ANNEX V_ RESULTS ANALYSIS FOR SEVERAL HIERARCHY TYPES

Let us review the results of each proposed aggregation function, by using them in several examples of hierarchies that represent various possible situations:

- Two-level hierarchies
  - Equally likely indicators [equally relevant]
  - Non-equally likely indicators [different relevance]
- Multilevel Hierarchies
  - Three levels hierarchy, with seven elementary indicators at the same level
  - Four levels hierarchy, with elementary basic indicators at different levels

For each of the four examples, we review the results obtained using three sets of 11 values each, with the following characteristics:

- Series 01: the values of the indicators grow or shrink evenly, keeping constant their arithmetic mean.
- Series 02: the values of the indicators are randomly chosen in the range 0.15 to 0.85[1]
- Series 03: the values of the indicators increase monotonically.

As a basis for the comparison we use the *Certainty Degree* function for two reasons:

- On one hand, numerous approaches to measure complexity rely on information measurement using Entropy formula.
- On the other hand, the Certainty Degree function informs us of the membership to certainty and uncertainty [complementary value] classes, whose importance has already been underlined.

And we will compare the aggregated values obtained using the Certainty Degree function against those provided by the other three proposed aggregation functions:

- $Rs[I]_{\%}$_ Resilience Degree
- GM_ Geometric Mean
- HM_ Harmonic Mean

In addition, we review three other aggregation functions, which are:

- MIN_ minimum
- AM_ Arithmetic mean
- Max_ Maximum

We introduce Minimum and Maximum because they are aggregation functions widely used in set theory, while Arithmetic Mean is commonly used for aggregating information [although we have indicated it cannot be a valid aggregating function for complexity manifestations since it does not meet Ax.00].

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[1] The range of values is limited to reduce the distortion introduced by the Geometric Mean and Harmonic Mean for values near to zero, and the *Certainty Degree* function for values equal to 1

A-V.1_ TWO LEVEL HIERARCHY WITH EQUIPROBABLE INDICATOR

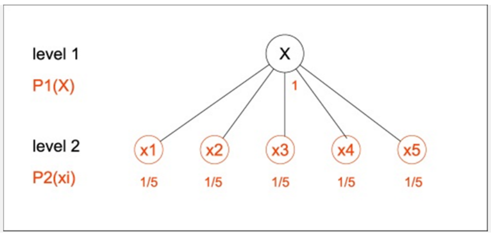
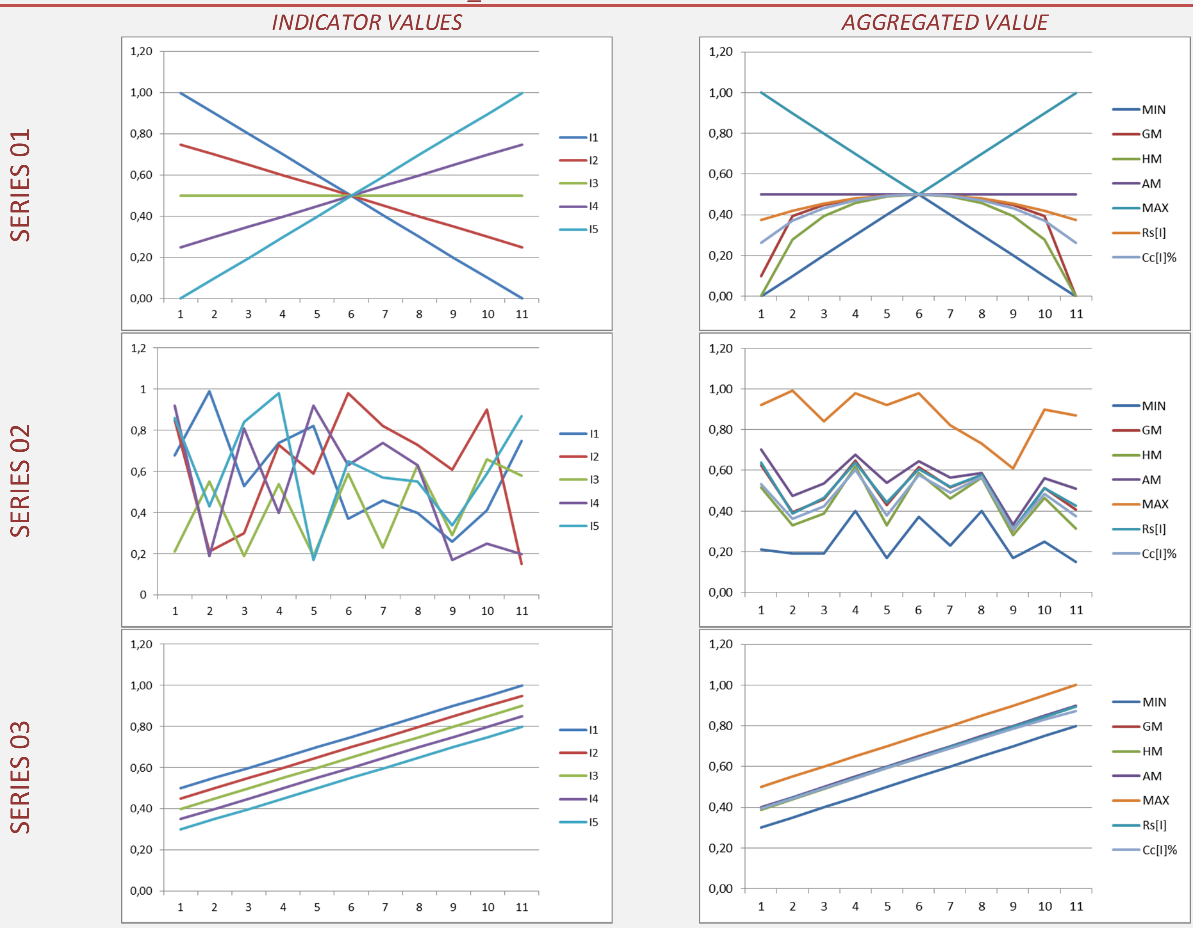


