NEW RESEARCH ON NEUTROSOPHIC ALGEBRAIC STRUCTURES

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EAN: 9781599733135

ISBN: 978-1-59973-313-5

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PREFACE

In this book, we define several new neutrosophic algebraic structures and their related properties. The main focus of this book is to study the important class of neutrosophic rings such as neutrosophic LA-semigroup ring, neutrosophic loop ring, neutrosophic groupoid ring and so on. We also construct their generalization in each case to study these neutrosophic algebraic structures in a broader sense. The indeterminacy element " I " gives rise to a more bigger algebraic structure than the classical algebraic structures. It mainly classifies the algebraic structures in three categories: such as neutrosophic algebraic structures, strong neutrosophic algebraic structures, and classical algebraic structures respectively. This reveals the fact that a classical algebraic structure is a part of the neutrosophic algebraic structures. This opens a new way for the researchers to think in a broader way to visualize these vast neutrosophic algebraic structures. This book is a small attempt to do this job.

This book is divided in ten chapter. Chapter one is about the introduction which will help the readers in later pursuit. In chapter two, we introduce neutrosophic Left Almost Semigroups abbreviated as neutrosophic LA-semigroups with its generalization and their core properties. Chapter three is entirely focuses on the neutrosophic soluble groups and neutrosophic nilpotent groups respectively which is basically based on the series of neutrosophic subgroups. We also give generalization of neutrosophic soluble groups and neutrosophic nilpotent groups respectively in the same chapter with some of their basic properties. In chapter four, we define neutrosophic Left Almost semigroup rings abbreviated as neutrosophic LA-semigroup rings and generalizes this concept to neutrosophic N-LA-semigroup N-rings. The fifth chapter is about to using this approach and defininig neutrosophic loop rings firstly and then giving its generalization with some of their basic properties. Similarly in chapter six, we construct the neutrosophic groupoid ring and its generalization. In chapter seven, we just generalize the neutrosophic rings and neutrosophic fields (since neutrosophic rings and neutrosophic fields were defined by W. B. V. Kandasamy and F. Smarandache in [165]). In chapter eight, we presents only the generalization of neutrosophic group rings (as neutrosophic group rings were defined already in [165]). Chapter nine is also about the generalization of semigroup rings (as these notions were defined by W. B. V. Kandasamy and F. Smarandache in [165]). In the final chapter ten, we present comprehensive amount of suggested problems and exercises for the interested readers to understand these neutrosophic algebraic structures.

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Chapter No. 1

Introduction

This is the introductory chapter of the book in which we have given some basic literature about the algebraic structures and neutrosophic algebraic structures which we used.

There are nine sections in this chapter. In the first section, we give some basic definitions and notions about the LA-semigroup. In section two, the literature about neutrosophic groupoids and their generalization is discuissed. Section three is about the basic definitions and their related notions of neutrosophic loops and their generalization. In the next section four, neutrosophic semigroups and their generalization is studied. In the further section five, we give some basic material about neutrosophic groups and their generalization. In section six, we studied the fundamental concpets and properties of soluble groups and nilpotent groups. In section seven, we give some basic definitions and notions of neutrosophic rings and neutrosophic fields. Section eight is about the basic definitons of neutrosophic group rings. In section nine, we give some basic and fundamental materials of neutrosophic semigroup rings.

We now proceed to define LA-semigroup and their related properties.

1.1 Left Almost Semigroups and their Properties

In this section, we give the basic definition of left almost semigroup which is abbrivated as LA-semigroup. LA-semigroup is basically a midway structure between a groupoid and a commutative semigroup. This structure is also termed as Abel-Grassmann's groupoid which is denoted as AG-groupoid. This is a non-associative and non-commutative algebraic structure which closely resembles to a commutative semigroup. The generalization of semigroup theory is an LA-semigroup which has a wide application in collaboration with semigroup. We now define an LA-semigroup as follows.

Definition 1.1.1: A groupiod $(S,*)$ is called a left almost semigroup abbreviated as LA-semigroup if the left invertive law holds, i.e.

$$(a*b)*c=(c*b)*a \text{ for all } a,b,c\in S.$$

Similarly $(S,*)$ is called right almost semigroup denoted as RA-semigroup if the right invertive law holds, i.e.

$$a*(b*c)=c*(b*a) \text{ for all } a,b,c\in S.$$

This situation can be explained in the following example.

Example 1.1.2: Let $S=\{1,2,3\}$ be a groupoid with an operation $*$. Then S is an LA-semigroup with the following table:

*	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Table 1.

We now give some properties which holds in an LA-semigroup.

Proposition 1.1.3: In an LA-semigroup S, the medial law holds. That is

$$(ab)(cd) = (ac)(bd) \text{ for all } a,b,c,d \in S.$$

Proposition 1.1.4: In an LA-semigrup S, the following statements are equivalent:

1) $(ab)c = b(ca)$
2) $(ab)c = b(ac)$, for all $a,b,c \in S$.

Theorem 1.1.5: An LA-semigroup S is a semigroup if and only if $a(bc) = (cb)a$, for all $a,b,c \in S$.

Theorem 1.1.6: An LA-semigroup with left identity satisfies the following Law,

$$(ab)(cd)=(db)(ca) \text{ forall } a,b,c,d \in S.$$

Theorem 1.1.7: In an LA-semigroup S, the following holds,

$$a(bc)=b(ac) \text{ for all } a,b,c \in S.$$

Theorem 1.1.8: If an LA-semigroup S has a right identity, then S is a commutative semigroup.

Definition 1.1.9: Let S be an LA-semigroup and H be a proper subset of S. Then H is called sub LA-semigroup of S if $H.H \subseteq H$.

We give an example for more explaination.

Example 1.1.10: Let $S=\{1,2,3\}$ be an LA-semigroup in example 1.1.2. Let $H=\{1,3\}$ be a proper subset of S. Then clearly H is a sub LA-semigroup of S.

Definition 1.1.11: Let S be an LA-semigroup and K be a subset of S. Then K is called Left (right) ideal of S if $SK \subseteq K$, $(KS \subseteq K)$.

If K is both left and right ideal, then K is called a two sided ideal or simply an ideal of S.

Lemma 1.1.12: If K is a left ideal of an LA-semigroup S with left identity e, then aK is a left ideal of S for all $a \in S$.

Definition 1.1.13: An ideal P of an LA-semigroup S with left identity e is called prime ideal if $AB \subseteq P$ implies either $A \subseteq P$ or $B \subseteq P$, where A, B are ideals of S.

Definition 1.1.14: An LA-semigroup S is called fully prime LA-semigroup if all of its ideals are prime ideals.

Definition 1.1.15: An ideal P is called semiprime ideal if $T.T \subseteq P$ implies $T \subseteq P$ for any ideal T of S.

Definition 1.1.16: An LA-semigroup S is called fully semiprime LA-semigroup if every ideal of S is semiprime ideal.

Definition 1.1.17: An ideal R of an LA-semigroup S is called strongly irreducible ideal if for any ideals H, K of S, we have $H \cap K \subseteq R$ implies $H \subseteq R$ or $K \subseteq R$.

Proposition 1.1.18: An ideal K of an LA-semigroup S is prime ideal if and only if it is semiprime and strongly irreducible ideal of S.

Definition 1.1.20: Let S be an LA-semigroup and Q be a non-empty subset of S. Then Q is called Quasi ideal of S if $QS \cap SQ \subseteq Q$.

Theorem 1.1.21: Every left $(right)$ ideal of an LA-semigroup S is a quasi-ideal of S.

Theorem 1.1.22: Intersection of two quasi ideals of an LA-semigroup is again a quasi ideal.

Definition 1.1.23: A sub LA-semigroup B of an LA-semigroup is called bi-ideal of S if $(BS)B \subseteq B$.

Definition 1.1.24: A non-empty subset A of an LA-semigroup S is termed as generalized bi-ideal of S if $(AS)A \subseteq A$.

Definition 1.1.25: A non-empty subset L of an LA-semigroup S is called interior ideal of S if $(SL)S \subseteq L$.

Theorem 1.1.26: Every ideal of an LA-semigroup S is an interior ideal.

We now give some basic definitions and notions about neutrosophic groupoids and their generalization.

1.2 Neutrosophic Groupoids, Neutrosophic N-groupoids and their Properties

In this section, we present the basic definitions of neutrosophic groupoids, neutrosophic bigroupoids and neutrosophic N-groupoids. We also give some properties of it.

1.2 .1 Neutrosophic Groupoids

Here we give the basic definitions and notions of neutrosophic groupoids and their related structural properties.

Definition 1.2.1.1: Let G be a groupoid, the groupoid generated by G and I i.e. $G \cup I$ is denoted by $\langle G \cup I \rangle$ is defined to be a neutrosophic groupoid where I is the indeterminacy element and termed as neutrosophic element element with property $I^2 = I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I = 0$.

I^{-1}, the inverse of I is not defined and hence does not exist.

The following examples illustrate this fact.

Example 1.2.1.2: Let

$$\langle Z_4 \cup I \rangle = \begin{Bmatrix} 0,1,2,3, I, 2I, 3I, 1+I, 1+2I, 1+3I \\ 2+I, 2+2I, 2+3I, 3+I, 3+2I, 3+3I \end{Bmatrix}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b (\text{mod } 4)$ for all $a, b \in \langle Z_4 \cup I \rangle$.

Example 1.2.1.3: Let

$$\langle Z_{10} \cup I \rangle = \begin{Bmatrix} 0, 1, 2, 3, \ldots, 9, I, 2I, \ldots, 9I, \\ 1+I, 2+I, \ldots, 9+9I \end{Bmatrix}$$

be a neutrosophic groupoid where $*$ is defined on $\langle Z_{10} \cup I \rangle$ by $a * b = 3a + 2b (\text{mod } 10)$ for all $a, b \in \langle Z10 \cup I \rangle$.

Definition 1.2.1.4: Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. A proper subset P of $\langle G \cup I \rangle$ is said to be a neutrosophic subgroupoid, if P is a neutrosophic groupoid under the operation of $\langle G \cup I \rangle$.

A neutrosophic groupoid $\langle G \cup I \rangle$ is said to have a subgroupoid if $\langle G \cup I \rangle$ has a proper subset which is a groupoid under the operations of $\langle G \cup I \rangle$.

Example 1.2.1.5: Let $\langle \mathbb{Z}_{10} \cup I \rangle$ be a neutrosophic groupoid in above example 1.2.3. Then $\{0, 5, 5\mathrm{I}, 5+5\mathrm{I}\}$, and $(\mathrm{Z}_{10}, *)$ are neutrosophic subgroupoids of $\langle \mathbb{Z}_{10} \cup I \rangle$.

Theorem 1.2.1.6: Let $\langle G \cup I \rangle$ be a neutrosophic groupoid. Suppose P_1 and P_2 be any two neutrosophic subgroupoids of $\langle G \cup I \rangle$, then $P_1 \cup P_2$, the union of two neutrosophic subgroupoids in general need not be a neutrosophic subgroupoid.

Definition 1.2.1.7: Let $\langle G \cup I \rangle$ be a neutrosophic groupoid under a binary operation $*$. P be a proper subset of $\langle G \cup I \rangle$. P is said to be a neutrosophic ideal of $\langle G \cup I \rangle$ if the following conditions are satisfied.

1. P is a neutrosophic groupoid.
2. For all $p \in P$ and for all $s \in \langle G \cup I \rangle$ we have $p * s$ and $s * p$ are in P.

We now extend the concept of neutrosophic groupoid to bigroupoid and give seom basic definitions and notions about neutrosophic bigroupoid.

1.2.2 Neutrosophic Bigroupoids

In this subsection, we present some fundamental concepts of neutrosophic bigroupoid which helps the readers in the later pursuit.

Definition 1.2.2.1: Let $(BN(\mathrm{G}), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{G}), *, \circ)$ is said to be a neutrosophic bigroupoid if $BN(\mathrm{G}) = P_1 \cup P_2$ where atleast one of $(P_1, *)$ or $(P_2, \circ)$ is a neutrosophic groupoid and other is just a groupoid. P_1 and P_2 are proper subsets of $BN(\mathrm{G})$.

If both $(P_1, *)$ and $(P_2, \circ)$ in the above definition are neutrosophic groupoids then we call $(BN(\mathrm{G}), *, \circ)$ a strong neutrosophic bigroupoid. All strong neutrosophic bigroupoids are trivially neutrosophic bigroupoids.

We show this fact as in the following example.

Example 1.2.2.2: Let $\{\mathrm{B}_N(\mathrm{G}), *, \circ\}$ be a neutrosophic groupoid with $\mathrm{B}_N(\mathrm{G}) = G_1 \cup G_2$, where $G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b(\mathrm{mod}\,10); a, b \in \langle Z_{10} \cup I \rangle\}$ and

$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b(\text{mod}\, 4); a, b \in \langle Z_4 \cup I \rangle\}$.

Definition 1.2.2.3: Let $(BN(\text{G}) = \text{P}_1 \cup P, *, \circ)$ be a neutrosophic bigroupoid. A proper subset $(T, \circ, *)$ is said to be a neutrosophic subbigroupoid of $BN(\text{G})$ if

1) $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2) At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic groupoid.

Definition 1.2.2.4: Let $(BN(\text{G}) = \text{P}_1 \cup P, *, \circ)$ be a neutrosophic strong bigroupoid. A proper subset T of $BN(S)$ is called the strong neutrosophic subbigroupoid if $T = T_1 \cup T_2$ with $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and if both $(T_1, *)$ and $(T_2, \circ)$ are neutrosophic subgroupoids of $(P_1, *)$ and $(P_2, \circ)$ respectively.

We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbigroupoid, if atleast one of $(T_1, *)$ or $(T_2, \circ)$ is a groupoid then $T = T_1 \cup T_2$ is only a neutrosophic subgroupoid.

Definition 1.2.2.5: Let $(BN(\text{G}) = \text{P}_1 \cup P_2, *, \circ)$ be any neutrosophic bigroupoid. Let J be a proper subset of $BN(\text{G})$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(\text{G})$.

Definition 1.2.2.6: Let $(BN(\text{G}), *, \circ)$ be a strong neutrosophic bigroupoid where $BN(S) = \text{P}_1 \cup P_2$ with $(P_1, *)$ and $(P_2, \circ)$ be any two neutrosophic groupoids. Let J be a proper subset of $BN(\text{G})$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic groupoids P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(\text{G})$.

Union of any two neutrosophic biideals in general is not a neutrosophic biideal. This is true of neutrosophic strong biideals.

We no give the basic definitions of the generalization of neutrosophic groupoids.

1.2.3. Neutrosophic N-groupoids

Here we introduced the basic definitons and properties of neutrosophic N-groupoids.

Definition 1.2.3.1: Let $\{N(G), *_1, \ldots, *_2\}$ be a non-empty set with N-binary operations defined on it. We call $N(G)$ a neutrosophic N-groupoid (N a positive integer), if the following conditions are satisfied.

1) $N(G) = G_1 \cup \ldots \cup G_N$ where each G_i is a proper subset of $N(G)$ i.e. $G_i \subset G_j$ or $G_j \subset G_i$ if $i \neq j$.
2) $(G_i, *_i)$ is either a neutrosophic groupoid or a groupoid for $i = 1, 2, 3, \ldots, N$.

If all the N-groupoids $(G_i, *_i)$ are neutrosophic groupoids (i.e. for $i = 1, 2, 3, \ldots, N$) then we call $N(G)$ to be a neutrosophic strong N-groupoid.

We no give example of neutrosophic N-groupoids.

Example 1.2.3.2: Let $N(G) = \{G_1 \cup G_2 \cup G_3, *_1, *_2, *_3\}$ be a neutrosophic 3-groupoid, where $G_1 = \{\langle Z_{10} \cup I \rangle \mid a * b = 2a + 3b (\text{mod} 10); a, b \in \langle Z_{10} \cup I \rangle\}$,

$G_2 = \{\langle Z_4 \cup I \rangle \mid a \circ b = 2a + b (\text{mod} 4); a, b \in \langle Z_4 \cup I \rangle\}$ and

$G_3 = \{\langle Z_{12} \cup I \rangle \mid a * b = 8a + 4b (\text{mod} 12); a, b \in \langle Z_{12} \cup I \rangle\}$.

Infact this is a strong neutrosophic 3-groupoid.

Definition 1.2.3.3: Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-groupoid. A proper subset $P = \{P_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic N-subgroupoid if $P_i = P \cap G_i, i = 1, 2, ..., N$ are subgroupiids of G_i in which atleast some of the subgroupoids are neutrosophic subgroupoids.

Definition 1.2.3.4: Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-groupoid. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic strong sub N-groupoid if each $(T_i, *_i)$ is a neutrosophic subgroupoid of $(G_i, *_i)$ for $i = 1, 2, ..., N$ where $T_i = G_i \cap T$.

If only a few of the $(T_i, *_i)$ in T are just subgroupoids of $(G_i, *_i)$, (i.e. $(T_i, *_i)$ are not neutrosophic subgroupoids then we call T to be a sub N-groupoid of $N(G)$.

Definition 1.2.3.5: Let $N(G) = \{G_1 \cup G_2 \cup .... \cup G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-groupoid. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N\}$ of $N(G)$ is said to be a neutrosophic N-subgroupoid, if the following conditions are true.

1. P is a neutrosophic sub N-groupoid of $N(G)$.
2. Each $P_i = G \cap P_i, i = 1, 2, ..., N$ is an ideal of G_i.

Then P is called or defined as the neutrosophic N-ideal of the neutrosophic N-groupoid $N(G)$.

Definition 1.2.3.6: Let $N(G) = \{G_1 \cup G_2 \cup .... G_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-groupoid. A proper subset $J = \{J_1 \cup J_2 \cup .... J_N, *_1, *_2, ..., *_N\}$ where

$J_t = J \cap G_t$ for $t = 1, 2, ..., N$ is said to be a neutrosophic strong N-ideal of $N(G)$ if the following conditions are satisfied.

1) Each it is a neutrosophic subgroupoid of $G_t, t = 1, 2, ..., N$ i.e. It is a neutrosophic strong N-subgroupoid of $N(G)$.
2) Each it is a two sided ideal of G_t for $t = 1, 2, ..., N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $N(G)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

In the next section, we give the basic concepts of neutrosophic loop and their generalization. We also give some properties of neutrosophic loops.

1.3 Neutrosophic Loops, Neutrosophic N-loops and their Properties

Here we give some basic material about neutrosophic loops.

1.3.1 Neutrosophic Loops

The definition and important notios of neutrosophic loops are presented here for the readers which will helps the readers for understanding this work easily. Neutrosophic loops are defined in the following manner.

Definition 1.3.1.1: A neutrosophic loop is generated by a loop L and I denoted by $\langle L \cup I \rangle$. A neutrosophic loop in general need not be a loop for

$I^2 = I$ and I may not have an inverse but every element in a loop has an inverse.

We now give some exaples of neutrosophic loops.

Example 1.3.1.2: Let $\langle L \cup I \rangle = \langle L_7(4) \cup I \rangle$ be a neutrosophic loop where $L_7(4)$ is a loop.

Example 1.3.1.3: Let $\langle L_5(3) \cup I \rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ be a neutrosophic loop, where $L_5(3)$ is a loop.

Definition 1.3.1.4: Let $\langle L \cup I \rangle$ be a neutrosophic loop. A proper subset $\langle P \cup I \rangle$ of $\langle L \cup I \rangle$ is called the neutrosophic subloop, if $\langle P \cup I \rangle$ is itself a neutrosophic loop under the operations of $\langle L \cup I \rangle$.

Definition 1.3.1.5: Let $(\langle L \cup I \rangle, \circ)$ be a neutrosophic loop of finite order. A proper subset P of $\langle L \cup I \rangle$ is said to be Lagrange neutrosophic subloop, if P is a neutrosophic subloop under the operation $\circ$ and $o(P) / \mathrm{o}\langle L \cup I \rangle$.

Definition 1.3.1.6: If every neutrosophic subloop of $\langle L \cup I \rangle$ is Lagrange, then we call $\langle L \cup I \rangle$ to be a Lagrange neutrosophic loop.

Definition 1.3.1.7: If $\langle L \cup I \rangle$ has no Lagrange neutrosophic subloop, then we call $\langle L \cup I \rangle$ to be a Lagrange free neutrosophic loop.

Definition 1.3.1.8: If $\langle L \cup I \rangle$ has atleast one Lagrange neutrosophic subloop, then we call $\langle L \cup I \rangle$ to be a weakly Lagrange neutrosophic loop.

1.3.2 Neutrosophic Biloops

The definition of neutrosophic biloops are presented here. It is defined as following.

Definition 1.3.2.1: Let $(\langle B \cup I \rangle, *_1, *_2)$ be a non-empty neutrosophic set with two binary operations $*_1, *_2$, $\langle B \cup I \rangle$ is a neutrosophic biloop if the following conditions are satisfied.

1. $\langle B \cup I \rangle = P_1 \cup P_2$ where P_1 and P_2 are proper subsets of $\langle B \cup I \rangle$.
2. $(P_1, *_1)$ is a neutrosophic loop.
3. $(P_2, *_2)$ is a group or a loop.

This situation can be explained in the following examples.

Example 1.3.2.2: Let $(\langle B \cup I \rangle, *_1, *_2) = (\{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\} \cup \{\text{g} : \text{g}^6 = e\}$ be a neutrosophic biloop.

Definition 1.3.2.3: Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop. A proper subset P of $\langle B \cup I \rangle$ is said to be a neutrosophic subbiloop of $\langle B \cup I \rangle$ if $(P_1, *_1, *_2)$ is itself a neutrosophic biloop under the operations of $\langle B \cup I \rangle$.

Definition 1.3.2.4: Let $B = (B_1 \cup B_2, *_1, *_2)$ be a finite neutrosophic biloop. Let $P = (\text{P}_1 \cup P_2, *_1, *_2)$ be a neutrosophic biloop. If $\text{o(P)} / \text{o(B)}$ then we call P, a Lagrange neutrosophic subbiloop of B.

Definition 1.3.2.5: If every neutrosophic subbiloop of B is Lagrange then we call B to be a Lagrange neutrosophic biloop.

Definition 1.3.2.6: If B has atleast one Lagrange neutrosophic subbiloop then we call B to be a weakly Lagrange neutrosophic biloop.

Definition 1.3.2.7: If B has no Lagrange neutrosophic subbiloops then we call B to be a Lagrange free neutrosophic biloop.

1.3.3 Neutrosophic N-loops

Here we introduced neutrosophic N-loops and their reated properties for the readers. Neutrosophic N-loops is defined as follows.

Definition 1.3.3.1: Let $S(B)=\{S(B_1)\cup S(B_2)\cup\ldots\cup S(B_n),*_1,*_2,\ldots,*_N\}$ be a non-empty neutrosophic set with N-binary operations. $S(B)$ is a neutrosophic N-loop if $S(B)=S(B_1)\cup S(B_2)\cup\ldots\cup S(B_n)$, $S(B_i)$ are proper subsets of $S(B)$ for $1\le i\le N$ and some of $S(B_i)$ are neutrosophic loops and some of the $S(B_i)$ are groups.

Definition 1.3.3.2: Let $S(B)=\{S(B_1)\cup S(B_2)\cup\ldots\cup S(B_n),*_1,*_2,\ldots,*_N\}$ be a neutrosophic N-loop. A proper subset $(P,*_1,*_2,\ldots,*_N)$ of $S(B)$ is said to be a neutrosophic sub N-loop of $S(B)$ if P itself is a neutrosophic N-loop under the operations of $S(B)$.

In the following examples, we explained this situation.

Example 1.3.3.3: Let $S(B)=\{S(B_1)\cup S(B_2)\cup S(B_3),*_1,*_2,*_3\}$ be a neutrosophic 3-loop, where $S(B_1)=\langle L_5(3)\cup I\rangle$, $S(B_2)=\{g:g^{12}=e\}$ and $S(B_3)=S_3$.

Definition 1.3.3.4: Let $(L=L_1\cup L_2\cup\ldots\cup L_N,*_1,*_2,\ldots,*_N)$ be a neutrosophic N-loop of finite order. Suppose P is a proper subset of L, which is a neutrosophic sub N -loop. If $o(P)/o(L)$ then we call P a Lagrange neutrosophic sub N -loop.

Definition 1.3.3.5: If every neutrosophic sub N -loop is Lagrange then we call L to be a Lagrange neutrosophic N -loop.

Definition 1.3.3.6: If L has atleast one Lagrange neutrosophic sub N -loop then we call L to be a weakly Lagrange neutrosophic N -loop.

Definition 1.3.3.7: If L has no Lagrange neutrosophic sub N -loop then we call L to be a Lagrange free neutrosophic N -loop.

In the next section, we give some basic material about neutrosophic semigrousps and their generalization.

1.4 Neutrosophic Semigroups, Neutrosophic N-semigroups and their Properties

The definitions and notions about neutrosophic loops are presented here.

1.4.1 Neutrosophic Semigroups

Here the definition of neutrosophic loops is given in this subsection. We also give some related properties of neutrosophic loops.

Definition 1.4.1.1: Let S be a semigroup, the semigroup generated by S and I i.e. $S \cup I$ is denoted by $\langle S \cup I \rangle$ is defined to be a neutrosophic semigroup where I is indeterminacy element and termed as neutrosophic element.

It is interesting to note that all neutrosophic semigroups contain a proper subset which is a semigroup.

This situation can be explained in the following example.

Example 1.4.1.2: Let $Z = \{$the set of positive and negative integers with zero$\}$, Z is only a semigroup under multiplication. Let $N(S) = \langle Z \cup I \rangle$ be the neutrosophic semigroup under multiplication. Clearly $Z \subset N(S)$ is a semigroup.

Definition 1.4.1.3: Let $N(S)$ be a neutrosophic semigroup. A proper subset P of $N(S)$ is said to be a neutrosophic subsemigroup, if P is a neutrosophic semigroup under the operations of $N(S)$.

A neutrosophic semigroup $N(S)$ is said to have a subsemigroup if $N(S)$ has a proper subset which is a semigroup under the operations of $N(S)$.

Theorem 1.4.1.4: Let $N(S)$ be a neutrosophic semigroup. Suppose P_1 and P_2 be any two neutrosophic subsemigroups of $N(S)$, then $P_1 \cup P_2$, the union of two neutrosophic subsemigroups in general need not be a neutrosophic subsemigroup.

Definition 1.4.1.5: A neutrosophic semigroup $N(S)$ which has an element e in $N(S)$ such that $e*s=s*e=s$ for all $s \in N(S)$, is called as a neutrosophic monoid.

Definition 1.4.1.6: Let $N(S)$ be a neutrosophic monoid under the binary operation $*$. Suppose e is the identity in $N(S)$, that is $s*e=e*s=s$ for all $s \in N(S)$. We call a proper subset P of $N(S)$ to be a neutrosophic submonoid if,

1. P is a neutrosophic semigroup under '$*$'.
2. $e \in P$, i.e. P is a monoid under '$*$'.

Definition 1.4.1.7: Let $N(S)$ be a neutrosophic semigroup under a binary operation $*$. P be a proper subset of $N(S)$. P is said to be a neutrosophic ideal of $N(S)$ if the following conditions are satisfied.

1. P is a neutrosophic semigroup.
2. For all $p \in P$ and for all $s \in N(S)$ we have $p*s$ and $s*p$ are in P.

Definition 1.4.1.8: Let $N(S)$ be a neutrosophic semigroup. P be a neutrosophic ideal of $N(S)$, P is said to be a neutrosophic cyclic ideal or neutrosophic principal ideal if P can be generated by a single element.

1.4.2 Neutrosophic Bisemigroups

In this subsection,we introduces neutrosophic bisemigroups and the related properties and notions are given.

Definition 1.4.2.1: Let $(BN(S),*,\circ)$ be a nonempty set with two binary operations $*$ and $\circ$. $(BN(S),*,\circ)$ is said to be a neutrosophic bisemigroup if $BN(S)=P_1\cup P_2$ where atleast one of $(P_1,*)$ or $(P_2,\circ)$ is a neutrosophic semigroup and other is just a semigroup. P_1 and P_2 are proper subsets of $BN(S)$.

If both $(P_1,*)$ and $(P_2,\circ)$ in the above definition are neutrosophic semigroups then we call $(BN(S),*,\circ)$ a strong neutrosophic bisemigroup.

All strong neutrosophic bisemigroups are trivially neutrosophic bisemigroups.

Lets take a look to the following example.

Example 1.4.2.2: Let $(BN(S),*,\circ)=\{0,1,2,3,I,2I,3I,S(3),*,\circ\}=(P_1,*)\cup(P_2,\circ)$ where $(P_1,*)=\{0,1,2,3,,2I,3I\}$ and $(P_2,\circ)=(S(3),\circ)$. Clearly $(P_1,*)$ is a neutrosophic semigroup under multiplication modulo 4. $(P_2,\circ)$ is just a semigroup. Thus $(BN(S),*,\circ)$ is a neutrosophic bisemigroup.

Definition 1.4.2.3: Let $(BN(S)=\mathrm{P}_1\cup P;:*,\circ)$ be a neutrosophic bisemigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic subbisemigroup of $BN(S)$ if

1. $T=T_1\cup T_2$ where $T_1=P_1\cap T$ and $T_2=P_2\cap T$ and
2. At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic semigroup.

Definition 1.4.2.4: Let $(BN(S)=\mathrm{P}_1\cup P,*,\circ)$ be a neutrosophic strong bisemigroup. A proper subset T of $BN(S)$ is called the strong neutrosophic subbisemigroup if $T=T_1\cup T_2$ with $T_1=P_1\cap T$ and $T_2=P_2\cap T$ and if both $(T_1,*)$ and $(T_2,\circ)$ are neutrosophic subsemigroups of $(P_1,*)$ and

$(P_2, \circ)$ respectively.

We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbisemigroup, if atleast one of $(T_1, *)$ or $(T_2, \circ)$ is a semigroup then $T = T_1 \cup T_2$ is only a neutrosophic subsemigroup.

Definition 1.4.2.5: Let $(BN(S) = \mathrm{P}_1 \cup P, *, \circ)$ be any neutrosophic bisemigroup. Let J be a proper subset of $BN(S)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

Definition 1.4.2.6: Let $(BN(S), *, \circ)$ be a strong neutrosophic bisemigroup where $BN(S) = \mathrm{P}_1 \cup P_2$ with $(P_1, *)$ and $(P_2, \circ)$ be any two neutrosophic semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic semigroups P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(S)$.

Union of any two neutrosophic biideals in general is not a neutrosophic biideal. This is true of neutrosophic strong biideals.

1.4.3 Neutrosophic N-semigroups

We now give the basic definition of neutrosophic N-semigroups which generalizes the concept of neutrosophic semigroups. Important terms and notions about neutrosophic N-semigroups are also given in this subsection.

Definition 1.4.3.1: Let $\{S(N), *_1, \ldots, *_2\}$ be a non empty set with N-binary operations defined on it. We call $S(N)$ a neutrosophic N-semigroup (N a positive integer) if the following conditions are satisfied.

1. $S(N) = S_1 \cup ... S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i \subset S_j$ or $S_j \subset S_i$ if $i \neq j$.
2. $(S_i, *_i)$ is either a neutrosophic semigroup or a semigroup for $i = 1, 2, 3, ..., N$.

If all the N-semigroups $(S_i, *_i)$ are neutrosophic semigroups (i.e. for $i = 1, 2, 3, ..., N$) then we call $S(N)$ to be a neutrosophic strong N-semigroup.

This can be shown in the following example.

Example 1.4.3.2: Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where
$S_1 = \{Z_{12}$, semigroup under multiplication modulo $12\}$,
$S_2 = \{1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo $4\}$, a neutrosophic semigroup.
$S_3 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication and
$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication.

Definition 1.4.3.3: Let $S(N) = \{S_1 \cup S_2 \cup .... S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic N-subsemigroup if $P_i = P \cap S_i, i = 1, 2, ..., N$ are subsemigroups of S_i in which atleast some of the subsemigroups are neutrosophic subsemigroups.

Definition 1.4.3.4: Let $S(N) = \{S_1 \cup S_2 \cup .... S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-semigroup. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N-semigroup if each $(T_i, *_i)$ is a neutrosophic subsemigroup of $(S_i, *_i)$ for $i = 1, 2, \ldots, N$ where $T_i = S_i \cap T$.

If only a few of the $(T_i, *_i)$ in T are just subsemigroups of $(S_i, *_i)$, (i.e. $(T_i, *_i)$ are not neutrosophic subsemigroups then we call T to be a sub N - semigroup of $S(N)$.

Definition 1.4.3.5: Let $S(N) = \{S_1 \cup S_2 \cup \ldots . S_N, *_1, *_2, \ldots, *_N\}$ be a neutrosophic N -semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \ldots . \cup P_N, *_1, *_2, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic N -subsemigroup, if the following conditions are true.

1. P is a neutrosophic sub N -semigroup of $S(N)$.
2. Each $P_i = S \cap P_i, i = 1, 2, \ldots, N$ is an ideal of S_i.

Then P is called or defined as the neutrosophic N -ideal of the neutrosophic N -semigroup $S(N)$.

Definition 1.4.3.6: Let $S(N) = \{S_1 \cup S_2 \cup \ldots . S_N, *_1, *_2, \ldots, *_N\}$ be a neutrosophic strong N -semigroup. A proper subset $J = \{J_1 \cup J_2 \cup \ldots . J_N, *_1, *_2, \ldots, *_N\}$ where $J_t = J \cap S_t$ for $t = 1, 2, \ldots, N$ is said to be a neutrosophic strong N -ideal of $S(N)$ if the following conditions are satisfied.

1. Each it is a neutrosophic subsemigroup of $S_t, t = 1, 2, \ldots, N$ i.e. It is a neutrosophic strong N-subsemigroup of $S(N)$.
2. Each it is a two sided ideal of S_t for $t = 1, 2, \ldots, N$.

Similarly one can define neutrosophic strong N -left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N -ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

We now introduce the important notions and definitions of neutrosophic groups, neutrosophic bigroups and neutrosophic N-groups.

1.5 Neutrosophic Groups, Neutrosophic N-groups and their Properties

In this section, we present the basic material about neutrosophic group and their generalization. These definitions are taken from [158].

1.5.1 Neutrosophic Groups

Neutrosophic groups are defined here in this subsection. Some of their core properties are also given here.

The definition of a neutrosophic group is as follows.

Definition 1.5.1.1: Let $(G,*)$ be a group . Then the neutrosophic group is generated by G and I under $*$ denoted by $N(G)=\{\langle G\cup I\rangle,*\}$.
I is called the indeterminate element with the property $I^2=I$. For an integer n , $n+I$ and nI are neutrosophic elements and $0.I=0$.
I^{-1} , the inverse of I is not defined and hence does not exist.

Next, we give examples of neutrosophic groups for further illustration.

Example 1.5.1.2: $(N(Z),+)$, $(N(Q),+)$ $(N(R),+)$ and $(N(C),+)$ are neutrosophic groups of integer, rational, real and complex numbers, respectively.

Example 1.5.1.3: Let $Z_7 = o,1,2,...,6$ be a group under addition modulo 7. $N\ G = \langle Z_7 \cup I \rangle,'+'\text{mod}\,ulo7$ is a neutrosophic group which is in fact a group. For $N\ G = a+bI : a,b \in Z_7$ is a group under ' $+$' modulo 7.

Definition 1.5.1.4: Let $N(G)$ be a neutrosophic group and H be a neutrosophic subgroup of $N(G)$. Then H is a neutrosophic normal subgroup of $N(G)$ if $xH = Hx$ for all $x \in N(G)$.

Definition 1.5.1.5: Let $N(G)$ be a neutrosophic group. Then center of $N(G)$ is denoted by $C(N(G))$ and defined as $C(N(G)) = \{x \in N(G) : ax = xa$ for all $a \in N(G)\}$.

1.5.2 Neutrosophic Bigroups and Neutrosophic N-groups

Here we give some baic concpets about neutrosophic bigroups and neutrosophic N-groups which we"ll used later in our work.

Definition 1.5.2.1: Let $B_N(G) = \{B(G_1) \cup B(G_2), *_1, *_2\}$ be a non empty subset with two binary operation on $B_N(G)$ satisfying the following conditions:

1. $B_N(G) = \{B(G_1) \cup B(G_2)\}$ where $B(G_1)$ and $B(G_2)$ are proper subsets of $B_N(G)$.

2. $(B(G_1),*_1)$ is a neutrosophic group.
3. $(B(G_2),*_2)$ is a group .

Then we define $(B_N(G),*_1,*_2)$ to be a neutrosophic bigroup.

Example 1.5.2.2: Let $B_N(G)=\{B(G_1)\cup B(G_2)\}$ where $B(G_1)=\{g / g^9=1\}$ be a cyclic group of order 9 and $B(G_2)=\{1,2,I,2I\}$ neutrosophic group under multiplication modulo 3. We call $B_N(G)$ a neutrosophic bigroup.

Example 1.5.2.3: Let $B_N(G)=\{B(G_1)\cup B(G_2)\}$ where $B(G_1)=\{1,2,3,4,I,2I,3I,4I\}$ a neutrosophic group under multiplication modulo 5 $B(G_2)=\{0,1,2,I,2I,1+I,2+I,1+2I,2+2I\}$ is a neutrosophic group under multiplication modulo 3. Clearly $B_N(G)$ is a strong neutrosophic bi group.

Definition 1.5.2.4: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. A proper subset $P=\{P_1\cup P_2,*_1,*_2\}$ is a neutrosophic subbigroup of $B_N(G)$ if the following conditions are satisfied $P=\{P_1\cup P_2,*_1,*_2\}$ is a neutrosophic bigroup under the operations $*_1,*_2$ i.e. $(P_1,*_1)$ is a neutrosophic subgroup of $(B_1,*_1)$ and $(P_2,*_2)$ is a subgroup of $(B_2,*_2)$. $P_1=P\cap B_1$ and $P_2=P\cap B_2$ are subgroups of B_1 and B_2 respectively.

If both of P_1 and P_2 are not neutrosophic then we call $P=P_1\cup P_2$ to be just a bigroup.

Definition 1.5.2.5: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. If both $B(G_1)$ and $B(G_2)$ are commutative groups then we call $B_N(G)$ to be a commutative bigroup.

Definition 1.5.2.6: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. If both $B(G_1)$ and $B(G_2)$ are cyclic, we call $B_N(G)$ a cyclic bigroup.

Definition 1.5.2.7: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic bigroup. $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ is said to be a neutrosophic normal subbigroup of $B_N(G)$ if $P(G)$ is a neutrosophic subbigroup and both $P(G_1)$ and $P(G_2)$ are normal subgroups of $B(G_1)$ and $B(G_2)$ respectively.

Definition 1.5.2.8: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be any neutrosophic bigroup. The neutrosophic bicenter of the bigroup $B_N(G)$ is denoted by $C(B_N(G))$ and is define to be $C(B_N(G))=\mathrm{C}(\mathrm{G}_1)\cup C(G_2)$, where $C(\mathrm{G}_1)$ is the center of $B(G_{1)}$ and $C(G_2)$ is the center of $B(G_2)$. If the neutrosophic bigroup is commutative, then $B_N(G)=C(B_N(G))$.

Definition 1.5.2.9: Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a nonempty set with N-binary operations defined on it. We say $\langle G\cup I\rangle$ is a strong neutrosophic N-group if the following conditions are true.

1. $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup...\cup\langle G_N\cup I\rangle$ where $\langle G_i\cup I\rangle$ are proper subsets of $\langle G\cup I\rangle$.
2. $(\langle G_i\cup I\rangle,*_i)$ is a neutrosophic group, $i=1,2,...,N$.

If in the above definition we have

a. $\langle G\cup I\rangle = G_1 \cup \langle G_2 \cup I\rangle \cup \ldots \cup \langle G_k \cup I\rangle \cup \langle G_{k+1} \cup I\rangle \cup \ldots \cup G_N$.

b. $(G_i, *_i)$ is a group for some i or $(\langle G_j \cup I\rangle, *_j)$ is a neutrosophic group for some j.

Then we call $\langle G \cup I\rangle$ to be a neutrosophic N-group.

This situation can be explained in the following example.

Example 1.5.2.10: Let $(\prec G\cup I \succ) = (\prec G_1 \cup I \succ \cup \prec G_2 \cup I \succ \cup \prec G_3 \cup I \succ \cup \prec G_4 \cup I \succ, *_1, *_2, *_3, *_4)$ be a neutrosophic 4 group where $\prec G_1 \cup I \succ = \{1,2,3,4,I,2I,3I,4I\}$ neutrosophic group under multiplication modulo 5.
$\prec G_2 \cup I \succ = \{0,1,2,I,2I,1+I,2+I,1+2I,2+2I\}$ a neutrosophic group under multiplication modulo 3, $\prec G_3 \cup I \succ = \prec Z \cup I \succ$, a neutrosophic group under addition and $\langle G_4 \cup I\rangle = \{(a,b) : a,b \in \{1,I,4,4I\}\}$, component-wise multiplication modulo $5\}$.
Hence $\langle G \cup I\rangle$ is a strong neutrosophic 4-group.

