Modelling and Simulation of Crowds

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Abstract

In this project, we extend the work already done in [1.] to include a generalised mathematical framework for studying and explaining the dynamics and behavior of crowds of humans. The method is both analytical and numerical. The numerical methods are used to solve the differential equations of crowds that are derived analytically. The analytical and numerical solutions are compared and their relevance is shown. In this project, we study mainly two types of responses of a crowd: **Position Response** and **Density Response.** The latter is formulated using stressors using an approach similar to [2.] which also enables us to derive the **General Adaptation Syndrome (GAS) Model in a very generalised form.** Finally, we extend stressors to define inter-crowd and intra-crowd interactions using a parameterisation linking it directly to a generalisation of the **stress-density equation.**

The detailed calculations are done in the later part of this paper. (Refer to Page 21 of this doucument).

Methodology

The method adopted in this project is standard to the differential analysis of any physical system using differential equations. This method is very frequently used in Physics to study various natural phenomena. The solutions of the differential equation of a system provides vital information about it. In this project, the differential equations are solved using both analytical and numerical methods and the solutions are carefully studied and interpreted. The analytical solutions utilise standard calculus techniques. For the numerical solutions,a C++ implementation of the classic 4th Order Runge-Kutta Method is used. GCC 4.8.0 is used for compling the code.
The data generated is fed into GNUPlot to produce the plots.
Throughout the project, a number of new terminology is used (with proper definition);the nomenclature arising from similar terms in Physics.

Ideas And Motivation

The main idea came from observation of crowd in everyday life followed by the inquisitiveness to understand it in a better way-which required the rigorous use of mathematical methods.
The starting point of the project was the aim to formulate a generalised model that could explain the dynamics and behavior of a variety of crowd types under various situations with reasonable accuracy.
This was backed by a thorough internet research on the work already done on crowd analysis. This revealed the efforts of the GAMMA

Group, University of North Carolina, Chapel Hill [A.] where the researchers showed innovative methods to simulate crowd behavior using multi-agent models and an "in-house" software developed by their team.
This, along with the studies in the field of Mathematical Sociology and Psychology in turn created the path for the actual development of the current project. Furthur help in modelling and testing was possible with availability of an open source multi-agent simulation platform like "Star Logo TNG" developed by MIT and distributed under the MIT License.
Finally, the open source nature of Python allowed the use of the Visual Module (Python 2.7.6) that hugely simplified the visualisation process as many multi-agent simulation algorithms quite readily are implementable in Python.

This model is based on some general observations that we see in our day to day life, such as while traveling in metros, observing a situition where
people are running in all different directions,or observing the gathering of a crowd(such as if a celebrety/public figure comes).

In the very first equation that we made is on the basis of basic fundamental concept that crowd can be represent as a form of set, this is because set is a collection of well defined objects, similarly in a group of people (crowd) all the members are well defined with respect to some parameter such as diatance form a particular object, rate of change of their position etc, or they can be defined by some other parameter.

The basic parameter that we take is the response potential, the idea behind taking this as a parameter is that we can predict the response

of a member of a crowd or simply the ability to response .
Then using this parameter we tried to make a funtion which describe the crowd, for wich we take onemore parameter is separation parameter.

Then we observe **different types of crowd** such as in [3.] to [8.] and beside this we also observe the crowd of a market, metro etc. And we obseve a common pattern that the overall movement of the crowd or the movement of a single member of a crowd is inversly proportional to the Inertia factor(.........).

$$S_i \propto \frac{dP_i}{dt}$$

so by observing the basic relation we made the 1st crowd eqation which is.....

$$s(f(\eta), r) = K(\frac{dP_i}{dt})$$

.

.

.Then for **alarm equation** we try to observe the effect i.e movevemt of crowd with repect to time, and we find that if the total time available for escaping from a particular situation is small then their movement is fast and vice versa., so on this basic fact of observation we tried to develop a eqation by relating the crowd equation and the dependence of time.

The idea that we got to develop the crowd density equation is mainly from the metro station, where people have to stand behind a particuar line before the metro train come. After the train come we saw that no. Of people passing/crossing the line is directrly depend upon the speed of people at which rate they are passing the line, and no of people per unit area, and using the factor we made the equations:

$$\frac{dN}{dt} = \alpha \bar{v} \frac{d}{dt} (\rho_A)$$

$$\rho'_A(\epsilon) = W\left(1 + \frac{(\varphi(\epsilon))}{(\psi(\epsilon))}\right) = W.R(\epsilon)$$

For crowd interaction the idea comes from by observing interview of different people,observing the gossip that we generaly made with our friends in day to day life.
The above case are govern by the psychological state of the members who are participating in the interaction, since there are lots of psychological parameters on which the inteaction depeds such as anger , clam ness, agrresion, ability to speak polietly, frankness etc. but we take only two parameters that are more general as compare to the other parameters i.e attractive nature of individual member, and repulsive nature of the same.
The interactions between the members can now be modelled using these 2 parameters with reasonable accuracy.

Crowd

A crowd is defined in general as a set of n-living objects (called members of the crowd) in a given region of space-time such that each member has a response potential(S) associated with it. For a given member, the response potential is a scalar that quantifies the effect of a given situation on that member.

In the most general form, a crowd is written as a finite set

$$C=\{M_1[S_1], M_2[S_2], ..., M_n[S_n]\}$$

where S is the response potential for a given member. The response potential is the value of a more general multivariate function called the Response Function and is given as

$s(D,r)=|S|$ and for the i^{th} member, $s(D,r)=|S|=S_i$

Here, 'D' is called the Defining Parameter and 'r' is called the Separation Parameter.

Conjecture 1: The response Potential is directly proportional to the change in position value of a given member w.r.t time. Mathematically,

$$S_i \propto \frac{dP_i}{dt}$$

Hence,

$$S_i = K\left(\frac{dP_i}{dt}\right)$$

this is the Crowd Equation

The Crowd Equation is a first order Ordinary Differential Equation. The constant of proportionality K is called the Crowd Inertia and is constant for a given crowd and its value is same for all members of crowd (this assumption holds for ideal crowds).
A more general form of the crowd equation can be obtained if we have

$$D = f(x, y, , , ,) = f(\eta)$$

This implies

$$s(D, r) = s(f(\eta), r)$$

Hence,

$$s(f(\eta), r) = K\left(\frac{dP_i}{dt}\right)$$ this is the generalized crowd equation

The above equation applies to any situation for which a Response Function can be written.
The crowd inertia is defined as "the property of a crowd by virtue of which it resists any change in the position of its members w.r.t time".

Conjecture 2: A crowd with a finite value of crowd inertia is affected by an external stimulus or situation that changes the position of its members according to a crowd equation derived from the generalized crowd equation.

We can write $\frac{dP_i}{dt} = \frac{S_i}{K}$

which implies that for a given situation corresponding to the response function s, a higher value of crowd inertia directly means that the position response (change in position of members due to the situation) of the crowd is low. In mathematical terms,

$$K=\frac{S_i}{\left(\frac{dP_i}{dt}\right)}$$

Classification of Crowds

To allow ease of study and analysis, a proper classification of crowds is required. We classify crowd on the basis of parameters defined by the variables in the crowd equation.

Static Crowd: A crowd in which the position of its members do not change with time is called static crowd. Example- audience
For a static crowd,

$$\frac{dP_i}{dt}=0$$

Therefore $S_i=0$

Also, a crowd is static if $K=\infty$ which is the condition for a rigid crowd which is a type of static crowd.
That is, rigid crowds are characterized by an infinite crowd inertia.

