Spin: Newton,Maxwell,Einstein,Dirac,Bell

In *'Quantum Spin and Local Reality'* (QSLR) I show that Bell suppressed key physical phenomena to arrive at his inequality. As a result Bell's conclusions are incorrect — his model fails to match reality. Bell's defense is based on quantum mechanical eigenvalue equations with reference to Dirac. I briefly review some issues in the history of spin, and analyze the non-relativistic Stern-Gerlach eigenvalue equation and the relativistic Dirac equation, and show their relevance to Bell.

As simple as possible, but no simpler.
Einstein

The Stern-Gerlach experiment, performed in 1922, was not a test of either Heisenberg's, Schrödinger's, or Dirac's quantum mechanics, which were three years in the future. Rather than *testing* quantum mechanics, the experiment served to *ground* quantum theory and tie it to spectroscopy and Zeeman shifts.

In 1964, when John Bell chose Stern-Gerlach as basis of his 'hidden variable' analysis, it was for a reason: he believed it to be a sufficiently simple system upon which one could test his thoughts on correlations, and he made it simple enough to "prove" his theorem. Physicists, from Newton through Dirac, have focused on angular momentum, from spinning buckets to intrinsic spin, but to understand Bell we must ask what $A = \pm 1$ means. So first, a brief review.

Newton: Local spin in a bucket

The topic of local spin is not a new one. Isaac Newton, ~1650, suspended a bucket of water from a twisted cord. As the bucket unwinds the water starts to spin and its surface takes on a concave shape. Though this phenomenon is local, observers in any reference frame will agree the surface is not flat. Issues of *spin, space, acceleration*, and *curvature* arise in Newton's spinning bucket. Centuries later Mach, Einstein, and others argued about "absolute rotation" and "absolute space". As these issues are still argued [1], we merely point out that Newton was opposing the view of René Descartes and Gottfried Leibniz that "empty space" exists only as a metaphysical "relation between things".

Maxwell: from spinning fluid to spinning field

Electricity and magnetism were essentially static phenomena until 1800 when Alexander Volta invented the chemical battery capable of producing continous current. Yet it was another 20 years until Oersted observed that an electric current affected a local magnet. In explaining why it took 20 years to place a magnetic compass near an electrical current, Ampere [2] said it was due to Coulomb's hypothesis on the nature of magnetic action:

"Everyone believed this hypothesis as though it was a fact.… Everyone resists changing ideas to which he is accustomed."

Today everyone believes Bell's 50-year-old hypothesis that local realism cannot produce quantum correlations, as though it is a fact. This paper and my longer '*Quantum Spin and Local Reality*' (QSLR) [3] examine this belief in detail. Some believe Bell's theorem has been proved by experiments. Not so. What *has* been proved by experiments is that Nature does not respect Bell's inequality, not that Bell's inequality actually precludes local realism.

The three primary forces of Nature – gravity, electricity, and static magnetism – pulled or pushed in straight lines, but Oersted determined that the force of the current on the compass acted in circles around the current. Faraday confirmed this and invented the electric motor, extracting circular motion from electrical current, and the (inductive) dynamo, extracting current from circular motion. This self-taught physicist was deficient in math, as he explained to Ampere: …not having the mathematical tools and power of entering with facility into abstract reasoning,

"I am obliged to feel my way by facts placed closely together."

Faraday conceived of lines of force – "*every electric current created a circular magnetic force around itself – a force that seemed to have a physical presence in the surrounding space.*" Fortunately, a physicist with superb mathematical skills, James Clerk Maxwell, formalized Faraday's concepts, and became convinced that Faraday was right – fields of force truly *existed* in space.

"Magnetic energy was akin to kinetic energy."

Maxwell transformed Faraday's *lines of force* in space to *fields existing in space* and formulated Maxwell's equations. Today the concept of field is so ingrained that hundreds of fields have been proposed as the basis of a multiverse [4], but only a very small number of physically real fields can actually be exhibited.

Einstein: there is no space "empty of field"

Maxwell's 1860s development of electromagnetic theory laid the grounds for today's physics. From a physics in which fields were unknown, and 'action-at-a-distance' prevailed, Maxwell prepared the way for Einstein [5] to state

"There is no such thing as empty space, i.e., a space without field."

Like Newton, Einstein did not view 'empty space' as a metaphysical or abstract relation between 'things'. He believed in physical reality, and in a 1935 paper, EPR [6], questioned whether quantum mechanics is a complete theory. His central point dealt with physical reality:

> *"If, without in any way disturbing a system, we can predict with certainty … the value of the physical quantity then there exists an element of physical reality corresponding to this physical quantity."*

This brief review establishes the key concepts of classical fields, as occupying space and having energy, and of having local physical reality. We now treat the topic of whether or not local reality is compatible with quantum mechanics.

Bell: No local realism model can produce quantum statistics.

Analyzing EPR in search of a classical explanation of quantum correlations Bell [7] formulated a model that fails to produce quantum correlations. Bell implied that he has explored 'all possible ways' such models could exist, and concluded that local causality (a.k.a. local reality) does not exist.

The current situation (17 Oct 2014) is summarized [30]:

> *"Though the original intent of EPR was to show that quantum mechanics is not complete, the standard present view is that entangled particles do experience nonlocal correlations."*

The following paper in the same journal [31] begins:

> *"The correlations resulting from local measurements on an entangled quantum state cannot be explained by local theory."*

QSLR focuses on deriving a local classical model that explains the correlations. But Bell's defenders challenge the model on quantum mechanical grounds. Analysis of, and answering, this challenge is the goal of this paper.

A brief discussion of the problem

In the October 2014 issue of *Physics Today*, Zurek [16] presents an article on *Quantum Darwinism* and mentions the *Quantum Credo*. Since a Credo is a statement of religious belief, that is not an entirely unrealistic description. A major belief in quantum mechanics is summed up in an email [11] as follows:

A. *Electron spin has two eigenvalues* ± 1
B. *In an idealized experiment the eigenvalues are determined exactly.*

A implies B.

There are several ways to address this but let me first put it in context. QSLR [3] contends that Bell suppressed the physics of the Stern-Gerlach experiment by replacing the force $F\cos\theta$ by $F\cos\theta/|\cos\theta|$. The paper presents and proves an *Energy-Exchange theorem* that says the energy of precession of the particle spin and the B-field will drain or dissipate into the deflection energy that is the output of the Stern-Gerlach experiment, and will do so in a θ-dependent way, where θ is the angle of precession. This should result in a θ-dependent deflection as appears to be the case in the iconic postcard. *QSLR* develops an ontological model of reality. Recall that spin magnetic moment $\vec{\mu}$ precesses in a homogeneous magnetic field B with frequency ω. Bell assumes that the angle of precession θ is constant and that spin projection is measured as ± 1 by σ_z (for $\vec{B} \| \vec{z}$). But Stern-Gerlach will not work in a homogeneous B-field, which would produce no deflection, a null result. Instead inhomogeneity is required, which produces a force $\vec{F} = \vec{\nabla}(\vec{\mu} \cdot \vec{B})$ with resultant energy $\vec{F} \cdot d\vec{z}$. Thus the appropriate particle energy is represented by

$$E = -\mu \cdot \vec{B} + \vec{\nabla}(\vec{\mu} \cdot \vec{B}) \cdot d\vec{z}$$

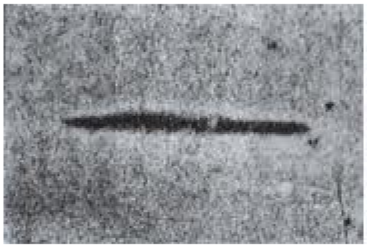
which, contrary to Bell, is θ-dependent, where θ is the angle of precession. The physics of this is described in *Quantum Spin and Local Reality* (QSLR).