Example 1.5.2.11: Let $(\langle G \cup I\rangle = \langle G_1 \cup I\rangle \cup \langle G_2 \cup I\rangle \cup G_3 \cup G_4, *_1, *_2, *_3, *_4)$ be a neutrosophic 4-group, where $\langle G_1 \cup I\rangle = \{1,2,3,4,I,2I,3I,4I\}$ is a neutrosophic group under multiplication modulo 5. $\langle G_2 \cup I\rangle = \{0,1,I,1+I\}$ is a neutrosophic group under multiplication modulo 2. $G_3 = S_3$ and $G_4 = A_5$ the alternating group. $\langle G \cup I\rangle$ is a neutrosophic 4-group.

Definition 1.5.2.12: Let $(\langle G \cup I\rangle = \langle G_1 \cup I\rangle \cup \langle G_2 \cup I\rangle \cup \ldots \cup \langle G_N \cup I\rangle, *_1, \ldots, *_N)$ be a neutrosophic N-group. A proper subset $(P, *_1, \ldots, *_N)$ is said to be a neutrosophic sub N-group or N-subgroup of $\langle G \cup I\rangle$ if $P = (P_1 \cup \ldots \cup P_N)$ and each $(P_i, *_i)$ is a neutrosophic subgroup (subgroup) of $(G_i, *_i), 1 \le i \le N$.

Definition 1.5.2.13: Let $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup...\cup\langle G_N\cup I\rangle,*_1,...,*_N)$ be a neutrosophic N -group. Suppose $H=\{H_1\cup H_2\cup...\cup H_N,*_1,...,*_N\}$ is a sub N -groups of $\langle G\cup I\rangle$, we say H is a normal N-subgroup of $N(G)$ if each H_i $(i=1,2,...,N)$ is a normal subgroup of G_i.

Definition 1.5.2.14: Let $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup...\cup\langle G_N\cup I\rangle,*_1,...,*_N)$ be any neutrosophic N-group. The neutrosophic N-center of the N-group $G(N)$ is denoted by $C(G(N))$ and is define to be $C(G(N))=\mathrm{C}(\mathrm{G}_1)\cup C(G_2)\cup...\cup\mathrm{C}(\mathrm{G}_n)$, where $C(\mathrm{G}_1)$ is the center of $\langle G_1\cup I\rangle$ and $C(G_2)$ is the center of $\langle G_2\cup I\rangle\dots$ $C(G_n)$ is the center of $\langle G_n\cup I\rangle$.

Note: If the neutrosophic N-group is commutative, then $G(N)=C(G(N))$. Nex we give some basic and fundamental material about soluble grousp and nilpotent groups.

1.6 Soluble Groups, Nilpotent Groups and their Properties

In this section, the definition and other related notions of soluble groups and nilpotent groups are present which we have used in later pursuit.

We now proceed to define these notions.

Definition 1.6.1: Let G be a group and $H_1,H_2,...,\mathrm{H}_n$ be the subgroups of G. Then

$$1=H_0\le H_1\le H_2\le...\le H_{n-1}\le H_n=G$$

is called subgroup series of G.

We just give an example to illustrate it.

Example 1.6.2: Let $G = \mathbb{Z}$ be the group of integers. Then the following is a subgroups series of the group G.

$$1 \le 4\mathbb{Z} \le 2\mathbb{Z} \le \mathbb{Z} \le \langle \mathbb{Z} \cup I \rangle .$$

Definition 1.6.3: Let G be a group. Then

$$1 = H_0 \triangleleft H_1 \triangleleft H_2 \triangleleft \ldots \triangleleft H_{n-1} \triangleleft H_n = G$$

is called subnormal series. That is H_j is normal subgroup of H_{j+1} for all j.

In the next examples, this situation can be shown.

Example 1.6.4: Let $G = A_4$ be an alternating subgroup of the permutation group S_4. Then the following is the subnormal series of the group G.

$$1 \triangleleft C_2 \triangleleft V_4 \triangleleft A_4 .$$

Definition 1.6.5: Let

$$1 = H_0 \triangleleft H_1 \triangleleft H_2 \triangleleft \ldots \triangleleft H_{n-1} \triangleleft H_n = G$$

be a subnormal series of G. If each H_j is normal in G for all j, then this subnormal series is called normal series.

Example 1.6.6: The series in above Example 1.6.4 is an example of a normal series.

Definition 1.6.7: A normal series

$$1 = H_0 \lhd H_1 \lhd H_2 \lhd \ldots \lhd H_{n-1} \lhd H_n = G$$

is called an abelian series if the factor group H_{j+1}/H_j is an abelian group.

Example 1.6.8: Let $G = A_4$ be an alternating subgroup of the permutation group S_4. Then the following is an abelian series of the group G.

$$1 \lhd C_2 \lhd V_4 \lhd A_4.$$

Definition 1.6.9: A group G is called a soluble group if G has an abelian series.

Example 1.6.10: Let $G = A_4$ be an alternating subgroup of the permutation group S_4. Then the following is an abelian series of the group G.

$$1 \lhd C_2 \lhd V_4 \lhd A_4.$$

Thus $G = A_4$ is a souble group.

Definition 1.6.11: Let G be a soluble group. Then length of the shortest abelian series of G is called derived length.

Definition 1.6.12: Let G be a group. The series

$$1 = H_0 \lhd H_1 \lhd H_2 \lhd \ldots \lhd H_{n-1} \lhd H_n = G$$

is called central series if $H_{j+1}/H_j \subseteq Z\left(G/H_j\right)$ for all j.

Definition 1.6.13: A group G is called a nilpotent group if G has a central series.

In the further section, we discuiss some elementary definitions and notions about neutrosophic rings and neutrosophic fields respectively.

1.7 Neutrosophic Rings, Neutrosophic Fields and their Properties

In this section, we give a brief description of neutrosophic rings and neutrosophic fields.

Definition 1.7.1: Let R be a ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operation of R, where I is called the neutrosophic element with property $I^2 = I$. For an integer n, $n + I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 1.7.2: Let $\mathbb{Z}$ be the ring of integers. Then $\langle \mathbb{Z} \cup I \rangle$ is the neutrosophic ring of integers.

Definition 1.7.3: Let $\langle R \cup I \rangle$ be a neutrosophic ring. A proper subset P of $\langle R \cup I \rangle$ is called a neutosophic subring if P itself a neutrosophic ring under the operation of $\langle R \cup I \rangle$.

Thus we can easily see that every ring is a neutrosophic subring of the neutrosophic ring.

Definition 1.7.4: Let T be a non-empty set with two binary operations $*$ and $\circ$. T is said to be a pseudo neutrosophic ring if

1. T contains element of the form $a+bI$ $(a,b$ are reals and $b \neq 0$ for atleast one value$)$.
2. $(T,*)$ is an abelian group.
3. $(T,\circ)$ is a semigroup.

Definition 1.7.5: Let $\langle R \cup I \rangle$ be a neutrosophic ring. A non-empty set P of $\langle R \cup I \rangle$ is called a neutrosophic ideal of $\langle R \cup I \rangle$ if the following conditions are satisfied.

1. P is a neutrosophic subring of $\langle R \cup I \rangle$, and
2. For every $p \in P$ and $r \in \langle R \cup I \rangle$, pr and $rp \in P$.

Definition 1.7.6: Let K be a field. The neutrosophic field generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Example 1.7.7: Let $\mathbb{R}$ be the field of real numbers. Then $\langle \mathbb{R} \cup I \rangle$ is the neutrosophic field of real numbers.

Similarly $\langle \mathbb{C} \cup I \rangle$ be the neutrosophic field of complex numbers.

Definition 1.7.8: Let $K(I)$ be a neutrosophic field. A proper subset P of $K(I)$ is called a neutrosophic sufield if P itself a neutrosophic field.

Next we introduce some basic literature about neutrosophic group rings.

1.8 Neutrosophic Group Rings and their Properties

In this section, we give a brief description of neutrosophic group rings and we also give some properties of neutrosophic group rings. These notions are taken from [165].

We now proceed to define the neutrosophic group ring.

Definition 1.8.1: Let $\langle G \cup I \rangle$ be any neutrosophic group. R be any ring with 1 which is commutative or field. We define the neutrosophic group ring $R\langle G \cup I \rangle$ of the neutrosophic group $\langle G \cup I \rangle$ over the ring R as follows:

1. $R\langle G \cup I \rangle$ consists of all finite formal sum of the form $\alpha = \sum_{i=1}^{n} r_i g_i$, $n < \infty$, $r_i \in R$ and $g_i \in \langle G \cup I \rangle$ $(\alpha \in R\langle G \cup I \rangle)$.

2. Two elements $\alpha = \sum_{i=1}^{n} r_i g_i$ and $\beta = \sum_{i=1}^{m} s_i g_i$ in $R\langle G \cup I \rangle$ are equal if and only if $r_i = s_i$ and $n = m$.
3. Let $\alpha = \sum_{i=1}^{n} r_i g_i, \beta = \sum_{i=1}^{m} s_i g_i \in R\langle G \cup I \rangle$; $\alpha + \beta = \sum_{i=1}^{n} (\alpha_i + \beta_i) g_i \in R\langle G \cup I \rangle$, as $\alpha_i, \beta_i \in R$, so $\alpha_i + \beta_i \in R$ and $g_i \in \langle G \cup I \rangle$.
4. $0 = \sum_{i=1}^{n} 0 g_i$ serve as the zero of $R\langle G \cup I \rangle$.
5. Let $\alpha = \sum_{i=1}^{n} r_i g_i \in R\langle G \cup I \rangle$ then $-\alpha = \sum_{i=1}^{n} (-\alpha_i) g_i$ is such that
$$\begin{aligned} &\alpha + (-\alpha) = 0 \\ &= \sum_{i=1}^{n} (\alpha_i + (-\alpha_i)) g_i \\ &= \sum 0 g_i \end{aligned}$$
Thus we see that $R\langle G \cup I \rangle$ is an abelian group under $+$.
6. The product of two elements α, β in $R\langle G \cup I \rangle$ is follows:
Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$. Then $\alpha . \beta = \sum_{\substack{1 \le i \le n \\ 1 \le j \le m}}^{n} \alpha_i . \beta_j g_i h_j$
$$= \sum_k y_k t_k$$
Where $y_k = \sum \alpha_i \beta_j$ with $g_i h_j = t_k$, $t_k \in \langle G \cup I \rangle$ and $y_k \in R$.
Clearly $\alpha . \beta \in R\langle G \cup I \rangle$.
7. Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$ and $\gamma = \sum_{k=1}^{p} \delta_k l_k$.
Then clearly $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and $(\beta + \gamma)\alpha = \beta\alpha + \gamma\alpha$ for all $\alpha, \beta, \gamma \in R\langle G \cup I \rangle$.

Hence $R\langle G \cup I \rangle$ is a ring under the binary operations $+$ and $\bullet$.

We now give examples of a neutrosophic group ring.

Example 1.8.2: Let $\mathbb{Q}\langle G\cup I\rangle$ be a neutrosophic group ring, where $\mathbb{Q}=$ field of rationals and $\langle G\cup I\rangle=\{1,\mathrm{g},g^2,g^3,g^4,g^5,I,gI,\ldots,g^5I:g^6=1,\mathrm{I}^2=I\}$.

Definition 1.8.3: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring and let P be a proper subset of $R\langle G\cup I\rangle$. Then P is called a subneutrosophic group ring of $R\langle G\cup I\rangle$ if $P=R\langle H\cup I\rangle$ or $S\langle G\cup I\rangle$ or $T\langle H\cup I\rangle$. In $P=R\langle H\cup I\rangle$, R is a ring and $\langle H\cup I\rangle$ is a proper neutrosophic subgroup of $\langle G\cup I\rangle$ or in $S\langle G\cup I\rangle$, S is a proper subring with 1 of R and $\langle G\cup I\rangle$ is a neutrosophic group and if $P=T\langle H\cup I\rangle$, T is a subring of R with unity and $\langle H\cup I\rangle$ is a proper neutrosophic subgroup of $\langle G\cup I\rangle$.

We can easily see it in the following example.

Example 1.8.4: Let $\mathbb{Q}\langle G\cup I\rangle$ be a neutrosophic group ring, where $\mathbb{Q}=$ field of rationals and $\langle G\cup I\rangle=\{1,\mathrm{g},g^2,g^3,g^4,g^5,I,gI,\ldots,g^5I:g^6=1,\mathrm{I}^2=I\}$. Let $\langle H_1\cup I\rangle=\{1,g^3:g^6=1\}$, $\langle H_2\cup I\rangle=\{1,g^3,\mathrm{I},\mathrm{g}^3\mathrm{I}:g^6=1,I^2=\mathrm{I}\}$, $\langle H_3\cup I\rangle=\{1,g^2,\mathrm{g}^4:g^6=1,I^2=\mathrm{I}\}$ and $\langle H_4\cup I\rangle=\{1,g^2,\mathrm{g}^4,\mathrm{I},\mathrm{g}^2\mathrm{I},\mathrm{g}^4\mathrm{I}:g^6=1,I^2=\mathrm{I}\}$. Then the following are subneutrosophic group rings of $R\langle G\cup I\rangle$, where

$$\mathbb{Q}\langle H_1\cup I\rangle,\mathbb{Q}\langle H_2\cup I\rangle,$$

$$\mathbb{Q}\langle H_3\cup I\rangle,\mathbb{Q}\langle H_4\cup I\rangle.$$

Definition 1.8.5: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G\cup I\rangle$ is called a neutrosophic subring if $P=\langle S\cup I\rangle$ where S is a subring of RG or R.

Definition 1.8.6: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset T of $R\langle G\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring.

Definition 1.8.7: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G\cup I\rangle$ is called a subgroup ring if $P=SH$ where S is a subring of R and H is a subgroup of G. SH is the group ring of the subgroup H over the subring S.

Definition 1.8.8: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G\cup I\rangle$ is called a subring but P should not have the group ring structure is defined to be a subring of $R\langle G\cup I\rangle$.

Definition 1.8.9: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G\cup I\rangle$ is called a neutrosophic ideal of $R\langle G\cup I\rangle$

1. if P is a neutrosophic subring or subneutrosophic group ring of $R\langle G\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle G\cup I\rangle$, αp and $p\alpha\in P$.

Definition 1.8.10: Let $R\langle G\cup I\rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G\cup I\rangle$ is called a neutrosophic ideal of $R\langle G\cup I\rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic group ring of $R\langle G\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle G\cup I\rangle$, αp and $p\alpha\in P$.

In the next final section of this introductory chapeter, the authors introduce some fundamental concepts about neutrosophic semigroup ring.

1.9 Neutrosophic Semigroup Rings and their Properties

In this section, the definitions and notions of neutrosophic semigroup ring are presented. Neutrosophic semigroup ring is defined analogously to neutrosophic group ring. These basic concepts are taken from [165].

The definition of neutrosophic semigroup ring is as follows.

Definition 1.9.1: Let $\langle S\cup I\rangle$ be any neutrosophic semigroup. R be any ring with 1 which is commutative or field. We define the neutrosophic semigroup ring $R\langle S\cup I\rangle$ of the neutrosophic semigroup $\langle S\cup I\rangle$ over the ring R as follows:

1. $R\langle S\cup I\rangle$ consists of all finite formal sum of the form $\alpha=\sum_{i=1}^{n} r_i g_i$, $n<\infty$, $r_i\in R$ and $g_i\in\langle S\cup I\rangle$ $(\alpha\in R\langle S\cup I\rangle)$.
2. Two elements $\alpha=\sum_{i=1}^{n} r_i g_i$ and $\beta=\sum_{i=1}^{m} s_i g_i$ in $R\langle S\cup I\rangle$ are equal if and only if $r_i=s_i$ and $n=m$.

3. Let $\alpha = \sum_{i=1}^{n} r_i g_i, \beta = \sum_{i=1}^{m} s_i g_i \in R\langle S \cup I \rangle$; $\alpha + \beta = \sum_{i=1}^{n} (\alpha_i + \beta_i) g_i \in R\langle S \cup I \rangle$, as $\alpha_i, \beta_i \in R$, so $\alpha_i + \beta_i \in R$ and $g_i \in \langle S \cup I \rangle$.
4. $0 = \sum_{i=1}^{n} 0 g_i$ serve as the zero of $R\langle S \cup I \rangle$.
5. Let $\alpha = \sum_{i=1}^{n} r_i g_i \in R\langle S \cup I \rangle$ then $-\alpha = \sum_{i=1}^{n} (-\alpha_i) g_i$ is such that

$$\alpha + (-\alpha) = 0$$
$$= \sum_{i=1}^{n} (\alpha_i + (-\alpha_i)) g_i$$
$$= \sum 0 g_i$$

   Thus we see that $R\langle S \cup I \rangle$ is an abelian group under $+$.
6. The product of two elements α, β in $R\langle S \cup I \rangle$ is follows:

   Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$. Then $\alpha.\beta = \sum_{\substack{1 \le i \le n \\ 1 \le j \le m}}^{n} \alpha_i . \beta_j g_i h_j$

$$= \sum_k y_k t_k$$

   Where $y_k = \sum \alpha_i \beta_j$ with $g_i h_j = t_k$, $t_k \in \langle S \cup I \rangle$ and $y_k \in R$.
   Clearly $\alpha.\beta \in R\langle S \cup I \rangle$.
7. Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$ and $\gamma = \sum_{k=1}^{p} \delta_k l_k$.

   Then clearly $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and $(\beta + \gamma)\alpha = \beta\alpha + \gamma\alpha$ for all $\alpha, \beta, \gamma \in R\langle S \cup I \rangle$.

Hence $R\langle S \cup I \rangle$ is a ring under the binary operations $+$ and $\bullet$.

This situation can be further explained in the following example.

Example 1.9.2: Let $\mathbb{Q}\langle Z^+ \cup \{0\} \cup \{I\} \rangle$ be a neutrosophic semigroup ring, where $\mathbb{Q} =$ field of rationals and $\langle S \cup I \rangle = \langle Z^+ \cup \{0\} \cup \{I\} \rangle$ be a neutrosophic semigroup under $+$. One can easily check all the conditions of neutrosophic semigroup ring.

Definition 1.9.3: Let $R\langle S \cup I \rangle$ be a neutrosophic semigroup ring and let P be a proper subset of $R\langle S \cup I \rangle$. Then P is called a subneutrosophic semigroup ring of $R\langle S \cup I \rangle$ if $P = R\langle H \cup I \rangle$ or $Q\langle S \cup I \rangle$ or $T\langle H \cup I \rangle$. In $P = R\langle H \cup I \rangle$, R is a ring and $\langle H \cup I \rangle$ is a proper neutrosophic subsemigroup of $\langle S \cup I \rangle$ or in $Q\langle S \cup I \rangle$, Q is a proper subring with 1 of R and $\langle S \cup I \rangle$ is a neutrosophic semigroup and if $P = T\langle H \cup I \rangle$, T is a subring of R with unity and $\langle H \cup I \rangle$ is a proper neutrosophic subsemigroup of $\langle S \cup I \rangle$.

Definition 1.9.4: Let $R\langle S \cup I \rangle$ be a neutrosophic semigroup ring. A proper subset P of $R\langle S \cup I \rangle$ is called a neutrosophic subring if $P = \langle S_1 \cup I \rangle$ where S_1 is a subring of RS or R.

Definition 1.9.5: Let $R\langle S \cup I \rangle$ be a neutrosophic semigroup ring. A proper subset T of $R\langle S \cup I \rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring.

Definition 1.9.6: Let $R\langle S \cup I \rangle$ be a neutrosophic group ring. A proper subset P of $R\langle G \cup I \rangle$ is called a subgroup ring if $P = SH$ where S is a subring of R and H is a subgroup of G. SH is the group ring of the subgroup H over the subring S.

Definition 1.9.7: Let $R\langle G \cup I\rangle$ be a neutrosophic semigroup ring. A proper subset P of $R\langle S \cup I\rangle$ is called a subring but P should not have the semigroup ring structure and is defined to be a subring of $R\langle S \cup I\rangle$.

Definition 1.9.8: Let $R\langle S \cup I\rangle$ be a neutrosophic semigroup ring. A proper subset P of $R\langle S \cup I\rangle$ is called a neutrosophic ideal of $R\langle S \cup I\rangle$

1. if P is a neutrosophic subring or subneutrosophic semigroup ring of $R\langle S \cup I\rangle$.
2. For all $p \in P$ and $\alpha \in R\langle S \cup I\rangle$, αp and $p\alpha \in P$.

Definition 1.9.9: Let $R\langle S \cup I\rangle$ be a neutrosophic semigroup ring. A proper subset P of $R\langle S \cup I\rangle$ is called a neutrosophic ideal of $R\langle S \cup I\rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic semigroup ring of $R\langle S \cup I\rangle$.
2. For all $p \in P$ and $\alpha \in R\langle S \cup I\rangle$ and $\alpha p, p\alpha \in P$.

Chapter No. 2

NEUTROSOPHIC LA-SEMIGROUPS AND THEIR GENERALIZATION

In this chapter, we introduce for the first time neutrosophic LA-semigroups and their generalization. Basically we define three type of strcututures such as neutrosophic LA-semigroups, neutrosophic bi-LA-semigroups and neutrosophic N-LA-semigroups respectively. There are three sections of this chapter. In first section, we introduced neutrosophic LA-semigroups and give some of their properties. In section two, we kept neutrosophic bi-LA-semigroups with its characterization. In the third section, neutrosophic N-LA-semigroups are introduced.

2.1 Neutrosophic LA-semigroups

In this section, neutrosophic LA-semigroups are defined in a natural way. We also defined several types of neutrosophic ideals to give some characterization of neutrosophic LA-semigroups.

Definition 2.1.1: Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a + bI : a, b \in S\}$. The neutrosophic LA-semigroup is generated by S and I under the operation $*$ which is denoted as $N(S) = \{\langle S \cup I \rangle, *\}$, where I is called the neutrosophic element with property $I^2 = I$. For an integer n, $n + I$ and nI are neutrosophic elements and $0.I = 0$.

I^{-1}, the inverse of I is not defined and hence does not exist.

The following example further explained this fact.

Example 2.1.2: Let $S = \{1,2,3\}$ be an LA-semigroup with the following table:

*	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Table 2.

Then $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	1	1	1	1I	1I	1I
2	3	3	3	3I	3I	3I
3	1	1	1	1I	1I	1I
1I	1I	1I	1I	1I	1I	1I
2I	3I	3I	3I	3I	3I	3I
3I	1I	1I	1I	1I	1I	1I

Table 3.

Similarly we can define neutrosophic RA-semigroup on the same lines.

Theorem 2.1.3: Let S be an LA-semigroup and $N(S)$ be the neutrosophic LA-semigroup. Then $S \subset N(S)$.

The proof is straightforward, so left as an exercise for the readers.

Proposition 2.1.4: In a neutrosophic LA-semigroup $N(S)$, the medial law holds. In other words the following holds.

$$(ab)(cd) = (ac)(bd) \text{ for all } a,b,c,d \in N(S).$$

Proposition 2.1.5: In a neutrosophic LA-semigrup $N(S)$, the following statements are equivalent.

1) $(ab)c = b(ca)$
2) $(ab)c = b(ac)$. For all $a,b,c \in N(S)$.

The proof is easy, so left as an exercise for the readers.

Theorem 2.1.6: A neutrosophic LA-semigroup $N(S)$ is a neutrosophic semigroup if and only if $a(bc) = (cb)a$, for all $a,b,c \in N(S)$.

The ineterested readers can easily prove it.

Theorem 2.1.7: Let $N(S_1)$ and $N(S_2)$ be two neutrosophic LA-semigroups. Then their cartesian product $N(S_1) \times N(S_2)$ is also a neutrosophic LA-semigroups.

Theorem 2.1.8: Let S_1 and S_2 be two LA-semigroups. If $S_1 \times S_2$ is an LA-semigroup, then $N(S_1) \times N(S_2)$ is also a neutosophic LA-semigroup.

The proof is straightforward. Therefore the reads can prove it easily.

Definition 2.1.9: Let $N(S)$be a neutrosophic LA-semigroup. An element $e \in N(S)$ is said to be left identity if $e * s = s$ for all $s \in N(S)$. Similarly e is called right identity if $s * e = s$.

The element e is called two sided identity or simply identity if e is left as well as right identity.

This can be shown in the following example.

Example 2.1.10: Let

$$N(S) = \langle S \cup I \rangle = \{1, 2, 3, 4, 5, 1I, 2I, 3I, 4I, 5I\}$$

with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	1I	2I	3I	4I	5I
1	4	5	1	2	3	4I	5I	1I	2I	3I
2	3	4	5	1	2	3I	4I	5I	1I	2I
3	2	3	4	5	1	2I	3I	4I	5I	1I
4	1	2	3	4	5	1I	2I	3I	4I	5I
5	5	1	2	3	4	5I	1I	2I	3I	4I
1I	4I	5I	1I	2I	3I	4I	5I	1I	2I	3I
2I	3I	4I	5I	1I	2I	3I	4I	5I	1I	2I
3I	2I	3I	4I	5I	1I	2I	3I	4I	5I	1I
4I	1I	2I	3I	4I	5I	1I	2I	3I	4I	5I
5I	5I	1I	2I	3I	4I	5I	1I	2I	3I	4I

Table 4.

Proposition 2.1.11: If $N(S)$ is a neutrosophic LA-semigroup with left identity e, then it is unique.

This is obvious, so the readers can prove it easily.

Theorem 2.1.12: A neutrosophic LA-semigroup with left identity satisfies the following Law,

$$(ab)(cd) = (db)(ca) \text{ for all } a,b,c,d \in N(S).$$

The proof is straight forward, so left as an exercise for the readers.

Theorem 2.1.13: In a neutrosophic LA-semigroup $N(S)$, the following holds,

$$a(bc) = b(ac) \text{ for all } a,b,c \in N(S).$$

By doing simple calculation, one can prove it.

Theorem 2.1.14: If a neutrosophic LA-semigroup $N(S)$ has a right identity, then $N(S)$ is a commutative semigroup.

Proof: Suppose that e be the right identity of $N(S)$. By definition $ae = a$ for all $a \in N(S)$. So

$$ea = (e.e)a = (a.e)e = a \text{ for all } a \in N(S).$$

Therefore e is the two sided identity. Now let $a,b \in N(S)$, then

$$ab = (ea)b = (ba)e = ba$$

and hence $N(S)$ is commutative.

Again let $a,b,c \in N(S)$, So

$$(ab)c = (cb)a = (bc)a = a(bc)$$

and hence $N(S)$ is commutative semigroup.

Now we define the substructures of the neutrosophic LA-semigroups.

Definition 2.1.15: Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

This situation can be explained in the following example.

Example 2.1.16: Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1\},\{1,3\},\{1,1I\},\{1,3,1I,3I\}$ etc are neutrosophic sub LA-semigroups but $\{2,3,2I,3I\}$ is not neutrosophic sub LA-semigroup of $N(S)$.

Now we give some characterization of the neutrosophic sub LA-semigroups.

Theorem 2.1.17: Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$is a neutrosophic sub LA-semigroup of $N(S)$ if $N(H).N(H) \subseteq N(H)$.

Theorem 2.1.18: Let H be a sub LA-semigroup of an LA-semigroup S, then $N(H)$ is aneutrosophic sub LA-semigroup of the neutrosophic LA-semigroup $N(S)$, where $N(H) = \langle H \cup I \rangle$.

Definition 2.1.19: A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$are neutrosophic elements.

In the following example, we present the strong or pure neutrosophic sub LA-semigroup of a neutrosophic LA-semigroup.

Example 2.1.20: Let $N(S) = \langle S \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1I, 3I\}$ is a strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup of $N(S)$.

Theorem 2.1.21: All strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups are trivially neutrosophic sub LA-semigroup but the converse is not true.

For the converse of this theorem, see the following example.

Example 2.1.22: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1\},\{1,3\}$ are neutrosophic sub LA-semigroups but not strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups of $N(S)$.

We now procced to define the ideal theory of the neutrosophic LA-semigroups.

Definition 2.1.23: Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if

$$N(S)N(K)\subseteq N(K),\ \{N(K)N(S)\subseteq N(K)\}.$$

If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

See the following example of neutrosophic ideal.

Example 2.1.24: Let $S=\{1,2,3\}$ be an LA-semigroup with the following table.

*	1	2	3
1	3	3	3
2	3	3	3
3	1	3	3

Table 5.

Then the neutrosophic LA-semigroup $N(S) = \langle S \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 6.

Then clearly $N(K_1) = \{3, 3I\}$ is a neutrosophic left ideal and $N(K_2) = \{1, 3, 1I, 3I\}$ is a neutrosophic left as well as right ideal.

Lemma 2.1.25: If $N(K)$ be a neutrosophic left ideal of a neutrosophic LA-semigroup $N(S)$ with left identity e, then $aN(K)$ is a neutrosophic left ideal of $N(S)$ for all $a \in N(S)$.

The proof is satraight forward. The readers can easily prove it.

Theorem 2.1.26: $N(K)$ is a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ if K is an ideal of an LA-semigroup S, where $N(K) = \langle K \cup I \rangle$.

Definition 2.1.27: A neutrosophic ideal $N(K)$ is called strong neutrosophic ideal or pure neutrosophic ideal if all of its elements are neutrosophic elements.

In the following example, we give strong or pure neutrosophic ideal.

Fxample 2.1.28: Let $N(S)$ be a neutrosophic LA-semigroup as in example (5), Then $\{1I, 3I\}$ and $\{1I, 2I, 3I\}$ are strong neutrosophic ideals or pure neutrosophic ideals of $N(S)$.

Theorem 2.1.29: All strong neutrosophic ideals or pure neutrosophic ideals are neutrosophic ideals but the converse is not true.

To see the converse part of above theorem, let us take the following example.

Example 2.1.30: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) . Then $N(K_1)=\{2,3,2I,3I\}$ and $N(K_2)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$ but clearly these are not strong neutrosophic ideals or pure neutrosophic ideals.

We now define different types of neutrosophic ideals.

Definition 2.1.31: A neutorophic ideal $N(P)$ of a neutrosophic LA-semigroup $N(S)$ with left identity e is called prime neutrosophic ideal if $N(A)N(B)\subseteq N(P)$ implies either $N(A)\subseteq N(P)$ or $N(B)\subseteq N(P)$, where $N(A),N(B)$ are neutrosophic ideals of $N(S)$.

This can be explained in the following example.

Example 2.1.32: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(A)=\{2,3,2I,3I\}$ and $N(B)=\{1,3,1I,3I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B)\subseteq N(P)$ implies $N(A)\subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a prime neutrosophic ideal of $N(S)$.

Theorem 2.1.33: Every prime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

Theorem2.1.34: If P is a prime ideal of an LA-semigoup S, Then $N(P)$ is prime neutrosophic ideal of $N(S)$ where $N(P)=\langle P\cup I\rangle$.

Definition 2.1.35: A neutrosophic LA-semigroup $N(S)$ is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

Definition 2.1.36: A prime neutrosophic ideal $N(P)$ is called strong prime neutrosophic ideal or pure neutrosophic ideal if x is neutrosophic element for all $x \in N(P)$.

See the following example for it.

Example 2.1.37: Let $N(S) = \langle S \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be as in example (5) and let $N(A) = \{2I, 3I\}$ and $N(B) = \{1I, 3I\}$ and $N(P) = \{1I, 3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B) \subseteq N(P)$ implies $N(A) \subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a strong prime neutrosophic ideal or pure neutrosophic ideal of $N(S)$.

Theorem 2.1.38: Every prime strong neutrosophic ideal or pure neutrosophic ideal is neutrosophic ideal but the converse is not true.

Theorem 2.1.39: Every prime strong neutrosophic ideal or pure neutrosophic ideal is a prime neutrosophic ideal but the converse is not true.

For converse, we take the following example.

Example 2.1.40: In example (6), $N(P)=\{1,3,1I,3I\}$ is a prime neutrosophic ideal but it is not strong neutrosophic ideal or pure neutrosophic ideal.

Definition 2.1.41: A neutrosophic ideal $N(P)$ is called semiprime neutrosophic ideal if $N(T).N(T)\subseteq N(P)$ implies $N(T)\subseteq N(P)$ for any neutrosophic ideal $N(T)$ of $N(S)$.

In the following example, we give this fact.

Example 2.1.42: Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1,1I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a semiprime neutrosophic ideal of $N(S)$.

Theorem 2.1.43: Every semiprime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

One can easily see the converse by the help of examples.

Definition 2.1.44: A neutrosophic semiprime ideal $N(P)$ is said to be strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal if every element of $N(P)$ is neutrosophic element.

This situation can be explained in the following example.

Example 2.1.45: Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1I,3I\}$ and $N(P)=\{1I,2I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal of $N(S)$.

Theorem 2.1.46: All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are trivially neutrosophic ideals but the converse is not true.

One can easily see the converse by the help of examples.

Theorem 2.1.47: All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are semiprime neutrosophic ideals but the converse is not true.

The converse can be seen easily by the help of examples.

Definition 2.1.48: A neutrosophic LA-semigroup $N(S)$ is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of $N(S)$ is semiprime neutrosophic ideal.

Definition 2.1.49: A neutrosophic ideal $N(R)$ of a neutrosophic LA-semigroup $N(S)$ is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals $N(H),N(K)$ of $N(S)$ $N(H)\cap N(K)\subseteq N(R)$ implies $N(H)\subseteq N(R)$ or $N(K)\subseteq N(R)$.

Lets see the following example to explain this fact.

Example 2.1.50: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(H)=\{2,3,2I,3I\}$, $N(K)=\{1I,3I\}$ and $N(R)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(H)\cap N(K)\subseteq N(R)$ implies $N(K)\subseteq N(R)$ but $N(H)$ is not contained in $N(R)$. Hence $N(R)$ is a strong irreducible neutrosophic ideal of $N(S)$.

Theorem 2.1.51: Every strongly irreducible neutrosohic ideal is a neutrosophic ideal but the converse is not true.

One can easily seen the converse of this theorem by the help of examples.

Theorem 2.1.52: If R is a strong irreducible neutrosophic ideal of an LA-semigoup S. Then $N(I)$ is a strong irreducible neutrosophic ideal of $N(S)$ where $N(R)=\langle R\cup I\rangle$.

Proposition 2.1.53: A neutrosophic ideal $N(I)$ of a neutrosophic LA-semigroup $N(S)$ is prime neutrosophic ideal if and only if it is semiprime and strongly irreducible neutrosophic ideal of $N(S)$.

The proof of this proposition is straightforward, so left as an exercise for the interested readers.

Definition 2.1.54: Let $N(S)$ be a neutrosophic ideal and $N(Q)$ be a non-empty subset of $N(S)$. Then $N(Q)$ is called quasi neutrosophic ideal of $N(S)$ if

$$N(Q)N(S) \cap N(S)N(Q) \subseteq N(Q).$$

In the following example, we show the quasi neutrosophic ideal of a neutrosophic LA-semigroup.

Example 2.1.55: Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be as in example (5) . Then $N(K) = \{3,3I\}$ be a non-empty subset of $N(S)$ and $N(S)N(K) = \{3,3I\}$, $N(K)N(S) = \{1,3,1I,3I\}$ and their intersection is $\{3,3I\} \subseteq N(K)$. Thus clearly $N(K)$ is quasi neutrosophic ideal of $N(S)$.

Theorem 2.1.56: Every left $(right)$ neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is a quasi neutrosophic ideal of $N(S)$.

Proof: Let $N(Q)$ be a left neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then $N(S)N(Q) \subseteq N(Q)$ and so $N(S)N(Q) \cap N(Q)N(S) \subseteq N(Q) \cap N(Q) \subseteq N(Q)$ which proves the theorem.

Theorem 2.1.57: Intersection of two quasi neutrosophic ideals of a neutrosophic LA-semigroup is again a quasi neutrosophic ideal.

The proof is straight forward, so it is left as an exercise for the readers.

Definition 2.1.58: A quasi-neutrosophic ideal $N(Q)$ of a neutrosophic LA-semigroup $N(Q)$ is called quasi-strong neutrosophic ideal or quasi-pure neutosophic ideal if all the elements of $N(Q)$ are neutrosophic elements.

Example 2.1.59: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) . Then $N(K)=\{1I,3I\}$ be a quasi-neutrosophic ideal of $N(S)$. Thus clearly $N(K)$ is a quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal of $N(S)$.

Theorem 2.1.60: Every quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal is qausi-neutrosophic ideal but the converse is not true.

We can easily establish the converse of this theorem by the help of examples.

Definition 2.1.61: A neutrosophic sub LA-semigroup $N(B)$ of a neutrosophic LA-semigroup is called bi-neutrosophic ideal of $N(S)$ if

$$\left(N(B)N(S)\right)N(B)\subseteq N(B).$$

Lets see the following example for further explaination.

Example 2.1.62: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1,3,1I,3I\}$ is a neutrosophic sub LA-semigroup of $N(S)$. Then Clearly $N(B)$ is a bi-neutrosophic ideal of $N(S)$.

Theorem 2.1.63: Let B be a bi-ideal of an LA-semigroup S, then $N(B)$ is bi-neutrosophic ideal of $N(S)$ where $N(B)=\langle B\cup I\rangle$.

Definition 2.1.64: A bi-neutrosophic ideal $N(B)$ of a neutrosophic LA-semigroup $N(S)$ is called bi-strong neutrosophic ideal or bi-pure neutrosophic ideal if every element of $N(B)$ is a neutrosophic element.

Example 2.1.65: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1I,3I\}$ is a bi-neutrosophic ideal of $N(S)$. Then Clearly $N(B)$ is a bi-strong neutrosophic ideal or bi-pure neutosophic ideal of $N(S)$.

Theorem 2.1.66: All bi-strong neutrosophic ideals or bi-pure neutrosophic ideals are bi-neutrosophic ideals but the converse is not true.

Definition 2.1.67: A non-empty subset $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is termed as generalized bi-neutrosophic ideal of $N(S)$ if

$$\left(N(A)N(S)\right)N(A) \subseteq N(A).$$

The following example illustrate this fact.

Example 2.1.68: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(A)=\{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(A)$ is a generalized bi-neutrosophic ideal of $N(S)$.

Theorem 2.1.69: Every bi-neutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-ideal but the converse is not true.

The converse is easily seen with the help of examples.

Definition 2.1.70: A generalized bi-neutrosophic ideal $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is called generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$ if all the elements of $N(A)$ are neutrosophic elements.

Example 2.1.71: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(A)=\{1I,3I\}$ is a generalized bi-neutrosophic ideal of $N(S)$. Then clearly $N(A)$ is a generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$.

Theorem 2.1.72: All generalized bi-strong neutrosophic ideals or generalized bi-pure neutrosophic ideals are generalized bi-neutrosophic ideals but the converse is not true.

Theorem 2.1.73: Every bi-strong neutrosophic ideal or bi-pure neutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal but the converse is not true.

Definition 2.1.74: A non-empty subset $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior neutrosophic ideal of $N(S)$ if

$$\big(N(S)N(L)\big)N(S) \subseteq N(L).$$

The example of an interior neutrosophic ideal is given below.

Example 2.1.75: Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(L) = \{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Theorem 2.1.76: Every neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior neutrosophic ideal.

Proof: Let $N(L)$ be a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then by definition $N(L)N(S) \subseteq N(L)$ and $N(S)N(L) \subseteq N(L)$. So clearly $\big(N(S)N(L)\big)N(S) \subseteq N(L)$ and hence $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Definition 2.1.77: An interior neutrosophic ideal $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior strong neutrosophic ideal or interior pure neutrosophic ideal if every element of $N(L)$ is a neutrosophic element.

Example 2.1.78: Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(L)=\{1I,3I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal of $N(S)$.

Theorem 2.1.79: All interior strong neutrosophic ideals or interior pure neutrosophic ideals are trivially interior neutrosophic ideals of a neutrosophic LA-semigroup $N(S)$ but the converse is not true.

We can easily establish the converse by the help of examples.

Theorem 2.1.80: Every strong neutrosophic ideal or pure neutosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal.

We now proceed on to define neutrosophic bi-LA-semigroup and give some characterization of it.

2.2 Neutrosophic Bi-LA-semigroups

In this section, we introduce neutrosophic bi-LA-semigroups. It is basically the generalization of neutrosophic LA-semigroups. We also give some properties of neutrosophic bi-LA-semigroups with the help of sufficient amount of examples.

Definition 2.2.1: Let $(BN(S),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. Then $(BN(S),*,\circ)$ is said to be a neutrosophic bi-LA-semigroup if $BN(S)=P_1\cup P_2$ where atleast one of $(P_1,*)$ or $(P_2,\circ)$ is a neutrosophic LA-semigroup and other is just an LA- semigroup. Here P_1 and P_2 are proper subsets of $BN(S)$.

Similarly we can define neutrosophic bi-RA-semigroup on the same lines.

Theorem 2.2.2: All neutrosophic bi-LA-semigroups contains the corresponding bi-LA-semigroups.

In the following examples, we showed a neutrosophic bi-LA-semigroup.

