Decoding the Nuclear Genome: Is there an Unambiguous and Precise way to Define the Current Quark Masses and Relate them to Nuclear Binding Energies and Mass Defects, and what is the Underlying Theory?

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Abstract: In several previous publications the author has presented the theory that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory and used that to deduce the up and down current quark masses with commensurately high precision from the tightly-known Q=0 empirical electron mass and the neutron minus proton mass difference. This is then used as a springboard to closely fit a wide range of empirical nuclear binding and fusion energy data and to obtain the proton and neutron masses themselves within all experimental errors. This paper systematically pulls all of this together and a) establishes that this way of defining current quark masses constitutes a valid measurement scheme, b) lays out the empirical support for this theory via observed nuclear binding and fusion energies as well as the proton and neutron masses themselves, c) solidifies the interface used to connect the theory to these empirical results and uncovers a mixing between the up and down current quark masses, and d) presents clearly how and why the underlying theory is very conservative, being no more and no less than a deductive mathematical synthesis of Maxwell's classical theory with both the electric and magnetic field equations merged into one, Yang-Mills gauge theory, Dirac fermion theory, the Fermi-Dirac-Pauli Exclusion Principle, and to get from classical chromodynamics to QCD, Feynman path integration.

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Contents

1. Introduction: Is there a Valid Method for Defining Quark Masses with High Precision?

In two earlier peer-reviewed publications [1], [2] the author demonstrated within parts per 10^5 AMU and better precision how the binding and fusion energies of the ^{2}H, ^{3}H, ^{3}He and ^{4}He light nuclides as well as the binding energy of ^{56}Fe could be explained as a function of *only two parameters*, namely, the current masses of the up and down quarks, found with extremely high precision in AMU to be m_u = 0.002 387 339 327 u and m_d = 0.005 267 312 526 u, see [10.3] and [10.4] and section 4 of [2] as well as section 12 of [1]. Using the conversion 1 u = 931.494 061(21) MeV [3] this equates with some loss of precision [4] to m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV, respectively. In an International Patent Application published at [5], this analysis was extended to ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N with equally-high precision. And in [6] this analysis was extended using the Fermi vev v_F=246.219651 GeV and the Cabibbo, Kobayashi and Maskawa (CKM) mass and mixing matrix as two additional parameters, to explain the proton and neutron masses M_N = 939.565379 MeV and M_P = 938.272046 MeV [7] *completely within all known experimental errors*.

Yet, there is one underlying point which has not been sufficiently explained in any of these prior papers: the Particle Data Group (PDG) lists these two current-quark masses to be to $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$ with large error bars of almost 20% for the down quark and almost 50% for the up quark, "in a mass-independent subtraction scheme such as $\overline{\mathrm{MS}}$ [modified minimal subtraction] at a scale $\mu \approx 2\mathrm{GeV}$." [8] (Here we shall use Q rather than μ.) In other words, the PDG values are extracted for a given renormalization scale Q and are actually a function of this scale and of the renormalization scheme. So although these m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV found by the author are well-placed near the center of these PDG error bars, the claimed precision raises the question: can we really talk about and understand these quark masses with such high precision in a fashion which is *independent* of renormalization scale and scheme? More plainly put: is there some sensible way to make the simple declarative statement that "the Q=0 up and down quark masses are X and Y," with X and Y being some mass-energy numbers which have an extremely small error bar due to nothing other than the accuracy of our measuring equipment? Is there a sensible, definite, unambiguous, very precise scheme we can use to define the current quark masses, consistent with empirical data, which scheme is renormalization scale-independent?

Specifically, the author's prior findings that m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV, which when represented in AMU has a precision close to a billion times as tight as the PDG error bars, even if *mathematically* correct in relation to the nuclear energies with which these quark masses are then interrelated, presuppose an understanding of how these quark masses are to be *physically* defined and measured. Without such an understanding, the author's prior work is incomplete, and to date, the author has not directly and plainly articulated this understanding.

The intention of the present paper is to remedy this deficiency by making clear that the mass defects found in nuclear weights which are related in a known way to nuclear binding and fusion / fission energies, are in fact a sort of "nuclear DNA" or "nuclear genome" the proper decoding of which teaches about nuclear and nucleon structure and the masses of the quarks in a way that has not to date been fully appreciated. In contrast to the *nuclear scattering schemes* presently used to establish quark masses, which are all based on renormalization-dependent, energy scale-dependent experiments involving scattering of nucleons and nuclei, the scheme which has been implicitly used by the author which this paper will now make explicit, is one in which the up and down current quark masses are defined at Q=0 directly in terms of the empirical Q=0 electron, proton and neutron (EPN) masses (really the electron mass and the neutron minus proton mass difference) via two "primary relationships" (3.1) and (3.2) infra, and thereafter enjoy very accurate relationships with a number of light nuclide binding and fusion energies and related defects in nuclear weights. In this "EPN scheme" which is supported by the observed mass defects, the up and down current quark masses are defined by and seen to be related to objective, very precise, experiment-independent, scale-independent, long-known energy numbers that have been experimentally found and catalogued for the nuclear mass defects, weights, binding energies, and fusion / fission energies.

The problem we confront, which we will elaborate in section 2, is that all scattering experiments essentially bombard a target and then use forensic analysis of the known bombardment and the found debris to learn about the nature of the target prior to bombardment. In contrast, nuclear mass defects require no bombardment of anything. They are no more and no less than an experiment-invariant expression of nuclear weights and of the energies which are missing from the nuclide weights against if one were to simply add up the weights of a nuclide's protons and neutrons when seen in a free state. In this context, the prevailing scheme for characterizing quark masses has wide error bars because it is based on "bombing" nucleons and nuclei and so yields results which depend on the specific bombing runs carried out, while the scheme to be elaborated here has very high precision because it is a "weighing" scheme which uses only nucleon weights and the free electron mass to define the current quark masses and so inherits the precision with which these weights are known and also inherits the benefit of not being experiment-dependent. So the scheme to be articulated here has very tight error bars because it is based on non-intrusive nuclear "weighing" rather than highly-intrusive nuclear "bombing," and because nuclear weights themselves are very precisely known and do not vary by experiment while scattering experiments introduce renormalization and scale issues which make it difficult to establish an approach for specifying the masses of confined quarks with the same precision as the masses of free particles. Before reviewing this problem more deeply in section 2, let us briefly summarize the remainder of this paper.

In section 3 we introduce two "primary relationships" emerging from the underlying theory that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory, through which the current quark masses are defined in the $Q\rightarrow 0$ limit based on the $Q\rightarrow 0$ electron rest mass and the $Q\rightarrow 0$ neutron minus proton mass difference (EPN scheme). We then lay out the three primary questions to be reviewed in the balance of the paper: 1) given the confinement of quarks which means that a free quark can never be directly measured in the $Q\rightarrow 0$ limit, is this a valid measurement scheme for defining current quark masses?; 2) if this is a valid measurement scheme, is there clear "secondary" support from other empirical data beyond the EPN masses, such as from nuclear weights and binding energies? and 3) is the theory that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory based on firm, conservative, well-tested and widely-regarded theoretical foundations, and does it provide a clear and precise interface between theory and experiment?

In section 4 we answer the first question, showing how this is indeed a legitimate and unambiguous measurement scheme. In sections 5 and 6 we answer the second question. Section 5 reviews how the empirical side of the theoretical-to-empirical interface leads to new understandings of phenomena such as quark confinement and nuclear binding, the binding energies of light Hydrogen and Helium nuclides, and the proton minus neutron mass difference which is then elevated to the primary relationship (3.2). Section 6 reviews the evidence that the primary relationships obtained from this theory garner precise secondary empirical support from a broad range of nuclear mass / energy data. In the remainder of the paper we answer the third question. Sections 7 and 8 review the interface between the underlying theory and its empirical validation with a degree of specificity not previously presented, and in section 8 this includes uncovering a form of mixing between the up and down current quark masses which does not appear to have previously been found. Section 9 contains a very concise review of how one gets to from the underlying theory to the theoretical side of the theoretical-to-empirical interface, and makes clear how this is not a new theory, but is a new yet fully-deductive and inexorable synthesis of Maxwell (both the magnetic and electric charge field equations), Yang-Mills, and Dirac theories, the Exclusion Principle for fermions, and to cross over from the classical to the quantum field theory, Feynman path integration. So for someone with requisite scientific skepticism to believe and accept that protons and neutrons and other baryons are Yang-Mills chromo-magnetic monopoles requires no more and no less from than the belief that all of these component theories are correct, the belief that when mathematics is correctly applied to combine input component theories which themselves are correct the result of that mathematical synthesis will be equally correct, and the belief that when the results of such a synthesis also find widespread empirical validation, the entire enterprise must be earnestly regarded.

We conclude by observing that what all this means, is that the magnetic monopoles which have been pursued ever since the time of James Clerk Maxwell are in fact hiding in plain sight, in Yang-Mills form, as the protons and neutrons and other baryons at the heart of our material universe.

2. Running Couplings, Vertical Confinement and Horizontal Freedom Asymptotes, Dimensional Transmutation, and the $Q \rightarrow 0$ Limit in QCD

The electromagnetic interaction and the electron which is a most important fermion source of this interaction furnish the best starting point for analyzing the questions about renormalization and quark mass definition posed in the introduction. Maxwell's electrodynamics when extended into non-abelian domains by Yang-Mills gauge theories and when $SU(3)_C$ is the particular Yang-Mills group chosen for consideration, is the template that one customarily uses to study strong chromodynamic interactions. And the electron which is an elementary spin ½ fermion subsisting in a $U(1)_{em}$ singlet following electroweak $SU(2)_W \times U(1)_Y$ symmetry breaking and which is observed as a *free* fermion, is the template best used to draw a contrast with quarks which also have spin ½, which are also regarded as "elementary" – at least to the same degree and in the same manner as electrons are elementary – but which form an $SU(3)_C$ color triplet and most importantly are not free but are *confined* within nucleons.

It is also important to keep in mind that Quantum Chromodynamics (QCD) is a branch of *elementary particle physics* insofar as it is used to describe the strong interactions between *colored* (R, G, B) quarks such as up and down quark flavors, via *bi-colored* (e.g., $\overline{\mathrm{R}}\mathrm{G}$) gluons, all confined within a baryon. Meanwhile, *nuclear physics* is used to describe *color-neutral* baryons such as the proton and neutron baryon flavors which are free hadrons with a wavefunction $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B} \equiv \mathrm{RGB} + \mathrm{GBR} + \mathrm{BRG} - \mathrm{RBG} - \mathrm{BGR} - \mathrm{GRB}$ that is *antisymmetric* under color interchange. And the nuclear interactions of these baryons are mediated via *color-neutral* mesons with a wavefunction $\overline{\mathrm{R}}\mathrm{R}+\overline{\mathrm{G}}\mathrm{G}+\overline{\mathrm{B}}\mathrm{B}$ that is *symmetric* under color interchange and which have short range but are also free hadrons and are not confined, such as the pion-flavored mesons originally predicted by Yukawa [9]. Although the elementary particle physics of confined colored quarks and bi-colored gluons and the nuclear physics of free antisymmetric color-neutral baryons and symmetric color-neutral mesons are often lumped together as one discipline in loose discourse, they are in fact distinct disciplines bridged via so-called hadronic physics in a fashion that to this date is still not fully understood. In many ways understanding baryons as the chromo-magnetic monopoles of Yang-Mills theory strengthens the understanding of this hadronic bridge between elementary chromodynamic particle physics and nuclear physics to advance unification among all of these physics disciplines by showing how the masses of quarks which are elementary and colored and confined are interrelated with the masses and binding energies of nucleons and nuclei which are not elementary and are color-neutral and are free.

It should also be kept in mind that the author's thesis first published in [1] that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is closely tied to the fact that baryons have a color wavefunction $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B}=\mathrm{R}\left[\mathrm{G},\mathrm{B}\right]+\mathrm{G}\left[\mathrm{B},\mathrm{R}\right]+\mathrm{B}\left[\mathrm{R},\mathrm{G}\right]$ which is *antisymmetric* under *color interchange*, while magnetic monopoles $\partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu}$ where the strength tensor $F_{\mu\nu} = -F_{\nu\mu}$ is antisymmetric whether abelian or non-abelian have a spacetime index symmetry $\sigma \wedge \mu \wedge \nu = \sigma\left[\mu,\nu\right]+\mu\left[\nu,\sigma\right]+\nu\left[\sigma,\mu\right]$ which is analogously *antisymmetric* under *spacetime index interchange*. In the former case there are three colors and in the latter three spacetime indexes, and in both cases the interchange symmetry is antisymmetric in identical fashion. The physically-meaningful link between these alike color and spacetime symmetries which demonstrates that baryons *are* the chromo-magnetic monopoles of non-Abelian gauge theory – i.e., the connection which advances us from like-symmetries to the *formal identification* of chromo-magnetic monopoles with baryons – is established in section 5 of [1] and deepened in section 10 of [10] through the application of the Fermi-Dirac-Pauli Exclusion Principle, and will be reviewed in section 9 here. What happens specifically is that the rank-3 of the differential three-form for $\partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu}$ is converted into the dimension-3 for the chromodynamics gauge group $SU(3)_C$ of strong interactions and the three colors of quark within each monopole / baryons are then naturally emergent rather than having to be postulated.

Now, when we talk about the electromagnetic interaction, we can readily state that the dimensionless "running" coupling of this interaction is measured to be the rather precise α_{em} = $e^2 / 4\pi\hbar c$ = 1/137.035 999 074 for low probe energies, where e is the electric charge strength, and specifically, that this "fine structure" number is the *horizontally-asymptotic* value of α_{em} as the renormalization scale $Q \to 0$ with Q plotted horizontally on the domain axis and the function

$\alpha_{em}(Q)$ plotted vertically on the range axis. We also know that as the renormalization scale Q is increased, so too is the strength of this interaction, which in quantum field theory is an important distinguishing feature between an abelian interaction and a non-abelian interaction. So for example, when $Q \approx M_W$, we also have the somewhat larger $\alpha_{em} \approx 1/128$. [3]

Likewise, when we talk about the mass of the electron, we can state that m_e = 0.510 998 928 ± 0.000 000 011 MeV, [11] which expresses an extremely high measurement precision limited only by the accuracy of our laboratory equipment. But just as the running coupling α_{em} is a function of renormalization scale Q so too is the measured electron mass m_e. So when we make the foregoing statement as to the energy number associated with the electron mass we are implicitly stating that this is the horizontally-asymptotic value of this mass for $Q \to 0$. At any deep probe scale, this mass is also expected to "run" just like the running coupling / charge strength. So whether stated explicitly or understood implicitly, we are *defining* the mass and electric charge strength of the electron based on what is asymptotically observed at $Q = 0$, and with this definition we are able to express both α_{em} and m_e with a high precision limited only by our measuring instrumentation. *But we are only able to do this because the natural world obliges us by providing a running electromagnetic coupling and a running electron mass which are in fact horizontally-asymptotic in the $Q \to 0$ limit, and an electron which is free and so can be observed in the $Q \to 0$ limit.*

So the question now arises: if we can define charge strength and mass in this way for electromagnetic interactions and electrons can we not do the same for strong interactions and quarks? That is, why can't we just define the running strong coupling α_s and the up and down and other quark masses based on their horizontally-asymptotic values as the renormalization scale $Q \to 0$?

The answer is evident from the very asking of this question: we cannot establish a definition for the quark charges and masses similar to that used for the electron charges and masses *precisely because quarks are confined and not free*. Quarks are not free particles in the same manner as electrons. They are only asymptotically free [12] deep inside a nucleon from which they can never be individually removed. Quantum Electrodynamics (QED) is abelian while QCD is non-abelian, so the running coupling curves are flipped in their qualitative features over the Q domain axis. In QCD the running coupling α_s and quark masses m_q approach a *horizontal* asymptote, not as $Q \to 0$, but as $Q \to \infty$, or at least as Q reaches some very large energy associated with the horizontal asymptotic freedom observed deep inside a nucleon via deep inelastic scattering (DIS). So notwithstanding their similarities because they are both rooted in Maxwell's electrodynamics, the confining nature of $SU(3)_C$ as a non-abelian interaction is what makes strong interactions *qualitatively different* from $U(1)_{em}$ electromagnetic interactions which are abelian. And notwithstanding the similarities of quarks to electrons as spin ½ fermions which are equally-elementary, the confinement of quarks within nucleons is what makes them *qualitatively different* from free electrons (and leptons generally).

The parameter Λ_{QCD} at which dimensional transmutation occurs in QCD provides a good quantitative vehicle to discuss these qualitative differences. Referring to Figure 9.4 of [13] reproduced as Figure 1 below for the reader's convenience, Λ_{QCD} specifies the energy-dimensioned domain value of a *vertical asymptote* approached by the dimensionless function $\alpha_s(Q)$ at $Q=\Lambda_{\text{QCD}}$ from right-to-left along the $Q>\Lambda_{\text{QCD}}$ domain. For example, for a six-flavor quark model in the $\overline{\text{MS}}$ scheme, as laid out in [9.24a] of [13] and the associated discussion, this vertical asymptote is determined to be situated at $\Lambda_{\text{QCD}}=90.6\pm3.4\,\text{MeV}$ which is one order of magnitude left of the leftmost domain of Figure 1. And as Q grows larger beyond the rightmost domain of Figure 1, there is *also a horizontal asymptote* associated with asymptotic freedom. So in contrast to an abelian interaction like QED the horizontal asymptote appears in the large-Q rather than the $Q\to0$ domain and so is qualitatively flipped. Via the conversion constant $\hbar c=.197\,326\,971\,8\,\text{GeV fm}$ [3] which in natural units $\hbar=c=1$ may be rewritten as $1\,\text{GeV}=5.067\,730\,939\,\text{fm}^{-1}$ one is able to deduce using the median value $\Lambda_{\text{QCD}}=.0906\,\text{GeV}$ that $\Lambda_{\text{QCD}}=.0906\,\text{GeV}=.0906\times5.0677\,\text{fm}^{-1}=.4591\,\text{fm}^{-1}=1/(2.1780\,\text{fm})$. So in the six-flavor quark model the deBroglie length associated with this vertical asymptote of confinement at Λ_{QCD} is $r_\Lambda\equiv\hbar/c\Lambda_{\text{QCD}}=2.1780\,\text{fm}$, i.e., just over 2 Fermi in length dimension.

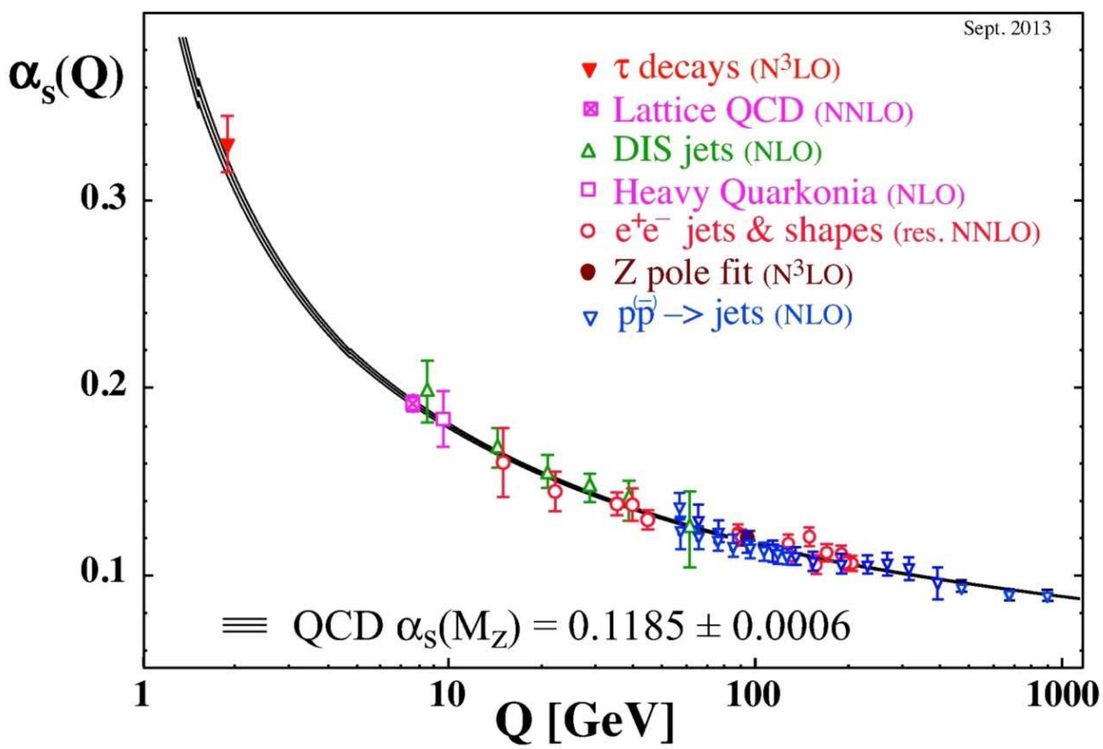

Figure 1: The Running Strong Coupling (reproduced from PDG's [13], Figure 9.4)

So while we are able in QCD to talk about the running of the strong coupling $\alpha_s=g_s^{\,2}/4\pi\hbar c$ and strong charge g_s acting between quarks for $Q>\Lambda_{\text{QCD}}$ as illustrated in Figure 1, it makes no sense to talk about the running of α_s for $Q<\Lambda_{\text{QCD}}$ or especially for $Q\to0$ as we are able to do for α_{em} in QED. In fact, when we do experiments in the low-energy $Q<\Lambda_{\text{QCD}}$ domain we are no longer observing *strong interactions between quarks* confined within a nucleon with a strength measured by α_s. Rather, we are observing *nuclear interactions between nucleons*. Further, these nuclear interactions are observed to have a very short range and with a strength exponentially diminishing to zero beyond separations of a few Fermi in length. For example, because of this exponential strength diminution, nuclei heavier than about ^{56}Fe start to manifest inherent instability because protons and neutrons a.k.a. nucleons within the same nucleus become situated far enough apart as to be beyond the range at which the nuclear force can hold them in the nucleus. So in contrast to the *strong* interaction between quarks in the six-quark model which has a short range on the order of $r_\Lambda = 2.1780\,\text{fm}$ which grows vertically-asymptotically stronger and becomes infinite so as to enforce confinement as $Q \to \Lambda_{\text{QCD}}$ from right-to-left, the *nuclear* interaction is short range because it grows exponentially-smaller for $Q < \Lambda_{\text{QCD}}$ from right-to-left and exponentially attenuates to zero strength beyond a distance of several Fermi. Thus, as we move laterally across the vertical asymptote at the energy Λ_{QCD} and its length equivalent r_Λ we are implicitly crossing the disciplinary boundary between the strong elementary particle physics of quarks and gluons, and the nuclear physics of nucleons and the assemblies thereof known as nuclei as well as mesons – collectively, hadrons. That is the boundary sought to be bridged by hadronic physics.

Consequently, while in QED we can *define* 1/137.035 999 074 as the dimensionless strength of α_{em} for $Q = 0$ because electrodynamics is an abelian interaction which thereby has a *horizontal* asymptote as $Q \to 0$, we cannot employ a similar definition in QCD. Because of QCD's non-abelian character the horizontal asymptote of QED as $Q \to 0$ is flipped to the horizontal asymptote of asymptotic freedom for $Q \gg \Lambda_{\text{QCD}}$, and the "low energy" domain is bounded on the left by a *vertical* asymptote at $Q = \Lambda_{\text{QCD}}$. *The $Q \to 0$ limit for α_s is effectively meaningless in QCD because as $Q \to 0$ the only pertinent interaction is the nuclear interaction between nucleons and not the strong interaction between quarks.* And that nuclear interaction, being short-range with exponential attenuation, has zero coupling strength at $Q = 0$ rather than a finite coupling like the meaningful α_{em} = 1/137.035 999 074 found in electrodynamics. Therefore, instead of characterizing the strong interaction strength starting with a *dimensionless range* value of $\alpha_s = 0$ at $Q = 0$ like we use α_{em} = 1/137.035 999 074 for QED, we define the strong interaction via the transmuted *energy-dimensioned domain* parameter Λ_{QCD} at which there is a vertical asymptote toward which $\alpha_s \to \infty$ from right to left as in Figure 1. And then for $Q > \Lambda_{\text{QCD}}$ α_s depends very definitively on the energy scale *Q* and in addition it depends on the specific renormalization scheme used to absorb the higher-order perturbative divergences.

In sum: The dimensionally-transmuted energy domain number $\Lambda_{\text{QCD}} = .0906\,\text{GeV}$ in six-quark QCD serves the exact same role for QCD as does the dimensionless range number α_{em} = 1/137.035 999 074 for QED in establishing the leftmost domain of the running couplings α_s and α_{em}. For QED, the "fine structure" number 1/137.035 999 074 tells us the dimensionless magnitude of α_{em} as $Q \to 0$ for which nature obliges us because the running coupling for an abelian interaction actually does approach a horizontal asymptote as $Q \to 0$. But nature does not similarly oblige us for a non-abelian interaction such as QCD. In QCD, at the low-energy boundary of the meaningful domain, for six quarks, there is a vertical asymptote for which $\alpha_s \to \infty$ at $\Lambda_{\text{QCD}} = .0906\,\text{GeV}$ and α_s has no meaning for $0 < Q < \Lambda_{\text{QCD}}$ because that is the domain of nuclear interactions between baryons not strong interactions between quarks. So we are compelled to use the energy dimensioned number $Q = \Lambda_{\text{QCD}} = .0906\,\text{GeV}$ to tell us the Q at which the dimensionless number α_s approaches its low-energy vertical asymptote. Therefore, while the $Q \to 0$ limit is meaningful for QED because $\alpha_{em} \to 1/137.035\,999$ in this limit the meaningful limit for six-quark QCD is $Q \to \Lambda_{\text{QCD}} = .0906\,\text{GeV}$ because $\alpha_s \to \infty$ in this limit. The $Q \to 0$ limit still does have meaning, but not for *strong interactions* between and among *quarks*. It has meaning for *nuclear interactions* between and among *baryons*, although at this limit there is no nuclear interaction because of the exponential attenuation of the nuclear interaction strength.

Now we have laid out sufficient background to return to the problem of whether, and if so, how it is possible within a consistent measurement scheme to define the up, down and other current quark masses in the $Q \to 0$ limit with a precision commensurate to that for the free-particle $Q \to 0$ electron, proton and neutron masses.

3. Primary Relationships among the Up and Down Current Quark Masses, and the Electron, Proton and Neutron Masses, and the Three Questions they Raise

In QED we are able to use the $Q \to 0$ limit to define the electron rest mass m_e = 0.510 998 928 ± 0.000000011 MeV because there is a horizontal asymptote at α_{em} = 1/137.035 999 in this limit and because electrons are free particles which can have their attributes such as mass and charge and spin measured directly and with precision. But in QCD the $Q \to 0$ limit appears to be taken off the table and the low-energy limit for meaningful discourse appears to be $Q = \Lambda_{\text{QCD}} = .0906\,\text{GeV}$ at which $\alpha_s \to \infty$ and quarks are confined. Plainly put: it is impossible to take a quark q out of a baryon and measure its mass m_q in the $Q \to 0$ limit in the same way that we would measure an electron mass. Thus, to try to define current quark masses based on their measured values $m_q(Q=0)$ would appear to make no sense because this is a measurement which it is physically impossible to ever take for an individual quark. How can we *define* a quark mass m_q based on its value at $Q = 0$ when it impossible to ever take such a measurement at $Q = 0$? We would be using a definition that seemingly can never be experimentally validated.

But as we do for free electrons it *is* possible to take $Q = 0$ mass measurements for baryons such as protons and neutrons, and indeed, we know very precise values for these measurements, namely M_P = 938.272046±0.000021 MeV and M_N = 939.565379±0.000021 MeV [7]. So while we certainly cannot *directly* measure quark masses $m_q(Q=0)$, we are able to directly measure baryon (B) masses $M_B(Q=0)$. And of course baryons contain quarks, and protons and neutrons which are the most abundant and stable flavors of baryon contain the up and down flavors of quark. So the question arises whether it might be possible to measure

$m_q(Q=0)$ not directly, but *indirectly by inference* from the direct measurements of $M_B(Q=0)$ which are well known with high precision, and whether this high precision might then be inherited by the indirectly-defined $m_q(Q=0)$.