Dynamic Crowd: A crowd in which the position of its members change with time is called a dynamic crowd. Example- pedestrians on a street
For a dynamic crowd,

$$\frac{dP_i}{dt}\neq 0$$

Single-level Crowd: A crowd extended in 2 or less spatial dimensions is called a single-level crowd.
Example- crowd gathered on a plane, a queue (one dimensional crowd)

Multilevel Crowd: A crowd extended in more that 2 dimensions are called multilevel crowd. Example- crowd in a whole building: on different floors.

Conjecture 3: Any multilevel crowd can be expressed as a collection of single-level crowds.

Mob (aggressive crowd): A crowd which has a large value of response potential and crowd inertia is called an aggressive crowd or Mob.

$S_i \gg 0 \wedge K \gg 0$ This implies $\frac{dP_i}{dt}$ has a low value close to one.

Characteristics of a Crowd

A close observation of human crowds: both physically and mathematically reveals some of their peculiar features. We briefly discuss them here.

(a) Existence of sub-crowds
We have defined a crowd C as a set of members with each member having a definite response potential. Now,
any finite subset of C is called a sub-crowd.

i.e. $A \subset C$ defines a sub-crowd

(b) Formation of Queues

$\forall c_1, c_2, \ldots.., c_n \in C$ we define a Queuing Set Q as

$$Q = \{c_1, c_{2,} \ldots.., c_n\}$$

If T is a relation such that $c_a T c_b$ implies that a queue is formed between c_a and c_b. We can write

$$c_a T c_b = T(c_a, c_b)$$

here T is called a Queuing Relation.
The complete Queuing Relation between 4 members(say) is thus

$$T = \{(c_a, c_b), (c_b, c_c), (c_c, c_d)\}$$

We define a Queue as a discrete structure $q(Q, T)$

It is evident from sections (a) and (b) that a Queuing Set can be treated as a sub-crowd and a queue 'q' is an ordered sub-crowd. We hence arrive at our 4th conjecture.

Conjecture 4: A queue is an ordered subset of a crowd.

It follows that a Queuing Set and a Sub-Crowd are essentially equivalent i.e.

$$Q \equiv A$$

This is called the **General Crowd Equivalence.**

(c) Effect of a situation on a crowd

In our current terminology, a situation maybe defined as something with a definite position that can affect the crowd generally by altering the position of its members. The existence of a situation is defined by a non zero value of the response function(s).

A situation can alter the crowd in a large variety of ways as can be deduced from the crowd equation.

Conjecture 5: A response function is uniquely determined for unique values of the Defining Parameter D and the Separation Parameter r

Conjecture 5 implies that the uniqueness of situation depends upon the values of D and r. The response function is a function of D and r. D is also a multivariate function as mentioned earlier.

The separation parameter for a particular member determine the proximity of the situation to that member. Clearly, for the i^{th} member,

$$r_i = r_S - P_i$$

where the terms in the RHS denote the positions of the situation and the member respectively.

By writing different forms of the response function in terms of D and r, we can calculate (after solving the crowd equation for the given response function) the change in position of members of the crowd.

In this paper, we consider the most basic solution of the crowd equation where

$$D \equiv D(t) \quad \text{and} \quad r \equiv r(t)$$

The crowd equation now becomes hugely simplified,

$$s(D(t), r(t)) = K\left(\frac{dP_i}{dt}\right) \quad \text{(equation for a Simple Crowd)}$$

Hence, the solution is

$$P_i = \frac{1}{k}\left(\int s(D(t), r(t))\,dt\right)$$

The Crowd Equation can be solved for more complicated situations for which the integral on the RHS may extend to multiple spatial dimensions and the solution of which may require numerical methods.

(I).<u>The General Crowd Equations</u>

$$\frac{dP}{dt} = \frac{1}{K} s(f(\eta, r)) \qquad (i)$$

$$\frac{d}{(d\epsilon)}(\rho) = W\left(1 + \frac{(\varphi + \Phi)}{\psi}\right) \qquad (ii)$$

Equation (i) is the Position Response Equation that describes the variation of position value w.r.t time

Equation (ii) is the Density Response Equation that describes the variation of crowd density.

Both the equations are in a very generalised form and are in true sense the crux of the whole project.

(II).Scalar Crowd Model(SCM)

The SCM is a mathematical model that is useful in explaining crowd behavior using standard mathematical tools.
It comprises of the following assumptions:

1. All members of have well defined positions on the cartesian plane.
2. Members move in the cartesian plane only i.e 2D motion
3. Each member follows a **movement function** in the plane.
4. The movement functions maybe continuous or discontinuous
5. Every member's x-coordinate varies with a step size "h" in the initial case (since in presence of a situation, h may vary)

The SCM can be used to derive both the general crowd equations.

(III).Solutions of Eq.(i)

Equation (i) can be used with different situations to calculate the position response of a crowd in the form of a modified movement function.

While solving equations of type(i), it is observed that

"Movement Functions are the solutions of Eq.(i) under various conditions".

Eq.(i) is solved analytically mainly using separation of variables for 1^{st} order ODEs though any other standard method can be used.

Some forms of Eq.(i) studied in this project are:

$$\frac{dP}{dt}=\frac{1}{(T-t)^2}\sqrt{((x-m)^2+(y-n)^2)}$$ **Alarm Equation**

This equation explains crowd behavior under a scenario where members must reach a certain location before passage total time T.

$$\frac{dP}{dt}=\frac{1}{K}\frac{a}{(\sqrt{(x^2+y^2)}t)}$$ **Gathering Equation**

This equation explains gathering and converegence of crowds due to a situation with a definite location in the cartesian plane.

Simulations of the above 2 scenarios are done by numerically solving the above 2 equations using classic 4^{th} Order Runge-Kutta Method implemented in C++(C++11 Standard).
The results are comparable to the analytical results in both cases.

If observed properly, both the scenarios have opposite nature.

Equation	**Nature**	**Solution Curve**
Alarm Equation	Diverging	exponential
Gathering Equation	Converging	logarithmic

The analytical and numerical results agree to this.

(IV).Crowd Density and its variation

Crowd density is defined simply as the number of members in a given region.

The crowd density can be represented as a density function. This can be calculated form Eq.(ii)

Eq.(ii) also enables us to directly study the variation of crowd density in the presence of stressors. But, another form of Eq.(ii) can be used if we confine ourselves to a given region 'A' in the plane. Then the density response in that region can be shown to be given by

$$\frac{dN}{dt} = \alpha \bar{v} \frac{d}{dt} (\rho_A)$$

where N is the Number Density i.e the members passing through a 1D region surrounded by A per unit time.
To calculate density responses under more complicated scenarios, stressors need to be used.

By definition and in our model, "***a stressor is anything that can induce mental stress in members directly or indirectly in a given region which CAN change the overall structure of the crowd".***
Or,
"A stressor is an object that can change the psychological state of a member at any instant directly or indirectly".

All the psychological states of members of a crowd and any stress in the cartesian plane can be defined using stressors. SCM uses 3 types of stressors:

1.Applied Stressor- defines stress produced by absolute presence of situation
2. Percieved Stressor-defines stress induced by the appplied stress
3.Interaction Stressor-defines stress produced by interaction of 2 or more members with each other (this is a type of applied stress).

The total stress is the sum of the three stresses.

The stressors are representable as functions that can be time-dependent,time-independent,spatially varying etc.

The stressor formulation allows us to derive the following relation for non-interacting crowds

$$R = 1 + \frac{\varphi}{\psi}$$

where R is the resistance to stress. This equation is the general equation of any Adaptation Syndrome Model that includes a resistance component like R.
For time- dependent stressors, models similar to GAS have the following form

$$R(t) = a + Bt^{(m_1 - m_2)}$$

where m1 and m2 are real numbers.
This result directly follows from the solutions of Eq.(ii)

(V)Solutions of Eq.(ii)- Crowd Density Simulations

In this project, two types of simulations are described

1. Position Response Simulations or PRS

2. Density Response Simulations or DRS

Both these simulations are done numerically using equations (i) and (ii) respectively.
PRS has been described in Section (III).

