Riemann Hypothesis

An Original Approach: A Ratio Method

Abstract

The simplest solution is usually the best solution---Albert Einstein

A Breath of Fresh Air

Assuming the sum of the original Riemann series is L, a ratio method was used to split-up the series equation into sub-equations and each sub-equation was solved in terms of L, and ratio terms. It is to be noted that unquestionably, each term of the series equation contributes to the sum, L, of the series. There are infinitely many sub-equations and solutions corresponding to the infinitely many terms of the series equation. After the sum, L, and the ratio terms have been determined and substituted in the corresponding equations, the Riemann hypothesis would surely be either proved or disproved, since the original equation is being solved. Solving the original series equation eliminates possible hidden flaws in derived equations and consequent solutions.

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Preliminaries

Here, one covers examples to illustrate the mathematical validity of how one splits up the series equation to form sub-equations.
Let one think like a child - Albert Einstein. Actually, one can think like an eighth or a ninth grader. Suppose one performs the following operations:

Example 1: $10+20+25=55$ (1)

$10=55\times\frac{10}{55}=55\times\frac{2}{11}$ (2)

$20=55\times\frac{20}{55}=55\times\frac{4}{11}$ (3)

$25=55\times\frac{25}{55}=55\times\frac{5}{11}$ (4)

Equations (2), (3), and (4) can be written as follows:

$10=55a$ (5)

$20=55b$ (6)

$25=55c$ (7)

One will call a, b and c ratio terms.

One is dividing 55 between 10, 20, and 25 in the ratio $a:b:c$.

Above, $a=\frac{2}{11}$, $b=\frac{4}{11}$, $c=\frac{5}{11}$. Observe also that $a+b+c=1$ $\left(\frac{2}{11}+\frac{4}{11}+\frac{5}{11}=\frac{11}{11}=1\right)$

One can conclude that the sum of the ratio terms is always 1.

Example 2: Solve the quadratic equation; $6x^2+11x-10=0$.

Method 1 (a common and straightforward method)

By factoring, $6x^2+11x-10=0$ $\quad(3x-2)(2x+5)=0$ and solving,

$(3x-2)=0$ or $(2x+5)=0$

$x=\frac{2}{3}, x=-\frac{5}{2}$. Solution set: $\{-\frac{5}{2}, \frac{2}{3}\}$

Method 2: One applies a **ratio method.**

Step 1**:** From $6x^2+11x-10=0$ (1)

$6x^2+11x=10$

$6x^2=10a$; (Here, a is a ratio term)

$3x^2=5a$ (2)

$11x=10b$ (Here, b is a ratio term)

$11x=10(1-a)$ $\quad (a+b)=1$

$11x=10-10a$

$x=\frac{10-10a}{11}$

$3(\frac{10-10a}{11})^2=5a$ (Substituting for x in (2)

$3(\frac{100-200a+100a^2)}{121}=5a$

Step 2: $300a^2-1205a+300=0$

$60a^2-241a+60=0$

$a=\frac{241\pm\sqrt{241^2-4(60)(60)}}{120}$

$a=\frac{241\pm\sqrt{43681}}{120}$

$a=\frac{241\pm 209}{120}$

$a=\frac{241\pm 209}{120}=\frac{241+209}{120}$ or $\frac{241-209}{120}$

$=\frac{450}{120}$ or $\frac{32}{120}$

$=\frac{15}{4}$ or $\frac{4}{15}$

Step 3: Since $a+b=1$, when $a=\frac{15}{4}$ or $3\frac{3}{4}$

$$b=1-3\tfrac{3}{4}=-2\tfrac{3}{4} \text{ or } -\frac{11}{4}$$

$$\text{when } a=\frac{4}{15},\ b=1-\frac{4}{15}=\frac{11}{15}$$

Step 4: When $b=-\frac{11}{4}$, $11x=10(-\frac{11}{4})$

$$x=-\frac{5}{2}$$

When $b=\frac{11}{15}$, $11x=10(\frac{11}{15})$

$$x=\frac{10}{11}(\frac{11}{15});\ x=\frac{2}{3}$$

Again, one obtains the same solution set $\{-\frac{5}{2}, \frac{2}{3}\}$ as by the factoring method.