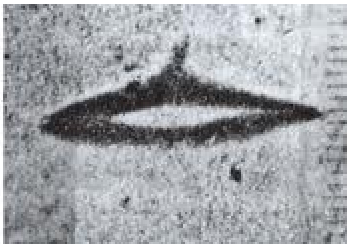
The spread in the data

Albert Messiah, in *Quantum Mechanics,* [18] [page 2]: The atom has a permanent magnetic moment $\vec{\mu}$, "considered as little gyroscopes of angular momentum $\vec{L}$ proportional to $\vec{\mu}$: $\vec{\mu} = m\vec{L}$." In magnetic field B, angular momentum executes a precession motion about B. If B is constant, the magnetic energy $-\vec{\mu} \cdot \vec{B}$ remains constant… If B is not constant, the center of mass of the atom is subject to a force $\vec{F} = \vec{\nabla}(\vec{\mu} \cdot \vec{B})$ and suffers a certain deflection [page 26]:

> *"The appearance on the screen of a more or less spread out distribution of impacts indicates that the atoms are not all in the same initial condition and that the dynamical variables defining the initial states are statistically distributed over a somewhat extended domain."*

This is exactly the problem addressed in *Quantum Spin and Local Reality.* In a nutshell, it is the problem. It is evident in the famous postcard that Stern and Gerlach sent to Bohr, showing the results of their experiment. The data are clearly spread out, not concentrated into a point or line, and the question is *why?* In my local realism model, these are the rational outputs derived from straightforward physics. But it is entirely possible this spread represents a spread in velocities, thermal noise, or other artifacts.





The statement of the problem

The problem seen by some Bell defenders [11] is as follows:

> *"A) electron spin has two eigenvalues* $\pm 1/2$
> *B) in an idealized experiment the eigenvalues are determined exactly."*

Further:

> *"A is a prediction of the Dirac equation and … B is part of the definition of an 'idealized experiment'. So yes, A implies B."*

The "So" implies that the argument clause is necessary to the implication. And "*B is a part of the definition of an ideal experiment*" is equivalent to *A implies B by definition*. Thus 'eigenvalues' is a key argument for attacking local realism:

A) Eigenvalues + B) idealized experiment $\Rightarrow$ A implies B

$$\text{Eigenvalues} \qquad \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix}$$

$$\text{Idealized experiment} \qquad \begin{pmatrix} - \\ - \\ - \\ - \end{pmatrix} \Rightarrow \begin{pmatrix} - \\ \\ \\ \end{pmatrix}$$

Thus, if I understand this correctly, my *local classical model* of Stern-Gerlach, which is *not* intended to be a quantum mechanical model, but only to produce quantum mechanical correlations, is seen to conflict with the basic credo of quantum mechanics. There are several ways to address this, and I do so now.

The issue of ideal experiments

First, the logic appears to be 'A implies B by definition'. While I don't know the definition of an *idealized experiment*, it is significant that the argument is about 'idealized experiment' rather than 'realized experiment'. An experiment, according to Bell, is based on [page 217]

> "*contriving artificially simple systems in which the number of factors involved is reduced to a minimum… But experiment is a tool, the aim remains: to understand the world.*"

For an excellent treatment of "idealized measurement", see pages 387-413 in Peres' [13] *Quantum Theory: Concepts and Methods* (including any necessary concepts treated in the preceding 386 pages). The statement B*: "in an idealized experiment the eigenvalues are determined exactly"*, is countered by Peres' treatment (see "*Fuzzy measurements*"). The contortions involved in analyzing these (see especially *Case Study: Stern-Gerlach Experiment* (page 402)) are of such nature that one cannot say in any reasonable sense "*A implies B*" from the perspective of a rigorous quantum mechanical treatment of measurement.

The issue of eigenvalues and eigenstates

Searching for a local explanation of quantum correlations, Bell chooses eigen-values as the meaning of $A(\vec{a},\lambda)=\pm 1$. Eigen-values are concepts defined by quantum operators operating on quantum states. But what is a quantum state? In 2014 we simply don't know. Leifer states [12]:

> *The status of the quantum state is one of the most controversial issues in the foundations of quantum theory. Is it a state of knowledge (an epistemic state), or a state of physical reality (an ontic state)?*
>
> *An ontological model for [...] experiments is an attempt to explain the quantum prediction in terms of some real physical properties… that exist independently of the experimenters,*

while an epistemic or 'knowledge' model leads to such questions as "*what is precessing*?" and to concepts such as *collapse of the wave function.* Einstein's belief was in "real physical properties" that exist independently, and our local model assumes the same.

Review of Quantum Mechanical model of spin

We first review Susskind's [8] nonrelativistic treatment of spin: We define a 2-dimensional spin state vector, $|u\rangle$ and $|d\rangle$ (*up* and *down*) with representation

$$|u\rangle=\begin{pmatrix}1\\0\end{pmatrix} \qquad |d\rangle=\begin{pmatrix}0\\1\end{pmatrix}$$

> *"We know σ_z has definite, unambiguous values for the states $|u\rangle$ and $|d\rangle$, and that the corresponding measurement values are $\sigma_z=+1$ and $\sigma_z=-1$."*

Here σ_z is the operator corresponding to the spin observable. Susskind states this as *Principle 2*:

> *"The eigenvectors of σ_z are $|u\rangle$ and $|d\rangle$. The corresponding eigenvalues are +1 and -1. We express this with the abstract equations*
>
> $$\sigma_z|u\rangle=+|u\rangle \qquad \sigma_z|d\rangle=-|d\rangle \qquad (3.12)$$

We combine these into

$$\sigma_z \,|\pm\rangle = \pm\,|\pm\rangle$$

This eigenvalue equation recalls the time-independent Schrödinger equation,

$$H\,|E_j\rangle = E_j\,|E_j\rangle$$

where H is the Hamiltonian operator, and the observable values of energy are just the eigenvalues, E_j, of H, with corresponding eigenvectors $|E_j\rangle$. For a magnetic field along the z-axis, the Hamiltonian is proportional to σ_z:

$$H = \frac{\hbar\omega}{2}\sigma_z \quad \sim \vec{\sigma}\cdot\vec{B} \quad = \quad \sigma_x B_x + \sigma_y B_y + \sigma_z B_z$$

Based on this, Bell [9] claims that the measurement results for any hidden variable model must be ± 1. He does so based on a model in which he has suppressed the relevant physics and assumed that the spin precesses as it traverses the apparatus. He expresses this requirement as $A(\vec{a},\lambda) = \pm 1$ and $B(\vec{b},\lambda) = \pm 1$ where A is Alice's measurement result based on her choice of setting $\vec{a}$ and the *hidden parameter* λ, and similarly for Bob. As a result [10],

> *"Bell proved that some predictions of quantum mechanics cannot be reproduced by <u>any</u> theory of local variables."*

Our emphasis is on *any*. *Physical Review Letters* is full of statements this general, with absolutely no qualifications. But Bell's local defenders do not really mean "any" theory. They mean only any theory that produces ± 1 results.

