On the Nature of the Planck Constants[1]

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A deeper understanding of why the reduced Planck constant and Planck constant ("Planck constants") have the values they have as determined by experiments is developed. New definitions of the Planck constants are arrived at using the speed of light in vacuum and geometric considerations. The kilogram SI base unit is found to be derived from the SI base units second and meter. The values of the Planck constants determined by experiments and published by CODATA (2010) are found to both have a relative accuracy error of 0.3552%. A new kilogram definition is proposed and it is argued that since the kilogram will then be a derived SI unit, the kilogram should not be considered an SI base unit anymore.

Keywords: planck constant, reduced planck constant

1 Introduction

The reduced Planck constant and Planck constant are of fundamental importance in quantum mechanics. The electromagnetic energy of a photon E is described by the equation $E = \hbar\omega$ where $\hbar$ is the reduced Planck constant and ω is the angular frequency of the photon. However the value of $\hbar$ has been determined by experiments and so far a deeper understanding of why it has the determined value has been lacking. In this paper I develop such a deeper understanding.

2 Derivation of the Planck constants

2.1 Geometric Planck constant equations

The published CODATA 2010 value of the Planck constant h is $6.62606957(29) \times 10^{-34}$ J s with a relative standard uncertainty $u_r = 4.4 \times 10^{-8}$ (TABLE XL) [1]. For the rest of this paper the above mentioned CODATA 2010 value is referred to as h_{exp}. The reduced Planck constant is defined as

$$\hbar = \frac{h}{2\pi}$$

The CODATA 2010 value of (hereafter referred to as $\hbar_{exp}$) is $1.054571726(47) \times 10^{-34}$ J s with a relative standard uncertainty $u_r = 4.4 \times 10^{-8}$ and the CODATA 2010 value of the speed of light in vacuum c_0 is exactly 299792458 m s^{-1} (TABLE XL) [1].

As a starting point, I notice that

$$\hbar_{exp} = \frac{k}{c_0^4}$$

where

$$k \approx \frac{8}{3\pi}$$

such that

$$\hbar_{exp} \approx \frac{8}{3\pi c_0^4} = 1.05083867430236 \times 10^{-34} \text{ J s}$$

I'm searching for numbers related to geometry in a 3D space and its subspaces such as lengths, areas, volumes and radians. There is a constant times π which may be related to the circumference of a circle or its higher dimension analogs. Not much geometry is clear from this equation. However if we split the 8 into two 4's, it starts to get interesting. Then

$$\hbar_{exp} \approx 2\frac{4}{3\pi c_0^4}$$

$\frac{4}{3}$ could be a factor in the volume equation for a 3D sphere which is

$$V = \frac{4}{3}\pi r^3$$

I now assume that this is the case to see where it leads me. After multiplying both top and bottom of the right side of the equation by π then

$$\hbar_{exp} \approx 2\frac{\frac{4}{3}\pi}{\pi^2 c_0^4}$$

There is also needed a radius raised to the third power. $r_1 = c_0 t_1$ with $t_1 = 1s$ seems like a great option as it is the radius of a 4D light cone cross-section which is a 3D sphere. Then

$$\hbar_{exp} \approx 2\frac{\frac{4}{3}\pi r_1^3}{\pi^2 r_1^3 c_0^4}$$

The meaning of the rest of the factors is still a bit fuzzy. Since $t_1 = 1s$ it can be multiplied with any of the existing 4 c_0's at the bottom to get lengths without changing the value. At the bottom there is one πr_1^2 factor which is a 2D area of a circle. Up to 3 such areas can be created with the available 7 c_0's at the bottom. After doing that

$$\frac{\hbar_{exp}}{\pi} \approx 2\frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3 c_0}$$

where both sides of the equation have been multiplied by π^{-1}. The remaining factor $\frac{2}{c_0}$ seems to not have a useful purpose on the right side of the equation at this point, so I multiply both sides by its inverse $(\frac{2}{c_0})^{-1} = \frac{c_0}{2}$ to remove the factor from the right side to get

$$\frac{\hbar_{exp} c_0}{2\pi} \approx \frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3}$$

Now at least the right side has only factors that are related to known geometry. The *geometric ħ equation* is

$$\frac{\hbar c_0}{2\pi} = \frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3} \tag{1}$$

and the *geometric h equation* is

$$\frac{h c_0}{4\pi^2} = \frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3} \tag{2}$$

where $r_1 = c_0 t_1$ with $t_1 = 1s$.

2.2 Geometric reduced Planck constant equation analysis

An analysis of the simpler geometric $\hbar$ equation (1) is in order. This equation says

$$\frac{hc_0}{2\pi} = \frac{\text{4D light cone unit cross-section volume}}{(\text{3D light cone unit cross-section volume})^3} \tag{3}$$

The top is the volume of a 3D sphere and the bottom is the product of three 2D circular areas. The light cone cross-section volumes are evaluated at time $t_1 = 1s$ which makes them light cone unit cross-sections – the unit being $r_1 = c_0 t_1$. Considering the inverse of (3)

$$(\frac{hc_0}{2\pi})^{-1} = \frac{(\text{3D light cone unit cross-section volume})^3}{\text{4D light cone unit cross-section volume}}$$

the right side looks like a 3D density, ie. $\frac{\text{something}}{m^3}$ and since $\hbar\omega = E$ the density may be related to electromagnetic energy. Electromagnetic energy is inversely proportional to the 3D light cone unit cross-section volume (2D area) and therefore also to the 3D light cone unit cross-section radius at time $t_1 = 1s$ which is $r_1 = c_0 t_1$. Similarly a larger 4D light cone unit cross-section volume imply smaller density and larger electromagnetic energy. This suggests that the right side is not an electromagnetic energy density. Rather it is an inverse electromagnetic energy density.