As we shall now start to explore this is indeed possible if as stated in the introduction we employ a scheme based on non-intrusive nuclear "weighing" rather than the highly-intrusive nuclear "bombing" of scattering experiments. Moreover, once we have defined the up and down current quark masses based on *indirect inference from nuclear weights* rather than direct inference from deep nuclear scattering it becomes possible with high precision to use these quark masses to also explain the empirical binding energy and nuclear weight and mass defect and fusion energy data of multiple light nuclides which data has heretofore never been given a satisfactory explanation. This in turn serves to validate the initial indirect inference of quark masses from nuclear weights. Theoretically, all of this is rooted in and emerges from the author's theory in [1] as further developed in [10] that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory.

In previous work by the author [2] the up and down quark masses are indirectly inferred from the $Q=0$ electron mass and from the $Q=0$ neutron minus proton mass difference using the following two relationships which for now will simply be stated and which we shall later explain and support in sections 7 and 8 based on the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory. First, as initially found in [11.23] of [1], the *difference* between the up and down current quark masses is related to the electron rest mass according to:

$$m_d - m_u = \frac{(2\pi)^{\frac{3}{2}}}{3} m_e . \tag{3.1}$$

Second, as initially found in [A15] and [7.2] and section 10 of [2], the *difference* between the neutron and proton masses is related to the up and down current quark masses and the electron mass, and via (3.1) through which we can eliminate m_e, *exclusively* to the up and down current quark masses according to:

$$M_N - M_P = m_u - m_e - \frac{2\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = m_u - \frac{3m_d - 3m_u + 2\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} . \tag{3.2}$$

We shall regard (3.1) and (3.2) above to be *exact* relationships not only at $Q = 0$ but for all Q, which is to say we shall take these to be both exact and Q-invariant. And we shall use these relationships as the starting point to obtain many other relationships – most very close to empirical data albeit still approximate beyond 10^{-5} to 10^{-7} AMU – intended to contradict or validate our treatment of (3.1) and (3.2) as exact Q-invariant relationships. For these reasons, simply to provide a shorthand for discourse we shall henceforth refer to (3.1) and (3.2) above as the "primary mass relationships" among the up and down current quark masses, and the electron, proton and neutron masses. It will be appreciated, because m_e in (3.1) is known with very high precision and because $M_N - M_P$ in (3.2) is known with similarly high precision, that when we take (3.1) and (3.2) together, and if we do regard these as exact Q-invariant relationships as just discussed, that we can combine these to deduce m_u and m_d with commensurately-high precision.

This calculation is performed in section 10 of [2] using the median empirical values $m_e = 0.000\,548\,579\,909\text{ u}$ [11], $M_N = 1.008\,664\,916\,0\text{ u}$ and $M_P = 1.007\,276\,466\,8\text{ u}$ [7] which all have been experimentally measured to ten or more digits of precision in AMU. So using these values in (3.1) and (3.2) above leads us to deduce in [10.3] and [10.4] of [2] to the same ten-digit precision as the proton and neutron masses that:

$$m_u = 0.002\,387\,339\,3\text{ u} = 2.223\ 792\ 40\text{ MeV}\ , \tag{3.3}$$

$$m_d = 0.005\,267\,312\,5\text{ u} = 4.906\ 470\ 34\text{ MeV}\ . \tag{3.4}$$

As noted in the introduction, the median electron mass to the same precision level in MeV is m_e = 0.510 998 93 MeV. Certainly (3.3) and (3.4) converted to MeV fit well within the PDG error bars which inform us that the empirical $m_u = 2.3^{+0.7}_{-0.5}\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\text{MeV}$ [8]. So we at least know that there is *no direct empirical contradiction* to these masses (3.3) and (3.4) from this particular empirical data.

Starting from (3.3) and (3.4) as deduced from the primary mass relationships (3.1) and (3.2) there are three questions which now need to be explored which will occupy the balance of the development in this paper:

1) Legitimate, Unambiguous Measurement Scheme?: Can we make such a precise statement about the masses of the up and down quarks, given: the wide PDG error bars $m_u = 2.3^{+0.7}_{-0.5}\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\text{MeV}$; that these error bars reflect that quark masses are thought to be dependent upon the renormalization scheme and the renormalization scale Q; that quarks are confined and so can never have their $Q=0$ masses *directly* measured in the same way we are able to measure the $Q=0$ electron or proton or neutron masses; and that the only domain within which it even starts to make sense to talk about directly measuring a quark mass is the domain where $Q \ge \Lambda_{\text{QCD}}$? Indeed, these wide error bars emerge because it is commonly perceived that $Q \ge \Lambda_{\text{QCD}}$ is the only domain in which it makes sense to talk about current quark masses and because as seen in Figure 1, measurement in this domain – invariably via scattering experiments at various Q-depths – is so highly-dependent upon the scale Q and the renormalization scheme we use. In short, can we use (3.3) and (3.4) as precise statements about the $Q=0$ up and down quark masses, in view of all these issues reviewed in section 2 and just summarized?

2) Clear Secondary Empirical Support?: If we can legitimately assert (3.3) and (3.4) to be the $Q=0$ up and down current quark masses by overcoming the "measurement" challenges of point 1 and section 2 above, are (3.3) and (3.4) supported by empirical particle data? This is a straightforward question as to whether nature supports (3.3) and (3.4) based on energies we observe when we do experiments. As noted the results m_u = 2.223 792 40 MeV and m_d = 4.906

470 34 MeV certainly are not contradicted by PDG's $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$; indeed they sit fairly near the mean of this data. But it would be desirable to see if (3.3) and (3.4) can be supported by *additional empirical data* beyond the electron, neutron and proton masses from which they were deduced via (3.1) and (3.2), via what we shall refer to as "secondary empirical relationships." Specifically, *if* (3.3) and (3.4) are indeed correct valuations for the up and down current quark masses on a $Q = 0$ scale, and because the neutron, proton and electron masses are already related to these via (3.1) and (3.2), it seems plausible that other energies of interest, namely the binding, fusion, mass defect and nuclear weight energies of light nuclides such as hydrogen and helium and lithium and beryllium, etc., might also be related to and be secondary functions of these exact same $Q = 0$ quark masses. In other words, if (3.3) and (3.4) are legitimately-defined $Q = 0$ quark masses then these masses will always be the $Q = 0$ quark masses whether these quarks are in a free proton are neutron or, for example, are in a proton or neutron inside of an alpha particle (^{4}He nucleus), or in a proton or neutron inside an ^{56}Fe nucleus, or are deep within the bowels of a lead or a uranium nucleus, etc. And that means that we *should* be able to specify the observed nuclear data for *any and all types of nuclei* solely as a function of these two quark masses. This provides ample latitude for empirical contradiction. But at the same time if a substantial number of nuclides can indeed have their nuclear data parameterized using secondary relationships based exclusively on the two masses (3.3) and (3.4), this would provide good empirical support for these results.

3) <u>Solid Theoretical Foundation and Clear Theoretical-to-Empirical Interface?</u>: If we can legitimately assert (3.3) and (3.4) to be the $Q = 0$ up and down quark masses and if we can find secondary support for these mass values from a broad array of empirical nuclear data then we get to the third question: what is the overarching theory, does that theory make sense within the overall framework of theoretical physics, and what is the interface by which we connect the theory to the means by which it can be empirically tested? As stated, the overarching theory first laid out in [1] and further developed and refined in [10] asserts that *baryons are the color-neutral chromo-magnetic monopoles of non-Abelian Yang-Mills gauge theory and that mesons are the non-vanishing magnetic field quanta which net flow across closed surfaces of these monopoles*. It is from this theory that the primary mass relationships (3.1) and (3.2) were initially discerned, and upon which the ^{2}H, ^{3}H, ^{3}He and ^{4}He [2] and ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N [5] binding energies can be explained *exclusively* as a function of the two masses (3.3) and (3.4), via a series of secondary relationships, to at least parts per hundred thousand AMU in all cases. And it is from this theory, once the Fermi vev v_F=246.219651 GeV and the Cabibbo, Kobayashi and Maskawa (CKM) mixing matrix are also admitted as parameters alongside of these two quark masses, that *the proton and neutron masses themselves can be fully explained within all known experimental errors*. [6]

So for the balance of this paper, we shall address each of these three questions in turn.

4. Does Deduction of Very Precise *Q* = 0 Up and Down Current Quark Masses from the *Q* = 0 Electron, Proton and Neutron (EPN) Masses Establish a Legitimate Measurement Scheme?

As discussed at the start of section 3, because quarks are confined it is impossible to ever measure their $Q=0$ masses *directly* because to access a quark in the six quark model (which clearly looks to be what nature chooses and which we shall henceforth regards as nature's choice) one must provide an impact energy at least on the order of $Q=\Lambda_{\mathrm{QCD}}=.0906\ \mathrm{GeV}$. In other words, to directly detect of *any attributes* of an individual quark – and indeed its very existence – one must supply an impact energy north of about 90 million electron volts. So whatever quark attributes we observe at *Q* = 90 MeV and higher – mass, coupling, spin, etc. – will *by definition* not be the $Q=0$ attributes of the quark. This is the measurement problem which leads to the large error bars $m_u=2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d=4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$ wherein the quark masses are dependent upon the chosen measurement scheme and once a scheme is chosen, upon the choice of *Q*, given that $Q=0$ quark attributes appear to not be measurable because quarks are confined, not free, particles. So it is supposed that we cannot *define* a *Q* = 0 quark mass because we can never *directly measure* a *Q* = 0 quark mass. This same problem may well be the root of the "proton spin crisis" as well. [14]

But in (3.1) and (3.2) we have chosen a measurement scheme by which the up and down quark masses are *inferred indirectly* from the $Q=0$ electron, proton and neutron masses. Just like minimal subtraction MS and modified minimal subtraction $\overline{\mathrm{MS}}$, (3.1) and (3.2) do represent a measurement scheme for quark masses albeit a different scheme from the usual. The question here is whether this different scheme is a *legitimate and unambiguous* measurement scheme.

As already noted in the introduction, any time that we do an experiment for which $Q>0$ we are necessarily doing a scattering experiment, which is to say we are bombarding a target in some fashion and discerning information about the target via forensic analysis of the post-bombardment debris coupled with knowledge of the bombardment we employed. No matter how it is couched in its specifics any experiment with $Q>0$ is *by definition* causing an impact with the target we seek to study and in the course of obtaining information about the target we are necessarily altering the target. Thus when we use several different *Q* at several different times we have to prepare for the possibility that what we are measuring about the target will take on several different values with no one particular value being any more correct or unique than any other value. Thus we will have error bars stemming from more than just the limitations of our measuring equipment, and that is what shows up in the PDG error bars. As said in section 1, such an experiment entails "bombing" the target not "weighing" the target.

Conversely, merely taking the weight of a body is *the quintessential* $Q=0$ *experiment*, whether that body is a person or a baseball, or an electron, proton or neutron. *Subject to the caveat in the next paragraph* we do not have to impact a body in order to weigh that body; we merely place it on a scale and then rely upon the equivalence of gravitational and inertial mass. So we are able to say that at $Q=0$ the mass of the electron is $m_e=0.000\,548\,579\,909\ \mathrm{u}$, period.

And we are similarly able to say that at $Q=0$ the masses of the proton and the neutron are $M_N = 1.008\,664\,916\,0\text{ u}$ and $M_P = 1.007\,276\,466\,8\text{ u}$, period. We do not need to talk about the measurement scheme and we do not need to talk about the renormalization scale Q other than to understand that by definition we are using $Q=0$, which we can do because the particles we are measuring are free. Of course we have the option if we wish to study how these masses may vary from their $Q=0$ values for various $Q\neq 0$. But $Q=0$ does provide a uniqueness which is not provided by any other Q, with the possible exception of $Q=\Lambda_{\text{QCD}}=.0906\text{ GeV}$ which happens to coincide with the confining $\alpha_s=\infty$ and so presents other measurement challenges because it is a divergent and highly non-perturbative region of the Q domain.

Now of course someone who is familiar with experiments used to obtain the above-recited electron, proton and neutron masses will understand the caveat that nobody can really put one of these particles on a scale and "weigh" that particle in the same manner that we can weigh ourselves or weigh a macroscopic object. The experiments used to establish these masses themselves do have some $Q\neq 0$ scattering aspect, if for no other reason that a particle cannot even be detected unless something else, such as some photons, interacts with that particle. However, the electron, proton and neutron are all free particles unlike quarks, and *their masses approach asymptotic values as* $Q\to 0$. So by doing enough experiments on these free particles – even with some impact – it is possible to deduce the asymptote that is approached by the masses of each of these particles. Therefore, the precision with which the experimental community has succeeded taking such asymptotic measurement is effectively expressed by the mass values and associated experimental errors for m_e, M_P and M_N given in [11] and [7]. The same can also be said for measurements of the masses of composite nuclides, such as ^{2}H, ^{3}H, ^{3}He, ^{4}He, etc.

So when we take the expressions (3.1) and (3.2), plug in the $Q=0$ "weights" of the electron, proton and neutron, and thereby deduce (3.3) and (3.4) for the up and down current quark masses, what we have discerned – albeit indirectly – must also be regarded as the $Q=0$ "weights" of these two quarks. This is a different scheme from the minimal subtraction schemes which are usually employed to specify quark masses and other running attributes of the quarks, but it is still a scheme and we need to determine if it is a *valid* scheme. So let us explore this.

Momentarily, suppose we were not aware of (3.1) and (3.2). Suppose simply that we were able – hypothetically – to establish *some unspecified pair of valid Q-invariant relations* which express the up and down quark masses in relation to the electron mass and the neutron minus proton mass difference such that these two quark masses were *uniquely fixed* once these other two numbers were fixed. Then by employing the $Q=0$ values of the electron and the neutron minus proton mass difference we would necessarily be deducing the $Q=0$ values of the up and down quark masses and we would have a legitimate measurement scheme. The point here is that this "weighing, not bombing" scheme is not wedded to the specifics of (3.1) and (3.2) but rather to the question whether *any valid relationships* which might *uniquely* output the up and down quark masses once the $Q=0$ electron, proton and neutron masses are given can *in principle* be said to yield legitimate values for the $Q=0$ quark masses.

Understood in this manner, it should be clear that it is perfectly legitimate *as a matter of defining a measurement scheme* to specify $Q=0$ confined quark masses in relation to the known masses of other particles which are free and which can be observed asymptotically in the low-Q energy domain, *if* such relationships exist and can be found and can be empirically confirmed by all of the other empirical data they affect. So the real question becomes *whether there do in fact exist some valid relations* in nature by which the up and down quark masses can be uniquely deduced from the electron, proton and neutron masses (or any other free particle $Q=0$ masses), and if so, what those relationships are and whether (3.1) and (3.2) are in fact those relationships.

We may also approach this same question by contradiction: To argue that a scheme in which Q=0 up and down current quark masses are defined using the Q=0 electron, proton and neutron (EPN) masses or any other free particle masses is *invalid in principle* one would have to argue that there are not and cannot exist in nature, any Q-invariant relationships whatsoever relating these up and down current quark masses to the EPN or other free particle masses. Current quark masses, one would have to argue, *cannot* bear any precise relationships to free particle masses because the former are confined and the latter are not. Strong and nuclear interactions *cannot* be unified, one would have to argue, because the former is about confined quarks and the latter is about free nucleons and nuclei. The current masses of the quarks inside a proton or neutron cannot bear any precise relationship to the proton or neutron masses themselves, or to the mass of an electron in the very same atom, one must argue. The logical culmination would have to be a “never the twain shall meet” argument that one cannot – even in principle – have relationships like (3.1) and (3.2) anywhere in nuclear and strong interaction physics. For, if such Q-invariant relationships were to be found, then the use of the Q=0 EPN masses in these relationships would necessarily yield the Q=0 up and down masses. This should make clear that *so long as valid relationships in the nature of (3.1) and (3.2) are possible* – and there appears no basis for stating that they are impossible – then a weight-based rather than scattering-based measurement scheme such as EPN would be valid, albeit different. The only question then left is whether (3.1) and (3.2) are indeed the actual relationships among the up and down masses and the EPN masses. That becomes an empirical question about how well these relationships and other related relationships match observed data, and a theoretical question about the basis upon which those relationships are rested.

If it should turn out that (3.1) and (3.2) are valid Q-invariant relationships, then (3.3) and (3.4) are indeed the $Q=0$ masses of the up and down quarks and a measurement scheme for defining these quark masses in this way is perfectly legitimate. Further, by having these two mass values (3.3) and (3.4) we would now know the quark masses with a precision that is *close to a billion times more precise* than what we learn from $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\mathrm{MeV}$ based the $\overline{\mathrm{MS}}$ scheme.

It is the foregoing elaboration of how the quark masses $m_u = 0.002\,387\,339\,3\ \mathrm{u}$ and $m_d = 0.005\,267\,312\,5\ \mathrm{u}$ can be *legitimately defined* from the electron, proton and neutron masses with a precision vastly exceeding the PDG data based on $\overline{\mathrm{MS}}$, which was absent from the author’s prior work. The forgoing discussion should remedy this deficiency. And it should also be very clear that a second mass-definition scheme which allows the quark masses to be defined close to a billion times more accurately than a first scheme is manifestly preferable to the first scheme, so long as that second scheme is unambiguous, uncontradicted by empirical data, and has solid theoretical roots.

Because this scheme *defines* $Q=0$ up and down current quark masses in (3.3) and (3.4) from the relationships (3.1) and (3.2) using the $Q=0$ electron (E), proton (P) and neutron (N) masses we shall refer to this as the EPN measurement scheme with an EPN-0 definition for the up and down quark masses. Of course relationships such as (3.1) and (3.2) should apply at all *Q*. So if one were to know how each of $m_e(Q)$, $M_P(Q)$ and $M_N(Q)$ run as a function of *Q*, one could then use (3.1) and (3.2) to further derive $m_u(Q)$ and $m_d(Q)$, or vice versa. In this way the EPN scheme provides a consistent and unambiguous basis for first defining the up and down quark masses at $Q=0$ based on three masses m_e, M_P and M_N which are each known at $Q=0$ with very high precision, and for then interrelating them in a *Q*-invariant manner as *Q* runs to higher energies. And it avoids the pitfalls and ambiguities of having to define quark masses based on scattering probes inside the nucleons which necessarily make these masses a function of our experiment.

Now, with the measurement question of how best to define the current quark masses addressed we next turn to question whether (3.3) and (3.4) are indeed the correct physical $Q=0$ quark masses. If they are then this in turn would validate the relationships (3.1) and (3.2) and the theory from which these are obtained. Certainly the fact that masses (3.3) and (3.4) fit well within PDG's $m_u=2.3^{+0.7}_{-0.5}\text{MeV}$ and $m_d=4.8^{+0.5}_{-0.3}\text{MeV}$ provides preliminary credence for these masses by failing to invalidate these masses. But this is a starting point not an endpoint. Now we arrive at the second question posed in section 3, whether the quark masses (3.3) and (3.4) have clear secondary empirical support from other nuclear data. As we shall now review in the next two sections, this empirical support is abundant.

5. Origins of the Primary Mass Relationships used in the EPN Measurement Scheme

In section 3, we simply stated the primary mass relationships (3.1) and (3.2). Now it is appropriate to begin discussing their physical origins which are found in the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory. For the moment, let us just lay out some general physics background which we will later apply in section 7.

It is well-known that $T^{\mu\nu}=\partial^\mu\phi\left(\partial\mathfrak{L}/\partial\left(\partial_\nu\phi\right)\right)-g^{\mu\nu}\mathfrak{L}$ is the canonical energy-momentum tensor for a given field φ with associated Lagrangian density $\mathfrak{L}$. If we require the spatially-integrated Lagrangian $L=\iiint\mathfrak{L}d^3x$ to be stationary under small field variations then the $\partial^\mu\phi\left(\partial\mathfrak{L}/\partial\left(\partial_\nu\phi\right)\right)$ term can be neglected and this becomes $T^{\mu\nu}=-g^{\mu\nu}\mathfrak{L}$. So in flat spacetime with $g^{00}=1$ we have $T^{00}=-\mathfrak{L}$. Therefore the total energy E of the system associated with $\mathfrak{L}$ will be $E=\iiint T^{00}d^3x=-\iiint \mathfrak{L}d^3x=-L$, and more simply, $E=-L$.

Now, in abelian electrodynamics the Lagrangian density associated with a pure gauge field $F^{\mu\nu}$ is given by $\mathfrak{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ and so $E=-L=-\iiint \mathfrak{L}d^3x=\iiint \frac{1}{4}F_{\mu\nu}F^{\mu\nu}d^3x$ will specify the energy arising from the pure gauge field terms. In Yang-Mills gauge theory the field strength may still be written with $F_{\mu\nu}$ as shorthand, but it contains additional internal symmetry structure which must be understood. Particularly, for any simple unitary gauge group SU(N) there are a set of Hermitian generators λ^i with $i=1...N^2-1$ forming a closed group and commuting according to $\left[\lambda^i,\lambda^j\right]=if^{ijk}\lambda^k$, conventionally normalized to $\mathrm{Tr}\lambda^{i2}=\frac{1}{2}$. Each of these generator matrices has rank 2 with an N×N dimensionality so to be fully explicit we must represent these Hermitian matrices by $\lambda^i{}_{AB}$ with $A,B=1...N$. So in reality the field strength $F_{\mu\nu}$ is a shorthand for $F_{\mu\nu AB}=\lambda^i{}_{AB}F^i{}_{\mu\nu}$, where the "adjoint form" $F^i{}_{\mu\nu}$ consists of N^2-1 individual 4×4 field strength tensors and the "matrix form" $F_{\mu\nu AB}$ is an N×N internal symmetry matrix of 4×4 field strength tensors. The pure-gauge field Lagrangian density represented in the matrix form is now $\mathfrak{L}=-\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$ with the doubling of the coefficient owing to the generator normalization and the trace arising because we need to obtain a scalar number from the internal symmetry matrices. So the energy for the pure Yang-Mills gauge field is $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$.

Now if we want to be as explicit as possible then rather than using the trace (Tr) notation we can use the matrix form $F_{\mu\nu AB}$ and explicitly show the index contractions which yield this trace, namely, $\mathfrak{L}=-\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)=-\frac{1}{2}F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$. That is, the trace is formed first by taking an *inner product* $F_{\mu\nu AB}F^{\mu\nu}{}_{BC}$ which yields a new N×N internal symmetry matrix. Then we contact the A and C indexes to obtain $F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$. It is by this latter contraction that we obtain the trace, and more specifically, the *inner product trace*. But mathematically there is a second trace available from $F_{\mu\nu}F^{\mu\nu}$ and that is the *outer product trace* which for any two matrices A and B is given by $\mathrm{Tr}(A\otimes B)=\mathrm{Tr}(A)\mathrm{Tr}(B)$. So using explicit indexes the outer product trace is $F_{\mu\nu AA}F^{\mu\nu}{}_{BB}$. Thus if we wish to be as general as possible we should entertain the possibility of constructing the pure Yang-Mills gauge field Lagrangian density using some linear combination of both the inner product trace $F_{\mu\nu AB}F^{\mu\nu}{}_{BA}$ and the outer product trace $F_{\mu\nu AA}F^{\mu\nu}{}_{BB}$.

With this general background in mind we start with an $F_{\mu\nu AB}$ which is carefully developed for the chromo-magnetic monopoles of Yang-Mills gauge theory in [10.1] of [1] and which is more deeply developed in [10.4] of [10]. This $F_{\mu\nu AB}$ employs the gauge group SU(3)$_C$ of strong chromodynamic interactions with colors R, G, B, which means that the internal symmetry matrices have a 3x3 dimensionality, see, e.g., the matrix [9.20] of [10] which explicitly shows this. We then represent a (duu) proton by assigning the R quark *color* to the down quark *flavor* and the G and B quark *colors* to the up quark *flavors* via the assignments $\mathrm{R} \to d; \mathrm{G} \to u; \mathrm{B} \to u$. We represent a (udd) neutron by an analogous assignment $\mathrm{R} \to u; \mathrm{G} \to d; \mathrm{B} \to d$. This is all detailed in sections 7 and 8 of [1] and the second half of section 10 in [10]. Finally, as laid out in sections 9, 11 and 12 of [1] we calculate an energy $E = \iiint \frac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} d^3x$ using the *outer product trace* $E = \iiint \frac{1}{2} F_{\mu\nu AA} F^{\mu\nu}{}_{BB} d^3x$ for each of the so-represented proton and neutron.

For the moment we simply show the result, and in sections 7 and 8 we shall show the calculations which lead to this result and how they fit within the overall theory. It turns out that these respective energies following calculation, showing both the matrix form and the scalar expression after the outer product trace is taken, see (12.4) and (12.5) of [1], are:

$$E_P = \frac{1}{(2\pi)^{\frac{3}{2}}} \mathrm{Tr} \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} = \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}} , \quad (5.1)$$

$$\equiv (2\pi)^{-\frac{3}{2}} \mathrm{Tr} K_P \otimes K_P = (2\pi)^{-\frac{3}{2}} K_{P\,AA} K_{P\,BB}$$

$$E_N = \frac{1}{(2\pi)^{\frac{3}{2}}} \mathrm{Tr} \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} = \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}} . \quad (5.2)$$

$$\equiv (2\pi)^{-\frac{3}{2}} \mathrm{Tr} K_N \otimes K_N = (2\pi)^{-\frac{3}{2}} K_{N\,AA} K_{N\,BB}$$

In the final lines of each of the above, we denote the matrix appearing twice in (5.1) as $K_{P\,AB}$ and twice in (5.2) as $K_{N\,AB}$. We also point out, as elaborated in sections 2 through 4 of [6], that these matrices K can be used to restate the Koide mass relationships [15] which is why we choose the symbol "*K*" for these. We further point out as elaborated in the rest of [6] that by supplementing the energy square roots $\sqrt{m_u}$ and $\sqrt{m_d}$ with $\sqrt{v_F}$ where v_F=246.219651 GeV is the Fermi vev one can make extended use of these "Koide matrices" to explain *the proton and neutron masses themselves*.

If we then take the *difference* $E_N - E_P$ between (5.2) and (5.1) the expression we get is

$$E_N - E_P = \frac{3}{(2\pi)^{\frac{3}{2}}} (m_d - m_u) \equiv m_e , \quad (5.3)$$

where we *define* (really, hypothesize) this to be equal to the electron rest mass. It will be seen that this is just another way of writing (3.1), which is the first primary mass relationship. Why do we make this hypothesis? The reasons are partly empirical and partly theoretical.

Originally in [1] the author approached (5.1) and (5.2) by calculating $E_N - E_P$ using the PDG data $m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ and found that $E_N - E_P = .476^{+.228}_{-.190}$ MeV which nicely contains the electron rest mass $m_e = .511\,\text{MeV}$ pretty much near the center of the error bar. This was the first plausible point of contact that was made from the theory that baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory to empirical data. This made theoretical sense because a neutron decaying into a proton via $n \to p^+ + e^- + \overline{\nu} + \text{Energy}$ and a down quark decaying into an up quark via $d \to u + e^- + \overline{\nu} + \text{Energy}$ would – at least at a "linear" or "lowest order" level – support a relationship of the form $E_N - E_P \propto m_d - m_u \propto m_e$ in (5.3). Which is to simply state that neglecting all the non-linear behaviors of nucleons, the difference between a proton and a neutron or between an up quark and a down quark is an electron. So given both this empirical concurrence and the $n - p^+ = e^- + \ldots$ and $d - u = e^- + \ldots$ decay sensibilities, (5.3) was elevated into a *hypothesized* relationship relating the electron rest mass to the down minus up quark current mass difference and to the difference between some neutron energy number and some proton energy number, all to be confirmed or contradicted based on additional empirical data. Subsequent theoretical development in section 9 of [10] demonstrated that (5.1) through (5.3) are in fact all relationships taken in the zero-order abelian field theory limit of classical Yang-Mills gauge theory. And subsequent empirical development which will be detailed below and in the next section appears to validate rather than refute (5.3) and to show that this abelian limit appears to govern what is observed in nuclear binding and fusion events and the nuclear mass defects.