The issue of eigenvalue versus counter value

But what is $A(\vec{a},\lambda) = \pm 1$? According to Bell, there are *two ways* to define this. His first way (page 14) is $A : \hat{\sigma}\cdot\vec{a} = \pm 1$ which is an eigenvalue-based result. His second way (page 84) states that A is a variable that takes values ± 1 according to whether counter 1 does or does not register (and $B = \pm 1$ for counter 2.) While his discussion of counter values is based on simulation, the fact is that *almost all* photon-based 'Bell test' experiments to date use *counter* values, not *eigen* values. We will consider both cases.

Assume, for purposes of discussion that Bell <u>is</u> based on spin eigenvalues, and that *ideal experiments* determine eigenvalues exactly. Then one must define how such eigenvalues are determined, and how they are measured. Susskind claims that experiments always yield ± 1, then proceeds to construct an eigen-value equation that yields the same. On this basis Bell's defenders use circular logic to insist that measurements *must be* ± 1, despite any evidence to the contrary. And on this basis Bell assumes locally causal hidden variables λ that determine A and B, and he assumes instrument settings $\vec{a}$ and $\vec{b}$, such that

$$P(a,b) = \overline{A(a,\lambda)B(b,\lambda)}$$

and shows that this cannot yield the quantum prediction

$$P(a,b) = \langle singlet \,|\, a\cdot\sigma(1)\, b\cdot\sigma(2) \,|\, singlet \rangle = -\cos(a,b)$$

which is not locally causal.

Although Bell extensively discusses Stern-Gerlach and spin, some argue [14] that "*Stern-Gerlach is not fundamental to Bell's work*", sometimes going so far as to say "physics has nothing to do with it" (it's just mathematics!). If Stern-Gerlach has nothing to do with it, then we should ignore Bell's discussion of the value $\hat{\sigma}\cdot\vec{a} = \pm 1$ in favor of his statement [page 84] that A is

> *"a variable which takes the values ± 1 according to whether counter 1 does or does not register." (and B the same).*

So does ± 1 represent an *eigen* value or a *counter* value? If one insists that Stern-Gerlach has nothing to do with Bell [I've heard this too many times to dismiss it] then the number of counts at a given angle (a,b) has nothing to do with eigenvalues, but only with how likely the particles are to trigger a counter for a given angle, a density matrix. Ideally, this likelihood will be encoded in a classical measurement. A probability distribution determines the counts at any given angle. That is why actual experiments produce the "correct" or quantum correlation, $-\vec{a}\cdot\vec{b}$, a correlation that cannot be explained by local causality via quantum mechanics, since QM includes *both remote results* in the one term as seen above, where $\theta = (a,b)$. In fact, the $-\cos(a,b)$ result falls out of what is essentially a *geometric formula,* having perhaps been derived from "eigenvalue" arguments, but having no necessary relation to eigenvalues in actuality [15].

In an excellent text [13] (page 160) Peres makes the following statement:

> *"Bell's* [paper] *is not about quantum mechanics. Rather, it is a general proof, independent of any specific physical theory, that there is an upper bound to the correlation of distant events, if one just assumes the validity of the principle of local causality."*

But Peres is mistaken. Bell's paper is about quantum mechanics. There is absolutely no other reason to impose the constraint $A(\vec{a},\lambda) = \pm 1$ on the results of measurement. It is strictly a quantum mechanical argument that Bell's defenders are making concerning eigenvalues and the quantum credo.

In fact, Peres [13] (p. 167) uses a table-based analysis to show that one cannot derive a transcendental function such as $-\cos(a,b)$ with only ± 1's. For actual experiments, this implies that ± 1 eigenvalue measurements *cannot* yield the correlation. This immediately makes one wonder how Bell experiments could actually produce the $-\vec{a}\cdot\vec{b}$ correlation, since they are supposedly measuring ± 1 eigenvalues. It is this that causes physicists to hypothesize the unintuitive (i.e., "weird") concept of 'entanglement'. But it is nevertheless hard to see how entanglement, if it exists, can change the calculations!

On the other hand, if, as Bell states (page 84), the ± 1 represents a *counter* value, (as is the case in all photon-based experiments) instead of an eigenvalue, and if the counter values are θ-dependent (as they are!) then the variation in counts, corresponding to different probability densities for different θ, *can* produce the desired correlation. This argument seems to suggest that the ±1 values are really counter values, not eigenvalues.

Because the quantum mechanical approach is unable to calculate individual results, but can only calculate probabilistic results *given nonlocal information*, quantum mechanics is incapable of describing local reality. Bell's 'hidden variable' model suppresses the actual local physics, compressing a range of measurements (deflections in Stern-Gerlach) into two values, +1 and -1.

The issue of throwing away local information

Does it make sense to require a classical deterministic solution (which is local) to throw away all information about what's really going on at each local experiment, simply because the quantum mechanical statistical formulation (which is not local) makes no use of this information? If so, then Bell's defenders are correct: one cannot achieve quantum correlations with a local model that is forced to throw away information. But what is the point of this? It makes no sense to me; after reading countless papers, a dozen or so quantum mechanics and quantum field theory texts, and after arguing for many hours, nothing has conveyed the sense to me of this approach. Was Bell searching for a classical explanation of quantum statistical results or was he simply playing games?

Some of us want to know how quantum results can be obtained with classical physics. We are not bound by faith to protect a particular interpretation of quantum mechanics. Why *not* look for a local physics model? As Susskind [17] just pointed out, quantum mechanics and gravity are *not* the final word.

The above discussion has taken the approach that "spin eigenvalues" are not really the issue, yet Bell's current defenders insist they are, referencing Dirac's equation as authority. So we now look at Dirac. Although a Stern-Gerlach experiment is non-relativistic, many consider spin to be fundamentally defined by Dirac. Our purpose is to address confusion, not to serve as text or tutorial on the Dirac equation, so I refer to standard texts for details. In discussing a particular text I use equation numbers used in that text.