For DRS, we require to solve eq.(ii) to get the density function in terms of the parameters. This can be done by doing the following integral

$$\rho = W \int \left(\frac{(\sigma(\epsilon))}{(\psi(\epsilon))} d\epsilon \right)$$

This can be solved numerically using standard numerical integration techniques like Simpson's Method, Trapezoid Method etc.

The stressor formulation gives us a serious advantage to perform a DRS simply by defining the stressors as functions to calculate the density function for a large number of scenarios for interacting and non-interacting crowds.

For interacting crowds, we have

$$R = \left(1 + \frac{(\varphi + \Phi)}{\psi} \right)$$

Interaction between 2 members can create applied stressor(s) in a region thereby modifying the crowd density.

Only the interactions between members of same crowd are considered in this project.

The following features are observed:

(a) Crowd Interactions can change overall crowd behavior only if the interaction value is large (large +ve or large -ve). This is seen by the following relation

$$i_{(M_1 M_2)} \propto \Phi$$

$$i_{(M_1, M_2)} = E\,\Phi$$

(b) Crowd Interactions can increase or decrease crowd density in a region

(c) Interactions are governed by the psychological parameters- $\tilde{a} \wedge \tilde{r}$ which define attractiveness and repulsiveness respectively of a member.

(d) The interaction value(which defines quantitatively the amount of interaction) between 2 members is

$$I_{(M_1, M_2)} = (\tilde{a}_1 - \tilde{r}_1) + (\tilde{a}_2 - \tilde{r}_2)$$

(e) The interaction values follow superposition principle i.e

$$\tilde{I}_1 = \sum_{j=2}^{k} i_{(M_1, M_j)}$$

for k members i.e $M_{2,.} \ldots, M_k$ in the interaction region of M_1

(f) A given member can interact with only those members that lie in a well defined circular area around that member (called its **interaction region**). The radius of the region is the **interaction radius.** Interaction regions of members can overlap.

The following pages describe the calculations and the procedures/ methods used to arrive at the results. This is done in order to make the sequence and thought process of the mathematical work more clear.

(Note:- This part describes the main calculatons only)

Scalar Crowd Model (SCM)

The SCM is a scalar model for explaining and analyzing crowds geometrically using the cartesian coordinate system.
This model is defined by the following points-

1. All members of have well defined positions on the cartesian plane.
2. Members move in the cartesian plane only i.e 2D motion
3. Each member follows a **movement function** of the form
   y=f1(t) and x=f2(t)
4. The movement functions maybe continuous or discontinuous
5. Every member moves with a step size of "h" units along x-axis in the initial case (since in presence of a situation, h may vary)

The movement function of a member is defined in the general form as

$$[mov_{((x),(y))}]=[x(t),y(t)]$$

This is the time-dependent form (or 1st Form) of the movement function.
The time independent form (or 2nd Form) is a regular function

$$y=f(x)$$

Since the time variable has linear variation, the form y=f(x) implies that the movement function is linear in X-direction and follows y=f(x)=f(t) in Y-direction.

Moreover if we assume that $\Delta x=\Delta t=h$ i.e $dx=dt$. Therefore,

$$x\equiv t$$

This is the **SCM Equivalence** and is useful in solving complicated crowd equations by reduction.
Note that SCM equivalence works only when the movement function along X-axis is linear in time.

Effect of situation(s)
Now we consider quantitatively the effect of a situation on the crowd.
We make the following assertion:

Assertion 1: "***The response of a member to a siuation is observable as a change in its movement function".***

We will first calculate the position response formula for finite intervals,then use it to analyse the change in movement function.
Consider the postion of a member is given at any instant as

$$P=\sqrt{(x^2+y^2)}$$

$$\frac{dP}{dt}=\frac{x}{(\sqrt{(x^2+y^2)})}\frac{dx}{dt}+\frac{y}{(\sqrt{(x^2+y^2)})}\frac{dy}{dt}$$

$$\frac{dP}{dt}=\frac{1}{(\sqrt{(x^2+y^2)})}(x\frac{dx}{dt}+y\frac{dy}{dt})$$

For finite intervals,

$$\frac{(\Delta P)}{(\Delta t)}=\frac{1}{(\sqrt{(x^2+y^2)})}(x\frac{(\Delta x)}{(\Delta t)}+y\frac{(\Delta y)}{(\Delta t)})$$

This is the **Position Response Formula** (PRF) for finite intervals. For infinitesimal intervals, the PRF is given by the crowd equation.

Now consider the initial case (in absence of a situation). The movement function is of the 2nd form

$$y=f(x)=x+7$$

$$\frac{dy}{dt}=\frac{dx}{dt}$$

The points are: (x,y)=(0,7),(1,8),(2,9),(3,10)
Using the PRF along with SCM equivalence, we get

$$\frac{(\Delta P)}{(\Delta t)}=\frac{(x+y)}{\sqrt{(x^2+y^2)}}$$

For different values of x and y,

$$(\frac{(\Delta P)}{(\Delta t)})_1\approx 1$$

Under the action of a situation, let the movement function be modified as

$$Y=f_2(x)=3\mathrm{x}+7$$

$$\frac{dY}{dt}=3\frac{dx}{dt}$$

Using the PRF along with SCM Equivalence,we get

$$\left(\frac{(\Delta P)}{(\Delta t)}\right)_2 = \frac{(x+3Y)}{\sqrt{(x^2+Y^2)}}$$

For different x and Y values,

$$\left(\frac{(\Delta P)}{(\Delta t)}\right)_2 \approx 3$$

Hence in this example,the rate of change of position of crowd members becomes thrice under the action of the situation.

Hence, for a general movement function of the form

$y = f(x) = mx + c$ ***(Linear Movement),***

$$\left(\frac{(\Delta P)}{(\Delta t)}\right)_2 \approx m$$

Note that the mathematical expression of the situation is still not available. We will define that in the general crowd equation and solve the ODE of the crowd analytically to examine the solutions.

We can observe the effect of the situation on the movement function directly from the crowd equation also. We will now do this derivation in a formal way.

The general crowd equation is

$$\frac{dP}{dt} = \frac{1}{K} s(f(\eta), r)$$

Where 's' defines the response function and K is the Crowd Inertia.

In absence of a situation, let the crowd be expressed by the equation

$$\frac{dP}{dt} = \frac{1}{K} \frac{[ax+by]}{\sqrt{(x^2+y^2)}}$$

clearly this is a simple Dynamic Crowd.
Replacing the LHS using the PRF, we have

$$x\frac{dx}{dt}+y\frac{dy}{dt}=\frac{(ax+by)}{K}$$

Solving this equation directly using separation of variables, we yield the following solutions

$$x(t)=at+c_1$$
$$y(t)=bt+c_2$$

These solutions are movement functions
Hence, ***movement functions are unique solutions of the crowd equation under given conditions***

Therefore, the effect of the situation must be seen in the solutions of the crowd equation when the situation is included.
Consider a situation defined by

$$s_2(f(\eta),r)=\frac{1}{(T-t)^2}\sqrt{(x^2+y^2)}$$

The crowd equation is thus

$$\frac{dP}{dt}=\frac{1}{K}[\frac{1}{(T-t)^2}\sqrt{(x^2+y^2)}] \qquad (1)$$

In general for total time T ,the crowd equation of form

$$\frac{dP}{dt}=\frac{1}{(T-t)^2}\sqrt{((x-m)^2+(y-n)^2)}$$

is called the **Alarm Response Equation.**

We will now solve this equation directly. Equaton (1) can be written as

$$x\frac{dx}{dt}+y\frac{dy}{dt}=\frac{(x^2+y^2)}{(K(T-t)^2)}$$

Solving this equation yields

$$x(t)=\exp\left(\frac{-1}{(K(T-t))}+c_1\right) \quad y(t)=\exp\left(\frac{-1}{(K(T-t))}+c_2\right)$$

This are also movement functions of 1st Form

Therefore, Assertion 1 is clearly justified

Other forms of the crowd equation

Equation (1) can be written in a compact and general form as

$$\frac{dP}{dt}=\frac{1}{K}F(x,y,t)$$

Also using the SCM Equivalence,

$$\frac{dP}{dt}=\frac{1}{K}F(y,t)$$

This is the reduced Crowd Equation and is very useful in Crowd Simulations and Numerical Computation using the Scalar Crowd Model.

