ANNEX VII_ NEARLY DECOMPOSABLE COMPLEX CONCEPTS

We reason in a fuzzy manner attributing meaning to objects and statements [conceptual objects], and conceptualizing complexity as information with meaning [meaningful information] implies that *we will be able to use the procedures and Axiomatic herein proposed to model any possible statement about an object.*

Any possible assertion relating to an object can be transformed in a *fuzzy proposition*[1] about said object, and Emergence Degree measures may be interpreted as measures of our 'degree of belief in the veracity of those claims' equivalent to the 'Emergence Degree of the meaning that such statement implies in an object'.

In this sense, proposed complexity measures have two possible readings:

- *One level emergence Complexity Degree* measures coincide with the logical statements implicit in the membership functions of Fuzzy Set Theory and Fuzzy Logic [Zadeh, 1965].
- *Several emergence levels Complexity Degree* coincide with the idea of Fuzzy Hierarchies [Fuzzy Sets Signatures] and Logical Decomposition [Alvira, 2014a]

While the first represent currently –more or less- resolved issues, the latter give us some key issues for modeling and aggregating information still unresolved in current models, especially aggregation formulations and the Axioms System.

Many of currently most important issues refer to concepts X whose degree of truth we cannot directly assess because it depends on many interdependent variables, but we can check if we analyze them; i.e., we decompose them into *logical hierarchies or systems of constraints.*

These hierarchies are composed of sub-concepts x_i which are also 'fuzzy', whose degree of truth sometimes we can assess directly but sometimes we cannot, so we need to break them down again. Hierarchies represent an *analysis or decomposition* of the initial concept into a structure we can *synthesize or reconstruct* following the proposals of the present theory.

This logical decomposition of concepts exists under different names in different theoretical bodies[2], announcing us a universality that allows us to analyze them transversely to the various scientific disciplines [i.e.: transdisciplinary] and alludes to two issues:

- To the qualities of these concepts, which present a similar structure, regardless of the chosen analytical prospect
- To our own way of thinking and reasoning.

The number and variety of concepts that fall into this category is very high including many commonly used in psychology [depression, happiness, talent, intelligence, ...], social science and economics

[1] Let us consider for this review that statements with fewer possible truth values [multivalued or binary logic], are simply particular cases [restricted] of fuzzy statements.

[2] Analytical Hierarchical Process [Decision Theory]; Nested Hierarchies [Systems and Hierarchy Theories]; Classifications [Taxonomy/Binary Logic]; Fuzzy Hierarchies [Fuzzy Signatures of Fuzzy Logic]; etc.

[wealth-poverty, development and underdevelopment, social cohesion, ..] biology [sustainability, resilience, etc...], politics [governance, democracy, ...], and etc...

Image 70: Freedom, Equality and Fraternity *French Second Republic converted in 1848 these three concepts [coined during the French Revolution] in a motto which is in fact a logical decomposition of its ideal of society. We could assess its 'success' in building that society from the degree to which each of these ideals is reached and their aggregation, taking into account it is a concept involving order or certainty.*

However, we have already noted that there are two types of essentially different concepts: those involving organization or certainty and those involving disorganization or uncertainty. And to be able to correctly model the truth degree of a concept [i.e., measuring it] it is necessary to first detect which of the two types it is.

The differences between the two types are very important, and to emphasize it, we will review a concept of each of these types which have special significance:

- the **utility or value of a decision,** as an example of a *concept that involves certainty*
- the **difficulty of performing a task,** as an example of a *concept that involves uncertainty*

The modeling of both concepts is of considerable importance today, although not to extend we briefly do this review.

A-VII.2.1_THE UTILITY OR VALUE OF A DECISION

We constantly make decisions. Almost every action we perform requires we have previously 'chosen' from a set of possible actions that we could have done.

The procedure we follow for making a decision is relatively simple. We start by selecting the set of possible options [this is our first decision]; we evaluate the 'utility' we would get from each of them; we sort them in descending order [of utility provided] and finally we chose the option or combination of options that provides us more utility.

Importantly, to make decisions generally *we do not calculate the Complexity Degree conditioned to concept 'utility', but to the concept that is going to provide that utility,* which must meet two conditions:

- **It must be a concept involving 'certainty';** since if a concept involving 'uncertainty' is modeled, the obtained order of preference will be the reverse [this does not exclude any concept from being calculated, since we can always model the opposite concept].
- **It must be a valid decision criterion in the area in which the decision is made;** the concept must refer to *desired/non-desired options or states* [otherwise the decision would not be of any interest]. Some examples of valid concepts for decision-making include: sustainability, fiscal stability, quality, satisfaction, etc.