Example 2.2.3: Let $BN(S)=\{\langle S_1\cup I\rangle\cup\langle S_2\cup I\rangle\}$ be a neutrosophic bi-LA-semigroup where $\langle S_1\cup I\rangle=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 7.

Also $\langle S_2 \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 8.

Definition 2.2.4: Let $(BN(S) = P_1 \cup P;: *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T, \circ, *)$ is said to be a neutrosophic sub bi-

LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic LA-semigroup.

Example 2.2.5: $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 1. Then $P = \{1,1I\} \cup \{3,3I\}$ and $Q = \{2,2I\} \cup \{1,1I\}$ are neutrosophic sub bi-LA-semigroups of $BN(S)$.

Theorem 2.2.6: Let $BN(S)$ be a neutrosophic bi-LA-semigroup and $N(H)$ be a proper subset of $BN(S)$. Then $N(H)$ is a neutrosophic sub bi-LA-semigroup of $BN(S)$ if $N(H).N(H) \subseteq N(H)$.

Definition 2.2.7: Let $(BN(S) = \mathrm{P}_1 \cup P, *, \circ)$ be any neutrosophic bi-LA-semigroup. Let J be a proper subset of $BN(S)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

This situation can be explained in the following example.

Example 2.2.8*. Let $BN(S) = \{\langle S_1 \cup I \rangle \cup \langle S_2 \cup I \rangle\}$ be a neutrosophic bi-LA-semigroup, where $\langle S_1 \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 9.

And $\langle S_2 \cup I \rangle = \{1, 2, 3, I, 2I, 3I\}$ be another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Table 10.

Then $P = \{1, 1\text{I}, 3, 3I\} \cup \{2, 2I\}$, $Q = \{1, 3, 1I, 3I\} \cup \{2, 3, 2\text{I}, 3\text{I}\}$ are neutrosophic biideals of $BN(S)$.

Proposition 2.2.9: Every neutrosophic biideal of a neutrosophic bi-LA-semigroup is trivially a Neutrosophic sub bi-LA-semigroup but the conver is not true in general.

One can easily see the converse by the help of example.

We now proceed on to define neutrosophic strong bi-LA-semigroups.

Definition 2.2.10: If both $(P_1,*)$ and $(P_2,\circ)$ in the Definition 2.2.1 are neutrosophic strong LA-semigroups then we call $(BN(S),*,\circ)$ is a neutrosophic strong bi-LA-semigroup.

In Example 2.2.8, the neutrosophic bi-LA-semigroup is basically a neutrosophic strong bi-LA-semigroup.

Definition 2.2.11: Let $(BN(S)=\mathrm{P}_1\cup P,*,\circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic strong sub bi-LA-semigroup of $BN(S)$ if

1. $T=T_1\cup T_2$ where $T_1=P_1\cap T$ and $T_2=P_2\cap T$ and
2. $(T_1,\circ)$ and $(T_2,*)$ are neutrosophic strong LA-semigroups.

The following is an example of a neutrosophic strong sub bi-LA-semigroup.

Example 2.2.12: Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 2.2.8. Then $P=\{1\mathrm{I},3I\}\cup\{2I\}$, and $Q=\{1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\}$ are neutrosophic strong sub bi-LA-semigroup of $BN(S)$.

Theorem 2.2.13: Every neutrosophic strong sub bi-LA-semigroup is a neutrosophic sub bi-LA-semigroup.

Definition 2.2.14: Let $(BN(S),*,\circ)$ be a strong neutrosophic bi-LA-semigroup where $BN(S)=\mathrm{P}_1\cup P_2$ with $(P_1,*)$ and $(P_2,\circ)$ be any two neutrosophic LA-semigroups. Let J be a proper subset of $BN(S)$ where $I=I_1\cup I_2$ with $I_1=I\cap P_1$ and $I_2=I\cap P_2$ are neutrosophic ideals of the neutrosophic LA-semigroups P_1 and P_2 respectively. Then I is called or defined as the neutrosophic strong biideal of $BN(S)$.

Theorem 2.2.15: Every neutrosophic strong biideal is trivially a neutrosophic sub bi-LA-semigroup.

Theorem 2.2.16: Every neutrosophic strong biideal is a neutrosophic strong sub bi-LA-semigroup.

Theorem 2.2.17: Every neutrosophic strong biideal is a neutrosophic biideal.

Example 2.2.18: Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 2.2.8. Then $P=\{1\mathrm{I},3I\}\cup\{2I\}$, and $Q=\{1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\}$ are neutrosophic strong biideal of $BN(S)$.

In the next section, we finally give the generalization of neutrosophic LA-semigroup to extend the theory of neutrosophic LA-semigroup to neutrosophic N-LA-semigroup.

2.3 Neutrosophic N-LA-semigroup

In this section, we introduce for the first time neutrosophic N-LA-semigroup and give some core properties of neutrosophic N-LA-semigroups with some necessary examples.

Definition 2.3.1: Let $\{S(N),*_1,...,*_2\}$ be a non-empty set with N -binary operations defined on it. We call $S(N)$ a neutrosophic N -LA-semigroup (N a positive integer) if the following conditions are satisfied.

1. $S(N)=S_1\cup ...S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i\subset S_j$ or $S_j\subset S_i$ if $i\neq j$.
2. $(S_i,*_i)$ is either a neutrosophic LA-semigroup or an LA-semigroup

   for $i=1,2,3,...,N$.

This situation can be explained in the following example.

Example 2.3.2: Let $S(N)=\{S_1\cup S_2\cup S_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup where $S_1=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 11.

and $S_2 = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 12.

And $S_3 = \{1, 2, 3, I, 2I, 3I\}$ is another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Table 13.

Theorem 2.3.3: All neutrosophic N-LA-semigroups contains the corresponding N-LA-semigroups.

Definition 2.3.4: Let $S(N) = \{S_1 \cup S_2 \cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic N-LA-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic sub N-LA-semigroup if $P_i = P \cap S_i, i = 1, 2, ..., N$ are sub LA-semigroups of S_i in which atleast some of the sub LA-semigroups are neutrosophic sub LA-semigroups.

This situation can be see in the example listed below.

Example 2.3.5: Let $S(N) = \{S_1 \cup S_2 \cup S_3, *_1, *_2, *_3\}$ be a neutrosophic 3-LA-semigroup in above Example 2.3.2. Then clearly
$P = \{1, 1I\} \cup \{2, 3, 3I\} \cup \{2, 2I\}$, $Q = \{2, 2I\} \cup \{1, 3, 1I, 3I\} \cup \{2, 3, 2I, 3I\}$, and

$R=\{4,4I\}\cup\{1I,3I\}\cup\{2I,3I\}$ are neutrosophic sub 3-LA-semigroups of $S(N)$.

Theorem 2.3.6: Let $N(S)$ be a neutrosophic N-LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is a neutrosophic sub N-LA semigroup of $N(S)$ if $N(H).N(H)\subseteq N(H)$.

Definition 2.3.7: Let $S(N)=\{S_1\cup S_2\cup....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N-LA-semigroup. A proper subset $P=\{P_1\cup P_2\cup....\cup P_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic N-ideal, if the following conditions are true.

1. P is a neutrosophic sub N-LA-semigroup of $S(N)$.
2. Each $P_i=S\cap P_i, i=1,2,...,N$ is an ideal of S_i.

Example 2.3.8: Consider Example 2.3.2. Then $I_1=\{1,1I\}\cup\{3,3I\}\cup\{2,2I\}$, and $I_2=\{2,2I\}\cup\{1I,3I\}\cup\{2,3,3I\}$ are neutrosophic 3-ideals of $S(N)$.

Theorem 2.3.9: Every neutrosophic N-ideal is trivially a neutrosophic sub N-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

We now proceed on to define neutrosophic strong N-LA-semigroups which are of pure neutrosophic character in nature.

Definition 2.3.10: If all the N-LA-semigroups $(S_i,*_i)$ in Definition ( ) are neutrosophic strong LA-semigroups (i.e. for $i=1,2,3,...,N$) then we call $S(N)$ to be a neutrosophic strong N-LA-semigroup.

The readers can easily construct many examples of neutrosophic strong N-LA-semigroups.

Definition 2.3.11: Let $S(N)=\{S_1 \cup S_2 \cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-LA-semigroup. A proper subset $T=\{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N-LA-semigroup if each $(T_i, *_i)$ is a neutrosophic strong sub LA-semigroup of $(S_i, *_i)$ for $i=1,2,...,N$ where $T_i = S_i \cap T$.

Theorem 2.3.12: Every neutrosophic strong sub N-LA-semigroup is a neutrosophic sub N-LA-semigroup.

Definition 2.3.13: Let $S(N)=\{S_1 \cup S_2 \cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-LA-semigroup. A proper subset $J=\{J_1 \cup J_2 \cup ....J_N, *_1, *_2, ..., *_N\}$ where $J_t = J \cap S_t$ for $t=1,2,...,N$ is said to be a neutrosophic strong N-ideal of $S(N)$ if the following conditions are satisfied.

1. Each it is a neutrosophic sub LA-semigroup of $S_t, t=1,2,...,N$ i.e. It is a neutrosophic strong N-sub LA-semigroup of $S(N)$.
2. Each it is a two sided ideal of S_t for $t=1,2,...,N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

Theorem 2.3.14: Every neutrosophic strong Nideal is trivially a neutrosophic sub N-LA-semigroup.

Theorem 2.3.15: Every neutrosophic strong N-ideal is a neutrosophic strong sub N-LA-semigroup.

Theorem 2.3.16: Every neutrosophic strong N-ideal is a N-ideal.

Chapter No. 3

NEUTROSOPHIC SOLUBLE GROUPS, NEUTROSOPHIC NILPOTENT GROUPS AND THEIR GENERALIZATION

In this chapter, the author has introduced for the first time neutrosophic soluble groups and neutrosophic nilpotent groups. Each neutrosophic soluble group contains a soluble group and similarly for a neutrosophic nilpotent group contained a nilpotent group respectively. Then generalization of neutrosophic soluble groups and nilpotent groups have been discussed in this chapter. Their basic properties and characterization is also presented with sufficient amount of illustrative examples.

We no proceed on to define neutrosophic soluble groups and nilpotent groups respectively.

3.1 Neutrosophic Soluble Groups

In this section, we introduced neutrosophic soluble groups and neutrosophic nilpotent groups respectively. These neutrosophic groups are defined on the basis of some kind of series. We also discuissed some fundamental properties of neutrosophic soluble groups and neutrosophic nilpotent groups.

We now proceed to define it.

Definition 3.1.1: Let $N(G)=\langle G\cup I\rangle$ be a neutrosophic group and let $H_1,H_2,...,\mathrm{H}_n$ be the neutrosophic subgroups of $N(G)$. Then a neutrosophic subgroup series is a chain of neutrosophic subgroups such that

$$1=H_0\leq H_1\leq H_2\leq...\leq H_{n-1}\leq H_n=N(G).$$

The following example illustrate this situation more explicitly.

Example 3.1.2: Let $N(G)=\langle\mathbb{Z}\cup I\rangle$ be a neutrosophic group of integers. Then the following are neutrosophic subgroups series of the neutrosophic group $N(G)$.

$$1\leq 4\mathbb{Z}\leq 2\mathbb{Z}\leq\langle 2\mathbb{Z}\cup I\rangle\leq\langle\mathbb{Z}\cup I\rangle,$$

$$1\leq\langle 4\mathbb{Z}\cup I\rangle\leq\langle 2\mathbb{Z}\cup I\rangle\leq\langle\mathbb{Z}\cup I\rangle,$$

$$1\leq 4\mathbb{Z}\leq 2\mathbb{Z}\leq\mathbb{Z}\leq\langle\mathbb{Z}\cup I\rangle.$$

Definition 3.1.3: Let $1=H_0\leq H_1\leq H_2\leq...\leq H_{n-1}\leq H_n=N(G)$ be a neutrosophic subgroup series of the neutrosophic group $N(G)$. Then this series of neutrosophic subgroups is called a strong neutrosophic subgroup series if each H_j is a neutrosophic subgroup of $N(G)$ for all j.

We can show it in the following example.

Example 3.1.4: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group. Then the following neutrosophic subgroup series of $N(G)$ is a strong neutrosophic subgroup series:

$$1\leq\langle 4\mathbb{Z}\cup I\rangle\leq\langle 2\mathbb{Z}\cup I\rangle\leq\langle \mathbb{Z}\cup I\rangle.$$

Theorem 3.1.5: Every strong neutrosophic subgroup series is trivially a neutrosophic subgroup series but the converse is not true in general.

One can easily see the converse by the help of example.

Definition 3.1.6: If some H_j's are neutrosophic subgroups and some H_k's are just subgroups of $N(G)$. Then the neutrosophic subgroups series is called mixed neutrosophic subgroup series.

The following example further explained this fact.

Example 3.1.7: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group. Then the following neutrosophic subgroup series of $N(G)$ is a mixed neutrosophic subgroup series:

$$1\leq 4\mathbb{Z}\leq 2\mathbb{Z}\leq\langle 2\mathbb{Z}\cup I\rangle\leq\langle \mathbb{Z}\cup I\rangle.$$

Theorem 3.1.8: Every mixed neutrosophic subgroup series is trivially a neutrosophic subgroup series but the converse is not true in general.

Definition 3.1.9: If H_j's in $1 = H_0 \leq H_1 \leq H_2 \leq ... \leq H_{n-1} \leq H_n = N(G)$ are only subgroups of the neutrosophic group $N(G)$, then the series is termed as subgroup series of the neutrosophic group $N(G)$.

For an instance, see the following example.

Example 3.1.10: Let $N(G) = \langle \mathbb{Z} \cup I \rangle$ be a neutrosophic group. Then the following neutrosophic subgroup series of $N(G)$ is just a subgroup series:

$$1 \leq 4\mathbb{Z} \leq 2\mathbb{Z} \leq \mathbb{Z} \leq \langle \mathbb{Z} \cup I \rangle.$$

Theorem 3.1.11: A neutrosophic group $N(G)$ has all three type of neutrosophic subgroups series.

Theorem 3.1.12: Every subgroup series of the group G is also a subgroup series of the neutrosophic group $N(G)$.

Proof: Since G is always contained in $N(G)$. This directly followed the proof.

Definition 3.1.13: Let $1 = H_0 \leq H_1 \leq H_2 \leq ... \leq H_{n-1} \leq H_n = N(G)$ be a neutrosophic subgroup series of the neutrosophic group $N(G)$. If

$$1 = H_0 \lhd H_1 \lhd H_2 \lhd ... \lhd H_{n-1} \lhd H_n = N(G) \ldots\ldots\ldots\ldots\ldots (1)$$

That is each H_j is normal in H_{j+1}. Then (1) is called a neutrosophic subnormal series of the neutrosophic group $N(G)$.

The following example shows this fact.

Example 3.1.14: Let $N(G) = \langle A_4 \cup I \rangle$ be a neutrosophic group, where A_4 is the alternating subgroup of the permutation group S_4. Then the following are the neutrosophic subnormal series of the group $N(G)$.

$$1 \triangleleft C_2 \triangleleft V_4 \triangleleft \langle V_4 \cup I \rangle \triangleleft \langle A_4 \cup I \rangle .$$

Definition 3.1.15: A neutrosophic subnormal series is called strong neutrosophic subnormal series if all H_j's are neutrosophic normal subgroups in (1) for all j.

For furthere explaination, see the following example.

Example 3.1.16: Let $N(G) = \langle \mathbb{Z} \cup I \rangle$ be a neutrosophic group of integers. Then the following is a strong neutrosophic subnormal series of $N(G)$.

$$1 \triangleleft \langle 4\mathbb{Z} \cup I \rangle \triangleleft \langle 2\mathbb{Z} \cup I \rangle \triangleleft \langle \mathbb{Z} \cup I \rangle .$$

Theorem 3.1.17: Every strong neutrosophic subnormal series is trivially a neutrosophic subnormal series but the converse is not true in general.

Definition 3.1.18: A neutrosophic subnormal series is called mixed neutrosophic subnormal series if some H_j's are neutrosophic normal subgroups in (1) while some H_k's are just normal subgroups in (1) for some j and k.

This can be shown in the following example.

Example 3.1.19: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group of integers. Then the following is a mixed neutrosophic subnormal series of $N(G)$.

$$1 \lhd 4\mathbb{Z} \lhd 2\mathbb{Z} \lhd \langle 2\mathbb{Z}\cup I\rangle \lhd \langle \mathbb{Z}\cup I\rangle .$$

Theorem 3.1.20: Every mixed neutrosophic subnormal series is trivially a neutrosophic subnormal series but the converse is not true in general.

The converse can be easily seen by the help of examples.

Definition 3.1.21: A neutrosophic subnormal series is called subnormal series if all H_j's are only normal subgroups in (1) for all j.

For this, we take the next example.

Example 3.1.22: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group of integers. Then the following is a subnormal series of $N(G)$.

$$1 \lhd 4\mathbb{Z} \lhd 2\mathbb{Z} \lhd \langle \mathbb{Z}\cup I\rangle .$$

Theorem 3.1.23: Every subnormal series of the group G is also a subnormal series of the neutrosophic group $N(G)$.

Definition 3.1.24: If H_j are all normal neutrosophic subgroups in $N(G)$. Then the neutrosophic subnormal series (1) is called neutrosophic normal series.

One can see it in this example.

Example 3.1.25: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group of integers. Then the following is a neutrosophic normal series of $N(G)$.

$$1 \lhd 4\mathbb{Z} \lhd 2\mathbb{Z} \lhd \langle \mathbb{Z}\cup I\rangle .$$

Theorem 3.1.26: Every neutrosophic normal series is a neutrosophic subnormal series but the converse is not true.

For the converse, see the following Example.

Example 3.1.27: Let $N(G)=\langle A_4\cup I\rangle$ be a neutrosophic group, where A_4 is the alternating subgroup of the permutation group S_4. Then the following are the neutrosophic subnormal series of the group $N(G)$.

$$1 \lhd C_2 \lhd V_4 \lhd \langle V_4\cup I\rangle \lhd \langle A_4\cup I\rangle .$$

This series is not neutrosophic normal series as C_2 (cyclic group of order 2) is not normal in V_4(Klein four group).

Similarly we can define strong neutrosophic normal series, mixed neutrosophic normal series and normal series respectively on the same lines of the neutrosophic group $N(G)$.

Definition 3.1.28: The neutrosophic normal series

$$1 = H_0 \lhd H_1 \lhd H_2 \lhd ... \lhd H_{n-1} \lhd H_n = N(G) \,.........\, (2)$$

is called neutrosophic abelian series if the factor group H_{j+1}/H_j are all abelian for all j.

This situation can be explained in the following example.

Example 3.1.29: Let $N(G) = \langle S_3 \cup I \rangle$ be a neutrosophic group, where S_3 is the permutation group. Then the following is the neutrosophic abelian series of the group $N(G)$.

$$1 \lhd A_3 \lhd \langle A_3 \cup I \rangle \lhd \langle S_3 \cup I \rangle.$$

We explain it as following:

Since $\langle S_3 \cup I \rangle / \langle A_3 \cup I \rangle \simeq \mathbb{Z}_2$ and $\mathbb{Z}_2$ is cyclic which is abelian. Thus $\langle S_3 \cup I \rangle / \langle A_3 \cup I \rangle$ is an abelian neutrosophic group.

Also,

$\langle A_3 \cup I\rangle / A_3 \simeq \mathbb{Z}_2$ and this is factor group is also cyclic and every cyclic group is abelian. Hence $\langle A_3 \cup I\rangle / A_3$ is also ablian group.

Finally,

$A_3 / I \simeq \mathbb{Z}_3$ which again abelian group.

Therefore the series is a neutrosophic abelian series of the group $N(G)$.

Thus on the same lines, we can define strong neutrosophic abelian series, mixed neutrosophic abelian series and abelian series of the neutrosophic group $N(G)$.

Definition 3.1.30: A neutrosophic group $N(G)$ is called neutrosophic soluble group if $N(G)$ has a neutrosophic abelian series.

The following is an example of a neutrosophic soluble group.

Example 3.1.31: Let $N(G) = \langle S_3 \cup I\rangle$ be a neutrosophic group, where S_3 is the permutation group. Then the following is the neutrosophic abelian series of the group $N(G)$.

$$1 \triangleleft A_3 \triangleleft \langle A_3 \cup I\rangle \triangleleft \langle S_3 \cup I\rangle .$$

Then clearly $N(G)$ is a neutrosophic soluble group.

Theorem 3.1.32: Every abelian series of a group G is also an abelian series of the neutrosophic group $N(G)$.

Theorem 3.1.33: If a group G is a soluble group, then the neutrosophic group $N(G)$ is also soluble neutrosophic group.

Proof: The proof is followed from above theorem 3.1.32.

Theorem 3.1.34: If the neutrosophic group $N(G)$ is an abelian neutrosophic group, then $N(G)$ is a neutrosophic soluble group.

Theorem 3.1.35: If $N(G) = C(N(G))$, then $N(G)$ is a neutrosophic soluble group.

Proof: Suppose the $N(G) = C(N(G))$. Then it follows that $N(G)$ is a neutrosophic abelian group. Hence by above Theorem 3.1.34, $N(G)$ is a neutrosophic soluble group.

Theorem 3.1.36: If the neutrosophic group $N(G)$ is a cyclic neutrosophic group, then $N(G)$ is a neutrosophic soluble group.

Definition 3.1.37: A neutrosophic group $N(G)$ is called strong neutrosophic soluble group if $N(G)$ has a strong neutrosophic abelian series.

The following example explain this fact.

Example 3.1.38: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group. Then the following is a strong neutrosophic abelian series of the group $N(G)$.

$$1 \triangleleft \langle 4\mathbb{Z}\cup I\rangle \triangleleft \langle 2\mathbb{Z}\cup I\rangle \triangleleft \langle \mathbb{Z}\cup I\rangle .$$

Then clearly $N(G)$ is a strong neutrosophic soluble group.

Theorem 3.1.39: Every strong neutrosophic soluble group $N(G)$ is trivially a neutrosophic soluble group but the converse is not true.

The converse of this theorem can be easily seen by the help of examples.

Definition 3.1.40: A neutrosophic group $N(G)$ is called mixed neutrosophic soluble group if $N(G)$ has a mixed neutrosophic abelian series.

We can easily seen it in the following example.

Example 3.1.41: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group. Then the following is a mixed neutrosophic abelian series of the group $N(G)$.

$$1 \triangleleft 4\mathbb{Z} \triangleleft 2\mathbb{Z} \triangleleft \langle 2\mathbb{Z}\cup I\rangle \triangleleft \langle \mathbb{Z}\cup I\rangle .$$

Then clearly $N(G)$ is a mixed neutrosophic soluble group.

Theorem 3.1.42: Every mixed neutrosophic soluble group $N(G)$ is trivially a neutrosophic soluble group but the converse is not true.

The converse of this theorem can be easily seen by the help of examples.

Definition 3.1.43: A neutrosophic group $N(G)$ is called soluble group if $N(G)$ has an abelian series.

See the example listed below.

Example 3.1.44: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic group. Then the following is an abelian series of the group $N(G)$.

$$1 \triangleleft 4\mathbb{Z} \triangleleft 2\mathbb{Z} \triangleleft \langle \mathbb{Z}\cup I\rangle .$$

Then clearly $N(G)$ is a soluble group.

Definition 3.1.45: Let $N(G)$ be a neutrosophic soluble group. Then length of the shortest neutrosophic abelian series of $N(G)$ is called derived length.

This example shows this fact.

Example 3.1.46: Let $N(G)=\langle \mathbb{Z}\cup I\rangle$ be a neutrosophic soluble group. The following is a neutrosophic abelian series of the group $N(G)$.

$$1 \triangleleft 4\mathbb{Z} \triangleleft 2\mathbb{Z} \triangleleft \langle 2\mathbb{Z} \cup I \rangle \triangleleft \langle \mathbb{Z} \cup I \rangle .$$

Then $N(G)$ has derived length 4.

Remark 3.1.47: Neutrosophic group of derive length zero is trivial neutrosophic group.

Proposition 3.1.48: Every neutrosophic subgroup of a neutrosophic soluble group is soluble.

Proposition 3.1.49: Quotient neutrosophic group of a neutrosophic soluble group is soluble.

3.2 Neutrosophic Soluble Bigroups

In this section, we are going towards the generalization of neutrosophic soluble groups and neutrosophic nilpotent groups respectively. This means we extend neutrosophic soluble groups to neutrosophic soluble bigroups. Similarly neutrosophic nilpotent groups is extend to neutrosophic nilpotent bigroups. Their related theorem and notions have been studied and many examples have constructed to understand it in a better and easy way.

Let’s proceed now to define neutrosophic soluble bigroups.

Definition 3.2.1: Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup and let $H_1,H_2,...,H_n$ be the neutrosophic subgroups of $B(G_1)$ and $K_1,K_2,...,K_n$ be the neutrosophic subgroups of $B(G_2)$ respectively. Then a neutrosophic bisubgroup series is a chain of neutrosophic bisubgroups such that

$$1=H_0\cup K_0\leq H_1\cup K_1\leq H_2\cup K_2\leq ...\leq H_{n-1}\cup K_{n-1}\leq H_n\cup K_n=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle=B_N(G),$$

where $H_s\leq H_{s+1}$ and $K_s\leq K_{s+1}$ for all s.

This situation can be easily seen in the following example.

Example 3.2.2: Let $B_N(G)=\langle\mathbb{Z}\cup I\rangle\cup\langle A_4\cup I\rangle$ be a neutrosophic bigroup. Then the following are the neutrosophic bisubgroups series of $B_N(G)$.

$$1\leq 4\mathbb{Z}\cup C_2\leq 2\mathbb{Z}\cup V_4\leq\langle 2\mathbb{Z}\cup I\rangle\cup\langle V_4\cup I\rangle\leq\langle\mathbb{Z}\cup I\rangle\cup\langle A_4\cup I\rangle=B_N(G),$$

$$1\leq\langle 4\mathbb{Z}\cup I\rangle\cup\langle C_2\cup I\rangle\leq\langle 2\mathbb{Z}\cup I\rangle\cup\langle V_4\cup I\rangle\leq\langle\mathbb{Z}\cup I\rangle\cup\langle A_4\cup I\rangle=B_N(G),$$

$$1\leq 4\mathbb{Z}\cup C_2\leq 2\mathbb{Z}\cup V_4\leq\mathbb{Z}\cup A_4\leq\langle\mathbb{Z}\cup I\rangle\cup\langle A_4\cup I\rangle=B_N(G).$$

Definition 3.2.3: Let

$$1=H_0\cup K_0\leq H_1\cup K_1\leq H_2\cup K_2\leq ...\leq H_{n-1}\cup K_{n-1}\leq H_n\cup K_n=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle=B_N(G)$$

be a neutrosophic bisubgroup series of the neutrosophic bigroup $B_N(G)$. Then this series is called a strong neutrosophic bisubgroup series if each $H_s\cup K_s$ is a neutrosophic bisubgroup of $B_N(G)$ for all s.

We can easily see it in the following example.

Example 3.2.4: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a strong neutrosophic bisubgroups series of $B_N(G)$.

$$1 \le \langle 4\mathbb{Z} \cup I \rangle \cup \langle C_2 \cup I \rangle \le \langle 2\mathbb{Z} \cup I \rangle \cup \langle V_4 \cup I \rangle \le \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.5: Every strong neutrosophic bisubgroup series is trivially a neutrosophic bisubgroup series but the converse is not true in general.

The converse of this theorem can be easily seen by the help of examples.

Definition 3.2.6: If some $H_s \cup K_s$ are neutrosophic bisubgroups and some $H_k \cup K_k$ are just bisubgroups of $B_N(G)$. Then the neutrosophic bisubgroups series is called mixed neutrosophic bisubgroup series.

This can be shown in the example below.

Example 3.2.7: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a mixed neutrosophic bisubgroups series of $B_N(G)$.

$$1 \le 4\mathbb{Z} \cup C_2 \le 2\mathbb{Z} \cup V_4 \le \langle 2\mathbb{Z} \cup I \rangle \cup \langle V_4 \cup I \rangle \le \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.8: Every neutrosophic mixed bisubgroup series is trivially a neutrosophic bisubgroup series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definiton 3.2.9: If all $H_s \cup K_s$ in

$$1 = H_0 \cup K_0 \le H_1 \cup K_1 \le H_2 \cup K_2 \le \ldots \le H_{n-1} \cup K_{n-1} \le H_n \cup K_n = \langle G_1 \cup I \rangle \cup \langle G_2 \cup I \rangle = B_N(G)$$

are only bisubgroups of the neutrosophic bigroup $B_N(G)$, then that series is termed as bisubgroup series of the neutrosophic bigroup $B_N(G)$.

One can see it in this example.

Example 3.2.10: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. The following is a bisubgroups series of $B_N(G)$.

$$1 \le 4\mathbb{Z} \cup C_2 \le 2\mathbb{Z} \cup V_4 \le \mathbb{Z} \cup A_4 \le \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.11: A neutrosophic bigroup $B_N(G)$ has all three type of neutrosophic bisubgroups series.

Theorem 3.2.12: Every bisubgroup series of the bigroup $B(G)$ is also a bisubgroup series of the neutrosophic bigroup $B_N(G)$.

Proof: Since $B(G)$ is always contained in $B_N(G)$. This the proof followed directly.

Definition 3.2.13: Let

$$1 = H_0 \cup K_0 \le H_1 \cup K_1 \le H_2 \cup K_2 \le \ldots \le H_{n-1} \cup K_{n-1} \le H_n \cup K_n = B_N(G)$$

be a neutrosophic bisubgroup series of the neutrosophic bigroup $B_N(G)$. If

$$1 = H_0 \cup K_0 \lhd H_1 \cup K_1 \lhd H_2 \cup K_2 \lhd \ldots \lhd H_{n-1} \cup K_{n-1} \lhd H_n \cup K_n = B_N(G) \ldots\ldots\ldots\ldots\ldots (1)$$

That is each $H_s \cup K_s$ is normal in $H_{s+1} \cup K_{s+1}$ such that H_s is normal in H_{s+1} and K_s is normal in K_{s+1} respectively. Then (1) is called a neutrosophic bi-subnormal series of the neutrosophic bigroup $B_N(G)$.

For an instance, see the following example.

Example 3.2.14: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a neutrosophic bi-subnormal series of $B_N(G)$.

$$1 \lhd 4\mathbb{Z} \cup C_2 \lhd 2\mathbb{Z} \cup V_4 \lhd \langle 2\mathbb{Z} \cup I \rangle \cup \langle V_4 \cup I \rangle \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Definition 3.2.15: A neutrosophic bi-subnormal series is called strong neutrosophic bi-subnormal series if all $H_s \cup K_s$ are neutrosophic normal subgroups in (1) for all s.

This situation can be shown in the following example.

Example 3.2.16: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. The following is a neutrosophic bi-subnormal series of $B_N(G)$.

$$1 \lhd \langle 4\mathbb{Z} \cup I \rangle \cup \langle C_2 \cup I \rangle \lhd \langle 2\mathbb{Z} \cup I \rangle \cup \langle V_4 \cup I \rangle \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.17: Every strong neutrosophic bi-subnormal series is trivially a neutrosophic bi-subnormal series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definition 3.2.18: A neutrosophic bi-subnormal series is called mixed neutrosophic bi-subnormal series if some $H_s \cup K_s$ are neutrosophic normal bisubgroups in (1) while some $H_k \cup K_k$ are just normal bisubgroups in (1) for some s and k.

Example 3.2.19: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the mixed neutrosophic bi-subnormal series of $B_N(G)$ is as following.

$$1 \le 4\mathbb{Z} \cup C_2 \le 2\mathbb{Z} \cup V_4 \le \langle 2\mathbb{Z} \cup I \rangle \cup \langle V_4 \cup I \rangle \le \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.20: Every mixed neutrosophic bi-subnormal series is trivially a neutrosophic bi-subnormal series but the converse is not true in general.

Definition 3.2.21: A neutrosophic bi-subnormal series is called bi-subnormal series if all $H_j \cup K_j$ are only normal bisubgroups in (1) for all j.

Example 3.2.22: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a bi-subnormal series of $B_N(G)$,

$$1 \lhd 4\mathbb{Z} \cup C_2 \lhd 2\mathbb{Z} \cup V_4 \lhd \mathbb{Z} \cup A_4 \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.23: Every bi-subnormal series of the bigroup $B(G)$ is also a bi-subnormal series of the neutrosophic bigroup $B_N(G)$.

Definition 3.2.24: If $H_s \cup K_s$ are all normal neutrosophic bisubgroups in $B_N(G)$, that is H_s is normal in $\langle G_1 \cup I \rangle$ and K_s is normal in $\langle G_2 \cup I \rangle$ respectively. Then the neutrosophic bi-subnormal series (1) is called neutrosophicbi- normal series.

Example 3.2.25: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a bi-normal series of $B_N(G)$,

$$1 \lhd 4\mathbb{Z} \cup C_2 \lhd 2\mathbb{Z} \cup V_4 \lhd \mathbb{Z} \cup A_4 \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Theorem 3.2.26: Every neutrosophic bi-normal series is a neutrosophic bi-subnormal series but the converse is not true.

For the converse, see the following Example.

Example 3.2.27: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a neutrosophic bi-subnormal series of $B_N(G)$.

$$1 \lhd C_2 \cup 4\mathbb{Z} \lhd V_4 \cup 2\mathbb{Z} \lhd \langle V_4 \cup I \rangle \cup \langle 2\mathbb{Z} \cup I \rangle \lhd \langle A_4 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle .$$

This series is not neutrosophic bi-normal series as in $C_2 \cup 4\mathbb{Z}$, the cyclic group C_2 is not normal in V_4(Klein four group).

Similarly we can define strong neutrosophic bi-normal series, mixed neutrosophic bi-normal series and bi-normal series respectively on the same lines.

Definition 3.2.28: The neutrosophic bi-normal series

$$1 = H_0 \cup K_0 \lhd H_1 \cup K_1 \lhd H_2 \cup K_2 \lhd ... \lhd H_{n-1} \cup K_{n-1} \lhd H_n \cup K_n = B_N(G) \,.........\, (2)$$

is called neutrosophic bi-abelian series if the factor group $H_{s+1} \cup K_{s+1} / H_s \cup K_s$ are all abelian for all s.

Clearly we have $H_{s+1} \cup K_{s+1} / H_s \cup K_s = H_{s+1} / H_s \cup K_{s+1} / K_s$.

For furthere explaination, see the following example.

Example 3.2.29: Let $B_N(G) = \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle$ be a neutrosophic bigroup, where S_3 is the permutation group. Then the following is the neutrosophic bi-abelian series of $B_N(G)$.

$$1 \lhd A_3 \cup 4\mathbb{Z} \lhd \langle A_3 \cup I \rangle \cup 2\mathbb{Z} \lhd \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle = B_N(G).$$

We explain it as following:

Since $\langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle \big/ \langle A_3 \cup I\rangle \cup \langle 2\mathbb{Z} \cup I\rangle \simeq \mathbb{Z}_2 \cup \mathbb{Z}_2$ and $\mathbb{Z}_2$ is cyclic which is abelian. Thus $\langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle \big/ \langle A_3 \cup I\rangle \cup \langle 2\mathbb{Z} \cup I\rangle$ is an abelian neutrosophic bigroup.

Also,

$\langle A_3 \cup I\rangle \cup 2\mathbb{Z} \big/ A_3 \cup 4\mathbb{Z} \simeq \mathbb{Z}_2 \cup \mathbb{Z}_2$ is also ablian bigroup.

Finally,

$A_3 \cup 4\mathbb{Z} \big/ I \simeq \mathbb{Z}_3 \cup \mathbb{Z}_4$ which is again abelian bigroup.

Therefore the series is a neutrosophic bi-abelian series of $B_N(\mathrm{G})$.

Thus on the same lines, we can define strong neutrosophic bi-abelian series, mixed neutrosophic bi-abelian series and bi-abelian series of the neutrosophic bigroup $B_N(G)$.

Definition 3.2.30: A neutrosophic bigroup $B_N(G)$ is called neutrosophic soluble bigroup if $B_N(G)$ has a neutrosophic bi-abelian series.

See the next example which illustrate this fact.

Example 3.2.31: Let $\mathrm{B}_N(G) = \langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle$ be a neutrosophic bigroup, where S_3 is the symmetric group. Then the following is the neutrosophic bi-abelian series of $B_N(G)$.

$$1 \lhd A_3 \cup 4\mathbb{Z} \lhd \langle A_3 \cup I\rangle \cup 2\mathbb{Z} \lhd \langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle = B_N(G).$$

Then clearly $B_N(G)$ is a neutrosophic soluble bigroup.

Theorem 3.2.32: Every bi-abelian series of a bigroup $B(G)$ is also a bi-abelian series of the neutrosophic bigroup $B_N(G)$.

Theorem 3.2.33: If a bigroup $B(G)$ is a soluble bigroup, then the neutrosophic bigroup $B_N(G)$ is also a neutrosophic soluble bigroup.

Proof: The proof is followed from above theorem 3.2.32.

Theorem 3.2.34: If the neutrosophic bigroup $B_N(G)$ is an abelian neutrosophic bigroup, then $B_N(G)$ is a neutrosophic soluble bigroup.

Theorem 3.2.35: If $\mathrm{B}_N(G) = \mathrm{C}(\mathrm{B}_N(G))$, where $\mathrm{C}(\mathrm{B}_N(G))$ is the neutrosophic bicenter of $\mathrm{B}_N(G)$, then $B_N(G)$ is a neutrosophic soluble bigroup.

Proof: Suppose the $\mathrm{B}_N(G) = \mathrm{C}(\mathrm{B}_N(G))$. Then it follows that $\mathrm{B}_N(G)$ is a neutrosophic abelian bigroup. Hence by above Theorem 3.2.34, $\mathrm{B}_N(G)$ is a neutrosophic soluble bigroup.

Definition 3.2.36: A neutrosophic bigroup $B_N(G)$ is called strong neutrosophic soluble bigroup if $B_N(G)$ has a strong neutrosophic bi-abelian series.

Example 3.2.37: Let $B_N(G)=\langle \mathbb{Z}\cup I\rangle \cup \langle A_4 \cup I\rangle$ be a neutrosophic bigroup. Then the following is a strong neutrosophic bi-abelian series of $B_N(G)$.

$$1 \lhd \langle 4\mathbb{Z}\cup I\rangle \cup \langle C_2 \cup I\rangle \lhd \langle 2\mathbb{Z}\cup I\rangle \cup \langle V_4 \cup I\rangle \lhd \langle \mathbb{Z}\cup I\rangle \cup \langle A_4 \cup I\rangle = B_N(G).$$

Clearly $B_N(G)$ is a strong neutrosophic soluble bigroup.

Theorem 3.2.38: Every strong neutrosophic soluble bigroup $\mathrm{B}_N(G)$ is trivially a neutrosophic soluble bigroup but the converse is not true.

The converse is left as an exercise for the interested readers.

Definition 3.2.39: A neutrosophic bigroup $B_N(G)$ is called mixed neutrosophic soluble bigroup if $B_N(G)$ has a mixed neutrosophic bi-abelian series.

Example 3.2.40: Let $\mathrm{B}_N(G)=\langle S_3 \cup I\rangle \cup \langle \mathbb{Z}\cup I\rangle$ be a neutrosophic bigroup, where S_3 is the symmetric group. Then the following is a mixed neutrosophic bi-abelian series of $B_N(G)$.

$$1 \lhd A_3 \cup 4\mathbb{Z} \lhd \langle A_3 \cup I\rangle \cup 2\mathbb{Z} \lhd \langle S_3 \cup I\rangle \cup \langle \mathbb{Z}\cup I\rangle = B_N(G).$$

Then clearly $B_N(G)$ is a mixed neutrosophic soluble bigroup.

Theorem 3.2.41: Every mixed neutrosophic soluble bigroup $B_N(G)$ is trivially a neutrosophic soluble bigroup but the converse is not true.

The converse is left as an exercise for the interested readers.

Definition 3.2.42: A neutrosophic bigroup $B_N(G)$ is called soluble bigroup if $B_N(G)$ has a bi-abelian series.

Example 3.2.43: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a bi-abelian series of $B_N(G)$,

$$1 \lhd 4\mathbb{Z} \cup C_2 \lhd 2\mathbb{Z} \cup V_4 \lhd \mathbb{Z} \cup A_4 \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Then clearly $B_N(G)$ is a soluble bigroup.

Definition 3.2.44: Let $B_N(G)$ be a neutrosophic soluble bigroup. Then length of the shortest neutrosophic bi-abelian series of $B_N(G)$ is called derived bi-length.

Example 3.2.45: Let $B_N(G) = \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle$ be a neutrosophic bigroup. Then the following is a neutrosophic bi-abelian series of $B_N(G)$.,

$$1 \leq 4\mathbb{Z} \cup C_2 \leq 2\mathbb{Z} \cup V_4 \leq \mathbb{Z} \cup A_4 \leq \langle \mathbb{Z} \cup I \rangle \cup \langle A_4 \cup I \rangle = B_N(G).$$

Then $B_N(G)$ has derived bi-length is 4.