Numerical Solutions of Crowd Equation

In this part, we consider the numerical solutions of the following crowd equations:

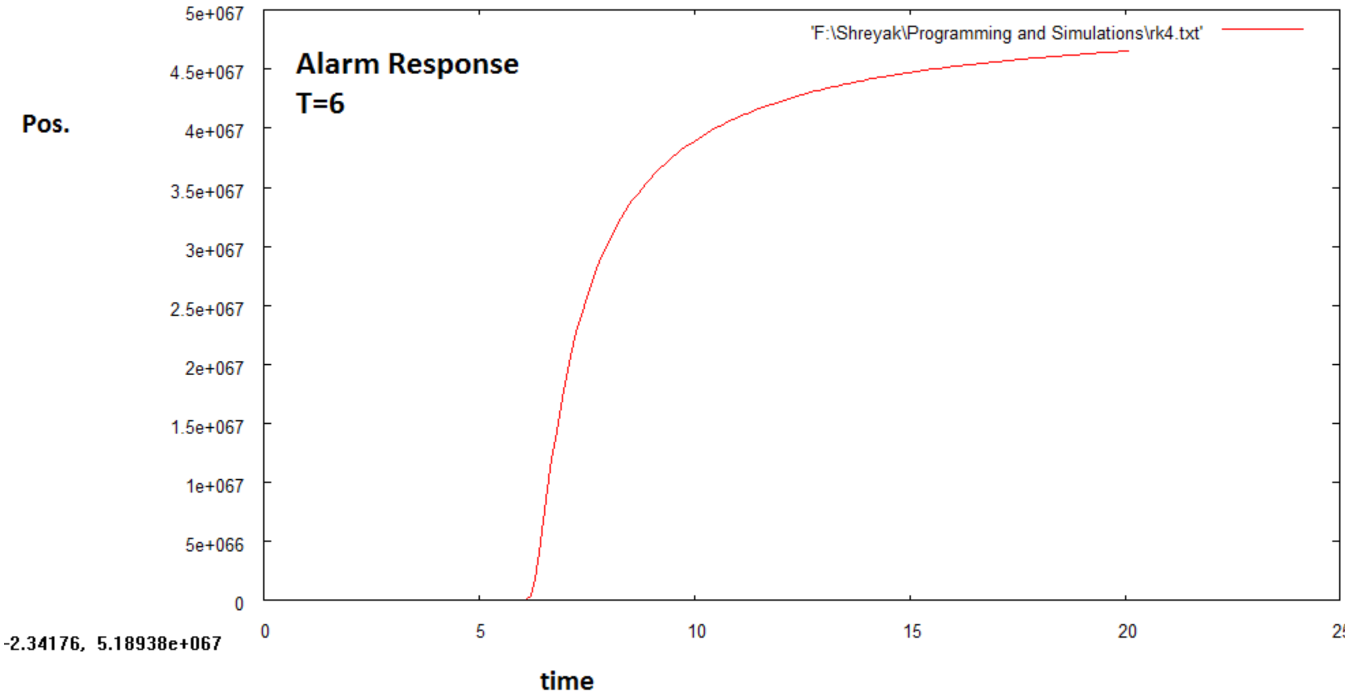
Alarm Response Equation : $$\frac{dP}{dt}=\frac{1}{K}[\frac{1}{(T-t)^2}\sqrt{(x^2+y^2)}]$$

Gathering Equation: $$\frac{dP}{dt}=\frac{1}{K}\frac{a}{(\sqrt{(x^2+y^2)}t)}$$

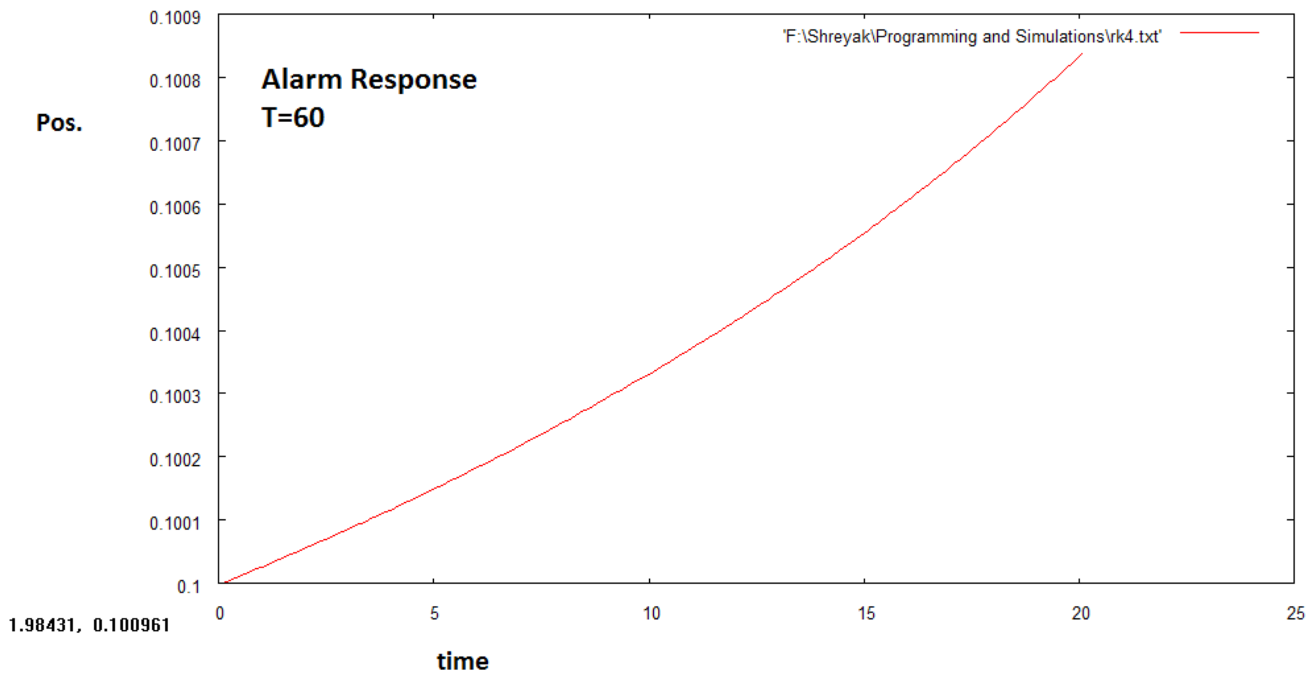
The algorithm used will be the classic 4[th] Order Runge-Kutta (RK) Method with the following attributes-

Implementation Language: C++11
Compiler : GCC 4.8.0 (open source)
IDE: MinGW Developer Studio
Plotter: GNUPlot (open source)