Image 71: Choosing a restaurant to have dinner *is a common decision and measurable in terms of utility / degree of truth. The phrase 'this is the restaurant I would rather go' is interchangeable with 'this is the restaurant which reports me more utility from all possible'. To choose the restaurant [making the decision] we assess different parameters; price, service, quality, location, environment, etc... We demand that all these factors present acceptable values, while one of them with a lousy value would have us discarding a restaurant, confirming 'utility' is a concept that implies certainty.*

If the above conditions are met, function $C_X[I]_{\%}$ possesses the **four qualities required of utility functions** [Von Neumann & Morgenstern 1944; Binmore 1994]:

- It is complete: either I is preferred, J is preferred or both options are equally preferred. It provides a value for each possible distribution of I_i.

$$\forall I, J \in \Omega: C_x[I]_{\%} < C_x[J]_{\%} \leftrightarrow I \prec J \quad (1)$$

- It is transitive: if J is preferred to I, and K is preferred to J, then K is preferred to I.
- It is continuous, providing a value in the range 0-1.
- It is independent of irrelevant options: if a state J is preferred to a state I, the emergence of another possible state K does not alter the order of preference between J and I.

Function $C_x[I]_{\%}$ provides a *Preference Order* on the possible decisions related to X. If $C_X[I]_{\%} < C_X[J]_{\%}$ then option J is more desired that I [i.e., it is the 'preferred option'] from X perspective [Certainty Degree function is excluded due to the issues already discussed]. Additionally $C_X[I]_{\%}$ function has two interesting features:

Firstly, **it can be considered as an *expected utility function*,** since its formulation involves multiplying *utilities* [values of each indicator] by *probabilities of occurrence* [position of each indicator in the hierarchy]. Any decision based on a utility function $C_X[I]_{\%}$, is then seeking to *maximize expected utility,* a requisite for rational decision making.

Secondly, **it values utility's decreasing marginality,** which implies that an increase of *$fx_i[I]$* value has an impact on the overall value of $C_X[I]_{\%}$ that is reduced as *$fx_i[I]$* increases its value, which we can graphically review:

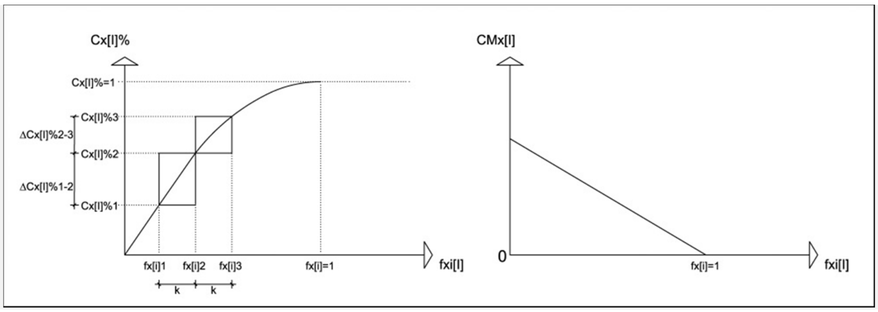

Figure 50: Graphical representation of the Complexity Degree increase for an indicator $fx_i[I]$

The graph on the right represents the slope of the curve [Marginal Complexity] which is decreasing until it reaches zero for $fx_i[I]$=1.

This is quite important for decision making, confirming an issue that standard practice had already shown us; **for a given effort to increase the utility provided by any object, the total obtained utility is greater if such effort is applied in the aspects/indicators with lower values.**

When the assessment relates to the current state of a system to be improved, the effort shall always be made first on those aspects that present worse values [i.e., for which further departure from optimal conditions is reached], a criterion that shall be applied in numerous fields ranging from politics, economy, health, etc...

Formulations of 'Several emergence levels Complexity' may be used for the design of utility functions for rational decision-making. And if the utility is assessed at the SES level [companies, cities, countries, etc...], they constitute collective utility functions which allow us to assess the Desirability Degree of the global state of a system for different courses of action possible, thus constituting a rational basis for decision making.