Review of Dirac equation

Srednicki, in *Quantum Field Theory*, [19] reminds us that special relativity tells us that *physics looks the same in all inertial frames*, and tests this claim with the relativistic Klein-Gordon equation

$$(\partial_\mu \partial^\mu + \kappa^2)\psi = 0 \quad \text{where} \quad \partial_\mu \partial^\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad \text{and} \quad \kappa = \frac{mc}{\hbar}.$$

He finds that Alice's form of this equation is equivalent to Bob's form of the equation *in every inertial frame*, but also that probability is not conserved. Thus, to preserve probability, Dirac derived another equation. Although treatments of Dirac often digress into a discussion of the interpretation of *negative energy states*, we will stay focused on spin. B.R. Martin [20] states:

> *The Dirac equation is of the form*
>
> $$i\hbar \frac{\partial \psi(\vec{x},t)}{\partial t} = H(x,\hat{p})\psi(\vec{x},t) \qquad (1.1)$$
>
> *where* $\hat{p} = -i\hbar\vec{\nabla}$ *is the usual quantum mechanical momentum operator and the Hamiltonian was postulated by Dirac to be*
>
> $$H = c\,\vec{\alpha}\cdot\hat{p} + \beta mc^2 \qquad (1.2)$$
>
> *The coefficients* $\vec{\alpha}$ *and* β *are determined by the requirement that the solutions of (1.1) are also solutions of the Klein-Gordon equation*
>
> $$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \psi + m^2 c^4 \psi\,. \qquad (1.3)$$
>
> *This leads to the conclusion that* $\vec{\alpha}$ *and* β *cannot be simple numbers; their simplest forms are 4 x 4 matrices. Thus the solutions of the Dirac equation are four component wave functions (called spinors) with the form*
>
> $$\psi(\vec{x},t) = \begin{pmatrix} \psi_1(\vec{x},t) \\ \psi_2(\vec{x},t) \\ \psi_3(\vec{x},t) \\ \psi_4(\vec{x},t) \end{pmatrix} \qquad (1.4)$$

Dirac: Spin in the relativistic equation

Steven Weinberg [21] notes: Dirac showed that

> *in a central field, the conservation of angular momentum takes the form*

$$[H, -i\hbar\vec{r}\times\vec{\nabla}+\hbar\,\hat{\sigma}/2]=0 \qquad (1.1.24)$$

where H is the matrix differential operator

$$H=-i\hbar c\,\vec{\alpha}\cdot\vec{\nabla}+\beta\,mc^2 \qquad (1.1.14)$$

and $\hat{\sigma}$ is the 4 x 4 version of the spin matrix introduced earlier by Pauli.

$$\hat{\sigma}=\begin{vmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{vmatrix}\hat{\alpha} \quad\rightarrow\quad \begin{vmatrix}\sigma&0\\0&\sigma\end{vmatrix}$$

Since each component of $\hat{\sigma}$ has eigenvalues ± 1, the presence of the extra term in (1.1.24)*shows that the electron has intrinsic angular momentum $\hbar/2$*

Dirac's equation so far agrees with QSLR, in which I remark that:

…the real quantized nature of the particle, the fact that the angular momentum is quantized in units of Planck's constant. That is the real phenomenon.

Weinberg continues: (page 10)

Dirac… obtained a second-order equation which turned out to have just the same form as the Klein-Gordon equation except for the presence on the right-hand side of two additional terms

$$[-e\hbar c\,\hat{\sigma}\cdot\vec{B}-ie\hbar c\,\hat{\alpha}\cdot\vec{E}]\psi\,. \qquad (1.1.26)$$

For a slowly moving electron, the first term dominates, and represents a magnetic moment in agreement with [1.1.8] *… this magnetic moment, together with the relativistic nature of the theory, guaranteed that this theory would give a fine structure splitting in agreement (to order $\alpha^4 mc^2$) with that found by Heisenberg, Jordan, and Darwin*

From $i\hbar\frac{\partial\psi}{\partial t}=H\psi$ where H is [1.1.14] we can derive a continuity equation

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\vec{J}=0 \qquad (1.1.28)$$

with $\rho=|\psi|^2$ and $\vec{J}=c\psi^{+}\alpha\psi$, so that $|\psi|^2$ can be interpreted as a probability. Weinberg notes that this theory achieves Dirac's primary aim: *a relativistic formalism with positive probabilities.*

For a given momentum $\vec{p}$, the wave equation (1.1.13) has four solutions of the plane wave function

$$\psi \propto \exp\left[\frac{i}{\hbar}\left(\vec{p}\cdot\vec{x} - Et\right)\right] . \qquad (1.1.30)$$

Two solutions with $E = +\sqrt{p^2c^2 + m^2c^4}$ correspond to the two spin states of an electron with $J_z = \pm\hbar/2$. The other two have $E = -\sqrt{p^2c^2 + m^2c^4}$, and no obvious physical solution.

Weinberg goes on to discuss the problems of negative energy states which are of no concern to us here.

But Dirac's equation has problems

Instead we use Schweber's [22] *An Intro to Relativistic Quantum Field Theory*. After he derives the solution to Dirac's equation, Schweber states (p 91)

Although we have derived many properties of the Dirac equation, we have not as yet given the physical interpretation to the operators appearing in the theory. The fact is that the Dirac equation in the form described above [the usual treatment of the Dirac equation] does not lend itself easily to simple interpretation. Consider for example the operator, $\dot{\vec{x}}$.

$$\dot{\vec{x}} = \frac{i}{\hbar}[H, x] = c\,\vec{\alpha}$$

which one might want call the velocity operator. Since ${\alpha_i}^2 = 1$ the absolute magnitude of the "velocity" in any given direction is always c, which is not physically reasonable. Furthermore, since $[\alpha_1, \alpha_2] \neq 0$, it would seem that, when the velocity in any one direction is defined, the velocity in the other two directions cannot be simultaneously defined. But this would deny the existence of velocity measurements. One must conclude that there must exist another representation of the Dirac equation in which the physical interpretation is more transparent.

In short, for a positive energy Dirac particle there are *two independent states associated with each value of the momentum*. These correspond to the two possible directions of the spin. There is a redundancy in the representation of these vectors in the usual form of the Dirac theory where the corresponding wave functions have four components.

The problem is that, when fields are present, the field can be regarded as a perturbation, which can cause transitions (at least virtual ones) between states with energies of opposite signs, so that a mixing of components seems to be inherent in the problem. Messiah in *Quantum Mechanics* [23] (p 940) notes:

> *Due to the coupling between the positive and negative components of the four-component Dirac wave function,* [Messiah's equation (XX.183)] *is, properly speaking, no longer an eigenvalue equation.*

This motivates the Foldy-Wouthuysen transformation, which allows one to approximate the four-component Dirac theory by a two component theory to any order in v/c, and thus remove the redundancy.

In the Dirac representation, the orbital angular momentum $\vec{r}\times\vec{p}$ and the spin angular momentum $\sigma/2$ are not separately constants of the motion, although their sum is. After the Foldy-Wouthuysen transformation these are decoupled and are separately constants of the motion. At this point the transformed operators representing physical quantities are in a one-to-one correspondence with the operators of the Pauli theory, thus linking the Dirac relativistic theory to the Pauli nonrelativistic theory addressed in QSLR.

