Anti CHSH - Refutation Of The CHSH Inequality

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Abstract: The principle of causality occupies an important place in the history of the philosophical interpretation of quantum mechanics from the beginning. In last consequence, today's Copenhagen dominated *acausal* interpretation of quantum mechanics casts doubt upon traditional views in the philosophy of nature and demands the revision of the principle of causality at a fundamental level of theoretical description in physics. Testing the mathematical and logical consistency of the quantum mechanical description of nature i. e. especially such as non-locality has become a subject of an ongoing dispute and research.

Today, quantum-mechanical concepts i. e. such as non-locality refer to some mathematical foundations, especially to Bell's inequality and the CHSH inequality. Experimental data, analyzed by the help of Bell's inequality or the CHSH inequality favor a quantum mechanical description of nature, over local hidden variable theories (often referred to as local realism). In general, the use of mathematically inconsistent methods can imply a waste of money, time and effort on this account. Under some certain conditions (*the assumption of independence*) Bell's theorem and the CHSH inequality are already refuted. The purpose of this publication is explore the terra incognita, the interior logic that may lie beyond Bell's original theorem and the CHSH-inequality and to refute both, Bell's original theorem and the CHSH-inequality under any circumstances by the proof that we can derive a logical contradiction out of Bell's inequalities. Thus far, accept Bell's theorem or the CHSH-inequality as correct, then you must accept too that **+0 = +1**, which is a logical contradiction. Bell's theorem and the CHSH-inequality are refuted in general. In this insight, it appears to be necessary to revisit the very foundations of quantum theory and of physics as such.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

In contrast to relativity theory, quantum mechanics is still not based on generally accepted principles or foundations. Theoretically, an axiomatic formalization of the mathematical foundations of quantum mechanics similar to Newton's principia is a possible approach to solve this problem but still not in sight.

The core of this problem is related to Einstein's Theory of Relativity. In general, Einstein's Special Theory of Relativity, supported by may experiments, require and obey something like a principle of locality [1], [2]. But due to Bell, "It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a

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distant system with which it has interacted in the past, that creates the essential difficulty." [3]. Thus far, a generally accepted Local Realistic interpretation of quantum mechanics is still not in sight. Meanwhile, this tension between the locality of Relativity Theory and the non-locality of quantum mechanics is favored by dozens of experiments performed; the philosophical implications of Bell's theorem and physical meaning are still a matter of discussion. Especially, due to the incorrect and bold position of d'Espagnat, we are forced to accept the following:

"The doctrine that the world is made up of objects whose existence is independent of human consciousness turns out to be in conflict with quantum mechanics." [4].

The many experiments performed reported dramatic violations of Bell's inequality. The most of these results are repeatedly regarded as being inconsistent with the assumption of local realism while consistent with quantum mechanics. But despite of the most of these experiments and despite of the different meanings associated with the notion of Locality, it is possible to drill very deep into details of this problem, even into the level of formal proof. However, in order to avoid misconceptions, the second difficulty is set aside, a formal definition of the notion Locality, Entanglement and Superposition will not be given.

Bell's theorem and the CHSH inequality are already refuted under some certain circumstances. The aim of this paper is to refute Bell's theorem and the CHSH inequality in general and under any circumstances. The purpose of this present paper is an attempt to recognize the logical structure behind the theorems above and to leave these incorrect theorems logically definitely behind us. In view of the extremely high precision by which some no-go theorems like Bell's theorem or the CHSH inequality have been experimentally confirmed, this appear to be not an easy task. In the following we will provide a mathematical and logical proof that neither Bell's theorem nor the CHSH inequality is logically consistent and mathematically valid. The concept of non-locality in quantum mechanics needs new mathematical foundations.

2. Material and Methods

The most important implication of Bell's Theorem and one striking aspect of today's quantum mechanics is non-locality. Today, the impossibility of a local realistic interpretation of quantum mechanics is back grounded by a family of inequalities known as Bell's theorem or as CHSH-inequality.