Figure 45: Hierarchical representation of the probabilities assignment [example 01]

It is equivalent to equally relevant rules/indicators for the emergence at the global level.

This is the simplest example of the four we are going to review. In this case, we obtain the following graphs:

EXAMPLE 01_GRAPHIC REPRESENTATION OF DATA



Source: Own elaboration. The codes mean the folowing: MIN [minimum], GM [Geometric Mean], HM [Harmonic Mean], AM [Arithmetic Mean], MAX [max]; Rs% [Resilience Degree]; Cc% [Certainty Degree].

And the following deviations and correlations related to the Certainty Degree:

COVARIATION RESULTS IN RELATION TO $C_C[I]_{\%}$

	SERIES 01			SERIES 02			SERIES 03		
	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION
MÍN	95,47%	3,65%	19,10%	80,47%	3,50%	18,70%	99,98%	0,79%	8,87%
GM	99,87%	0,02%	1,44%	97,54%	0,16%	3,99%	99,98%	0,01%	0,99%
HM	99,83%	0,21%	4,62%	99,51%	0,02%	1,25%	99,99%	0,01%	0,82%
AM	-0,02%	0,50%	7,07%	91,19%	0,67%	8,20%	99,98%	0,02%	1,28%
MÁX	-95,47%	10,67%	32,67%	77,69%	11,79%	34,33%	99,98%	1,25%	11,16%
$Rs[I]_{\%}$	99,94%	0,07%	2,65%	97,64%	0,13%	3,62%	99,98%	0,01%	0,87%

SOURCE: Own elaboration.

The data in the above tables allow us to check functions' compliance with the Axioms:

- A function does not meet Ax.00_ Non-linearity:
  - Arithmetic Mean function.
- No function meets at the same time Axioms 04 [Non-Emergence] and 05 [Emergence]:
  - Arithmetic mean, Maximum and Resilience Degree $Rs[I]_{\%}$, provide an aggregated value different from 0 if any indicator has a non-zero value.
  - Minimum, Geometric Mean and Harmonic Mean provide a zero aggregated value if any indicator value is zero.
  - Certainty Degree $C_c[I]_{\%}$ does not differentiate between indicators' values 0 and 1, providing a zero value in both cases.
- Three functions do not meet the Ax.08_ Monotonicity:
  - Minimum, if the minimum is not modified, the aggregated value does not modify.
  - Maximum, if the maximum does not change, the aggregated value does not modify.
  - Geometric and Harmonic Averages if any indicator is 0.

Non-compliance with the Axiomatic System by formulations Resilience Degree $Rs[I]_{\%}$, Geometric Mean [GM] and Harmonic Mean [HM] is easily solved by simply choosing one formulation or another depending of the type of indicators included in each aggregation subsystem.