Now, we turn to explain the origins of the second primary relationship (3.2) and for this we must begin to discuss nuclear binding energies. While (5.3) was the first plausible point of contact between theory and experiment uncovered by the author it was (5.1) and (5.2) themselves which opened up fertile new vistas via some extremely compelling connections to nuclear binding energies. We now explain how this is developed.

If (5.1) and (5.2) represent some to-be-determined energies associated with the proton and neutron then it is certainly a good idea to calculate these energies. We may do so using $m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}$ and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ from PDG which is what the author first did in [12.4] and [12.5] of [1]. But rather than retread this same ground let us use the much-more-precise masses (3.3) and (3.4) which are the correct quark masses *if (3.1) and (3.2) are valid relationships*, since that is what we are testing at present. So if we use (3.3) and (3.4) in each of (5.1) and (5.2) and then also apply 1 u = 931.494 061(21) MeV we may calculate to ten significant digits in AMU and seven significant digits in less-precise MeV [4] that:

$$E_P = \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}} = 0.001\,837\,399\,7\ \text{u} = 1.711\,526\,9\ \text{MeV}\,, \tag{5.4}$$

$$E_N = \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}} = 0.002\,387\,693\,9\ \text{u} = 2.224\,122\,7\ \text{MeV}\,. \tag{5.5}$$

Now at first sight, these energies are a bit mysterious. After all M_N = 939.565379 MeV and M_P = 938.272046 MeV so these energies are certainly not the proton and neutron masses themselves. But we know that the proton and neutron contain three quarks each, that the current masses of the quarks contribute only slightly to the overall proton and neutron masses, and that the remainder of the mass is generated through extensive non-linear interactions involving quarks and gluons. So let us strip out all of these interactions and focus solely on the current quark masses which, when properly summed together, should represent something of a “zero order” value for the proton and neutron masses. Continuing to use the masses (3.3) and (3.4), the sums Σ of these current quark masses, for the duu proton and udd neutron respectively, are:

$$\Sigma_P = 2m_u + m_d = \mathrm{Tr}\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} = \mathrm{Tr} K_P \cdot K_P = K_{PAB} K_{PBA} , \qquad (5.6)$$
$$= 0.010\,023\,9911\,\mathrm{u} = 9.337\,288\,2\ \mathrm{MeV}$$

$$\Sigma_N = 2m_d + m_u = \mathrm{Tr}\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} = \mathrm{Tr} K_N \cdot K_N = K_{NAB} K_{NBA} . \qquad (5.7)$$
$$= 0.0129129643\,\mathrm{u} = 12.0283496\ \mathrm{MeV}$$

We note that these sums $\Sigma_P = 2m_u + m_d = \mathrm{Tr} K_P \cdot K_P$ and $\Sigma_N = 2m_d + m_u = \mathrm{Tr} K_N \cdot K_N$ employ the *inner product trace* of the *same Koide matrices* for which the outer product trace was taken in (5.1) and (5.2).

These energy numbers deepen the mystery further because one would expect the predicted energies (5.4) and (5.5) to at least be as much as the mass sums (5.6) and (5.7) and yet they are substantially less. That is, some of the mass we expect to see in (5.6) and (5.7) is “missing” from (5.4) and (5.5), in much the same way some of the mass one might expect to see by fusing two nuclides if we naively add their separate masses together goes missing in the mass defect and is released as fission energy. So now the question becomes: how much mass has gone missing in (5.5)? We can easily calculate this missing energy difference $\Delta = \Sigma - E$ for each of the proton and neutron by subtracting (5.4) from (5.6) and (5.5) from (5.7) as was first done using the PDG data in [12.6] and [12.7] of [1], but is now done using (3.3) and (3.4), to obtain:

$$\Delta_P = \Sigma_P - E_P = 2m_u + m_d - \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}} = 0.008\,186\,591\,4\,\mathrm{u} = 7.625\,761\,3\ \mathrm{MeV} , \qquad (5.8)$$
$$= \mathrm{Tr} K_P \cdot K_P - (2\pi)^{-\frac{3}{2}} \mathrm{Tr} K_P \otimes K_P$$

$$\Delta_N = \Sigma_N - E_N = 2m_d + m_u - \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}} = 0.010\,525\,270\,4\ \mathrm{u} = 9.804\,226\,8\ \mathrm{MeV} \\ = \mathrm{Tr}K_N \cdot K_N - (2\pi)^{-\frac{3}{2}}\,\mathrm{Tr}K_N \otimes K_N \quad . \quad (5.9)$$

We see that these missing masses Δ combine *both the inner and outer product traces* of the 3x3 Koide matrices in (5.1), (5.2), (5.6) and (5.7).

We may then easily calculate that the average of these two missing energies $\frac{1}{2}(\Delta_P + \Delta_N) = 8.714\,994\,1\ \mathrm{MeV}$. *It is this number which starts to reveal some very deep empirical connections with nuclear physics.* For, if we refer to the well-known empirical curve for the binding energy per nucleon which is reproduced below as Figure 2, and if we keep in mind that most nuclides have roughly the same number of protons as neutrons but with larger proportion of neutrons over protons as the nuclides get heavier, we see that this number is very close to the peak per-nucleon energy at about 8.75 MeV per nucleon. In particular we know that the heaviest nuclides do give up approximately 8.75 MeV per nucleon in order to bind together which very closely tracks the missing energy $\frac{1}{2}(\Delta_P + \Delta_N) = 8.714\,994\,1\ \mathrm{MeV}$. Plainly put: (5.9) predicts that about 8.75 MeV of energy goes missing on average from a nucleon and Figure 2 tells us that about 8.75 MeV of energy really is empirically missing on average from nucleons near the peak of the nuclear binding table. Both energies are just about the same, and both energies are "missing" energies, and really, *binding energies*.

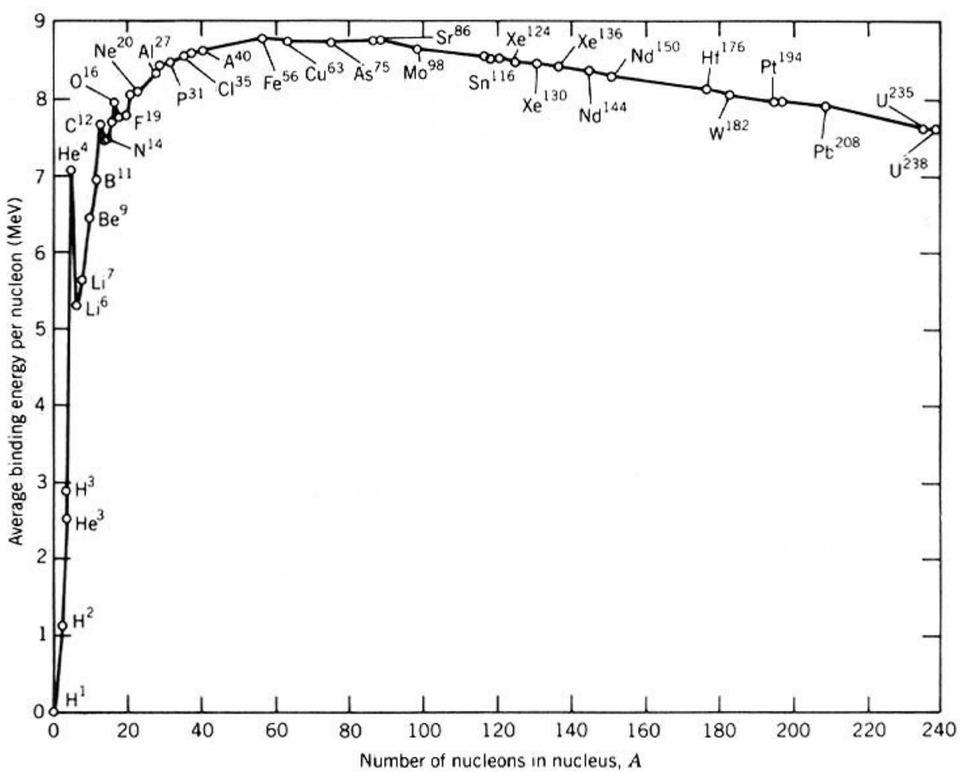

Figure 2: Empirical Binding Energy per Nucleon

It is this observation, first reported in section 12 of [1], which caused the author to initially suspect that these missing masses are very closely related to nuclear binding. And to be clear, the author had no *a priori* suspicion that these missing masses might be related to nuclear binding. This was just an exploratory exercise. Had the result of the foregoing calculation been $\frac{1}{2}(\Delta_P + \Delta_N) = 20\,\text{MeV}$, or $\frac{1}{2}(\Delta_P + \Delta_N) = 3\,\text{MeV}$, or some other number, then this would not have implicated nuclear binding and mass defects as the source of this missing mass. *It is only because the missing mass was theoretically predicted to be* $\frac{1}{2}(\Delta_P + \Delta_N) = 8.714\,994\,1\,\text{MeV}$ *and this is so close to the peak of the nuclear binding curve, that these missing masses were first suspected to be related to the mass defect and nuclear binding.* So here, the matching of a theoretical prediction to empirical data gave birth to a new theoretical pursuit that was unanticipated at the outset and that was driven by empirical numerical energy data.

Once this connection is discerned, one is highly motivated to use (5.8) and (5.9) to examine the binding energies of nuclides right near the peak of Figure 1. The two best examples are ^{56}Fe and ^{62}Ni which have two of the highest per-nucleon binding energies of all the nuclides in nature. The former has 26 protons plus 30 neutrons with an empirical binding energy of 492.253892 MeV [16] at the median of the error range, and the latter has 28 protons and 34 neutrons with an empirical binding energy of 545.2590 MeV (calculated from [17]) at the median. So if we use (5.8) and (5.9) to ascertain how much energy is "missing" from each of these nuclides we find that:

$$\Delta\left({}^{56}\text{Fe}\right) = 26\Delta_P + 30\Delta_N = 492.396\,598\,5\,\text{MeV} \quad \text{versus} \quad 492.253\,892\,\text{MeV observed}, \tag{5.10}$$

$$\Delta\left({}^{62}\text{Ni}\right) = 28\Delta_P + 34\Delta_N = 546.865\,028\,4\,\text{MeV} \quad \text{versus} \quad 545.259\,0\,\text{MeV observed}. \tag{5.11}$$

So for ^{56}Fe the observed binding energy is 99.9710% of the theoretical missing energy $\Delta\left({}^{56}\text{Fe}\right)$ and for ^{62}Ni this same percentage is 99.7063%. And if one does a similar calculation for all of the other nuclides near ^{56}Fe and ^{62}Ni it turns out – very, very importantly – that *no nuclide reaches or exceeds 100%* and that the very highest percentage is the one just shown for ^{56}Fe. This means that (5.8) and (5.9) – in some manner that needs to be understood – are establishing the upper empirical per-nucleon limit which is observed in the nuclear binding curve in Figure 1. Clearly then, the results in (5.10) and (5.11) validate that (5.8) and (5.9) are revealing something very real and very deep about nuclear binding, which gives further credence to the validity of the relationships (5.1) and (5.2) and thus the primary mass relationship (3.1) a.k.a. (5.3) with which these are integrally interconnected.

From here, we shall avoid repetition and instead refer the reader to the primary reference [2] in which the author first deciphers and explores the meaning of these results in detail. But the most important highlights which do need to be conveyed in the context of the present paper, specifically to explain the origins of the primary mass relationship (3.2) presently under consideration for the neutron minus proton mass difference, are the following:

1) Nuclear Binding and Quark Confinement: The energies (5.8) and (5.9), in physical reality, are "latent binding energies," or "energies available for nuclear binding," of the *free* proton and neutron, respectively. What does this mean? When a proton or a neutron is *free*, i.e.,

not bound to any other nucleon, then the entirety of this latent binding energy is used to confine quarks within the nucleon. But when a proton or neutron is *fused and bound* into a nucleus with at least one other nuclide, some – but never all – of the latent binding energy in (5.8) / (5.9) is released as fusion energy, the mass of the fused nucleus as a whole becomes less than the sum of the masses of all its separate nucleons, this is what underlies the mass defect, and this lost mass / energy goes into the binding energy fusing together the nucleus, all in a sort of energetic nuclear "see saw" between confinement and binding. So the quarks inside *free* nucleons are most tightly confined because these nucleons are unbound and so none of their latent binding energy is diverted from quark confinement to nuclear binding. At the other end of the see saw, the quarks inside nucleons inside tightly bound nuclei such as ^{56}Fe are least-tightly confined, because almost all (99.9710% in the case of ^{56}Fe) of the latent binding energy is used for nuclear binding and not confinement. But these quarks are still confined nonetheless because *no nuclide ever gives up more than 99.9710% of its total latent binding energy for binding*. There is always at least 0.00290% of the latent binding energy which remains behind to confine the quarks. Understood in this way, confinement is empirically manifest by the fact that *no nuclide ever reaches or exceeds 100% usage of its latent binding energy for actual nuclear binding*. For ^{56}Fe, the total 0.00290% which is held in reserve for confinement and not channeled into nuclear binding amounts to a scant 0.142706 MeV which is less than 1/3 the mass of a single electron. But that is still enough to keep the quarks confined. Because *no nuclide* ever uses up more than 100% of its latent binding energies for actual binding, but always reserves at least some energy for confinement, quarks are always confined. Quarks inside the nucleons of ^{56}Fe are less-tightly confined than the quarks inside any other nuclide (which is a basis for understanding the "first EMC effect" [18]), but they do assuredly remain confined. The peak in Figure 2 at ^{56}Fe which sits at 99.9710% of what it would take to de-confine quarks, is one very direct way in which nature displays confinement. Indeed, the fact that the observed binding energies in (5.10) and (5.11) and any other nuclides are *always* less than the total latent binding energies reveals the energy-based explanation for why *quarks always remain confined*.

2) Observed and Latent Nuclear Binding Energies: In general, for a nuclide with Z protons and N neutrons hence $A = Z + N$ nucleons, the latent binding energy which we denote by ${}_Z^A B$ is calculated from (5.8) and (5.9) using:

$$ {}_Z^A B = Z \cdot \Delta_P + N \cdot \Delta_N \, . \tag{5.12} $$

So for example, (5.10) and (5.11) may be represented as specific applications of this formula for ${}_{26}^{56} B = \Delta\left({}^{56}\mathrm{Fe}\right)$ and ${}_{28}^{62} B = \Delta\left({}^{62}\mathrm{Ni}\right)$. And if we denote the *observed* empirical binding energies generally as ${}_Z^A B_0$ with the 0 subscript, then the percentage ratios discussed earlier are ${}_{26}^{56} B_0 \,/\, {}_{26}^{56} B = 99.9710\%$ and ${}_{28}^{62} B_0 \,/\, {}_{28}^{62} B = 99.7063\%$. These latent binding energies ${}_Z^A B$ thereby establish *upper limits* for the empirical binding energies. But as ^{56}Fe demonstrates, these limits are never reached or exceeded, that is, ${}_Z^A B_0 < {}_Z^A B$, or alternatively, ${}_Z^A B_0 \,/\, {}_Z^A B < 100\%$, *always*. So this now leads us to ask how to explain the specific *observed* binding energies ${}_Z^A B_0$ for *all* the nuclides. This is especially of interest for the lightest nuclides which have the lowest ${}_Z^A B_0 \,/\, {}_Z^A B$

ratios and for which the observed binding energies to date have not yet been satisfactorily explained. So, what do we now know to help us figure this out?

3) <u>The Binding and Fusion Energy "Toolkit"</u>: We know that the latent binding energies ${}_{Z}^{A}B = Z \cdot \Delta_P + N \cdot \Delta_N$ employ linear combinations of (5.8) and (5.9) and these in turn involve inner and outer product traces of the matrices (5.1), (5.2), (5.6) and (5.7). The elements of these matrix products in turn are very limited to only the energy numbers m_u, m_d, $\sqrt{m_u m_d}$, the foregoing divided by $(2\pi)^{\frac{3}{2}}$, and *integer multiples* of all these. We take the conservative and very stringent view that *every single observed nuclear binding energy* ${}_{Z}^{A}B_0$ must be constructed out of some combination of the foregoing energy number "toolkit" and "structurally sensible" integer multiples thereof which in turn means that the observed ${}_{Z}^{A}B_0$ must *all* be functions of the $Q = 0$ up and down quark masses (3.3) and (3.4) *and nothing else*. This is stringent because it gives us no room to adjust anything. If we cannot consistently construct the observed binding energies from these energy numbers with some fairly high degree of precision, which means as functions of the up and down quark masses – viewed as parameters – and nothing more, then this approach is contradicted. But if we can construct a fair number of observed binding energies in this way then that would lend solid empirical support to this approach. We know that the latent binding energies ${}_{Z}^{A}B = Z \cdot \Delta_P + N \cdot \Delta_N$ come readily packaged so for any given nuclide we should consider both adding to and subtracting from a pertinent ${}_{Z}^{A}B$, i.e., we should ask how much its binding energy either exceeds or falls below some ${}_{Z}^{A}B$. That is, how much is released for nuclear binding, and how much is held in reserve for quark confinement? We should also sensibly include in our "toolkit" scalar traces of the Koide matrices, namely, $\mathrm{Tr}K_P = \sqrt{m_d} + 2\sqrt{m_u}$ and $\mathrm{Tr}K_N = \sqrt{m_u} + 2\sqrt{m_d}$ multiplied by $\sqrt{m_u}$ or $\sqrt{m_d}$. Finally, to extend this approach we should consider matching these energy numbers not only to binding energies but also to the energies released during various fusion or fission and other decay reactions. From here, with toolkit assembled, the task of characterizing individual observed binding energies ${}_{Z}^{A}B_0$ involves elbow grease, a good spreadsheet or computer program, and educated trial and error. In this venture, one is using empirical data in combination with the foregoing toolkit to try to discern systematic but hidden theoretical patterns in the nuclear binding energies – in broad scope, seeking to "decode" and "map" the nuclear "genome."

4) <u>Hydrogen-2</u>: The easiest place to start is with the ^{2}H deuteron consisting of one proton and one neutron. In AMU the observed binding energy is ${}_{1}^{2}B_0 = 0.002\,388\,170\,100\ \mathrm{u}$. We then refer to our energy number "toolkit" m_u, m_d, $\sqrt{m_u m_d}$, the foregoing divided by $(2\pi)^{\frac{3}{2}}$, and integer multiples of these. But we need not search very far. From (3.3) the mass of the up quark is $m_u = 0.002\,387\,339\,3\ \mathrm{u}$. The difference is ${}_{1}^{2}B_0 - m_u = 8.308 \times 10^{-7}\ \mathrm{u}$, which is to say, the accuracy is to better *eight parts per ten million AMU*. It should be pointed out that in [1] the author originally *hypothesized* that the deuteron binding energy is *exactly equal to* the up quark mass due to how close they in fact appeared to be. That is, the author originally employed ${}_{1}^{2}B_0 = m_u$ rather than (3.2) as a primary mass relationship in combination with (3.1). Then on

this basis, over the course of the development in sections 1 through 9 of [2] the author was able for the first time to derive the primary mass relationship (3.2) for the neutron minus proton mass difference with eight parts per ten million AMU accuracy. Once this (3.2) had been derived, for the reasons elaborated at length in section 10 of [2], the author shifted hypotheses and advanced (3.2) to a primary, exact mass relationship while withdrawing ${}^{2}_{1}B_0 = m_u$, so that the sub-parts-per-million AMU error was shifted from (3.2) to ${}^{2}_{1}B_0$ and the original hypotheses retreated to ${}^{2}_{1}B_0 \approx m_u$ within less than one part per million AMU. It must also be pointed out that this error is *outside* of experimental error margins because ${}^{2}_{1}B_0$ is known with greater than ten-digit (parts per ten billion) accuracy. So as close as these results may be, the difference beyond parts per million AMU still need to be understood. Nonetheless, the match is surely close enough to warrant attention.

5) <u>Helium-3 and Helium-4</u>: From there we seek to likewise explain some other light nuclide binding energies based on the foregoing toolkit, particularly hydrogen and helium isotopes. For the highly stable alpha particle – the ^{4}He nucleus – it was found through trial and error that the observed binding energy ${}^{4}_{2}B_0 = 0.030\,376\,586\,5\ \mathrm{u}$ is less than the latent binding energy ${}^{4}_{2}B = 2\cdot\Delta_P + 2\cdot\Delta_N = 0.037\,465\,212\,2\ \mathrm{u}$ by approximately $2\sqrt{m_u m_d}$. So we then calculate $2\cdot\Delta_P + 2\cdot\Delta_N - 2\sqrt{m_u m_d} = 0.030\,373\,002\,0\ \mathrm{u} \approx {}^{4}_{2}B_0$, to find that this differs from the observed alpha binding energy by under *four parts per million AMU*. The integer factor 2 used with $\sqrt{m_u m_d}$ is "structurally sensible" because the alpha particle has 2 protons and 2 neutrons, i.e., 2 neutron / proton pairs and one might "read out" $2\sqrt{m_u m_d}$ as saying that "the alpha has two protons with an extra up quark and two protons with an extra down quark." And this overall expression for ${}^{4}_{2}B$ is structurally sensible because just like the alpha particle itself, it is completely symmetric under both $P \leftrightarrow N$ and $u \leftrightarrow d$ interchange. This is first developed in detail in section 5 of [2] and the numerical results are recalibrated in section 10 of [2] after (3.2) is used to replace ${}^{2}_{1}B_0 = m_u$ as a primary mass relationship.

For the ^{3}He nucleus (helion) with observed binding energy ${}^{3}_{2}B_0 = 0.008\,285\,602\,8\ \mathrm{u}$ we calculate $\sqrt{m_u}\,\mathrm{Tr}K_P = 2m_u + \sqrt{m_u m_d} = 0.008\,320\,783\,9 \approx {}^{3}_{2}B_0$ by employing the trace of the Koide proton matrix $\mathrm{Tr}K_P = \sqrt{m_d} + 2\sqrt{m_u}$ from our toolkit. Having $\sqrt{m_d} + 2\sqrt{m_u}$ involved here is "structurally sensible" because ^{3}He has one neutron (one extra down quark) and two protons (two extra up quarks). This was first developed in detail in section 6 of [2] and differs from the empirical data by *under four parts per hundred thousand AMU* after recalibration in section 10 of [2].

6) <u>Hydrogen-3 and the Neutron minus Proton Mass Difference</u>: It was in the course of attempting to obtain a binding energy for the ^{3}H triton that the author finally discovered the mass relationship (3.2) which was then advanced to a *primary* exact relationship in section 10 of [2]. While ${}^{2}_{1}B_0 \approx m_u$, ${}^{4}_{2}B_0 \approx 2\cdot\Delta_P + 2\cdot\Delta_N - 2\sqrt{m_u m_d}$ and ${}^{3}_{2}B_0 \approx 2m_u + \sqrt{m_u m_d}$ for ^{2}H, ^{4}He and ^{3}He respectively could be ferretted out relatively straightforwardly using binding energies, latent

binding energies (5.12) and the toolkit from point 3, finding ${}^{3}_{1}B_0$ for ^{3}H proved to be impossible working with binding energies alone. So at that point in time as detailed in the appendix of [2] we begin to consider certain nuclear fusion reactions to see if the energies released in these reactions might provide a close empirical connection to the point 3 toolkit. And we also began to make use of the general mass defect relationship

$$ {}^{A}_{Z}B_0 = Z \cdot M_P + N \cdot M_N - {}^{A}_{Z}M_0 \tag{5.13}$$

which relates the observed binding energy ${}^{A}_{Z}B_0$ to the observed nuclear mass (weight) ${}^{A}_{Z}M_0$ for any nuclide with *Z* protons, *N* neutrons and $A = Z + M$ nucleons. (Note: the free proton mass $M_P = {}^{1}_{1}M$ and the free neutron mass $M_N = {}^{1}_{0}M$.)

First, because our goal is ^{3}H, we consider the fusion ${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + e^{+} + \nu + \text{Energy}$ of a proton and a deuteron into a triton and ask: how much energy is released? Empirically, neglecting the neutrino, what is observed is $\text{Energy} = {}^{1}_{1}M + {}^{2}_{1}M - {}^{3}_{1}M - m_e = 0.004\,780\,386\,2 \text{ u}$. Dipping into the toolkit we find a close connection using $2m_u = 0.004\,774\,678\,6 \text{ u}$ which differs from the observed fusion energy by 5.7076×10^{-6} u, i.e., just under *six parts per million AMU*. And the factor of 2 makes some structural sense because we are fusing two nuclides. So we make the close association $\text{Energy}\left({}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + ...\right) \approx 2m_u$. After some calculations using (5.13) and leading to [A9] in [2] we obtain the expression ${}^{3}_{1}B_0 \approx M_N - M_P + 3m_u + m_e$ for the ^{3}H binding energy. But this violates the stringent toolkit rule: we must be able to closely fit *each and every nuclide* to nothing other than the up and down quark masses. But this expression contains the neutron minus proton mass difference $M_N - M_P$ which is the primary relationship (3.2). To stay true to this stringency, we must now find a way to express $M_N - M_P$ itself *exclusively* as a function of the up and down quark masses.

For this we do a second study, this time of the most elementary fusion ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu + \text{Energy}$ of two protons into a deuteron. Again we ask: how much energy is released? The observed empirical energy is $\text{Energy} = 2M_P - {}^{2}_{1}M - m_e = 0.000\,451\,141\,0 \text{ u}$. We again return to trial and error with the toolkit, this time dipping into the $(2\pi)^{\frac{3}{2}}$ divisor to find that $2\sqrt{m_\mu m_d}\,/\,(2\pi)^{\frac{3}{2}} = 0.000\,450\,424\,1 \text{ u}$. This differs from the empirical fusion energy by 7.169×10^{-7} u and so has an accuracy of *better than one part per million AMU.* So we make the close association $\text{Energy}\left({}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + ...\right) \approx 2\sqrt{m_\mu m_d}\,/\,(2\pi)^{\frac{3}{2}}$. The coefficient 2 makes structural sense because we are fusing two protons. Thereafter, we arrive in [A15] of [2] at $M_N - M_P = m_u - m_e - 2\sqrt{m_\mu m_d}\,/\,(2\pi)^{\frac{3}{2}} = m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/\,(2\pi)^{\frac{3}{2}}$ solely by deductive calculation, which is the primary mass relationship (3.2). With this we have completed the explanation of how the second primary relationship (3.2) for the neutron minus proton mass difference is obtained.

Of course, when (3.2) was first obtained in [A15] of [2] this was as an intermediate step that was necessitated to reduce ${}^{3}_{1}B_0 \approx M_N - M_P + 3m_u + m_e$ to obtain the binding energy for the ^{3}H triton, which has the empirical value ${}^{3}_{1}B_0 = 0.009\,105\,585\,4\text{ u}$. So we then completed the calculations in the appendix of [2] using all of these results to arrive in [A17] at the approximate expression $4m_u - 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.009\,099\,047\,1\text{ u} \approx {}^{3}_{1}B_0$ for the triton bending energy, which differs from the observed value by 6.5383×10^{-6} u, *just under seven parts per million AMU.*

7) <u>Recalibration of Mass Relationships</u>: As just discussed, the primary mass relationship (3.2) was first uncovered as a byproduct of pursuing the triton binding energy. But based on the initial hypothesis in place at the time that ${}^{2}_{1}B_0 = m_u$, this relationship (3.2) itself predicted a neutron minus proton mass difference which was off by a few parts per ten million AMU. Then, for the reasons detailed in section 10 of [2] the author withdrew ${}^{2}_{1}B_0 = m_u$ as a primary relationship and in its place hypothesized (3.2) to be a primary, exact relationship among the electron, proton and neutron masses, and the up and down quark masses. It is with this hypothesis that (3.2) joined (3.1) as a “primary mass relationship” which was then used in accordance with the EPN-0 quark mass definition to deduce very precise quark masses (3.3) and (3.4) which have been used in the development here ever since. With this shift in hypothesis, all other mass / energy relationships previously developed were recalibrated to reflect this revised hypothesis.