But as George Trigg notes [24] (p 284), in *Quantum Mechanics,* this decoupling comes at a cost. The Foldy-Wouthuysen is not a point transformation but an integral transformation. As noted, $\hat{\sigma}$ and $L=\vec{r}\times\vec{p}/\hbar$ are *not* constants of the motion under H, but *are* under the transformed Hamiltonian. Because $\Psi'(\vec{r})$ includes the contribution from values of $\Psi(\vec{r}\,')$ for all $\vec{r}\,'$ in the neighborhood of $\vec{r}$, whose extent is of order $\hbar/mc$, the transformed spin operator is called the 'mean' or 'average' spin Σ. For an integral transformation in coordinate space, the transformed state vector involves contributions from an extended region in the original description.

> *The particle described by the transformed Hamiltonian is therefore 'smeared out' and interacts not only with the potential at the mean position, but with the average of the potential over the region it 'occupies'.*

The nonlocal FW transformation yielding spinor $\Psi'(\vec{r})$ is obtained by averaging over ψ values in a volume about $\vec{r}$ whose linear dimensions are the order of the Compton wavelength of the particle (electron $=2.4\times10^{-12}$ m.). Thus the spin of the 2-component Bell theory (versus the 4-component Dirac theory) does *not* correspond to the spin $\vec{\sigma}$ of the Dirac theory, but to the 'average' spin $\vec{\Sigma}$.

In Dirac's theory, the particle interaction with the electromagnetic potential is a local interaction, acting at the location of the particle, $\vec{r}$. In FW representation this interaction is transformed into a nonlocal interaction over a region $\sim(\hbar/mc)^3$

Most energy/momentum equations in quantum mechanics have a counterpart in classical mechanics, but observables in classical mechanics are represented by operators in quantum mechanics with consequential complexity, such as the requirement that operators be Hermitian in order to produce 'real' results from complex eigenvectors. The underlying classical reality, if any, is obscured by a complex formalism, and made even more obscure by 'second quantization' of quantum field theory. Nevertheless Weinberg points out [Vol. 1 p. 49]

> *Quantum field theory is the way it is because (…) this is the only way to reconcile quantum mechanics with special relativity. (…) Quantum field theory is based on the same quantum mechanics that was invented by Schrödinger, Heisenberg, Pauli, Born, and others in 1925-26…*

Spin Projection in Dirac's equation

Schweber [22] (page 82): the Hamiltonian operator $H = c\vec{\alpha}\cdot\hat{p} + \beta mc^2$ commutes with Hermitian operator

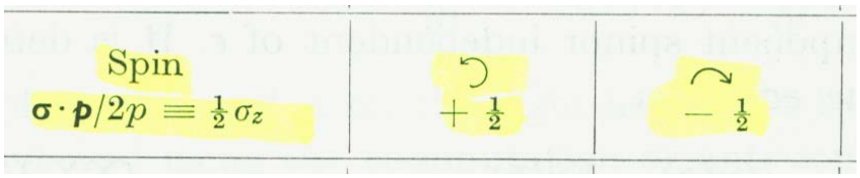
$$s(\vec{p}) = \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|} \qquad \text{where} \qquad \Sigma = \begin{pmatrix} \hat{\sigma} & 0 \\ 0 & \hat{\sigma} \end{pmatrix},$$

where $\hat{\sigma}$ is the Pauli matrix operator, and $s(\vec{p})$ is called the *helicity operator* or simply the *helicity* of the particle and physically corresponds to the spin of the particle parallel to the direction of motion. Solutions can be chosen to be simultaneous eigen-functions of H and $s(\vec{p})$. Since $s^2(\vec{p}) = 1$, the eigenvalues of $s(\vec{p})$ are ± 1. For a given momentum and sign of the energy, the solutions can be classified according to the eigenvalues +1 or -1 of $s(\vec{p})$. A similar treatment of intrinsic spin is given by Cottingham and Greenwood [25]. After formulating the free space solution of the Dirac equation they derive the intrinsic spin of a Dirac particle in a frame where *the particle is at rest*. They then transform to a frame in which the particle has velocity v. Their *helicity operator*, useful in classifying plane wave states is again $\vec{\Sigma}\cdot\vec{p}/|\vec{p}|$. The expectation value of this operator in a given state is a measure of the alignment of a particle's intrinsic spin with the direction of motion in that state. Roman [26] considers, in an *arbitrary representation*, the spin operator $s = \sigma_i p_i / |p|$, with $s^2 = 1$, eigenvalues of s are ± 1, and s commutes with the Hamiltonian ($H = c\alpha_i p_i + \beta m_0 c^2$) — hence energy states are simultaneous eigenstates of s. The operator represents the normalized projection of the spin onto the direction of the momentum.

We see that Dirac's equation can be solved for a particle at rest in free space. The magnitude of the spin is $\hbar/2$. The direction of the spin is undefined. It makes no physical sense to "*quantize the projection of spin in a given direction*". There are no meaningful directions defined for such a particle, other than the spin axis of the particle, which should essentially be in a random direction. Thus if Goudsmit and Uhlenbeck claimed [27] that

> *the spin projection on any axis is* ± 1,

this can only mean that a particle's spin establishes its own direction when at rest in free space. But if the particle is transformed to a frame in which it is in motion, a natural direction appears: the direction of the velocity, a meaningful direction to which to relate spin.



Spin		
$\boldsymbol{\sigma}\cdot\boldsymbol{p}/2p \equiv \frac{1}{2}\sigma_z$	$+\frac{1}{2}$	$-\frac{1}{2}$

But how is the spin direction related to the particle's direction of motion? Assume that the directions are aligned. A particle approaches an observer, who observes clockwise or anti-clockwise spin; there are no other choices. As the *magnitude* of the spin does not vary, spin variation is completely specified by the *direction* of the rotation. It is this dichotomy that is the basis of the two eigenvalues associated with spin. In fact, Griffiths, [28] (page 221) points out that these Dirac spin eigenvalues have nothing to do with 'up' and 'down':

> *Thus the four solutions are* [equation (7.46)]. *You might guess that* $u^{(1)}$ *describes an electron with spin up,* $u^{(2)}$ *an electron with spin down, but this is not quite the case. For Dirac particles the spin matrices … are*
>
> $$\vec{s} = \frac{\hbar}{2}\hat{\Sigma} \qquad \textit{with} \qquad \hat{\Sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \qquad (7.48)$$
>
> *and it's easy to check that* $u^{(1)}$*, for instance, is not an eigenstate of* Σ_z*.*
>
> *However, if we orient the z-axis so that it points along the direction of motion (in which case* $p_x = p_y = 0$*) then* $u^{(1)}$*,* $u^{(2)}$*,* $u^{(3)}$ *and* $u^{(4)}$ *are eigen-spinors of* s_z*:* $u^{(1)}$ *and* $u^{(3)}$ *are spin up, and* $u^{(2)}$ *and* $u^{(4)}$ *are spin down.*

Thus FW-Dirac yields energy/momentum $\vec{p}$ and spin $\vec{\Sigma}\cdot\vec{p}/|\vec{p}|$ valid in all frames, at rest or in motion, in free space or in a field. They represent the physical fact that a particle has energy/momentum and spin, and the further fact that the spin has two possible values relative to the momentum. This is the only "spin projection" fact that is derived from Dirac's equation. Hence the belief that *spin up* and *spin down* are intrinsically meaningful is mistaken.