Non-compliance with the Axiomatic System of Certainty Degree $C_c[I]_{\%}$ cannot be solved, and we dismiss it as aggregating function[2]. However, noncompliance is limited to a very few specific cases, so we use it for the following comparison.

Non-compliance of Ax.00 by Arithmetic Mean [AM] excludes its use as aggregation function in complex concepts/phenomena.

Non-compliance with the Axiomatic System of Minimum [Min] and Maximum functions [Max] cannot be solved, and we dismiss these formulations as aggregating functions.

Additionally, the comparison of the values obtained using the different formulations shows high covariation between formulations Resilience Degree $R_s[I]_{\%}$, Certainty Degree $C_c[I]_{\%}$, Geometric Mean and Harmonic Mean. This allows us to state their validity as formulations for calculating Complexity Degree [with the $C_c[I]_{\%}$ exception commented above].

A-V.2_ TWO LEVEL HIERARCHY WITH NON-EQUIPROBABLE INDICATORS

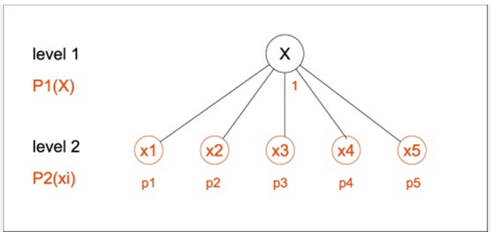


Figure 46: Hierarchical representation of the probabilities assignment [example 02]

It is equivalent to non-equally relevant rules/indicators for the emergence at the global level.

[2] Non-compliance of Certainty Degree with the System of Axioms has in fact a conceptual origin. While our complete certainty on the veracity of an assertion X implies denying the opposite assertion non-X; our complete uncertainty relating the veracity of X does not imply asserting non-X. Ax.04 can never be satisfied by this function, except in the case x=certainty/uncertainty, in which it is a valid aggregation function.

This is the second example of formulation application, in which the following graphs have been obtained:

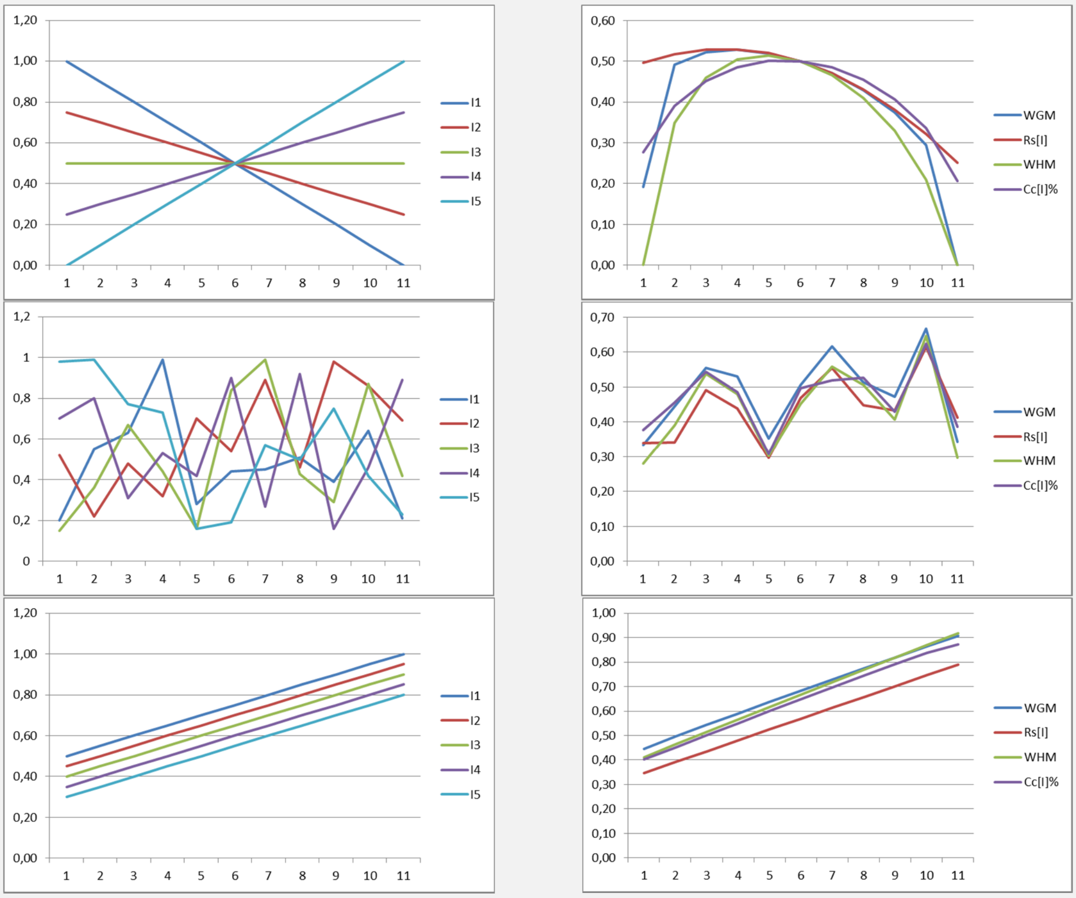
EXAMPLE 02_GRAPHIC REPRESENTATION OF DATA