6. Is there Clear Secondary Empirical Support for the Deduced $Q = 0$ Up and Down Current Quark Masses?

Having shown how the primary mass relationships (3.1) and (3.2) are obtained we now return to the second of the three questions posed in section 3, namely whether these primary mass relationships (3.1) and (3.2) and the very precise $Q = 0$ up and down current quark masses (3.3) and (3.4) deduced therefrom can be supported by other “secondary relationships” rooted in nuclear data, or whether there are contradictions to be found.

When discussing in general whether a theory is “valid” or has “support” one must keep in mind that for scientific work, one can never truly “validate” a theory. One can simply show that at multiple places where the theory might be open to contradiction, no contradiction is found. This takes place at two levels: the empirical level, and the theoretical level.

At the empirical level, the question is whether efforts to make contact with empirical data are contradicted or not contradicted: do the experiments rule out the theory, or do they fail to rule out the theory? If a sufficient number of efforts are made to contradict and no contradictions are found then the weight of those “failures to contradict” start to translate into “empirical support” for the theory. But there is no objective, scientific measurement as to when there are enough failures to contradict so as to constitute theoretical validation, other than perhaps trying to assess the probability that multiple failures to contradict are only just coincidence and deciding that at some level, say, 6-sigma, the threshold is crossed from careful skepticism to acceptance. But at bottom, this is a subjective judgment which must first be made by individual scientists and then, eventually, by the scientific community as a whole.

At the theoretical level, the question is whether a proposed theory is consistent with, i.e., not contradictory to, other settled theories and theoretical elements which have advanced to the point of having gained wide acceptance in the scientific community based on multiple failures to contradict those settled theories. There are other corollary questions related to this: whether the theory is economical, which in a conservative view of science might be reframed as whether the theory requires brand new notions to be injected into the theoretical discourse of the community, or whether the theory can be rested solely on a novel synthesis of well-established and well-settled theories and theoretical elements to uniquely and unambiguously deduce new results and new explanations for previously-unexplained observational data. From a conservative scientific stance the latter (synthesis of settled science) is preferable, while the former (brand new notions) is not ruled out but should only be used as a last resort when there is no apparent way to succeed by restricting oneself to synthesizing known theory elements in novel ways.

In this section, we shall discuss empirical support, which is the second of the three questions posed in section 3. In the final three sections we shall discuss theoretical support, which is the third and final of the three questions posed in section 3

The findings regarding the ^{56}Fe and ^{62}Ni latent binding energies (5.11) and (5.12) and the fitting of the mass number "toolkit" to the ^{2}H,^{3}H, ^{3}He and ^{4}He binding and fusion energies in section 5 appear to provide preliminary secondary support for the view that (3.3) and (3.4) are correct quark masses and therefore (3.1) and (3.2) are correct primary relationships, as well as for the view that the "toolkit" energies can in fact be used to fit observed nuclear binding and fusion energies. Specifically, we hypothesized that the latent binding energies (5.8) and (5.9) and toolkit components thereof should be able to provide the *exclusive* basis for fitting empirical binding and fusion energy observational data. Then when we applied this hypothesis to ^{2}H, ^{3}H, ^{3}He and ^{4}He we were indeed able to fit energy numbers for all four of these nuclides to better than parts per hundred thousand AMU, which means that this hypothesis was uncontradicted by these four nuclides' binding and fusion energies. While we do not attempt to calculate a probability for this, it does seem that the probability of mere coincidence that all four of these binding energies do not contradict this hypothesis beyond parts per 10^5 AMU or better apiece is quite low. Now we shall review this empirical support together with additional empirical support, as catalogued below.

Thus far, we started out by hypothesizing (3.1) and (3.2) to be valid, exact, Q-invariant relationships, and thereby hypothesizing (3.3) and (3.4) to be valid, very precise up and down $Q=0$ quark masses. Based on this, the author has to date been able to deduce the following non-contradictory, supporting empirical results:

1) <u>Hyrdrogen-2 and -3, Helium-3 and -4 Binding Energies</u>: Secondary relationships for the ^{2}H, ^{3}H, ^{3}He and ^{4}He (1s shell) nuclide binding energies strictly terms of m_u and m_d with very close matches to parts per 10^5, 10^6 or even 10^7 AMU. Respectively, these secondary relationships are: ${}^{2}_{1}B_0 \approx m_u$ (section 5, point 4); ${}^{3}_{1}B_0 \approx 4m_u - 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}}$ (section 5, point 6); ${}^{3}_{2}B_0 \approx 2m_u + \sqrt{m_u m_d}$ (section 5, point 5); and in view of the latent binding energies (5.8) and

(5.9), ${}_{2}^{4}B_{0}\approx 2\cdot\Delta_{P}+2\cdot\Delta_{N}-2\sqrt{m_{u}m_{d}}$ (section 5, point 5). This means that ^{2}H, ^{3}H and ^{3}He respectively *release* energies of about m_u, $4m_u-2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}$ and $2m_u+\sqrt{m_u m_d}$ from quark confinement to nuclear binding, while ^{4}He *retains* an energy of about $2\sqrt{m_u m_d}$ for quark confinement and releases all the remaining latent binding energy for nuclear binding.

2) Deuteron and Triton Fusion Energies: Interrelated to the point 1 secondary relationships and the primary relationship (3.2) for M_N-M_P, an $\text{Energy}\left({}_{1}^{1}H+{}_{1}^{2}H\rightarrow{}_{1}^{3}H+...\right)\approx 2m_u$ for the fusion energy released when a fusing proton and a deuteron into a triton and an $\text{Energy}\left({}_{1}^{1}H+{}_{1}^{1}H\rightarrow{}_{1}^{2}H+...\right)\approx 2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}$ for the fusion energy released when fusing two protons into a deuteron (section 5, point 6).

3) The Nuclear Binding Peak near 8.75 MeV: The relationships (5.8) and (5.9) for Δ_P and Δ_N which represent "missing energy" and which have a value of $\frac{1}{2}\left(\Delta_P+\Delta_N\right)=8.714\,994\,1\,\text{MeV}$ which is right at the peak of the empirical nuclear binding curve in Figure 2, which likewise represents a "missing energy" from composite nuclides.

4) Iron-56 and other Tightly-Bound Nuclides: Based on (5.8) and (5.9), the relationship $\Delta\left({}^{56}\text{Fe}\right)=26\Delta_P+30\Delta_N=492.396\,598\,5\,\text{MeV}$ in (5.10) which is *extremely close* to the empirical ${}_{26}^{56}B_0=492.253\,892\text{ MeV}$, such that ${}_{26}^{56}B_0\,/\,{}_{26}^{56}B=99.9710\%$. This, and other relationships such as (5.11) which are deduced via (5.12) provide the basis for recognizing that Δ_P and Δ_N are latent energies available to be used for binding which confine quarks in free nucleons but which are partially released as fusion energies for nuclear binding in a percentage that varies for each type of nuclide but never exceeds 100% and is greater for ^{56}Fe than for any other nuclide. This enables us to understand quark confinement on an energetic basis and possibly explain the first EMC effect [18] whereby quarks inside bound nuclei are observed to be less-confined than those in free nucleons.

All of the foregoing provide secondary empirical validation to the view that (3.1) and (3.2) are empirically-valid relationships, and that (3.3) and (3.4) are therefore empirically-valid quarks masses. But there are further supporting empirical results as well:

5) Solar Fusion: By combining the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding results in point 1 above with $\text{Energy}\left({}_{1}^{1}H+{}_{1}^{2}H\rightarrow{}_{1}^{3}H+...\right)\approx 2m_u$ and $\text{Energy}\left({}_{1}^{1}H+{}_{1}^{1}H\rightarrow{}_{1}^{2}H+...\right)\approx 2\sqrt{m_\mu m_d}\,/\left(2\pi\right)^{\frac{3}{2}}$ for the fusion events in point 2 above, it is possible as detailed in section 9 of [2] to accurately express the 26.73 MeV energy observed to be released during a single solar fusion event by the relationship [9.8] of [2]:

$$\begin{aligned}&\text{Energy}\left(4\cdot{}_{1}^{1}H+2e^{-}\rightarrow{}_{2}^{4}He+\gamma(12.79MeV)+2\gamma(5.52MeV)+2\gamma(.42MeV)+4\gamma(e)+2\nu\right)\\&=4m_u+6m_d-2\sqrt{m_u m_d}+\left(2m_d-22m_u-12\sqrt{m_u m_d}\right)/\left(2\pi\right)^{\frac{3}{2}}=26.73\,MeV\end{aligned}\quad. \tag{6.1}$$

Like the other binding and fusion results this is also expressed wholly and exclusively in terms of the same two parameters: the up quark mass (3.3) and the down quark mass (3.4). Most of the energy we encounter in the world had its origin in an event (6.1) on the sun, so this is certainly an empirically-validated result.

6) Stable Neutron-Rich Nuclides: The fact that the latent binding energy of the neutron in (5.9) is greater than that of the proton in (5.8) by a factor of $\Delta_N / \Delta_P = 1.284\,295\,230\,4$ teaches that a neutron inherently carries 28.43% more latent binding energy than does a proton. This explains the clear empirical evidence that for all nuclei heavier than helium the stable isotopes *always* have either equal numbers of protons and neutrons *N*=*Z* or are neutron-rich *N*>*Z*. If one has a given nucleus and seeks to fuse on an extra proton or neutron, it is clear that a neutron which can contribute more latent energy which can be used for nuclear binding will have an easier time becoming and staying bound than a proton which contributes less such energy.

7) Lithium-6 and -7 and Beryllium-7 and -8: Thus far we have only examined the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding energies. But there is further support available from some heavier nuclides as well. To date, the author has characterized eleven additional nuclides ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N with equally-high precision, exclusively as a function of the up and down quark masses, via the toolkit of section 5 point 3. All of these derivations are detailed at length in [5], so we shall simply summarize them here.

The detailed derivations for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, which are 2s shell nuclides, are contained in section 13 of [5] and are exceptionally revealing in terms of the requirement that the integer multiples of the m_u, m_d, $\sqrt{m_u m_d}$ and these divided by $(2\pi)^{\frac{3}{2}}$ must be "structurally sensible." We have already applied this in points 5 and 6 of section 5 for the hydrogen and helium derivations, but when applied to Li and Be, this requirement provides deep empirical support and is quite intriguing.

The respective binding energies for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be are found in [13.21] and [13.12] of [5] to be:

$$ {}^{6}_{3}B_0 \approx 7m_u + 6m_d - 2\sqrt{m_u m_d} + \left(-10m_u - 10m_d - 9\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.034\,336\,427\,2 \text{ u} . \qquad (6.2) $$

$$ {}^{7}_{3}B_0 \approx 8m_u + 6m_d - 2\sqrt{m_u m_d} + \left(2m_u + 2m_d - 11\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.042\,105\,716\,0 \text{ u} . \qquad (6.3) $$

$$ {}^{7}_{4}B_0 \approx 7m_u + 6m_d - 2\sqrt{m_u m_d} + \left(-10m_u + 8m_d - 9\sqrt{m_u m_d}\right) \Big/ (2\pi)^{\frac{3}{2}} = 0.040\,356\,362\,0 \text{ u} . \qquad (6.4) $$

$$ {}^{8}_{4}B_0 \approx 4 \cdot \Delta E_P + 4 \cdot \Delta E_N - 2\sqrt{m_u m_d} - 32\sqrt{m_u m_d} \,/\, (2\pi)^{1.5} = 0.060\,633\,250\,9 \text{ u} . \qquad (6.5) $$

The respective *empirical* values out to seven digits are ${}^{6}_{3}B_0 = 0.034\,347\,1$ u (difference of -1.07×10^{-5} u); ${}^{7}_{3}B_0 = 0.042\,130\,3$ u (difference of -2.45×10^{-5} u); ${}^{7}_{4}B_0 = 0.040\,365\,1$ u (difference of -8.74×10^{-6} u), and ${}^{8}_{4}B_0 = 0.060\,654\,8$ u (difference of -2.16×10^{-5} u). So as with H and He, these all have accuracy to parts in 10^5 or 10^6 AMU.

Now, while the existence of the coefficients 6, 7 and 8 multiplying the quark masses provides some "structural sensibility" for nuclides with 6, 7 or 8 nucleons, the deep and striking structural sensibility emerges from the fusion relationships which were used in section 13 of [5] to establish (6.2) through (6.4) above. Specifically, to arrive at (6.2) for ^{6}Li we used the fusion reaction ${}^{4}_{2}He + 2p \to {}^{6}_{3}Li + e^+ + \nu + \text{Energy}$ for which the empirical energy to seven digits is $0.002\,033\,5$ u. And, after using the toolkit and "structurally-sensible" integer multiples, it is found in [13.3] of [5] that:

$$\text{Energy}\left({}^{4}_{2}He + 2p \to {}^{6}_{3}Li + e^+ + \nu + \text{Energy}\right) \approx 9\sqrt{m_u m_d}\,/\,(2\pi)^{1.5} = 0.002\,026\,4\ \text{u}\,, \tag{6.5}$$

which has the coefficient 9 and differs by -7.1×10^{-6} u. To arrive at (6.3) for ^{7}Li we developed the β^+ decay reaction ${}^{7}_{4}Be + e \to {}^{7}_{3}Li + \nu + \text{Energy}$ for which the empirical energy is $0.000\,925\,3$ u. Using the toolkit and "structurally-sensible" integer multiples, we found in [13.9] of [5] that:

$$\text{Energy}\left({}^{7}_{4}Be + e \to {}^{7}_{3}Li + \nu + \text{Energy}\right) \approx 6m_u\,/\,(2\pi)^{1.5} = 0.000\,909\,5\ \text{u}\,, \tag{6.6}$$

which has the coefficient 6 and differs by -1.58×10^{-5} u. And to arrive at (6.4) for ^{7}Be we worked with the reaction ${}^{6}_{3}Li + p \to {}^{7}_{4}Be + \text{Energy}$ which has an empirical energy of $0.006\,018\,0$ u. Here, we found in [13.6] of [5] that:

$$\text{Energy}\left({}^{6}_{3}Li + p \to {}^{7}_{4}Be + \text{Energy}\right) \approx 18m_d\,/\,(2\pi)^{1.5} = 0.006\,019\,9\ \text{u}\,, \tag{6.7}$$

which has the coefficient 18 and differs by 1.9×10^{-6} u. These three coefficients, 9, 6 and 18 not only yield very close results to parts per 10^5 or 10^6 but also provide structural sensibility and begin to teach us deeply about nuclear structure and the "nuclear genome." Let's take a closer look.

When we build the ^{6}Li nucleus by fusing 2 nucleons with an alpha particle in (6.5), we are creating a nucleus with 9 up quarks and 9 down quarks, i.e., with 9 up / down quark pairs. And what is the toolkit number that gets us from ^{4}He to ^{6}Li? $9\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}$. How better to formally state that there are 9 up / down quark pairs than with $9\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}$, and to state that both the beginning and end-products ^{4}He and ^{6}Li are absolutely symmetric under $P \leftrightarrow N$ and $u \leftrightarrow d$ interchange. In (6.6) we have the isotopic β^+ decay from unstable proton-rich ^{7}Be to stable neutron-rich ^{7}Li for which the toolkit gives us $6m_u\,/\,(2\pi)^{1.5}$. (Keep in mind point 6 where we explained based on latent binding energies why nature favors extra neutrons over extra protons for anything heavier than He.) In this reaction a proton is being traded for a neutron, but the unchanging nucleus during this reaction is the underlying stable ^{6}Li nucleus which is an isotope of ^{7}Li and an isotone of ^{7}Be. The structural piece of the nucleus which does not change is the underlying ^{6}Li with 6 nucleons. So what is the coefficient here? Why, it is 6. In (6.7) we are adding a proton to ^{6}Li to obtain ^{7}Be, and the toolkit yields $18m_d/(2\pi)^{1.5}$. Why 18? The nucleus at the root of this fusion event is ^{6}Li which contains 18 quarks. It is also interesting to observe that the three main toolbox elements $\sqrt{m_u m_d}$, m_u and m_d are each used in these decays via $9\sqrt{m_u m_d}/(2\pi)^{1.5}$, $6m_u/(2\pi)^{1.5}$ and $18m_d/(2\pi)^{1.5}$ and that the ^{6}Li nucleus common to all three reactions with 9 quark pairs, 6 nucleons and 18 quarks appears to drive these coefficients.

All of this suggests that when any nuclear transition occurs and some energy is being released there is definitive set of energy "dosages" which are released or otherwise used in the process, and which are allocated discretely to each of the quarks or quark pairs or nucleons, etc. So for ${}_{2}^{4}He+2p\rightarrow {}_{3}^{6}Li+...$ with $9\sqrt{m_u m_d}/(2\pi)^{1.5}$, each of the nine quark pairs gives up a single energy dosage $\sqrt{m_u m_d}/(2\pi)^{1.5}$ to be able to establish the ^{6}Li with the start of new proton and neutron shells overlaid on the alpha nucleus, that is, to "entice" an extra proton and neutron to join the alpha core. For ${}_{4}^{7}Be+e\rightarrow {}_{3}^{7}Li+...$ with $6m_u/(2\pi)^{1.5}$ each of the six nucleons – three protons and three neutrons – in the ^{6}Li core gives up a single energy dosage $m_u/(2\pi)^{1.5}$ to the β^{+} decay. And for ${}_{3}^{6}Li+p\rightarrow {}_{4}^{7}Be+...$ with $18m_d/(2\pi)^{1.5}$, every single quark in the ^{6}Li core needs to give up a single $m_d/(2\pi)^{1.5}$ energy dosage to "entice" the new proton into the core.

Applying this new understanding retrospectively to point 2, we now see that to create a deuteron which is symmetric under $P\leftrightarrow N$ and $u\leftrightarrow d$ interchange, via the most basic fusion reaction $\text{Energy}\left(p+p\rightarrow {}_{1}^{2}H+...\right)\approx 2\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}}$, each proton has to contribute a $\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}}$ dosage of energy which dosage is similarly symmetric. And to create a triton via $\text{Energy}\left(p+{}_{1}^{2}H\rightarrow {}_{1}^{3}H+...\right)\approx 2m_u$ each of the proton and the deuteron must contribute an energy dose valued at m_u. This all provides a deeper picture of what it means to say that the "toolbox" elements need to be used with coefficients which are "structurally sensible." We come to understand that when we observe some fusion or fission energy released during some reaction, this energy originates from a collection of discrete "dosages" of the toolbox energies which bear a real relation to the structural elements of the involved nuclei.

We also see that the method of fitting the toolkit to observed fusion or β-decay energies (versus fitting to binding energies) is extremely important in building up larger nuclides. In section 13 of [5], we started with the ^{4}He nucleus and built that into ^{6}Li which is diagonally-adjacent upper left to lower right in the nuclide table, per (6.5). Then we added a proton as in (6.7) and built this into its isotone ^{7}Be. Then we diagonally beta-decayed this upper right to lower left into ^{7}Li as in (6.6). Once lighter nuclides are so-characterized, we have the ability to "weave" over from one nuclide to horizontally or vertically-adjacent nuclides by examining their decay energies, and then convert over to binding energies via (5.13). This stepwise approach to building up nuclei with guidance from the nuclear structure at each step provides some sense of confidence that the binding energies obtained are validly-related to real physical energy events.

Further, we see from the ^{4}He binding energy $^4_2B_0 \approx 2\cdot\Delta_P + 2\cdot\Delta_N - 2\sqrt{m_u m_d}$ and from the ^{8}Be binding energy $^8_4B_0 \approx 4\cdot\Delta E_P + 4\cdot\Delta E_N - 2\sqrt{m_u m_d} - 32\sqrt{m_u m_d}/(2\pi)^{1.5}$ that the $Z = N = \text{even}$ nuclides appear to form something of a nuclear "backbone" which are $N \leftrightarrow P$ and $u \leftrightarrow d$ invariant, and that their binding energies are perhaps best uncovered by first using (5.12) ascertain their latent binding energies, then using the toolkit to see how much of this latent energy is retained for confinement, and throughout being guided by the $N \leftrightarrow P$ and $u \leftrightarrow d$ symmetry of these nuclides.

So the basic approach to "decoding the nuclear genome" is to first establish the diagonal $Z = N = \text{even}$ "backbone" nuclides which have full nuclear shells, and then branch over to nearby nuclides. For the backbone nuclides we first calculate the latent binding energy via (5.12) which uses (5.8) and (5.9),and we take advantage of the $u \leftrightarrow d$ and $N \leftrightarrow P$ symmetry. Then we use the toolkit to find out how much of this latent binding energy (5.12) goes unused for nuclear binding and is instead reserved for quark confinement. Once we have established a backbone nuclide we then "weave" our way over to nearby nuclides using pertinent fusion reactions while making use of the various emergent integer dosage coefficients which bear relations to and provide clues about the nuclear substructure and which elements within the nucleus are contributing what energy dosages.

8) <u>Stability of Helium-4 over Beryllium-8</u>: By now having close fits for both ^{8}Be and ^{4}He with the ratio $^8_4B / ^4_2B = 1.996\,759\,9$ based on (6.5) and point 5 of section 5, we implicitly explain why ^{8}Be is energetically unstable and always decays rapidly into two ^{4}He nuclei which are energetically stable. This is another important empirical feature of nuclear physics which does not contradict this approach.

9) <u>Boron-10</u>: Further empirical validation is obtained through characterizing the ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N nuclides as the author has previously done in section 14 of [5]. We shall not repeat those derivations here because they are available at the original source [5]. But the patterns which started to emerge for ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be do appear for some of these even-heavier nuclides, and deepen the empirical confirmation of this approach.

An excellent example of this is the $^8_4Be + 2p \rightarrow {}^{10}_5B + e^+ + \nu + \text{Energy}$ reaction, which is analogous to $^4_2He + 2p \rightarrow {}^6_3Li + e^+ + \nu + \text{Energy}$ summarized in (6.5). The empirically-released energy in this reaction is $0.006\,921\,0\text{ u}$. And as found in [14.3] of [5]:

$$\text{Energy}\left({}^8_4Be + 2p \rightarrow {}^{10}_5B + e^+ + \nu + \text{Energy}\right) = \sqrt{m_u m_d} + 15\sqrt{m_u m_d}/(2\pi)^{1.5} = 0.006\,923\,4\text{ u}, \quad (6.8)$$

which differs from the empirical energy by 2.4×10^{-6} u and is which is symmetric under $u \leftrightarrow d$ interchange as expected for any $Z = N$ nuclides. What is extremely striking is that the creation of ${}^{6}_{3}Li$ with 9 up / down quark pairs from ${}^{4}_{2}He$ contained a $9\sqrt{m_u m_d}/(2\pi)^{1.5}$ term shown in (6.5) with the coefficient 9, and the creation of ${}^{10}_{5}B$ with 15 up / down quark pairs from ${}^{8}_{4}Be$ contains exactly the same term, but now $15\sqrt{m_u m_d}/(2\pi)^{1.5}$ with the coefficient 15. *This cannot be mere coincidence. This reveals a very definite and meaningful data pattern.* As with ${}^{4}_{2}He + 2p \rightarrow {}^{6}_{3}Li + \ldots$, each quark pair in the ${}^{8}_{4}Be + 2p \rightarrow {}^{10}_{5}B + \ldots$ contributes a single $\sqrt{m_u m_d}/(2\pi)^{1.5}$ energy dosage, except now there are more quark pairs – 15 rather than 9 – to make such a contribution. But the new feature in (6.8) is that there is also a single overall $\sqrt{m_u m_d}$ dosage. Because structural sensibility is important in discerning which possible relationships are true signals of physical reality and which are merely misleading noise, we need to closely look at the structure of the nuclides involved. Earlier, ${}^{6}_{3}Li$ opened up a new 2s shell for a protons and a neutrons alike, but in 2s, the orbital angular momentum is *l*=0 as it is for 1s. Now, however, ${}^{10}_{5}B$ is opening up a new 2p shell for a proton and a neutron, and these shells have *l*=1, for the first time. So to create this shell, and particularly to sustain both a proton (extra up quark) and a neutron (extra down quark) in an *l*=1 state, we need some additional energy. The $\sqrt{m_u m_d}$ term appears to tell us that the *l*=1 proton contributes the m_u and the *l*=1 neutron contributes the m_d to this $\sqrt{m_u m_d}$ energy dose as the price for entry into the ^{10}B nuclide *and maintenance in an orbital state*. That is, in ^{10}B there is one proton and one neutron in an orbital state, and the energetic price for this is $\sqrt{m_u m_d}$, contributed based on the extra up and extra down quark in each of the *l*=1 nucleons that open up 2p shells.

In sum: equation (6.8) is telling us that to create ^{10}B from ^{8}Be plus two nucleons, each of the 15 up/down quark pairs in the target ^{10}B must contribute an $\sqrt{m_u m_d}/(2\pi)^{1.5}$ dosage and the neutron / proton pair which opens up 2p must further contribute $\sqrt{m_u m_d}$ to maintain an orbital angular momentum. This is *identical* to what happens to create ^{6}Li from ^{4}He plus two nucleons, except that ^{10}B needs some additional energy to fill an *l*=1 orbital while ^{6}Li does not. Again: decoding the nuclear genome.

10) Carbon-12: The ^{12}C nuclide is seat of biological life and the chosen standard of nuclear weight measurement with an isotopic mass exactly equal to 12 u by definition. It is also of keen interest in for confirming certain patterns already seen for the ^{4}He and ^{8}Be which are the first two nuclides with $Z = N = \text{even}$. This ^{12}C sits on the nuclear backbone and so following the basic approach stated at the end of point 7 above we go straight to (5.12) with $Z = N = 6$ to obtain the latent binding energy and then see how much is subtracted away, i.e., held in reserve to confine quarks rather than bind the nucleus. The empirical binding energy ${}^{12}_{6}B_0 = 0.098\,939\,8\;u$. What we discern in [14.30] of [5] is that:

$$ {}^{12}_{6}B_0 \approx 6\cdot\Delta E_P + 6\cdot\Delta E_N - (m_u + m_d) - 12(m_u + m_d)/(2\pi)^{1.5} = 0.098\,908\,7\text{ u}\,. \tag{6.9} $$

The empirical difference is -3.10508×10^{-5} u . Thus far the $u \leftrightarrow d$ -symmetric energy number we have used is $\sqrt{m_u m_d}$, yet the above makes clear that $m_u + m_d$ is a good tool to add to the toolkit (by corollary it is already there because m_u and m_d are already there, but it helps to be cognizant of the equally-weighted sum $m_u + m_d$ especially for $u \leftrightarrow d$ -symmetric nuclides). The coefficient 12 clearly makes structural sense: there are after all, 12 nucleons in ^{12}C, so each nucleon is responsible for one of the $\left(m_u + m_d\right)/\left(2\pi\right)^{1.5}$ energy dosages. But like ^{10}B, ^{12}C has nucleons in the 2p shell and so must sustain yet another proton and neutron in an *l*=1 orbital state. So in the same way that $\sqrt{m_u m_d}$ sustained the first proton / neutron pair in the *l*=1 orbital for ^{10}B in (6.8), $m_u + m_d$ sustains the second proton / neutron pair in the *l*=1 orbital for ^{12}C in (6.9). *This also establishes a very definite and meaningful data pattern.* Physically, this motivates us to now think of the toolkit as representing actual energy dosages contributed by different component parts of the nuclei in order to bind everything together and enable various nuclear reactions to take place. As we map the nuclear genome, the dosages we uncover are telling us which quarks, quark pairs, protons and neutrons, proton / neutron pairs, shells etc. are contributing energy, and how much energy they are contributing, and *what that energy is used for*, e.g., to do basic binding, to maintain an orbital *l* quantum state, or, presumably, to maintain a not-yet-examined magnetic *m* quantum state.

For the remaining ^{9}Be, ^{10}Be, ^{11}B, ^{11}C and ^{14}N nuclides which the author has also characterized, we will take no further space here, but refer the reader to section 14 of [5].

11) <u>Masses of the Proton and Neutron, and the Constituent Mass Contributions by the Up and Down Quarks</u>: A very important empirical validation comes through using an extension of the foregoing approaches to explain the observed proton and neutron masses M_N = 939.565379 MeV and M_P = 938.272046 MeV themselves, in relation to these very same quark masses, *within all experimental errors.* This was the central result in [6], which will be summarized here.