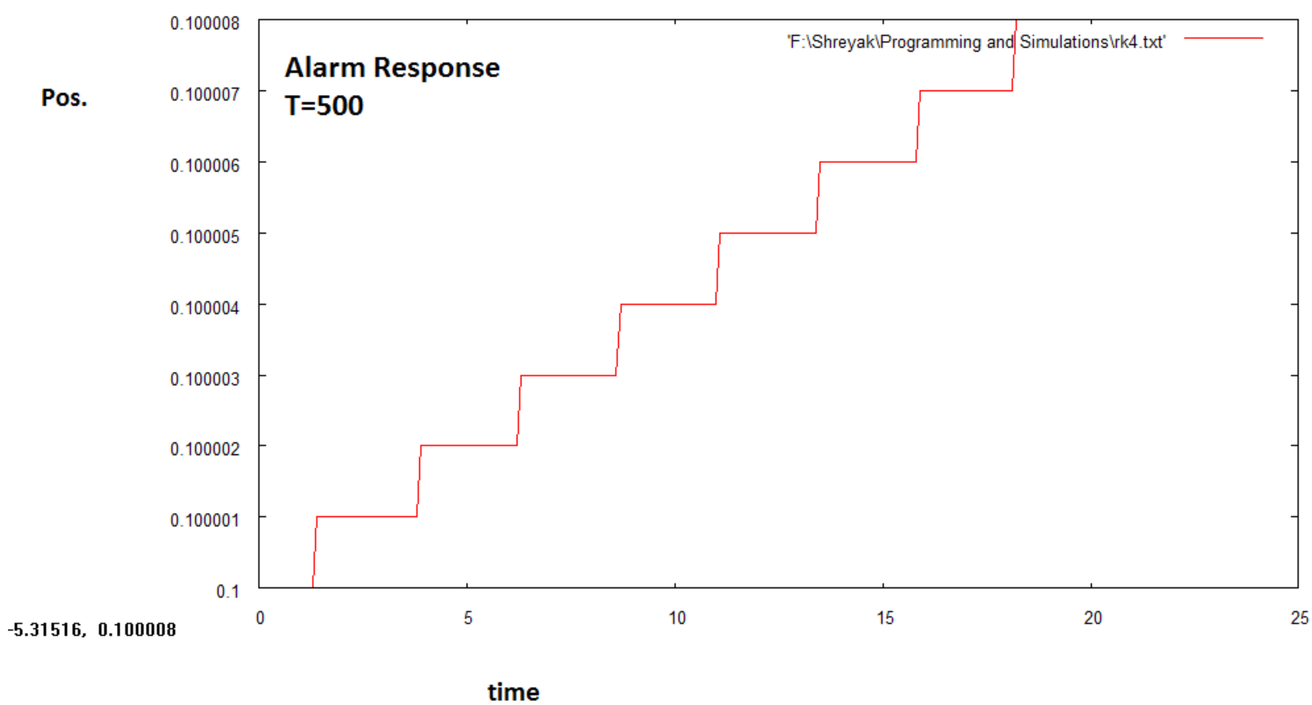
The graphs are plotted for different values of Total Time (T).



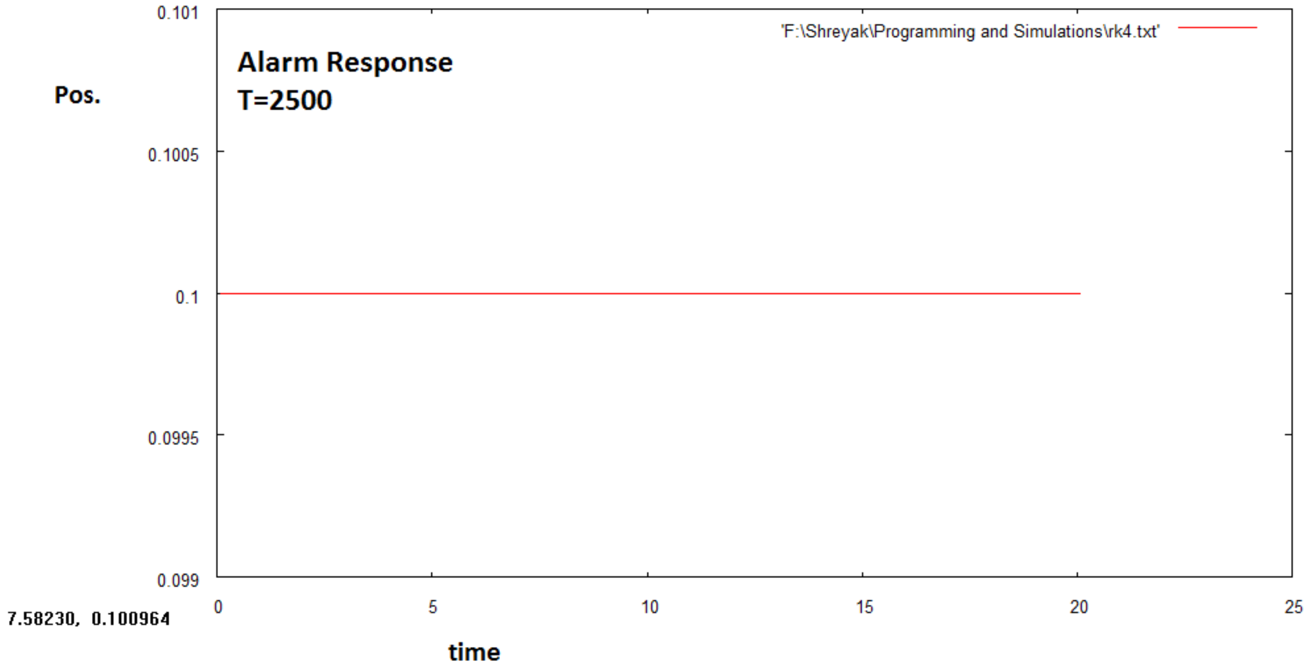
Alarm Response
T=6
Pos.
'F:\Shreyak\Programming and Simulations\rk4.txt'
5e+067
4.5e+067
4e+067
3.5e+067
3e+067
2.5e+067
2e+067
1.5e+067
1e+067
5e+066
0
0
5
10
15
20
25
-2.34176, 5.18938e+067
time



Pos.
0.1009
0.1008
0.1007
0.1006
0.1005
0.1004
0.1003
0.1002
0.1001
0.1
1.98431, 0.100961
0
5
10
15
20
25
time
'F:\Shreyak\Programming and Simulations\rk4.txt'
Alarm Response
T=60



Pos.
0.100008
0.100007
0.100006
0.100005
0.100004
0.100003
0.100002
0.100001
0.1
-5.31516, 0.100008
0
5
10
15
20
25
time
'F:\Shreyak\Programming and Simulations\rk4.txt'
Alarm Response
T=500



<u>The Gathering Equation</u>

The *Gathering Scenario* can be considered to be the reverse of the *Alarm Scenario*.

Here, a member located at O(0,0) attracts a number of people to gather around itself in a way that a definite structural formarion is seen in the crowd.

