On quasi-normal modes, area quantization and Bohr correspondence principle

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Abstract

In Int. Journ. Mod. Phys. D 14, 181 (2005) Khriplovich verbatim claims that "the correspondence principle does not dictate any relation between the asymptotics of quasinormal modes and the spectrum of quantized black holes" and that "this belief is in conflict with simple physical arguments". In this paper we stress that Khriplovich's criticisms work only for the original proposal by Hod, while they do not work for the improvements suggested by Maggiore and recently finalized by the author and collaborators through a connection between Hawking radiation and black hole (BH) quasi-normal modes (QNMs). Thus, QNMs can be really interpreted as BH quantum levels.

1 Introduction

BH QNMs are frequencies of radial spin $j = 0, 1, 2$ for scalar, vector and gravitational perturbation respectively, obeying a time independent Schröedinger-like equation [1, 2]. Such BH modes of energy dissipation have a frequency which is allowed to be complex [1, 2]. In a remarkable paper [3], York proposed the intriguing idea to model the quantum BH in terms of BH QNMs. More recently, by using Bohr's Correspondence Principle, Hod proposed that QNMs should release information about the area quantization as QNMs are associated to absorption of particles [4, 5]. Hod's work was improved by Maggiore [6] who solved some important problems. On the other hand, as QNMs are countable frequencies, ideas on the continuous character of Hawking radiation did not agree with attempts to interpret QNMs in terms of emitted quanta, preventing to associate QNMs modes to Hawking radiation [1]. Recently, Zhang, Cai, Zhan and You [7, 8, 9, 10] and the author and collaborators [11, 12, 13, 14] observed that the non-thermal spectrum by Parikh and Wilczek [21, 22] also implies the countable character o subsequent emissions of Hawking quanta. This issue enables a natural correspondence between QNMs and Hawking radiation, permitting to interpret QNMs also in terms of emitted energies [11, 12, 13, 14].

For Schwarzschild BH and in strictly thermal approximation, QNMs are usually labelled as ω_{nl}, where n and l are the "overtone" and the angular momentum quantum numbers [1, 2, 4, 5, 6]. For each $l\geq 2$ for BH perturbations, we have a countable infinity of QNMs, labelled by n ($n = 1, 2, ...$) [1, 2, 4, 5, 6]. Working with $G = c = k_B = \hbar = \frac{1}{4\pi\epsilon_0} = 1$ (Planck units), for large n BH QNMs become independent of l having the structure [1, 2, 4, 5, 6]

$$\begin{array}{c}
\omega_n = \ln 3 \times T_H + 2\pi i(n + \frac{1}{2}) \times T_H + \mathcal{O}(n^{-\frac{1}{2}}) = \\
= \frac{\ln 3}{8\pi M} + \frac{2\pi i}{8\pi M}(n + \frac{1}{2}) + \mathcal{O}(n^{-\frac{1}{2}}).
\end{array} \tag{1}$$

In a famous paper, Bekenstein [15] showed that the area quantum of the Schwarzschild BH is $\triangle A = 8\pi$ (we recall that the *Planck distance* $l_p = 1.616 \times 10^{-33}$ cm is equal to one in Planck units). Using properties of the spectrum of Schwarzschild BH QNMs a different numerical coefficient has been found by Hod in [4, 5]. Hod's analysis started by observing that, as for the Schwarzschild BH the *horizon area* A is related to the mass through the relation $A = 16\pi M^2$, a variation $\triangle M$ in the mass generates a variation

$$\triangle A = 32\pi M \triangle M \tag{2}$$

in the area. By considering a transition from an unexcited BH to a BH with very large n, Hod [4, 5] assumed *Bohr's correspondence principle* (which states that transition frequencies at large quantum numbers should equal classical oscillation frequencies) [16, 17, 18] to be valid for large n and enabled a semiclassical description even in absence of a complete theory of quantum gravity. Hence, from Eq. (1), the minimum quantum which can be absorbed in the transition is [4, 5]

$$\triangle M = \omega = \frac{\ln 3}{8\pi M}. \tag{3}$$

This gives $\triangle A = 4 \ln 3$. The presence of e of the numerical factor $4 \ln 3$ stimulated possible connections with loop quantum gravity [19].