Remark 3.2.46: Neutrosophic bigroup of derive bi-length zero is trivial neutrosophic bigroup.

Proposition 3.2.47: Every neutrosophic sub-bigroup of a neutrosophic soluble bigroup is soluble.

Proposition 3.2.48: Quotient neutrosophic bigroup of a neutrosophic soluble bigroup is soluble.

We now generalize this concept upto neutrosophic soluble N-groups.

3.3 Neutrosophic Soluble N-groups

This section is about the generalization of neutrosophic soluble groups and neutrosophic nilpotent groups. In this section, we define neutrosophic soluble N-groups and neutrosophic nilpotent N-groups respectively.

Definition 3.3.1: Let $G(N)=\{G_1\cup G_2\cup....\cup G_N,*_1,*_2,...,*_N\}$ be a neutrosophic N-group and let $H_1,H_2,...,H_n$ be the neutrosophic subgroups of $B(G_1)$ and $K_1,K_2,...,K_n$ be the neutrosophic subgroups of $B(G_2)$ … $Y_1,Y_2,...,Y_n$ be the neutrosophic subgroups of G_N respectively. Then a neutrosophic N-subgroup series is a chain of neutrosophic N-subgroups such that

$$1 = H_0 \cup K_0 \cup ... \cup Y_0 \leq H_1 \cup K_1 \cup ... \cup Y_1 \leq H_2 \cup K_2 \cup ... \cup Y_2 \leq ...$$
$$... \leq H_{n-1} \cup K_{n-1} \cup ... \cup Y_{n-1} \leq H_n \cup K_n \cup ... \cup Y_n = G(N)$$

where $H_s \leq H_{s+1}$, $K_s \leq K_{s+1}$, …, $Y_s \leq Y_{s+1}$ for all s.

This situation can be furthere explained in the following example.

Example 3.3.2: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4$ be a neutrosophic 3-group. Then the following are the neutrosophic 3-subgroups series of $G(N)$.

$$1 \leq 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \leq 2\mathbb{Z} \cup \{\pm 1, \pm \mathrm{i}\} \cup V_4 \leq \langle 2\mathbb{Z} \cup I \rangle \cup Q_8 \cup A_4 \leq \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N)$$

$$1 \leq 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \leq 2\mathbb{Z} \cup \{\pm 1, \pm j\} \cup V_4 \leq \mathbb{Z} \cup Q_8 \cup A_4 \leq \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N).$$

Definition 3.3.3: Let

$$1 = H_0 \cup K_0 \cup ... \cup Y_0 \leq H_1 \cup K_1 \cup ... \cup Y_1 \leq H_2 \cup K_2 \cup ... \cup Y_2 \leq ...$$
$$... \leq H_{n-1} \cup K_{n-1} \cup ... \cup Y_{n-1} \leq H_n \cup K_n \cup ... \cup Y_n = G(N)$$

be a neutrosophic N-subgroup series of the neutrosophic N-group $G(N)$. Then this series is called a strong neutrosophic N-subgroup series if each $H_s \cup K_s \cup ... \cup Y_s$ is a neutrosophic N-subgroup of $G(N)$ for all s.

Example 3.3.4: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle$ be a neutrosophic 3-group. Then the following is a strong neutrosophic 3-subgroups series of $G(N)$.

$$1 \le \langle 6\mathbb{Z} \cup I \rangle \cup \{I, iI\} \cup \langle C_2 \cup I \rangle \le \langle 4\mathbb{Z} \cup I \rangle \cup \{1, \mathrm{i}, I, iI\} \cup \langle V_4 \cup I \rangle$$
$$\le \langle 2\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm i, \pm I, \pm iI\} \cup \langle A_4 \cup I \rangle \le \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle = G(N)$$

Theorem 3.3.5: Every strong neutrosophic N-subgroup series is trivially a neutrosophic N-subgroup series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definition 3.3.6: If some $H_s \cup K_s \cup ... \cup Y_s$ are neutrosophic N-subgroups and some $H_k \cup K_k \cup ... \cup Y_k$ are just N-subgroups of $G(N)$. Then the neutrosophic N-subgroups series is called mixed neutrosophic N-subgroup series.

Example 3.3.7: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle$ be a neutrosophic 3-group. Then the following is a mixed neutrosophic 3-subgroups series of $G(N)$.

$$1 \le 4\mathbb{Z} \cup \{I, iI\} \cup \langle C_2 \cup I \rangle \le 2\mathbb{Z} \cup \{1, \mathrm{i}, I, iI\} \cup \langle V_4 \cup I \rangle$$
$$\le \langle 2\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm i, \pm I, \pm iI\} \cup \langle A_4 \cup I \rangle \le \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle = G(N)$$

Theorem 3.3.8: Every neutrosophic mixed N-subgroup series is trivially a neutrosophic N-subgroup series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definiton 3.3.9: If all $H_s \cup K_s \cup ... \cup Y_s$ in

$$1 = H_0 \cup K_0 \cup ... \cup Y_0 \leq H_1 \cup K_1 \cup ... \cup Y_1 \leq H_2 \cup K_2 \cup ... \cup Y_2 \leq ...$$
$$... \leq H_{n-1} \cup K_{n-1} \cup ... \cup Y_{n-1} \leq H_n \cup K_n \cup ... \cup Y_n = G(N)$$

are only N-subgroups of the neutrosophic N-group $G(N)$, then the series is said to be N-subgroup series of the neutrosophic N-group $G(N)$.

Example 3.3.10: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4$ be a neutrosophic 3-group. Then the following is a 3-subgroups series of $G(N)$,

$$1 \leq 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \leq 2\mathbb{Z} \cup \{\pm 1, \pm j\} \cup V_4 \leq \mathbb{Z} \cup Q_8 \cup A_4 \leq \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N).$$

Theorem 3.3.11: A neutrosophic N-group $G(N)$ has all three type of neutrosophic N-subgroups series.

Theorem 3.3.12: Every N-subgroup series of the N-group is also a N-subgroup series of the neutrosophic N-group $G(N)$.

Definition 3.3.12: Let

$$1 = H_0 \cup K_0 \cup ... \cup Y_0 \leq H_1 \cup K_1 \cup ... \cup Y_1 \leq H_2 \cup K_2 \cup ... \cup Y_2 \leq ...$$
$$... \leq H_{n-1} \cup K_{n-1} \cup ... \cup Y_{n-1} \leq H_n \cup K_n \cup ... \cup Y_n = G(N)$$

be a neutrosophic N-subgroup series of the neutrosophic N-group $G(N)$. If

$$1 = H_0 \cup K_0 \cup ... \cup Y_n \lhd H_1 \cup K_1 \cup ... \cup Y_1 \lhd H_2 \cup K_2 \cup ... \cup Y_n \lhd ...$$
$$... \lhd H_{n-1} \cup K_{n-1} \cup ... \cup Y_{n-1} \lhd H_n \cup K_n \cup ... \cup Y_n = G(N) \quad (1)$$

That is each $H_s \cup K_s \cup ... \cup Y_n$ is normal in $H_{s+1} \cup K_{s+1} \cup ... \cup Y_{n+1}$ such that H_s is normal in H_{s+1} and K_s is normal in K_{s+1} … Y_s is normal in Y_{s+1} respectively. Then (1) is called a neutrosophic N-subnormal series of the neutrosophic N-group $G(N)$.

Example 3.3.14: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4$ be a neutrosophic 3-group. Then the following is a neutrosophic 3-subnormal series of $G(N)$,

$$1 \leq 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \leq 2\mathbb{Z} \cup \{\pm 1, \pm j\} \cup V_4 \leq \mathbb{Z} \cup Q_8 \cup A_4 \leq \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N).$$

Definition 3.3.15: A neutrosophic N-subnormal series is called strong neutrosophic N-subnormal series if all $H_s \cup K_s \cup ... \cup Y_n$ are neutrosophic normal N-subgroups in (1) for all s.

Example 3.3.16: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle$ be a neutrosophic 3-group. Then the following is a strong neutrosophic 3-subnormal series of $G(N)$.

$$\begin{aligned} &1 \lhd \langle 6\mathbb{Z} \cup I \rangle \cup \{I, iI\} \cup \langle C_2 \cup I \rangle \lhd \langle 4\mathbb{Z} \cup I \rangle \cup \{1, \mathrm{i}, I, iI\} \cup \langle V_4 \cup I \rangle \\ &\lhd \langle 2\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm i, \pm I, \pm iI\} \cup \langle A_4 \cup I \rangle \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle = G(N) \end{aligned}$$

Theorem 3.3.17: Every strong neutrosophic N-subnormal series is trivially a neutrosophic N-subnormal series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definition 3.3.18: A neutrosophic N-subnormal series is called mixed neutrosophic N-subnormal series if some $H_s \cup K_s \cup ... \cup Y_n$ are neutrosophic normal N-subgroups in (1) while some $H_k \cup K_k \cup ... \cup Y_k$ are just normal N-subgroups in (1) for some s and k.

Example 3.3.19: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle$ be a neutrosophic 3-group. Then the following is a mixed neutrosophic 3-subnormal series of $G(N)$.

$$1 \lhd 4\mathbb{Z} \cup \{I, iI\} \cup \langle C_2 \cup I \rangle \lhd 2\mathbb{Z} \cup \{1, \mathrm{i}, I, iI\} \cup \langle V_4 \cup I \rangle$$
$$\lhd \langle 2\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm i, \pm I, \pm iI\} \cup \langle A_4 \cup I \rangle \lhd \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup \langle S_4 \cup I \rangle = G(N)$$

Theorem 3.3.20: Every mixed neutrosophic N-subnormal series is trivially a neutrosophic N-subnormal series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Definition 3.3.21: A neutrosophic N-subnormal series is called N-subnormal series if all $H_j \cup K_j \cup ... \cup Y_j$ are only normal N-subgroups in (1) for all j.

Example 3.3.22: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4$ be a neutrosophic 3-group. Then the following is a 3-subnormal series of $G(N)$,

$$1 \le 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \le 2\mathbb{Z} \cup \{\pm 1, \pm j\} \cup V_4 \le \mathbb{Z} \cup Q_8 \cup A_4 \le \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N).$$

Theorem 3.3.23: Every N-subnormal series of the N-group $G(N)$ is also a N-subnormal series of the neutrosophic N-group $G(N)$.

Definition 3.3.24: If $H_s \cup K_s \cup ... \cup Y_s$ are all normal neutrosophic N-subgroups in $G(N)$, that is H_s is normal in the neutrosophic group G_1 and K_s is normal in the neutrosophic group G_2 … Y_s is normal in the neutrosophic group G_n respectively. Then the neutrosophic N-subnormal series (1) is called neutrosophic N- normal series.

Example 3.3.25: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup Q_8 \cup S_3$ be a neutrosophic 3-group. Then the following is a neutrosophic 3-normal series of $G(N)$.

$$1 \lhd \langle 2\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm i\} \cup A_3 \lhd \langle \mathbb{Z} \cup I \rangle \cup Q_8 \cup S_3 = G(N).$$

Theorem 3.3.26: Every neutrosophic N-normal series is a neutrosophic N-subnormal series but the converse is not true.

For the converse, see the following Example.

Example 3.3.27: Let $G(N) = \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4$ be a neutrosophic 3-group. Then the following are the neutrosophic 3-subnormal series of $G(N)$.

$$1 \le 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \le 2\mathbb{Z} \cup \{\pm 1, \pm \mathrm{i}\} \cup V_4 \le \langle 2\mathbb{Z} \cup I \rangle \cup Q_8 \cup A_4 \le \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N)$$

$$1 \le 4\mathbb{Z} \cup \{\pm 1\} \cup C_2 \le 2\mathbb{Z} \cup \{\pm 1, \pm j\} \cup V_4 \le \mathbb{Z} \cup Q_8 \cup A_4 \le \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle \cup S_4 = G(N).$$

These series are not neutrosophic N-normal series as in $C_2 \cup 4\mathbb{Z}$, the cyclic group C_2 is not normal in V_4 (Klein four group).

Similarly we can define strong neutrosophic N-normal series, mixed neutrosophic N-normal series and N-normal series respectively on the same lines.

Definition 3.3.28: The neutrosophic N-normal series

$$\begin{aligned} &1 = H_0 \cup K_0 \cup \ldots \cup Y_n \lhd H_1 \cup K_1 \cup \ldots \cup Y_1 \lhd H_2 \cup K_2 \cup \ldots \cup Y_n \lhd \ldots \\ &\ldots \lhd H_{n-1} \cup K_{n-1} \cup \ldots \cup Y_{n-1} \lhd H_n \cup K_n \cup \ldots \cup Y_n = G(N) \end{aligned} \quad (2)$$

is called neutrosophic N-abelian series if the factor group $H_{s+1} \cup K_{s+1} \cup \ldots \cup Y_{s+1} / H_s \cup K_s \cup \ldots \cup Y_s$ are all abelian for all s.

Clearly we have $H_{s+1} \cup K_{s+1} \cup \ldots \cup Y_{s+1} / H_s \cup K_s \cup \ldots \cup Y_s = H_{s+1}/H_s \cup K_{s+1}/K_s \cup \ldots \cup Y_{s+1}/Y_s$

Example 3.3.29: Let $G(N) = \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is a neutrosophic 3-abelian series of $G(N)$.

$$1 \lhd A_3 \cup 4\mathbb{Z} \cup \{\pm 1\} \lhd \langle A_3 \cup I \rangle \cup 2\mathbb{Z} \cup \{\pm 1, \pm j\} \lhd \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8 = G(N).$$

Thus on the same lines, we can define strong neutrosophic N-abelian series, mixed neutrosophic N-abelian series and N-abelian series of the neutrosophic N-group $G(N)$.

Definition 3.3.30: A neutrosophic N-group $G(N)$ is called neutrosophic soluble Ngroup if $G(N)$ has a neutrosophic N-abelian series.

Example 3.3.31: Let $G(N) = \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is a neutrosophic 3-abelian series of $G(N)$.

$$1 \lhd A_3 \cup 4\mathbb{Z} \cup \{\pm 1\} \lhd \langle A_3 \cup I \rangle \cup 2\mathbb{Z} \cup \{\pm 1, \pm j\} \lhd \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8 = G(N).$$

Then clearly $G(N)$ is a neutrosophic soluble 3-group.

Theorem 3.3.32: Every N-abelian series of a N-group is also an N-abelian series of the neutrosophic N-group $G(N)$.

Theorem 3.3.33: If an N-group is a soluble N-group, then the neutrosophic N-group is also a neutrosophic soluble N-group.

Proof: The proof is followed from above theorem 3.3.32.

Theorem 3.3.34: If the neutrosophic N-group is an abelian neutrosophic N-group, then the neutrosophic N-group is neutrosophic soluble N-group.

Theorem 3.3.35: If $G(N) = \mathrm{C}(G(N))$, where $\mathrm{C}(G(N))$ is the neutrosophic N-center of $G(N)$, then $G(N)$ is a neutrosophic soluble N-group.

Definition 3.3.36: A neutrosophic N-group $G(N)$ is called strong neutrosophic soluble N-group if $G(N)$ has a strong neutrosophic N-abelian series.

This situation can be explained in the following example.

Example 3.3.37: Let $G(N) = \langle C_{12} \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is a strong neutrosophic 3-abelian series of $G(N)$.

$$\begin{aligned} &1 \lhd \langle C_2 \cup I \rangle \cup \langle 4\mathbb{Z} \cup I \rangle \cup \{\pm 1, \pm I\} \lhd \langle C_6 \cup I \rangle \cup \langle 2\mathbb{Z} \cup I \rangle \{\pm 1, \pm j, \pm I, \pm jI\} \\ &\lhd \langle C_{12} \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup \langle Q_8 \cup I \rangle = G(N) \end{aligned}.$$

Then clearly $G(N)$ is a strong neutrosophic soluble 3-group.

Theorem 3.3.38: Every strong neutrosophic soluble Ngroup $G(N)$ is trivially a neutrosophic soluble N-group but the converse is not true.

The converse is left as an exercise for the interested readers.

Definition 3.3.39: A neutrosophic N-group $G(N)$ is called mixed neutrosophic soluble N-group if $G(N)$ has a mixed neutrosophic N-abelian series.

Example 3.3.40: Let $G(N) = \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is a mixed neutrosophic 3-abelian series of $G(N)$.

$$1 \triangleleft A_3 \cup 4\mathbb{Z} \cup \{\pm 1\} \triangleleft \langle A_3 \cup I \rangle \cup 2\mathbb{Z} \cup \{\pm 1, \pm j\} \triangleleft \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8 = G(N).$$

Then clearly $G(N)$ is a mixed neutrosophic soluble 3-group.

Theorem 3.3.41: Every mixed neutrosophic soluble N-group $G(N)$ is trivially a neutrosophic soluble N-group but the converse is not true.

The converse is left as an exercise for the interested readers.

Definition 3.3.42: A neutrosophic N-group $G(N)$ is called soluble 3-group if $G(N)$ has a N-abelian series.

The following example explain this fact.

Example 3.3.43: Let $G(N) = \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is an 3-abelian series of $G(N)$.

$$1 \triangleleft A_3 \cup 2\mathbb{Z} \cup \{\pm 1, \pm j\} \triangleleft \langle S_3 \cup I \rangle \cup \langle \mathbb{Z} \cup I \rangle \cup Q_8 = G(N).$$

Then clearly $G(N)$ is a soluble 3-group.

Definition 3.3.44: Let $G(N)$ be a neutrosophic soluble N-group. Then length of the shortest neutrosophic N-abelian series of $G(N)$ is called derived N-length.

Let's take a look to the following example for derived N-length of a neutrosophic soluble N-group.

Example 3.3.45: Let $G(N)=\langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle \cup Q_8$ be a neutrosophic 3-group, where S_3 is the permutation group. Then the following is an 3-abelian series of $G(N)$.

$$1 \lhd A_3 \cup 2\mathbb{Z} \cup \{\pm 1, \pm j\} \lhd \langle S_3 \cup I\rangle \cup \langle \mathbb{Z} \cup I\rangle \cup Q_8 = G(N).$$

Then clearly $G(N)$ is a soluble 3-group and $G(N)$ has derived 3-length is 2.

Remark 3.3.46: Neutrosophic N-group of derive N-length zero is trivial neutrosophic N-group.

Proposition 3.3.47: Every neutrosophic N-subgroup of a neutrosophic soluble N-group is soluble.

Proposition 3.3.48: Quotient neutrosophic N-group of a neutrosophic soluble N-group is soluble.

We now define neutrosophic nilpotent groups and their related properties.

3.4 Neutrosophic Nilpotent Groups, Neutrosophic Nilpotent N-groups and their Properties

In this section, we introduced neutrosophic nilpotent groups and their generalization. We also discuissed some of their properties.

3.4.1 Neutrosophic Nilpotent Groups

In this section, the neutrosophic nilpotent groups are introduced. We also studies some of their core properties.

We now proceed on to define neutrosophic nilpotent groups.

Definition 3.4.1.1: Let $N(G)$ be a neutrosophic group. The series

$$1 = H_0 \lhd H_1 \lhd H_2 \lhd \ldots \lhd H_{n-1} \lhd H_n = N(G) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

is called neutrosophic central series if $H_{j+1}/H_j \subseteq C\left(N(G)/H_j\right)$ for all j.

Definition 3.4.1.2: A neutrosophic group $N(G)$ is called a neutrosophic nilpotent group if $N(G)$ has a neutrosophic central series.

Theorem 3.4.1.3: Every neutrosophic central series is a neutrosophic abelian series.

The proof is straightforward and so left as an exercise for the interested readers.

Theorem 3.4.1.4: If $N(G) = \mathrm{C}(N(G))$, then $N(G)$ is a neutrosophic nilpotent group.

It is a routine of matter for the readers to prove this theorem.

Theorem 3.4.1.5: Every neutrosophic nilpotent group $N(G)$ is a neutrosophic soluble group.

The proof is straightforward, so left as an exercise for the interested readers.

Theorem 3.4.1.6: All neutrosophic abelian groups are neutrosophic nilpotent groups.

Theorem 3.4.7: All neutrosophic cyclic groups are neutrosophic nilpotent groups.

Theorem 3.2.1.8: The direct product of two neutrosophic nilpotent groups is nilpotent.

Definition 3.4.1.9: Let $N(G)$ be a neutrosophic group. Then the neutrosophic central series (3) is called strong neutrosophic central series if all H_j's are neutrosophic normal subgroups for all j.

Theorem 3.4.1.10: Every strong neutrosophic central series is trivially a neutrosophic central series but the converse is not true in general.

Theorem 3.4.1.11: Every strong neutrosophic central series is a strong neutrosophic abelian series.

Definition 3.4.1.12: A neutrosophic group $N(G)$ is called strong neutrosophic nilpotent group if $N(G)$ has a strong neutrosophic central series.

Theorem 3.4.1.13: Every strong neutrosophic nilpotent group is trivially a neutrosophic nilpotent group.

Theorem 3.4.1.14: Every strong neutrosophic nilpotent group is also a strong neutrosophic soluble group.

Definition 3.4.1.15: Let $N(G)$ be a neutrosophic group. Then the neutrosophic central series (3) is called mixed neutrosophic central series if some $H_j's$ are neutrosophic normal subgroups while some $H_k's$ are just normal subgroups for j,k.

Theorem 3.4.1.16: Every mixed neutrosophic central series is trivially a neutrosophic central series but the converse is not true in general.

The converse can be easily seen by the help of examples.

Theorem 3.4.1.17: Every mixed neutrosophic central series is a mixed neutrosophic abelian series.

Definition 3.4.1.18: A neutrosophic group $N(G)$ is called mixed neutrosophic nilpotent group if $N(G)$ has a mixed neutrosophic central series.

Theorem 3.4.1.19: Every mixed neutrosophic nilpotent group is trivially a neutrosophic nilpotent group.

Theorem 3.4.1.20: Every mixed neutrosophic nilpotent group is also a mixed neutrosophic soluble group.

Definition 3.4.1.21: Let $N(G)$ be a neutrosophic group. Then the neutrosophic central series (3) is called central series if all H_j's are only normal subgroups for all j.

Theorem 3.4.1.22: Every central series is an abelian series.

Definition 3.4.1.23: A neutrosophic group $N(G)$ is called nilpotent group if $N(G)$ has a central series.

Theorem 3.4.1.24: Every nilpotent group is also a soluble group.

Theorem 3.4.1.25: If G is nilpotent group, then $N(G)$ is also a neutrosophic nilpotent group.

3.4.2 Neutrosophic Nilpotent Bigroups

In this section, we present neutosophic nilpotent groups and their basic properties and characterization.

Neutrosophic nilpotent groups can be defined as follows.

Definition 3.4.2.1: Let $B_N(G)$ be a neutrosophic bigroup. The series

$$1 = H_0 \cup K_0 \lhd H_1 \cup K_1 \lhd H_2 \cup K_2 \lhd \ldots \lhd H_{n-1} \cup K_{n-1} \lhd H_n \cup K_n = B_N(G) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

is called neutrosophic bi-central series if
$H_{j+1} \cup K_{j+1} / H_j \cup K_j \subseteq C\left(\mathrm{B}_N(G) / H_j \cup K_j \right)$ for all j, that is $H_{j+1} / H_j \subseteq C\left(\langle G_1 \cup I \rangle / H_j \right)$ and $K_{j+1} / K_j \subseteq C\left(\langle G_2 \cup I \rangle / K_j \right)$.

Definition 3.4.2.2: A neutrosophic bigroup $B_N(G)$ is called a neutrosophic nilpotent bigroup if $B_N(G)$ has a neutrosophic bi-central series.

Theorem 3.4.2.3: Every neutrosophic bi-central series is a neutrosophic bi-abelian series.

Theorem 3.4.2.4: If $\mathrm{B}_N(G) = \mathrm{C}(\mathrm{B}_N(G))$, then $B_N(G)$ is a neutrosophic nilpotent bigroup.

Theorem 3.4.2.5: Every neutrosophic nilpotent bigroup $\mathrm{B}_N(G)$ is a neutrosophic soluble bigroup.

Theorem 3.4.2.6: All neutrosophic abelian bi-groups are neutrosophic nilpotent bi-groups.

Theorem 3.4.2.7: The direct product of two neutrosophic nilpotent bigroups is nilpotent.

Definition 3.4.2.8: Let $B_N(G)$ be a neutrosophic bigroup. Then the neutrosophic bi-central series (3) is called strong neutrosophic bi-central series if all $H_j \cup K_j's$ are neutrosophic normal sub-bigroups for all j.

Theorem 3.4.2.9: Every strong neutrosophic bi-central series is trivially a neutrosophic bi-central series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Theorem 3.4.2.10: Every strong neutrosophic bi-central series is a strong neutrosophic bi-abelian series.

Definition 3.2.4.11: A neutrosophic bigroup $B_N(G)$ is called strong neutrosophic nilpotent bigroup if $B_N(G)$ has a strong neutrosophic bi-central series.

Theorem 3.4.2.12: Every strong neutrosophic nilpotent bigroup is trivially a neutrosophic nilpotent bigroup.

Theorem 3.2.4.13: Every strong neutrosophic nilpotent bigroup is also a strong neutrosophic soluble bigroup.

Definition 3.4.2.14: Let $B_N(G)$ be a neutrosophic bigroup. Then the neutrosophic bi-central series (3) is called mixed neutrosophic bi-central series if some $H_j \cup K_j's$ are neutrosophic normal sub-bigroups while some $H_k \cup K_k's$ are just normal sub-bigroups for j,k.

Theorem 3.4.2.15: Every mixed neutrosophic bi-central series is trivially a neutrosophic bi-central series but the converse is not true in general.

Theorem 3.4.2.16: Every mixed neutrosophic bi-central series is a mixed neutrosophic bi-abelian series.

Definition 3.4.2.17: A neutrosophic bigroup $B_N(G)$ is called mixed neutrosophic nilpotent bigroup if $B_N(G)$ has a mixed neutrosophic bi-central series.

Theorem 3.4.2.18: Every mixed neutrosophic nilpotent bigroup is trivially a neutrosophic nilpotent bi-group.

Theorem 3.4.2.19: Every mixed neutrosophic nilpotent bigroup is also a mixed neutrosophic soluble bigroup.

Definition 3.4.2.20: Let $B_N(G)$ be a neutrosophic bigroup. Then the neutrosophic bi-central series (3) is called bi-central series if all $H_j \cup K_j's$ are only normal sub-bigroups for all j.

Theorem 3.4.2.21: Every bi-central series is a bi-abelian series.

Definition 3.4.2.23: A neutrosophic bigroup $B_N(G)$ is called nilpotent bigroup if $B_N(G)$ has a bi-central series.

Theorem 3.4.2.24: Every nilpotent bi-group is also a soluble bi-group.

Theorem 3.4.2.25: If $B(G)$ is a nilpotent bigroup, then $B_N(G)$ is also a neutrosophic nilpotent bigroup.

Finally we define neutrosophic nilpotent N-groups in this chapter and this is basically a generalization of neutrosophic nilpotent groups.

3.4.4 Neutrosophic Nilpotent N-groups

In this section, we introduced neutrosophic nilpotent N-groups which is basically the union of neutrosophic finite nilpotent groups. We also studied some of their basic results which holds in neutrosophic nilpotent N-groups.

We now proceed on to define neutrosophic nilpotent N-groups.

Definition 3.4.3.1: Let $G(N)$ be a neutrosophic N-group. The series

$$\begin{aligned}&1 = H_0 \cup K_0 \cup \ldots \cup Y_n \lhd H_1 \cup K_1 \cup \ldots \cup Y_1 \lhd H_2 \cup K_2 \cup \ldots \cup Y_n \lhd \ldots \\ &\ldots \lhd H_{n-1} \cup K_{n-1} \cup \ldots \cup Y_{n-1} \lhd H_n \cup K_n \cup \ldots \cup Y_n = G(N)\end{aligned} \quad (3)$$

is called neutrosophic N-central series if

$$H_{j+1} \cup K_{j+1} \cup \ldots \cup Y_{j+1} / H_j \cup K_j \cup \ldots \cup Y_j \subseteq C\left(G(N) / H_j \cup K_j \cup \ldots \cup Y_j \right) \text{ for all } j,$$

that is $H_{j+1}/H_j \subseteq C\left(G_1/H_j\right)$, $K_{j+1}/K_j \subseteq C\left(G_2/K_j\right) \ldots Y_{j+1}/Y_j \subseteq C\left(G_n/Y_j\right)$.

Definition 3.4.3.2: A neutrosophic N-group $G(N)$ is called a neutrosophic nilpotent N-group if $G(N)$ has a neutrosophic N-central series.

Theorem 3.4.3.3: Every neutrosophic N-central series is a neutrosophic N-abelian series.

Theorem 3.4.3.4: If $G(N) = C(G(N))$, then $G(N)$ is a neutrosophic nilpotent N-group.

Theorem 3.4.3.5: Every neutrosophic nilpotent N-group $G(N)$ is a neutrosophic soluble N-group.

Theorem 3.4.3.6: All neutrosophic abelian N-groups are neutrosophic nilpotent N-groups.

Theorem 3.4.3.7: The direct product of two neutrosophic nilpotent N-groups is nilpotent.

Definition 3.4.3.8: Let $G(N)$ be a neutrosophic N-group. Then the neutrosophic N-central series (3) is called strong neutrosophic N-central series if all $H_j \cup \mathrm{K}_j \cup ... \cup \mathrm{Y}_j{}'s$ are neutrosophic normal N-subgroups for all j.

Theorem 3.4.3.9: Every strong neutrosophic N-central series is trivially a neutrosophic N-central series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Theorem 3.4.3.10: Every strong neutrosophic N-central series is a strong neutrosophic N-abelian series.

Definition 3.4.3.11: A neutrosophic N-group $G(N)$ is called strong neutrosophic nilpotent N-group if $G(N)$ has a strong neutrosophic N-central series.

Theorem 3.4.3.12: Every strong neutrosophic nilpotent N-group is trivially a neutrosophic nilpotent N-group.

Theorem 3.4.3.13: Every strong neutrosophic nilpotent N-group is also a strong neutrosophic soluble N-group.

Definition 3.4.3.14: Let $G(N)$ be a neutrosophic N-group. Then the neutrosophic N-central series (3) is called mixed neutrosophic N-central series if some $H_j \cup \mathrm{K}_j \cup ... \cup \mathrm{Y}_j's$ are neutrosophic normal N-subgroups while some $H_k \cup \mathrm{K}_k \cup ... \cup \mathrm{Y}_k's$ are just normal N-subgroups for j,k.

Theorem 3.4.3.15: Every mixed neutrosophic N-central series is trivially a neutrosophic N-central series but the converse is not true in general.

The converse is left as an exercise for the interested readers.

Theorem 3.4.3.16: Every mixed neutrosophic N-central series is a mixed neutrosophic N-abelian series.

Definition 3.4.3.17: A neutrosophic N-group $G(N)$ is called mixed neutrosophic nilpotent N-group if $G(N)$ has a mixed neutrosophic N-central series.

Theorem 3.4.3.18: Every mixed neutrosophic nilpotent N-group is trivially a neutrosophic nilpotent N-group.

Theorem 3.4.3.19: Every mixed neutrosophic nilpotent N-group is also a mixed neutrosophic soluble N-group.

Definition 3.4.3.20: Let $G(N)$ be a neutrosophic N-group. Then the neutrosophic N-central series (3) is called N-central series if all $H_j \cup K_j \cup ... \cup Y_j's$ are only normal N-subgroups for all j.

Theorem 3.4.3.21: Every N-central series is an N-abelian series.

Definition 3.4.3.22: A neutrosophic N-group $G(N)$ is called nilpotent N-group if $G(N)$ has a N-central series.

Theorem 3.4.3.23: Every nilpotent N-group is also a soluble N-group.

Theorem 3.4.3.24: If G_N is a nilpotent N-group, then $G(N)$ is also a neutrosophic nilpotent N-group.

Chapter No. 4

NEUTROSOPHIC LA-SEMIGROUP RINGS AND THEIR GENERALIZATION

In this chapter, we introduce for the first time neutrosophic LA-semigroup rings and their generalization. Basically we define three type of neutrosophic strcutures such as neutrosophic LA-semigroup rings, neutrosophic bi-LA-semigroup birings and neutrosophic N-LA-semigroup N-rings respectively. There are three sections of this chapter. In first section, we introduced neutrosophic LA-semigroup rings and give some of their basic properties. In section two, we kept neutrosophic bi-LA-semigroup birings with its characterization. In the third section, neutrosophic N-LA-semigroup rings are introduced.

4.1 Neutrosophic LA-semigroup Rings

In this section, we introduced the new notion of neutrosophic LA-semigroup ring in a natural way. We also give certain basic cocepts and other properties of neutrosophic LA-semigroup ring with many illustrative exmaples.

We now proceed on to define neutrosophic LA-semigroup ring.

Definition 4.1.1: Let $\langle S \cup I \rangle$ be any neutrosophic LA-semigroup. R be any ring with 1 which is commutative or field. We define the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ of the neutrosophic LA-semigroup $\langle S \cup I \rangle$ over the ring R as follows:

1. $R\langle S \cup I \rangle$ consists of all finite formal sum of the form $\alpha = \sum_{i=1}^{n} r_i g_i$, $n < \infty$, $r_i \in R$ and $g_i \in \langle S \cup I \rangle$ $\left(\alpha \in R\langle S \cup I \rangle\right)$.
2. Two elements $\alpha = \sum_{i=1}^{n} r_i g_i$ and $\beta = \sum_{i=1}^{m} s_i g_i$ in $R\langle S \cup I \rangle$ are equal if and only if $r_i = s_i$ and $n = m$.
3. Let $\alpha = \sum_{i=1}^{n} r_i g_i, \beta = \sum_{i=1}^{m} s_i g_i \in R\langle S \cup I \rangle$; $\alpha + \beta = \sum_{i=1}^{n} (\alpha_i + \beta_i) g_i \in R\langle S \cup I \rangle$, as $\alpha_i, \beta_i \in R$, so $\alpha_i + \beta_i \in R$ and $g_i \in \langle S \cup I \rangle$.
4. $0 = \sum_{i=1}^{n} 0 g_i$ serve as the zero of $R\langle S \cup I \rangle$.
5. Let $\alpha = \sum_{i=1}^{n} r_i g_i \in R\langle S \cup I \rangle$ then $-\alpha = \sum_{i=1}^{n} (-\alpha_i) g_i$ is such that

$$\alpha + (-\alpha) = 0$$
$$= \sum_{i=1}^{n} (\alpha_i + (-\alpha_i)) g_i$$
$$= \sum 0 g_i$$

   Thus we see that $R\langle S \cup I \rangle$ is an abelian group under $+$.
6. The product of two elements α, β in $R\langle S \cup I \rangle$ is follows:

   Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$. Then $\alpha.\beta = \sum_{\substack{1 \le i \le n \\ 1 \le j \le m}} \alpha_i.\beta_j g_i h_j$

$$= \sum_k y_k t_k$$

Where $y_k = \sum \alpha_i \beta_j$ with $g_i h_j = t_k$, $t_k \in \langle S \cup I \rangle$ and $y_k \in R$.

Clearly $\alpha.\beta \in R\langle S \cup I \rangle$.

7. Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$ and $\gamma = \sum_{k=1}^{p} \delta_k l_k$.

Then clearly $\alpha(\beta+\gamma) = \alpha\beta + \alpha\gamma$ and $(\beta+\gamma)\alpha = \beta\alpha + \gamma\alpha$ for all $\alpha, \beta, \gamma \in R\langle S \cup I \rangle$, that is the distributive law holds.

Hence $R\langle S \cup I \rangle$ is a ring under the binary operations $+$ and $\bullet$. We call $R\langle S \cup I \rangle$ as the neutrosophic LA-semigroup ring.

Similarly on the same lines, we can define neutrosophic Right Almost semigroup ring abbrivated as neutrosophic RA-semigroup ring.

Let's see the following example for further explaination.

Example 4.1.2: Let $\mathbb{R}$ be the ring of real numbers and let $N(S) = \{1,2,3,4,1I,2I,3I,4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 14.

Then $\mathbb{R}\langle S \cup I \rangle$ is a neutrosophic LA-semigroup ring.

We now give some characterization of neutrosophic LA-semigroup rings.

Theorem 4.1.3: Let $\langle S \cup I \rangle$ be a neutrosophic LA-semigroup and $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring such that $R\langle S \cup I \rangle$ is a neutrosophic LA-semigroup ring over R. Then $\langle S \cup I \rangle \subseteq R\langle S \cup I \rangle$.

The proof is straight forward, so left as an exercise for the readers.

Proposition 4.1.4: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring over the ring R. Then $R\langle S \cup I \rangle$ has non-trivial idempotents.

Remark 4.1.5: The neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ is commutative if and only if $\langle S \cup I \rangle$ is commutative neutrosophic LA-semigroup.

Remark 4.1.6: The neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$ has finite number of elements if both R and $\langle S \cup I \rangle$ are of finite order.

We just give the following examples for it.

Example 4.1.7: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example 4.1.2. Then $R\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of infinite order.

Example 4.1.8: Let $\langle S\cup I\rangle=\{1,2,3,4,5,1I,2I,3I,4I,5I\}$ with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
1	4	5	1	2	3	$4I$	$5I$	$1I$	$2I$	$3I$
2	3	4	5	1	2	$3I$	$4I$	$5I$	$1I$	$2I$
3	2	3	4	5	1	$2I$	$3I$	$4I$	$5I$	$1I$
4	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
5	5	1	2	3	4	$5I$	$1I$	$2I$	$3I$	$4I$
$1I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$
$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$
$3I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$
$4I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$
$5I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$

Table 15.

Let $\mathbb{Z}_2$ be the ring of two elements. Then $\mathbb{Z}_2\langle S\cup I\rangle$ is a neutrosophic LA-semigroup ring of finite order.

Theorem 4.1.9: Every neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ contains atleast one proper subset which is an LA-semigroup ring.

Proof: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. Then clearly $RS\subseteq R\langle S\cup I\rangle$. Thus $R\langle S\cup I\rangle$ contains an LA-semigroup ring.

Definition 4.1.10: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let P be a proper subset of $R\langle S\cup I\rangle$. Then P is called a subneutrosophic LA-semigroup ring of $R\langle S\cup I\rangle$ if $P=R\langle H\cup I\rangle$ or $Q\langle S\cup I\rangle$ or $T\langle H\cup I\rangle$.

In $P=R\langle H\cup I\rangle$, R is a ring and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$ or in $Q\langle S\cup I\rangle$, Q is a proper subring with 1 of R and $\langle S\cup I\rangle$ is a neutrosophic LA-semigroup and if $P=T\langle H\cup I\rangle$, T is a subring of R with unity and $\langle H\cup I\rangle$ is a proper neutrosophic sub LA-semigroup of $\langle S\cup I\rangle$.

This situation can be explained in the following example.

Example 4.1.11: Let $\langle S\cup I\rangle$ and $\mathbb{R}\langle S\cup I\rangle$ be as in example 4.1.2. Let $H_1=\{1,3\}, H_2=\{1,1I\}$ and $H_3=\{1,3,1I,3I\}$ are neutrosophic sub LA-semigroups. Then $\mathbb{Q}\langle S\cup I\rangle$, $\mathbb{R}H_1$, $\mathbb{Z}\langle H_2\cup I\rangle$ and $\mathbb{Q}\langle H_3\cup I\rangle$ are all subneutrosophic LA-semigroup rings of $\mathbb{R}\langle S\cup I\rangle$.

Definition4.1.12: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a neutrosophic subring if $P=\langle S_1\cup I\rangle$ where S_1 is a subring of RS or R.

See the next example for explaination.

Example 4.1.13: Let $R\langle S\cup I\rangle=\mathbb{Z}_2\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring in Example 4.1.8. Then clearly $\langle\mathbb{Z}_2\cup I\rangle$ is a neutrosophic subring of $\mathbb{Z}_2\langle S\cup I\rangle$.

Theorem 4.1.14: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring R. Then $R\langle S\cup I\rangle$ always has a nontrivial neutrosophic subring.

Proof: Let $\langle R\cup I\rangle$ be the neutrosophic ring which is generated by R and I. Clearly $\langle R\cup I\rangle\subseteq R\langle S\cup I\rangle$ and this guaranteed the proof.

Definition 4.1.15: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset T of $R\langle S\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring of $R\langle S\cup I\rangle$.

Example 4.1.16: Let $\mathbb{Z}_6\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup $\langle S\cup I\rangle$ over $\mathbb{Z}_6$. Then $T=\{0,3I\}$ is a proper subset of $\mathbb{Z}_6\langle S\cup I\rangle$ which is a pseudo neutrosophic subring of $\mathbb{Z}_6\langle S\cup I\rangle$.

Definition 4.1.17: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S\cup I\rangle$ is called a sub LA-semigroup ring if $P=R_1H$

where R_1 is a subring of R and H is a sub LA-semigroup of S. SH is the LA-semigroup ring of the sub LA-semigroup H over the subring R_1.