Considering this and turning back attention to (3). $\frac{E}{V}$ is an electromagnetic energy density where electromagnetic energy E is present within a volume V of 3-dimensional spacial extents. Here is $\frac{V}{A^3}$ where A is a 2-dimensional circular area. As a simple *spacetime spin toy model* A is considered a measure of *spacetime spin* and one can think of this spacetime spin as being similar to the intrinsic spin of some elementary particles of the standard model of particle physics. Spacetime spin is considered to be spin of some currently unknown substance around orthogonal axes of all 3 observed spacial dimensions thereby avoiding dimensional degeneracy. Then $\frac{V}{A^3}$ can be interpreted as a volume V (4D spacetime: 3D sphere) embedding a 3-spacetime spin volume set A^3 (4D spacetime: three 2D circular areas). Also $(\frac{V}{A^3})^{-1} = \frac{A^3}{V}$ can be interpreted as a *3-spacetime spin volume set* A^3 within a volume V. $\frac{V}{A^3}$ is then a *spacetime spin specific volume* (inverse density like specific volume of thermodynamics) that specify volume per unit spacetime spin volume set. Spacetime spin specific volume is directly proportional to electromagnetic energy as required by the equation $\hbar\omega = E$.

2.3 Definitions of the Planck constants

The definition of the reduced Planck constant follows from the geometric $\hbar$ equation (1) as

$$\begin{aligned}\hbar &= 2\pi\frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3 c_0} \\ &= 1.05083867430236 \times 10^{-34}\ \text{s m}^{-4}\end{aligned} \tag{4}$$

and the definition of the Planck constant follows from the geometric h equation (2) as

$$\begin{aligned}h = 2\pi\hbar &= 4\pi^2\frac{\frac{4}{3}\pi r_1^3}{(\pi r_1^2)^3 c_0} \\ &= 6.60261411859264 \times 10^{-34}\ \text{s m}^{-4}\end{aligned} \tag{5}$$

where $r_1 = c_0 t_1$ with $t_1 = 1s$. The reduced Planck constant is more fundamental as it is defined by the simpler of the two equations (4) and (5).

2.4 Units of the Planck constants and the kilogram

Both $\hbar_{exp}$ and h_{exp} have unit

$$\mathrm{J\ s} = \mathrm{kg\ m^2\ s^{-1}}$$

while the theoretical reduced Planck constant has unit

$$\mathrm{s\ m^{-4}} = \mathrm{s^2\ m^{-6}\ m^2\ s^{-1}}$$

This implies that the kilogram SI base unit is derived from the SI base units second and meter

$$\mathrm{s^2\ m^{-6}} \tag{6}$$

since then

$$\mathrm{s^2\ m^{-6}\ m^2\ s^{-1}} = \mathrm{kg\ m^2\ s^{-1}} = \mathrm{J\ s}$$

which is consistent.

2.5 Experimental Planck constant accuracy

Now I turn to the accuracy of the experimental values relative to the theoretical values. With a theoretical value given by (4) the absolute accuracy error of the experimental value is

$$\hbar_{exp} - \hbar = 3.73305169764349 \times 10^{-37}\ \mathrm{J\ s}$$

which results in relative accuracy error 0.3552%. Similarly with a theoretical value given by (5) the absolute accuracy error of the experimental value is

$$h_{exp} - h = 2.34554514073594 \times 10^{-36}\ \mathrm{J\ s}$$

which results in relative accuracy error 0.3552%. These differences cannot be fully accounted for by the CODATA 2010 relative standard uncertainties (TABLE XL) [1] and this suggest a need for investigation of their sources.

2.6 A proposed new kilogram definition

The General Conference on Weights and Measures (CGPM) has issued draft chapters of the 9th SI brochure [2]. The brochure states a proposed new definition of the kilogram SI base unit as

"The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be exactly $6.62606957 \times 10^{-34}$ *when it is expressed in the SI unit for action* $\mathrm{J\ s} = \mathrm{kg\ m^2\ s^{-1}}$."

The new definition of the reduced Planck constant presented in this paper enable a redefinition of the kilogram such that the kilogram magnitude is fixed by the speed of light in vacuum. This is possible since the Planck constant is now a derived constant rather than a fundamental constant. The proposed new definition of kilogram presented next avoids fixing the numerical value of the Planck constant. The proposed new definition of kilogram is

The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be exactly 299 792 458 when it is expressed in the SI unit for speed $\mathrm{m\ s^{-1}}$.

Since the kilogram will then be a derived SI unit, the kilogram should not be considered an SI base unit anymore.

3 Conclusion

I started by deriving geometric $\hbar$ and h equations using the speed of light in vacuum and geometric considerations. An analysis of the simpler geometric $\hbar$ equation revealed a deeper understanding of why the Planck constants have the values they have and enabled new definitions of the constants (4) and (5). I argued that the reduced Planck constant is more fundamental as it is defined by the simpler of the two equations. I found that the kilogram SI base unit is derived from the SI base units second and meter (6).

The values of the Planck constants determined by experiments and published by CODATA (2010) were found to both have a relative accuracy error of 0.3552%. I also found that these differences cannot be fully accounted for by the CODATA 2010 relative standard uncertainties and that this suggest a need for investigation of their sources.

Finally I proposed a new kilogram definition and argued that since the kilogram will then be a derived SI unit, the kilogram should not be considered an SI base unit anymore.

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References

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