And Michael Scadron in *Advanced Quantum Theory* [29] (p 68 and 74) notes that

> φ_R *is polarized right-handed, corresponding to helicity* $\lambda = +1/2$*, and* φ_L *is polarized left-handed, with* $\lambda = -1/2$ *…. In particular, we further specify these two component spinors as helicity eigenstates* $\varphi^{(\lambda)}(\hat{p}) = |\,\hat{p}\lambda\rangle$ *and* $\chi^{(\lambda)}(\hat{p})$ *[…] obeying the eigenvalue equation,*

$$\hat{\sigma} \cdot \hat{p}\, \varphi^{(\lambda)}(\hat{p}) = \lambda\, \varphi^{(\lambda)}(\hat{p}) \tag{5.62}$$

where $\lambda = \pm 1$, and I have suppressed the phase factor associated with $\chi^{(\lambda)}$ represented by the rotated spinors (3.91) [...] The $s = 1/2$ helicity rest-frame eigen-spinors for massive particles, $\varphi^{(\lambda)}(\hat{p}) = |\,\hat{p}\lambda\rangle$, obeying the above equation must also transform according to the irreducible representations of O(3). By going to Dirac's equation for spin, we have thus discovered the FW-based 'intrinsic' eigenvalue equation, which is the helicity eigenvalue equation:

$$\hat{\sigma} \cdot \hat{p}\,|\,\hat{p}\lambda\rangle = \lambda\,|\,\hat{p}\lambda\rangle$$

This is the fundamental spin eigenvalue equation for a relativistic particle. As Griffiths points out, it has nothing to do with 'up' or 'down'-spin unless one chooses up and down in the direction of the momentum vector. It exhibits *projection* in the direction of momentum, and explicitly shows that *spin states have two values* because they are helicity eigenstates, which appear clockwise or counterclockwise with respect to the momentum vector. *This* is the intrinsic eigenvalue equation that is being confused with the deflection measured by the Stern-Gerlach experiment.

The quantum machinery of spin in a local field

The machinery of quantum mechanics is well adapted to specific field configurations. For example, one atom in the near field of another (a diatomic molecule) forms a harmonic oscillator with quantized energy states described by an eigen value equation. And a magnetic moment precessing in a *constant* magnetic field will possess two quantized energy levels, $\pm \hbar\omega/2$. So Bell can legitimately treat *precession in a constant field* as a 'quantum entity' in which $\sigma \cdot B$ is meaningful and thus represent a magnetic moment precessing in a constant field as a quantum mechanical eigenvalue problem in which $A = \hat{\sigma} \cdot \vec{B} = \pm 1$.

But Trigg [24] (page 285) also states

> *The case of a constant* [...] *magnetic field is exceptional, since such a field does not exchange energy with the particle and the classification of the solution according to the sign of the energy remains valid. Apart from this exception, it is not clear that a single (closed) transformation can ever be sufficient.*

But in Stern-Gerlach the field is not constant, so Bell's insistence on $A = \pm 1$ is incorrect, and it is the reason that he is unable to find a local classical system that yields quantum correlations. In the QSLR perspective, precession energy is not an eigenvalue; the particle exchanges energy among two energy modes: *precession* and *deflection*.

Although Bell's defenders reference *electron* spin, electrons will not work in the Stern-Gerlach apparatus; the charge of the particle must be neutral, so the electron is embedded in an atom, originally a silver atom. To this we add a constant magnetic field. Despite that we believe we are measuring the spin of the electron (the object of Dirac's equation) we are in reality measuring the interaction of electron with the nucleus and closed shells of other electrons and the interaction with the magnetic field, all of which fulfills the definition of a 'quasi-particle', although it is not usually labeled such.

While the term *quasi-particle* normally refers to a large number of particles [32], Lev Landau, ~1930, suggested physicists could

> "*combine a particle and its interactions into one composite quasi-particle.*"

So we can create *another* quantum system—effectively a *quasi-particle*—by treating the atom in a *constant* B field. Since the particle precesses in this constant field, and can be shown to have two energy states via a photon $\hbar\omega$, we can construct, per Susskind, *another* eigenvalue equation

$$\Sigma\,|\pm\rangle = \pm\,|\pm\rangle$$

This is the eigenvalue equation describing the *precession states* $|+\rangle$ and $|-\rangle$, <u>not</u> the particle *spin (helicity) states* per se. It is the equation that Bell limits his consideration to, by ignoring the field gradient, $\vec{\nabla}(\vec{\mu}\cdot\vec{B})$, and, thus, the θ-dependent force on the particle in the Stern-Gerlach apparatus.

The two spin-based eigenvalue equations

Having reviewed Dirac's theory for a particle at rest, a particle in motion, a particle in empty space (ignoring the field of gravity) and a particle in a field, we see that for a particle at rest ($\vec{p}=0$) in a vacuum the magnitude of spin is $\hbar/2$ but there is no direction to which to relate the spin; it defines its own direction by its existence. If $\vec{p}=0$ then the energy term is βmc^2, and spin exists trivially. If we transform to a frame in which the particle is in motion ($\vec{p}\neq 0,\ \hat{p}=\vec{p}/|\vec{p}|$) then the projection eigenvalue equation is dichotomous,

Helicity eigenvalue equation: $\quad \hat{\sigma}\cdot\hat{p}\,|\,\vec{p}\lambda\rangle = \lambda\,|\,\vec{p}\lambda\rangle$

representing the fact that only two spin eigenstates have meaning, clockwise with respect to momentum or counterclockwise. <u>This</u> is the fundamental quantum mechanical eigenvalue equation for a particle with spin.

Now add a constant field ($\vec{B}\neq 0,\ \hat{B}=\vec{B}/|\vec{B}|$) with which the particle interacts. We now have a <u>new</u> dichotomous eigenvalue equation,

Precession eigenvalue equation: $\quad \hat{\sigma}\cdot\hat{B}\,|\pm\rangle = \pm\,|\pm\rangle$

representing the experimentally verified fact that the precession can be in state $+\vec{\mu}\cdot\vec{B}$ or $-\vec{\mu}\cdot\vec{B}$. In other words, when we add a constant field for the particle to interact with, we have a *quasi-particle*, as defined. Although this usage is unorthodox, it is important to see that *particle plus constant field* is essentially a *quasi-particle*, with a new eigenvalue equation describing its quantum states.