A-VII.2.2_THE DIFFICULTY OF PERFORMING A TASK

It is another concept whose review is of great interest, since it is an issue that we continuously assess, and it can refer to many different matters: the implementation of a project, the difficulty of managing a computer program, etc...

From the perspective of emergence, the complexity of performing a task occurs due to two types of interactions inherent to their 'dual' character as:

- *Information system* that involves processing different types of interacting information which shape the *overall meaning.*
- *System of actions* [or decisions] that involves the *coordination of different actions.*

And in most cases we find a combination of both. There are sets of information of different types that need to be processed in an interrelated way and it is necessary to provide answers or decisions involving coordination of actions[3].

Therefore, it usually shall be modeled as '**Several emergence levels Complexity',** since different information or actions may have different significance level for the emergence of the *global difficulty.* And it is a concept involving Uncertainty, which can be justified from two perspectives:

- From the **perspective of meaning, it is a concept involving Uncertainty or Ignorance;** when we increase our knowledge about a certain task, its difficulty decreases[4].

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[3] Skating, driving in traffic [or in a mountain road], managing a project, ... fiddling. They all are examples in which the coordination of actions becomes so important as the management of different types of information that are simultaneously received.

[4] For example, the difficulty of understanding a literary text; of piloting a sailboat, etc... In some cases the tasks require physical dexterity, which does not match the definition of 'knowledge' but it does the definition of 'control'.

- **From a *thermodynamics* perspective, it is a concept involving approach to *thermal equilibrium.*** As the difficulty of an action increases, it becomes more improbable and total inactivity [no-activity] may be reached if *maximum difficulty or impossibility is reached.*

This means that for modeling it, we must take into account that there are variables that may make the execution of a task very difficult [or impossible] by themselves, while there is no variable which can make the task very easy by itself[5].

And as a concept that involves uncertainty, sometimes it may be easier to measure the opposite concept [the ease of performing the task] and calculate the difficulty of performing the task as the complementary value.

A-VII.2.3_ UTILITY vs DISUTILITY: PRINCIPLE OF MARGINAL INCREASING DISUTILITY

We have seen that there are two kinds of concepts, and for each concept of a type there is a complementary concept, which information aggregation conditions are also complementary.

For every possible concept, we can state a non-concept that implies the opposite qualities. And if we refer this to Utility, it allows us to state that *to the [in the absence of the unusual] 'decreasing marginality' of utility [Bernoulli, 1738] necessarily an 'increasing marginality' of disutility shall oppose,* an issue we can graphically review:

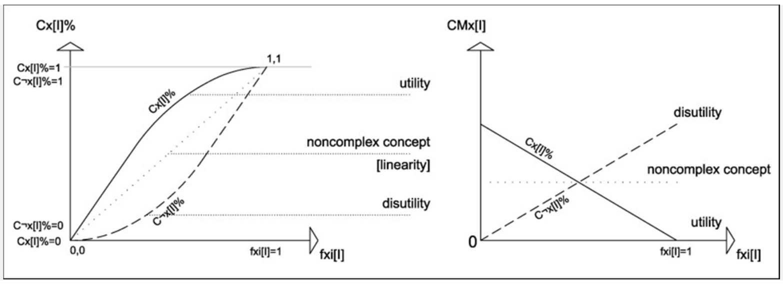

Figure 51: Comparison of utility-disutility obtained for an indicator $fx_i[I]$

The hatched curve is the Disutility which 'complementary' to the Utility [solid curve] presents increasing marginality.

If Utility / Disutility were linear, they would follow the dotted lines.

And the review of the variation of each indicator's influence on the aggregated value shows that **'decreasing marginality' is not only a property of 'utility', but of any concept involving certainty:**

- **Concepts involving Certainty present *decreasing marginality*;** the increase of the degree of global emergence of the concept that produces the same increase in an indicator is lower the higher the value of the indicator is.
- **And consequently, concepts involving Uncertainty present *increasing marginality*;** the increase of the degree of emergence of an overall concept that produces the same increase in an indicator is greater, the higher the value of the indicator is.

To the 'diminishing marginality' of concepts involving certainty it opposes the 'increasing marginality' of any concept involving Uncertainty.

[5] Performing a task is a clear example that the 'average compliance degree' with the different task requirements guarantees an 'average' performance, while failure in any of the requirements may lead to the impossibility of performing it [e.g. if someone does not have balance, he cannot skate even if he acquires all necessary knowledge]