In the above problem, one could say that if $6x^2$ and $11x$ are in the ratio $a:b$ and the sum of $6x^2$ and $11x$ is 10, then $6x^2=10a$ and $11x=10b$. (Divide 10 between $6x^2$ and $11x$ in the ratio $a:b$) Also, finding the zeros of $f(x)=6x^2+11x-10$ is equivalent to solving $6x^2+11x=10$ for x.

Example 3: A grandmother left \$45,000 in her will to be divided between eight grandchildren, Betsy, Comfort, Elaine, Ingrid, Elizabeth, Maureen, Ramona, Marilyn, in the ratio $\frac{1}{36}:\frac{1}{18}:\frac{1}{12}:\frac{1}{9}:\frac{5}{36}:\frac{1}{6}:\frac{7}{36}:\frac{2}{9}$. (**Note**: $\frac{1}{36}+\frac{1}{18}+\frac{1}{12}+\frac{1}{9}+\frac{5}{36}+\frac{1}{6}+\frac{7}{36}+\frac{2}{9}=1$)
How much does each receive?

Solution:

Betsy's share of \$45,000 $=\frac{1}{36}\times\$45{,}000=\$1{,}250$

Comfort's share of \$45,000 $=\frac{1}{18}\times\$45{,}000=\$2{,}500$

Elaine's share of \$45,000 $=\frac{1}{12}\times\$45{,}000=\$3{,}750$

Ingrid's share of \$45,000 $=\frac{1}{9}\times\$45{,}000=\$5{,}000$

Elizabeth's share of \$45,000 $=\frac{5}{36}\times\$45{,}000=\$6{,}250$

Maureen's share of \$45,000 $=\frac{1}{6}\times\$45{,}000=\$7{,}500$

Ramona's share of \$45,000 $=\frac{7}{36}\times\$45{,}000=\$8{,}750$

Marilyn's share of \$45,000 $=\frac{2}{9}\times\$45{,}000=\$10{,}000$

Check: Sum of shares $\boxed{=\$45{,}000}$
Sum of the fractions = 1

Example 4: The returns on investments A, B, C, D are in the ratio $a:b:c:d$. If the total return on these four investments is P dollars, what is the return on each of these investments? $(a+b+c+d=1)$

Solution Return on investment $A=aP$ dollars
Return on investment $B=bP$ dollars
Return on investment $C=cP$ dollars
Return on investment $D=dP$ dollars

Check: $aP+bP+cP+dP=P$

$$P(a+b+c+d)=P$$

$$a+b+c+d=1 \quad \text{(dividing both sides by } P\text{)}$$

Example 5: The returns on investments $\frac{1}{2^z}$, $\frac{1}{3^z}$, $\frac{1}{4^z}$, $\frac{1}{5^z}$, are in the ratio $\lambda_2 : \lambda_3 : \lambda_4 : \lambda_5$.

If the total return on these four investments is L, what is the return on each of these investments? ($\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1$).

Solution Return on investment $\frac{1}{2^z} = \lambda_2 L$

Return on investment $\frac{1}{3^z} = \lambda_3 L$

Return on investment $\frac{1}{4^z} = \lambda_4 L$

Return on investment $\frac{1}{5^z} = \lambda_5 L$

Check: $\lambda_2 L + \lambda_3 L + \lambda_4 L + \lambda_5 L = L$

$L(\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) = L$

($\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1$) (dividing both sides by L)

Example 6: Sir Isaac Newton left ρg_x units in his will to be divided between $-\mu\frac{\partial^2 v_x}{\partial x^2}$, $-\mu\frac{\partial^2 v_x}{\partial y^2}$, $-\mu\frac{\partial^2 v_x}{\partial z^2}$, $\frac{\partial p}{\partial x}$, $\rho\frac{\partial v_x}{\partial t}$, $\rho V_x\frac{\partial v_x}{\partial x}$, $\rho V_y\frac{\partial v_x}{\partial y}$, $\rho V_z\frac{\partial v_x}{\partial z}$ in the ratio $a:b:c:d:f:h:m:n$.

where $a+b+c+d+f+h+m+n=1$. How much does each receive?