The difference: Fundamental vs. Provisional

The significance of this is that the quantum mechanical description of quasi-particles is provisional. If the interactions vanish, the fundamental particle still exists, and is still described by the *helicity eigenvalue equation*, but the interacting system (of particle plus field) no longer exists, and thus

> *the precession eigenvalue equation vanishes!*

The next thing to understand is that we can remove the constant field either by deleting the B-field entirely or by making it a non-constant or inhomogeneous field. In either case *the dichotomous precession eigenvalue equation vanishes.*

> *Bell and his defenders wrongly equate the fundamental helicity eigenvalue equation with the provisional precession eigenvalue equation, and so insist that the result of the Stern-Gerlach experiment must be A.) the eigenvalue, that is expected from B.) the idealized experiment. But the provisional precession eigenvalue equation ceases to be relevant (as a dichotomous equation implying* ± 1*) when the particle traverses an inhomogeneous field, therefore the belief that the output must be* ± 1 *is mistaken.*

Thus it is useful to consider the particle and its interactions with the constant magnetic field as a *quasi-particle,* to distinguish it from a fundamental particle. The quasi-particle has two states $|+\rangle$ and $|-\rangle$ and an eigenvalue equation:

$$\hat{\sigma} \cdot \vec{B} \,|\pm\rangle = \pm |\pm\rangle .$$

We call this the provisional *precession eigenvalue equation*, and contrast it with the fundamental *helicity eigenvalue equation:*

$$\hat{\sigma} \cdot \hat{p} |\, \hat{p}\lambda \rangle = \lambda |\, \hat{p}\lambda \rangle ,$$

which is the FW-Dirac equation for a fundamental particle with spin.

Another way of seeing this is simply to recognize that the energy term $-\vec{\mu} \cdot \vec{B}$ applies only to the particle in a homogeneous field. The Hamiltonian for an inhomogeneous field requires another energy term, $\vec{\nabla}(\vec{\mu} \cdot \vec{B}(\vec{x})) \cdot d\vec{x}$. This term is position dependent and will not yield simple ± 1 eigenvalues. It is missing in Bell's equation because he chose to suppress all θ-dependent physics.

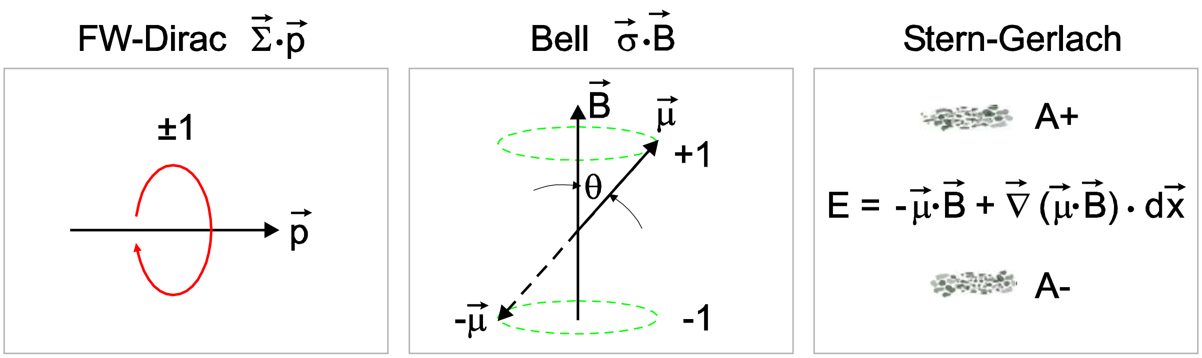
The logic of the 'eigenvalue' argument

So if we assume that angle θ changes as the particle traverses the non-uniform field, the quasiparticle eigenvalue equation, describing precession in a constant field, is *not* preserved, as the system it is describing (precessing moment plus field) undergoes change. Thus the argument that deflection of particles in a Stern-Gerlach apparatus must be ± 1 because of 'eigenvalues' is false.

It is perfectly encapsulated in Susskind's treatment of spin:

1. *The measured values are always* ± 1
2. *We can construct an eigenvalue equation:* $\Sigma \,|\pm\rangle = \pm\,|\pm\rangle$
3. *Therefore the measured values must be* ± 1

This is circular logic which collapses if θ changes, as the *Energy-Exchange theorem* says θ must. But the root problem is confusion of this provisional precession eigenvalue equation with the fundamental FW-Dirac helicity eigenvalue equation.



$\hat{\sigma}\cdot\hat{p}\,|\,\hat{p}\lambda\rangle = \lambda\,|\,\hat{p}\lambda\rangle$	$\hat{\sigma}\cdot\hat{B}\,|\pm\rangle = \pm\,|\pm\rangle$	no eigenvalue equation
Fundamental particle: Helicity eigenvalues	Quasi-particle: Precession eigenvalues	helicity not relevant and precession varies

Summary: 50 years of Bell's theorem

On the 50 year anniversary of Bell's theorem, in a special issue, [33], Wiseman proclaims:

> *"Bell's theorem is the most profound ramification of quantum theory that has been experimentally confirmed."*

Yet Brukner notes "*there still remains a controversy about its implications.*" Nevertheless, all authors appear to accept that Bell proved that

> *"No local deterministic hidden variable theory can reproduce all the experimental predictions of quantum mechanics."*,

specifically, statistical predictions, $-\vec{a}\cdot\vec{b}$. However, as Werner points out

> *"When my interpretation of quantum mechanics forces me to introduce theoretical entities about which nothing concrete and nothing intelligent can be said, I would take this is a hint that something might be wrong with that interpretation."*

These brief quotes are intended to convey the significance of Bell's theorem to physics and the fact that "*50 years later, not all issues are settled.*"

I have noted that some claim Stern-Gerlach is not fundamental to Bell, but Bertlmann (of the singlet socks) states:

> *"In such a Bohm-EPR set up…the spin measurement… performed by a Stern-Gerlach magnet along some direction $\vec{a}$ is described by the operator $\vec{\sigma}_A \cdot \vec{a}$ and yields the values ± 1. … Consider a spin measurement along some direction $\vec{n}$*
>
> $$\vec{\sigma} \cdot \vec{n} \, | \pm \vec{n} \rangle = \pm \, | \pm \vec{n} \rangle$$
>
> *where $| \pm \vec{n} \rangle$ are the eigenstates of operator $\vec{\sigma} \cdot \vec{n}$ measuring the spin."*

In QSLR [3] I analyze the fact that Bell suppressed the physics of Stern-Gerlach by removing the θ-dependence, specifically replacing $F \cos\theta$ by $F \cos\theta / |\cos\theta|$. I present and prove the *Energy-Exchange theorem* that implies the precession energy will contribute to deflection energy in θ-dependent fashion, and yields variable Stern-Gerlach outputs not equal to ± 1, which aligns the spin with the local field. This is *not* the usual interpretation of spin dynamics and leads to a local model that violates Bell's theorem.

Whereas I claim that a local deterministic (classical) model should not be constrained by quantum conditions ($A = \pm 1$), Bell's defenders insist that even a classical model must satisfy the quantum eigenvalue equation above, and reference Dirac's equation as the quantum 'source' of spin.

The goal of this paper is to refute this argument, which I do by analyzing both relativistic (Dirac) and nonrelativistic (Stern-Gerlach) eigenvalue equations. By reviewing standard QM and QFT texts I establish that the Dirac equation, per se, does not lead to a spin eigenvalue equation, due to mixing of the four component wave function. But the Foldy-Wouthuysen (integral) transformation leads to a fundamental two component helicity equation

$$\hat{\sigma} \cdot \hat{p} \, | \pm \vec{p} \rangle = \pm \, | \pm \vec{p} \rangle ,$$

with two eigenvalues, ± 1. But these eigenvalues have nothing to do with 'spin up' or 'spin down' unless 'up' is chosen as the particle momentum direction.