2 Criticisms by Khriplovich

Hod's approach has been criticized by Khriplovich [20], who claims that properties of ringing frequencies cannot be related directly to Bohr correspondence principle. Here we show that this criticism works only for the original proposal by Hod [4, 5], while they do not work for the improvements suggested by Maggiore [6] and recently finalized by the author and collaborators [11, 12, 13, 14] through a connection between Hawking radiation and BH QNMs. Let us see this issue in detail. The criticisms by Khriplovich [20] are essentially the following:

1. The exact meaning of Bohr correspondence principle is the following. In quantized systems, the energy jump $\triangle E$ between two neighbouring levels with large quantum numbers i. e. between levels with n and $n+1$, being $n \gg 1$, is related to the classical frequency ω of the system by the formula [20]
$$\triangle E = \omega. \tag{4}$$
Thus, in a semiclassical approximation with$n \gg 1$, the frequencies which corresponds to transitions between energy levels with $\triangle n \ll n$ are integer multiples of the classical frequency ω. Khriplovich concludes by claiming that contrary to the assumption by Hod [4, 5], in the discussed problem of a BH, large quantum numbers n of Bohr correspondence principle are unrelated to the asymptotics (3) of QNMs, but are quantum numbers of the BH itself.

2. The real part QNMs does not differ appreciably from its asymptotic value (3) in the whole numerically investigated range of n, starting from $n \sim 1$. Meanwhile, the imaginary part grows as $n + \frac{1}{2}$, and together with it the spectral width of QNMs (in terms of common frequencies) also increases linearly with n. In this situation, the idea that the resolution of a QNM becomes better and better with the growth of n, and that in the limit $n \to \infty$ this mode resolves an elementary edge (or site) of a quantized surface, is not reasonable.

3 Clarifications

One can easily check that criticisms in point 2 above have been well addressed by the observation by Maggiore [6], who suggested that one must take

$$\triangle M = \omega = (\omega_0)_n - (\omega_0)_{n-1}, \tag{5}$$

where $(\omega_0)_n \equiv |\omega_n|$, instead of the value (3) proposed in [4, 5]. In fact, the imaginary part becomes dominant for large n and, in turn the idea that the resolution of a QNM becomes better and better with the growth of n, and that in the limit $n \to \infty$ this mode resolves an elementary edge (or site) of a quantized surface works. We will indeed show a quantitative analysis of this important issue in the following, and this will also help to well address the criticisms of point 1. above.

Let us return on the connection between BH QNMs and Hawking radiation. Working in strictly thermal approximation, one writes down the probability of emission of Hawking quanta as [21, 22, 23]

$$\Gamma \sim \exp(-\frac{\omega}{T_H}), \tag{6}$$

being $T_H \equiv \frac{1}{8\pi M}$ is the Hawking temperature and ω the energy-frequency of the emitted radiation respectively.
The important correction by Parikh and Wilczek, due to the BH back reaction yields [21, 22]

$$\Gamma \sim \exp[-\frac{\omega}{T_H}(1-\frac{\omega}{2M})]. \tag{7}$$

This result takes into account the BH varying geometry and adds the term $\frac{\omega}{2M}$ like correction [21, 22]. We have improved the Parikh and Wilczek framework by showing that the probability of emission (7) is indeed associated to the two *non* strictly thermal distributions [24]

$$< n >_{boson}= \frac{1}{\exp\left[-4\pi n\left(M-\omega\right)\omega\right]-1}, \;\; < n >_{fermion}= \frac{1}{\exp\left[-4\pi n\left(M-\omega\right)\omega\right]+1}, \tag{8}$$

for bosons and fermions respectively.
It is well known that in various fields of physics and astrophysics the deviation of the spectrum of an emitting body from the strict black body spectrum is taken into account by introducing an *effective temperature,* which represents the temperature of a black body emitting the same total amount of radiation. The effective temperature, which is a frequency dependent quantity, can be introduced in BH physics too [11, 12, 13, 14, 24] as

$$T_E(\omega) \equiv \frac{2M}{2M-\omega}T_H = \frac{1}{4\pi(2M-\omega)}. \tag{9}$$

Defining $\beta_E(\omega) \equiv \frac{1}{T_E(\omega)}$, one rewrites eq. (7) in Boltzmann-like form as [11, 12, 13, 14, 24]

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp(-\frac{\omega}{T_E(\omega)}), \tag{10}$$

where one introduces the *effective Boltzmann factor* $\exp[-\beta_E(\omega)\omega]$ appropriate for a BH having an inverse effective temperature $T_E(\omega)$ [11, 12, 13, 14, 24]. Then, the ratio $\frac{T_E(\omega)}{T_H} = \frac{2M}{2M-\omega}$ represents the deviation of the BH radiation spectrum from the strictly thermal character [11, 12, 13, 14, 24]. In correspondence of $T_E(\omega)$ one can also introduce the *effective mass* and of the *effective horizon* [11, 12, 13, 14, 24]

$$M_E \equiv M - \frac{\omega}{2}, \;\; r_E \equiv 2M_E \tag{11}$$

of the BH *during* the emission of the particle, i.e. *during* the BH contraction phase [11, 12, 13, 14, 24]. Such quantities are average values of the mass and the horizon *before* and *after* the emission [11, 12, 13, 14, 24].
The correction to the thermal spectrum is also very important for the physical interpretation of BH QNMs, and, in turn, is very important to realize the underlying quantum gravity theory as BHs represent theoretical laboratories

for developing quantum gravity and BH QNMs are the best candidates like quantum levels [11, 12, 13, 14, 24].