Theorem 4.1.18: All neutrosophic LA-semigroup rings have proper sub LA-semigroup rings.

Definition 4.1.19: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S \cup I \rangle$ is called a subring but P should not have the LA-semigroup ring structure and is defined to be a subring of $R\langle S \cup I \rangle$.

Definition 4.1.20: Let $R\langle S \cup I \rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S \cup I \rangle$ is called a neutrosophic ideal of $R\langle S \cup I \rangle$,

1. if P is a neutrosophic subring or subneutrosophic LA-semigroup ring of $R\langle S \cup I \rangle$.
2. For all $p \in P$ and $\alpha \in R\langle S \cup I \rangle$, αp and $p\alpha \in P$.

One can easily define the notions of left or right neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

We can show it in the following example.

Example 4.1.21: Let $\langle S \cup I \rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	$1I$	$2I$	$3I$
1	3	3	3	$3I$	$3I$	$3I$
2	3	3	3	$3I$	$3I$	$3I$
3	1	3	3	$1I$	$3I$	$3I$
$1I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$2I$	$3I$	$3I$	$3I$	$3I$	$3I$	$3I$
$3I$	$1I$	$3I$	$3I$	$1I$	$3I$	$3I$

Table 16.

Let $R = \mathbb{Z}$ be the ring of integers. Then $\mathbb{Z}\langle S \cup I\rangle$ is a neutrosophic LA-semigroup ring of the neutrosophic LA-semigroup over the ring $\mathbb{Z}$. Thus clearly $P = 2\mathbb{Z}\langle S \cup I\rangle$ is a neutrosophic ideal of $R\langle S \cup I\rangle$.

Definition 4.1.22: Let $R\langle S \cup I\rangle$ be a neutrosophic LA-semigroup ring. A proper subset P of $R\langle S \cup I\rangle$ is called a pseudo neutrosophic ideal of $R\langle S \cup I\rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic LA-semigroup ring of $R\langle S \cup I\rangle$.
2. For all $p \in P$ and $\alpha \in R\langle S \cup I\rangle$, αp and $p\alpha \in P$.

Definition 4.1.23: Let $R\langle S \cup I\rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S \cup I\rangle$ such that R_1 is an ideal over P i.e, $rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S \cup I\rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S\cup I\rangle$.

Definition 4.1.24: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 4.1.25: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S\cup I\rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 41.26: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1\neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P,R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

Definition 4.1.27: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle S\cup I\rangle$ such that R_1 is an ideal over P i.e, $rs, sr\in R_1$ for all $p\in P$ and $r\in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle S\cup I\rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle S\cup I\rangle$.

Definition 41.28: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 4.1.29: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup. If every subring R_1 of $R\langle S\cup I\rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$ to be a loyal neutrosophic LA-semigroup ring.

Definition 4.1.30: Let $R\langle S\cup I\rangle$ be a neutrosophic LA-semigroup ring. If for R_1, a subring P is another subring $(R_1\neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P,R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic LA-semigroup ring $R\langle S\cup I\rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

One can define pseudo quasi neutrosophic ideal, pseudo loyal quasi neutrosophic ideal and pseudo bounded quasi neutrosophic ideals of a neutrosophic LA-semigroup ring $R\langle S \cup I \rangle$.

Definition 4.1.31: Let S be an LA-semigroup and $\langle R \cup I \rangle$ be a commutative neutrosophic ring with unity. $\langle R \cup I \rangle[S]$ is defined to be the LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle R \cup I \rangle$ and $s_i \in S$.

This LA-semigroup neutrosophic ring is defined analogous to the group ring or semigroup ring.

This can be shown in the following example.

Example 4.1.32: Let $\langle \mathbb{Z}_2 \cup I \rangle = \{0, 1, I, 1+I\}$ be the neutrosophic ring and let $S = \{1, 2, 3\}$ be an LA-semigroup with the following table:

$*$	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Table 17.

Then $\langle \mathbb{Z}_2 \cup I \rangle [S]$ is an LA-semigroup neutrosophic ring.

Definition 4.1.33: Let $\langle S \cup I \rangle$ be a neutrosophic LA-semigroup and $\langle K \cup I \rangle$ be a neutrosophic field or a commutative neutrosophic ring with unity. $\langle K \cup I \rangle [\langle S \cup I \rangle]$ is defined to be the neutrosophic LA-semigroup neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle K \cup I \rangle$ and $s_i \in S$.

This can be seen in the following example.

Example 4.1.34: Let $\langle \mathbb{Z} \cup I \rangle$ be the ring of integers and let $N(S) = \{1,2,3,4,1I,2I,3I,4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 18.

Then $\langle \mathbb{Z} \cup I \rangle \langle S \cup I \rangle$ is a neutrosophic LA-semigroup neutrosophic ring.

Theorem 4.1.35: Every neutrosophic LA-semigroup neutrosophic ring contains a proper subset which is a neutrosophic LA-semigroup ring.

Proof: Let $\langle R \cup I \rangle \langle S \cup I \rangle$ be a neutrosophic LA-semigroup neutrosophic ring and let $T = R\langle S \cup I \rangle$ be a proper subset of $\langle R \cup I \rangle \langle S \cup I \rangle$. Thus clearly $T = R\langle S \cup I \rangle$ is a neutrosophic LA-semigroup ring.

We now generalize the concept of neutrosophic LA-semigroup rings to extend it to neutrosophic bi-LA-semigroup biring.

4.2 Neutrosophic Bi-LA-semigroup Birings

In this section, we define neutrosophic bi-LA-semigroup birings with some of their basic and core properties and characterization.

Definition 4.2.1: Let $R_B\langle S \cup I \rangle = \{R_1\langle S_1 \cup I \rangle \cup R_2\langle S_2 \cup I \rangle, *_1, *_2\}$ be a non-empty set with two binary operations on $R_B\langle S \cup I \rangle$. Then $R_B\langle S \cup I \rangle$ is called a neutrosophic bi-LA-semigroup biring if

1. $R_B\langle S \cup I \rangle = R_1\langle S_1 \cup I \rangle \cup R_2\langle S_2 \cup I \rangle$, where $R_1\langle S_1 \cup I \rangle$ and $R_2\langle S_2 \cup I \rangle$ are proper subsets of $R_B\langle S \cup I \rangle$.
2. $(R_1\langle S_1 \cup I \rangle, *_1, *_2)$ is a neutrosophic LA-semigroup ring and

3. $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$ is an LA-semigroup ring.

If both $R_1\langle S_1 \cup I\rangle$ and $R_2\langle S_2 \cup I\rangle$ are neutrosophic LA-semigroup rings in the above definition. Then we call it strong neutrosophic bi-LA-semigroup biring.

For further explaination, one can seen the following example.

Example 4.2.2: Let $R_B\langle S \cup I\rangle = \{\mathbb{R}_1\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle, *_1, *_2\}$, where $\mathbb{R}_1\langle S_1 \cup I\rangle$ be a neutrosophic LA-semigroup ring such that $\mathbb{R}_1$ is the ring of real numbers and let $\langle S \cup I\rangle = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 19.

and $\mathbb{Z}_2\langle S_2 \cup I\rangle$ be a neutrosophic LA-semigroup ring where $\mathbb{Z}_2 = \{0,1\}$ and $\langle S_2 \cup I\rangle$ is a neutrosophic LA-semigroup with the following table:

.	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
1	4	5	1	2	3	$4I$	$5I$	$1I$	$2I$	$3I$
2	3	4	5	1	2	$3I$	$4I$	$5I$	$1I$	$2I$
3	2	3	4	5	1	$2I$	$3I$	$4I$	$5I$	$1I$
4	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
5	5	1	2	3	4	$5I$	$1I$	$2I$	$3I$	$4I$
$1I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$
$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$
$3I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$
$4I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$
$5I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$

Table 20.

Thus $R_B\langle S \cup I\rangle = \{\mathbb{R}_1\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle, *_1, *_2\}$ is a neutrosophic bi-LA-semigroup biring.

Theorem 4.2.3: All strong neutrosophic bi-LA-semigroup birings are trivially neutrosophic bi-LA-semigroup birings.

Definition 4.2.4: Let $R_B\langle S \cup I\rangle = \{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic bi-LA-semigroup biring and let $P = \{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle\}$ be a proper subset of $R_B\langle S \cup I\rangle$. Then P is called subneutrosophic bi-LA-semigroup biring if $(R_1\langle H_1 \cup I\rangle, *_1, *_2)$ is a subneutrosophic LA-semigroup ring of $\left(R_1\langle S_1 \cup I\rangle, *_1, *_2\right)$ and $\left(R_2\langle H_2 \cup I\rangle, *_1, *_2\right)$ is a sub LA-semigroup ring of $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$.

If in the above definition, both $\left(R_1\langle H_1 \cup I\rangle, *_1, *_2\right)$ and $\left(R_2\langle H_2 \cup I\rangle, *_1, *_2\right)$ are subneutrosophic LA-semigroup rings. Then $P=\left\{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle\right\}$ is said to be strong subneutrosophic bi-LA-semigroup ring.

The next exmaple shows this fact.

Example 4.2.5: Let $R_B\langle S \cup I\rangle=\left\{\mathbb{R}_1\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle, *_1, *_2\right\}$ be a neutrosophic bi-LA-semigroup biring in Example (1). Let $P=\{\mathbb{R}_1 H_1 \cup \mathbb{Z}_2 H_2\}$ be a proper subset of $R_B\langle S \cup I\rangle$ such that $H_1=\{1,2,3,4\}$ and $H_2=\{1,2,3,4\}$. Then clearly P is a subneutrosophic bi-LA-semigroup biring of $R_B\langle S \cup I\rangle$.

Theorem 4.2.6: All strong subneutrosophic bi-LA-semigroup birings are trivially subneutrosophic bi-LA-semigroup birings.

Definition 4.2.7: Let $R_B\langle S \cup I\rangle=\left\{R_1\langle S_1 \cup I\rangle \cup R_2 \mathrm{S}_2, *_1, *_2\right\}$ be a neutrosophic bi-LA-semigroup biring and let $P=\{\langle R_1 \cup I\rangle \cup R_2\}$ be a proper subset of $R_B\langle S \cup I\rangle$. Then P is called neutrosophic subbiring if $\left(\langle R_1 \cup I\rangle, *_1, *_2\right)$ is a neutrosophic subring of $\left(R_1\langle S_1 \cup I\rangle, *_1, *_2\right)$ and $(R_2, *_1, *_2)$ is a subring of $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$.

If both R_1 and R_2 are neutrosophic subrings, then we call P to be a strong neutrosophic subbiring.

Exmple 4.2.8: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}_1\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bi-LA-semigroup biring in above Example (1). Then clearly $P=\{\langle\mathbb{R}_1\cup I\rangle\cup\mathbb{Z}_2\}$ is a neutrosophic subbiring of $R_B\langle S\cup I\rangle$ and $P=\{\langle\mathbb{R}_1\cup I\rangle\cup\langle\mathbb{Z}_2\cup I\rangle\}$ is a strong neutrosophic subbiring of $R_B\langle S\cup I\rangle$.

Theorem 42.9: If $R_B\langle S\cup I\rangle$ is a strong neutrosophic bi-LA-semigroup biring, then P is also a strong neutrosophic subbiring of $R_B\langle S\cup I\rangle$.

Definition 4.2.10: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bi-LA-semigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called pseudo neutrosophic subbiring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a pseudo neutrosophic subring of $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is also a pseudo neutrosophic subring of $(R_2\langle S_2\cup I\rangle,*_1,*_2)$.

Definition 4.2.11: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bi-LA-semigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called neutrosophic sub bi-LA-semigroup biring if $(R_1 \mathrm{H}_1,*_1,*_2)$ is a neutrosophic sub bi-LA-semigroup ring of $\left(R_1\langle S_1 \cup I\rangle, *_1, *_2\right)$ and $(R_2 \mathrm{H}_2, *_1, *_2)$ is also a sub bi-LA-semigroup ring of $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$.

See the following example.

Exmple 4.2.12: Let $R_B\langle S \cup I\rangle = \{\mathbb{R}_1\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic bi-LA-semigroup biring in above Example (1). Then clearly $P = \{\mathbb{R}_1 S_1 \cup \mathbb{Z}_2 S_2\}$ is a neutrosophic sub bi-LA-semigroup biring of $R_B\langle S \cup I\rangle$.

Definition 4.2.13: Let $R_B\langle S \cup I\rangle = \{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic bi-LA-semigroup biring and let $P = \{P_1 \cup P_2\}$ be a proper subset of $R_B\langle S \cup I\rangle$. Then P is called subbiring if $(P_1, *_1, *_2)$ is a subring of $\left(R_1\langle S_1 \cup I\rangle, *_1, *_2\right)$ and $(P_2, *_1, *_2)$ is also a subring of $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$.

Exmple 4.2.14: Let $R_B\langle S \cup I\rangle = \{\mathbb{R}_1\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic bi-LA-semigroup biring in above Example (1). Then clearly $P = \{\mathbb{R}_1 \cup \mathbb{Z}_2\}$ is a subbiring of $R_B\langle S \cup I\rangle$.

Definition 4.2.15: Let $R_B\langle S \cup I\rangle = \{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic bi-LA-semigroup biring and let $J = \{J_1 \cup J_2\}$ be a proper subset of $R_B\langle S \cup I\rangle$. Then J is called neutrosophic biideal if $(J_1, *_1, *_2)$ is a neutrosophic ideal of $\left(R_1\langle S_1 \cup I\rangle, *_1, *_2\right)$ and $(J_2, *_1, *_2)$ is just an ideal of $\left(R_2\langle S_2 \cup I\rangle, *_1, *_2\right)$.

If both J_1 and J_2 are strong neutrosophic ideals. Then J is called to be strong neutrosophic biideal of the neutrosophic bi-LA-semigroup biring $R_B\langle S\cup I\rangle$.

This can be further explained in the next example.

Example 4.2.16. Let $R_B\langle S\cup I\rangle=\{\mathbb{C}\langle S_1\cup I\rangle\cup\mathbb{Z}\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bi-LA-semigroup biring,where $\langle S_1\cup I\rangle=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 21.

and $\langle S_2\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 22.

Then $J=\{J_1\cup J_2\}$ is a neutrosophic biideal of $R_B\langle S\cup I\rangle$, where $J_1=\mathbb{R}\langle S_1\cup I\rangle$ and $J_2=3\mathbb{Z}S_2$.

Theorem 4.2.17: Every neutrosophic biideal of a neutrosophic bi-LA-semigroup biring $R_B\langle S\cup I\rangle$ is trivially a neutrosophic sub bi-LA-semigroup biring.

Theorem 4.2.18: Every strong neutrosophic biideal of a neutrosophic bi-LA-semigroup biring is trivially a neutrosophic biideal of $R_B\langle S\cup I\rangle$.

Definition 4.2.19: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bi-LA-semigroup biring and let $J=\{J_1\cup J_2\}$. Then J is called pseudo neutrosophic biideal if $(J_1,*_1,*_2)$ is a pseudo neutrosophic ideal of $\left(R_1\langle S_1\cup I\rangle,*_1,*_2\right)$ and $(J_2,*_1,*_2)$ is also a pseudo neutrosophic ideal of $\left(R_2\langle S_2\cup I\rangle,*_1,*_2\right)$.

In the next section, we introduced neutosophic N-semigroup N-ring to generalize this concept.

4.3 Neutrosophic N-LA-semigroup N-rings

In this section, we finally extend neutrosophic LA-semigroup rings to neutrosophic N-LA-semigroup N-rings and develop some interesting theory on it.

Neutrosophic N-LA-semigroup N-ring can be defined as follows.

Definition 4.3.1: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a non-empty set with n binary operations on $N\left(R\langle S\cup I\rangle\right)$. Then $N\left(R\langle S\cup I\rangle\right)$ is called a neutrosophic N-LA-semigroup N-ring if

1. $N\left(R\langle S\cup I\rangle\right)=R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup R_n\langle S_n\cup I\rangle$, where $R_i\langle S_i\cup I\rangle$ is a proper subset of $N\left(R\langle S\cup I\rangle\right)$ for all i.
2. Some of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic LA-semigroup ring for some i.
3. Some of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ are just LA-semigroup ring for some i.

If all $(R_i\langle S_i \cup I\rangle, *_i, *_i)$ are neutrosophic LA-semigroup rings, then we call $N(R\langle S \cup I\rangle)$ to be a strong neutrosophic LA-semigroup ring.

In the following example, we give the neutrosophic N-LA-semigroup N-ring.

Example 4.3.2: Let $N(R\langle S \cup I\rangle) = \{\mathbb{R}\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle \cup \mathbb{Z}\langle S_3 \cup I\rangle\}$, where $\mathbb{R}\langle S_1 \cup I\rangle$ be a neutrosophic LA-semigroup ring such that $\mathbb{R}$ is the ring of real numbers and let $\langle S \cup I\rangle = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Table 23.

and $\mathbb{Z}_2\langle S_2 \cup I\rangle$ be a neutrosophic LA-semigroup ring where $\mathbb{Z}_2 = \{0,1\}$ and $\langle S_2 \cup I\rangle$ is a neutrosophic LA-semigroup with the following table:

.	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
1	4	5	1	2	3	$4I$	$5I$	$1I$	$2I$	$3I$
2	3	4	5	1	2	$3I$	$4I$	$5I$	$1I$	$2I$
3	2	3	4	5	1	$2I$	$3I$	$4I$	$5I$	$1I$
4	1	2	3	4	5	$1I$	$2I$	$3I$	$4I$	$5I$
5	5	1	2	3	4	$5I$	$1I$	$2I$	$3I$	$4I$
$1I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$
$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$
$3I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$
$4I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$
$5I$	$5I$	$1I$	$2I$	$3I$	$4I$	$5I$	$1I$	$2I$	$3I$	$4I$

Table 24.

and $\mathbb{Z}\langle S_3 \cup I\rangle$ be an LA-semigroup ring, with $\langle S_3 \cup I\rangle = \{1, 2, 3, 1I, 2I, 3I\}$ be another neutrosophic LA-semigroup with the following table.

#	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Table 25.

Thus $N(R\langle S\cup I\rangle)=\{\mathbb{R}\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle\cup\mathbb{Z}\langle S_3\cup I\rangle\}$ is a neutrosophic 3-LA-semigroup 3-ring.

Theorem 4.3.3: All strong neutrosophic N-LA-semigroup N-rings are trivially neutrosophic N-LA-semigroup N-rings.

Definition 4.3.4: Let $N(R\langle S\cup I\rangle)=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle H_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a proper subset of $N(R\langle S\cup I\rangle)$. Then P is called subneutrosophic N-LA-semigroup N-ring if some $(R_i\langle H_i\cup I\rangle,*_i,*_i)$ are subneutrosophic LA-semigroup ring of $N(R\langle S\cup I\rangle)$ for some i and the remaining are just sub LA-semigroups for some i.

If all $(R_i\langle H_i\cup I\rangle,*_i,*_i)$ are neutrosophic sub LA-semigroups. Then P is called strong subneutrosophic N-LA-semigroup N-ring of $N(R\langle S\cup I\rangle)$ for all i.

This can be shown in the next example.

Example 4.3.5: Let $N(R\langle S\cup I\rangle)=\{\mathbb{R}\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle\cup\mathbb{Z}\langle S_3\cup I\rangle\}$ be a neutrosophic 3-LA-semigroup 3-ring in Example 4.3.2. Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\cup\mathbb{Z}\langle H_3\cup I\rangle\}$ be a proper subset of $N(R\langle S\cup I\rangle)$ such that

$H_1 = \{1,2,3,4\}$ and $H_2 = \{1,2,3,4\}$ and $\langle H_3 \cup I \rangle = \{1,2,1I,2I\}$. Then clearly P is a subneutrosophic 3-LA-semigroup 3-ring of $N(R\langle S \cup I \rangle)$.

Theorem 4.3.6: All strong subneutrosophic N-LA-semigroup N-rings are trivially subneutrosophic N-LA-semigroup N-rings.

Definition 4.3.7: Let $N\left(R\langle S \cup I \rangle\right) = \{R_1\langle S_1 \cup I \rangle \cup R_2\langle S_2 \cup I \rangle \cup ... \cup \mathrm{R}_n\langle S_n \cup I \rangle, *_1, *_2, ..., *_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $P = \{\langle P_1 \cup I \rangle \cup \langle P_2 \cup I \rangle \cup ... \cup \langle P_n \cup I \rangle\}$. Then P is called neutrosophic sub N-ring if $(\langle P_i \cup I \rangle, *_i, *_i)$ is a neutrosophic sub N-ring of $(R_i\langle S_i \cup I \rangle, *_i, *_i)$ for all i.

Theorem 4.3.8: If $N(R\langle S \cup I \rangle)$ is a strong neutrosophic N-LA-semigroup N-ring, then P is also a strong neutrosophic sub N-ring of $N(R\langle S \cup I \rangle)$.

Definition 4.3.9: Let $N\left(R\langle S \cup I \rangle\right) = \{R_1\langle S_1 \cup I \rangle \cup R_2\langle S_2 \cup I \rangle \cup ... \cup \mathrm{R}_n\langle S_n \cup I \rangle, *_1, *_2, ..., *_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $P = \{R_1\langle H_1 \cup I \rangle \cup R_2\langle H_2 \cup I \rangle \cup ... \cup R_n\langle H_n \cup I \rangle\}$. Then P is called pseudo neutrosophic subbiring if $(R_i\langle H_i \cup I \rangle, *_i, *_i)$ is a pseudo neutrosophic sub N-ring of $(R_i\langle S_i \cup I \rangle, *_i, *_i)$ for all i.

Definition 4.3.10: Let $N\left(R\langle S \cup I \rangle\right) = \{R_1\langle S_1 \cup I \rangle \cup R_2\langle S_2 \cup I \rangle \cup ... \cup \mathrm{R}_n\langle S_n \cup I \rangle, *_1, *_2, ..., *_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $P = \{R_1H_1 \cup R_2H_2 \cup ... \cup \mathrm{R}_n H_n\}$. Then P is called a neutrosophic sub N-LA-semigroup N-ring if $(R_i \mathrm{H}_i, *_i, *_i)$ is a neutrosophic sub N-LA-semigroup N-ring of $(R_i\langle S_i \cup I\rangle, *_i, *_i)$ for all i.

Definition 4.3.11: Let $N(R\langle S \cup I\rangle) = \{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle \cup ... \cup \mathrm{R}_n\langle S_n \cup I\rangle, *_1, *_2, ..., *_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $P = \{P_1 \cup P_2 \cup ... \cup P_n\}$. Then P is called sub N-ring if $(P_i, *_i, *_i)$ is a subring of $(R_i\langle S_i \cup I\rangle, *_i, *_i)$ for all i.

Definition 4.3.12: Let $N(R\langle S \cup I\rangle) = \{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle \cup ... \cup \mathrm{R}_n\langle S_n \cup I\rangle, *_1, *_2, ..., *_n\}$ be a neutrosophic N-LA-semigroup N-ring and let $J = \{J_1 \cup J_2 \cup ... \cup J_n\}$. Then J is called neutrosophic N-ideal if $(J_i, *_i, *_i)$ is a neutrosophic ideal of $(R_i\langle S_i \cup I\rangle, *_i, *_i)$ for all i.

If $(J_i, *_i, *_i)$ are neutrosophic ideals for all I, then J is said to be a strong neutrosophic N-ideal of $N(R\langle S \cup I\rangle)$.

Theorem 4.3.13: Every neutrosophic N-ideal of a neutrosophic N-LA-semigroup N-ring $N(R\langle S \cup I\rangle)$ is trivially a neutrosophic sub N-LA-semigroup N-ring.

Theorem 4.3.14: Every strong neutrosophic N-ideal of a neutrosophic N-LA-semigroup N-ring is trivially a neutrosophic N-ideal of $N\left(R\langle S\cup I\rangle\right)$.

Definition 4.3.15: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-LA-semigroup N-ring and let $J=\{J_1\cup J_2\cup\ldots J_n\}$. Then J is called pseudo neutrosophic N-ideal if $(J_i,*_i,*_i)$ is a pseudo neutrosophic ideal of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ for all i.

Chapter No. 5

NEUTROSOPHIC LOOP RINGS AND THEIR GENERALIZATION

In this chapter, the authors defined neutrosophic loop rings with their generalization. There are three section in this chapter. In first section, we discuissed about neutrosophic loop rings with their basic and core properties. In section two, we extend neutrosophic loop rings to neutrosophic biloop birings and give some exciting results of it. In the last section, the theory of neutrosophic loop rings is extended to neutrosophic N-loop N-rings which is basically the generalization of neutrosophic loop rings. Their related properties have been also discuissed in this section.

5.1 Neutrosophic Loop Rings

In this section, we introduced the new notion of neutrosophic loop ring. It is basically a combination of neutrosophic loops and rings (neutrosophic rings) in the form of finite formal sum which satisfy the conditions of a ring.

We now proceed on to define neutrosophic loop ring.

Definition 5.1.1: Let $\langle L \cup I \rangle$ be a neutrosophic loop and R be any ring with 1 which is commutative or field. We define the neutrosophic loop ring $R\langle L \cup I \rangle$ of the neutrosophic loop $\langle L \cup I \rangle$ over the ring R as follows:

1. $R\langle L \cup I \rangle$ consists of all finite formal sum of the form $\alpha = \sum_{i=1}^{n} r_i g_i$, $n < \infty$, $r_i \in R$ and $g_i \in \langle L \cup I \rangle$, where $(\alpha \in R\langle L \cup I \rangle)$.
2. Two elements $\alpha = \sum_{i=1}^{n} r_i g_i$ and $\beta = \sum_{i=1}^{m} s_i g_i$ in $R\langle L \cup I \rangle$ are equal if and only if $r_i = s_i$ and $n = m$.
3. Let $\alpha = \sum_{i=1}^{n} r_i g_i, \beta = \sum_{i=1}^{m} s_i g_i \in R\langle L \cup I \rangle$; $\alpha + \beta = \sum_{i=1}^{n} (\alpha_i + \beta_i) g_i \in R\langle L \cup I \rangle$, as $\alpha_i, \beta_i \in R$, so $\alpha_i + \beta_i \in R$ and $g_i \in \langle L \cup I \rangle$.
4. $0 = \sum_{i=1}^{n} 0 g_i$ serve as the zero of $R\langle L \cup I \rangle$.
5. Let $\alpha = \sum_{i=1}^{n} r_i g_i \in R\langle L \cup I \rangle$. Then $-\alpha = \sum_{i=1}^{n} (-\alpha_i) g_i$ is such that

$$\begin{aligned} \alpha + (-\alpha) &= 0 \\ &= \sum_{i=1}^{n} (\alpha_i + (-\alpha_i)) g_i \\ &= \sum 0 g_i \end{aligned}$$

   Thus we see that $R\langle L \cup I \rangle$ is an abelian group under $+$.
6. The product of two elements α, β in $R\langle L \cup I \rangle$ is follows:

   Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$. Then $\alpha.\beta = \sum_{\substack{1 \le i \le n \\ 1 \le j \le m}} \alpha_i . \beta_j g_i h_j$

$$= \sum_k y_k t_k$$

   Where $y_k = \sum \alpha_i \beta_j$ with $g_i h_j = t_k$, $t_k \in \langle L \cup I \rangle$ and $y_k \in R$.
   Clearly $\alpha.\beta \in R\langle G \cup I \rangle$.

7. Let $\alpha = \sum_{i=1}^{n} \alpha_i g_i$ and $\beta = \sum_{j=1}^{m} \beta_j h_j$ and $\gamma = \sum_{k=1}^{p} \delta_k l_k$.

Then clearly $\alpha(\beta+\gamma) = \alpha\beta + \alpha\gamma$ and $(\beta+\gamma)\alpha = \beta\alpha + \gamma\alpha$ for all $\alpha, \beta, \gamma \in R\langle L \cup I \rangle$, that is the distributive law holds.

Hence $R\langle L \cup I \rangle$ is a ring under the binary operations $+$ and $\bullet$. We call $R\langle L \cup I \rangle$ as the neutrosophic loop ring.

Lets take the following example for further explaination.

Example 5.1.2: Let $\mathbb{R}\langle L_7(4) \cup I \rangle$ is a neutrosophic loop ring, where $\mathbb{R}$ is the ring of real numbers and $\langle L_7(4) \cup I \rangle = \{e, 1, 2, 3, 4, 5, 6, 7, eI, 1I, 2I, 3I, 4I, 5I, 6I, 7I\}$ is a neutrosophic loop.

We now give some characterization of neutrosophic loop rings.

Theorem 5.1.3: Let $\langle L \cup I \rangle$ be a neutrosophic loop and $R\langle L \cup I \rangle$ be a neutrosophic loop ring such that $R\langle L \cup I \rangle$ is a neutrosophic loop ring over R. Then $\langle L \cup I \rangle \subseteq R\langle L \cup I \rangle$.

It is a matter of routine to prove this theorem.

Proposition 5.1.4: Let $R\langle L \cup I \rangle$ be a neutrosophic loop ring over a ring R. Then $R\langle L \cup I \rangle$ has non-trivial idempotents.

Remark 5.1.5: The neutrosophic loop ring $R\langle L \cup I\rangle$ is commutative if and only if $\langle L \cup I\rangle$ is commutative neutrosophic loop.

The proof is straightforward, so left as an exercise.

Remark 5.1.6: The neutrosophic loop ring $R\langle L \cup I\rangle$ has finite number of elements if both R and $\langle L \cup I\rangle$ have finite order.

For this, we take the following examples.

Example 5.1.7: Let $R\langle L \cup I\rangle = \mathbb{R}\langle L_7(4) \cup I\rangle$ be a neutrosophic loop ring in Example 5.1.2. Then $R\langle L \cup I\rangle$ is a neutrosophic loop ring of infinite order.

Example 5.1.8: Let $R\langle L \cup I)\rangle = \mathbb{Z}_2\langle L_7(4) \cup I\rangle$ be a neutrosophic loop ring over $\mathbb{Z}_2 = \{0,1\}$.

Then $\mathbb{Z}_2\langle L_7(4) \cup I\rangle$ is a neutrosophic loop ring of finite order.

Theorem 5.1.9: Every neutrosophic loop ring $R\langle L \cup I\rangle$ contains atleast one proper subset which is a loop ring.

Proof: Let $R\langle L \cup I\rangle$ be a neutrosophic loop ring. Then clearly $RL \subseteq R\langle L \cup I\rangle$. Thus $R\langle L \cup I\rangle$ contains a loop ring.

Definition 5.1.10: Let $R\langle L \cup I\rangle$ be a neutrosophic loop ring and let P be a proper subset of $R\langle L \cup I\rangle$. Then P is called a subneutrosophic loop ring of $R\langle L \cup I\rangle$ if $P = R\langle H \cup I\rangle$ or $Q\langle L \cup I\rangle$ or $T\langle H \cup I\rangle$.

In $P = R\langle H \cup I\rangle$, R is a ring and $\langle H \cup I\rangle$ is a proper neutrosophic subloop of $\langle L \cup I\rangle$ or in $Q\langle L \cup I\rangle$, Q is a proper subring with 1 of R and $\langle L \cup I\rangle$ is a neutrosophic loop and if $P = T\langle H \cup I\rangle$, T is a subring of R with unity and $\langle H \cup I\rangle$ is a proper neutrosophic subloop of $\langle L \cup I\rangle$.

For furthere explaination, we take the following example.

Example 5.1.11: Let $\langle L_7(4) \cup I\rangle = \{e,1,2,3,4,5,6,7,eI,1I,2I,3I,4I,5I,6I,7I\}$

be a neutrosophic loop and $\mathbb{C}$ is the field of complex numbers. Then $\mathbb{C}\langle L \cup I\rangle$ is a neutrosophic loop ring over $\mathbb{C}$. Let $L_1 = \{\mathrm{e}, \mathrm{I}, 2, 2I,\}$ and $H_2 = \{e,3\}$ are neutrosophic subloops. Then $\mathbb{Q}\langle L \cup I\rangle$, $\mathbb{R}L_1$ and $\mathbb{Z}L_2$ are all subneutrosophic loop rings of $\mathbb{C}\langle L \cup I\rangle$.

Definition 5.1.12: Let $R\langle L \cup I\rangle$ be a neutrosophic loop ring. A proper subset P of $R\langle L \cup I\rangle$ is called a neutrosophic subring if $P = \langle R_1 \cup I\rangle$ where R_1 is a subring of RL or R.

This situation can be explained in this example.

Example 5.1.13: Let $R\langle L \cup I\rangle = \mathbb{Z}_2\langle L_7(4) \cup I\rangle$ be a neutrosophic loop ring in Example 5.1.8. Then clearly $\langle \mathbb{Z}_2 \cup I\rangle$ is a neutrosophic subring of $\mathbb{Z}_2\langle L_7(4) \cup I\rangle$.

Theorem 5.1.14: Let $R\langle L\cup I\rangle$ be a neutrosophic loop ring of a neutrosophic loop over a ring R. Then $R\langle L\cup I\rangle$ always has a nontrivial neutrosophic subring.

Proof: Let $\langle R\cup I\rangle$ be the neutrosophic ring which is generated by R and I. Clearly $\langle R\cup I\rangle \subseteq R\langle L\cup I\rangle$ and this guaranteed the proof.

Definition 5.1.15: Let $R\langle L\cup I\rangle$ be a neutrosophic loop ring. A proper subset T of $R\langle L\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring of $R\langle L\cup I\rangle$.

See the following example for it.

Example 5.1.16: Let $\mathbb{Z}_6\langle L_7(4)\cup I\rangle$ be a neutrosophic loop ring of a neutrosophic loop $\langle L_7(4)\cup I\rangle$ over $\mathbb{Z}_6$. Then $T=\{0,3I\}$ is a proper subset of $\mathbb{Z}_6\langle G\cup I\rangle$ which is a pseudo neutrosophic subring of $\mathbb{Z}_6\langle L_7(4)\cup I\rangle$.

Definition 5.1.17: Let $R\langle L\cup I\rangle$ be a neutrosophic loop ring. A proper subset P of $R\langle L\cup I\rangle$ is called a subgroupoid ring if $P=R_1H$ where R_1 is a subring of R and H is a subloop of L. R_1H is the loop ring of the subloop H over the subring R_1.

One can construct several exmaples for it but here we give one example for more explaination.

Example 5.1.18: Let $\mathbb{C}\langle L_7(4)\cup I\rangle$ be a neutrosophic loop ring in Example 5.1.11. Then $P=\mathbb{Q}H$ is a subloop ring of $\mathbb{C}\langle L_7(4)\cup I\rangle$, where $\mathbb{Q}$ is a subring of $\mathbb{C}$ and $H=\{\text{e},2,eI,2I\}$ is a subloop of $\langle L_7(4)\cup I\rangle$.

Theorem 5.1.19: All neutrosophic loop rings have proper subloop rings.

Definition 5.1.120: Let $R\langle L\cup I\rangle$ be a neutrosophic loop ring. A proper subset P of $R\langle L\cup I\rangle$ is called a subring but P should not have the loop ring structure and is defined to be a subring of $R\langle L\cup I\rangle$.

Definition 5.1.21: Let $R\langle L\cup I\rangle$ be a neutrosophic loop ring over R. A proper subset P of $R\langle L\cup I\rangle$ is called a neutrosophic ideal of $R\langle L\cup I\rangle$,

1. if P is a neutrosophic subring or subneutrosophic loop ring of $R\langle L\cup I\rangle$.
2. For all $p\in P$ and $\alpha\in R\langle L\cup I\rangle$, αp and $p\alpha\in P$.

One can easily define the notions of left or right neutrosophic ideal of the neutrosophic loop ring $R\langle L\cup I\rangle$.

Example 5.1.22: Let $\langle L_7(4)\cup I\rangle$ be a neutrosophic loop in Example 5.1.11 and let $R=\mathbb{Z}$ be the ring of integers. Then $\mathbb{Z}\langle L_7(4)\cup I\rangle$ is a neutrosophic loop ring of the neutrosophic loop over the ring $\mathbb{Z}$. Thus clearly $P = 3\mathbb{Z}\langle L_7(4) \cup I \rangle$ is a neutrosophic ideal of $\mathbb{Z}\langle L_7(4) \cup I \rangle$.

Theorem 5.1.23: Every neutrosophic ideal is trivially a subneutrosophic loop ring.

Definition 5.1.24: Let $R\langle L \cup I \rangle$ be a neutrosophic loop ring. A proper subset P of $R\langle L \cup I \rangle$ is called a pseudo neutrosophic ideal of $R\langle L \cup I \rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic loop ring of $R\langle L \cup I \rangle$.
2. For all $p \in P$ and $\alpha \in R\langle L \cup I \rangle$, αp and $p\alpha \in P$.

Definition 5.1.25: Let $R\langle L \cup I \rangle$ be a neutrosophic loop ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle L \cup I \rangle$ such that R_1 is an ideal over P i.e, $rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle L \cup I \rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle L \cup I \rangle$.

Definition 5.1.26: Let $R\langle L \cup I \rangle$ be a neutrosophic loop ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 5.1.27: Let $R\langle L \cup I\rangle$ be a neutrosophic loop ring. If every subring R_1 of $R\langle L \cup I\rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic loop ring $R\langle L \cup I\rangle$ to be a loyal neutrosophic loop ring.

Definition 5.1.28: Let $R\langle L \cup I\rangle$ be a neutrosophic loop ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic loop ring $R\langle L \cup I\rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

One can define pseudo quasi neutrosophic ideal, pseudo loyal quasi neutrosophic ideal and pseudo bounded quasi neutrosophic ideals of a neutrosophic loop ring $R\langle L \cup I\rangle$.

Definition 5.1.29: Let L be a loo and $\langle R \cup I\rangle$ be a commutative neutrosophic ring with unity. $\langle R \cup I\rangle[L]$ is defined to be the loop neutrosophic ring which consist of all finite formal sums of the form

$\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle R \cup I \rangle$ and $s_i \in L$. This loop neutrosophic ring is defined analogous to the group ring or semigroup ring or LA-semigroup ring or loop ring.

This can be shown in the following example.

Example 5.1.30: Let $\langle \mathbb{Z}_2 \cup I \rangle = \{0, 1, I, 1+I\}$ be a neutrosophic ring and let $L_5(3) = \{e, 1, 2, 3, 4, 5\}$ be a loop. Then $\langle \mathbb{Z}_2 \cup I \rangle [L_5(3)]$ is a loop neutrosophic ring.

Definition 5.1.31: Let $\langle L \cup I \rangle$ be a neutrosophic loop and $\langle K \cup I \rangle$ be a neutrosophic field or a commutative neutrosophic ring with unity. $\langle K \cup I \rangle [\langle L \cup I \rangle]$ is defined to be the neutrosophic loop neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle K \cup I \rangle$ and $s_i \in L$.

Example 5.1.32: Let $\langle \mathbb{Z} \cup I \rangle$ be the ring of integers and let $\langle L_5(3) \cup I \rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ be a neutrosophic loop.Then $\langle \mathbb{Z} \cup I \rangle \langle L_5(3) \cup I \rangle$ is a neutrosophic loop neutrosophic ring.

Theorem 5.1.33: Every neutrosophic loop neutrosophic ring contains a proper subset which is a neutrosophic loop ring.

Proof: Let $\langle R \cup I \rangle \langle L \cup I \rangle$ be a neutrosophic loop neutrosophic ring and let $T = R\langle L \cup I \rangle$ be a proper subset of $\langle R \cup I \rangle \langle L \cup I \rangle$. Thus clearly $T = R\langle L \cup I \rangle$ is a neutrosophic loop ring.

In the next section, we discuiss neutrosophic biloop birings and give some of their basic results.

5.2 Neutrosophic Biloop Birings

Neutrosophic biloop birings are introduced in this section. Actually we extend the theory of neutrosophic loop rings to neutrosophic biloop birings.

Definition 5.2.1: Let $R_B\langle L \cup I \rangle = \{R_1\langle L_1 \cup I \rangle \cup R_2\langle L_2 \cup I \rangle, *_1, *_2\}$ be a non-empty set with two binary operations on $R_B\langle L \cup I \rangle$. Then $R_B\langle L \cup I \rangle$ is called a neutrosophic biloop biring if

1. $R_B\langle L \cup I \rangle = R_1\langle L_1 \cup I \rangle \cup R_2\langle L_2 \cup I \rangle$, where $R_1\langle L_1 \cup I \rangle$ and $R_2\langle L_2 \cup I \rangle$ are proper subsets of $R_B\langle L \cup I \rangle$.
2. $(R_1\langle L_1 \cup I \rangle, *_1, *_2)$ is a neutrosophic loop ring and
3. $(R_2\langle L_2 \cup I \rangle, *_1, *_2)$ is just a loop ring.

If both $R_1\langle L_1 \cup I\rangle$ and $R_2\langle L_2 \cup I\rangle$ are neutrosophic loop rings in the above definition. Then we call $R_B\langle L \cup I\rangle$ to be a strong neutrosophic biloop biring.

In the following example, we illustrate this fact.