INDICATOR VALUES *AGGREGATED VALUE*

SERIES 01

SERIES 02

SERIES 03



Source: Own elaboration. Codes are WGM [Weighted Geometric Mean] and WHM [Weighted Harmonic Mean].

And the following deviations and correlations related to the Certainty Degree:

COVARIATION RESULTS IN RELATION TO $C_C[I]_{\%}$

	SERIES 01			SERIES 02			SERIES 03		
	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION
WGM (1)	94,42%	0,23%	4,81%	96,16%	0,16%	4,03%	99,98%	0,13%	3,61%
WHM (1)	97,98%	0,29%	5,41%	93,67%	0,16%	4,06%	99,96%	0,05%	2,30%
Rs[I]%	69,07%	0,29%	5,36%	95,32%	0,31%	5,54%	99,96%	0,60%	7,77%

SOURCE: Own elaboration.

(1) Geometric and Harmonic Mean are weighted by the probabilities of each indicator.

(2) Probabilities considered in Series 01 and 02 are p1=0.33; p2=0.25; p3, p4 and p5=0.14. In Series 02 the value is randomly generated for each case establishing a possible range p1=0.15-0.50; p2=(1-p1)/2 and p3=(1-p1-p2)/3.

(3) It is a bit surprising $Rs[I]_{\%}$ deviation in Series 01 [somewhat higher than desirable], motivated by the fact that the highest probability assignments meet the highest values in the initial cases of such Series [left side of graphic]

Data still show high correlation and low deviation of the revised three functions in relation to Certainty Degree Cc[I]%.

A-V.3_ THREE LEVEL HIERARCHY

Let us review a three-level hierarchy in which elementary indicators are grouped into three subsystems located at the same level.

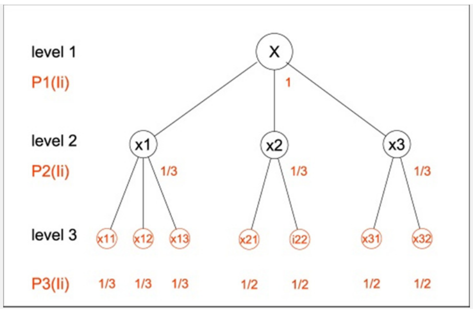


Figure 47: Hierarchical representation of the probabilities assignment [example 03]

Calculation has been made according to the explained procedure, obtaining the following graphs:

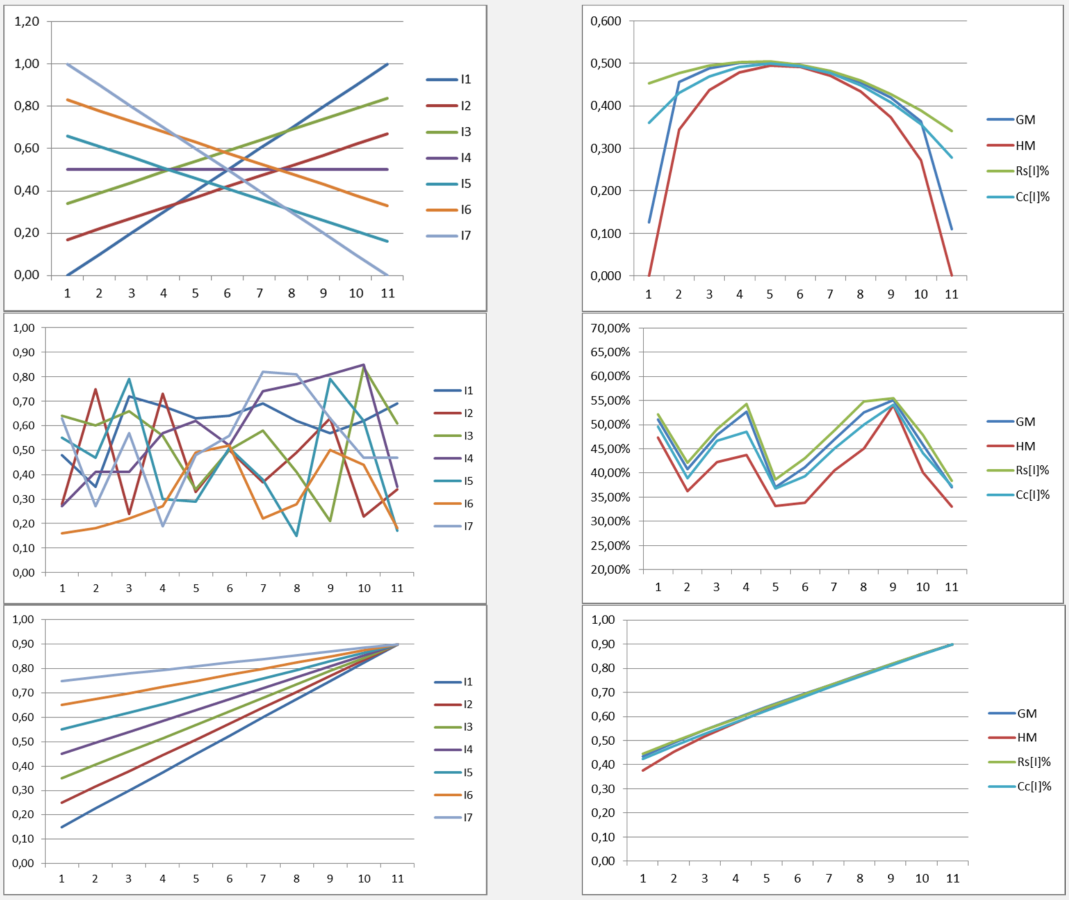
EXAMPLE 03_GRAPHIC REPRESENTATION OF DATA

INDICATOR VALUES | *AGGREGATED VALUE*

SERIES 01

SERIES 02

SERIES 03



Source: Own elaboration

And the following deviations and correlations related to the Certainty Degree:

COVARIATION RESULTS IN RELATION TO $C_C[I]_{\%}$

	SERIES 01			SERIES 02			SERIES 03		
	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION
GM	99,45%	0,03%	1,75%	98,43%	0,01%	0,93%	99,98%	0,01%	1,05%
HM	99,49%	0,05%	2,33%	97,93%	0,04%	1,98%	99,79%	0,03%	1,68%
Rs[I]%	98,50%	0,08%	2,87%	93,69%	0,16%	3,95%	100,00%	0,01%	1,19%

SOURCE: Own elaboration

Data still show high correlation and low deviation of the revised three functions in relation to Certainty Degree $Cc[I]_{\%}$.

A-V.4_ FOUR LEVEL HIERARCHY

This is an example of hierarchy with indicators distributed into various levels and subsystems, leading to coexistence at each level of elementary and aggregated indicators. It has considerable interest, since its 'structure' is similar to those we find in reality.

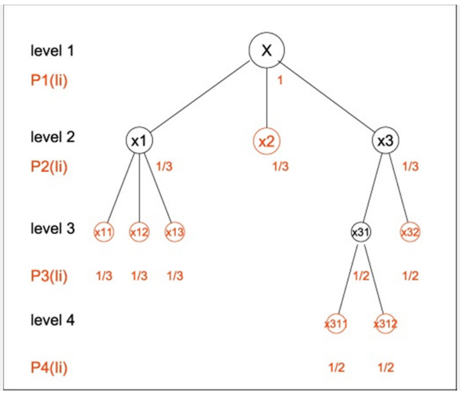


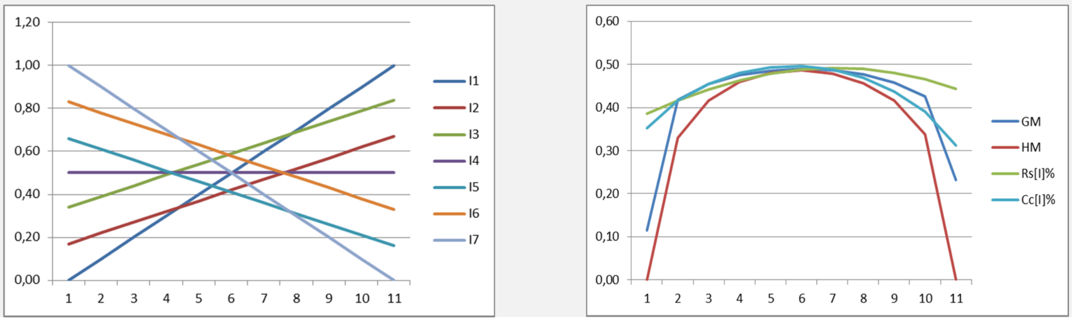
Figure 48: Hierarchical representation of the probabilities assignment [example 04]