Solution $-\mu\frac{\partial^2 v_x}{\partial x^2}$'s share of ρg_x units $= a\rho g_x$ units

$-\mu\frac{\partial^2 v_x}{\partial y^2}$'s share of ρg_x units $= b\rho g_x$ units

$-\mu\frac{\partial^2 v_x}{\partial z^2}$'s share of ρg_x units $= c\rho g_x$ units

$\frac{\partial p}{\partial x}$'s share of ρg_x units $= d\rho g_x$ units

$\rho\frac{\partial v_x}{\partial t}$'s share of ρg_x units $= f\rho g_x$ units

$\rho V_x\frac{\partial v_x}{\partial x}$'s share of ρg_x units $= h\rho g_x$ units

$\rho V_y\frac{\partial v_x}{\partial y}$'s share of ρg_x units $= m\rho g_x$ units

$\rho V_z\frac{\partial v_x}{\partial z}$'s share of ρg_x units $= n\rho g_x$ units

Sum of shares = $\boxed{\rho g_x \text{ units}}$ **Note**: $a+b+c+d+f+h+m+n=1$

Example 7: Professor Bernhard Riemann left the sum, L, of the comvergent series, $\sum_{n=1}^{\infty}\frac{1}{n^z}$ in his will to be divided between the terms, $\frac{1}{2^z}, \frac{1}{3^z}, \frac{1}{4^z}, \frac{1}{5^z}$, ... of the series in the ratio $\lambda_2 : \lambda_3 : \lambda_4 : \lambda_5$,... where the sum of the $\lambda' s$ equals 1.
How much does each term receive?

Solution $\frac{1}{2^z}' s$ share of $L = \lambda_2 L$

$\frac{1}{3^z}'$s share of $L = \lambda_3 L$

$\frac{1}{4^z}' s$ share of $L = \lambda_4 L$

$\frac{1}{5^z}' s$ share of $L = \lambda_5 L$

....

. ...

Sum of the terms = L **Note**: The sum of the $\lambda' s$ .equals 1.

The objective of presenting examples 1, 2, 3, 4, 5, and 6 was to convince the reader that the principles to be used in splitting the terms of the convergent Riemann series, are valid.
In Example 2, one could have used the quadratic formula directly to solve for x, without finding a and b first. The objective was to convince the reader that the introduction of a and b did not change the solution set of the original equation.

Nota bene

1. Finding the zeros of $f(x) = 6x^2 + 11x - 10$ is equivalent to solving $6x^2 + 11x = 10$ for x.
   (Note: $f(x) = 6x^2 + 11x - 10 = 0$)

2. Similarly, finding the zeros of $f(z) = \sum_{n=1}^{\infty}\frac{1}{n^z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + ...$ is

   equivalent to solving $\sum_{n=1}^{\infty}\frac{1}{n^z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + ... = L$ for z, where L is the sum of the series.

   (Note: $\sum_{n=1}^{\infty}\frac{1}{n^z} - L = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + ... - L = 0$)

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Given: $f(z)=\sum_{n=1}^{\infty}\frac{1}{n^z}=\frac{1}{1^z}+\frac{1}{2^z}+\frac{1}{3^z}+...\quad \text{Re}(z)=x>1$

$$z=x+iy$$

Required: To find the zeros of $f(z)$.

Plan: One will assume the sum of the above series is L, and equivalently solve $\sum_{n=1}^{\infty}\frac{1}{n^z}=\frac{1}{1^z}+\frac{1}{2^z}+\frac{1}{3^z}+...=L$ for z, where L is the sum of the series.
One will split-up the series equation to form sub-equations using ratio terms and then solve for z.

Let the sum, L, of the convergent series be divided between the terms, $\frac{1}{2^z}$, $\frac{1}{3^z}$, $\frac{1}{4^z}$, $\frac{1}{5^z}$, ... of the series in the ratio $\lambda_2:\lambda_3:\lambda_4:\lambda_5,...$ where the sum of the $\lambda's$ equals 1.
Two types of ratio terms will be used, namely, primary ratio terms and secondary ratio terms. The primary ratio terms will be symbolized by $\lambda's$, and the secondary ratio terms will be symbolized by $\beta's$.

(Note that unquestionably, each term of the series contributes to the sum, L, of the series.)

Each sub-equation from the series yields a relation, and the relations for which $\text{Re}(z)=x>1$ will be accepted as solutions. From the terms of the series and the ratio terms, one obtains the following sub-equations which are for the first four terms, skipping $n=1$. There are infinitely many sub-equations and solutions corresponding to the infinitely many terms of the series.