Mathematically, according to SCM

$$S \propto \frac{1}{\sqrt{(x^2+y^2)}}, S \propto \frac{1}{bt}$$

$$S(x,y,t) = \frac{a}{(\sqrt{(x^2+y^2)}t)}, where \frac{1}{b} = a$$

Hence,

$$\frac{dP}{dt}=\frac{1}{K}\frac{a}{\left(\sqrt{\left(x^2+y^2\right)}t\right)}$$

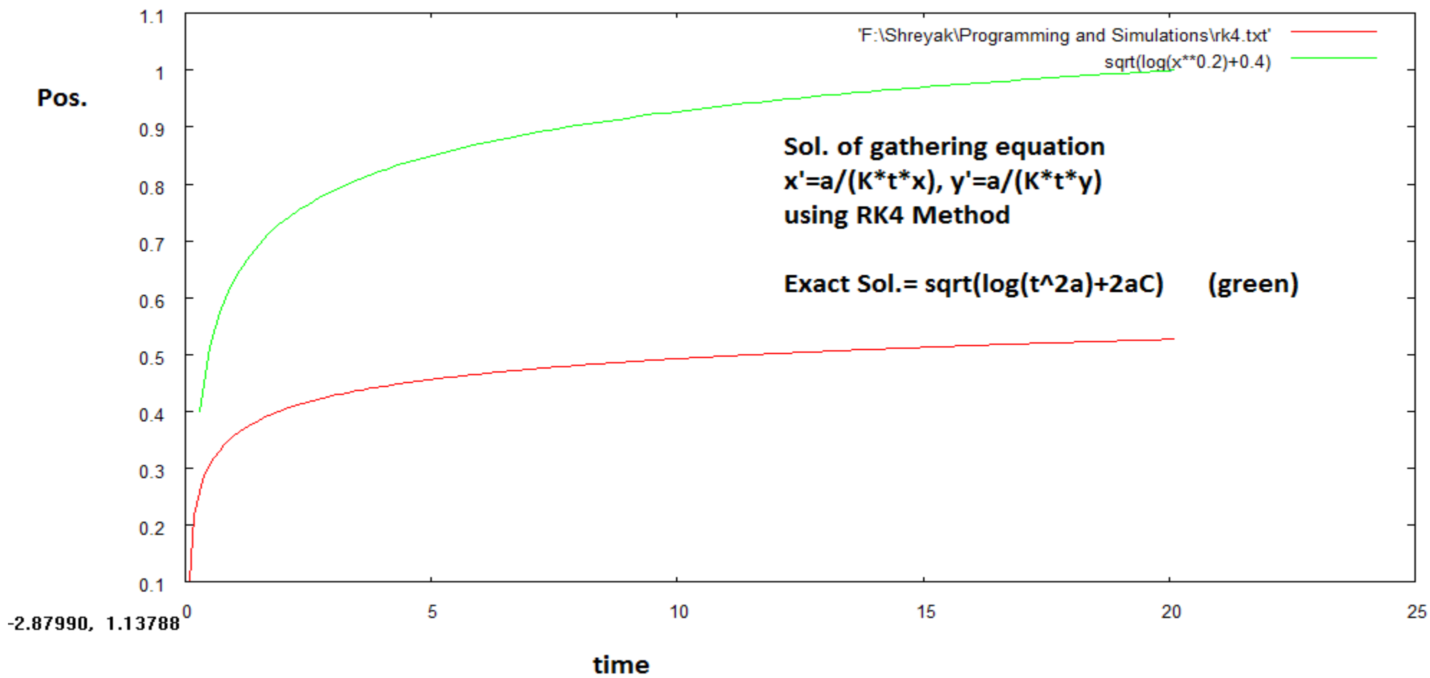
this is the Gathering Equation.
It can be solved analytically by considering

$$x\frac{dx}{dt}+y\frac{dy}{dt}=\frac{a}{Kt}=\frac{\left(a_1+a_2\right)}{Kt} \qquad (1)$$

The solutions of which are the folowing movement functions

$$x=\sqrt{\left(\log\left(t^{(2a_1/K)}\right)+2\lambda_1\right)}$$
$$y=\sqrt{\left(\log\left(t^{(2a_2/K)}\right)+2\lambda_2\right)}$$

Equation (1) can also be solved numerically using RK4 Method.
The following result is generated-



From the above graph, it can be observed that the gathering equation generates a logarithmic plot while the result generated by alarm response equation is overall exponential (in graph with T=6, the graph is to be read in reverse since the member that gathers other members exists at (0,0)).

This clearly indicates the reverse nature of the two scenarios as mentioned earlier.

We now define some terms.

Crowd Density- It is defined as the number of members(%mu) in a region(%tau).

$$\rho_\tau = \frac{\mu}{\tau}$$

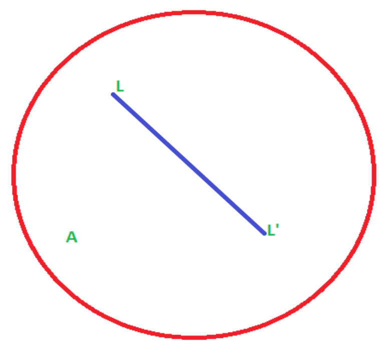
<u>Number Density</u>- It is defined as the number of members crossing a region(1D) in a given time. Denoted by N

$$N = \frac{M}{T}$$

M is the no. Of members passing through the 1D region.
We now try to find the relation between number density and crowd density of a crowd.

Consider a 1D region LL' surrounded by a region of area A. Clearly,

$$\rho_A = \frac{\mu_A}{A}$$



By the definition of M,

$$M \propto \bar{v}$$
$$M \propto \rho_A$$

where $\bar{v} = \frac{(v_1 + v_2 + \ldots.. + v_n)}{n}$

$$M = \lambda \bar{v} \rho_A \Rightarrow N = \frac{(\lambda \bar{v} \rho_A)}{T}$$

Therefore, $\frac{dN}{dt} = (\lambda \frac{\bar{v}}{T}) \frac{d}{dt} (\rho_A)$ for a scalar %lambda

Assuming, $\alpha = \frac{\lambda}{T}$

$$\frac{dN}{dt} = \alpha \bar{v} \frac{d}{dt} (\rho_A)$$

this is the variation of Number Density.
It can be noted that variations in number density and crowd density are related by a factor known as scale factor as clearly visible in above equation. Using dimensional analysis, it can be shwn that %alpha indeed has the dimensions of length i.e.

$$[\frac{\lambda}{T}]=[M^0 L^1 T^0]=[L]=[\alpha]$$

Stressor formulation of SCM

In the regime of psychology and sociology, the term 'stressor' commonly refers to something that can induce stress in the members of a crowd.
In SCM, a stressor or a 'stress function' is a function which when defined in a region, its nonzero value changes the crowd density in that region.

Consider a region A in the cartesian plane. The crowd density is

$$\rho_A=\frac{\mu_A}{A}$$

A stress is applied in the plane (i.e. a stressor exists) that is time dependent. This applied stress induces stress in the members of crowd in the region A. This is called 'induced stress' or 'percieved stress'. We can write

Total Stress= Applied Stress + Induced Stress

Mathematically,

$$\sigma(\epsilon)=\varphi(\epsilon)+\psi(\epsilon)$$

where %sigma, %phi and %psi are all Stressors
The percieved stress can also be written as
$\psi(\epsilon)=k(I(\epsilon))^n$ for a scale factor k and intensity of stress I

According to SCM,

$$\rho'_A(\epsilon) \propto \sigma(\epsilon)$$

$$\rho'_A(\epsilon) \propto \frac{1}{(\psi(\epsilon))}$$

Hence,

$$\rho'_A(\epsilon)=W\left(1+\frac{(\varphi(\epsilon))}{(\psi(\epsilon))}\right)=W.R(\epsilon)$$

for a scalar W and Stress Resistance R.
this is the general form of the **Stress-Density Relation.**

For time-dependent stressors, the stress-density relation modifies to

$$\rho'_A(t)=W\left(1+\frac{(\varphi(t))}{(\psi(t))}\right)=W.R(t)$$

In the trivial case, the stress resistance R is

$$R=1+\frac{\varphi}{\psi}$$

Now, we can derive the General Adaptation Syndrome(GAS) Model from the Scalar Crowd Model(SCM).