Example 5.2.2: Let $R_B\langle L \cup I\rangle = \{\mathbb{R}\langle L_5(3) \cup I\rangle \cup \mathbb{Z}_2\langle L_7(4) \cup I\rangle, *_1, *_2\}$, where $\mathbb{R}\langle L_5(3) \cup I\rangle$ is a neutrosophic loop ring such that $\mathbb{R}$ is the ring of real numbers and $\langle L_5(3) \cup I\rangle$ is a neutrosophic loop.
Also $\mathbb{Z}_2\langle L_7(4) \cup I\rangle$ is a neutrosophic loop ring where $\mathbb{Z}_2 = \{0,1\}$ and $\langle L_7(4) \cup I\rangle$ is a neutrosophic loop.

Thus $R_B\langle L \cup I\rangle = \{\mathbb{R}\langle L_5(3) \cup I\rangle \cup \mathbb{Z}_2\langle L_7(4) \cup I\rangle, *_1, *_2\}$ is a neutrosophic biloop biring.

In fact $R_B\langle L \cup I\rangle$ is a strong neutrosophic biloop biring.

We are now giving some characterization of neutrosophic biloop birings.

Theorem 5.2.3: All strong neutrosophic biloop birings are trivially neutrosophic biloop birings.

Definition 5.2.4: Let $R_B\langle L \cup I\rangle = \{R_1\langle L_1 \cup I\rangle \cup R_2\langle L_2 \cup I\rangle, *_1, *_2\}$ be a neutrosophic biloop biring and let $P = \{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle\}$ be a proper subset of $R_B\langle L \cup I\rangle$. Then P is called subneutrosophic biloop biring if $(R_1\langle H_1 \cup I\rangle, *_1, *_2)$ is a subneutrosophic loop ring of $(R_1\langle L_1 \cup I\rangle, *_1, *_2)$ and $(R_2\langle H_2 \cup I\rangle, *_1, *_2)$ is just a subloop ring of $(R_2\langle L_2 \cup I\rangle, *_1, *_2)$.

If in the above definition, both $\left(R_1\left\langle H_1\cup I\right\rangle,*_1,*_2\right)$ and $\left(R_2\left\langle H_2\cup I\right\rangle,*_1,*_2\right)$ are subneutrosophic loop rings. Then $P=\left\{R_1\left\langle H_1\cup I\right\rangle\cup R_2\left\langle H_2\cup I\right\rangle\right\}$ is said to be strong subneutrosophic biloop biring.

This can be shown in the following example.

Example 5.2.5: Let $R_B\left\langle L\cup I\right\rangle=\left\{\mathbb{R}\left\langle L_5(3)\cup I\right\rangle\cup\mathbb{Z}_2\left\langle L_7(4)\cup I\right\rangle,*_1,*_2\right\}$ be a neutrosophic biloop biring in Example 5.2.2. Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\}$ be a proper subset of $R_B\left\langle L\cup I\right\rangle$ such that $H_1=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}$ and $H_2=\{\mathrm{e},2,eI,2\mathrm{I}\}$. Then clearly P is a subneutrosophic biloop biring of $R_B\left\langle L\cup I\right\rangle$.

Theorem 5.2.6: All strong subneutrosophic biloop birings are trivially subneutrosophic biloop birings.

Definition 5.2.7: Let $R_B\left\langle L\cup I\right\rangle=\left\{R_1\left\langle L_1\cup I\right\rangle\cup R_2L_2,*_1,*_2\right\}$ be a neutrosophic biloop biring and let $P=\left\{\left\langle R_1\cup I\right\rangle\cup R_2\right\}$ be a proper subset of $R_B\left\langle L\cup I\right\rangle$. Then P is called neutrosophic subbiring if $\left(\left\langle R_1\cup I\right\rangle,*_1,*_2\right)$ is a neutrosophic subring of $\left(R_1\left\langle L_1\cup I\right\rangle,*_1,*_2\right)$ and $\left(R_2,*_1,*_2\right)$ is a subring of $\left(R_2\left\langle L_2\cup I\right\rangle,*_1,*_2\right)$.

If both R_1 and R_2 are neutrosophic subrings, then we call P to be a strong neutrosophic subbiring.

For this, one can see the following example.

Exmple 5.2.8: Let $R_B\left\langle L\cup I\right\rangle=\left\{\mathbb{R}\left\langle L_5(3)\cup I\right\rangle\cup\mathbb{Z}_2\left\langle L_7(4)\cup I\right\rangle,*_1,*_2\right\}$ be a neutrosophic biloop biring in above Example 5.2.2. Then clearly

$P=\{\langle\mathbb{R}\cup I\rangle\cup\mathbb{Z}_2\}$ is a neutrosophic subbiring of $R_B\langle L\cup I\rangle$ and $P=\{\langle\mathbb{R}\cup I\rangle\cup\langle\mathbb{Z}_2\cup I\rangle\}$ is a strong neutrosophic subbiring of $R_B\langle L\cup I\rangle$.

Theorem 5.2.9: If $R_B\langle L\cup I\rangle$ is a strong neutrosophic biloop biring, then P is also a strong neutrosophic subbiring of $R_B\langle L\cup I\rangle$.

Definition 5.2.10: Let $R_B\langle L\cup I\rangle=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle L\cup I\rangle$. Then P is called pseudo neutrosophic subbiring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a pseudo neutrosophic subring of $(R_1\langle L_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is also a pseudo neutrosophic subring of $(R_2\langle L_2\cup I\rangle,*_1,*_2)$.

Definition 5.2.11: Let $R_B\langle L\cup I\rangle=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring and let $P=\{R_1H_1\cup R_2H_2\}$ be a proper subset of $R_B\langle L\cup I\rangle$. Then P is called neutrosophic sub biloop biring if $(R_1\mathrm{H}_1,*_1,*_2)$ is a neutrosophic subloop ring of $(R_1\langle L_1\cup I\rangle,*_1,*_2)$ and $(R_2\mathrm{H}_2,*_1,*_2)$ is a subloop ring of $(R_2\langle L_2\cup I\rangle,*_1,*_2)$.

This situation can be furthere seen in the following example.

Exmple 5.2.12: Let $R_B\langle L\cup I\rangle=\{\mathbb{R}\langle L_5(3)\cup I\rangle\cup\mathbb{Z}_2\langle L_7(4)\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring in Example 5.2.2. Then clearly $P=\{\mathbb{R}L_5(3)\cup\mathbb{Z}_2L_7(4)\}$ is a neutrosophic sub biloop biring of $R_B\langle L\cup I\rangle$.

Definition 5.2.13: Let $R_B\langle L\cup I\rangle=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring and let $P=\{P_1\cup P_2\}$ be a proper subset of $R_B\langle L\cup I\rangle$. Then P is called subbiring if $(P_1,*_1,*_2)$ is a subring of $(R_1\langle L_1\cup I\rangle,*_1,*_2)$ and $(P_2,*_1,*_2)$ is also a subring of $(R_2\langle L_2\cup I\rangle,*_1,*_2)$.

We now take an example for it.

Exmple 5.2.14: Let $R_B\langle L\cup I\rangle=\{\mathbb{R}\langle L_5(3)\cup I\rangle\cup\mathbb{Z}_2\langle L_7(4)\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring in Example 5.2.2. Then clearly $P=\{\mathbb{R}\cup\mathbb{Z}_2\}$ is a subbiring of $R_B\langle L\cup I\rangle$.

Definition 5.2.15: Let $R_B\langle L\cup I\rangle=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring and let $J=\{J_1\cup J_2\}$ be a proper subset of $R_B\langle L\cup I\rangle$. Then J is called neutrosophic biideal if $(J_1,*_1,*_2)$ is a neutrosophic ideal of $(R_1\langle L_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is just an ideal of $(R_2\langle L_2\cup I\rangle,*_1,*_2)$.

If both J_1 and J_2 are neutrosophic ideals. Then J is called to be strong neutrosophic biideal of the neutrosophic biloop biring $R_B\langle L\cup I\rangle$.

This is shown in the example below.

Example 5.2.16: Let $R_B\langle L\cup I\rangle=\{\mathbb{R}\langle L_5(3)\cup I\rangle\cup\mathbb{Z}_2\langle L_7(4)\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring. Then $J=\{J_1\cup J_2\}$ is a neutrosophic biideal of $R_B\langle L\cup I\rangle$, where $J_1=\mathbb{Q}\langle L_5(3)\cup I\rangle$ and $J_2=3\mathbb{Z}\langle L_7(4)\cup I\rangle$.

Theorem 5.2.17: Every neutrosophic biideal of a neutrosophic biloop biring $R_B\langle L\cup I\rangle$ is trivially a neutrosophic sub biloop biring.

Theorem 5.2.18: Every strong neutrosophic biideal of a neutrosophic biloop biring $R_B\langle L\cup I\rangle$ is trivially a neutrosophic biideal of $R_B\langle L\cup I\rangle$.

Definition 5.2.19: Let $R_B\langle L\cup I\rangle=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic biloop biring and let $J=\{J_1\cup J_2\}$. Then J is called pseudo neutrosophic biideal of $R_B\langle L\cup I\rangle$ if $(J_1,*_1,*_2)$ is a pseudo neutrosophic ideal of $(R_1\langle L_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is also a pseudo neutrosophic ideal of $(R_2\langle L_2\cup I\rangle,*_1,*_2)$ respectively.

We now give the generalization of neutrosophic loop rings.

5.3 Neutrosophic N-loop N-rings

In this section, we present neutrosophic N-loop N-rings which is basically the generalization of neutrosophic loop rings. We also present some of their basic properties with many illustrative examples.

We now define neutrosophic N-loop N-ring in the following way.

Definition 5.3.1: Let $N\left(R\langle L\cup I\rangle\right)=\left\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a non-empty set with n binary operations on $N\left(R\langle L\cup I\rangle\right)$. Then $N\left(R\langle L\cup I\rangle\right)$ is called a neutrosophic N-loop N-ring if

1. $N\left(R\langle L\cup I\rangle\right)=R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup\ldots\cup R_n\langle L_n\cup I\rangle$, where each $R_i\langle L_i\cup I\rangle$ is a proper subset of $N\left(R\langle L\cup I\rangle\right)$ for all i.
2. Some of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic loop rings for some i.
3. Some of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ are just loop rings for some i.

If all $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic loop rings, then we call $N\left(R\langle L\cup I\rangle\right)$ to be a strong neutrosophic N-loop N-ring.

This situation can be explaine in the following example.

Example 5.3.2: Let $N(R\langle L\cup I\rangle)=\left\{\mathbb{R}\langle L_5(3)\cup I\rangle\cup\mathbb{Z}_2\langle L_7(4)\cup I\rangle\cup\mathbb{Z}\mathrm{L}_3,*_1,*_2,*_3\right\}$, where $\mathbb{R}\langle L_5(3)\cup I\rangle$ be a neutrosophic loop ring such that $\mathbb{R}$ is the ring of real numbers and $\langle L_5(3)\cup I\rangle=\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ is a neutrosophic loop.
Also $\mathbb{Z}_2\langle L_7(4)\cup I\rangle$ is a neutrosophic loop ring where $\mathbb{Z}_2=\{0,1\}$ and $\langle L_7(4)\cup I\rangle=\{e,1,2,3,4,5,6,7,eI,1I,2I,3I,4I,5I,6I,7I\}$ is a neutrosophic loop ring and $\mathbb{Z}L_3$ is a loop ring, where $\mathbb{Z}$ is a ring integers and $L_3=\{1,2,1I,2I\}$.

Thus $N(R\langle G\cup I\rangle)=\left\{\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle\cup\mathbb{Z}\langle\mathbb{Z}_{12}\cup I\rangle\right\}$ is a neutrosophic 3-loop 3-ring.

We now give some characterization of it.

Theorem 5.3.3: All strong neutrosophic N-loop N-rings are trivially neutrosophic N-loop N-rings.

Definition 5.3.4: Let $N\left(R\langle L\cup I\rangle\right)=\left\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-loop N-ring and let $P=\left\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle H_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a proper subset of $N\left(R\langle L\cup I\rangle\right)$. Then P is called subneutrosophic N-loop N-ring if some $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ are subneutrosophic loop ring of $N\left(R\langle L\cup I\rangle\right)$ for some i and the remaining are just subloop rings for some i.

If all $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic subloop rings. Then P is called strong subneutrosophic N-loop N-ring of $N\left(R\langle L\cup I\rangle\right)$ for all i.

For this, see the following example.

Example 5.3.5: Let $N(R\langle L\cup I\rangle)=\left\{\mathbb{R}\langle L_5(3)\cup I\rangle\cup\mathbb{Z}_2\langle L_7(4)\cup I\rangle\cup\mathbb{Z}\mathrm{L}_3,*_1,*_2,*_3\right\}$ be a neutrosophic 3-loop 3-ring in Example 5.3.2. Let $P=\{\mathbb{R}_1L_1\cup\mathbb{Z}_2L_2\cup\mathbb{Z}L_3\}$ be a proper subset of $N(R\langle L\cup I\rangle)$ such that $L_1=\{\mathrm{e},2,\mathrm{eI},2\mathrm{I}\}$, $L_2=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}$ and $L_3=\{1,1\mathrm{I}\}$. Then clearly P is a subneutrosophic 3-loop 3-ring of $N(R\langle L\cup I\rangle)$.

Theorem 5.3.6: All strong subneutrosophic N-loop N-rings are trivially subneutrosophic N-loop N-rings.

Definition 5.3.7: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup...\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,...,*_n\}$ be a neutrosophic N-loop N-ring and let $P=\{\langle P_1\cup I\rangle\cup\langle P_2\cup I\rangle\cup...\cup\langle P_n\cup I\rangle\}$. Then P is called neutrosophic sub N-ring if $(\langle P_i\cup I\rangle,*_i,*_i)$ is a neutrosophic sub N-ring of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

Theorem 5.3.8: If $N(R\langle L\cup I\rangle)$ is a strong neutrosophic N-loop N-ring, then P is also a strong neutrosophic sub N-ring of $N(R\langle L\cup I\rangle)$.

Definition 5.3.9: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup...\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,...,*_n\}$ be a neutrosophic N-loop N-ring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup...\cup R_n\langle H_n\cup I\rangle\}$. Then P is called pseudo neutrosophic subbiring if $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ is a pseudo neutrosophic sub N-ring of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

Definition 5.3.10: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup...\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,...,*_n\}$ be a neutrosophic N-loop N-ring and let $P=\{R_1H_1\cup R_2H_2\cup...\cup \mathrm{R}_n\, H_n\}$. Then P is called a neutrosophic sub N-loop N-ring if $(R_i\,\mathrm{H}_i,*_i,*_i)$ is a neutrosophic sub N-loop N-ring of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

Definition 5.3.11: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup...\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,...,*_n\}$ be a neutrosophic

N-loop N-ring and let $P=\{P_1\cup P_2\cup\ldots\cup P_n\}$. Then P is called sub N-ring if $(P_i,*_i,*_i)$ is a subring of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

Definition 5.3.12: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a neutrosophic N-loop N-ring and let $J=\{J_1\cup J_2\cup\ldots\cup J_n\}$. Then J is called neutrosophic N-ideal if $(J_i,*_i,*_i)$ is a neutrosophic ideal of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

If $(J_i,*_i,*_i)$ are neutrosophic ideals for all I, then J is said to be a strong neutrosophic N-ideal of $N\left(R\langle L\cup I\rangle\right)$.

Theorem 5.3.13: Every neutrosophic N-ideal of a neutrosophic N-loop N-ring $N\left(R\langle L\cup I\rangle\right)$ is trivially a neutrosophic sub N-loop N-ring.

Theorem 5.3.14: Every strong neutrosophic N-ideal of a neutrosophic N-group N-ring is trivially a neutrosophic N-ideal of $N\left(R\langle L\cup I\rangle\right)$.

Definition 5.3.15: Let $N\left(R\langle L\cup I\rangle\right)=\{R_1\langle L_1\cup I\rangle\cup R_2\langle L_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle L_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a neutrosophic N-loop N-ring and let $J=\{J_1\cup J_2\cup\ldots J_n\}$. Then J is called pseudo neutrosophic N-ideal if $(J_i,*_i,*_i)$ is a pseudo neutrosophic ideal of $\left(R_i\langle L_i\cup I\rangle,*_i,*_i\right)$ for all i.

Chapter No. 6

NEUTROSOPHIC GROUPOID RINGS AND THEIR GENERALIZATION

In this chapter, we introduced for the first time neutrosophic groupoid rings. These neutrosophic groupoid rings satisfies the ring conditions in natural way as these are the finite formal sum. The organization of this chapter is as follows.
This chapter is divided into three sections. In section one, we presentet neutrosophic groupoid rings and their properties. In second section, neutrosophic bigroupoid birings are introduced with some of their core properties. In section three, we define neutrosophic N-groupoid N-rings and give some of their characterization.

6.1 Neutrosophic Groupoid Rings

In this section, the notion of neutrosophic groupoid ring is introduced. Neutrosophic groupoid ring ha basically a ring structure in the form of finite formal sum. We also establish some basic properties of neutrosophic groupoid rings.

We now proceed on to define neutrosophic groupoid rings.

Definition 6.1.1: Let $\langle G \cup I \rangle$ be any neutrosophic groupoid. R be any ring with 1 which is commutative or field. We define the neutrosophic groupoid ring $R\langle G\cup I\rangle$ of the neutrosophic groupoid $\langle G\cup I\rangle$ over the ring R as follows:

1. $R\langle G\cup I\rangle$ consists of all finite formal sum of the form $\alpha=\sum_{i=1}^{n} r_i g_i$, $n<\infty$, $r_i\in R$ and $g_i\in\langle G\cup I\rangle$, where $\left(\alpha\in R\langle G\cup I\rangle\right)$.
2. Two elements $\alpha=\sum_{i=1}^{n} r_i g_i$ and $\beta=\sum_{i=1}^{m} s_i g_i$ in $R\langle G\cup I\rangle$ are equal if and only if $r_i=s_i$ and $n=m$.
3. Let $\alpha=\sum_{i=1}^{n} r_i g_i, \beta=\sum_{i=1}^{m} s_i g_i\in R\langle G\cup I\rangle$; $\alpha+\beta=\sum_{i=1}^{n}(\alpha_i+\beta_i)g_i\in R\langle G\cup I\rangle$, as $\alpha_i,\beta_i\in R$, so $\alpha_i+\beta_i\in R$ and $g_i\in\langle G\cup I\rangle$.
4. $0=\sum_{i=1}^{n} 0 g_i$ serve as the zero of $R\langle G\cup I\rangle$.
5. Let $\alpha=\sum_{i=1}^{n} r_i g_i\in R\langle G\cup I\rangle$ then $-\alpha=\sum_{i=1}^{n}(-\alpha_i)g_i$ is such that

$$\begin{aligned}&\alpha+(-\alpha)=0\\&=\sum_{i=1}^{n}(\alpha_i+(-\alpha_i))g_i\\&=\sum 0 g_i\end{aligned}$$

   Thus we see that $R\langle G\cup I\rangle$ is an abelian group under $+$.
6. The product of two elements α,β in $R\langle G\cup I\rangle$ is follows:

   Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$. Then $\alpha.\beta=\sum_{\substack{1\le i\le n\\1\le j\le m}}\alpha_i.\beta_j g_i h_j$

$$=\sum_k y_k t_k$$

   Where $y_k=\sum\alpha_i\beta_j$ with $g_i h_j=t_k$, $t_k\in\langle G\cup I\rangle$ and $y_k\in R$. Clearly $\alpha.\beta\in R\langle G\cup I\rangle$.
7. Let $\alpha=\sum_{i=1}^{n}\alpha_i g_i$ and $\beta=\sum_{j=1}^{m}\beta_j h_j$ and $\gamma=\sum_{k=1}^{p}\delta_k l_k$.

Then clearly $\alpha(\beta+\gamma)=\alpha\beta+\alpha\gamma$ and $(\beta+\gamma)\alpha=\beta\alpha+\gamma\alpha$ for all $\alpha,\beta,\gamma\in R\langle G\cup I\rangle$, that is the distributive law holds.

Hence $R\langle G\cup I\rangle$ is a ring under the binary operations $+$ and $\bullet$. We call $R\langle G\cup I\rangle$ as the neutrosophic groupoid ring.

We now give example to explain this fact.

Example 6.1.2: Let $\mathbb{R}$ be the ring of real numbers and let

$$\langle \mathbb{Z}_{10}\cup I\rangle=\{0,\ 1,\ 2,3,\ldots,9,\mathrm{I},2\mathrm{I},\ldots,9\mathrm{I},1+\mathrm{I},\ldots,9+\mathrm{I}\}$$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined on $\langle Z_{10}\cup I\rangle$ by $a*b=3a+2b(\bmod 10)$ for all $a,b\in\langle Z10\cup I\rangle$.

Then $\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic groupoid ring.

We now give some characterization of neutrosophic groupoid rings.

Theorem 6.1.3: Let $\langle G\cup I\rangle$ be a neutrosophic groupoid and $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring such that $R\langle G\cup I\rangle$ is a neutrosophic groupoid ring over R. Then $\langle G\cup I\rangle\subseteq R\langle G\cup I\rangle$.

The proof is straightforward, so left as an exercise for the readers.

Proposition 6.1.4: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring over a ring R. Then $R\langle G\cup I\rangle$ has non-trivial idempotents.

Remark 6.1.5: The neutrosophic groupoid ring $R\langle G\cup I\rangle$ is commutative if and only if $\langle G\cup I\rangle$ is commutative neutrosophic groupoid.

Remark 6.1.6: The neutrosophic groupoid ring $R\langle G\cup I\rangle$ has finite number of elements if both R and $\langle G\cup I\rangle$ have finite order.

In the following example, we further explain this.

Example 6.1.7: Let $R\langle G\cup I\rangle = \mathbb{R}\langle \mathbb{Z}_{10}\cup I\rangle$ be a neutrosophic groupoid ring in Example 6.1.2. Then $R\langle G\cup I\rangle$ is a neutrosophic groupoid ring of infinite order.

Example 6.1.8: Let $R\langle G\cup)\rangle = \mathbb{Z}_2\langle \mathbb{Z}_{10}\cup I\rangle$ be a neutrosophic groupoid ring over $\mathbb{Z}_2$, where $\langle \mathbb{Z}_{10}\cup I\rangle = \{0,\ 1,\ 2,3,\ldots,9,\mathrm{I},2\mathrm{I},\ldots,9\mathrm{I},1+\mathrm{I},\ldots,9+\mathrm{I}\}$ be a neutrosophic groupoid and $*$ is defined on $\langle Z_{10}\cup I\rangle$ by $a*b = 3a+2b(\mathrm{mod}10)$ for all $a,b\in\langle Z10\cup I\rangle$.
Then $\mathbb{Z}_2\langle \mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic groupoid ring of finite order.

Theorem 6.1.9: Every neutrosophic groupoid ring $R\langle G\cup I\rangle$ contains atleast one proper subset which is a groupoid ring.

Proof: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring. Then clearly $RG\subseteq R\langle G\cup I\rangle$. Thus $R\langle G\cup I\rangle$ contains a groupoid ring.

Definition 6.1.10: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring and let P be a proper subset of $R\langle G\cup I\rangle$. Then P is called a subneutrosophic groupoid ring of $R\langle G\cup I\rangle$ if $P=R\langle H\cup I\rangle$ or $Q\langle G\cup I\rangle$ or $T\langle H\cup I\rangle$. In $P=R\langle H\cup I\rangle$, R is a ring and $\langle H\cup I\rangle$ is a proper neutrosophic subgroupoid of $\langle G\cup I\rangle$ or in $Q\langle G\cup I\rangle$, Q is a proper subring with 1 of R and $\langle G\cup I\rangle$ is a neutrosophic groupoid and if $P=T\langle H\cup I\rangle$, T is a subring of R with unity and $\langle H\cup I\rangle$ is a proper neutrosophic subgroupoid of $\langle G\cup I\rangle$.

This situation can be explained in the following example.

Example 6.1.11: Let $\langle Z_4\cup I\rangle=\left\{\begin{matrix}0,1,2,3,I,2I,3I,1+I,1+2I,1+3I\\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I\end{matrix}\right\}$

be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined a $a*b=2a+b(\text{mod}\,4)$ for all $a,b\in\langle Z_4\cup I\rangle$ and $\mathbb{C}$ be a ring of complex numbers. Then $\mathbb{C}\langle G\cup I\rangle$ is a neutrosophic groupoid ring over $\mathbb{C}$. Let $H_1=\{0,2,2I,2+2I\}$ and $H_2=\{0,2+2I\}$ are neutrosophic subgroupoids. Then $\mathbb{Q}\langle G\cup I\rangle$, $\mathbb{R}H_1$ and $\mathbb{Z}\langle H_2\cup I\rangle$ are all subneutrosophic groupoid rings of $\mathbb{C}\langle G\cup I\rangle$.

Definition 6.1.12: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring. A proper subset P of $R\langle G\cup I\rangle$ is called a neutrosophic subring if $P=\langle G_1\cup I\rangle$ where G_1 is a subring of RG or R.

See the following example for this.

Example 6.1.13: Let $R\langle G\cup I\rangle = \mathbb{Z}_2\langle G\cup I\rangle$ be a neutrosophic groupoid ring in Example 6.1.8. Then clearly $\langle \mathbb{Z}_2 \cup I\rangle$ is a neutrosophic subring of $\mathbb{Z}_2\langle G\cup I\rangle$.

Theorem 6.1.14: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring of a neutrosophic groupoid over a ring R. Then $R\langle G\cup I\rangle$ always has a nontrivial neutrosophic subring.

Proof: Let $\langle R\cup I\rangle$ be the neutrosophic ring which is generated by R and I. Clearly $\langle R\cup I\rangle \subseteq R\langle G\cup I\rangle$ and this guaranteed the proof.

Definition 6.1.15: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring. A proper subset T of $R\langle G\cup I\rangle$ which is a pseudo neutrosophic subring. Then we call T to be a pseudo neutrosophic subring of $R\langle G\cup I\rangle$.

For an instance, see the following example.

Example 6.1.16: Let $\mathbb{Z}_6\langle G\cup I\rangle$ be a neutrosophic groupoid ring of a neutrosophic groupoid $\langle G\cup I\rangle$ over $\mathbb{Z}_6$. Then $T=\{0,3I\}$ is a proper subset of $\mathbb{Z}_6\langle G\cup I\rangle$ which is a pseudo neutrosophic subring of $\mathbb{Z}_6\langle G\cup I\rangle$.

Definition 6.1.17: Let $R\langle G\cup I\rangle$ be a neutrosophic groupoid ring. A proper subset P of $R\langle G\cup I\rangle$ is called a subgroupoid ring if $P=R_1H$ where

R_1 is a subring of R and H is a subgroupoid of G. R_1H is the groupoid ring of the subgroupoid H over the subring R_1.

One can see this example for furthere explaination.

Example 6.1.18: Let $\mathbb{C}\langle \mathbb{Z}_4 \cup I \rangle$ be a neutrosophic groupoid ring in Example 6.1.11. Then $P = \mathbb{Q}H$ is a subgroupoid ring of $\mathbb{C}\langle \mathbb{Z}_4 \cup I \rangle$, where $\mathbb{Q}$ is a subring of $\mathbb{C}$ and $H = \{0,2\}$ is a subgroupoid of $\langle \mathbb{Z}_4 \cup I \rangle$.

Theorem 6.1.19: All neutrosophic groupoid rings have proper subgroupoid rings.

Definition 6.1.20: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring. A proper subset P of $R\langle G \cup I \rangle$ is called a subring but P should not have the groupoid ring structure and is defined to be a subring of $R\langle G \cup I \rangle$.

Definition 6.1.21: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring over R. A proper subset P of $R\langle G \cup I \rangle$ is called a neutrosophic ideal of $R\langle G \cup I \rangle$,

1. if P is a neutrosophic subring or subneutrosophic groupoid ring of $R\langle G \cup I \rangle$.
2. For all $p \in P$ and $\alpha \in R\langle G \cup I \rangle$, αp and $p\alpha \in P$.

One can easily define the notions of left or right neutrosophic ideal of the neutrosophic groupoid ring $R\langle G \cup I \rangle$.

We take this example for an instance.

Example 6.1.22: Let $\langle \mathbb{Z}_4 \cup I \rangle$ be a neutrosophic groupoid in Example 6.1.11 and let $R = \mathbb{Z}$ be the ring of integers. Then $\mathbb{Z}\langle \mathbb{Z}_4 \cup I \rangle$ is a neutrosophic groupoid ring of the neutrosophic groupoid over the ring $\mathbb{Z}$. Thus clearly $P = 2\mathbb{Z}\langle \mathbb{Z}_4 \cup I \rangle$ is a neutrosophic ideal of $\mathbb{Z}\langle \mathbb{Z}_4 \cup I \rangle$.

Theorem 6.1.23: Every neutrosophic ideal is trivially a subneutrosophic groupoid ring.

Definition 6.1.24: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring. A proper subset P of $R\langle G \cup I \rangle$ is called a pseudo neutrosophic ideal of $R\langle G \cup I \rangle$

1. if P is a pseudo neutrosophic subring or pseudo subneutrosophic groupoid ring of $R\langle G \cup I \rangle$.
2. For all $p \in P$ and $\alpha \in R\langle G \cup I \rangle$, αp and $p\alpha \in P$.

Definition 6.1.25: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring and let R_1 be any subring (neutrosophic or otherwise). Suppose there exist a subring P in $R\langle G \cup I \rangle$ such that R_1 is an ideal over P i.e, $rs, sr \in R_1$ for all $p \in P$ and $r \in R$. Then we call R_1 to be a quasi neutrosophic ideal of $R\langle G \cup I \rangle$ relative to P.

If R_1 only happens to be a right or left ideal, then we call R_1 to be a quasi neutrosophic right or left ideal of $R\langle G \cup I \rangle$.

Definition 6.1.26: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring. If for a given R_1, we have only one P such that R_1 is a quasi neutrosophic ideal relative to P and for no other P. Then R_1 is termed as loyal quasi neutrosophic ideal relative to P.

Definition 6.1.26: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring. If every subring R_1 of $R\langle G \cup I \rangle$ happens to be a loyal quasi neutrosophic ideal relative to a unique P. Then we call the neutrosophic groupoid ring $R\langle G \cup I \rangle$ to be a loyal neutrosophic groupoid ring.

Definition 6.1.27: Let $R\langle G \cup I \rangle$ be a neutrosophic groupoid ring. If for R_1, a subring P is another subring $(R_1 \neq P)$ such that R_1 is a quais neutrosophic ideal relative to P. In short P happens to be a quasi neutrosophic ideal relative to R_1. Then we call (P, R_1) to be a bounded quasi neutrosophic ideal of the neutrosophic groupoid ring $R\langle G \cup I \rangle$.

Similarly we can define bounded quasi neutrosophic right ideals or bounded quasi neutrosophic left ideals.

One can define pseudo quasi neutrosophic ideal, pseudo loyal quasi neutrosophic ideal and pseudo bounded quasi neutrosophic ideals of a neutrosophic groupoid ring $R\langle G \cup I \rangle$.

Definition 6.1.28: Let G be a groupoid and $\langle R \cup I \rangle$ be a commutative neutrosophic ring with unity. $\langle R \cup I \rangle[G]$ is defined to be the groupoid neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle R \cup I \rangle$ and $s_i \in G$. This groupoid neutrosophic ring is defined analogous to the group ring or semigroup ring or LA-semigroup ring.

The following example illustrate this fact.

Example 6.1.29: Let $\langle \mathbb{Z}_2 \cup I \rangle = \{0,1,I,1+I\}$ be a neutrosophic ring and let $\mathbb{Z}_4 = \{0,1,2,3\}$ be a groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b (\text{mod}\, 4)$ for all $a, b \in \mathbb{Z}_4$.
Then $\langle \mathbb{Z}_2 \cup I \rangle[\mathbb{Z}_4]$ is a groupoid neutrosophic ring.

Definition 6.1.30: Let $\langle G \cup I \rangle$ be a neutrosophic groupoid and $\langle K \cup I \rangle$ be a neutrosophic field or a commutative neutrosophic ring with unity. $\langle K \cup I \rangle[\langle G \cup I \rangle]$ is defined to be the neutrosophic groupoid neutrosophic ring which consist of all finite formal sums of the form $\sum_{i=1}^{n} r_i s_i$; $n < \infty$, $r_i \in \langle K \cup I \rangle$ and $s_i \in G$.

For an instance, see the following example.

Example 6.1.31: Let $\langle \mathbb{Z} \cup I \rangle$ be the ring of integers and let $\langle \mathbb{Z}_4 \cup I \rangle$ be a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a * b = 2a + b (\mathrm{mod}\, 4)$ for all $a, b \in \mathbb{Z}_4$.
Then $\langle \mathbb{Z} \cup I \rangle \langle \mathbb{Z}_4 \cup I \rangle$ is a neutrosophic groupoid neutrosophic ring.

Theorem 6.1.32: Every neutrosophic groupoid neutrosophic ring contains a proper subset which is a neutrosophic groupoid ring.

Proof: Let $\langle R \cup I \rangle \langle G \cup I \rangle$ be a neutrosophic groupoid neutrosophic ring and let $T = R\langle G \cup I \rangle$ be a proper subset of $\langle R \cup I \rangle \langle G \cup I \rangle$. Thus clearly $T = R\langle G \cup I \rangle$ is a neutrosophic groupoid ring.

In the next section, we define neutrosophic bigroupoid birings.

6.2 Neutrosophic Bigroupoid Birings

In this section, we extend the theory of neutrosophic groupoid rings to neutrosophic bigroupoid birings. It is basically a generalization of neutrosophic bigroupoid biring. We als present their related theory and notions in this section.

Definition 6.2.1: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a non-empty set with two binary operations on $R_B\langle G\cup I\rangle$. Then $R_B\langle G\cup I\rangle$ is called a neutrosophic bigroupoid biring if,

1. $R_B\langle G\cup I\rangle=R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle$, where $R_1\langle G_1\cup I\rangle$ and $R_2\langle G_2\cup I\rangle$ are proper subsets of $R_B\langle G\cup I\rangle$.
2. $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ is a neutrosophic groupoid ring and
3. $(R_2\langle G_2\cup I\rangle,*_1,*_2)$ is just a groupoid ring.

If both $R_1\langle G_1\cup I\rangle$ and $R_2\langle G_2\cup I\rangle$ are neutrosophic groupoid rings in the above definition. Then we call $R_B\langle G\cup I\rangle$ to be a strong neutrosophic bigroupoid biring.

The following example is given for furhtere explaination.

Example 6.2.2: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$, where $\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic groupoid ring such that $\mathbb{R}$ is the ring of real numbers and let $\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined on $\langle Z_{10}\cup I\rangle$ by $a*b=3a+2b(\bmod 10)$ for all $a,b\in\langle Z10\cup I\rangle$ .Also $\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle$ is a neutrosophic groupoid ring where $\mathbb{Z}_2=\{0,1\}$ and $\langle\mathbb{Z}_4\cup I\rangle$ is a neutrosophic groupoid with respect to the operation $*$ where $*$ is defined as $a*b=2a+b(\bmod 4)$ for all $a,b\in\mathbb{Z}_4$.

Thus $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle,*_1,*_2\}$ is a neutrosophic bigroupoid biring.

Infact it is a strong neutrosophic bigroupoid biring.

Theorem 6.2.3: All strong neutrosophic bigroupoid birings are trivially neutrosophic bigroupoid birings.

Definition 6.2.4: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle G\cup I\rangle$. Then P is called subneutrosophic bigroupoid biring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a subneutrosophic groupoid ring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is just a subgroupoid ring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If in the above definition, both $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ are subneutrosophic groupoid rings. Then $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ is said to be strong subneutrosophic bigroupoid ring.

The next example shows this fact.

Example 6.2.5: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring in Example 6.2.2. Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\}$ be a proper subset of $R_B\langle G\cup I\rangle$ such that $H_1=\{0,5,5\mathrm{I},5+5\mathrm{I}\}$ and $H_2=\{0,2+2\mathrm{I}\}$. Then clearly P is a subneutrosophic bigroupoid biring of $R_B\langle G\cup I\rangle$.

Theorem 6.2.6: All strong subneutrosophic bigroupoid birings are trivially subneutrosophic bigroupoid birings.

Definition 6.2.7: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2G_2,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $P=\{\langle R_1\cup I\rangle\cup R_2\}$ be a proper subset of $R_B\langle G\cup I\rangle$. Then P is called neutrosophic subbiring if $(\langle R_1\cup I\rangle,*_1,*_2)$ is a neutrosophic subring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(R_2,*_1,*_2)$ is a subring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If both R_1 and R_2 are neutrosophic subrings, then we call P to be a strong neutrosophic subbiring.

For an instance, we can see the following example.

Exmple 6.2.8: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring in above Example 6.2.2. Then clearly $P=\{\langle\mathbb{R}\cup I\rangle\cup\mathbb{Z}_2\}$ is a neutrosophic subbiring of $R_B\langle G\cup I\rangle$ and $P=\{\langle\mathbb{R}\cup I\rangle\cup\langle\mathbb{Z}_2\cup I\rangle\}$ is a strong neutrosophic subbiring of $R_B\langle G\cup I\rangle$.

Theorem 6.2.9: If $R_B\langle G\cup I\rangle$ is a strong neutrosophic bigroupoid biring, then P is also a strong neutrosophic subbiring of $R_B\langle G\cup I\rangle$.

Definition 6.2.10: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle G\cup I\rangle$. Then P is called pseudo neutrosophic subbiring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a pseudo neutrosophic subring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is also a pseudo neutrosophic subring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

Definition 6.2.11: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoig biring and let $P=\{R_1H_1\cup R_2H_2\}$ be a proper subset of $R_B\langle G\cup I\rangle$. Then P is called neutrosophic sub bigroupoid biring if $(R_1\mathrm{H}_1,*_1,*_2)$ is a neutrosophic subgroupoid ring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(R_2\mathrm{H}_2,*_1,*_2)$ is a subgroupoid ring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

One can easily see it in the following example.

Exmple 6.2.12: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle \mathbb{Z}_{10}\cup I\rangle\cup \mathbb{Z}_2\langle \mathbb{Z}_4\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring in Example 6.2.2. Then clearly $P=\{\mathbb{R}\mathbb{Z}_{10}\cup \mathbb{Z}_2\mathbb{Z}_4\}$ is a neutrosophic sub bigroupoid biring of $R_B\langle G\cup I\rangle$.

Definition 6.2.13: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $P=\{P_1\cup P_2\}$ be a proper subset of $R_B\langle G\cup I\rangle$. Then P is called subbiring if $(P_1,*_1,*_2)$ is a subring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(P_2,*_1,*_2)$ is also a subring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

This situation can be furthere explained in the following example.

Exmple 6.2.14: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle \mathbb{Z}_{10}\cup I\rangle\cup \mathbb{Z}_2\langle \mathbb{Z}_4\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring in Example 6.2.2. Then clearly $P=\{\mathbb{R}\cup \mathbb{Z}_2\}$ is a subbiring of $R_B\langle G\cup I\rangle$.

Definition 6.2.15: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $J=\{J_1\cup J_2\}$ be a proper subset of

$R_B\langle G\cup I\rangle$. Then J is called neutrosophic biideal if $(J_1,*_1,*_2)$ is a neutrosophic ideal of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is just an ideal of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If both J_1 and J_2 are neutrosophic ideals. Then J is called to be strong neutrosophic biideal of the neutrosophic bigroupoid biring $R_B\langle G\cup I\rangle$.

Neutrosophic biideals can be shown in the following example.

Example 6.2.16: Let $R_B\langle G\cup I\rangle=\{\mathbb{C}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}\langle\mathbb{Z}_4\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring. Then $J=\{J_1\cup J_2\}$ is a neutrosophic biideal of $R_B\langle G\cup I\rangle$, where $J_1=\mathbb{Q}\langle\mathbb{Z}_{10}\cup I\rangle$ and $J_2=3\mathbb{Z}\mathbb{Z}_4$.

Theorem 6.2.17: Every neutrosophic biideal of a neutrosophic bigroupoid biring $R_B\langle G\cup I\rangle$ is trivially a neutrosophic sub bigroupoid biring.

Theorem 6.2.18: Every strong neutrosophic biideal of a neutrosophic bigroupoid biring is trivially a neutrosophic biideal of $R_B\langle G\cup I\rangle$.

Definition 6.2.19: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroupoid biring and let $J=\{J_1\cup J_2\}$. Then J is called pseudo neutrosophic biideal of $R_B\langle G\cup I\rangle$ if $(J_1,*_1,*_2)$ is a pseudo neutrosophic ideal of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is also a pseudo neutrosophic ideal of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

In the next section, we present the generalization of neutrosophic groupoid rings in a similar fassion.

6.3 Neutrosophic N-groupoid N-rings

In this section, we define neutrosophic N-groupoid N-rings in a natural way. It is the generalization of neutrosophic groupoid ring which is a kind of extension of it. We have also constructed the related theory and other notions as well.