It will be understood from basic algebra that if we know the difference *A-B* between any two numbers *A* and *B* and also know their sum *A*+*B* then we can then deduce these two separate numbers. Because we already know the neutron minus proton mass difference $M_N - M_P$ in relation to the up and down quark masses from the primary relationship (3.2), we are one step away from knowing the proton and neutron masses themselves if we can also determine $M_N + M_P$. So the objective is to deduce this sum of these two masses. Once that is the objective, there is an important symmetry benefit that we have already seen with the $Z = N$ nuclides: we expect that $M_N + M_P$ which represents baryons with a combined total of three up and three down quarks must be symmetric under $u \leftrightarrow d$ interchange. This greatly restricts the toolkit elements we may use to either $m_u m_d$ products or $m_u + m_d$ sums (or perhaps $\sqrt{m_u{}^2 + m_d{}^2}$ which will make its first natural appearance in (7.19)).

The problem we have, however, is that the proton and neutron masses are well over two orders of magnitude larger than m_u = 2.223 792 40 MeV and m_d = 4.906 470 34 MeV, so the "sensible integer multiples" approach does not help us here. But we know from electroweak theory that the Fermi vev v_F=246.219651 GeV is used to set the mass scale for certain observed masses, notably the masses for the *W* and *Z* bosons, and we might expect on general principles including the recently-confirmed Higgs field theory that this vev will also turn up in the proton and neutron masses. So knowing that we are going to need $u \leftrightarrow d$ symmetric constructs such as $\sqrt{m_u m_d}$ to obtain $M_N + M_P$, and entertaining the possibility of employing $\sqrt{v_F}$ as an additional energy square root to supplement $\sqrt{m_u}$ and $\sqrt{m_d}$ which we are already using, we perform an exploratory calculation in [3.8] of [19] to encouragingly find that the construct $\sqrt{v \cdot \sqrt{m_u m_d}} = 901.835259\text{ MeV}$ lands within about 3% of the actual proton and neutron masses. To use a golf analogy, this lands the ball on the green; now we need to figure out how to hit it into the cup.

The next step is to employ $\text{diag}(\Phi_F) = v_F \text{diag}Q = v_F\left(0, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -1, -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ which is a Fermi vacuum in the adjoint representation that the author had used in [19] to break the electroweak symmetry for elementary fermions which were grouped into an $\left(\nu, (u_R, d_G, d_B), e, (d_R, u_G, u_B)\right)$ octet in the fundamental representation of an SU(8) Grand Unified Theory (GUT) which naturally explained the existence of three fermion generations and CKM mixing and so answered Rabi's long ago quip about the muon, "who ordered that?" Plainly put: the electric charges $Q = +\frac{2}{3}, -\frac{1}{3}$ of the up and down quarks needed to enter $\sqrt{v \cdot \sqrt{m_u m_d}} = 901.835259\text{ MeV}$ in the form of $v_F Q$.

So supplementing the Koide matrices *K* which were first discussed at (5.1) and (5.2) above with the quark electric charge *magnitudes* via Φ_F, the author in [5.8] of [6] constructed and then calculated the following inner product trace between a first Koide-type matrix with the duu (proton) charges and mass, and a second matrix with the udd (neutron) charges and masses:

$$\text{Tr}\begin{pmatrix} \sqrt[4]{\frac{1}{3} v_F m_d} & 0 & 0 \\ 0 & \sqrt[4]{\frac{2}{3} v_F m_u} & 0 \\ 0 & 0 & \sqrt[4]{\frac{2}{3} v_F m_u} \end{pmatrix} \cdot \begin{pmatrix} \sqrt[4]{\frac{2}{3} v_F m_u} & 0 & 0 \\ 0 & \sqrt[4]{\frac{1}{3} v_F m_d} & 0 \\ 0 & 0 & \sqrt[4]{\frac{1}{3} v_F m_d} \end{pmatrix} = 3 \cdot \sqrt[4]{\frac{2}{9} {v_F}^2 m_u m_d} \tag{6.10}$$
$$= 1857.570635\text{ MeV}$$

which was understood to apply to all but the current quark mass sum $3m_u + 3m_d$ associated with $M_N + M_P$. Upon adding this sum $3(m_u + m_d)$ to (6.10) it was found in [5.10] of [6] that:

$$M_N + M_P \approx 3\left(\sqrt[4]{\frac{2}{9} {v_F}^2 m_u m_d} + m_u + m_d\right) = 1878.961\,415\text{ MeV}. \tag{6.11}$$

which differs from the observed $M_N + M_P = 1877.837\,425\text{ MeV}$ by a scant 0.0599%. This now moves the golf ball to inches from the cup.

The balance section 6 of [6] was devoted to closing this final gap. In sum, it was found in [6.6] of [6] (see also [5.14] of [6]) that the *exact* $M_N + M_P$ includes a mixing angle θ_1 and a phase δ parameter which also need to be in (6.11) growing out of the fact that the up and down quarks have oppositely signed electric charges neglected when we only used magnitudes in (6.10), and that the complete expression is:

$$M_N + M_P = 3\left(\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d}\,\exp(i\delta) + (m_u + m_d)\cos\theta_1\right). \tag{6.12}$$

In [6] it was then deduced in [6.28] from the *empirical* $M_N + M_P$ that $\cos\theta_1 = 0.9474541242$ and in [6.30] that $\delta = 0$ *by mathematical identity*. The latter result tells us that there are no CP-violating effects associated with neutron and proton, which is validated by the empirical data that the mass of the antiproton is equal to that of the proton, and similarly for the neutron, see, e.g., [20], [21]. The former result boils down and bundles up the problem of explaining the proton and neutron masses within all experimental errors, to the problem of explaining the value of this deduced "nucleon fitting angle" $\cos\theta_1 = 0.9474541242$ within all experimental errors.

Because this θ_1 and the phase δ emerged from matrices with were *mathematically* the same as the CKM mixing matrices, it made sense to see if $\cos\theta_1 = 0.9474541242$ could be related in some way to the observed CKM mixing angles themselves. Equations [11.2], [11.3] and [11.27] (for empirical magnitude-only data) of PDG's [22] coupled with [23] tell us that:

$$\begin{aligned}
V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \\
&= \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ -0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ -0.00867^{+0.00029}_{-0.00031} & -0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}
\end{aligned} \tag{6.13}$$

and the Jarlskog determinant which is a phase-convention-independent measure of CP violation is $J = 2.96^{+0.20}_{-0.16} \times 10^{-5}$. A comparison of the empirical data with $\cos\theta_1 = 0.9474541242$ suggests that the *determinant* $|V|$ might be of help. We see from the product of three separate matrices in the first line above that $|V| = V_{ud}V_{cs}V_{tb} + V_{us}V_{cb}V_{td} + V_{ub}V_{cd}V_{ts} - V_{ub}V_{cs}V_{td} - V_{us}V_{cd}V_{tb} - V_{ud}V_{cb}V_{ts} = 1$, *by construction*. So there is no empirical information to be gleaned from the entire $|V| = 1$. But this determinant has two parts which we call the "major" and "minor" determinants

$|V|_+ = V_{ud}V_{cs}V_{tb} + V_{us}V_{cb}V_{td} + V_{ub}V_{cd}V_{ts}$ and $|V|_- = V_{ub}V_{cs}V_{td} + V_{us}V_{cd}V_{tb} + V_{ud}V_{cb}V_{ts}$, such that $|V| = |V|_+ - |V|_- = 1$, and these numbers are physically interesting and each are constructed from all nine entries in *V*. From the median empirical magnitude-only data, we calculate $|V|_+ = 0.947535$ and $|V|_- = -0.052355$ thus $|V| = |V|_+ - |V|_- = 0.999889$, while the CP violating phase aspects of *V* are captured by $J = 2.96^{+0.20}_{-0.16} \times 10^{-5}$. Then, comparing the data number $\cos\theta_1 = 0.9474541242$ with $|V|_+ = 0.947535$, it begins to appears as if $\cos\theta_1$ may in fact be synonymous with $|V|_+$. In fact, when considering the experimental errors in (6.13), we find in [7.4] of [6] that $|V|_+ = 0.947454^{+0.000400}_{-0.000262}$, i.e., that $0.947273 < |V|_+ < 0.947935$. This places the nucleon fitting angle $\cos\theta_1 = 0.9474541242$ predicted from the actual proton and neutron masses *well within the experimental errors for* $|V|_+$.

So, once again driven by empirical data concurrence within the known errors, we define $\cos\theta_1 \equiv |V|_+$ by hypothesis, and this connects the CKM matrix with the nucleon fitting angle. Also using the identity $\delta = 0$ which makes anti-nucleon masses the same as nucleon masses, we then rewrite (6.12) as:

$$M_N + M_P = 3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + (m_u + m_d)|V|_+\right). \tag{6.14}$$

Now, this proton plus neutron mass sum becomes specified *within all experimental errors*. When (6.14) is then solved together with the primary relationship (3.2) for $M_N - M_P$ we obtain theoretical values for the proton and neutron masses which are a function of only four parameters: m_u and m_d from (3.3) and (3.4), the Fermi vev, and the major determinant $|V|_+$ obtained from the CKM mixing matrix. Solving in combination with the mass difference of the primary relationship (3.2) then yields the separate masses in [6.31] and [7.6] of [6], namely (it is also convenient at times to employ the shorthand $\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} \equiv \sqrt{M_u M_d}$, see [5.14] of [6]):

$$M_N = \tfrac{1}{2}\left(3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + |V|_+ (m_u + m_d)\right) + m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/(2\pi)^{\frac{3}{2}}\right), \tag{6.15}$$

$$M_P = \tfrac{1}{2}\left(3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + |V|_+ (m_u + m_d)\right) - m_u + \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/(2\pi)^{\frac{3}{2}}\right), \tag{6.16}$$

These are not just approximations. They are exact within all experimental errors. This then provides the basis in [8.3] through [8.6] of [6] for obtaining the so-called “constituent” quark masses (which we shall refer to as “contributive” quark masses) in which the current quark masses are combined with all of their associated non-linear behaviors to specify their separate contributions on the order of 310 to 320 MeV to the overall observed free nucleon masses.

12) Charm, Strange, Top and Bottom-Flavored Baryon Masses: If the proton and neutron can be expressed in terms of the up and down current quark masses as we see in (6.14) through

(6.16), then this suggests that other flavors of baryon containing c, s, t and b quarks can similarly be expressed once these second and third generation quark flavors are included. In this regard, the culmination of the development leads in [6.17] of [6] to a "mass and mixing matrix":

$$\Theta = 27\begin{pmatrix} \begin{matrix} -m_u\sqrt{m_s m_c}\sqrt{m_b m_t}\,\mathrm{c}_1\,\mathrm{s}_2\,\mathrm{s}_3 \\ +\sqrt{M_u M_d}\,m_s m_b\,\mathrm{c}_2\,\mathrm{c}_3\,\mathrm{e}^{i\delta} \end{matrix} & \begin{matrix} m_u\sqrt{m_s m_c}\,m_t\,\mathrm{c}_1\,\mathrm{s}_2\,\mathrm{c}_3 \\ +\sqrt{M_u M_d}\,m_s\sqrt{m_b m_t}\,\mathrm{c}_2\,\mathrm{s}_3\,\mathrm{e}^{i\delta} \end{matrix} & \sqrt{m_u m_d}\sqrt{m_s m_c}\sqrt{M_t M_b}\,\mathrm{s}_1\,\mathrm{s}_2 \\ \begin{matrix} -m_u m_c\sqrt{m_b m_t}\,\mathrm{c}_1\,\mathrm{c}_2\,\mathrm{s}_3 \\ -\sqrt{M_u M_d}\sqrt{m_s m_c}\,m_b\,\mathrm{s}_2\,\mathrm{c}_3\,\mathrm{e}^{i\delta} \end{matrix} & \begin{matrix} m_u m_c m_t\,\mathrm{c}_1\,\mathrm{c}_2\,\mathrm{c}_3 \\ -\sqrt{M_u M_d}\sqrt{m_s m_c}\sqrt{m_b m_t}\,\mathrm{s}_2\,\mathrm{s}_3\,\mathrm{e}^{i\delta} \end{matrix} & \sqrt{m_u m_d}\,m_c\sqrt{M_t M_b}\,\mathrm{s}_1\,\mathrm{c}_2 \\ \sqrt{m_u m_d}\sqrt{M_c M_s}\sqrt{m_b m_t}\,\mathrm{s}_1\,\mathrm{s}_3 & -\sqrt{m_u m_d}\sqrt{M_c M_s}\,m_t\,\mathrm{s}_1\,\mathrm{c}_3 & m_d\sqrt{M_c M_s}\sqrt{M_t M_b}\,\mathrm{c}_1 \end{pmatrix} \quad (6.17)$$

which includes the shorthand definitions $M_{u,c,t} \equiv \sqrt{\frac{2}{3} v_F m_{u,c,t}}$ and $M_{d,s,b} \equiv \sqrt{\frac{1}{3} v_F m_{d,s,b}}$ for "vacuum-amplified" quark masses containing the current quark masses amplified by the Fermi vev and attenuated by their electric charge magnitudes. The mathematics in the above was developed in the original parameterization of the Kobayashi and Maskawa matrices but can be developed if desired in the standard parameterization appearing in (6.13). If we examine the special case for which we set the c, s, t, b masses equal to 1, set $s_2 = s_3 = 0$ and take the trace, then in view of the above shorthands for $M_{u,c,t}$ and $M_{d,s,b}$ we obtain $\frac{1}{9}\mathrm{Tr}\Theta = 3\left(\sqrt[4]{\frac{2}{9} v_F{}^2 m_u m_d}\,\exp(i\delta) + (m_u + m_d)\cos\theta_1\right) = M_N + M_P$. This is identical to the $M_N + M_P$ sum in (6.12), and *this means that the proton plus neutron mass sum is embedded in* Θ *as a special case*. Thus, it must be considered that upon further study, this matrix may help provide an explanation of the various c, s, t and b flavored baryon masses.

13) <u>Who Ordered That? Why are there Three Fermion Generations?</u>: Having just discussed the second and third generation of quarks and baryons it is worth now going back to Rabi's original quip "who ordered that?" about the muon. While the second and third generation quarks and leptons and their mixing properties have been well-characterized since then, Rabi's question remains unanswered to this day. Nobody has yet shown the *theoretical imperative* for having three generations, or for the mixing of these generations. These have been *described*, but why nature manifests itself in this way remains unexplained. The author in [19] shows how three stages of symmetry breaking of the SU(8) octuplet $\left(\nu,(u_R,d_G,d_B),e,(d_R,u_G,u_B)\right)$ already mentioned in point 11 above and integrally used in deriving the proton and neutron masses, *leads inexorably to the appearance of three generations of CKM-type quark and lepton mixing*.

Briefly: it is well understood that spontaneous symmetry breaking breaks the symmetry between *particles* in a multiplet, for example, electroweak symmetry breaking separates electrons from neutrinos, and GUT symmetry breaking is widely thought to separate quarks from leptons (lepto-quark symmetry breaking). But is also possible to "break" a symmetry between *generators* (not just between particles) whereby some generators become separated ("fractured" or "stranded") from the remaining generators and so must either "disappear" or become "horizontal symmetry" generators. In this SU(8) GUT one starts with the seven (7) generators of SU(8), but by the time we reach low energies and electroweak symmetry is broken, (linear combinations of) only five of these seven SU(8) generators are required to describe the strong, weak and electromagnetic interactions in both left- and right-chiral states, and the other two of the original seven generators have become stranded, i.e., they are unneeded and unused. These remaining two generators cannot just disappear, however. They still provide two degrees of freedom and thus give rise to three horizontal states for all the fermions, and these are what become associated with three fermion generations. Moreover, when this is carefully considered together with the particle multiplets for which symmetry is broken, *not only the existence of three generations but also the observed CKM mixing of both left-chiral quarks and leptons naturally emerges and is fully explained, deductively and inexorably*. In retrospect, it was the author's unfortunate omission not to reference this finding as to the theoretical imperative for three fermion generations and CKM mixing in the title of [19]. Unlike what has been discussed in points 1 through 12, this is a *qualitative*, not quantitative concurrence with empirical data. But it is equally important because although these three generations and their mixing is well-characterized, the *raison d'etre* for the existence of three fermion generations with observed mixing has, until now, remained one the great unexplained empirical mysteries of nature.

14) <u>Resonant Nuclear Fusion</u>: All fundamental science has technological implications which may be developed over time, and the foregoing is no exception. Protons and neutrons bind together to form nuclei. When they do so they release fusion energies and the fused nuclei harbor mass defects which are very precise energy numbers which never vary from one experiment to the next. There must be an explanation why, for example, the deuteron *always* has a binding energy of 2.224 52 ± 0.00020 MeV, each and every time, and indeed, why all the binding energies shown in Figure 1 and all the energies of the fusion and fission events related to these are as they are. As we have now seen, the explanation rests in the current masses of the up and down quarks which these nucleons contain. Stepping back and applying hindsight, there is little else that *could* account for these energies, because protons and neutrons are no more and no less than systems containing up and down quarks and their highly-non-linear interactions. But if that is the case, then as pointed out in section 9 of [2] and more completely elaborated in [5], the binding and fusion energy "toolkit" discussed in point 3 of section 5 which specifies the most elemental energy dosages released during a fusion event may be not only a theoretical toolkit, but also a *technological* one.

Nikola Tesla, who possessed one of the greatest historical aptitudes for extracting technology from science, once stated "if you want to find the secrets of the universe, think in terms of energy, frequency and vibration." So if the secret we wish to extract from nature is how to extract energy via nuclear fusion in the best way possible, and if we think about applying vibrations to nuclei and nucleons in resonance with certain energies and frequencies that might facilitate fusion better than can be done absent applying this vibration, then the foregoing toolkit energies which explain the nuclear binding and fusion data provide possible guidance. A good precedent for this line of reasoning is the use of microwaves or radio waves to excite atoms into higher energy states (Hertzian resonances) which formed the basis for lasers and other optical "pumping" devices. It is on this basis that the author has proposed and filed the international patent application [5] for catalyzing "resonant nuclear fusion" by bathing a nuclear fuel in gamma radiation at energies established by the discrete energies in the dosage toolkit. This needs to be tested and if viable, developed. But the testing is very simple: In experiment 1 carry out a given fusion reaction in the "usual" and "ordinary" way and carefully assemble and monitor all of the variables, e.g., temperature, power, density etc. which are involved as an experimental "control." Then in experiment 2 apply gamma radiation proximate the toolkit frequencies which are pertinent to that fusion reaction, and change nothing else. Make certain that the only difference is that in experiment 2 the gamma radiation is applied and in experiment 1 it is not. See if the fusion moves any of the key variables in a "fusion-favorable" direction. If it does, then the further development of those results may provide the path for more practical and widespread applications of nuclear fusion to produce commercial energy. And, any favorable change based on using the toolkit energies would be a further empirical validation of these scientific results.

So for example, consider the simplest fusion event $2p \to {}^2_1H + e^+ + \nu + \text{Energy}$. We found in section 5 point 6 that this releases an energy $2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.000\,450\,424\,1\,\text{u}$ which differs from the empirical $0.000\,451\,141\,0\,\text{u}$ by less than 1 part per million AMU. Using the nuclear structure insights obtained above from Lithium and Boron fusion, this means that each of the protons must contribute a single energy dosage $\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.000\,225\,155\,1\,\text{u}$, which is about 0.210 MeV, to enable this fusion to occur. So what we should try to determine is whether, if we bathe the hydrogen fuel in gamma radiation near 0.210 MeV, this energy bath will provide the protons with just the necessary energy they need to contribute to catalyze this fusion more favorably than if we do not provide this bath, and whether with proper technological development the fusion energy output can be made to exceed catalytic gamma radiation input.

15) Decoding the Nuclear Genome: The many ways, the fundamental purpose of this paper is to present empirical evidence for the viewpoint that there is in fact a nuclear genome which needs to be decoded if humankind is to advance its understanding of nuclear and elementary particle physics beyond where it stands at present. This nuclear genome is physically manifest through multiple relationships in which the nuclear masses and mass defects and binding and fusion / fission energies are expressed in terms of current quark masses (and for proton and neutrons and other baryons themselves, additionally the Fermi vev and the CKM quark generation mixing matrices) which quark masses can be established with the same level of precision as the free particles and observed energies to which they are related. And, all of this can be achieved using an unambiguous electron-proton-neutron (EPN) measurement system for defining the $Q \to 0$ up and down quark masses notwithstanding the fact that quarks are confined and so can never be *directly* observed in their quiescent $Q = 0$ states of being.

This exposition began with the postulated "primary mass relationships" (3.1) and (3.2) from which we then deduced $Q = 0$ up and down quark masses with a high precision inherited from the EPN masses. Then we posed the three questions 1) whether it is legitimate and unambiguous as a measurement system to establish $Q = 0$ quark masses in this way, 2) whether such an approach relating the quark masses to nuclear masses and energies could be validated by empirical data and 3) whether and how the thesis that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory provides a firm theoretical foundation upon which all of this may be supported, and what the interface is between theory and experiment.

The evidence presented in this section of parts-per 10^5, 10^6 and even 10^7 AMU empirical fits between the up and down current quark masses as parameters and multiple light nuclide binding energies for ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be, ^{10}B, ^{9}Be, ^{10}Be, ^{11}B, ^{11}C, ^{12}C and ^{14}N, very tightly-bound nuclides like ^{56}Fe, and even the proton and neutron masses themselves within all experimental errors, demonstrate that there really do exist definitive relationships in nature between the up and down current quark masses and a plethora of energies observed in the nuclear world. This evidence thus suggests that the Q=0 up and down quark masses are indeed the masses deduced in (3.3) and (3.4) with a precision close to a billion times better than anything that has been achieved to date by defining quark masses from the results of nuclear scattering experiments. If our purpose was to validate the primary relationships (3.1) and (3.2) and thus the up and down quark masses (3.3) and (3.4) by showing that *if* these relationships and masses are regarded as true many other nuclear energies could also be similarly-related to these masses, then every single one of points 1 through 11 of this section contain further examples of secondary nuclear energy relationships which can be closely expressed in terms of the up and down current quark masses, just like the primary relationships (3.1) and (3.2), thus providing clear empirical validation, a.k.a. consistent non-contradiction. Point 12 suggests possible additional validation (or contradiction) through the study of other baryon masses, and it is also very important as we are reminded of in point 13, that this approach allows us to finally answer Rabi's questions about the higher fermion generations, "who ordered that?" Per point 14, the ability to better develop nuclear fusion technology could be a potent practical benefit, and if testing shows this to be feasible, this would provide additional validation of the underlying theoretical science.

So at this point, the primary relationships (3.1) and (3.2) have been amply validated by empirical data, and this validation also demonstrates that the EPN-0 measurement system laid out here yields sensible and unambiguous results which connect without contradiction of a substantial range of empirical nuclear data. Finally then, the time has arrived to summarize the theoretical considerations from which the author originally deduced the mass / energy relationships (3.1), (5.1) and (5.2) from which all of the other empirical connections elaborated here were developed via comparison with empirical data. The underlying theory, of course, is that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory as originally presented by the author in [1], and thereafter more-deeply developed in [10] which for the first time fully lays out the quantum field theory for this via an exact, recursive, non-linear path integration of classical Yang-Mills gauge theory. Thus, we now turn to the third question from section 3: is there a firm theoretical foundation upon which all of this may be supported, and what is the interface which connects theory to experiment? We now review this question throughout the remainder of this paper.

7. Merged Magnetic and Electric Maxwell, Yang-Mills, Dirac, Exclusion, Feynman, and the Theoretical / Empirical Interface

The author's thesis that the observed baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is what initially led following development in [1] and later deeper elaboration in [10] to equations (5.1) and (5.2) and then by subtraction of (5.1) from (5.2), to equation (3.1). These three equations, in turn, became the foundation for all of the empirical connections elaborated in the last section which cumulatively provide substantial evidence for the validity of the underlying theory, as has been reviewed here. So it is equations (5.1) and (5.2) which are the "interface" between the underlying theory and the ability to prove that theory by reference to empirical data. In the interest of economy we shall leave out those details of the underlying theory which can be readily found in the original source materials [1] and [10], and focus on how it is that the interface equations (5.1) and (5.2) ultimately derive from that theory.

We start by returning to the question posed in point 3 of section 3: "If we can legitimately assert (3.3) and (3.4) to be the $Q=0$ up and down quark masses and if we can find secondary support from a broad array of nuclear data [which has now been done], then we get to the third question: what is the overarching theory, does that theory make sense within the overall framework of theoretical physics, and what is the interface by which we connect the theory to the means by which it can be empirically tested?"

As to theoretical sensibility, the thesis that the observed protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory is in fact exceptionally conservative, and is grounded solely in widely-accepted, highly-settled, thoroughly-tested science. Its novelty rests in its deductive synthesis of known, accepted and well-validated scientific theories and theoretical elements to uniquely and unambiguously deduce new results and new explanations for previously-unexplained observational data, such as what was reviewed in the last section. As suggested near the start of section 6, while brand new ideas ought not to be ruled out out-of-hand, a synthesis of settled science and scientific elements is preferable, and brand new notions should only be used as a last resort when there is no apparent way to succeed by restricting oneself to combining known elements in unknown ways. This theory follows the preferable and more conservative path by synthesizing in new ways, what is known and well-tested and settled.

Specifically, separately from the empirical validations already reviewed, in order to accept this theory from a *theoretical standpoint*, one is required simply to believe and accept no more and no less than: a) that Maxwell's electrodynamics which includes (vanishing) magnetic monopoles is a correct theory of nature; b) that Yang-Mills gauge theory which extends Maxwell's electrodynamics to non-abelian domains is a correct theory of nature; c) that Dirac's theory is a correct theory of nature particularly insofar as it relates fermion wavefunctions to current densities via $J^{\sigma}=\overline{\psi}\gamma^{\sigma}\psi$; d) that Dirac-Fermi-Pauli were correct when they asserted that multiple fermions within a single system must occupy exclusive states distinguished from one another by one or more quantum numbers (the "Exclusion Principle"); and e) for the quantum theory of chromodynamics QCD, believing that Feynman's method of path integration is the correct way to start with a classical field equation in spacetime (configuration space) for a field φ with source *J* and its related Lagrangian density $\mathfrak{L}(\varphi,J)$ and action $S(\varphi,J)=\int d^4x\mathfrak{L}(\varphi,J)$, and convert this over to a quantum field theory by performing the integration $Z=\exp iW(J)=\mathcal{C}\int D\varphi\exp iS(\varphi,J)$ and then extracting the quantum field $W(J)$ in (Fourier-transformed) momentum space. And to cross the threshold from theory to empirical confirmation by obtaining the interface equations (5.1) and (5.2), one also needs to believe and accept f) that the quarks inside a baryon, although confined, are asymptotically free and can thus be treated *at least in an approximate manner* as free fermions with wavepackets having close to minimal Heisenberg uncertainty.

If one accepts and believes a) through d), then the inexorable result of *merely synthesizing all of these together* leads one to conclude that the classical magnetic monopoles of Yang-Mills gauge theory – specifically the sources of a non-vanishing magnetic field flux $\oiint F \neq 0$ across closed spatial surfaces – do indeed have the earlier noted antisymmetric $\mathrm{R} \wedge \mathrm{G} \wedge \mathrm{B}$ color symmetry of a baryon, and that this $\oiint F \neq 0$ has the symmetric $\overline{\mathrm{R}}\mathrm{R}+\overline{\mathrm{G}}\mathrm{G}+\overline{\mathrm{B}}\mathrm{B}$ color symmetry of a meson, all as established in detail in Part I of [10]. This synthesis also teaches that employing $SU(3)_C$ as the color group of chromodynamics is not a *choice*, but is *required* (the only choice is how to name the three mandated eigenstates). *So chromodynamics is not a theory of first principle, but is a corollary theory* emerging inexorably from the synthesis of a) through d). And if one further accepts and believes e), then the quantum theory which emerges via theoretical deduction following path integration leads to a running QCD coupling which matches up to Figure 1 above within experimental errors, as established generally in section 18 and specifically in [18.22] and Figure 14 of [10]. Finally, if one accepts f), then it becomes possible to use this theory to obtain (5.1) and (5.2) which is the bridge to empirical testing. But the fact that (5.1) and (5.2) and their offspring (3.1) lead to all of the empirical confirmations already enumerated here provides some confidence that this treatment of quarks inside a baryon as approximately-free particles is approximately empirically-valid to parts in at least 10^5 AMU based on the energies it predicts. So let us now turn as directly as possible to how the interface equations (5.1) and (5.2) are obtained and then work backwards to place that in the overall theoretical context.