Thus Dirac's helicity eigenvalue equation has a superficial resemblance to Pauli's spin eigenvalue equation, but is, in reality, quite different. To see this we note that helicity is a <u>fundamental</u> characteristic of particle spin, with or without fields, whereas Bell's eigenstates are provisional "precession" eigenstates that have meaning only in a constant magnetic field. This precession eigenvalue equation is

$$\hat{\sigma}\cdot\hat{B}\,|\pm\vec{B}\rangle = \pm\,|\pm\vec{B}\rangle$$

which is mathematically identical to the helicity equation but is physically significantly different. The difference is summarized as *fundamental* versus *provisional*. Helicity is intrinsic to all particles with spin one-half. Precession is a stable phenomenon that exists only when such particles are in a constant magnetic field. Bell, by suppressing the θ-dependence, assumes that

$$E = -\mu\cdot\vec{B} + \vec{\nabla}(\vec{\mu}\cdot\vec{B})\cdot d\vec{x} \quad \Rightarrow \quad -\vec{\mu}\cdot\vec{B} \;\cong\; \hat{\sigma}\cdot\hat{B}$$

But this condition is incompatible with Stern-Gerlach, as the deflection derives only from the $\vec{\nabla}(\vec{\mu}\cdot\vec{B})$ term. The consequence of Bell's erroneous assumption is the simple fact that when the constant field is removed (or made non-constant) the provisional precession eigenvalue equation ceases to have two eigenvalues, ± 1, and Bell's theorem is falsified, by a classical (local realism) model that does produce quantum correlation $-\vec{a}\cdot\vec{b}$.

Bertlmann notes that, after studying Bohm's quantum mechanics,

> *"John told me once:*
>
> *'At the beginning I just played around to get simple relations which would give a local account for the quantum correlations but everything I tried didn't work. So I felt it couldn't be done and then I constructed an impossibility proof."*

This is consistent with Bertlmann's claim that Bell's maxim was:

> *"Always test your general reasoning against simple models."*

But it is not compatible with Einstein's maxim:

> *"As simple as possible, but no simpler."*

Bell oversimplified by suppressing the physics of the phenomena and by ignoring the physical difference between the *fundamental helicity* and the *provisional precession*:

$$\hat{\sigma}\cdot\hat{p}\,|\pm\vec{p}\rangle = \pm\,|\pm\vec{p}\rangle \qquad \neq \qquad \hat{\sigma}\cdot\hat{B}\,|\pm\vec{B}\rangle = \pm\,|\pm\vec{B}\rangle$$

The apparent *mathematical* equivalence (for constant $\vec{p}$ and constant $\vec{B}$) does *not* imply *physical* equivalence; one is fundamental, one is provisional.

For fifty years physicists have believed that local realism (classical) models cannot reproduce quantum statistical correlations. In my next paper I will exhibit a counterexample: a local realism model that *does* yield $-\vec{a}\cdot\vec{b}$.

Acknowledgments

I thank Dick Zacher for being willing to play the part of *chief Bell defender*, Monty Frost, for being willing and able to defend my theory, Bob Rader, who has searched for precedents and has so far found none, and Gordon Watson, who has developed the most relevant formalism to link quantum states with classical precursors.

References

1 D Janzen, 2014, '*Einstein's Two Minds'*, and *arXiv:1402.3212v1 , 13 Feb 2014*
2 N Forbes and B Mahon, 2014, *Faraday, Maxwell, and the Electromagnetic Field*, Prometheus books, New York
3 EE Klingman, '*Quantum Spin and Local Reality'*, http://vixra.org/abs/1408.0005
4 L Susskind, Cosmology Lecture, 2013, Stanford University
5 A Einstein, 1952, *Relativity*, Crown publishers, New York
6 Einstein, Podolsky, Rosen, 1935, "*Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", Physical Review* 47, 15 May 1935
7 JS Bell, Physics 1,195 (1964) reprinted in [9]
8 L Susskind and Friedman, 2014, *Quantum Mechanics*, Basic Books
9 JS Bell. 1987. *Speakable and Unspeakable in Quantum Mechanics*, Cambridge
10 Navascues and Popescu, 2014, *"How Energy Conservation Limits Our Measurements*", *Phys Rev Lett*, 112, 140502 (11 Apr 14)
11 Susskind modern-physics Google Group email: 20 Oct 2014 @ 10:30 PM
12 MS Leifer, 2014 "*ψ -epistemic models are exponentially bad at explaining the distinguishability of quantum states*", *Phys Rev Lett* 112, 160404, (25 Apr 14)
13 A Peres, 1995, *Quantum Theory: Concepts and Methods*, Kluwer Pub.
14 Private Communications
15 JJ Sakurai, 1994 , *Modern Quantum Theory*, Addison-Wesley Pub, 1994
16 WH Zurek, 2014, '*Quantum Darwinism*', *Physics Today*, Oct 2014
17 L Susskind, 2014, Stanford video, 27 Oct 2014
18 A Messiah, 1961, *Quantum Mechanics, Vol. I*, John Wiley, New York
19 M Srednicki, 2007, *Quantum Field Theory*, Cambridge
20 BR Martin, 2006, *Nuclear and Particle Physics*, Wiley
21 S Weinberg, 2005, The *Quantum Theory of Fields, Vol I*,, Cambridge U.
22 S Schweber, 1989, *An Intro. to Relativistic Quantum Field Theory*, Dover
23 A Messiah, 1961, *Quantum Mechanics, Vol. II*, John Wiley, New York
24 GL Trigg, 1963, *Quantum Mechanics*, then Nostrand Co., Princeton
25 W Cottingham and D Greenwood, 2007, *The Standard Model of Particle Physics, 2nd Ed.*, Cambridge
26 P Roman, 1965, *Advanced Quantum Theory*, Addison Wesley, Palo Alto
27 JQPMaster, 2010, *"The Phenomenal Physics of Spin"* [*eg.bucknell.edu/~koutslts/Ph332/Text/Chap6_JQPMaster.pdf*]
28 D Griffiths, 2004, *Introduction to Elementary Particles*, John Wiley-VCH
29 MD Scadron, 1991, *Advanced Quantum Theory*, Springer Verlag, Berlin
30 Moreau, Devaux, and Lantz , 2014, *'Einstein-Podolsky-Rosen Paradox in Twin Images'*, *Phys Rev Lett, 113,* 160401 (17 Oct 2014)
31 Quintino, Vertesi, and Brunner, 2014, '*Joint Measureability, Einstein-Podolsky-Rosen Steering, and Bell Nonlocality*', *Phys Rev Lett, 113,* 160402 (17 Oct 2014)
32 A Grant, 2014, '*Not-So-Real Particles*', *Science News*, 18 Oct 2014
33 http://iopscience.iop.org/1751-8121/47/42 , 2014, J.Phys. A: Math. Theor. 47