In the appendix of [13] we have rigorously shown that, if one takes into account the deviation to the strictly thermal behavior of the spectrum, eq. (1) must be replaced with

$$\begin{array}{c} \omega_n = \ln 3 \times T_E(\omega_n) + 2\pi i(n + \frac{1}{2}) \times T_E(\omega_n) + \mathcal{O}(n^{-\frac{1}{2}}) = \\ = \frac{\ln 3}{4\pi[2M - (\omega_0)_n]} + \frac{2\pi i}{4\pi[2M - (\omega_0)_n]}(n + \frac{1}{2}) + \mathcal{O}(n^{-\frac{1}{2}}). \end{array} \tag{12}$$

In other words, the Hawking temperature T_H is replaced by the effective temperature T_E in eq. (1). Strictly speaking, eqs. (1) and (12) are corrected only for scalar and gravitational perturbations. On the other hand, for large n eq. (12) is well approximated by (we consider the leading term in the imaginary part of the complex frequencies)

$$\omega_n \simeq \frac{2\pi i n}{4\pi\left[2M - (\omega_0)_n\right]}, \tag{13}$$

and we have shown in the appendix of [13] that the behavior (13) also holds for $j = 1$ (vector perturbations). In complete agreement with Bohr's correspondence principle, it is trivial to adapt the analysis in [1] in the sense of the Appendix of [13] and, in turn, to show that the behavior (13) holds if j is a half-integer too. In [11, 12, 13] we have shown that the physical solution of (13), is

$$(\omega_0)_n = M - \sqrt{M^2 - \frac{n}{2}}. \tag{14}$$

Now, we clarify how the correspondence between QNMs and Hawking radiation works. One considers a BH original mass M. After an high number of emissions of Hawking quanta and eventual absorptions, because neighboring particles can, in principle be captured by the BH, the BH is at an excited level $n-1$ and its mass is $M_{n-1} \equiv M - (\omega_0)_{n-1}$ where $(\omega_0)_{n-1}$, is the absolute value of the frequency of the QNM associated to the excited level $n-1$. $(\omega_0)_{n-1}$ is interpreted as the total energy emitted at that time. The BH can further emit a Hawking quantum to jump to the subsequent level: $\triangle M_n = (\omega_0)_{n-1} - (\omega_0)_n$. Now, the BH is at an excited level n and the BH mass is

$$\begin{array}{c} M_n \equiv M - (\omega_0)_{n-1} + \triangle M_n = \\ = M - (\omega_0)_{n-1} + (\omega_0)_{n-1} - (\omega_0)_n = M - (\omega_0)_n. \end{array} \tag{15}$$

The BH can, in principle, return to the level $n-1$ by absorbing an energy $-\triangle M_n = (\omega_0)_n - (\omega_0)_{n-1}$. By using eq. (14) one gets immediately [11, 12, 13]

$$\triangle M = \omega = (\omega_0)_{n-1} - (\omega_0)_n = -f_n(M,n) \tag{16}$$

with [11, 12, 13]

$$f_n(M,n) \equiv \sqrt{M^2 - \frac{1}{2}(n-1)} - \sqrt{M^2 - \frac{n}{2}}. \tag{17}$$

One can easily check that in the limit $n \to \infty$ one gets $f_n(M, n) \to \frac{1}{4M}$. Thus, by using eq. (2) one gets immediately that for $n \to \infty$ two adjacent QNMs resolve an elementary edge (or site) of a quantized surface $\triangle A \to 8\pi$, which corresponds to the famous historical result by Bekenstein [15] and this cannot be a coincidence. Then, the quantum levels are equally spaced for both emissions and absorptions being $\frac{1}{4M}$ the jump between two adjacent levels, and this also clarify falsifies the criticism 1. by Khriplovich because in our semiclassical approximation with $n \gg 1$, the frequencies which corresponds to transitions between energy levels with $\triangle n \ll n$ are integer multiples of the classical frequency $\omega = \frac{1}{4M}$.

4 Conclusion remarks

Khriplovich [20] verbatim claimed that "the correspondence principle does not dictate any relation between the asymptotics of quasinormal modes and the spectrum of quantized black holes" and that "this belief is in conflict with simple physical arguments". In this paper we have shown that the criticisms in [20] work only for the original proposal by Hod, while they do not work for the improvements suggested by Maggiore and recently finalized by the author and collaborators through a connection between Hawking radiation and BH QNMs. Thus, QNMs can be really interpreted as BH quantum levels.

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