The starting point for deriving the interface equations (5.1) and (5.2) in the original formulation of the baryon / monopole thesis was equation [11.2] of [1]. In the later formulation presented in [10] which includes a complete non-linear development of the chromo-electric charge equation, the equivalent starting point is equation [10.4], which is reproduced below:

$$\begin{aligned} & i\mathrm{Tr}\Sigma F_{\mathrm{eff}\,\mu\nu}\left(\left(0\right)\right)_0 = \mathrm{Tr}\Sigma\left[G_\mu, G_\nu\right]\left(\left(0\right)\right)_0 \\ &= \overline{\psi_R}\gamma_{[\mu}\left(\not{p}_R - m_R\right)^{-1}\gamma_{\nu]}\psi_R + \overline{\psi_G}\gamma_{[\mu}\left(\not{p}_G - m_G\right)^{-1}\gamma_{\nu]}\psi_G + \overline{\psi_B}\gamma_{[\mu}\left(\not{p}_B - m_B\right)^{-1}\gamma_{\nu]}\psi_B \end{aligned} \quad . \tag{7.1}$$

The notation in $\Sigma F_{\mathrm{eff}\,\mu\nu}\left(\left(0\right)\right)_0$ is a bit cumbersome so let us simplify this a bit, and also remind the reader what this means. The Σ in (7.1) simply reminds us of the use of the spin sum $\Sigma_{\mathrm{spins}} u\overline{u} = N^2 / \left(E+m\right)\left(\not{p}+m\right)$ during the course of the derivation starting with [9.12] of [10]. If we simply keep in mind that a spin sum was used to get to that point then we can drop the Σ from the notation. So for the normalization, e.g., $N^2 = \left(E+m\right)$, we may write the spin sum as $u\overline{u} = \left(\not{p}+m\right)$ with the sum being mentally noted, and follow suit for any downstream results.

The $((0))_0$ notation developed in section 8 of [10] tells us that that (7.1) is taken in the abelian limit of non-abelian gauge theory for which $G_\mu((0))_0 = (k_\tau k^\tau - m^2 + i\varepsilon)^{-1} J_\mu$ and in which we have not recursed G_μ into itself at all. As shown in section 7 of [10], a natural consequence of the non-linearity of Yang-Mills gauge theory is that when we invert the classical Maxwell chromo-electric charge equation between G_μ and J_μ, we find that $G_\mu(G_\mu, J_\mu)$ is a function of itself along with J_μ. So if we recurse *n* time before cutting off then we denote this as $G_\mu((0))_n$. To simplify, we shall simply keep the subscript "0" as a reminder that $F_{\mu\nu}$ above is taken at the zero recursive order which is the abelian limit, and drop the nested parenthesis.

Finally, the "eff" subscript for "effective" in (7.1) is used to denote that this is the portion of the field strength tensor $F_{\mu\nu}$ which actually net-flows $\oiint F = \oiint F_{\text{eff}} = -i\oiint [G,G] \neq 0$ across the closed spatial surfaces surrounding the "faux" magnetic sources $P' = -id[G,G] = -i[dG,G]$ of Yang-Mills gauge theory. This is because the term dG in the complete field strength $F = dG - i[G,G]$ identically drops out of any expression for $\oiint F$. This is because the exterior derivative of an exterior derivative is zero in differential geometry, $ddG = 0$. And $ddG = 0$ is why in electrodynamics $\oiint F = 0$, which combines Gauss' law for magnetism and Faraday's law for induction and via Gauss states that there are no magnetic charges. This is the heart of how baryons are initially developed theoretically from the monopoles of Yang-Mills gauge theory by deductively combining points a) and b) above, that Maxwell and Yang-Mills are both correct theories of nature. So if one were to start with $\oiint F_{\text{eff}} = -i\oiint [G,G] \neq 0$ and simply equate the integrands, $F_{\text{eff}} = -i[G,G] \neq 0$ would be the result. This is not the complete Yang-Mills field strength $F = dG - i[G,G]$, but when it comes to net surface fluxes, $\oiint dG = 0$ for all the reasons just noted that there are no magnetic monopoles at all in electrodynamics. Thus we shall retain the "eff" subscript as a reminder of this, and shall refer to $F_{\text{eff}} = -i[G,G] \neq 0$ as the "monopole-net-flux-effective" field strength. Therefore, $\Sigma F_{\text{eff}\,\mu\nu}((0))_0$ above shall now be denoted simply $F_{\text{eff}\,0\mu\nu}$ to mean the net-flowing $\oiint F \neq 0$ portion of F in the abelian zero-recursive order of Yang-Mills gauge theory.

The final aspect of (7.1) which we have not yet discussed is that this is a trace equation. If we backtrack to an earlier equation such as [9.20] of [1] from which this is descended to write this in matrix form prior to taking the trace, then (7.1) can be put in its matrix form:

$$F_{\text{eff}\,0\mu\nu} = -i\begin{pmatrix} \overline{\psi_R}\gamma_{[\mu}(\not{p}_R - m_R)^{-1}\gamma_{\nu]}\psi_R & 0 & 0 \\ 0 & \overline{\psi_G}\gamma_{[\mu}(\not{p}_G - m_G)^{-1}\gamma_{\nu]}\psi_G & 0 \\ 0 & 0 & \overline{\psi_B}\gamma_{[\mu}(\not{p}_B - m_B)^{-1}\gamma_{\nu]}\psi_B \end{pmatrix}. \quad (7.2)$$

This is the formal starting point via $E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ using both inner and outer product traces as reviewed in this paper near the start of section 5, for deriving (5.1) and (5.2) which are the interface equations leading to all the empirical connections reviewed in section 6. So let us proceed to show how this connection is made. This will essentially review section 11 of [1], but with the revelation of a type of up and down quark mass mixing not previously elaborated, and with the additional clarity and perspective the author has gained in the two years elapsed since first deriving this result. We begin by looking at the generic expression $\overline{\psi}\gamma_{[\mu}\left(\not{p}-m\right)^{-1}\gamma_{\nu]}\psi$ in (7.2) which is replicated three times for each of the three colors of quark.

First, we separate propagators $\left(\not{p}-m\right)^{-1}=\left(\not{p}+m\right)/\left(p^2-m^2\right)$ into two parts and write:

$$\overline{\psi}\gamma_{[\mu}\left(\not{p}-m\right)^{-1}\gamma_{\nu]}\psi=\frac{\overline{\psi}\gamma_{[\mu}\left(\not{p}+m\right)\gamma_{\nu]}\psi}{p^2-m^2}=\frac{m\overline{\psi}\left[\gamma_\mu,\gamma_\nu\right]\psi}{p^2-m^2}+\frac{\overline{\psi}\gamma_{[\mu}\not{p}\gamma_{\nu]}\psi}{p^2-m^2}. \tag{7.3}$$

Now we expand out the numerator in the latter term using $\not{p}=p^\sigma\gamma_\sigma$, as such:

$$\overline{\psi}\gamma_{[\mu}\not{p}\gamma_{\nu]}\psi=p^\sigma\overline{\psi}\gamma_{[\mu}\gamma_\sigma\gamma_{\nu]}\psi=p^0\overline{\psi}\gamma_{[\mu}\gamma_0\gamma_{\nu]}\psi+p^1\overline{\psi}\gamma_{[\mu}\gamma_1\gamma_{\nu]}\psi+p^2\overline{\psi}\gamma_{[\mu}\gamma_2\gamma_{\nu]}\psi+p^3\overline{\psi}\gamma_{[\mu}\gamma_3\gamma_{\nu]}\psi. \tag{7.4}$$

We evaluate each of the independent components $\mu\nu=01,02,03,12,23,31$ and apply the Dirac relation $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$ in various combinations to terms which do not drop out via the $\left[\mu,\nu\right]$ commutator. Using $g_{\mu\nu}=\eta_{\mu\nu}$ for flat spacetime, one may summarize the result by:

$$\overline{\psi}\gamma_{[\mu}\not{\rho}\,\gamma_{\nu]}\psi=2i\varepsilon_{\mu\nu\alpha\beta}\rho^{[\alpha}\overline{\psi}\gamma^{\beta]}\gamma^5\psi \tag{7.5}$$

So we use this as well as the Dirac covariant $\left[\gamma_\mu,\gamma_\nu\right]=-2i\sigma_{\mu\nu}$ to rewrite (7.3) as:

$$\overline{\psi}\gamma_{[\mu}\left(\not{p}-m\right)^{-1}\gamma_{\nu]}\psi=-2i\frac{m\overline{\psi}\sigma_{\mu\nu}\psi}{p^2-m^2}+2i\frac{\varepsilon_{\mu\nu\alpha\beta}\rho^{[\alpha}\overline{\psi}\gamma^{\beta]}\gamma^5\psi}{p^2-m^2}. \tag{7.6}$$

We see therefore that this generic expression contains both a second rank antisymmetric tensor $\overline{\psi}\sigma_{\mu\nu}\psi$ which is a fermion polarization and magnetization bivector, and a first rank *axial* vector $\overline{\psi}\gamma^\beta\gamma^5\psi$. Using chirality language, this means that $F_{\mathrm{eff}\,0\mu\nu}$ in (7.2) admits to a vector (*V*) and axial (*A*) separation, $F_{\mathrm{eff}\,0\mu\nu}=F_{V\,\mathrm{eff}\,0\mu\nu}+F_{A\mathrm{eff}\,0\mu\nu}$.

Let us now set aside the axial term $F_{A\mathrm{eff}\,0\mu\nu}$ and focus on the chiral vector term $F_{V\,\mathrm{eff}\,0\mu\nu}$ in the $p^2\to 0$ limit for which the propagators disappear and the interactions essentially occur at a point. We refer, e.g., to [24] at p. 257 for a similar analysis explaining how the Fermi coupling constant G_F really is a point-interaction manifestation of a *W* vector boson propagator

$\left(g_{\mu\nu} - k_\mu k_\nu / M_W{}^2\right) / \left(k^2 - M_W{}^2\right)^{-1}$ in the $k^2 \to 0$ limit for which $G_F / \sqrt{2} = g_w{}^2 / 8M_W{}^2$, connecting the modern understanding of weak interactions with Fermi's original conception of β-decay modelled on electromagnetic interactions. Using the chiral vector V portion of (7.6) in (7.2) for $p^2 \to 0$ allows us to now write this matrix as:

$$F_{V\,\mathrm{eff}\,0\mu\nu} = 2\begin{pmatrix} \dfrac{\overline{\psi}_R \sigma_{\mu\nu} \psi_R}{m_R} & 0 & 0 \\ 0 & \dfrac{\overline{\psi}_G \sigma_{\mu\nu} \psi_G}{m_G} & 0 \\ 0 & 0 & \dfrac{\overline{\psi}_B \sigma_{\mu\nu} \psi_B}{m_B} \end{pmatrix}. \tag{7.7}$$

It is important to see that the trace of the above is

$$\tfrac{1}{2}\mathrm{Tr} F_{V\,\mathrm{eff}\,0\mu\nu} = \frac{\overline{\psi}_R \sigma_{\mu\nu} \psi_R}{m_R} + \frac{\overline{\psi}_G \sigma_{\mu\nu} \psi_G}{m_G} + \frac{\overline{\psi}_B \sigma_{\mu\nu} \psi_B}{m_B} \tag{7.8}$$

which has the requisite $\overline{R}R + \overline{G}G + \overline{B}B$ color wavefunction of a meson.

It is this matrix (7.7) which is the theoretical point of departure, i.e., the interface for connecting the underlying theory with the electron rest mass in (3.1) a.k.a. (5.3) and the various nuclear energies elaborated in sections 5 and 6 of this paper starting with (5.1) and (5.2). So now, with the benefit of two years of retrospective perspective including the many empirical connections enumerated in section 6, we shall elucidate this connection which was originally uncovered in sections 11 and 12 of [1], between (7.7) and observational mass and energy data.

As reviewed at the start of section 5, the energy of pure gauge fields in Yang-Mills theory may be deduced by taking $E = \iiint d^3x \tfrac{1}{2}\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$, and $\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$ may be taken via both an outer and an inner product. We now have an $F_{V\,\mathrm{eff}\,0\mu\nu}$ in (7.7) above which flows from the thesis that baryons are the chromo-magnetic monopoles of Yang-Mills and specifically from synthesizing Maxwell and Yang-Mills and Dirac Theories and Fermi-Dirac-Pauli Exclusion. So we shall use this to deduce the associated energy E.

First, based on (7.7), we form the outer product trace:

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{eff}\,0\mu\nu}\otimes F_{V\,\mathrm{eff}\,0}{}^{\mu\nu}$$
$$=2\begin{pmatrix}\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}+\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}\\ +2\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+2\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}+2\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}\end{pmatrix}. \tag{7.9}$$

It will be appreciated that this includes the inner product trace, which consists only of the top parenthetical line in the above:

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{eff}\,0\mu\nu}\cdot F_{V\,\mathrm{eff}\,0}{}^{\mu\nu}=2\left(\frac{\overline{\psi}_R\sigma_{\mu\nu}\psi_R}{m_R}\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{m_R}+\frac{\overline{\psi}_G\sigma_{\mu\nu}\psi_G}{m_G}\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{m_G}+\frac{\overline{\psi}_B\sigma_{\mu\nu}\psi_B}{m_B}\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{m_B}\right). \tag{7.10}$$

So the inner product has pure-color RR, GG and BB products of the rank-2 $\overline{\psi}\sigma_{\mu\nu}\psi$ tensors while the outer product supplements these with RG, GB and BR cross-color products.

Next, we refer to sections 7 and 8 of [1] as also reviewed in section 10 of [10] whereby for the proton, the RGB colors of quark are respectively assigned to and have the appropriate flavor generators for the duu flavors of quark and for the neutron these same colors are assigned to and have generators for the udd flavors of quark. That is, $\mathrm{RGB}\to duu$ for the proton and $\mathrm{RGB}\to udd$ for the neutron. Therefore, (7.7) with these assignments is used to specify the chiral vector "V" portion for both a proton (*P*) and a neutron (*N*) field strength:

$$F_{V\,P\,\mathrm{eff}\,0\mu\nu}=2\begin{pmatrix}\frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d} & 0 & 0\\ 0 & \frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u} & 0\\ 0 & 0 & \frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\end{pmatrix}, \tag{7.11}$$

$$F_{V\,N\,\mathrm{eff}\,0\mu\nu}=2\begin{pmatrix}\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u} & 0 & 0\\ 0 & \frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d} & 0\\ 0 & 0 & \frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d}\end{pmatrix}. \tag{7.12}$$

This is the first place at which the up and down current quark masses and wavefunctions enter the picture. This means that the outer product traces:

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{Peff}\,0\mu\nu}\otimes F_{V\,\mathrm{Peff}\,0}{}^{\mu\nu}=2\left(\frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}+4\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}+4\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_u\sigma^{\mu\nu}\psi_u}{m_u}\right),\quad(7.13)$$

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu}\otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu}=2\left(\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_u\sigma^{\mu\nu}\psi_u}{m_u}+4\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}+4\frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}\right).\quad(7.14)$$

So if we subtract (7.13) for the proton from (7.14) for the neutron, we find that the difference:

$$\tfrac{1}{2}\mathrm{Tr}F_{V\,N\mathrm{eff}\,0\mu\nu}\otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu}-\tfrac{1}{2}\mathrm{Tr}F_{V\,\mathrm{Peff}\,0\mu\nu}\otimes F_{V\,\mathrm{Peff}\,0}{}^{\mu\nu}=2\left(3\frac{\overline{\psi}_d\sigma_{\mu\nu}\psi_d}{m_d}\frac{\overline{\psi}_d\sigma^{\mu\nu}\psi_d}{m_d}-3\frac{\overline{\psi}_u\sigma_{\mu\nu}\psi_u}{m_u}\frac{\overline{\psi}_u\sigma^{\mu\nu}\psi_u}{m_u}\right).\quad(7.15)$$

It is (7.13) which eventually turns into $E_P=\left(m_d+4\sqrt{m_u m_d}+4m_u\right)/\left(2\pi\right)^{\frac{3}{2}}$ in (5.1); (7.14) which turns into $E_N=\left(m_u+4\sqrt{m_u m_d}+4m_d\right)/\left(2\pi\right)^{\frac{3}{2}}$ in (5.2); and finally, (7.15) which turns into $E_N-E_P=3\left(m_d-m_u\right)/\left(2\pi\right)^{\frac{3}{2}}\equiv m_e$ in (5.3) a.k.a. the primary relationship (3.1). The reader should closely make these respective comparisons, because these is how the structure of the theory that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory bleeds through to (5.1), (5.2) and (5.3) which become the basis for all of the other empirical relationships heretofore reviewed.

Specifically, as will now be reviewed, when we use (7.13) to (7.15) in $E=\iiint\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$, carry out the integration, and then establish the normalization of the Dirac spinors by comparing the theoretical energy results to empirical data ("empirical normalization," see [1] after [11.29]), we uncover term mappings $\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_u\sigma_{\mu\nu}\psi_u/m_u{}^2\Rightarrow m_u$, $\overline{\psi}_d\sigma_{\mu\nu}\psi_d\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_d{}^2\Rightarrow m_d$ and $\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_u m_d\Rightarrow\sqrt{m_u m_d}$, together with the $\left(2\pi\right)^{\frac{3}{2}}=\sqrt{2\pi}^3$ divisor which emerges from the $E=\iiint\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ integral over three space dimensions. Let us now detail how this is done.

All of (7.13), (7.14) and (7.15) when used as integrands in $E=\iiint\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ will yield combinations of three distinct terms: $\tfrac{1}{2}E_{uu}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_u\sigma_{\mu\nu}\psi_u/m_u{}^2$ which is a pure up / up term, $\tfrac{1}{2}E_{dd}=\iiint d^3x\overline{\psi}_d\sigma_{\mu\nu}\psi_d\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_d{}^2$ which is a pure down / down term, and $\tfrac{1}{2}E_{ud}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_u m_d$ which is a mixed up / down term. The factor of $\tfrac{1}{2}$ is to account for the overall factors of 2 in (7.13) through (7.15) so we are comparing energy numbers on an apples-to-apples basis. These terms are then weighted within the overall energies

$E=\iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ via the constant coefficients 1, 3 and 4 variously appearing in (7.13), (7.14) and (7.15). And these also become the "energy dosages" in the "toolkit" first referred to after (6.7) which physically are later understood to be the energy dosages emitted from nuclei during fusion events. So, for example, we earlier spoke after (6.8) of how nine (9) energy dosages $9\sqrt{m_u m_d}/(2\pi)^{1.5}$ are emitted as energy when ^{4}He is fused with two protons to create ^{6}Li with the same number of nine (9) up / down quark pairs, and of how fifteen (15) energy dosages $15\sqrt{m_u m_d}/(2\pi)^{1.5}$ are emitted when ^{8}Be is fused with two protons to create ^{10}Li with the same number of fifteen (15) up / down quark pairs. What we were really saying when more formally-specified in terms of the underlying theoretical physics, is that in the former case ${}^4_2He+2p\rightarrow {}^6_3Li+e^++\nu+\text{Energy}$ there are nine (9) and in the latter case ${}^8_4Be+2p\rightarrow {}^{10}_5B+e^++\nu+\text{Energy}$ there fifteen (15) emissions of the energy dosage $\frac{1}{2}E_{ud}=\iiint d^3x\overline{\psi}_u\sigma_{\mu\nu}\psi_u\overline{\psi}_d\sigma_{\mu\nu}\psi_d/m_u m_d$, one such dosage associated with each pair of up and down quarks. So now, let us review how this connection gets made.

We start with the generic expression $\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi/m^2$ for a fermion wavefunction $\psi(\mathbf{x})$ and take this to be representative of the up or down quark when used in the "pure" terms mentioned just above. Now, any spatial dependence for this integral over d^3x is contained in $\psi(\mathbf{x})$ so to go any further with this calculation we must make some supposition about the spatial-dependency of $\psi(\mathbf{x})$. We can choose from a number of possible functions, e.g., Lorentzian, exponential, Gaussian, etc. Indeed, any function may be used, whether or not it is radially symmetric, provided it is renormalizable and so finitely integrates when placed in $\frac{1}{2}E=\iiint d^3x\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi/m^2$. As an *ansatz* to be able to perform *some* numeric calculation, and without limitation as to any other *ansatz* that another may choose, the author at [9.9] of [1] chose the radially-symmetric Gaussian wavefunction $\psi(r)=u(p)\left(\pi/m^2\right)^{-.75}\exp\left(-\frac{1}{2}m^2(r-r_0)^2\right)$ where *m* generically needs to be a number with mass dimensionality and r_0 is the radial coordinate of the center peak of the Gaussian. Further, to give *m* some meaning in relation to the physics being studied, *m* is chosen in this *ansatz* to be equal to the rest mass of the fermion. Again, this is done simply to be able to do an integral calculation over d^3x with the hope that energy numbers which makes sense in relation to something observed might emerge from this calculation. Again, other exploratory choices for $\psi(\mathbf{x})$ are also possible.

Now, a Gaussian is the standard expression used to represent a minimum-uncertainty wave-packet $\sigma_x\sigma_p=\hbar/2$ (not >) and thus is associated with free particles. So, one may ask whether this "freedom" is suitable for quarks which are confined. But quarks *are* in fact *asymptotically free*, so aside from the "edge" region of a nucleon near $Q=\Lambda_{\mathrm{QCD}}$ as discussed in section 2, a free-particle Gaussian could be a good approximation to an "approximately free" fermion such as an asymptotically-free quark. Also, wave-packets such as the foregoing Gaussian with a standard deviation comparable to their Compton wavelength $\lambda\!\!\!^{-}=\hbar/mc$ contain negative-energy amplitudes indicating the presence of antiparticles. But we know that nucleons are teeming with quark / antiquark states, exhibited no more clearly than through the manifold of $\overline{q}q$ meson jets emitted under any substantial scattering impact. Finally, the Compton wavelengths of the current quark masses are on the order of 40 Fermi for the down quark and 85 Fermi for the up quark, which exceeds ~ 2 Fermi length scale $r_\Lambda \equiv \hbar / c\Lambda_{\text{QCD}} = 2.1780\,\text{fm}$ of Λ_{QCD} by more than a full order of magnitude and so "bleeds out" from the proton and neutron even though the quarks are confined. But as noted at the end of section 11 in [1], see also after (6.16), the Compton wavelengths for the constituent i.e. contributive quark masses are less than 1 Fermi which places them well within the r_Λ length scale. And what we learn in sections 5 and 6 is that although the current quarks are confined, their mass values are the central drivers of the energies which do pass in and out of nuclides and nucleons during fusion and fission events. So while nucleons do confine quarks, *they do not confine energies*, and the energies they release are driven directly by the current quark masses. Thus one can acquire some qualitative comfort with a Compton wavelength that extends beyond r_Λ by over 1 order of magnitude given that the same wavelength drives the energies which also bleed out from the nucleons. So we cease playing "Hamlet" over what $\psi(\mathbf{x})$ to use, we keep in mind that different $\psi(\mathbf{x})$ can be tried and that this might be an interesting exercise, and we go into $\frac{1}{2}E = \iiint d^3x \overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi / m^2$ with a radially-symmetric Gaussian for which $\sigma_x \sigma_p = \hbar / 2$ (or slightly larger if not a perfect Gaussian for a perfectly free fermion) and with the Compton wavelengths of the current quark masses setting the spatial spread, simply to see what comes out. If the results make some approximate empirical sense to some degree, then what we have done will be seen to be approximately correct to the same degree. If the results are contradicted, then we must try something else.

For this Gaussian *ansatz*, $\rho = J^0 = \overline{\psi}\gamma^0\psi = \psi^\dagger\psi = \left(m^3 / \pi^{\frac{3}{2}}\right)\exp\left(-m^2\left(r-r_0\right)^2\right)u^\dagger u$ is the probability density. The Gaussian integral $\iiint d^3x\left(m^3 / \pi^{\frac{3}{2}}\right)\exp\left(-m^2\left(r-r_0\right)^2\right) = 1$ tells us that the spatial dependency integrates to unity, so that $\iiint d^3x\rho = \overline{u}\gamma^0 u = u^\dagger u$. Therefore, we now set $\psi(r) = u(p)\left(\pi / m^2\right)^{-.75}\exp\left(-\frac{1}{2}m^2\left(r-r_0\right)^2\right)$ in $\frac{1}{2}E = \iiint d^3x \overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi / m^2$ four times which yields fourth powers of the terms inside $\psi(r)$, and we remove the space-independent terms from the integral. We then make use of the mathematical solution $\iiint d^3x\exp\left(-2m^2\left(r-r_0\right)^2\right) = \left(\pi / 2\right)^{\frac{3}{2}} / m^3$ for the Gaussian integral, and finally reduce. Thus:

$$\begin{aligned}\frac{1}{2}E &= \iiint d^3x \frac{\overline{\psi}\sigma_{\mu\nu}\psi\overline{\psi}\sigma_{\mu\nu}\psi}{m^2} = \frac{1}{m^2}\left(\frac{\pi}{m^2}\right)^{-3}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u\iiint d^3x\exp\left(-2m^2\left(r-r_0\right)^2\right) \\ &= \frac{1}{m^2}\left(\frac{m^2}{\pi}\right)^3\left(\frac{\pi}{2}\right)^{\frac{3}{2}}\frac{1}{m^3}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u = \frac{m}{\left(2\pi\right)^{\frac{3}{2}}}\overline{u}\sigma_{\mu\nu}u\overline{u}\sigma_{\mu\nu}u\end{aligned} \quad . \tag{7.16}$$

So we see how this integration converts the pure terms and also injects a $(2\pi)^{\frac{3}{2}}$ divisor via $\overline{\psi}_u \sigma_{\mu\nu} \psi_u \overline{\psi}_u \sigma_{\mu\nu} \psi_u / m_u{}^2 \Rightarrow m_u / (2\pi)^{\frac{3}{2}}$ and $\overline{\psi}_d \sigma_{\mu\nu} \psi_d \overline{\psi}_d \sigma_{\mu\nu} \psi_d / m_d{}^2 \Rightarrow m_d / (2\pi)^{\frac{3}{2}}$. The $(2\pi)^{\frac{3}{2}}$ which was laced throughout the empirical calculations in sections 3 through 6 is therefore seen to have its fundamental mathematical origins in $\iiint d^3x \exp(-2Ar^2) = (\pi/2A)^{\frac{3}{2}}$ which is the three-space Gaussian integral, and in the normalization of the space-dependency. And we see that for some different, not-Gaussian, normalizable $\psi(\mathbf{x})$ with fourth-power integral $\iiint d^3x f(\mathbf{x}) = M$, whatever factor appears in place of $(2\pi)^{\frac{3}{2}}$ would be driven by M and the normalization of the spatially-integrated probability density to unity.