Consider
$\varphi(t)=b_1 t \qquad \psi(t)=b_2 t^m$
Using the stress-density relation, we get

$$R(t)=1+\left(\frac{b_1}{b_2}\right)t^{(1-m)}=1+Bt^{(1-m)}$$

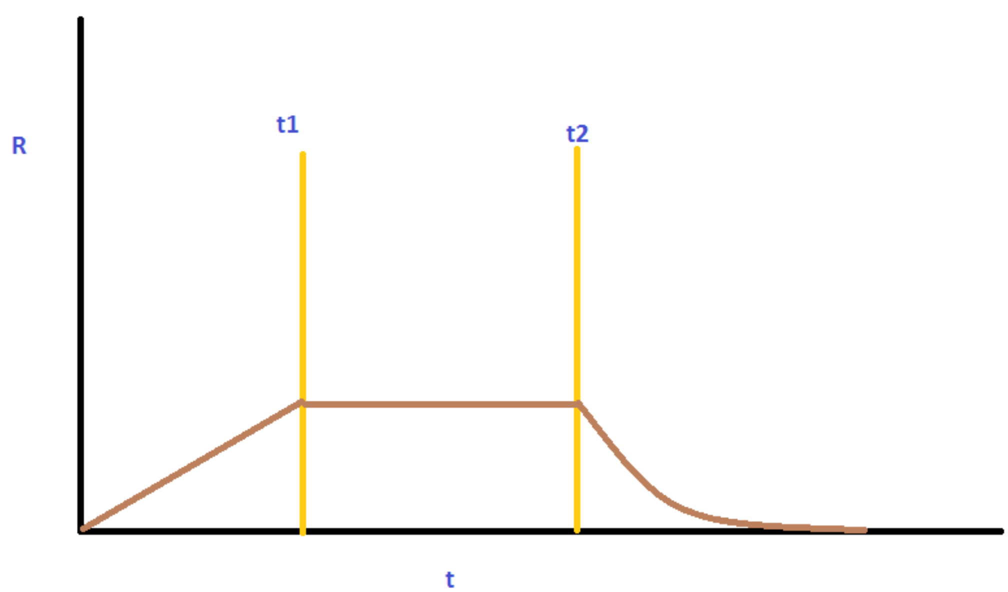
this gives the Resistance Function for the GAS Model by SCM. This can also be expressed for different values of m as

$$R(t)=\begin{matrix} 1+Bt & ,m=0 & ,0<t<t_1 \\ 1+B & ,m=1 & ,t_1<t<t_2 \\ 1+Bt^{(1-m)} & ,m>1 & ,t_2<t \end{matrix}$$

In a more general form, for Adpatation Models with time-dependent stressors,

$$R(t)=a+Bt^{(m_1-m_2)}$$

This when plotted generates the rcognizable graph from Standard GAS Models i.e of Stress Resistance (R) with time (t).



Crowd Interactions

In general, a crowd can be sudivided or partitioned into sub-crowds such that member of the same subcrowd shows identical behavior. This is important for Mulri-Agent Simulations.
The interactions between sub-crowds or 2 individual members of a crowd are called **intra-crowd interactions.**
Interactions between 2 or more crowd are called **inter-crowd interactions**(out of our current scope).

Mathematically, the interaction between 2 member is measured by a scalar quantity called interaction value denoted by 'i'.
According to SCM, a region exists around each member such that the member can interact with only those members of the crowd that lie in that region. Such a region is generally circular and defined as **interaction radius** or **interaction range**. Interaction radii of 2 members can be same or different.
Interaction radius exists for every member and is a general characterstic of a crowd.

For a member M1 with k members in its interaction range, the interaction for M1 is the sum

$$\tilde{I}=i_{(M_1 m_1)}+i_{(M_2 m_2)}+....+i_{(M_1 m_k)}$$

Or,

$$\tilde{I}=\sum_{j=1}^{k} i_{(M_n m_j)}$$

The total interacton for the crowd of n-members is

$$\tilde{I}=\tilde{I}_1+\tilde{I}_2+...+\tilde{I}_n$$

We now use the stressor formulation to derive few results related to crowd interactions.

We know that under applied stress %sigma and percieved stress %psi, the density response is given by

$$\frac{d}{dt}(\rho)=W\frac{(\sigma(t))}{(\psi(t))}$$

Stress-density relation in above form is important to study crowd interactions.
Since,

$$R(t)=\frac{(\sigma(t))}{(\psi(t))} \quad \text{Therefore,} \quad \begin{array}{c} \sigma(t)=R(t).\,\psi(t) \\ |\sigma(t)|\propto|\psi(t)| \end{array}$$

This is the **stress proportionality** in SCM.

For a more complete description, we consider a given interaction value 'i'.
If i<0, the interaction is unfriendly.
If i>0, the interacion is friendly.

If the value of i is very large (large +ve or large -ve), only then there is n appreciable change in crowd density due to the interaction(s).
We can show that mathematically using a parameterization.

Let us assume that 2 parameters are associated with every member.
We dub these as **Psychological Paramters** since they define the external psychology of the member at a given instant.
The parameters are:

$\tilde{a}$ - attracting or attractive parameter
(tends to increase crowd density)

$\tilde{r}$ - repelling or repulsive parameter
(tends to decrease crowd density)

If a1,r1 and a2,r2 are the paramters of M1 and M2 resp. Then the interaction value between them is given as

$$I_{(M_1 M_2)} = (\tilde{a}_1 - \tilde{r}_1) + (\tilde{a}_2 - \tilde{r}_2)$$

Thus,

$$\tilde{I}_1 = \sum_{j=2}^{k} i_{(M_1 M_j)}$$

for k members i.e $M_{2,..} \ldots, M_k$ in the interaction region of M_1

Hence-
"The interaction value follows the superposition principle".

The interaction between members also produces an applied stress called the **Interaction Stress** which is proportional to the interaction value.

$$i_{(M_1 M_2)} \propto \Phi$$

$$i_{(M_1 M_2)} = E\Phi$$

where E is a constant that depends on the surrounding environment.

Thus the total stress in general is
$$\sigma = \varphi + \psi + \Phi = (\varphi + \Phi) + \psi$$

Therefore, the stress-density relation can be written in a more general form as

$$\frac{d}{dt}(\rho) = W\left(1 + \frac{(\varphi + \Phi)}{\psi}\right)$$

The above equation is applicable to any crowd (interacting or non-interacting).

For interacting crowds, $\Phi \neq 0$
For non-interacting crowds, $\Phi = 0$

<u>Numerical Simulation of Crowds using Stressor formulation of Scalar Crowd Model</u>

The scalar crowd model allows to calculate both the desnity response and the position response of a crowd.

Using stressors (applied, percieved and interaction), we can numerically compute the density response of a crowd using the integral.

$$\rho = W \int \left(\frac{(\sigma(\epsilon))}{(\psi(\epsilon))} d\epsilon\right)$$

This can be solved numerically using the approximation:

$$\int_a^b f(x)\,dx \approx \sum_a^b f(x)\,\Delta x$$

Other numerical methods like Newton-Cotes, Trapezoid, Simpson's can also be used to solve the integral numerically.

For a member with movement function $y = f(x)$, SCM can be used to simulate movement of members using the foolowing algorithm-

```
Start:

FUNCTION f(var x)
{}
FUNCTION ends

dx=dt=h
INPUT h

LOOP over t:
x=x+dx
y=f(x)
OUTPUT x,y
t=t+dt
LOOP ends

End
```

This is the **SCM Algorithm for Position Response Simulation**

To numerically compute the density response using SCM, the following algorithm is used

```
Start:
// It is assumed that the stressors are time dependent only

INPUT rho
dx=dt=h

FUNCTION applied_stress(var t)
FUNCTION ends

FUNCTION percieved_stress(var t)
FUNCTION ends

FUNCTION interaction_stress(var t)
FUNCTION ends

LOOP over t FROM t=0 TO t=T

F=W*(applied_stress(t)+percieved_stress(t)+interaction
stress(t))/percieved stress(t)

rho=rho+F*dx
OUTPUT rho

LOOP ends

End
```