2.1. Bell's theorem

There are several different, mathematical formulations of Bell's theorem. In 1964 John S. Bell, a native of Northern Ireland, published his theorem in the form of a non-strict inequality [5] in the short-lived journal *Physics* as

$$1+E(b,c)\geq \left|E(a,b)-E(a,c)\right| \tag{1}$$

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where *a, b* and *c* are *the local detector settings* of the apparatus and E(a, b), E(a, c), E(b, c) denote the expectation values. Thus far, the values measured by observers (i. e. Alice or Bob) are only functions of the local detector settings and the hidden parameter. The value observed by Bob with detector setting b is B(b, λ), the value observed by Alice with detector setting a is A(a, λ).

Further, Bell's other vital assumption is that [6]

$$A(a,\lambda)=\pm 1 \text{ and } \mathrm{B}(b,\lambda)=\pm 1. \tag{2}$$

Bell's approach to the calculation of quantum mechanical expectation values must ensure the equality of the quantum mechanical expectation values calculated to his proposal with the expectation values as calculated due to probability theory and mathematical statistic. So it is no surprise that there was no difficulty to show, that this is not the case. Using Bell's coding for spin-up and spin-down, incorrect [7] quantum mechanical expectation values will be calculated. In general, following Bell's theorem, we must accept too, that

$$1+E(b,c)=\left|E(a,b)-E(a,c)\right| \tag{3}$$

or equally that

$$1+E(b,c)>\left|E(a,b)-E(a,c)\right|. \tag{4}$$

2.2. CHSH - Inequality

Due to Bell's theorem, predictions of quantum mechanics are inconsistent or in disagreement with the assumption of locality. Based on Bell's contribution, Clauser, Horne, Shimony and Holt presented "a generalization of Bell's theorem which applies to realizable experiments" [8] and derived another inequality, the so called CHSH – Inequality as

$$2\geq\left|\,E(a,b)-E(a\,',b)\right|+\left|\,E(a,b\,')+E(a\,',b\,')\right| \tag{5}$$

where a, a', b and b' are the local detector settings and E(a, b), E(a', b), E(a, b') and E(a', b') denote the expectation values. There is an extensive literature supporting the validity of the CHSH – Inequality.

In general, the authors of the CHSH-inequality are coding spin-up and spin-down, similar to Bell, in a mathematically and logically inconsistent way. "... henceforth interpret A(a)=±1 and B(b)= ±1 ..." [9]

Another far reaching and striking aspect of the CHSH – inequality is that the number 2 is defined as

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$$2 = \left|E(a,b) - E(a',b)\right| + \left|E(a,b') + E(a',b')\right| \tag{6}$$

and that the number 2 is in the same respect defined as a strict inequality as

$$2 > \left|E(a,b) - E(a',b)\right| + \left|E(a,b') + E(a',b')\right|. \tag{7}$$

TABLE I: CHSH-Inequality and Bell's Theorem

"Bell's theorem has profound implications in that it points to a decisive experimental test of the entire family of local hidden-variable theories." [10]

3. Results

3.1. Refutation of Bell's theorem in general

Claim.
Bell's theorem is logically and mathematically not correct. Thus far, accept Bell's theorem as valid then you must accept too that

$$+0 = +1. \tag{8}$$

Proof by contradiction.
The technique of a proof by contradiction is widely used in physics, mathematics and philosophy. Thus far, in opposite to our claim above, we are sure that Bell's theorem is mathematically valid and accepting too that Bell's theorem is logically consistent. Consequently, we are not able to derive any logical contradiction out of Bell's theorem. Bell's theorem is valid and correct and formulated in the form of a non-strict inequality as

$$1 + E(b,c) \geq \left|E(a,b) - E(a,c)\right| \tag{9}$$

The same theorem is logically and mathematically correct if

$$1 + E(b,c) = \left|E(a,b) - E(a,c)\right| \tag{10}$$

and equally if

$$1 + E(b,c) > \left|E(a,b) - E(a,c)\right|. \tag{11}$$

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Clearly, both implications of Bell's theorem may not be correct at the same time, but this does not make Bell's theorem logically and mathematically inconsistent as such. The following table illustrates the last relationship.