1. $\frac{1}{2^z}=\lambda_2 L$; **2.** $\frac{1}{3^z}=\lambda_3 L$; **3**. $\frac{1}{4^z}=\lambda_4 L$; 4. $\frac{1}{5^z}=\lambda_5 L$. ($\lambda_2,\lambda_3,\lambda_4$ and λ_5 are ratio terms)

Solutions of the Sub-equations

Case 1: For $n=2$, $\frac{1}{2^z}=\lambda_2 L$

Step 1:	Step 2:
Let $\sum_{n=1}^{\infty}\frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{2^z}=\lambda_2 L$ (λ_2 is a ratio term) $2^{-z}=\lambda_2 L$ $\log 2^{-z}=\log\lambda_2 L$ $-z\log 2=\log\lambda_2 L$ $z\log 2=-\log\lambda_2 L$ $z=-\frac{\log\lambda_2 L}{\log 2}$	$z=x+iy=-\frac{\log\lambda_2 L}{\log 2}$ $x=-\beta_2\frac{\log\lambda_2 L}{\log 2}$ (β_2 is a secondary ratio term) $iy=-\beta_3\frac{\log\lambda_2 L}{\log 2}$ (β_3 is a secondary ratio term) ($\beta_2+\beta_3=1$) $y=i\beta_3\frac{\log\lambda_2 L}{\log 2}$

Step 3: Determine the sum L of the series; determine λ_2, β_2, β_3 and substitute.
To be continued.

Case 2: For $n = 3, \frac{1}{3^z} = \lambda_3 L$

Step 1: Let $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{3^z} = \lambda_3 L$ (λ_3 is a ratio term) $3^{-z} = \lambda_3 L$ $\log 3^{-z} = \log \lambda_3 L$ $-z \log 3 = \log \lambda_3 L$ $z \log 3 = -\log \lambda_3 L$ $z = -\frac{\log \lambda_3 L}{\log 3}$	**Step 2:** $z = x + iy = -\frac{\log \lambda_3 L}{\log 3}$ $x = -\beta_4 \frac{\log \lambda_3 L}{\log 3}$ (β_4 is a secondary ratio term) $iy = -\beta_5 \frac{\log \lambda_3 L}{\log 3}$ (β_5 is a secondary ratio term) $(\beta_4 + \beta_5 = 1)$ $y = i\beta_5 \frac{\log \lambda_3 L}{\log 3}$
Step 3 Determine λ_3, β_4, β_5 and substitute. (Also substitute for L from Case 1) To be continued.	

Case 3: For $n = 4 \; \frac{1}{4^z} = \lambda_4 L$

Step 1: Let $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{4^z} = \lambda_4 L$ (λ_4 is a ratio term) $4^{-z} = \lambda_4 L$ $\log 4^{-z} = \log \lambda_4 L$ $-z \log 4 = \log \lambda_4 L$ $z \log 4 = -\log \lambda_4 L$ $z = -\frac{\log \lambda_4 L}{\log 4}$	**Step 2:** $z = x + iy = -\frac{\log \lambda_4 L}{\log 4}$ $x = -\beta_6 \frac{\log \lambda_4 L}{\log 4}$ (β_6 is a secondary ratio term) $iy = -\beta_7 \frac{\log \lambda_4 L}{\log 4}$ (β_7 is a secondary ratio term) $(\beta_6 + \beta_7 = 1)$ $y = i\beta_7 \frac{\log \lambda_4 L}{\log 4}$
Step 3 Determine λ_4, β_6, β_7 and substitute. (Also substitute for L from Case 1) To be continued.	

Case 4: For $n = 5$, $\frac{1}{5^z} = \lambda_5 L$

Step 1:	**Step 2:**
Let $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{5^z} = \lambda_5 L$ (λ_5 is a ratio term) $5^{-z} = \lambda_5 L$ $\log 5^{-z} = \log \lambda_5 L$ $-z \log 5 = \log \lambda_5 L$ $z \log 5 = -\log \lambda_5 L$ $z = -\frac{\log \lambda_5 L}{\log 5}$	$z = x + iy = -\frac{\log \lambda_5 L}{\log 5}$ $x = -\beta_8 \frac{\log \lambda_5 L}{\log 5}$ (β_8 is a secondary ratio term) $iy = -\beta_9 \frac{\log \lambda_5 L}{\log 5}$ (β_9 is a secondary ratio term) $(\beta_8 + \beta_9 = 1)$ $y = i\beta_9 \frac{\log \lambda_5 L}{\log 5}$

Step 3

Determine λ_5, β_8, β_9 and substitute. (Also substitute for L from Case 1)
To be continued.

