

The published CODATA 2010 value of the Newton gravitational constant G is $6.67384(80) \times 10^{-11}$ m^3 kg^{-1} s^{-2} with a relative standard uncertainty $u_r = 1.2 \times 10^{-4}$ (TABLE XL) [1]. For the rest of this paper that value is referred to as G_{exp}.

The Planck mass is

$$m_P = \sqrt{\frac{\hbar c_0}{G}} \tag{1}$$

The product of two Planck point masses is then

$$m_{01} m_{02} = m_P^2 = \sqrt{\frac{\hbar c_0}{G}}^2 = \frac{\hbar c_0}{G}$$

After expansion of $\hbar$ using the definition from [2]

$$m_{01} m_{02} = \frac{\left(2\pi \frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3 c_0}\right) c_0}{G}$$

Reorganize

$$m_{01} m_{02} = 2\pi \frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3 G}$$

Here there is 2π times a spacetime spin specific volume divided by G when analyzed using the spacetime spin toy model of [2]. The special theory of relativity lets us know that the mass of a body increase with increased speed of movement of the body relative to a reference body. Therefore I expect mass contributions from relative spacetime spin, relative orbital motion, and relative linear motion to be part of the equation and this require three factors. If the 3-spacetime spin volume set is expanded

$$m_{01} m_{02} = 2\pi \frac{\frac{4}{3}\pi r_1^3}{\sqrt[4]{(\pi r_1^2)^3 (\pi r_1^2)^3 (\pi r_1^2)^3 (\pi r_1^2)^3} G}$$

then the individual mass contributions can be seen. The fourth factor seems misplaced thereby illustrating the somewhat weird division of geometric information that is partially encoded in $\hbar$ and G and that complicates interpretation. However this fourth factor does give a hint as to the structure of the G definition equation.G must remove the factor. Let us for the moment assume that G also removes the 2π factor then

$$G = 2\pi \sqrt[4]{\frac{1}{(\pi r_1^2)^3}} k$$

where k is some factor such that the experimentally determined value G_{exp} is within an acceptable relative accuracy error. The constraints are satisfied for

$$k \approx \sqrt[4]{c_0}$$

A definition of G is then

$$\begin{aligned} G &= 2\pi \sqrt[4]{\frac{c_0}{(\pi r_1^2)^3}} \\ &= 6.74981057667161 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}} \end{aligned} \tag{2}$$

However the additional factor

$$\sqrt[4]{c_0}$$

must be accounted for in the mass product equation under derivation

$$m_{01} m_{02} = \frac{\frac{4}{3}\pi r_1^3}{\sqrt[4]{(\pi r_1^2)^3 (\pi r_1^2)^3 (\pi r_1^2)^3 c_0}} \tag{3}$$

2.2 Experimental Newton gravitational constant accuracy

Accuracy of the experimental values of G relative to the theoretical value is considered. With a theoretical value given by (2) the relative accuracy error of several experimentally determined values are listed in table 1. There is a general tendency of the experimentally determined values to be smaller than the theoretical value. I have included the value from some experiments that have significantly lower relative accuracy error than the CODATA 2010 value.

3 Mass product equations

3.1 Rest mass product equation

The centripetal acceleration is

$$a_c = \frac{v^2}{r} = r\omega^2 \tag{4}$$

Equating the gravitational force between two bodies to the centripetal force using (4)

Experiment / source	Experiment G-value (10^{-11} m^3 kg^{-1} s^{-2})	Relative accuracy error
CODATA 2010 [1]	6.67384	−1.1255%
Atom interf. #1 [3]	6.693	−0.8417%
Atom interf. #2 [4]	6.67191	−1.1541%
PTB experiment [5]	6.71540	−0.5098%

Table 1: Some experimentally determined values of G and their relative accuracy error.

$$\frac{G m_{01} m_{02}}{r^2} = m_{01} a_c = m_{01} r\omega^2$$

and rearranging then

$$m_{01} m_{02} = \frac{m_{01}\omega^2 r^3}{G} \tag{5}$$

Comparing (3) to (5), I notice that ω^2 is missing at the right side of (3). This relative angular frequency squared modifier must describe both relative spacetime spin and orbital motion of a two-body point mass system since those are the two components of rest mass. When split into its components

$$\omega^2 = \omega_s \omega_o$$

where ω_s is relative spacetime spin frequency and ω_o is relative orbital frequency of the two bodies. Then

$$m_{01} m_{02} = \frac{\frac{4}{3}\pi r_1^3 \omega_s \omega_o}{\sqrt[4]{(\pi r_1^2)^3 (\pi r_1^2)^3 (\pi r_1^2)^3 c_0}}$$

Reorganize and the rest mass product equation is

$$m_{01} m_{02} = \frac{\frac{4}{3}\pi r_1^3}{\sqrt[4]{\omega_s^{-4} (\pi r_1^2)^3 \omega_o^{-4} (\pi r_1^2)^3 (\pi r_1^2)^3 c_0}} \tag{6}$$

where ω_s is relative spacetime spin frequency and ω_o is relative orbital frequency of the two bodies.