This is the **SCM Algorithm for Density Response Simulation**

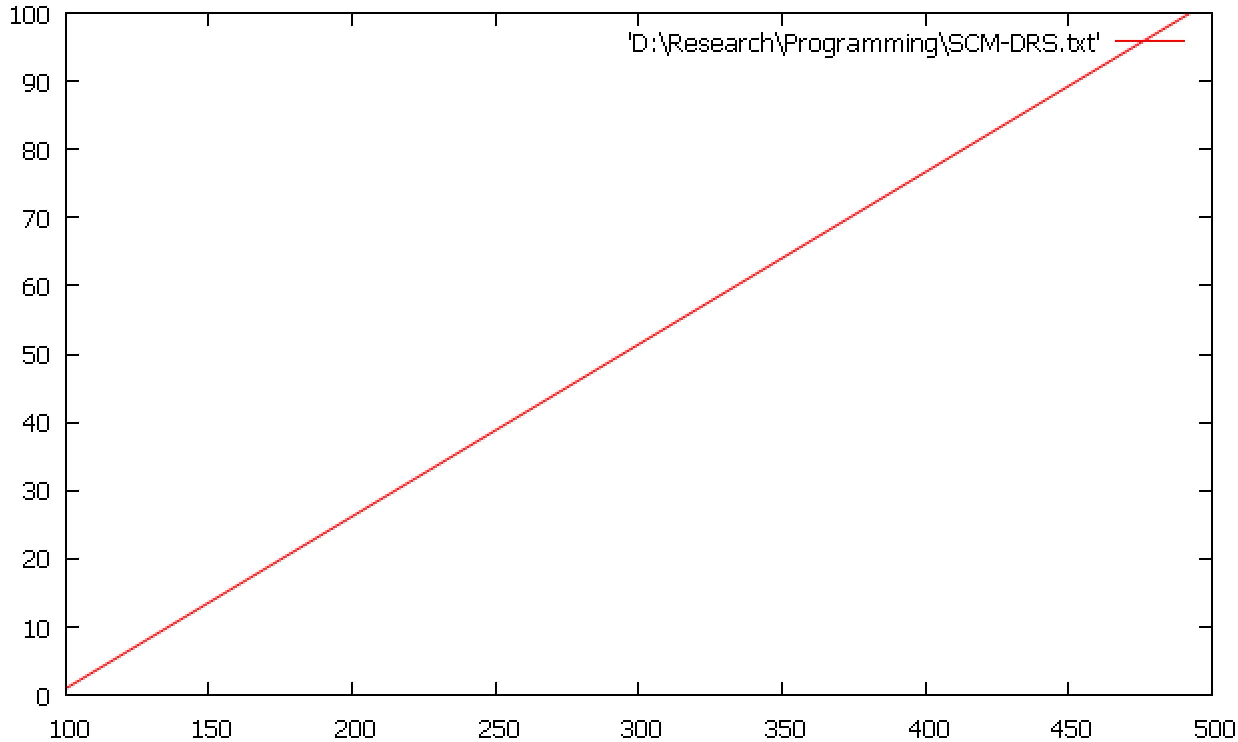
Two Density Response Simulations are done using the above algorithm's C++ implementation with following parameters

```
float AS(float t)
{
return (5*t);
}


float PS(float t)
{
return (2.5*t);   // This is because PS(t)=(1/R)*AS(t)
}

float IS(float t,float E)
{
return (E*(a-r)*t);
}
```

Parameters:
a=10
r=6
W=10
E=0.6
T=100
rho=100

$$\rho_1(t) = 10 \int \left(\frac{(5\mathrm{t} + 2.5\mathrm{t} + 2.4\mathrm{t})}{2.5\mathrm{t}} dt \right) = 3.96\mathrm{t}$$

The numerical solution is



which is clearly linear. This was obvious too since all the stressors are linear.

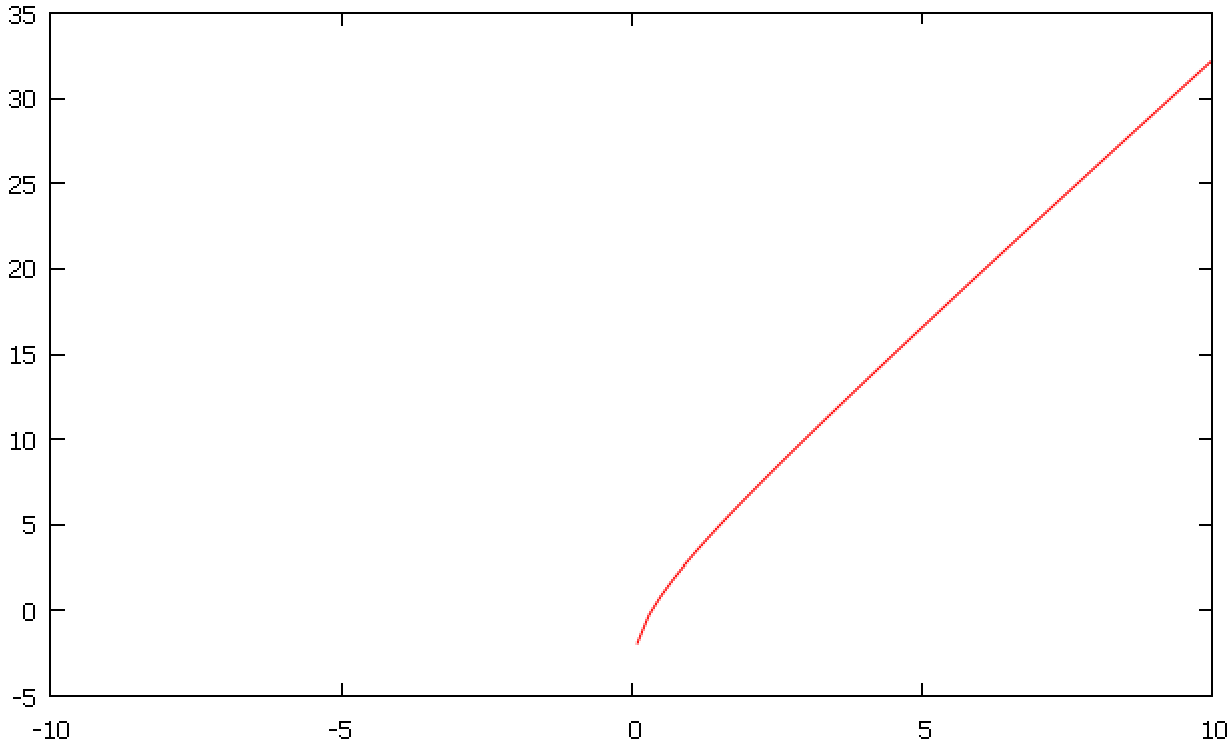
Next, if the Interaction Stressor is modified as

```
float IS(float t,float E)
{
return (E*(a-r));
}
```

We can now predict that the density function is

$$\rho_2(t) = 10 \int \left(\frac{(5t + 2.5t + 2.4)}{2.5t} dt \right) = 3t + 0.96 \log(t)$$

The numerical solution is



These algorithms can be extended to MAS with large number of agents/members by using a 1D-array of objects of type

```
struct Member
{
float x,y;  // Spatial Coordinates
float a,r;  //  Psychological Parameters
}
```

The array is declared as

```
Member all[1000];     //1001 Agents
```

The member at any index of the array can now be controlled

Conclusion

The relevance of this project lies in its ability to mathematically explain the general crowd behavior seen almost daily in any region of the world inhabited by humans. As a further step, studying crowd gives us an insight into collective human behavior and general psychology of humans and the society as a whole. A proper mathematical model for the same would hugely benifit us in managing large number of people in a systematic,well defined and reasonably predictable manner. The current progress in areas of mathematical sociology and mathematical psychology have incorporated crowd analysis at its roots.

With this project, an attempt has been made to develop and the areas considered to be largely non mathematical into a common mathematical framework resulting in quite accurate predictions about collective human behavior from the base of mathematical ideas. Furthur research on this project and its extension into biological and sociological regimes will be highly interesting and will enlighten us in our journey to unravel the secrets of human psychology.

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