Because Dirac spinors u are a function only of $u(m,\mathbf{p})$ and not $\mathbf{x}$, the final term $\overline{u}\sigma_{\mu\nu} u \overline{u} \sigma_{\mu\nu} u$ in (7.16) above is a function only of mass m and momentum $\mathbf{p}$. These Dirac spinors are subject to normalization and this normalization can be *chosen*. So we should choose the spinor normalization such that the energy number in the resultant $\frac{1}{2}E = m/(2\pi)^{\frac{3}{2}} \cdot \overline{u}\sigma_{\mu\nu} u \overline{u} \sigma_{\mu\nu} u$ makes sense in relation to an observed energy or energies. So we return to (7.15) which contains only pure up / up and down / down terms, and because of this we can now use (7.16). Specifically, combining (7.15) and (7.16) enables us to write:

$$\begin{aligned} E_\Delta &\equiv E_{V\,N\mathrm{eff}\,0} - E_{V\,N\mathrm{eff}\,0} = \iiint d^3x \tfrac{1}{2} \mathrm{Tr} F_{V\,N\mathrm{eff}\,0\mu\nu} \otimes F_{V\,N\mathrm{eff}\,0}{}^{\mu\nu} - \iiint d^3x \tfrac{1}{2} \mathrm{Tr} F_{V\,P\mathrm{eff}\,0\mu\nu} \otimes F_{V\,P\mathrm{eff}\,0}{}^{\mu\nu} \\ &= 2\left[3\iiint d^3x \left(\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} \right) - 3\iiint d^3x \left(\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} \right) \right] \\ &= 2\left(\frac{3}{(2\pi)^{\frac{3}{2}}} m_d \overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d - \frac{3}{(2\pi)^{\frac{3}{2}}} m_u \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u \right) \end{aligned} \quad . \tag{7.17}$$

This E_Δ represents the energy difference between $E_{V\mathrm{eff}\,0} = \iiint d^3x \frac{1}{2} \mathrm{Tr} F_{V\mathrm{eff}\,0\mu\nu} \otimes F_{V\mathrm{eff}\,0}{}^{\mu\nu}$ for the neutron and proton chiral-vector, monopole-net-flux-effective, zero-recursive-order pure field strengths (7.12) and (7.11). And it will be seen that if we normalize the Dirac spinors *such that* $\overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d = \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u = \frac{1}{2}$, (7.17) will reduce to:

$$E_\Delta = E_{V\,N\mathrm{eff}\,0} - E_{V\,P\mathrm{eff}\,0} = \frac{3}{(2\pi)^{\frac{3}{2}}} (m_d - m_u) . \tag{7.18}$$

This is (5.3) a.k.a. (3.1), the first of the two primary relationships upon which all of the empirical results from section 3 onward were based.

Now, as was stated after (5.3), and as may be reviewed in section 11 and specifically [11.21] of [1], the author first evaluated (7.17) and (7.18) using the PDG data $m_u = 2.3^{+0.7}_{-0.5}\,\mathrm{MeV}$

and $m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}$ and its error bar ranges to deduce that $.286\,\text{MeV}<\text{E}_\Delta < .704\,\text{MeV}$, with a median value of $\text{E}_\Delta = .495\,\text{MeV}$ which is only about 3% off from the electron rest mass based on PDG data with error bars much larger than 3%. The author then hypothesized subject to further independent confirmation which was subsequently successful in the other ways enumerated section 6, that *this energy* $E_\Delta = E_{V\,N\text{eff}\,0} - E_{V\,P\text{eff}\,0}$ *is in fact equal to the electron rest mass* because in the zero-recursion abelian limit where $G_\mu\left((0)\right)_0 = \left(k_\tau k^\tau - m^2 + i\varepsilon\right)^{-1} J_\mu$, all of the interactions which gives rise to the observed neutron minus proton mass difference have been turned off. Thus (7.18) is a relationship which contains only a "signal" from bare current quark masses without gluonic interactive "noise." And with only signal and no noise, it is sensible that the neutron "signal mass" would differ from the proton "signal mass" by precisely the mass of the electron, as has been discussed earlier in some depth following (5.3).

So this data concurrence is what motivated the author to set $m_e \equiv E_\Delta = E_{V\,N\text{eff}\,0} - E_{V\,P\text{eff}\,0}$ by definitional hypothesis, which then mandates $\overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d = \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u = \frac{1}{2}$ for normalization because this is what reduces (7.17) to (7.18) which then enables the empirically-accurate definition $m_e \equiv E_\Delta = E_{V\,N\text{eff}\,0} - E_{V\,P\text{eff}\,0}$. When we then calculate out the consequence of this "empirical normalization" we find in [11.29] of [1] that the dimensionless quark normalization coefficient has the form $N^2 = \frac{1}{\sqrt{4!}}\left(E+m\right)/2m$, and specifically, that $N_u{}^2 = \frac{1}{\sqrt{4!}}\left(E_u + m_u\right)/2m_u$ and $N_d{}^2 = \frac{1}{\sqrt{4!}}\left(E_d + m_d\right)/2m_d$ for the up and down quark spinors respectively based on the conventional spinor definition $u^{(s)T} \equiv N\left(\chi^{(s)} \quad \chi^{(s)} \boldsymbol{\sigma}\cdot\mathbf{p}/\left(E+m\right)\right)$. It is also of interest as discussed in Figure 3 of [1] that when we empirically match up (7.18) with the electron via $m_e \equiv E_\Delta$, the deduced 4! constant in the divisor of the fourth power normalization coefficient happens to coincide with the precise number of fermions known in nature: 4=3+1 colors of quark plus lepton (lepto-quark in GUT parlance) times 3 generations times 2 isospin states up and down. Which is to say, it makes independent sense for each of 24 flavor / color / generation fermion types in nature to carry a 1/24 coefficient in its fourth order normalization. It is highly intriguing that by normalizing (7.17) to have (7.18) empirically match the electron rest mass, this 4! is precisely the divisor that one deduces.

Now, if $E_\Delta = E_{V\,N\text{eff}\,0} - E_{V\,P\text{eff}\,0}$ appears to produce a close empirical result, one might expect each of the neutron and proton signal energies $E_{V\,N\text{eff}\,0}$ and $E_{V\,P\text{eff}\,0}$ to also have some meaning in relation to something what is observed. So the next step is to study these energies. But as noted after (7.15), the mixed energy $\frac{1}{2}E_{ud} = \iiint d^3x \overline{\psi}_u \sigma_{\mu\nu} \psi_u \overline{\psi}_d \sigma_{\mu\nu} \psi_d / m_u m_d$ needs to now be calculated because these mixed up / down integrands appear in (7.13) and (7.14) for the proton and neutron field strengths. So similarly to (7.16), we employ $\psi_{u,d}\left(r\right) = u_{u,d}(p)\left(\pi/m_{u,d}{}^2\right)^{-.75} \exp\left(-\frac{1}{2}m_{u,d}{}^2\left(r-r_0\right)^2\right)$ now explicitly quark-labelled because we need to distinguish up from down quarks to calculate the mixed energy. Here, after solving the Gaussian and reducing and separately isolating a term $\sqrt{m_u m_d}$ with mass dimensionality of +1 we obtain:

$$\begin{aligned}
\tfrac{1}{2}E_{ud} &= \iiint d^3x \frac{\overline{\psi}_u \sigma_{\mu\nu}\psi_u \overline{\psi}_d \sigma_{\mu\nu}\psi_d}{m_u m_d} \\
&= \frac{1}{m_u m_d}\left(\frac{\pi}{m_u{}^2}\right)^{-\frac{3}{2}}\left(\frac{\pi}{m_d{}^2}\right)^{-\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d \iiint d^3x \exp\left(-\left(m_u{}^2+m_d{}^2\right)\left(r-r_0\right)^2\right) \\
&= \frac{1}{m_u m_d}\left(\frac{m_u{}^2}{\pi}\right)^{\frac{3}{2}}\left(\frac{m_d{}^2}{\pi}\right)^{\frac{3}{2}}\left(\frac{\pi}{m_u{}^2+m_d{}^2}\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d \\
&= \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}\left(\frac{m_u m_d}{m_u{}^2+m_d{}^2}\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d
\end{aligned} \quad . \qquad (7.19)$$

Solving this Gaussian starts with the mathematical solution $\iiint d^3x \exp\left(-m^2\left(r-r_0\right)^2\right) = \pi^{\frac{3}{2}}/m^3$ from which we obtain $\iiint d^3x \exp\left(-\left(m_u{}^2+m_d{}^2\right)\left(r-r_0\right)^2\right) = \pi^{\frac{3}{2}}/\left(m_u{}^2+m_d{}^2\right)^{\frac{3}{2}}$ by the variable substitution $m^2 \to m_u{}^2+m_d{}^2$ thus $m^3 \to \left(m_u{}^2+m_d{}^2\right)^{\frac{3}{2}}$, i.e., scaling the coefficient. As a check on the calculation we see that in the special case where $m_u = m_d \equiv m$ the result in (7.19) will coincide identically that in (7.16).

Now, the dimensionless term $\left(m_u m_d/\left(m_u{}^2+m_d{}^2\right)\right)^{\frac{3}{2}}$ from which we have separated the +1 dimensional $\sqrt{m_u m_d}$ looks a bit complicating at first. But any time an a^2+b^2 shows up somewhere in a mathematical expression we immediately know we can place vectors with lengths *a* and *b* at right angles to one another, specify an angle $\tan\theta = a/b$, and use a^2+b^2 as the "invariant" hypotenuse. So it looks like there is some angle $\theta = \arctan\left(m_u/m_d\right)$ which needs to be understood. Importantly, we also recall that in electroweak theory there emerge similar expressions of the form $m_u m_d/\left(m_u{}^2+m_d{}^2\right)$. Specifically, we recall that $g_w \sin\theta_W = g_y \cos\theta_W = e$ where *e* is the electric charge, g_w is the weak charge, g_y is the weak hypercharge and θ_W is the weak mixing angle. And we recall that in the course of calculating from this one arrives at $\sin\theta_W \cos\theta_W = g_w g_y/\left(g_w{}^2+g_y{}^2\right)$ where $g_Z{}^2 = g_w{}^2+g_y{}^2$ is the charge strength of the Z boson with a mass $M_Z = \tfrac{1}{2}v_F g_Z$ where v_F is the Fermi vev which enters by via of spontaneous symmetry breaking using the Higgs field. So the $m_u m_d/\left(m_u{}^2+m_d{}^2\right)$ appearing in (7.19) seems suggestive that there is a mixing angle analogous to the weak mixing angle which rotates between the up and down quark masses. Let us now explore this connection *which the author has not presented explicitly in any earlier papers*. As the discussion of this angle proceeds, the reader may find it helpful to refer to Figure 3 following (8.22) below.

8. First Generation Quark Mass Mixing

Analogously to electroweak theory, we postulate a first generation quark mass mixing angle θ and mass m_1 *defined* such that:

$$m_d \sin\theta \equiv m_u \cos\theta \equiv m_1 . \tag{8.1}$$

So immediately, because $\tan\theta = m_u / m_d$, we may draw a right triangle with m_u on the leg opposite and m_d on the leg adjacent θ, and thus with $\sqrt{m_u^{\,2} + m_d^{\,2}}$ on the hypotenuse. Therefore $\sin\theta = m_u / \sqrt{m_u^{\,2} + m_d^{\,2}}$, $\cos\theta = m_d / \sqrt{m_u^{\,2} + m_d^{\,2}}$ and thus:

$$\sin\theta\cos\theta = \frac{m_u m_d}{m_u^{\,2} + m_d^{\,2}} = \frac{m_u m_d}{m_\zeta^{\,2}} \tag{8.2}$$

which is identical to the factor to the 3/2 power which appeared in (7.19). In the above we have defined $m_\zeta^{\,2} \equiv m_u^{\,2} + m_d^{\,2}$ simply for convenience, and used the Greek zeta to remind us of the analogy to the electroweak $g_Z^{\,2} = g_w^{\,2} + g_y^{\,2}$. So we can use (8.2) to remove the masses from this factor and instead express it in terms of θ. Thus, using (8.2) in (7.19) we have:

$$\tfrac{1}{2} E_{ud} = \iiint d^3x \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u \overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_u m_d} = \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}} \left(\sin\theta\cos\theta\right)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d . \tag{8.3}$$

If (3.3) and (3.4) are indeed the empirical $Q = 0$ quark masses in the EPN measurement scheme discussed section 4, then these can be used to deduce tan θ = 0.453 236 693, therefore the mixing angle θ = 24.381 777 8°. Additionally, $m_\zeta = \sqrt{m_u^{\,2} + m_d^{\,2}} = 0.005\,783\,076\ \mathrm{u} = 5.386\,90110\ \mathrm{MeV}$ may be deduced.

At this point, we have all that we need to return to (7.13) and (7.14), use them as integrands in $E = \iiint d^3x \tfrac{1}{2} \mathrm{Tr} F_{\mu\nu} \otimes F^{\mu\nu}$ for each of the proton $F_{V\,P\mathrm{eff}\,0}$ and the neutron $F_{V\,N\mathrm{eff}\,0}$, and thereby calculate associated energies $E_{V\,P\mathrm{eff}\,0}$ and $E_{V\,P\mathrm{eff}\,0}$. Inserting (7.16) for both the up and down quarks and the mixed-quark (8.3) into (7.14) and (7.15), then forming $E = \iiint d^3x \tfrac{1}{2} \mathrm{Tr} F_{\mu\nu} \otimes F^{\mu\nu}$, we obtain:

$$E_{V\,P\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,P\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,P\,\mathrm{eff}\,0}{}^{\mu\nu}$$
$$= 2\iiint d^3x \left(\frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4 \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4 \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} \right) \quad ,(8.4)$$
$$= 2\left(\frac{m_d}{(2\pi)^{\frac{3}{2}}} \overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d + 4 \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}} (\sin\theta\cos\theta)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d + 4 \frac{m_u}{(2\pi)^{\frac{3}{2}}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u \right)$$

$$E_{V\,N\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,N\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,N\,\mathrm{eff}\,0}{}^{\mu\nu}$$
$$= 2\iiint d^3x \left(\frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_u \sigma^{\mu\nu} \psi_u}{m_u} + 4 \frac{\overline{\psi}_u \sigma_{\mu\nu} \psi_u}{m_u} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} + 4 \frac{\overline{\psi}_d \sigma_{\mu\nu} \psi_d}{m_d} \frac{\overline{\psi}_d \sigma^{\mu\nu} \psi_d}{m_d} \right) \quad .(8.5)$$
$$= 2\left(\frac{m_u}{(2\pi)^{\frac{3}{2}}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u + 4 \frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}} (\sin\theta\cos\theta)^{\frac{3}{2}} \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d + 4 \frac{m_d}{(2\pi)^{\frac{3}{2}}} \overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d \right)$$

Next we apply the empirical normalization $\overline{u}_d \sigma_{\mu\nu} u_d \overline{u}_d \sigma_{\mu\nu} u_d = \overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_u \sigma_{\mu\nu} u_u = \tfrac{1}{2}$ used after (7.18) to associate the deduced energy difference $E_\Delta = E_{V\,N\,\mathrm{eff}\,0} - E_{V\,P\,\mathrm{eff}\,0}$ with the electron rest mass via $m_e \equiv E_\Delta$ which results in $N_u{}^2 = \tfrac{1}{\sqrt{4!}}(E_u + m_u)/2m_u$ and $N_d{}^2 = \tfrac{1}{\sqrt{4!}}(E_d + m_d)/2m_d$. So this means that in the mixed term $N_{ud}{}^2 = \tfrac{1}{\sqrt{4!}}\sqrt{(E_u + m_u)(E_d + m_d)/(2m_u)(2m_d)}$ turns out to be the normalization which emerges from the square root of the product of these individual quark normalizations via (8.3), and this in turn means that there is a like-normalization $\overline{u}_u \sigma_{\mu\nu} u_u \overline{u}_d \sigma_{\mu\nu} u_d = \tfrac{1}{2}$ for the mixed term found in (8.3). Applying all of these normalizations in (8.4) and (8.5) now leads us to:

$$E_{V\,P\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,P\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,P\,\mathrm{eff}\,0}{}^{\mu\nu} = \frac{m_d}{(2\pi)^{\frac{3}{2}}} + 4\frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}(\sin\theta\cos\theta)^{\frac{3}{2}} + 4\frac{m_u}{(2\pi)^{\frac{3}{2}}}, \quad (8.6)$$
$$E_{V\,N\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,N\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,N\,\mathrm{eff}\,0}{}^{\mu\nu} = \frac{m_u}{(2\pi)^{\frac{3}{2}}} + 4\frac{\sqrt{m_u m_d}}{\pi^{\frac{3}{2}}}(\sin\theta\cos\theta)^{\frac{3}{2}} + 4\frac{m_d}{(2\pi)^{\frac{3}{2}}}. \quad (8.7)$$

For the special case $\theta = \pi/4 = 45°$, we have $(\sin\theta\cos\theta)^{\frac{3}{2}} = 1/2^{\frac{3}{2}}$, and these will reduce to:

$$E_{V\,P\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,P\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,P\,\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d} + 4m_u\right), \quad (8.8)$$
$$E_{V\,N\,\mathrm{eff}\,0} = \iiint d^3x\, \tfrac{1}{2}\,\mathrm{Tr} F_{V\,N\,\mathrm{eff}\,0\,\mu\nu} \otimes F_{V\,N\,\mathrm{eff}\,0}{}^{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u + 4\sqrt{m_u m_d} + 4m_d\right). \quad (8.9)$$

These are now identical with (5.1) and (5.2), which then led in (5.8) and (5.9) to the missing mass average $\frac{1}{2}\left(\Delta_P+\Delta_N\right)=8.714\,9941\,\text{MeV}$ at the empirical peak in the nuclear binding curve of Figure 2 and the 99.9710% match to the ^{56}Fe binding energy and an understanding of how this relates to quark confinement and nuclear binding and to the toolkit masses m_u, m_d, $\sqrt{m_u m_d}$ and the foregoing divided by $\left(2\pi\right)^{\frac{3}{2}}$. This then exploded into the plethora of empirical matches enumerated in section 6 culminating in the neutron minus proton mass difference in (3.2) which was then elevated into a primary relationship and used in combination with (3.1) to deduce the very precise up and down quark masses (3.3) and (3.4). And this further led once the Fermi vev v_F and the CKM mixing matrix are brought to bear, to the proton and neutron masses themselves *within all experimental errors*. So it is abundantly clear that (8.8) and (8.9) can be connected tightly with and indeed are the springboard to a whole wealth of nuclear energy data, and thus are empirically-accurate relationships to high degrees of precision. But there is only one problem: to get from (8.6) and (8.7) to the empirically-validated (8.8) and (8.9) we employed $\theta=\pi/4=45°$. But from the definitions (8.1) and (8.2) and the quark masses (3.3) and (3.4) which are one of the consequences of (8.8) and (8.9), we found that θ = 24.381 777 8°, not 45°. So what do we do?

We *defined* θ in (8.1) in a manner which ensured based on the current quark masses (3.3) and (3.4) that it would be equal to θ = 24.381 777 8°. But as we see from (8.8) and (8.9) and all the development in sections 5 and 6, it is $\theta=\pi/4=45°$ which in fact matches the empirical data. So if θ so-defined *does not match* the empirical data, but if we also now know that the up and down quark masses *do mix* over a circle with a hypotenuse radius $m_\zeta=\sqrt{{m_u}^2+{m_d}^2}=5.386\,90110\,\text{MeV}$ and that $\sqrt{m_u m_d}$ *is in general multiplied by the factor* $\left(\sin\theta\cos\theta\right)^{\frac{3}{2}}$ which specializes to $\left(\sin\theta\cos\theta\right)^{\frac{3}{2}}=1/2^{\frac{3}{2}}$ for $\theta=\pi/4=45°$, then that means that we need to retain the mass mixing over the circle with mass radius m_ζ but change (rotate) the definition of our angle to match the empirical data. That is, the empirical data suggests that we are correct that there is a mixing of the up and down masses via a mixing angle, but are incorrect about how we defined this angle in (8.1). So we now need to redefine our angle to match the empirical data. How?

In addition to θ, let us now introduce a new angle ϕ, defined such $\phi=0$ when the current quark masses are (3.3) and (3.4). That is, we define $\phi\equiv 0$ to be the mixing angle associated with the $Q=0$ current quark masses (3.3) and (3.4), which we now denote by $m_u\left(0\right)$ and $m_d\left(0\right)$ to indicate Q=0. So likewise by implication, $\phi=0$ is the associated angle for all of the empirical data developed and enumerated in sections 3 through 6. Then, because (8.2) and (8.3) teach that there *is* a rotation occurring between the up and down quark masses which maintains a $m_\zeta=5.386\,9011\,\text{MeV}$ hypotenuse, we shall define ϕ in terms of the Q=0 up and down current quark masses by way of the mixing relationship:

$$\begin{pmatrix} m'_u \\ m'_d \end{pmatrix}\equiv\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} m_u\left(0\right) \\ m_d\left(0\right) \end{pmatrix}. \tag{8.10}$$

As specified, for $\phi=0$ this definition produces $m'_u = m_u$ and $m'_d = m_d$ which are also the Q=0 quark masses. *This now replaces the definition of θ in (8.1), which we now withdraw in favor of (8.10).* There is, of course, still a rotation between the quark masses of the exact same form produced by (8.1), and $m_\zeta = 5.386\,9011\,\text{MeV}$ is still maintained as the hypotenuse of rotation. But we are no longer tied to a tan θ = 0.453 236 693 and θ = 24.381 777 8° which is a mismatch with the empirical data. In fact, as we shall shortly elaborate after some further mathematical development, it seems that both θ and ϕ need to be understood not as fixed angles, but as *variable angles with run with Q* , i.e., as $\theta(Q)$ and $\phi(Q)$, which thus help to specify the behaviors of *all* of the empirical data previously developed as a running function of Q for Q>0.

Now, with the definitions (8.1) and thus the constraint θ = 24.381 777 8° no longer in force, we revert to (8.6) and (8.7) keeping in mind that $\theta=\pi/4=45°$ leads to (8.8) and (8.9) and many correct empirical matches. So we now define $\theta \equiv \pi/4+\phi$ as the general relationship between θ and ϕ in each of (8.6) and (8.7), which is to say, we simply define ϕ to be equal to θ less 45 degrees. Via basic trigonometric angle addition formulae we find that $\sin(\pi/4+\phi)=\tfrac{1}{\sqrt{2}}(\cos\phi+\sin\phi)$ and $\cos(\pi/4+\phi)=\tfrac{1}{\sqrt{2}}(\cos\phi-\sin\phi)$ and therefore that $\sin\theta\cos\theta=\sin(\pi/4+\phi)\cos(\pi/4+\phi)=\tfrac{1}{2}(\cos^2\phi-\sin^2\phi)$. Consequently, we may use $(\sin\theta\cos\theta)^{\frac{3}{2}}=\left(1/2^{\frac{3}{2}}\right)(\cos^2\phi-\sin^2\phi)^{\frac{3}{2}}$ in (8.6) and (8.7) to write:

$$E_{V\,P\text{eff}\,0}=\iiint d^3x\,\tfrac{1}{2}\,\text{Tr}F_{V\,P\text{eff}\,0\mu\nu}\otimes F_{V\,P\text{eff}\,0}{}^{\mu\nu}=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d+4\sqrt{m_u m_d}\,(\cos^2\phi-\sin^2\phi)^{\frac{3}{2}}+4m_u\right), \tag{8.11}$$

$$E_{V\,N\text{eff}\,0}=\iiint d^3x\,\tfrac{1}{2}\,\text{Tr}F_{V\,N\text{eff}\,0\mu\nu}\otimes F_{V\,N\text{eff}\,0}{}^{\mu\nu}=\frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u+4\sqrt{m_u m_d}\,(\cos^2\phi-\sin^2\phi)^{\frac{3}{2}}+4m_d\right). \tag{8.12}$$

Here the empirically-supported (8.8) and (8.9) are more transparently visible, and when $\phi=0$, these will reduce identically to (8.8) and (8.9), by design.

Now that we have simply used a different angle ϕ rotated clockwise by 45° from θ in the formulae for $E_{V\,P\text{eff}\,0}$ and $E_{V\,N\text{eff}\,0}$ to translate (8.6) and (8.7) into the more-transparent (8.11) and (8.12), we could, if we wish, go back to reintroduce the withdrawn definition (8.1) slightly differently, by defining yet a third angle η in the form of

$$m_d\sin\eta\equiv m_u\cos\eta\equiv m_1\,, \tag{8.13}$$

with the consequence that $\tan\eta=m_u/m_d$ and η = 24.381 777 8°, compare after (8.3). This η is a different angle from $\theta\equiv\pi/4+\phi$, and it does specify the empirical m_u/m_d ratio for the Q=0 up and down current quark masses. Thus:

$$\frac{m_u m_d}{m_u{}^2 + m_d{}^2} = \frac{m_u m_d}{m_\zeta{}^2} = \sin\eta\cos\eta \tag{8.14}$$

now replaces (8.2), and η = 24.381 777 8° which is the magnitude previously assigned to θ from the initial, now replaced, definition (8.1).

Then, to see how this η definition transforms as function of θ we would transform $m_d \sin\eta \equiv m_u \cos\eta \equiv m_1$ to $m'_d \sin\eta' \equiv m'_u \cos\eta' \equiv m'_1$ and use (8.10) to substitute m'_u, and m'_d. To relate back to the redefined angle θ we may then also use $\phi = \theta - \pi/4$, apply the angle difference identities and consolidate. All this teaches that:

$$\begin{aligned} &m_d \sin\eta = m_u \cos\eta \Rightarrow m'_d \sin\eta' = m'_u \cos\eta' \\ &= (m_d \cos\phi - m_u \sin\phi)\sin\eta' = (m_u \cos\phi + m_d \sin\phi)\cos\eta' \\ &= \tfrac{1}{\sqrt{2}}\big((m_d - m_u)\sin\theta + (m_d + m_u)\cos\theta\big)\sin\eta' = \tfrac{1}{\sqrt{2}}\big((m_d + m_u)\sin\theta - (m_d - m_u)\cos\theta\big)\cos\eta' \end{aligned} \tag{8.15}$$

Therefore, the mass ratio angle η transforms $\eta \to \eta'$ with changing ϕ and θ and so also runs with Q according to:

$$\tan\eta = \frac{m_u}{m_d} \to \tan\eta' = \frac{m'_u}{m'_d} = \frac{m_u \cos\phi + m_d \sin\phi}{m_d \cos\phi - m_u \sin\phi} = \frac{(m_u - m_d)\cos\theta + (m_d + m_u)\sin\theta}{(m_d + m_u)\cos\theta + (m_d - m_u)\sin\theta} \tag{8.16}$$

Finally, to complete this development so we may turn back from mathematics to physics, we may also use (8.10) in (8.11) and (8.12) to represent the transformation of the proton and neutron energies, $E_{VP\text{eff}\,0}(0) \to E'_{VP\text{eff}\,0}$ and $E_{VN\text{eff}\,0}(0) \to E'_{VN\text{eff}\,0}$, noting that in the unprimed Q=0 state, $\phi = 0$ so $\cos^2\phi - \sin^2\phi = 1$:

$$\begin{aligned} &E_{VP\text{eff}\,0}(0) = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d}\left(\cos^2\phi - \sin^2\phi\right)^{\frac{3}{2}} + 4m_u\right) \\ &\to E'_{VP\text{eff}\,0} = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m'_d + 4\sqrt{m'_u m'_d}\left(\cos^2\phi - \sin^2\phi\right)^{\frac{3}{2}} + 4m'_u\right) \\ &= \frac{1}{(2\pi)^{\frac{3}{2}}}\left(\begin{aligned} &m_d(\cos\phi + 4\sin\phi) + 4\sqrt{m_u m_d\left(\cos^2\phi - \sin^2\phi\right) + \left(m_d{}^2 - m_u{}^2\right)\sin\phi\cos\phi}\left(\cos^2\phi - \sin^2\phi\right)^{\frac{3}{2}} \\ &+ m_u(4\cos\phi - \sin\phi) \end{aligned}\right) \end{aligned} \quad ,(8.17)$$

$$E_{V\,N\mathrm{eff}\,0}\left(0\right)=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_u+4\sqrt{m_u m_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m_d\right)$$
$$\rightarrow E'_{V\,N\mathrm{eff}\,0}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}}\left(m'_u+4\sqrt{m'_u m'_d}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}}+4m'_d\right) \qquad .(8.18)$$
$$=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}}\left(\begin{array}{l} m_u\left(\cos\phi-4\sin\phi\right)+4\sqrt{m_u m_d\left(\cos^2\phi-\sin^2\phi\right)+\left(m_d{}^2-m_u{}^2\right)\sin\phi\cos\phi}\left(\cos^2\phi-\sin^2\phi\right)^{\frac{3}{2}} \\ +m_d\left(4\cos\phi+\sin\phi\right) \end{array}\right)$$

Similarly, we may also examine how the electron rest mass $m_e=E_\Delta$ in (7.18) a.k.a. (5.3) a.k.a. the primary relationship (3.1) transforms $m_e\rightarrow m'_e$ with ϕ. Here, we just use (8.10) in (7.18):

$$m_e\left(0\right)=\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_d-m_u\right)\rightarrow m'_e=\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}\left(m'_d-m'_u\right)=\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_d\left(\cos\phi-\sin\phi\right)-m_u\left(\sin\phi+\cos\phi\right)\right) \qquad (8.19)$$

So now we can finally go directly to the relationships (5.1), (5.2) and (3.1) which were the springboard for all of the other empirical connections outlined earlier. We start with m_u and m_d which by definition are the $Q=0$ quark masses which also by the definition (8.10) correspond to $\phi=0$. So we first ask: what happens when we set $\phi=0$? By (8.10) $m'_u=m_u$ and $m'_d=m_d$, so (8.17) through (8.19) immediately reduce to:

$$E'_{V\,P\mathrm{eff}\,0}=E_{V\,P\mathrm{eff}\,0}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_d+4\sqrt{m_u m_d}+4m_u\right), \qquad (8.20)$$
$$E'_{V\,N\mathrm{eff}\,0}=E_{V\,N\mathrm{eff}\,0}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_u+4\sqrt{m_u m_d}+4m_d\right), \qquad (8.21)$$
$$m'_e=m_e=\frac{3}{\left(2\pi\right)^{\frac{3}{2}}}\left(m_d-m_u\right). \qquad (8.22)$$

These are the foundational relationships upon which all of the empirical connections in sections 5 and 6 are based. But there is still a rotation which can occur through a non-zero angle ϕ which first appeared in (8.3) as $\theta=\pi/4+\phi$. And in the more general case, the $Q=0$ quark masses can be rotated via (8.10) through a circle with a mass hypotenuse m_ζ, the proton and neutron and electron energies transform via (8.17) through (8.19), and the mass ratio angle η transforms via (8.15) and (8.16).