TABLE II: Bell's Theorem

$1+E(b,c)$	$>$	$\lvert E(a,b)-E(a,c)\rvert$

This is a strict inequality which must obey the needs of an equality too and this with any contradictions or without any loss of generality. Since the value of the term $\lvert E(a,b)-E(a,c)\rvert$ is non-negative, we must accept too, that the term $1+E(b,c)$ is non-negative too. On this account, the straightforward consequence is that there is a Bell's term (may be of unknown magnitude) denoted as B, which itself has to be non-negative too and greater than zero or | B > 0|. Bell's term assures that the above strict Bell's inequality is compatible with an equality too. The following table illustrates the last relationship.

TABLE III: Bell's Theorem

		$\lvert B>0\rvert$
$1+E(b,c)$	$=$	$\lvert E(a,b)-E(a,c)\rvert$

In other words, as may readily be verified, Bell's inequality, treated as logically and mathematically correct demands thus far equally that

$$1+E(b,c)=\lvert E(a,b)-E(a,c)\rvert+\lvert B>0\rvert . \tag{12}$$

At the same time, Bell's inequality demands that

$$1+E(b,c)=\lvert E(a,b)-E(a,c)\rvert . \tag{13}$$

It is the same $1+E(b,c)$ which is defined in two different ways. We equate (12) and (13), one then finds straightforwardly that

$$\lvert E(a,b)-E(a,c)\rvert=\lvert E(a,b)-E(a,c)\rvert+\lvert B>0\rvert . \tag{14}$$

Rearranging (14), we obtain

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$$+0 = +|B > 0| . \tag{15}$$

Dividing (15) by (|B > +0|), it is

$$\frac{+0}{+|B > 0|} = \frac{+|B > 0|}{+|B > 0|} . \tag{16}$$

or

$$+0 = +1. \tag{17}$$

Quod erat demonstrandum.

Thus far, contrary to our starting point and in contrast to our expectation, a logical contradiction can be derived out of Bell's theorem. It is not true that + 0 = + 1. Following Bell's reasoning we must accept something, which is obviously incorrect. A logical contradiction is something we try to avoid. Thus far, it appears to be very difficult to convince the scientific community that our world is grounded on a logical contradiction's which is exactly what Bell's theorem demands. Bell's theorem leads to logical contradiction's and is based on logical contradictions. Consequently, Bell's theorem is refuted in general.

3.2. Refutation of the CHSH-Inequality

Under some known circumstances, the CHSH inequality is already refuted [11]. In the following, we will refute the CHSH inequality under any circumstances.

Claim.
The CHSH inequality is logically and mathematically incorrect. If you accept the CHSH inequality as generally valid then you must accept too that

$$+0 = +1. \tag{18}$$

Proof by contradiction.
In general, the CHSH inequality can be formulated as [12]

$$2 \geq \left| E(a,b) - E(a',b) \right| + \left| E(a,b') + E(a',b') \right| \tag{19}$$

As a non-strict inequality, the CHSH inequality is determined by an equality as

$$2 = \left| E(a,b) - E(a',b) \right| + \left| E(a,b') + E(a',b') \right| \tag{20}$$

and by an strict inequality as

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$$2 > \left|E(a,b) - E(a',b)\right| + \left|E(a,b') + E(a',b')\right| . \tag{21}$$

Both forms of the CHSH inequality are defining the number 2. The strict form of the CHSH inequality can be illustrated as follows.

TABLE IV: The strict Form of the CHSH inequality

+2	>	$\left\|E(a,b) - E(a',b)\right\|$ + $\left\|E(a,b') + E(a',b')\right\|$

The CHSH inequality as a strict inequality can and must be compatible with the CHSH equality too without any loss of generality. To proceed further, we can transfer the strict from of the CHSH inequality into an equality by adding a CHSH term (may be of unknown magnitude) denoted as C, which itself has to be greater than zero or +C > 0. This CHSH term assures the compatibility of CHSH inequality with the CHSH equality. The following picture illustrates this far reaching relationship once again to achieve another point of view.