Definition 6.3.1: Let $N\left(R\langle G\cup I\rangle\right)=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a non-empty set with n binary operations on $N\left(R\langle G\cup I\rangle\right)$. Then $N\left(R\langle G\cup I\rangle\right)$ is called a neutrosophic N-groupoid N-ring if

1. $N\left(R\langle G\cup I\rangle\right)=R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup R_n\langle G_n\cup I\rangle$, where each $R_i\langle G_i\cup I\rangle$ is a proper subset of $N\left(R\langle G\cup I\rangle\right)$ for all i.
2. Some of $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic groupoid rings for some i.
3. Some of $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ are just groupoid rings for some i.

If all $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic groupoid rings, then we call $N\left(R\langle G\cup I\rangle\right)$ to be a strong neutrosophic N-groupoid N-ring.

For an instance, we take the following example for neutrosophic groupoid ring.

Example 6.3.2: Let $N(R\langle G\cup I\rangle)=\{\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle\cup\mathbb{Z}\langle\mathbb{Z}_{12}\cup I\rangle\}$, where $\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle$ be a neutrosophic groupoid ring such that $\mathbb{R}$ is the ring of real numbers and

$G_1=\{\langle Z_{10}\cup I\rangle \mid a*b=2a+3b(\text{mod}\,10); a,b\in\langle Z_{10}\cup I\rangle\}$. Also $\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle$ is a neutrosophic groupoid ring where $\mathbb{Z}_2=\{0,1\}$ an

$G_2=\{\langle Z_4\cup I\rangle \mid a\circ b=2a+b(\text{mod}\,4); a,b\in\langle Z_4\cup I\rangle\}$ and

$\mathbb{Z}\langle\mathbb{Z}_{12}\cup I\rangle$ is a neutrosophic groupoid ring, where $\mathbb{Z}$ is a ring integers and

$G_3=\{\langle Z_{12}\cup I\rangle \mid a*b=8a+4b(\text{mod}\,12); a,b\in\langle Z_{12}\cup I\rangle\}$.

Thus $N(R\langle G\cup I\rangle)=\{\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle\cup\mathbb{Z}_2\langle\mathbb{Z}_4\cup I\rangle\cup\mathbb{Z}\langle\mathbb{Z}_{12}\cup I\rangle\}$ is a neutrosophic 3-groupoid 3-ring.

We now present some characterization of neutrosophic groupoid rings.

Theorem 6.3.3: All strong neutrosophic N-groupoid N-rings are trivially neutrosophic Ngroupoid N-rings.

Definition 6.3.4: Let

$N\left(R\langle G\cup I\rangle\right)=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a neutrosophic N-groupoid N-ring and let

$P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle H_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a proper subset of $N\left(R\langle G\cup I\rangle\right)$. Then P is called subneutrosophic N-groupoid N-ring if some $\left(R_i\langle H_i \cup I\rangle, *_i, *_i\right)$ are subneutrosophic groupoid ring of $N\left(R\langle G \cup I\rangle\right)$ for some i and the remaining are just subgroupoid rings for some i.

If all $\left(R_i\langle H_i \cup I\rangle, *_i, *_i\right)$ are neutrosophic subgroupoid rings. Then P is called strong subneutrosophic N-groupoid N-ring of $N\left(R\langle G \cup I\rangle\right)$ for all i.

For this, we see the following example.

Example 6.3.5: Let $N(R\langle G \cup I\rangle) = \{\mathbb{R}\langle \mathbb{Z}_{10} \cup I\rangle \cup \mathbb{Z}_2\langle \mathbb{Z}_4 \cup I\rangle \cup \mathbb{Z}\langle \mathbb{Z}_{12} \cup I\rangle\}$ be a neutrosophic 3-groupoid 3-ring in Example 6.3.2. Let $P = \{\mathbb{R}_1 H_1 \cup \mathbb{Z}_2 H_2 \cup \mathbb{Z} H_3\}$ be a proper subset of $N(R\langle G \cup I\rangle)$ such that $H_1 = \{0, 5, 5\mathrm{I}, 5 + 5\mathrm{I}\}$, $H_2 = \{0, 2, 2\mathrm{I}, 2 + 2\mathrm{I}\}$ and $H_3 = \{0, 2\}$. Then clearly P is a subneutrosophic 3-groupoid 3-ring of $N(R\langle G \cup I\rangle)$.

Theorem 6.3.6: All strong subneutrosophic N-groupoid N-rings are trivially subneutrosophic N-groupoid N-rings.

Definition 6.3.7: Let $N\left(R\langle G \cup I\rangle\right) = \{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup \mathrm{R}_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\}$ be a neutrosophic N-groupoid N-ring and let $P = \{\langle P_1 \cup I\rangle \cup \langle P_2 \cup I\rangle \cup \ldots \cup \langle P_n \cup I\rangle\}$. Then P is called neutrosophic sub N-ring if $\left(\langle P_i \cup I\rangle, *_i, *_i\right)$ is a neutrosophic sub N-ring of $\left(R_i\langle G_i \cup I\rangle, *_i, *_i\right)$ for all i.

Theorem 6.3.8: If $N(R\langle G \cup I\rangle)$ is a strong neutrosophic N-groupoid N-ring, then P is also a strong neutrosophic sub N-ring of $N(R\langle G \cup I\rangle)$.

Definition 6.3.9: Let $N\left(R\langle G \cup I\rangle\right) = \left\{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup \mathrm{R}_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\right\}$ be a neutrosophic N-groupoid N-ring and let $P = \left\{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle \cup \ldots \cup R_n\langle H_n \cup I\rangle\right\}$. Then P is called pseudo neutrosophic subiring if $\left(R_i\langle H_i \cup I\rangle, *_i, *_i\right)$ is a pseudo neutrosophic sub N-ring of $\left(R_i\langle G_i \cup I\rangle, *_i, *_i\right)$ for all i.

Definition 6.3.10: Let $N\left(R\langle G \cup I\rangle\right) = \left\{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup \mathrm{R}_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\right\}$ be a neutrosophic N-groupoid N-ring and let $P = \left\{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle \cup \ldots \cup R_n\langle H_n \cup I\rangle\right\}$. Then P is called neutrosophic sub N-groupoid N-ring if $\left(R_i\langle H_i \cup I\rangle, *_i, *_i\right)$ is a neutrosophic subgroupoid ring of $\left(R_i\langle G_i \cup I\rangle, *_i, *_i\right)$ for all i.

Definition 6.3.11: Let $N\left(R\langle G \cup I\rangle\right) = \left\{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup \mathrm{R}_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\right\}$ be a neutrosophic N-groupoid N-ring and let $P = \left\{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle \cup \ldots \cup R_n\langle H_n \cup I\rangle\right\}$. Then P is called neutrosophic N-ideal if $\left(R_i\langle H_i \cup I\rangle, *_i, *_i\right)$ is a neutrosophic ideal of $\left(R_i\langle G_i \cup I\rangle, *_i, *_i\right)$ for all i.

Definition 6.3.12: Let $N\left(R\langle G\cup I\rangle\right)=\left\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-groupoid N-ring and let $P=\left\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup R_n\langle H_n\cup I\rangle\right\}$. Then P is called pseudo neutrosophic N-ideal if $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ is a pseudo neutrosophic ideal of $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ for all i.

Chapter No. 7

GENERALIZATION OF NEUTROSOPHIC RINGS, NEUTROSOPHIC FIELDS AND THEIR PROPERTIES

In this chapter, we only introduced the notions of neuturosophic birings, neutrosophic N-rings, neutrosophic bifields, and neutrosophic N-fields. It is basically the generalization of neutrosophic rings and neutrosophic fields respectively as neutrosophic rings and neutrosophic fields were already discuissed by W. B. V. Kandasamy and F. Smarandache in [165]. We also constructed the related theory and other important notion for the readers.
This chapter has three sections. In first section, we introduced neutrosophic birings and their properties. In section two, neutrosophic N-rings are defined with their core properties. In third section, we put neutrosophic bifields, neneutrosophic N-fields and their related theory and notions.

7.1 Neutrosophic Birings

Here we introduce the important notion of neutrosophic birings. Actually we extend the theory of neutrosophic rings to neutrosophic birings. It is a kind of generalization. We also give some of their exciting properties with the help of many illustrative examples.

We now proceed on to define neutrosophic birings.

Definition 7.1.1. Let $(BN(\mathrm{R}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{R}),*,\circ)$ is said to be a neutrosophic biring if $BN(\mathrm{Rs}) = R_1 \cup R_2$ where atleast one of $(\mathrm{R}_1,*,\circ)$ or $(\mathrm{R}_2,*,\circ)$ is a neutrosophic ring and other is just a ring. R_1 and R_2 are proper subsets of $BN(\mathrm{R})$.

The following example explain this fact.

Example 7.1.2: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring.

We now give some characterization of neutrosophic birings.

Theorem 7.1.3: Every neutrosophic biring contains a corresponding biring.

Definition 7.1.4: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a commutative neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a commutative neutrosophic ring.

For instance, see the following example.

Example 7.1.5: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a commutative neutrosophic ring and $(\mathrm{R}_2,*,\circ)$ is also a commutative ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a commutative neutrosophic biring.

Definition 7.1.6: Let $BN(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a pseudo neutrosophic biring if each $(\mathrm{R}_1, *, \circ)$ and $(\mathrm{R}_2, *, \circ)$ is a pseudo neutrosophic ring.

We show this in the next example.

Example 7.1.7: Let $BN(\mathrm{R}) = (\mathrm{R}_1, +, \times) \cup (\mathrm{R}_2, +, \circ)$ where $(\mathrm{R}_1, +, \times) = \{0, I, 2I, 3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4 and $(\mathrm{R}_2, +, \times) = \{0, \pm 1\mathrm{I}, \pm 2\mathrm{I}, \pm 3\mathrm{I}, \ldots\}$ is another pseudo neutrosophic ring. Thus $(BN(\mathrm{R}), +, \times)$ is a pseudo neutrosophic biring.

Theorem 7.1.8: Every pseudo neutrosophic biring is trivially a neutrosophic biring but the converse may not be true.

Definition 7.1.9: Let $(BN(\mathrm{R}) = R_1 \cup R_2; *, \circ)$ be a neutrosophic biring. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbiring of $BN(\mathrm{R})$ if,

1. $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and
2. At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic ring.

In this example, we give the neutrosophic subbiring of a neutrosophic biring.

Example 7.1.10: Let $BN(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ)$ where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$ and $(\mathrm{R}_2, *, \circ) = (\mathbb{C}, +, \times)$. Let $P = P_1 \cup P_2$ be a proper subset of $BN(R)$**,** where $P_1 = (\mathbb{Q}, +, \times)$ and $P_2 = (\mathbb{R}, +, \times)$. Clearly $(P, +, \times)$ is a neutrosophic subbiring of $BN(R)$.

Definition 7.1.11: If both $(\mathrm{R}_1, *)$ and $(\mathrm{R}_2, \circ)$ in the above definition 7.1.1 are neutrosophic rings then we call $(BN(\mathrm{R}), *, \circ)$ to be a strong neutrosophic biring.

One can see the example below for it.

Example 7.1.12: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\langle \mathbb{Q} \cup I \rangle,+,\times)$. Clearly R_1 and R_2 are neutrosophic rings under addition and multiplication. Thus $(BN(\mathrm{R}),*,\circ)$ is a strong neutrosophic biring.

Theorem 7.1.13: All strong neutrosophic birings are trivially neutrosophic birings but the converse is not true in general.

To see the converse, we take the following Example.

Example 7.1.14: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring but not a strong neutrosophic biring.

Remark 7.1.15: A neutrosophic biring can have subbirings, neutrosophic subbirings, strong neutrosophic subbirings and pseudo neutrosohic subbirings.

Definition 7.1.16: Let $(BN(\mathrm{R}) = R_1 \cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a neutrosophic biideal of $BN(R)$ if,

1. $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and

2. At least one of $(T_1,*,\circ)$ or $(T_2,*,\circ)$ is a neutrosophic ideal.

If both $(T_1,*,\circ)$ and $(T_2,*,\circ)$ in the above definition are neutrosophic ideals, then we call $(T,*,\circ)$ to be a strong neutrosophic biideal of $BN(R)$.
We take this example to show the strong neutrosophic biideal.

Example 7.1.17: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}_{12}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Z}_{16},+,\times)$. Let $P=P_1\cup P_2$ be a neutrosophic subbiring of $BN(R)$, where $P_1=\{0,6,2I,4I,6I,8I,10I,6+2I,...,6+10I\}$ and $P_2=\{02I,4I,6I,8I,10I,12I,14I\}$. Clearly $(P,+,\times)$ is a neutrosophic biideal of $BN(R)$.

Theorem 7.1.18: Every neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.1.19: Every strong neutrosophic biideal is trivially a neutrosophic biideal but the converse may not be true.

By taking example, one can easily verify the converse.

Theorem 7.1.20: Every strong neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.1.21: Every strong neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Definition 7.1.22: Let $(BN(\mathrm{R})=R_1\cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a pseudo neutrosophic biideal of $BN(R)$ if

1. $T=T_1\cup T_2$ where $T_1=R_1\cap T$ and $T_2=R_2\cap T$ and
2. $(T_1,*,\circ)$ and $(T_2,*,\circ)$ are pseudo neutrosophic ideals.

Theorem 7.1.23: Every pseudo neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

One can easily verify the converse by the help of examples.

Theorem 7.1.24: Every pseudo neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.1.25: Every pseudo neutrosophic biideal is trivially a neutrosophic biideal but the converse may not be true.

One can easily see the converse by the help of example.

Theorem 7.1.26: Every pseudo neutrosophic biideal is trivially a strong neutrosophic biideal but the converse may not be true.

The converse is straight forward, so left as an exercise for the interested readers.

In the next section, we present the generalization of neutrosophic rings.

7.2 Neutrosophic N-rings

In this section, we extend neutrosophic rings to neutrosophic N-rings. This is the generalization of neutrosophic rings. We also give some characterization of neutrosophic N-rings.

Definition 7.2.1: Let $\{N(R), *_1, ..., *_2, \circ_1, \circ_2, ..., \circ_N\}$ be a non-empty set with two N-binary operations defined on it. We call $N(R)$ a neutrosophic N-ring (N a positive integer) if the following conditions are satisfied.

1. $N(R) = R_1 \cup R_2 \cup ... \cup R_N$ where each R_i is a proper subset of $N(R)$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2. $(R_i, *_i, \circ_i)$ is either a neutrosophic ring or a ring for $i = 1, 2, 3, ..., N$.

This situation can be shown in the following example.

Example 7.2.2: Let $N(R) = (R_1, *, \circ) \cup (R_2, *, \circ) \cup (R_3, *, \circ)$ where $(R_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(R_2, *, \circ) = (\mathbb{Q}, +, \times)$ and $(R_3, *, \circ) = (Z_{12}, +, \times)$. Thus $(N(R), *, \circ)$ is a neutrosophic N-ring.

Theorem 7.2.3: Every neutrosophic N-ring contains a corresponding N-ring.

Definition 7.2.4: Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. Then $N(R)$ is called a pseudo neutrosophic N-ring if each $(R_i, *_i)$ is a pseudo neutrosophic ring where $i = 1, 2, ..., N$.

For an instance, take the following example.

Example 7.2.5: Let $N(\mathrm{R}) = (\mathrm{R}_1,+,\times) \cup (\mathrm{R}_2,+,\times) \cup (\mathrm{R}_3,+,\times)$ where $(\mathrm{R}_1,+,\times) = \{0, I, 2I, 3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4, $(\mathrm{R}_2,+,\times) = \{0, \pm 1\mathrm{I}, \pm 2\mathrm{I}, \pm 3\mathrm{I}, \ldots\}$ is a pseudo neutrosophic ring and $(\mathrm{R}_3,+,\times) = \{0, \pm 2\mathrm{I}, \pm 4\mathrm{I}, \pm 6\mathrm{I} \ldots\}$. Thus $(N(\mathrm{R}),+,\times)$ is a pseudo neutrosophic 3-ring.

Theorem 7.2.6: Every pseudo neutrosophic N-ring is trivially a neutrosophic N-ring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Definition 7.2.7: If all the N-rings $(\mathrm{R}_i, *_i)$ in definition * are neutrosophic rings (i.e. for $i = 1, 2, 3, \ldots, N$) then we call $\mathrm{N(R)}$ to be a neutrosophic strong N-ring.

Example 7.2.8: Let $N(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ) \cup (\mathrm{R}_3,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$, $(\mathrm{R}_2,*,\circ) = (\langle \mathbb{Q} \cup I \rangle,+,\times)$ and $(R_3,*,\circ) = (\langle \mathbb{Z}_{12} \cup I \rangle,+,\times)$. Thus $(N(\mathrm{R}),*,\circ)$ is a strong neutrosophic N-ring.

Theorem 7.2.9: All strong neutrosophic N-rings are neutrosophic N-rings but the converse may not be true.

The converse is left as an exercise for the readers.

Definition 7.2.10: Let $\mathrm{N(R)} = \{\mathrm{R}_1 \cup R_2 \cup \ldots. \cup \mathrm{R}_N, *_1, *_2, \ldots, *_N, \circ_1, \circ_2, \ldots, \circ_N\}$ be a neutrosophic N-ring. A proper subset $P=\{\mathrm{P}_1 \cup P_2 \cup \ldots . \mathrm{P}_N, *_1, *_2, \ldots, *_N\}$ of $\mathrm{N(R)}$ is said to be a neutrosophic N-subring if $P_i = P \cap R_i, i=1,2,\ldots,N$ are subrings of R_i in which atleast some of the subrings are neutrosophic subrings.

One can see the following example for furthere illustration.

Example 7.2.11: Let $N(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)\cup(\mathrm{R}_3,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{R}\cup I\rangle,+,\times)$, $(\mathrm{R}_2,*,\circ)=(\mathbb{C},+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathrm{Z}_{10},+,\times)$ Let $P=P_1\cup P_2\cup P_3$ be a proper subset of $N(R)$, where $P_1=(\mathbb{Q},+,\times)$, $P_2=(\mathbb{R},+,\times)$ and $(\mathrm{R}_3,*,\circ)=\{0,2,4,6,8,\mathrm{I},2\mathrm{I},4\mathrm{I},6\mathrm{I},8\mathrm{I}\}$. Clearly $(P,+,\times)$ is a neutrosophic sub 3-ring of $N(R)$.

Definition 7.2.12: Let $\mathrm{N(R)}=\{\mathrm{R}_1\cup R_2\cup\ldots.\cup\mathrm{R}_N,*_1,*_2,\ldots,*_N,\circ_1,\circ_2,\ldots,\circ_N\}$ be a neutrosophic N-ring. A proper subset $T=\{\mathrm{T}_1\cup T_2\cup\ldots.\cup\mathrm{T}_N,*_1,*_2,\ldots,*_N,\circ_1,\circ_2,\ldots,\circ_N\}$ of $N(R)$ is said to be a neutrosophic strong sub N-ring if each $(T_i,*_i)$ is a neutrosophic subring of $(\mathrm{R}_i,*_i,\circ_i)$ for $i=1,2,\ldots,N$ where $T_i = R_i \cap T$.

Remark 7.2.13: A strong neutrosophic su N-ring is trivially a neutrosophic sub N-ring but the converse is not true.

The converse is left as an exercise for the interested readers.

Remark 7.2.14: A neutrosophic N-ring can have sub N-rings, neutrosophic sub N-rings, strong neutrosophic sub N-rings and pseudo neutrosohic sub N-rings.

Definition 7.2.15: Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1. Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.
2. Each it is a two sided ideal of R_t for $t = 1, 2, ..., N$.

If $(P_i, *_i, \circ_i)$ in the above definition are neutrosophic ideals, then we call $(P_i, *_i, \circ_i)$ to be a strong neutrosophic N-ideal of $N(R)$.

Theorem 7.2.16: Every neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.2.17: Every strong neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

The converse is left as an exercise for the interested readrs.

Theorem 7.2.18: Every strong neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.2.19: Every strong neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Definition 7.2.20: Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a pseudo neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1. Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.
2. Each $(P_i, *_i, \circ_i)$ is a pseudo neutrosophic ideal.

Theorem 7.2.21: Every pseudo neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

The converse is left as an exercise for the readers by taking examples.

Theorem 7.2.22: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic sub N-ring but the converse may not be true.

The converse is left as an exercise for the interested readers.

Theorem 7.2.23: Every pseudo neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

The convers can be easily seen by the help of examples.

Theorem 7.2.24: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic N-ideal but the converse may not be true.

The converse is left as an exercise for the interested readers.

We now proceed on to construct the generalization of neutrosophic fields.

7.3 Neutrosophic Bi-fields and Neutrosophic N-fields

In this section, we present the generalization of neutrosophic fields. Basically we define neutrosophic bifields and neutrosophic N-fields respectively. We aslo give some characterization of it.

Definition 7.3.1. Let $(BN(\mathrm{F}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. Then $(BN(\mathrm{F}),*,\circ)$ is said to be a neutrosophic bifield if $BN(\mathrm{F}) = F_1 \cup F_2$ where atleast one of $(\mathrm{F}_1,*,\circ)$ or $(\mathrm{F}_2,*,\circ)$ is a neutrosophic field and other is just a field. F_1 and F_2 are proper subsets of $BN(\mathrm{F})$.

If in the above definition both $(\mathrm{F}_1,*,\circ)$ and $(\mathrm{F}_2,*,\circ)$ are neutrosophic fields, then we call $(BN(\mathrm{F}),*,\circ)$ to be a strong neutrosophic bifield.

We now give an example to illustrate this fact.

Example 7.3.2: Let $BN(\mathrm{F}) = (\mathrm{F}_1,*,\circ) \cup (\mathrm{F}_2,*,\circ)$ where $(\mathrm{F}_1,*,\circ) = (\langle \mathbb{C} \cup I \rangle,+,\times)$ and $(\mathrm{F}_2,*,\circ) = (\mathbb{Q},+,\times)$. Clearly $(\mathrm{F}_1,*,\circ)$ is a neutrosophic field and $(\mathrm{F}_2,*,\circ)$ is just a field. Thus $(BN(\mathrm{F}),*,\circ)$ is a neutrosophic bifield.

Theorem 7.3.3: All strong neutrosophic bifields are trivially neutrosophic bifields but the converse is not true.

Definition 7.3.4: Let $BN(\mathrm{F}) = (\mathrm{F}_1 \cup F_2, *, \circ)$ be a neutrosophic bifield. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbifield of $BN(\mathrm{F})$ if,

1. $T = T_1 \cup T_2$ where $T_1 = F_1 \cap T$ and $T_2 = F_2 \cap T$ and
2. At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic field and the other is just a field.

For an instance, one can see the following example.

Example 7.3.5: Let $BN(\mathrm{F}) = (\mathrm{F}_1, *, \circ) \cup (\mathrm{F}_2, *, \circ)$ where $(\mathrm{F}_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$ and $(\mathrm{F}_2, *, \circ) = (\mathbb{C}, +, \times)$. Let $P = P_1 \cup P_2$ be a proper subset of $BN(\mathrm{F})$**,** where $P_1 = (\mathbb{Q}, +, \times)$ and $P_2 = (\mathbb{R}, +, \times)$. Clearly $(P, +, \times)$ is a neutrosophic subbifield of $BN(\mathrm{F})$.

We now define neutrosophic N-fields with some of their basic properties.

Definition 7.3.6: Let $\{\mathrm{N(F)}, *_1, ..., *_2, \circ_1, \circ_2, ..., \circ_N\}$ be a non-empty set with two N-binary operations defined on it. We call $N(F)$ a neutrosophic N-field (N a positive integer) if the following conditions are satisfied.

1. $\mathrm{N(F)} = F_1 \cup F_2 \cup ... \cup F_N$ where each F_i is a proper subset of $\mathrm{N(F)}$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2. $(F_i, *_i, \circ_i)$ is either a neutrosophic field or just a field for $i = 1, 2, 3, ..., N$.

If in the above definition each $(F_i, *_i, \circ_i)$ is a neutrosophic field, then we call $N(F)$ to be a strong neutrosophic N-field.

Theorem 7.3.7: Every strong neutrosophic N-field is obviously a neutrosophic field but the converse is not true.

One can easily check the converse by the help of examples.

Definition 7.3.5: Let $N(F) = \{F_1 \cup F_2 \cup .... \cup F_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-field. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ of $N(F)$ is said to be a neutrosophic N-subfield if each $(T_i, *_i)$ is a neutrosophic subfield of $(F_i, *_i, \circ_i)$ for $i = 1, 2, ..., N$ where $T_i = F_i \cap T$.

Chapter No. 8

GENERALIZATION OF NENTROSOPHIC GROUP RINGS AND THEIR PROPERTIES

The notion of neutrosophic group ring have been discuissed in [165] by W. B. V. Kandasamy and F. Smarandache. In this chapter, we present neutrosophic bigroup birings and neutrosophic N-group N-rings respectively. It is the generalization of neutrosophic group ring. We also give some of their basic and fundamental properites of neutrosophic bigroup birings and neutrosophic N-group N-rings.

8.1 Neutrosophic Bigroup Birings

In this section, we introduce neutrosophic bigroup birings and discuiss some of their basic properties and characteristics.

We now proceed on to define neutrosophic bigroup biring.

Definition 8.1.1: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a non-empty set with two binary operations on $R_B\langle G\cup I\rangle$. Then $R_B\langle G\cup I\rangle$ is called a neutrosophic bigroup biring if

1. $R_B\langle G\cup I\rangle=R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle$, where $R_1\langle G_1\cup I\rangle$ and $R_2\langle G_2\cup I\rangle$ are proper subsets of $R_B\langle G\cup I\rangle$.
2. $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ is a neutrosophic group ring and
3. $(R_2\langle G_2\cup I\rangle,*_1,*_2)$ is just a group ring.

If both $R_1\langle G_1\cup I\rangle$ and $R_2\langle G_2\cup I\rangle$ are neutrosophic group rings in the above definition. Then we call $R_B\langle G\cup I\rangle$ to be a strong neutrosophic bigroup biring.

Now we give some examples of neutrosophic bigroup birings.

Example 8.1.2: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$, where $\mathbb{R}\langle G_1\cup I\rangle=\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic group ring such that $\mathbb{R}$ is the ring of real numbers and $\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic group undr addition modulo 10 and $\mathbb{Z}_2\langle\mathbb{Z}\cup I\rangle$ is a neutrosophic group ring, where $\mathbb{Z}_2=\{0,1\}$ is a field of two elements with $\langle\mathbb{Z}\cup I\rangle$ is a neutrosophic group with respect to +.

Thus $R_B\langle G\cup I\rangle$ is a strong neutrosophic group ring.

Example 8.1.3: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\mathrm{G}_2,*_1,*_2\}$, where $\mathbb{R}\langle G_1\cup I\rangle=\mathbb{R}\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic group ring such that $\mathbb{R}$ is the ring of real numbers and $\langle\mathbb{Z}_{10}\cup I\rangle$ is a neutrosophic group undr addition modulo 10 and $\mathbb{Z}_2\mathbb{Z}$ is a group ring, where $\mathbb{Z}_2=\{0,1\}$ is a field of two elements with $\mathbb{Z}$ is a group with respect to +.

Thus $R_B\langle G\cup I\rangle$ is a neutrosophic group ring.

We now give some characterization of neutrosophic bigroup birings.

Theorem 8.1.4: All strong neutrosophic bigroup birings are trivially neutrosophic bigroup birings.

Definition 8.1.5: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$. Then P is called subneutrosophic bigroup biring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a subneutrosophic group ring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is also a subneutrosophic group ring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If in the above definition, both $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ are subneutrosophic group rings. Then $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ is said to be strong subneutrosophic bigroup ring.

For an instance, take the following example.

Example 8.1.6: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring in Example 8.1.2. Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\}$ be a proper subset of $R_B\langle G\cup I\rangle$ such that $H_1=\{0,5,5\mathrm{I},5+5\mathrm{I}\}$ and $H_2=2\mathbb{Z}$. Then clearly P is a subneutrosophic bigroup biring of $R_B\langle G\cup I\rangle$.

Theorem 8.1.7: All strong subneutrosophic bigroup birings are trivially subneutrosophic bigroup birings.

Definition 8.1.8: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{\langle S_1\cup I\rangle\cup\langle S_2\cup I\rangle\}$. Then P is called neutrosophic subbiring if $(\langle S_1\cup I\rangle,*_1,*_2)$ is a neutrosophic subring of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(\langle S_2\cup I\rangle,*_1,*_2)$ is also a neutrosophic subring of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If both R_1 and R_2 are neutrosophic subrings, then we call P to be a strong neutrosophic subbiring.

The following example furthere explain this fact.

Exmple 8.1.9: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring in above Example 8.1.2. Then clearly $P=\{\langle\mathbb{R}\cup I\rangle\cup\mathbb{Z}_2\}$ is a neutrosophic subbiring of $R_B\langle G\cup I\rangle$ and $P=\{\langle\mathbb{R}\cup I\rangle\cup\langle\mathbb{Z}_2\cup I\rangle\}$ is a strong neutrosophic subbiring of $R_B\langle G\cup I\rangle$.

Theorem 8.1.10: If $R_B\langle G\cup I\rangle$ is a strong neutrosophic bigroup biring, then P is also a strong neutrosophic subbiring of $R_B\langle G\cup I\rangle$.

Definition 8.1.11: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$. Then P is called pseudo neutrosophic subbiring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a pseudo neutrosophic subring of $\left(R_1\langle G_1\cup I\rangle,*_1,*_2\right)$ and $\left(R_2\langle H_2\cup I\rangle,*_1,*_2\right)$ is also a pseudo neutrosophic subring of $\left(R_2\langle G_2\cup I\rangle,*_1,*_2\right)$.

Definition 8.1.12: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$. Then P is called neutrosophic subbigroup ring if $(R_1\mathrm{H}_1,*_1,*_2)$ is a neutrosophic subgroup ring of $\left(R_1\langle G_1\cup I\rangle,*_1,*_2\right)$ and $(R_2\mathrm{H}_2,*_1,*_2)$ is also a neutrosophic subgroup ring of $\left(R_2\langle G_2\cup I\rangle,*_1,*_2\right)$.

Let’s take a look to the following example.

Exmple 8.1.13: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring in Example 8.1.2. Then clearly $P=\{\mathbb{R}G_1\cup\mathbb{Z}_2G_2\}$ is a neutrosophic sub bigroup biring of $R_B\langle G\cup I\rangle$.

Definition 8.1.14: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{P_1\cup P_2\}$. Then P is called subbiring if $(P_1,*_1,*_2)$ is a subring of $\left(R_1\langle G_1\cup I\rangle,*_1,*_2\right)$ and $(P_2,*_1,*_2)$ is also a subring of $\left(R_2\langle G_2\cup I\rangle,*_1,*_2\right)$.

One can see it in this example.

Exmple 8.1.15: Let $R_B\langle G\cup I\rangle=\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring in Example 8.1.2. Then clearly $P=\{\mathbb{R}\cup\mathbb{Z}_2\}$ is a subbiring of $R_B\langle G\cup I\rangle$.

Definition 8.1.16: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $J=\{J_1\cup J_2\}$. Then J is called neutrosophic biideal if $(J_1,*_1,*_2)$ is aneutrosophic ideal of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is also a neutrosophic ideal of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

If both J_1 and J_2 are neutrosophic ideals. Then J is called to be strong neutrosophic biideal of the neutrosophic bigroup biring $R_B\langle G\cup I\rangle$.

In the following example, we give neutrosophic biideal of a neutrosophic group ring.

Example 8.1.17: Let $R_B\langle G\cup I\rangle=\{\mathbb{C}\langle G_1\cup I\rangle\cup\mathbb{Z}\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring. Then $J=\{J_1\cup J_2\}$ is a neutrosophic biideal of $R_B\langle G\cup I\rangle$, where $J_1=\mathbb{Q}\langle G_1\cup I\rangle$ and $J_2=3\mathbb{Z}G_2$.

Theorem 8.1.18: Every neutrosophic biideal of a neutrosophic bigroup biring $R_B\langle G\cup I\rangle$ is trivially a neutrosophic sub bigroup biring.

Theorem 8.1.19: Every strong neutrosophic biideal of a neutrosophic bigroup biring is trivially a neutrosophic biideal of $R_B\langle G\cup I\rangle$.

Definition 8.1.20: Let $R_B\langle G\cup I\rangle=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $J=\{J_1\cup J_2\}$. Then J is called pseudo neutrosophic biideal if $(J_1,*_1,*_2)$ is apseudo neutrosophic ideal of $(R_1\langle G_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is also a pseudo neutrosophic ideal of $(R_2\langle G_2\cup I\rangle,*_1,*_2)$.

We now proceed to define neutrosophic N-group N-rings.

8.2 Neutrosophic N-group N-rings

In this section, we finally present the generalization of neutrosophic group ring and define neutrosophic N-group N-rings. We also give some important results about neutrosophic N-group N-rings.

Definition 8.2.1: Let $N(R\langle G\cup I\rangle)=\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\}$ be a non-empty set with n binary operations on $N(R\langle G\cup I\rangle)$. Then $N(R\langle G\cup I\rangle)$ is called a neutrosophic N-group N-ring if

1. $N(R\langle G\cup I\rangle)=R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup R_n\langle G_n\cup I\rangle$, where each $R_i\langle G_i\cup I\rangle$ is a proper subset of $N(R\langle G\cup I\rangle)$ for all i.
2. Some of $(R_i\langle G_i\cup I\rangle,*_i,*_i)$ are neutrosophic group rings for some i.
3. Some of $(R_i\langle G_i\cup I\rangle,*_i,*_i)$ are just group rings for some i.

If all $\left(R_i\langle G_i \cup I\rangle, *_i, *_i\right)$ are neutrosophic group rings, then we call $N\left(R\langle G \cup I\rangle\right)$ to be a strong neutrosophic N-group N-ring.

We now give some examples to illustrate this fact.

Example 8.2.2: Let $N(R\langle G \cup I\rangle) = \left\{\mathbb{R}\left\langle G_1 \cup I\right\rangle \cup \mathbb{Z}_2\left\langle G_2 \cup I\right\rangle \cup \mathbb{Z}\left\langle G_3 \cup I\right\rangle, *_1, *_2, *_3\right\}$, where $\mathbb{R}\langle G_1 \cup I\rangle = \mathbb{R}\langle \mathbb{Z}_{10} \cup I\rangle$ is a neutrosophic group ring such that $\mathbb{R}$ is the ring of real numbers and $\langle \mathbb{Z}_{10} \cup I\rangle$ is a neutrosophic group undr addition modulo 10 and $\mathbb{Z}_2\langle \mathbb{Z} \cup I\rangle$ is a neutrosophic group ring, where $\mathbb{Z}_2 = \{0,1\}$ is a field of two elements with $\langle \mathbb{Z} \cup I\rangle$ is a neutrosophic group with respect to +.
Also $\mathbb{Z}\langle G_3 \cup I\rangle$ is a neutrosophic group ring with $\mathbb{Z}$ is the ring of integers and $\langle G_3 \cup I\rangle = \langle C_6 \cup I\rangle$ is a neutrosophic cyclic group.

Thus $N(R\langle G \cup I\rangle)$ is a strong neutrosophic 3-group 3-ring.

Example 8.2.3: Let $N(R\langle G \cup I\rangle) = \left\{\mathbb{R}\left\langle G_1 \cup I\right\rangle \cup \mathbb{Z}_2\left\langle G_2 \cup I\right\rangle \cup \mathbb{Z}G_3, *_1, *_2, *_3\right\}$, where $\mathbb{R}\langle G_1 \cup I\rangle = \mathbb{R}\langle \mathbb{Z}_{10} \cup I\rangle$ is a neutrosophic group ring such that $\mathbb{R}$ is the ring of real numbers and $\langle \mathbb{Z}_{10} \cup I\rangle$ is a neutrosophic group undr addition modulo 10 and $\mathbb{Z}_2\mathbb{Z}$ is a group ring, where $\mathbb{Z}_2 = \{0,1\}$ is a field of two elements with $\mathbb{Z}$ is a group with respect to +.
Also $\mathbb{Z}G_3$ is a group ring with $\mathbb{Z}$ is the ring of integers and $G_3 = C_6$ is a cyclic group of order 6.

Thus $N(R\langle G \cup I\rangle)$ is a neutrosophic 3-group 3-ring.

Theorem 8.2.4: All strong neutrosophic N-group N-rings are trivially neutrosophic Ngroup N-rings.

Definition 8.2.5: Let $N\left(R\langle G\cup I\rangle\right)=\left\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup R_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-group N-ring and let $P=\left\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup R_n\langle H_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a proper subset of $N\left(R\langle G\cup I\rangle\right)$. Then P is called subneutrosophic N-group N-ring if $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ is a subneutrosophic group ring of $N\left(R\langle G\cup I\rangle\right)$ for all i.

If all $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic subgroup rings. Then P is called strong subneutrosophic N-group N-ring of $N\left(R\langle G\cup I\rangle\right)$ for all i.

This situation can be explained in the following example.

Example 8.2.6: Let $N(R\langle G\cup I\rangle)=\left\{\mathbb{R}\langle G_1\cup I\rangle\cup\mathbb{Z}_2\langle G_2\cup I\rangle\cup\mathbb{Z}G_3,*_1,*_2,*_3\right\}$ be a neutrosophic 3-group 3-ring in Example 8.2.2. Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\cup\mathbb{Z}H_3\}$ be a proper subset of $R_B\langle G\cup I\rangle$ such that $H_1=\{0,5,5\mathrm{I},5+5\mathrm{I}\}$ and $H_2=2\mathbb{Z}$ and $H_3=C_3$. Then clearly P is a subneutrosophic bigroup biring of $R_B\langle G\cup I\rangle$.

Theorem 8.2.7: All strong subneutrosophic N-group N-rings are trivially subneutrosophic N-group N-rings.

Definition 8.2.8: Let $N\left(R\langle G\cup I\rangle\right)=\left\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup R_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-group N-ring and let $P=\left\{\langle S_1\cup I\rangle\cup\langle S_2\cup I\rangle\cup\ldots\cup\langle S_n\cup I\rangle\right\}$.

Then P is called neutrosophic sub N-ring if $(\langle S_i \cup I\rangle, *_i, *_i)$ is a neutrosophic sub N-ring of $(R_i\langle G_i \cup I\rangle, *_i, *_i)$ for all i.

Theorem 8.2.9: If $N(R\langle G \cup I\rangle)$ is a strong neutrosophic N-group N-ring, then P is also a strong neutrosophic sub N-ring of $N(R\langle G \cup I\rangle)$.

Definition 8.2.10: Let $N(R\langle G \cup I\rangle) = \{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup R_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\}$ be a neutrosophic N-group N-ring and let $P = \{R_1\langle H_1 \cup I\rangle \cup R_2\langle H_2 \cup I\rangle \cup \ldots \cup R_n\langle H_n \cup I\rangle\}$. Then P is called pseudo neutrosophic subbiring if $(R_i\langle H_i \cup I\rangle, *_i, *_i)$ is a pseudo neutrosophic sub N-ring of $(R_i\langle G_i \cup I\rangle, *_i, *_i)$ for all i.

Definition 8.2.11: Let $N(R\langle G \cup I\rangle) = \{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup R_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\}$ be a neutrosophic N-group N-ring and let $P = \{R_1 H_1 \cup R_2 H_2 \cup \ldots \cup R_n H_n\}$. Then P is called neutrosophic sub N-group ring if $(R_i H_i, *_i, *_i)$ is a neutrosophic subgroup ring of $(R_i\langle G_i \cup I\rangle, *_i, *_i)$ for all i.

Definition 8.2.12: Let $N(R\langle G \cup I\rangle) = \{R_1\langle G_1 \cup I\rangle \cup R_2\langle G_2 \cup I\rangle \cup \ldots \cup R_n\langle G_n \cup I\rangle, *_1, *_2, \ldots, *_n\}$ be a neutrosophic N-group N-ring and let $P = \{P_1 \cup P_2 \cup \ldots \cup P_n\}$. Then P is called sub N-ring if $(P_i, *_i, *_i)$ is a subring of $(R_i\langle G_i \cup I\rangle, *_i, *_i)$ for all i.

Definition 8.2.13: Let $N\left(R\langle G\cup I\rangle\right)=\left\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-group N-ring and let $J=\{J_1\cup J_2\cup\ldots\cup J_n\}$. Then J is called neutrosophic N-ideal if $(J_i,*_i,*_i)$ is a neutrosophic ideal of $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ for all i.

If $(J_i,*_i,*_i)$ are neutrosophic ideals for all I, then J is said to be a strong neutrosophic N-ideal of $N\left(R\langle G\cup I\rangle\right)$.

Theorem 8.2.14: Every neutrosophic N-ideal of a neutrosophic N-group N-ring $N\left(R\langle G\cup I\rangle\right)$ is trivially a neutrosophic sub N-group N-ring.