All of the foregoing definitions of the angles ϕ, θ and η and the interrelationships of these angles with one another as well as with the quark masses m_u and m_d and the circle radius $m_\zeta=\sqrt{m_u{}^2+m_d{}^2}$ and the renormalization energy Q as will be discussed further momentarily, are illustrated in Figure 3 below:

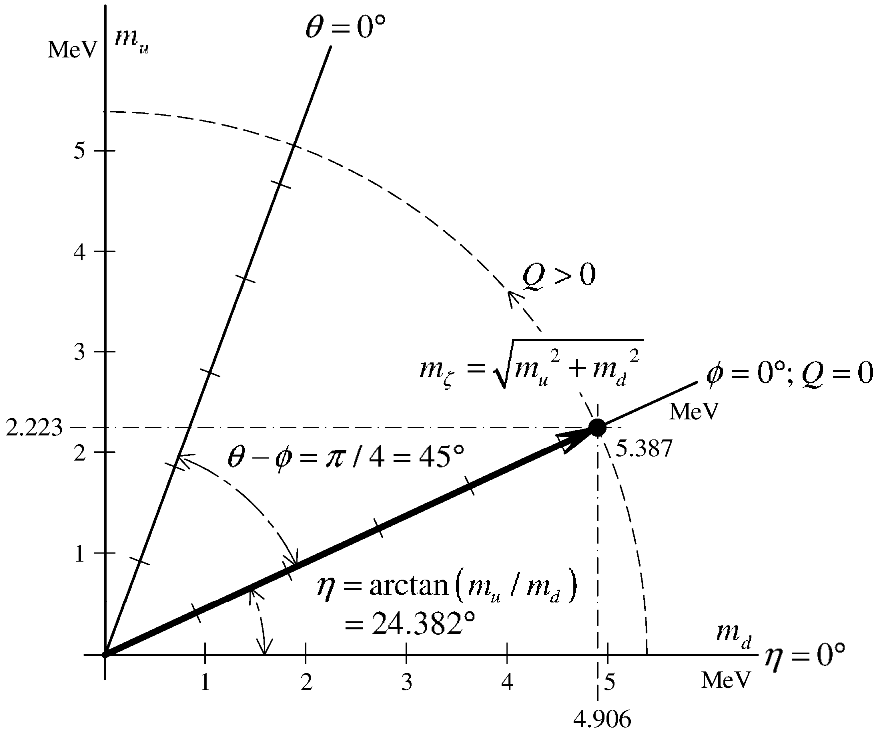

Figure 3: First Generation Quark Mass Mixing

Now let's briefly review what may be learned from (8.1) through (8.22), and then let's talk about the broader physics within which all of this fits.

By noticing that the $m_u m_d / \left(m_u^2 + m_d^2\right)$ term which first emerged in (7.19) is analogous to a like-term $\sin\theta_W \cos\theta_W = g_w g_y / \left(g_w^2 + g_y^2\right)$ which emerges in electroweak theory once we specify $g_w \sin\theta_W = g_y \cos\theta_W = e$, we are noticing that there is a similar type of mixing occurring between m_d and m_u via some angle θ as there is between g_w and g_y via the electroweak mixing angle θ_W in electroweak theory. In (8.3) we see how this mixing enters in the form of the $\left(\sin\theta\cos\theta\right)^{\frac{3}{2}}$ factor. But we see in (8.8) and (8.9) that $\theta = \pi / 4 = 45°$ is the specific angle which matches the empirical data, which contradicts the definition (8.1) from which we deduce θ = 24.381 777 8° from all of the empirical evidence reviewed earlier. So something must give, and in science, empirical validation certainly takes precedence over how we first define an angle which definition can readily be rotated to match what is observed.

So to explicitly and concretely reconcile both ends of this seeming contradiction, we separate the appearance of $\sin\theta\cos\theta$ in (8.6) and (8.7) from its connection (8.2) to the quark masses because the empirically-accurate results differ from (8.6) and (8.7) simply by a rotation in the definition of the mixing angle against the quark masses. In other words, we treat $\sin\theta\cos\theta$ as being independent of its original moorings in (8.2), and allow its relationship to the quark masses to be redefined *without at the same time touching the other quark mass terms* m_d,

$\sqrt{m_u m_d}$ *and* m_u *appearing in (8.6) and (8.7)*, so long as the redefinition takes place somewhere on the circle of radius $m_\zeta = \sqrt{m_u^{\,2} + m_d^{\,2}}$ which we now know exists mathematically. So we retain the rotations with radius m_ζ which we are tipped off about per above, and we use a new angle $\phi \equiv \theta - \pi / 4$ to define rotations from the observed current quark masses via (8.10) which then enters (8.11) and (8.12) in a fashion that is more transparent in relation to the empirical nuclear springboards (8.8) and (8.9). The original $m_u m_d / \left(m_u^{\,2} + m_d^{\,2}\right)$ which tipped us off to all of this now is redefined in (8.14) in terms of a new η = 24.381 777 8° angle.

But now let us talk about these angles themselves, because there looks to be some interesting physics here, which seems to bring us back full circle to the start of this paper when we first asked whether there was some sensible way to define Q=0 masses for the up and down current quarks when the quarks are confined and so can never be directly observed without applying a Q>0, and indeed, roughly a $Q > \Lambda_{\mathrm{QCD}}$. We established in section 4 how this could be done with the Electron, Proton and Neutron (EPN) scheme, but have never reached the question – even with Q=0 masses properly established – how these masses might run as we move up the Q scale.

When we first defined θ in (8.1), we were defining a simple ratio $\tan\theta = m_u / m_d$ of the up quark to the down quark mass at Q=0. There was nothing in this definition which might tell us how these masses run with Q. But we also saw in (8.3) and especially (8.6) and (8.7) that there is some mass mixing going on. And we know that in the two other known instances of mass mixing – via the weak mixing angle θ_W, and via the CKM quark and lepton mixing matrices which are shown in (6.13) – these angles are understood to be *running functions of Q*. So we should suspect that the angle θ in (8.6) and (8.7) is a function of Q as well, and we need to be alert for ways that this running Q might enter these equations.

The empirically-driven need to withdraw the definition (8.1) and its implied (8.2) and replace it with (8.10) solves two problems at once: It enables angles to be defined in relation to the up and down current quark masses to match up with the empirical data, and at the same time it takes advantage of the rotation first noticed from $m_u m_d / \left(m_u^{\,2} + m_d^{\,2}\right)$ to explicitly start with the EPN-defined $m_u(0)$ and $m_d(0)$ Q=0 quark masses and then rotate them to m'_u and m'_d as in (8.10). So what do these "primed" masses, and the many other "primed" nuclear energies that are functions of these masses, represent? Since there must be only one unique Q=0 mass for a quark or an electron or a proton or a neutron or a nucleus, once we now have an m'_u and m'_d which are *different* from $m_u(0)$ and $m_d(0)$, they can no longer be the Q=0 masses. So it would appear that all these can be are the $Q \neq 0$ masses, that is, these must be $m'_u = m_u(Q)$ and $m'_d = m_d(Q)$. This gives us a way to parameterize via ϕ, how these masses and indeed all of the empirical data run with the energy scale Q. This is highlighted especially by (8.16) in which we have defined η to replace what was the original role of θ right after (8.1) as the arctangent of the up-to-down mass ratio. We see in (8.16) that η is a running ratio of the quark masses, but is *not* the driving parameter as to running with Q. It is no different in this way from the electron or proton or neutron masses, or form the nuclide masses. Rather, it is $\phi(Q)$ and $\theta(Q)=\pi/4+\phi(Q)$ which are the parameters which directly drive the running. So the redefinition to match the empirical data also spawns a running ratio angle η which runs with Q but is not the underlying parameter for running, and two angles θ and ϕ which are in lockstep with one another differing by a constant $\pi/4$ which are the underlying driving parameters for the Q-running of everything else. We do not in this paper seek to ascertain how, precisely, these angles θ and ϕ run with Q. That is the subject of some different sets of inquiries. We merely wish make clear that it appears likely that they do. This apparent running behavior emerges from the redefinition of angles in (8.10), and the ability to match up the empirical data likewise emerges from this same redefinition. Two problems are simultaneously solved.

One other point must be noted as well. The fact that the up and down quark masses appear to be rotated via (8.10) based on what now appears to be some to-be-determined function of $\phi(Q)$ suggests that $m_\zeta=\sqrt{m_u{}^2+m_d{}^2}=5.386\,90110\,\text{MeV}$ is an invariant of this rotation, i.e., that $m'_\zeta=m_\zeta(Q)=m_\zeta(0)$ at all Q. And we have mentioned on several occasions in this section that m_ζ is the hypotenuse of this rotation, i.e., the radius of the circle of rotation. But we need to be very careful, because our discussion here is limited to the first quark generation which contains the up and down quarks. When we expand our view to the second and third generations and the CKM mixing of these generations, we must keep in mind that the CKM angles $\theta_{12}(Q)$, $\theta_{13}(Q)$, $\theta_{23}(Q)$ and phase $\delta(Q)$ are also expected to run with Q, and *can also shift mass from one generation to another*. So if we rewrite m_ζ by $m_{\zeta 1}$ to denote that this is the mass radius / hypotenuse for the first generation rotation, one should consider the prospect that there are two other $m_{\zeta 2}$ and $m_{\zeta 3}$ radii for the second and third generation with some presently unknown relationships among all of them. (See, however, section 3 of [6] which discusses the Koide relationships which provide the best insights known to date for how to characterize the inter-generational empirical fermion masses, and relates these to matrices displayed here in (5.1) and (5.2) which are also another way to express (8.20) and (8.21).) And one should expect that as Q increases, not only does the angle η = 24.381 777 8° change, *but so too does the* $m_{\zeta 1}$ *radius*. Thus, as among the three generations, we might envision three circles of radii $m_{\zeta 1}$, $m_{\zeta 2}$ and $m_{\zeta 3}$ such that as the angles η_1, η_2 and η_3 for each generation are rotated due to changing Q, so too do the radii change, and as one or two of the radii expand, the third one compensates by contracting, all in some presently-unknown interrelationship. So these may be less circles than spirals or perhaps some mathematically conic section, which likely converge in some way at GUT and higher-Q scales.

9. Theoretical Foundations: How the Magnetic Monopoles of Yang-Mills Gauge Theory are Populated with Quarks to Reveal Baryons and Mesons

What we have detailed in sections 8 and 9 is that (7.7) for $F_{\text{eff}\,\mu\nu 0}$, which is obtained as a direct deductive consequence of the thesis that protons and neutrons and other baryons are the chromo-magnetic monopoles of Yang-Mills gauge theory, is the theoretical expression which provides the "interface" to be able to make empirical predictions. One then uses (7.7) in $E = \iiint \frac{1}{2}\text{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ to be able to deduce energies, and after a full test calculation using a Gaussian *ansatz* for an approximately-free fermion as explained after (7.15), and the discovery and interpretation of intra-generational mixing between the up and down current quark masses reviewed in section 8, one arrives at (8.20) through (8.22) which form the basis for the broad range of empirically-accurate relationships developed and enumerated in sections 5 and 6. This is how the theoretical results captured in the monopole-net-flux-effective, zero-recursion field strength $F_{\text{eff}\,\mu\nu 0}$ connect to expressions which can be used for empirical validation via certain predicted energies driven by the current quark masses. So in effect, this paper has now shown the manner in which (7.7) for $F_{\text{eff}\,\mu\nu 0}$ leads to multiple empirical concurrences with a range of nuclear energies which connections have never been known before. So now, having largely worked backwards from measurement definitions to empirical results to the theory-to-experiment interface, we come to the final question as to the theoretical origins and foundations for $F_{\text{eff}\,\mu\nu 0}$ in (7.7).

The fundamental theoretical starting point is to recognize that in classical Yang-Mills gauge theory there is inherently a non-vanishing net flux $\oiint F \neq 0$ of "magnetic fields" across closed spatial surfaces, as first communicated in [5.6] of [1] and thereafter reiterated in [3.3] of [10]. This is in contrast to electrodynamics for which $\oiint F = 0$ and so for which there is no net magnetic field flux across closed surfaces. In classical electrodynamics, electric fields terminate at an electric charge and magnetic fields are aterminal closed loops. As was initially made clear in [2.4] and [2.5] of [10], when expressed in differential forms, just as *ddA* = 0 in electrodynamics where *A* is the vector potential one-form associated in QED with photons, *DDG* = 0 in Yang-Mills theory where *G* is the Yang-Mills vector potential one-form which in QCD becomes associated with the gluons and *D* is the gauge-covariant extension of the exterior derivative. The former is an identity of differential forms geometry; the latter a Jacobian identity. So formally speaking there are still no *elementary* magnetic monopoles in Yang-Mills theory either. But there is a non-vanishing "faux" monopole density three-form $P' = -id[G,G] = -i[dG,G]$ which arises exclusively as a composite object via the non-commuting nature of Yang-Mills theory. This does not exist, i.e., it is zero by identity, in electrodynamics. (Reference [10] in present draft states that $P' = -idGG$; this is an error which will be corrected before this paper goes to formal publication.) So when expressed in the integral formulations of Gauss and Stokes, this becomes $\oiint F = -i\oiint [G,G] = -i\iiint [dG,G] \neq 0$, which is non-vanishing. This means that these magnetic field analogs which we have referred to throughout as the "chromo-magnetic monopoles" of Yang-Mill gauge theory, *do exhibit a net flux across closed surfaces*. In electrodynamics everything commutes, so the analogous expression $\oiint F = -i\oiint[A,A] = -i\iiint[dA,A] = 0$. This is why classical Yang-Mills theory gives us $\oiint F \neq 0$ while electrodynamics gives us $\oiint F = 0$.

So if one believes in Maxwell and one believes in Yang-Mills as correct, empirically-validated theories of nature, then because their logical synthesis inexorably leads to a faux magnetic charge density $P' = -id[G,G] = i[G,dG] \neq 0$ and an associated $\oiint F \neq 0$ which do not appear in Maxwell's abelian theory alone, one must believe that these non-abelian $P' \neq 0$ and $\oiint F \neq 0$ exhibit *some manifestation in the physical universe*. The only question is how these are manifest. The author's fundamental thesis is that $\iiint[dG,G] \neq 0$ manifests as baryons and $\oiint F = -i\oiint[G,G]$ manifests as the meson and energy fluxes in and out of baryons, for example, through all of the nuclear binding and fusion energies reviewed in section 6 here. It is the field strength F appearing in $\oiint F \neq 0$ which eventually becomes the $F_{\text{eff}\,\mu\nu 0}$ for which we then calculate energies $E = \iiint \frac{1}{2}\text{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$ for both the proton in (5.1) and neutron in (5.2) as well as the difference between the two which becomes the electron rest mass in (5.3). And it is from these energies that the empirical connections elaborated throughout this paper ultimately then emerge.

So now the question becomes how to "populate" these non-vanishing faux monopole entities $\oiint F = -i\oiint[G,G] = -i\iiint[dG,G] \neq 0$ with quarks and show that they manifest through baryons and mesons. Referring back to section 7 here, while a) Maxwell for *magnetic* charges and b) Yang-Mills get us to these net-flowing magnetic fields $\oiint F \neq 0$, it is a) Maxwell for *electric* charges, c) Dirac theory and d) Dirac-Fermi-Pauli Exclusion which when deductively synthesized with the foregoing, demonstrate that these magnetic monopole entities have the correct color attributes of baryons and mesons. This was originally communicated in section 5 of [1]. It was later elaborated in section 9 of [10] to establish all of the non-linear features of these monopoles and at the same time show the monopole behaviors in the abelian limit as discussed following (7.1) here. Let us now review how the composite *faux* magnetic sources $P' = -id[G,G] = -i[dG,G]$ of these magnetic fluxes $\oiint F \neq 0$ become populated in the abelian limit with exactly three fermions which have the color symmetries of quarks.

Briefly, while the classical field equation for Yang-Mills *electric* charge $*J = D*F = D*DG$ expresses the current density differential three-form $*J$ as a function of the gauge field G, namely $J(G)$, it is desirable to invert this field equation to instead express G as a function of J, i.e., as the function $G(J)$. By way of contrast, in electrodynamics the abelian equation is the three-form $*J = d*F = d*dG$, and the often- written inverse, which is the zero-recursion inverse in Yang-Mills gauge theory as noted after (7.1), is $G_{\mu 0} = \left(k_\tau k^\tau - m^2 + i\varepsilon\right)^{-1} J_\mu$. Then, by what is effectively *a <u>merger</u> of both of Maxwell's classical <u>magnetic and electric field</u>*

<u>*equations*</u> *into a single equation* one can advance $\oiint F = -i\oiint[G,G] = -i\iiint[dG,G] \neq 0$ to $\oiint F = -i\oiint\left[G(J),G(J)\right] = -i\iiint\left[dG(J),G(J)\right] \neq 0$ for the Yang-Mills non-vanishing net monopole fluxes, which we refer to as "Merged-Maxwell." But by Dirac, we know that current densities may in turn be expressed in terms of fermion wavefunctions $J(\psi)$ via $J^\sigma = \overline{\psi}\gamma^\sigma\psi$. So now we advance to $\oiint F = -i\oiint\left[G(\psi),G(\psi)\right] = -i\iiint\left[dG(\psi),G(\psi)\right] \neq 0$, *and the monopole entities contain fermions* arrived at via Merged-Maxwell, Yang-Mills and Dirac.

How many fermions? In the abelian linear limit, each faux monopole entity contains precisely *three* fermion eigenstates. At bottom, this emerges from the fact that the faux magnetic charge density $P' = -id[G,G] = i[G,dG] \neq 0$ (with $G\left(J(\psi)\right)$ after advancement) is a differential *three-form*. So if this monopole "system" contains precisely three fermion eigenstates in its linear limit, then by the Exclusion Principle, we must place these fermions into three distinct eigenstates. We then use the gauge group SU(3) to enforce Exclusion, and now the only question is what to name these distinct eigenstates, which naming is arbitrary. So we choose R, G and B, call this color, and now *the SU(3)$_C$ color group of chromodynamics naturally emerges as a corollary to merely synthesizing Merged-Maxwell, Yang-Mills, Dirac and the Exclusion Principle together all at once.* The rank-3 of the monopole three-form once populated with fermions via the inverse of the electric charge equation (again, a merging of *both* of Maxwell's equations into a single equation plus the use of Dirac theory) converts over into the dimension-3 of the chromodynamic gauge group, and SU(3)$_C$ is seen not as a fundamental theory but as a corollary theory rooted in Merged-Maxwell-Yang-Mills-Dirac-Exclusion.

Once color is assigned, as first communicated in section 5 of [1] and thereafter in section 10 of [10], the faux monopole three form P' has the $R \wedge G \wedge B$ color symmetry of a baryon and the $\mathrm{Tr}\Sigma iF_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0 = \mathrm{Tr}\Sigma\left[G_\mu, G_\nu\right]\left((0)\right)_0$ entity has the color wavefunction $\overline{R}R + \overline{G}G + \overline{B}B$ of a meson. And in equation [10.4] of [10] for $F_{\mathrm{eff}\,\mu\nu}\left((0)\right)_0$ where this $\overline{R}R + \overline{G}G + \overline{B}B$ meson wavefunction first becomes clear, reproduced earlier as equation (7.1) here (see also [5.6] of [1]), we also obtain the starting point for connecting the theory to its means of empirical confirmation by calculating the energies $E = \iiint \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}d^3x$. The very same equations (7.7) and (7.8) which reveal to us the color wavefunction $\overline{R}R + \overline{G}G + \overline{B}B$ for the mesons which flow in and out of baryons and hold together the nuclei, also give us the basis for quantitatively studying the energies which fuse and bind the nucleons into nuclei. To see this, just go to (7.7) from which we used $E = \iiint d^3x \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$ to derive energies which led to the empirical results in sections 5 and 6, and look at its $\overline{R}R + \overline{G}G + \overline{B}B$ color-neutral trace in (7.8). This is the main crossroads between theory and experiment.

The one other important finding which emerges in the process of all this, is that because of the non-linear features of Yang-Mills gauge theory, when we attempt to express G as a function of J we are unable to obtain a simple $G(J)$ *except in the abelian limit of Yang-Mills* *gauge* theory, which $G(J)$ is $G_{\mu 0} = \left(k_\tau k^\tau - m^2 + i\varepsilon\right)^{-1} J_\mu$. In general G is a function not only of J but also of itself, $G(J,G)$. So if we are looking for an expression $G(J)$ which does not self-feed via $G(J,G)$, then as first detailed in section 8 of [10], we need to treat $G(J,G)$ *recursively*. We feed $G(J,G)$ into itself as many times as we wish – anywhere from zero times to an infinite number of times – and then cut off any further feeds by setting a perturbation V to zero. Doing this "zero times" expresses the abelian limit which puts the "0" subscript in $F_{\mathrm{eff}\,\mu\nu 0}$. On the other hand, self-feeding an infinite number of times is the behavior ascribed to nature. For human beings and their computers doing non-linear analytical or numerical calculations to some acceptable level of precision, one would recurse a finite number of times, whether 1 or 2 or 5 or 10, or 100 etc. and then study those results. So this recursive approach enables us to as detailed in section 9 of [10] to describe these baryon monopoles in terms of their natural condition with infinite recursion, and to also take the abelian limit of zero recursion, as well as to do in-between calculation and analysis. The empirical connections we have developed here to nuclear binding energies are all developed from the zero-recursion limit, and their close concurrence with empirical data informs us that the observed nuclear binding and fusion energies are expressing "abelian signals" from the nucleons which need to be "decoded" as in sections 5 and 6 to teach us about the "nuclear genome." On the other hand, the complete proton and neutron masses and the constituent / contributive quark masses discussed in point 11 in section 6 tell us about all of the non-abelian "noise" which then overlays upon these abelian signals in the infinite recursion limit to exhibit the observed masses and other properties of nucleons as complete nucleons.

It will be appreciated that all of the foregoing makes use only of the *classical Yang-Mills field theory*. We have not yet discussed or resorted to *quantum* Yang-Mills field theory. But because Merged-Maxwell-Yang-Mills-Dirac-Exclusion merely implies *classical* $SU(3)_C$ and this is what gets us to the results in sections 5 and 6, this means we have not yet needed quantum but only classical chromodynamics to obtain all the these results. So while one might approach the empirical concurrences we have laid out in sections 5 and 6 here under the assumption that they cannot be obtained except by a quantum field theory, the results here reveal – perhaps surprisingly – that this is a false assumption. All of the empirical results enumerated in sections 5 and 6 are based on *classical*, not *quantum* Yang-Mills field theory. When we finally do wish to study *quantum* Yang-Mills field theory, the recursion just discussed is an *indispensable* element. For, when we finally bring Feynman-path integration into the mix as laid out in point e) near the start of section 7, we run into the long-standing *mathematical problem* of how to exactly and analytically (not numerically) calculate a path integral for a non-linear classical Yang-Mills field theory, which is a close cousin to the so-called φ^4 problem for scalar fields. As demonstrated in section 11 of [10], this recursion is the precise aspect of Yang-Mills field theory which enables us to finally solve this important mathematical problem and perform an analytically exact path integration to prove the existence of a non-trivial quantum Yang–Mills field theory on R^4 for any simple gauge group G, see the Yang-Mills and Mass Gap Problem [25] at page 6.

Once this is achieved, it is possible to obtain the quantum field equations of Yang-Mills QCD which are [13.21] of [10] and thereafter to derive the running QCD curve of Figure 1 here within all experimental errors, see section 18 and especially Figure 14 of [10]. So in the simplest terms, QCD may now be thought of as no more and no less than Merged-Maxwell-Yang-Mills-Dirac-Exclusion-Feynman, where it is Feynman via path integration that finally takes a classical chromodynamic theory which properly explains a wide range of nuclear energy data including confinement when expressed in terms of nuclear energies as in point 1 of section 5, over to a quantum QCD theory which explains the running QCD curve which is the fundamental quantum evidence of confinement. All of this combines to provide clear evidence that the non-vanishing flows $\oiint F \neq 0$ of chromo-magnetic fields across closed spatial surfaces in Yang-Mills gauge theory are in fact synonymous with the existence of baryons, including the protons and neutrons from which all of the atomic nuclei are constructed.

10. Conclusion: Hiding in Plain Sight -- A Century and a Half after Maxwell, Protons and Neutrons and other Baryons are Finally Understood to be the Chromo-Magnetic Monopoles of Yang-Mills Gauge Theory

During the century and a half since Maxwell and Heaviside first taught that there are no magnetic monopoles in electrodynamics these monopoles have been an endless source of fascination for physicists wondering whether the natural world contains magnetic monopoles in some form, and if so, what form those monopoles might take. At the same time, although Rutherford and Chadwick established the existence of protons and neutrons almost a century ago, and while protons and neutrons and their other baryon cousins have been well-characterized since, there remains to date no convincing *theoretical* explanation of *what a baryon actually is* beyond it being some confining bound state of three quarks teeming with gluons and highly-non-linear quantum interactions. To this very date, Rabi's immortal quip, "who ordered that?" remains an unanswered question for protons and neutrons.

The answer to Rabi's question is that the protons and neutrons and other baryons were ordered by a deductive synthesis of Merged-Maxwell-Yang-Mills-Dirac-Exclusion-Feynman, with "merged Maxwell" being the synthesis of Maxwell's electric and magnetic charge equations into a single equation for a baryon, and with the Exclusion Principle being the combined effort of Fermi-Dirac-Pauli. The cast of characters who placed this order, and the highly-settled and thoroughly-validated nature of the theories which they used to do so, make clear that the author's thesis that baryons are Yang-Mills chromo-magnetic monopoles is a highly conservative thesis, grounded in a synthesis of some of the most fundamental, widely-accepted and extensively-tested scientific theories. To believe and accept this thesis requires nothing more than a belief that all of these theories offer correct descriptions of nature, and a belief that when the power of mathematics is correctly applied to combine correct and well-tested input component theories, the result of that mathematical synthesis will be equally correct and should be able to withstand its own extensive testing. The empirical proof enumerated in section 6 appears to validate this belief.

So it is perhaps with a touch of irony that when future generations look back on the century and a half from Maxwell's time to the present during which scientists passionately pursued magnetic monopoles and wondered aloud if they exist in some form in the material world and how they would present themselves if they did, they may chuckle over the fact these monopoles in Yang-Mills form were mocking our inquiries and hiding in plain sight all along, as the protons and neutrons and other baryons at the heart of the material universe.

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