3.2 Relativistic mass product equation

Let γ be the Lorentz factor. Then

$$\gamma m_{01} \gamma m_{02} = \gamma^2 m_{01} m_{02} = \frac{\gamma^2 \frac{4}{3}\pi r_1^3}{\sqrt[4]{\omega_s^{-4} (\pi r_1^2)^3 \omega_o^{-4} (\pi r_1^2)^3 (\pi r_1^2)^3 c_0}}$$

Reorganize and the relativistic mass product equation is

$$m_1 m_2 = \frac{\frac{4}{3}\pi r_1^3}{\sqrt[4]{\omega_s^{-4}(\pi r_1^2)^3 \omega_o^{-4}(\pi r_1^2)^3 \gamma^{-8}(\pi r_1^2)^3 c_0}}$$

where ω_s is relative spacetime spin frequency, ω_o is relative orbital frequency of the two bodies and γ is the Lorentz factor describing relative linear speed of the two bodies.

4 The Planck mass

The Planck mass can be found using (6) as

$$\begin{aligned} m_P &= \sqrt{\frac{\frac{4}{3}\pi r_1^3}{\sqrt[4]{\omega_s^{-4}(\pi r_1^2)^3 \omega_o^{-4}(\pi r_1^2)^3 (\pi r_1^2)^3 c_0}}} \\ &= 2.16039211144077 \times 10^{-8}\,\mathrm{kg} \end{aligned}$$

where

$$\omega_s \omega_o = \omega_{planck}^2 = 1\,\mathrm{s}^{-2}$$

The published CODATA 2010 value of the Planck mass is $2.17651(13) \times 10^{-8}$ kg with a relative standard uncertainty $u_r = 6.0 \times 10^{-5}$ (TABLE XLI) [1]. This result in a relative accuracy error of the CODATA 2010 Planck mass value of 0.7461%. The error is attributed to use of the different definition of $\hbar$ from [2].

5 Kilogram unit definition

Consider the unit of (6)

$$\mathrm{m}^{\frac{12}{8}}\mathrm{m}^{-\frac{18}{8}}\mathrm{s}^{-\frac{7}{8}}\mathrm{m}^{-\frac{1}{8}}$$

Then the unit of mass derived from (6) is

$$\mathrm{m}^{-\frac{7}{8}}\mathrm{s}^{-\frac{7}{8}} = \mathrm{kg} \tag{7}$$

which is different from what was derived from the reduced Planck constant in [2]

$$\mathrm{s}^2\mathrm{m}^{-6} = \mathrm{kg} \tag{8}$$

Consider the unit of G derived from (2)

$$\mathrm{m}^{\frac{1}{4}}\mathrm{s}^{-\frac{1}{4}}\mathrm{m}^{-\frac{6}{4}}$$

Reorganize

$$\mathrm{m}^{\frac{12}{4}}\mathrm{s}^{\frac{7}{4}}\mathrm{m}^{-\frac{17}{4}}\mathrm{s}^{-\frac{8}{4}} = \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$$

Then the unit of mass derived from (2) is

$$\mathrm{m}^{\frac{17}{4}}\mathrm{s}^{-\frac{7}{4}} = \mathrm{kg} \tag{9}$$

which is different from both (7) and (8). $\hbar$ and G constants have their unit defined such that (1) gives kilogram unit, but that the kilogram unit of the constants do not correspond to the same physics. When the constants are defined using the speed of light in vacuum, the derived kilogram unit definition candidates are inconsistent. (7) is the only kilogram definition candidate that has equal second and meter exponents suggesting a kind of symmetry. I consider (7) to be the nominal kilogram unit definition and (8) and (9) to be artifacts of the $\hbar$ and G constants.

6 Conclusion

I derived a definition of G using the product of two Planck point masses and a definition of $\hbar$ based on the speed of light in vacuum and geometry as (2). The theoretical value of G was found to be

$$6.74981057667161 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$$

The relative accuracy error of the CODATA 2010 G-value is -1.1255%. One experiment resulted in a value with a smaller relative accuracy error than the CODATA 2010 G-value of -0.5098%.

Both rest and relativistic mass product equations were derived. These equations relate the relative spacetime spin frequency ω_s, the relative orbital frequency ω_o and (relativistic equation only) the Lorentz factor γ describing relative linear speed of two bodies to the mass product.

The Planck mass is a special case mass with

$$\omega_s \omega_o = \omega_{planck}^2 = 1\,\mathrm{s}^{-2}$$

The theoretical value of the Planck mass was found to be

$$2.16039211144077 \times 10^{-8}\,\mathrm{kg}$$

The relative accuracy error of the CODATA 2010 Planck mass value is 0.7461%. This error is attributed to use of the different definition of $\hbar$.

When derived from both $\hbar$ and G constants as well as the rest mass product equation, the three kilogram unit definition candidates are all inconsistent. The candidate derived from the rest mass product equation is the only candidate that has equal second and meter exponents suggesting a kind of symmetry. This definition is considered the nominal kilogram unit definition

$$\mathrm{kg} = \mathrm{m}^{-\frac{7}{8}}\mathrm{s}^{-\frac{7}{8}}$$

The other two kilogram unit definition candidates are considered to be artifacts of the $\hbar$ and G constants.

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