TABLE V: The CHSH equality

+2	=	**+C > 0**
		$\left\|E(a,b) - E(a',b)\right\|$ + $\left\|E(a,b') + E(a',b')\right\|$

Consequently, the CHSH inequality may be written as the superposition of a known and an unknown term as

$$2 = \left|E(a,b) - E(a',b)\right| + \left|E(a,b') + E(a',b')\right| + (C > 0) \tag{22}$$

From the above, we must equally accept that

$$2 = \left|E(a,b) - E(a',b)\right| + \left|E(a,b') + E(a',b')\right| \tag{23}$$

The number 2 is defined at least in two different ways. Substituting this relationship into the equation before, we obtain

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$$\left|E(a,b)-E(a',b)\right|+\left|E(a,b')+E(a',b')\right| = \left|E(a,b)-E(a',b)\right|+\left|E(a,b')+E(a',b')\right|+(C>0) \tag{24}$$

Subtracting $\left|E(a,b)-E(a',b)\right|+\left|E(a,b')+E(a',b')\right|$ and collecting together terms, we obtain

$$0=(C>0) \tag{25}$$

Dividing by (C > 0) we must accept that

$$\frac{0}{+(C>0)}=\frac{+(C>0)}{+(C>0)} \tag{26}$$

or

$$+0=+1. \tag{27}$$

Quod erat demonstrandum.

In other words, as already verified by our direct proof before, the CHSH inequality leads to a logical contradiction and is based on a logical contradiction. The CHSH inequality, the general form of Bell's theorem, is refuted.

4. Discussion

The results of this publication are based on the law of non-contradiction (LNC) as one of the foremost among the principles of science. In particular, in our today's understanding of the foundations of science as such, logical contradictions cannot be accepted as the foundation of science or of our thinking. Thus far, a seemingly sound piece of reasoning (i. e. Bell's theorem or the CHSH inequality) based on apparently true assumptions cannot lead to any logical contradictions, especially it is difficult to convince the scientific community that **+0 = +1**. Scientist and philosophers have to draw a sharp line between correct and not correct to assure that logical contradictions are avoided. So it is no surprise that generations of experimentalists and theoreticians have often accused Bell's theorem and the CHSH inequality of being incorrect but without a definite refutation in sight. In contrast to our expectation, Bell's theorem and the CHSH inequality contain some serious mathematical and physical deficiencies which leads to a logical contradiction when we try to determine whether the same are mathematically true or not which proofs them as invalid. Bell's theorem and the CHSH inequality are refuted in general.
Most recently, Bell's theorem and the CHSH inequality were refuted [13] under certain conditions. Meanwhile, even Heisenberg's uncertainty principle [14] together with the several different, mathematical re-formulations of the same like Ozawa's "universally valid uncertainty relation" is proofed as not correct [15] and as no longer valid [16].

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According to the Copenhagen dominated acausal interpretation of quantum mechanics, the principle of causality occupied an important place in the history of the philosophical interpretation of quantum mechanics from the beginning [17]. Finally, some basic pillars of today's Copenhagen dominated acausal interpretation of quantum mechanics are describing fundamental quantum-mechanical phenomena in a logically inconsistent way. Consequently, a new and unique principle as the starting point for the unification of quantum and relativity theory appears to be necessary. In contrast to the Copenhagen dominated acausal interpretation of quantum mechanics, **the principle of causality**, valid and correct since thousands of years, can be such a principle.

5. Conclusion

Bell's theorem and the CHSH inequality are refuted in general. With this, the anti-causal [18], anti-deterministic [19] strictly non-local [20], [21] Copenhagen dominated approach to elementary quantum-mechanical phenomena such as superposition, non-locality and entanglement has lost its mathematical foundation.

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Appendix

None.

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