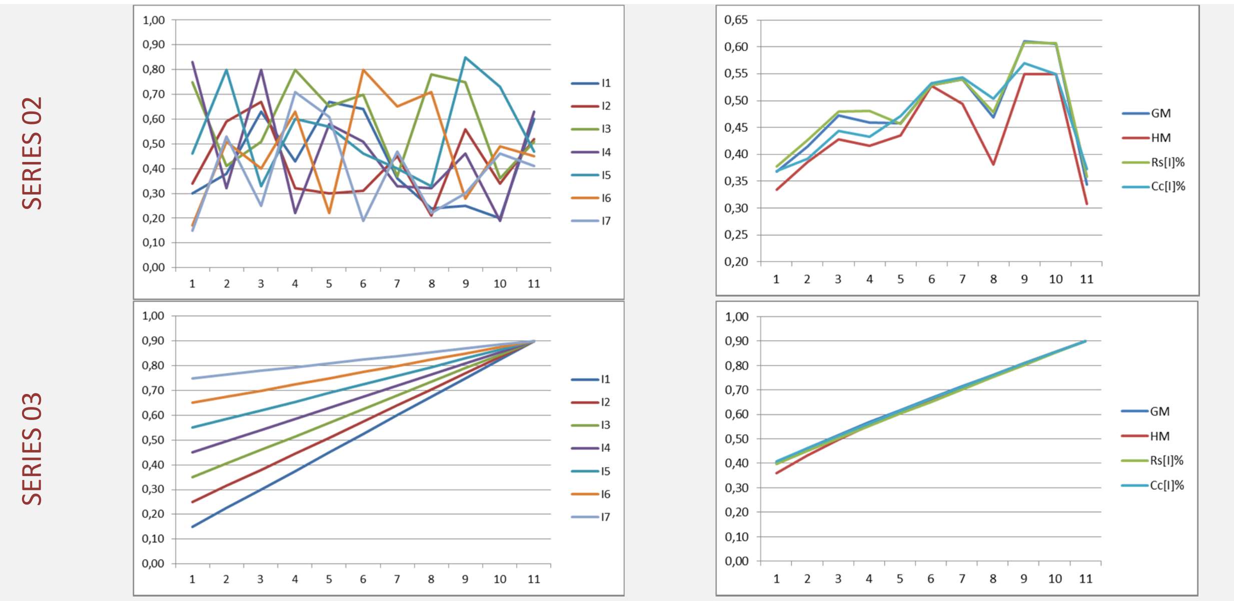
The calculation has been done in an identical manner to the previous case, obtaining the following graphs:

EXAMPLE 04_GRAPHIC REPRESENTATION OF DATA

SERIES 01

INDICATOR VALUES | *AGGREGATED VALUE*





Source: Own elaboration

And the following deviations and correlations related to the Certainty Degree:

COVARIATION RESULTS IN RELATION TO $C_C[I]_{\%}$

	SERIES 01			SERIES 02			SERIES 03		
	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION	CORRELATION	VARIANCE	DEVIATION
GM	95,14%	0,02%	1,46%	98,08%	0,07%	2,73%	99,99%	0,00%	0,23%
HM	96,22%	0,15%	3,82%	96,14%	0,29%	5,37%	99,88%	0,03%	1,81%
Rs[I]%	67,31%	0,25%	3,14%	97,29%	0,08%	2,74%	99,99%	0,01%	0,95%

FUENTE: Own elaboration

(1) We see that Series 01 again shows a less desirable correlation between Cx[I]% and Rs [I]% but it is corrected for the other two series, therefore it is not so relevant due to the 'uniqueness' of Series 01.

The review of data still show high correlations and low deviation of function $C_c[I]_{\%}$ in relation to the other three revised functions, confirming the validity of the four formulations as functions for measuring objects' Complexity Degree in the stated conditions.