Other solutions

Similarly, as in cases 1-4, one can continue solving the sub-equations from the series for $n = 6$, and so on. There will be infinitely many solutions.

General Case : $\boxed{\frac{1}{n^z} = \lambda_n L}$

Step 1:	**Step 2:**
Let $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{n^z} = \lambda_n L$ (λ_n is a ratio term) $n^{-z} = \lambda_n L$ $\log n^{-z} = \log \lambda_n L$ $-z \log n = \log \lambda_n L$ $z \log n = -\log \lambda_n L$ $z = -\frac{\log \lambda_n L}{\log n}$	$z = x + iy = -\frac{\log \lambda_n L}{\log n}$ $x = -\beta_{2n-2} \frac{\log \lambda_n L}{\log n}$ (β_{2n-2} is a secondary ratio term) $iy = -\beta_{2n-1} \frac{\log \lambda_n L}{\log n}$ (β_{2n-1} is a secondary ratio term) $(\beta_{2n-2} + \beta_{2n-1} = 1)$ $y = i\beta_{2n-1} \frac{\log \lambda_n L}{\log n}$

Step 3

Determine λ_n, β_{2n-2}, β_{2n-1} and substitute. (Also substitute for L from Case 1)
To be continued.

Case Extra: For $n = 10^{12}$ $\dfrac{1}{10^{12z}} = \lambda_{10^{12}} L$

Step 1:	Step 2:
Let $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converge to the limit L. Then $\frac{1}{10^{12z}} = \lambda_{10^{12}} L$ ($\lambda_{10^{12}}$ is a ratio term) $10^{-12z} = \lambda_{10^{12}} L$ $\log 10^{-12z} = \log \lambda_{10^{12}} L$ $-12z \log 10 = \log \lambda_{10^{12}} L)$ $z = -\frac{\log \lambda_{10^{12}} L}{12 \log 10}$	$z = x + iy = -\frac{\log \lambda_{10^{12}} L}{12 \log 10}$ $x = -\beta_{2(10^{12})-2} \frac{\log \lambda_{10^{12}} L}{12 \log 10}$ ($\beta_{2(10^{12})-2}$ is a ratio term $iy = -\beta_{2(10^{12})-1} \frac{\log \lambda_{10^{12}} L}{12 \log 10}$ ($\beta_{2(10^{12})-1}$ is a ratio term) ($\beta_{2(10^{12})-2} + \beta_{2(10^{12})-1} = 1$) $y = i\beta_{2(10^{12})-1} \frac{\log \lambda_{10^{12}} L}{12 \log 10}$

Step 3

Determine $\lambda_{10^{12}}$, $\beta_{10^{12}}$, $\beta_{10^{12}+1}$ and substitute. (Also substitute for L from Case 1)

To be continued.

Summary for the First Four Solutions ($n = 2, 3, 4, 5$)

$$\begin{bmatrix} x = -\beta_2 \dfrac{\log \lambda_2 L}{\log 2} \quad (\beta_2 \text{ is a secondary ratio term}) \\ y = i\beta_3 \dfrac{\log \lambda_2 L}{\log 2} \end{bmatrix}$$

$$\begin{bmatrix} x = -\beta_4 \dfrac{\log \lambda_3 L}{\log 3} \quad (\beta_4 \text{ is a secondary ratio term}) \\ y = i\beta_5 \dfrac{\log \lambda_3 L}{\log 3} \end{bmatrix}$$

$$\begin{bmatrix} x = -\beta_6 \dfrac{\log \lambda_4 L}{\log 4} \quad (\beta_6 \text{ is a secondary ratio term}) \\ y = i\beta_7 \dfrac{\log \lambda_4 L}{\log 4} \end{bmatrix}$$

$$\begin{bmatrix} x = -\beta_8 \dfrac{\log \lambda_5 L}{\log 5} \quad (\beta_8 \text{ is a secondary ratio term}) \\ y = i\beta_9 \dfrac{\log \lambda_5 L}{\log 5} \end{bmatrix}$$

Observation of the above solutions shows that for the real parts to be equal,
$\beta_2(\log \lambda_2 L)(\log 3) = \beta_4(\log \lambda_3 L)(\log 2)$ (for $n = 2, 3$)
To be continued.