Theorem 8.2.15: Every strong neutrosophic N-ideal of a neutrosophic N-group N-ring is trivially a neutrosophic N-ideal of $N\left(R\langle G\cup I\rangle\right)$.

Definition 8.2.16: Let $N\left(R\langle G\cup I\rangle\right)=\left\{R_1\langle G_1\cup I\rangle\cup R_2\langle G_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle G_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-group N-ring and let $J=\{J_1\cup J_2\cup\ldots J_n\}$. Then J is called pseudo neutrosophic N-ideal if $(J_i,*_i,*_i)$ is a pseudo neutrosophic ideal of $\left(R_i\langle G_i\cup I\rangle,*_i,*_i\right)$ for all i.

Chapter No. 9

GENERALIZATION OF NEUTROSOPHIC SEMIGROUP RINGS AND THEIR PROPERTIES

In this chapter, we give the generalization of neutrosophic semigroup rings as neutrosophic semigroup rings were defined by W. B. V. Kandasamy and F. Smarandache in [165]. Here we construct neutrosophic bisemigroup birings and neutrosophic N-semigroup N-rings respectively. The organization of this chapter is as follows. There are totoally two section in this chapter. In first section, we present neutrosophic bisemigroup birings and its related theory. In section two, we introduce neutrosophic N-semigroup N-rings and their properties.

9.1 Neutrosophic Bisemigroup Birings

In this section, we just presented the notion of neutrosophic bisemigroup biring and gave some of their fundamental properties and other related theory.

We now proceed on to define neutrosophic bisemigroup biring.

Definition 9.1.1: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a non-empty set with two binary operations on $R_B\langle S\cup I\rangle$. Then $R_B\langle S\cup I\rangle$ is called a neutrosophic bisemigroup biring if

1. $R_B\langle S\cup I\rangle=R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle$, where $R_1\langle S_1\cup I\rangle$ and $R_2\langle S_2\cup I\rangle$ are proper subsets of $R_B\langle S\cup I\rangle$.
2. $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ is a neutrosophic semigroup ring and
3. $(R_2\langle S_2\cup I\rangle,*_1,*_2)$ is a neutrosophic semigroup ring.

If both $R_1\langle S_1\cup I\rangle$ and $R_2\langle S_2\cup I\rangle$ are neutrosophic semigroup rings in the above definition. Then we call it strong neutrosophic bisemigroup biring.

This situation can be explained in the following example.

Example 9.1.2: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}_1\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle,*_1,*_2\}$, where $\mathbb{R}_1\langle S_1\cup I\rangle=\mathbb{R}\langle\mathbb{Z}\cup I\rangle$ be a neutrosophic semigroup ring such that $\mathbb{R}_1$ is the ring of real numbers and $\langle\mathbb{Z}\cup I\rangle$ is a neutrosophic semigroup with respect to +. Also $\mathbb{Z}\langle\mathbb{Z}_3\cup I\rangle$ is a neutrosophic semigroup ring where $\mathbb{Z}$ is the ring of integers and $\langle\mathbb{Z}_3\cup I\rangle$ is a neutrosophic semigroup under multiplication modulo 3.
Thus $R_B\langle S\cup I\rangle$ is a neutrosophic bisemigroup biring.

Theorem 9.1.3: All strong neutrosophic bisemigroup birings are trivially neutrosophic bisemigroup birings.

Definition 9.1.4: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called subneutrosophic bisemigroup biring if $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ is a subneutrosophic semigroup ring of $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ is also a subneutrosophic semigroup ring of $(R_2\langle S_2\cup I\rangle,*_1,*_2)$.

If in the above definition, both $(R_1\langle H_1\cup I\rangle,*_1,*_2)$ and $(R_2\langle H_2\cup I\rangle,*_1,*_2)$ are subneutrosophic semigroup rings. Then $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ is said to be strong subneutrosophic bisemigroup biring.

For this, see the following example.

Example 9.1.5: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}_1\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring in Example (1). Let $P=\{\mathbb{R}_1H_1\cup\mathbb{Z}_2H_2\}$ be a proper subset of $R_B\langle S\cup I\rangle$ such that $H_1=2\mathbb{Z}$ and $H_2=\mathbb{Z}_3$. Then clearly P is a subneutrosophic bisemigroup biring of $R_B\langle S\cup I\rangle$.

Theorem 9.1.6: All strong subneutrosophic bisemigroup birings are trivially subneutrosophic bisemigroup birings.

Definition 9.1.7: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{\langle S_1\cup I\rangle\cup\langle S_2\cup I\rangle\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called neutrosophic subbiring if

$(\langle S_1 \cup I \rangle, *_1, *_2)$ is a neutrosophic subring of $(R_1 \langle S_1 \cup I \rangle, *_1, *_2)$ and $(\langle S_2 \cup I \rangle, *_1, *_2)$ is also a neutrosophic subring of $(R_2 \langle S_2 \cup I \rangle, *_1, *_2)$.

If both R_1 and R_2 are neutrosophic subrings, then we call P to be a strong neutrosophic subbiring.

For an instance, we give the following example.

Exmple 9.1.8: Let $R_B \langle S \cup I \rangle = \{\mathbb{R}_1 \langle S_1 \cup I \rangle \cup \mathbb{Z}_2 \langle S_2 \cup I \rangle, *_1, *_2\}$ be a neutrosophic bisemigroup biring in above Example (1). Then clearly $P = \{\langle \mathbb{R}_1 \cup I \rangle \cup \mathbb{Z}_2\}$ is a neutrosophic subbiring of $R_B \langle S \cup I \rangle$ and $P = \{\langle \mathbb{R}_1 \cup I \rangle \cup \langle \mathbb{Z}_2 \cup I \rangle\}$ is a strong neutrosophic subbiring of $R_B \langle S \cup I \rangle$.

Theorem 9.1.9: If $R_B \langle S \cup I \rangle$ is a strong neutrosophic bisemigroup biring, then P is also a strong neutrosophic subbiring of $R_B \langle S \cup I \rangle$.

Definition 9.1.10: Let $R_B \langle S \cup I \rangle = \{R_1 \langle S_1 \cup I \rangle \cup R_2 \langle S_2 \cup I \rangle, *_1, *_2\}$ be a neutrosophic bisemigroup biring and let $P = \{R_1 \langle H_1 \cup I \rangle \cup R_2 \langle H_2 \cup I \rangle\}$ be a proper subset of $R_B \langle S \cup I \rangle$. Then P is called pseudo neutrosophic subbiring if $(R_1 \langle H_1 \cup I \rangle, *_1, *_2)$ is a pseudo neutrosophic subring of $(R_1 \langle S_1 \cup I \rangle, *_1, *_2)$ and $(R_2 \langle H_2 \cup I \rangle, *_1, *_2)$ is also a pseudo neutrosophic subring of $(R_2 \langle S_2 \cup I \rangle, *_1, *_2)$.

Definition 9.1.11: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bigroup biring and let $P=\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called neutrosophic subbigroup ring if $(R_1 \mathrm{H}_1,*_1,*_2)$ is a neutrosophic subgroup ring of $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ and $(R_2 \mathrm{H}_2,*_1,*_2)$ is also a neutrosophic subgroup ring of $(R_2\langle S_2\cup I\rangle,*_1,*_2)$.

This can be furthere explained in the following example.

Exmple 9.1.12: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}_1\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring in above Example (1). Then clearly $P=\{\mathbb{R}_1S_1\cup\mathbb{Z}_2S_2\}$ is a neutrosophic sub bisemigroup biring of $R_B\langle S\cup I\rangle$.

Definition 9.1.13: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring and let $P=\{P_1\cup P_2\}$ be a proper subset of $R_B\langle S\cup I\rangle$. Then P is called subbiring if $(P_1,*_1,*_2)$ is a subring of $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ and $(P_2,*_1,*_2)$ is also a subring of $(R_2\langle S_2\cup I\rangle,*_1,*_2)$.

See the following example.

Exmple 9.1.14: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}_1\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring in above Example (1). Then clearly $P=\{\mathbb{R}_1\cup\mathbb{Z}_2\}$ is a subbiring of $R_B\langle S\cup I\rangle$.

Definition 9.1.15: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring and let $J=\{J_1\cup J_2\}$ be a proper subset of

$R_B\langle S\cup I\rangle$. Then J is called neutrosophic biideal if $(J_1,*_1,*_2)$ is a neutrosophic ideal of $(R_1\langle S_1\cup I\rangle,*_1,*_2)$ and $(J_2,*_1,*_2)$ is also a neutrosophic ideal of $(R_2\langle S_2\cup I\rangle,*_1,*_2)$.

If both J_1 and J_2 are strong neutrosophic ideals. Then J is called to be strong neutrosophic biideal of the neutrosophic bisemigroup biring $R_B\langle S\cup I\rangle$.

This situation can be given in the following example.

Exmple 9.1.16: Let $R_B\langle S\cup I\rangle=\{\mathbb{R}\langle S_1\cup I\rangle\cup\mathbb{C}\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring. Then $J=\{J_1\cup J_2\}$ is a neutrosophic biideal of $R_B\langle S\cup I\rangle$, where $J_1=\mathbb{Q}\langle S_1\cup I\rangle$ and $J_2=\mathbb{Z}S_2$.

Theorem 9.1.17: Every neutrosophic biideal of a neutrosophic bisemigroup biring $R_B\langle S\cup I\rangle$ is trivially a neutrosophic sub bisemigroup biring.

Theorem 9.1.18: Every strong neutrosophic biideal of a neutrosophic bisemigroup biring is trivially a neutrosophic biideal of $R_B\langle S\cup I\rangle$.

Definition 9.1.19: Let $R_B\langle S\cup I\rangle=\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle,*_1,*_2\}$ be a neutrosophic bisemigroup biring and let $J=\{J_1\cup J_2\}$. Then J is called pseudo neutrosophic biideal if $(J_1,*_1,*_2)$ is a pseudo neutrosophic ideal of

$\left(R_1\langle S_1 \cup I\rangle,*_1,*_2\right)$ and $(J_2,*_1,*_2)$ is also a pseudo neutrosophic ideal of $\left(R_2\langle S_2 \cup I\rangle,*_1,*_2\right)$.

We now present the notions of neutrosophic N-semigroup N-riings.

9.2 Neutrosophic N-semigroup N-rings

In this section, we introduce the generalization of neutrosophic semigroup rings and thus define neutrosophic N-semigroup N-rings to extend the theory of neutrosophic semigroup ring. We also gove some characterization of neutrosophic N-semigroup N-rings.

We now proceed to define neutrosophic N-semigroup N-ring.

Definition 9.2.1: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle \cup R_2\langle S_2\cup I\rangle \cup ... \cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,...,*_n\right\}$ be a non-empty set with n binary operations on $N\left(R\langle S\cup I\rangle\right)$. Then $N\left(R\langle S\cup I\rangle\right)$ is called a neutrosophic N-LA-semigroup N-ring if

1. $N\left(R\langle S\cup I\rangle\right)=R_1\langle S_1\cup I\rangle \cup R_2\langle S_2\cup I\rangle \cup ... \cup R_n\langle S_n\cup I\rangle$, where $R_i\langle S_i\cup I\rangle$ is a proper subset of $N\left(R\langle S\cup I\rangle\right)$ for all i.
2. Some of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ are neutrosophic LA-semigroup ring for some i.
3. Some of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ are just LA-semigroup ring for some i.

If all $\left(R_i\langle S_i \cup I\rangle, *_i, *_i\right)$ are neutrosophic LA-semigroup rings, then we call $N\left(R\langle S \cup I\rangle\right)$ to be a strong neutrosophic LA-semigroup ring.

The following example furthere illustrated this fact.

Example 9.2.2: Let $N(R\langle S \cup I\rangle) = \left\{\mathbb{R}\langle S_1 \cup I\rangle \cup \mathbb{Z}_2\langle S_2 \cup I\rangle \cup \mathbb{Z}\langle S_3 \cup I\rangle \cup \mathbb{C}S_4\right\}$, where $\mathbb{R}\langle S_1 \cup I\rangle$ be a neutrosophic semigroup ring such that $\mathbb{R}$ is the ring of real numbers, $\mathbb{Z}_2\langle S_2 \cup I\rangle$ is another neutrosophic semigroup ring with $\mathbb{Z}_2 = \{0,1\}$ is a field of two elements, $\mathbb{Z}\langle S_3 \cup I\rangle$ is a also a neutrosophic semigroup ring with $\mathbb{Z}$ is the ring of inetegrs and $\mathbb{C}S_4$ is just a semigroup ring with $\mathbb{C}$ is the field of complex numers such that $S_1 = \{Z_{12},$ semigroup under multiplication modulo $12\}$, $S_2 = \{0,1,2,3,I,2I,3I,4I$, semigroup under multiplication modulo $4\}$, a neutrosophic semigroup, $S_3 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \langle R \cup I\rangle\right\}$, neutrosophic semigroup under matrix multiplication and $S_4 = \langle Z \cup I\rangle$, neutrosophic semigroup under multiplication.

Thus $N(R\langle S \cup I\rangle)$ is a neutrosophic 4-semigroup 4-ring.

We now give some characterization of neutrosophic N-semigroup N-rings.

Theorem 9.2.3: All strong neutrosophic N-semigroup N-rings are trivially neutrosophic N-semigroup N-rings.

Definition 9.2.4: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $P=\left\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle H_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a proper subset of $N\left(R\langle S\cup I\rangle\right)$. Then P is called subneutrosophic N-semigroup N-ring if $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ is a subneutrosophic semigroup ring of $N\left(R\langle S\cup I\rangle\right)$ for all i.

The readers can see it in this example.

Example 9.2.5: Let $N(R\langle S\cup I\rangle)=\left\{\mathbb{R}\langle S_1\cup I\rangle\cup\mathbb{Z}_2\langle S_2\cup I\rangle\cup\mathbb{Z}\langle S_3\cup I\rangle\cup\mathbb{C}S_4\right\}$ be a neutrosophic 4-semigroup 4-ring in Example (**). Let $P=\left\{\mathbb{R}H_1\cup\mathbb{Z}_2H_2\cup\mathbb{Z}\langle S_3\cup I\rangle\cup\mathbb{Z}S_4\right\}$ be a proper subset of $N(R\langle S\cup I\rangle)$ such that $H_1=\{0,4,8\}$ and $H_2=\{0,2,\mathrm{I},2\mathrm{I}\}$ and $H_4=\langle 2\mathbb{Z}\cup I\rangle$.

Then clearly P is a subneutrosophic 4-semigroup 4-ring of $N(R\langle S\cup I\rangle)$.

Theorem 9.2.6: All strong subneutrosophic N-semigroup N-rings are trivially subneutrosophic N-semigroup N-rings.

Definition 9.2.7: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $P=\left\{\langle P_1\cup I\rangle\cup\langle P_2\cup I\rangle\cup\ldots\cup\langle P_n\cup I\rangle\right\}$. Then P is called neutrosophic sub N-ring if $\left(\langle P_i\cup I\rangle,*_i,*_i\right)$ is a neutrosophic sub N-ring of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ for all i.

Theorem 9.2.8: If $N(R\langle S\cup I\rangle)$ is a strong neutrosophic N-semigroup N-ring, then P is also a strong neutrosophic sub N-ring of $N(R\langle S\cup I\rangle)$.

Definition 9.2.9: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $P=\left\{R_1\langle H_1\cup I\rangle\cup R_2\langle H_2\cup I\rangle\cup\ldots\cup R_n\langle H_n\cup I\rangle\right\}$. Then P is called pseudo neutrosophic subbiring if $\left(R_i\langle H_i\cup I\rangle,*_i,*_i\right)$ is a pseudo neutrosophic sub N-ring of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ for all i

Definition 9.2.10: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $P=\{R_1H_1\cup R_2H_2\cup\ldots\cup \mathrm{R}_n H_n\}$. Then P is called neutrosophic sub N-semigroup ring if $(R_i \mathrm{H}_i,*_i,*_i)$ is a neutrosophic subgroup ring of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ for all i.

Definition 9.2.11: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $P=\{P_1\cup P_2\cup\ldots\cup P_n\}$. Then P is called sub N-ring if $(P_i,*_i,*_i)$ is a subring of $\left(R_i\langle S_i\cup I\rangle,*_i,*_i\right)$ for all i.

Definition 9.2.12: Let $N\left(R\langle S\cup I\rangle\right)=\left\{R_1\langle S_1\cup I\rangle\cup R_2\langle S_2\cup I\rangle\cup\ldots\cup \mathrm{R}_n\langle S_n\cup I\rangle,*_1,*_2,\ldots,*_n\right\}$ be a neutrosophic N-semigroup N-ring and let $J=\{J_1\cup J_2\cup\ldots\cup J_n\}$. Then J is called neutrosophic N-ideal if $(J_i, *_i, *_i)$ is a neutrosophic ideal of $\left(R_i\langle S_i \cup I\rangle, *_i, *_i\right)$ for all i.

If $(J_i, *_i, *_i)$ are neutrosophic ideals for all I, then J is said to be a strong neutrosophic N-ideal of $N\left(R\langle S \cup I\rangle\right)$.

Theorem 9.2.13: Every neutrosophic N-ideal of a neutrosophic N-semigroup N-ring $N\left(R\langle S \cup I\rangle\right)$ is trivially a neutrosophic sub N-semigroup N-ring.

Theorem 9.2.14: Every strong neutrosophic N-ideal of a neutrosophic N-semigroup N-ring is trivially a neutrosophic N-ideal of $N\left(R\langle S \cup I\rangle\right)$.

Definition 9.2.15: Let $N\left(R\langle S \cup I\rangle\right) = \left\{R_1\langle S_1 \cup I\rangle \cup R_2\langle S_2 \cup I\rangle \cup \ldots \cup \mathrm{R}_n\langle S_n \cup I\rangle, *_1, *_2, \ldots, *_n\right\}$ be a neutrosophic N-semigroup N-ring and let $J = \{J_1 \cup J_2 \cup \ldots J_n\}$. Then J is called pseudo neutrosophic N-ideal if $(J_i, *_i, *_i)$ is a pseudo neutrosophic ideal of $\left(R_i\langle S_i \cup I\rangle, *_i, *_i\right)$ for all i.

Chapter No. 10

SUGGESTED PROBLEMS AND EXERCISES

In this chapter, we present 581 number of suggested problems and exercises for the readers which will help in better understanding of the book.

1. How one can define neutrosophic right almost semigroup? Give some examples of neutrosophic right almost semigroup?

2. Define neutrosophic bi-right almost semigroup and neutrosophic N-right almost semigroup with examples?

3. Construct the whole theory for neutrosophic right almost semigroup and for its generalization?

4. When will be a neutrosophic left almost semigroup is neutrosophic

right almost semigroup? Give the condition briefly?

5. Give some example of a neutrosophic LA-semigroup which also a neutrosophic RA-semigroup.

6. What is the difference between a neutrosophic semigroup and a neutrosophic LA-semigroup? Discuiss it?

7. When will be a neutrosophic LA-semigroup is a neutrosophic semigroup?

8. Give 3 examples in each case for the following.
   (1). Neutrosophic LA-semigroups (2). Neutrosophic semigroups which are also neutrosophic LA-semigroups.

9. Give some examples of a neutrosophic sub LA-semigroups which are not neutrosophic ideals?

10. Prove the medial law in a neutrosophic LA-semigroup as well as in a neutrosophic RA-semigroup?

11. What is a strong neutrosophic LA-semigroup? Discuiss with illustrative examples?
12. What is a strong neutrosophic RA-semigroup? Discuiss with examples?
13. What is the difference between a strong neutrosophic LA-semigroup and a neutrosophic LA-semigroup?
14. Give different examples of strong neutrosophic LA-semigroups?
15. Give different examples of strong neutrosophic RA-semigroup?
16. What is a strong neutrosophic bi-LA-semigroup? Give some examples?
17. What is a strong neutrosophic N-LA-semigroup?
18. Can one define strong neutrosophic bi-RA-semigroup and strong neutrosophic N-RA-semigroup?
19. Give some concrete examples of strong neutrosophic bi-RA-

semigroups and strong neutrosophic N-RA-semigroups?

20. Can one define commutative neutrosophic RA-semigroup? Also give example to explain it?

21. What strong neutrosophic sub-LA-semigroup?

22. What is strong neutrosophic ideal of a neutrosophic LA-semigroup?

23. Give 4 examples of strong neutrosophic ideal?

24. Can you define neutrosophic sub bi-RA-semigroup? Give examples for illustration?

25. How can you define neutrosophic bi-ideal of a neutrosophic bi-RA-semigroup? Explain it with examples?

26. What is the difference between strong neutrosophic bi-ideal and neutrosophic bi-ideal of a neutrosophic LA-semigroup?

27. What is prime neutrosophic ideal of a neutrosophic RA-semigroup?

28. Give some example of prime neutrosophic ideal?
29. What is meant by strong prime neutrosophic ideal of an LA-semigroup? Give some examples?

30. What is a semi-prime neutrosophic ideal of a neutrosophic LA-semigroup?

31. Give examples of semi-prime neutrosophic ideals of a neutrosophic LA-semigroup?

32. Discuiss the semi-prime neutrosophic bi-ideal of a neutrosophic bi-LA-semigroup?

33. What is strong semi-prime neutrosophic ideal?

34. Give some examples of semi-prime bi-ideal of a neutrosophic bi-LA-semigroup?

35. Give 3 example of strong semi-prime neutrosophic ideal?

36. Can one define prime neutrosophic bi-ideal of a neutrosophic bi-

LA-semigroup? Explain it with some examples?

37. What is strong prime neutrosophic bi-ideal?

38. Give some examples of strong prime neutrosophic ideal?

39. What is the main difference between a prime neutrosophic ideal and a semi-prime neutrosophic ideal of a neutosophic LA-semigroup?

40. Breifly discuiss prime neutrosophic N-ideal of a neutrosophic N-LA-semigroup?

41. Give some examples of prime neutrosophic N-ideal?

42. What is prime strong neutrosophic N-ideal of a neutrosophic N-LA-semigroup?

43. Give examples of a neutrosophic LA-semigroup which has prime neutrosophic ideals?

44. Give examples of a neutrosophic LA-semigroup which has semi-

prime neutrosophic ideals?

45. What is semi-prime neutrosophic N-ideal of a neutrosophic N-LA-semigroup?

46. Give some examples of semi-prime neutrosophic ideal?

47. What is semi-prime strong neutrosophic N-ideal? Give examples?

48. What is fully prime neutrosophic LA-semigroup? Discuiss it with examples?

49. What is meant by fully semi-prime neutrosophic LA-semigroup?

50. Give examples of semi-prime neutrosophic LA-semigroup?

51. What is meant by lagrange neutrosophic sub-LA-semigroup?

52. Give examples of lagrange neutrosophic sub-LA-semigroup?

53. Discuiss lagrange neutrosophic LA-semigroup with a few

examples?

54. What is weakly lagrange neutrosophic LA-semigroup?

55. Give some exmples of weakly lagrange neutrosophic LA-semigroups?

56. What are lagrange free neutrosohic LA-semigroups. Give some examples?

57. What is meant by lagrange neutrosophic sub bi-LA-semigroup?

58. Give some examples of lagrange neutrosophic sub bi-LA-semigroup?

59. Discuiss lagrange neutrosophic bi-LA-semigroup with examples?

60. What is a weakly lagrange neutrosophic bi-LA-semigroup?

61. Give some exmples of weakly lagrange neutrosophic bi-LA-semigroups?

61. What are lagrange free neutrosohic bi-LA-semigroups. Give some examples?

62. What is meant by lagrange neutrosophic sub N-LA-semigroup?

63. Give examples of lagrange neutrosophic sub N-LA-semigroup?

64. Discuiss lagrange neutrosophic N-LA-semigroup with a few examples?

65. What is weakly lagrange neutrosophic N-LA-semigroup?

66. Give some exmples of weakly lagrange neutrosophic N-LA-semigroups?

67. What are the lagrange free neutrosohic N-LA-semigroups. Give some examples?

68. Can you define lagrange strong neutrosophic sub-LA-semigroup? Discuiss it with examples?

69. How can one define lagrange strong neutosophic sub bi-LA-semigroup?

70. Give some examples of it?

71. What is a lagrnage strong neutrosophic sub N-LA-semigroup? Breifly discuiss it with examples?

72. What are strong irreducible neutrosophic ideals?

73. Give some examples of strong irreducible neutrosophic ideals?

74. What is strongly irreducible neutrosophic bi-ideal of a neutrosophic bi-LA-semigroup? Explain it with examples?

75. Can you define strongly irreducible neutrosophic N-ideal of a neutrosophic N-LA-semigroup?

76. Give 3 examples of strongly irreducible neutrosophic N-ideal?

77. What is meant by quasi neutrosophic ideal? Give examples of neutrosophic quasi ideal of neutrosophic LA-semigroup?

78. Can one define quasi neutrosophic bi-ideal?

79. Give some concrete and clear examples of quasi neutrosophic bi-ideals?

80. What are quasi neutrosophic N-ideals? Explain it with examples?

81. Do you know about bi-neutrosophic ideal of a neutrosophic LA-semigroup? Provide examples of bi-neutrosophic ideals?

82. What is a bi-neutrosophic bi-ideal of a neutrosophic bi-LA-semigroup? Breifly explain with examples?

83. What are bi-neutrosophic N-ideals of a neutrosophic N-LA-semigroup?

84. Give examples in each case of 81, 82 and 83?

85. What is meant by generalized bi-neutrosophic ideal?

86. Give some examples of generalized bi-neutrosophic ideals?

87. What is the main difference between generalized bi-neutrosophic ideal and bi-neutrosophic ideal of a neutrosophic LA-semigroup? Explain it with example?

88. What are generalized bi-neutrosophic bi-ideals of neutrosophic bi-LA-semigroups?

89. Give different examples for illustration?

90. Can one define generalized bi-neutrosophic N-ideals of neutrosophic N-LA-semigroups. Discuiss it?

91. What is meant by strong generalized bi-neutrosophic ideal?

92. Breifly state the notion of neutrosophic interior ideal of a neutrosophic LA-semigroup?

93. Give some examples of neutrosophic interior ideals?

94. Can we define neutrosophic interior bi-ideal of a neutrosophic bi-LA-semigroup?

95. Give some suitable examples of neutrosophic interior ideal?

96. What is meant by neutrosophic interior N-ideal of a neutrosophic N-LA-semigroup?

97. Give some examples of neutrosohic interior N-ideals?

98. What are strong neutrosophic interior ideal?

99. What are strong neutrosophic interior bi-ideals, strong neutrosohic interior N-ideals?

100. Give examples in each case of 98 and 99?

101. What is the difference between neutrosophic subnormal series and neutrosophic normal series?

102. Give examples of neutrosophic subnormal series and neutrosophic normal series?

103. Give an examples of a neutrosophic subnormal series which is not neutrosophic normal?

104. Can one point out the main differences among neutrosophic subgroup series, neutrosophic subnormal series and neutrosophic normal series?

105. What are strong neutrosophic subgroup series, mixed neutrosophic subgroup series and subgroup seires? Give examples in each case?

106. Identify the main differences among strong neutrosophic subgroup series, mixed neutrosophic subgroup series and subgroup seires?

107. What are strong neutrosophic subnormal series, mixed neutrosophic subnormal series and subnorma seires?

108. Give examples in each case?

109. Identify the main differences among strong neutrosophic subnormal series, mixed neutrosophic subnormal series and subnormal seires?

110. What are strong neutrosophic normal series, mixed neutrosophic normal series and normal seires? Give examples in each case?

111. Identify the main differences among strong neutrosophic normal series, mixed neutrosophic normal series and normal seires?

112. What is a neutrosophic abelian series? Give some examples of neutrosohic abelian series?

113. Breifly state strong neutrosophic abelian series with illustrative examples?

114. Can you give examples of mixed neutrosophic abelian series with examples?

115. What is an abelian seires? Give examples of it?

116. What are neutrosophic soluble groups?

117. Give 5 different examples of neutrosophic soluble groups, 4 examples of soluble groups? Identify the connection between neutosophic soluble groups and soluble groups?

118. Breifly discuiss mixed neutrosophic soluble groups? Give different examples of it?

119. Construct some examples of soluble groups with some other properties?

120. What are the conditions for a neutrosophic group to be a neutrosophic soluble groups? Discuiss in details?

121. What is strong neutrosophic soulbe group? Give some examples?

122. What is mixed neutrosophic soluble group? Give some examples?

123. What is the conncetion between a neutrosophic soluble group and a soluble group?

124. Define derived length of a neutrosophic souble group?

125. Give the derive length of some neutrosophic soluble groups?

126. What is a neutrosophic central series? Give some examples of it.

127. What are neutrosophic nilpotent groups? Discuiss in details with examples?

128. What are the conditions for a neutrosophic group to be a neutrosophic nilpotent group?

129. What is a strong neutrosophic central series, mixed neutrosophic central series and central series? Discuiss in detail and find the relationship among them?

130. Give examples of each in above 129.

131. Give examples of strong neutrosophic nilpotent groups, mixed neutrosophic nilpotent groups and nilpotent groups? Find their

relation among each other?

132. What is the difference between neutrosophic bi-subnormal series and neutrosophic bi-normal series?

133. Give examples of neutrosophic bi-subnormal series and neutrosophic bi-normal series?

134. Give an example of a neutrosophic bi-subnormal series which is not neutrosophic bi-normal?

135. Can someone point out the main differences among neutrosophic bisubgroup series, neutrosophic bi-subnormal series and neutrosophic bi-normal series?

136. What are strong neutrosophic bisubgroup series, mixed neutrosophic bisubgroup series and bisubgroup seires? Give examples in each case?

137. Identify the main differences among strong neutrosophic bisubgroup series, mixed neutrosophic bisubgroup series and

bisubgroup seires?

138. What are strong neutrosophic bi-subnormal series, mixed neutrosophic bi-subnormal series and bi-subnormal seires?

139. Give examples in each case?

140. Identify the main differences among strong neutrosophic bi-subnormal series, mixed neutrosophic bi-subnormal series and bi-subnormal seires?

141. What are strong neutrosophic bi-normal series, mixed neutrosophic bi-normal series and bi-normal seires? Give examples in each case?

142. Identify the main differences among strong neutrosophic bi-normal series, mixed neutrosophic bi-normal series and bi-normal seires?

143. What is a neutrosophic bi-abelian series? Give some examples of neutrosohic bi-abelian series?

144. Breifly state strong neutrosophic bi-abelian series with illustrative examples?

145. Can you give examples of mixed neutrosophic bi-abelian series with examples?

146. What is a bi-abelian seires? Give examples of it?

147. What are neutrosophic soluble bigroups?

148. Give 5 different examples of neutrosophic soluble bigroups, 4 examples of soluble bigroups? Identify the connection between neutosophic soluble bigroups and soluble bigroups?

149. Breifly discuiss mixed neutrosophic soluble bigroups? Give different eamples of it?

150. Construct some examples of soluble bigroups with some other properties?

151. What are the conditions for a neutrosophic bigroup to be a

neutrosophic soluble bigroups? Discuiss in details?

152. What is strong neutrosophic souble bigroup? Give some examples?

153. What is mixed neutrosophic soluble bigroup? Give some examples?

154. What is the conncetion between a neutrosophic soluble bigroup and a soluble bigroup?

155. Define derived length of a neutrosophic souble bigroup?

156. Give the derive length of some neutrosophic soluble bigroups?

157. What is a neutrosophic bi-central series? Give some examples of it.

158. What are neutrosophic nilpotent bigroups? Discuiss in details with examples?

159. What are the conditions for a neutrosophic bigroup to be a neutrosophic nilpotent bigroup?

160. What is a strong neutrosophic bi-central series, mixed neutrosophic bi-central series and bi-central series? Discuiss in detail and find the relationship among them?

161. Give examples of each in above 160.

162. Give examples of strong neutrosophic nilpotent bigroups, mixed neutrosophic nilpotent bigroups and nilpotent bigroups? Find their relation among each other?

163. What is the difference between neutrosophic N-subnormal series and neutrosophic N-normal series?

164. Give examples of neutrosophic N-subnormal series and neutrosophic N-normal series?

165. Give an example of a neutrosophic N-subnormal series which is not neutrosophic N-normal?

166. Can one point out the main differences among neutrosophic N-subgroup series, neutrosophic N-subnormal series and

neutrosophic N-normal series?

167. What are strong neutrosophic N-subgroup series, mixed neutrosophic N-subgroup series and N-subgroup seires? Give examples in each case?

168. Identify the main differences among strong neutrosophic N-subgroup series, mixed neutrosophic N-subgroup series and N-subgroup seires?

169. What are strong neutrosophic N-subnormal series, mixed neutrosophic N-subnormal series and N-subnormal seires?

170. Give examples in each case?

171. Identify the main differences among strong neutrosophic N-subnormal series, mixed neutrosophic N-subnormal series and N-subnormal seires?

172. What are strong neutrosophic N-normal series, mixed neutrosophic N-normal series and N-normal seires? Give examples in each

case?

173. Identify the main differences among strong neutrosophic N-normal series, mixed neutrosophic N-normal series and N-normal seires?

174. What is a neutrosophic N-abelian series? Give some examples of neutrosohic N-abelian series?

175. Breifly state strong neutrosophic N-abelian series with illustrative examples?

176. Can you give examples of mixed neutrosophic N-abelian series with examples?

177. What is an N-abelian seires? Give examples of it?

178. What are neutrosophic soluble N-groups?

179. Give 5 different examples of neutrosophic soluble N-groups, 4 examples of soluble N-groups? Identify the connection between neutosophic soluble N-groups and soluble N-groups?

180. Breifly discuiss mixed neutrosophic soluble N0groups? Give different eamples of it?

181. Construct some examples of soluble N-groups with some other properties?

182. What are the conditions for a neutrosophic N-group to be a neutrosophic soluble N-groups? Discuiss in details?

183. What is strong neutrosophic souble N-group? Give some examples?

183. What is a mixed neutrosophic soluble N-group? Give some examples?

184. What is the conncetion between a neutrosophic soluble N-group and a soluble N-group?

185. Define derived length of a neutrosophic souble N-group?

186. Give the derive length of some neutrosophic soluble N-groups?

187. What is a neutrosophic N-central series? Give some examples of it.

188. What are neutrosophic nilpotent Nogroups? Discuiss in details with examples?

189. What are the conditions for a neutrosophic N-group to be a neutrosophic nilpotent N-group?

190. What is a strong neutrosophic N-central series, mixed neutrosophic N-central series and N-central series? Discuiss in detail and find the relationship among them?

191. Give examples of each in above 190.

192. Give examples of strong neutrosophic nilpotent N-groups, mixed neutrosophic nilpotent N-groups and nilpotent N-groups? Find their relation among each other?

193. How one can define neutrosophic RA-semigroup ring? Give

examples to illustrate it.

194. Give some different examples of the neutrosophic LA-semigroup ring?

195. Does there exist any relation between the neutrosophic LA-semigroup ring and an LA-semigroup ring? Justify your answer with the help of examples?

196. How one can define zero divisors in a neutrosophic LA-semigroup ring?

197. Give some examples of the neutrosophic LA-semigroup ring in which zero divisors are exist?

198. How one can define neutrosophic principal ideal of a neutrosophic LA-semigroup ring?

199. What is the difference between a neutrosophic semigroup ring and a neutrosophic LA-semigroup ring? Explain it with the help of examples?

200. When a neutrosophic LA-semigroup ring is a neutrosophic RA-semigroup ring?

201. Which one is more generalize in the following.
    1). Neutrosophic semigroup ring. 2). Neutrosophic LA-semigroup ring.

202. Can one define strong neutrosophic LA-semgroup ring?

203. Discuis in details different types of neutrosophic sub LA-semigroup rings?

204. Give examples of neutrosophic sub LA-semigroup rings?

205. Discuiss different kinds of strong neutrosophic sub LA-semigroup rings?

206. Can one define neutrosophic minimal ideal of a neutrosophic LA-semigroup ring?

207. Give 4 examples of neutrosophic minimal ideals of a neutrosophic

LA-semigroup ring if there exist?

208. How one can define neutrosophic maximal ideal of a neutrosophic LA-semigroup ring? Give examples to illustrate it?

209. Does there exist strong neutrosophic minimal ideal and strong neutrosophic maximal ideal of a neutrosophic LA-semigroup?

210. Give some examples in each case of above?

211. How one can define lagrange neutrosophic sub LA-semigroup ring?

212. Does there exist lagrange neutrosophic LA-semigroup ring?

213. If yes, give some examples of lagrnage neutrosophic LA-semigroup ring?

214. What is meant by weakly lagrange neutrosophic LA-semigroup ring?

215. Does there exist weakly lagrange neutrosophic LA-semigroup ring?

216. Give some examples of weakly lagrange neutrosophic LA-semigroup ring?

217. Define lagrange free neutrosophic LA-semigroup ring?

218. Give examples of lagrange free neutrosophic LA-semigroup ring?

219. Discuiss about pseudo neutrosophic ideals with examples?

220. Give some examples loyal neutrosophic ideals?

221. What are quasi loyal neutrosophic ideals? Give some examples?

222. Discuiss about bounded quasi neutrosophic ideals in detail? Give some examples of bounded quasi neutrosophic ideal?

223. Can one define neutrosophic ideal of an LA-semigroup neutrosophic ring? Give examples of it?

224. How one can define neutrosophic bi-RA-semigroup biring? Give examples to illustrate it.

225. Give some different examples of the neutrosophic bi-LA-semigroup biring?

226. Does there exist any relation between the neutrosophic bi-LA-semigroup ring and a bi-LA-semigroup biring? Justify your answer with the help of examples?

227. How one can define zero divisors in a neutrosophic bi-LA-semigroup biring?

228. Give some examples of the neutrosophic bi-LA-semigroup biring in which zero divisors are exist?

229. How one can define neutrosophic principal bi-ideal of a neutrosophic bi-LA-semigroup biring?

230. What is the difference between a neutrosophic bisemigroup biring

and a neutrosophic bi-LA-semigroup biring? Explain it with the help of examples?

231. When a neutrosophic bi-LA-semigroup ring is a neutrosophic bi-RA-semigroup biring?

232. Which one is more generalize in the following.

1). Neutrosophic bisemigroup biring. 2). Neutrosophic bi-LA-semigroup biring.

233. Can one define strong neutrosophic bi-LA-semgroup biring?

234. Discuis in details different types of neutrosophic sub bi-LA-semigroup birings?

235. Give examples of neutrosophic sub bi-LA-semigroup birings?

236. Discuiss different kinds of strong neutrosophic sub bi-LA-semigroup birings?

237. Can one define neutrosophic minimal bi-ideal of a neutrosophic

LA-semigroup biring?

238. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic bi-LA-semigroup biring if there exist?

239. How one can define neutrosophic maximal biideal of a neutrosophic bi-LA-semigroup biring? Give examples to illustrate it?

240. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic bi-LA-semigroup biring?

241. Give some examples in each case of above?

242. How one can define lagrange neutrosophic sub bi-LA-semigroup biring?

243. Does there exist lagrange neutrosophic bi-LA-semigroup biring?

244. If yes, give some examples of such lagrnage neutrosophic bi-LA-

semigroup biring?

245. What is meant by weakly lagrange neutrosophic bi-LA-semigroup biring?

246. Does there exist weakly lagrange neutrosophic LA-semigroup biring?

247. Give some examples of weakly lagrange neutrosophic bi-LA-semigroup biring?

248. Define lagrange free neutrosophic bi-LA-semigroup biring?

249. Give examples of lagrange free neutrosophic bi-LA-semigroup biring?

250. Discuiss about pseudo neutrosophic bi-ideals with examples?

251. Give some examples loyal neutrosophic bi-ideals?

252. What are quasi loyal neutrosophic bi-ideals? Give some examples?

253. Discuiss about bounded quasi neutrosophic bi-ideals in detail? Give some examples of bounded quasi neutrosophic bi-ideal?

254. Can one define neutrosophic bi-ideal of a bi-LA-semigroup neutrosophic biring? Give examples of it?

255. How one can define neutrosophic N-RA-semigroup N-ring? Give examples to illustrate it.

256. Give some different examples of the neutrosophic N-LA-semigroup N-ring?

257. Does there exist any relation between the neutrosophic N-LA-semigroup N-ring and an N-LA-semigroup N-ring? Justify your answer with the help of examples?

258. How one can define neutrosophic principal N-ideal of a neutrosophic N-LA-semigroup N-ring?

259. What is the difference between a neutrosophic N-semigroup N-ring

and a neutrosophic N-LA-semigroup N-ring? Explain it with the help of examples?

260. When a neutrosophic N-LA-semigroup N-ring is a neutrosophic N-RA-semigroup N-ring?

261. Can one define strong neutrosophic N-LA-semgroup N-ring?

262. Discuis in details different types of neutrosophic sub N-LA-semigroup N-rings?

263. Give examples of neutrosophic sub N-LA-semigroup N-rings?

264. Discuiss different kinds of strong neutrosophic sub N-LA-semigroup N-rings?

265. Can one define neutrosophic minimal N-ideal of a neutrosophic N-LA-semigroup N-ring?

266. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-LA-semigroup N-ring if there exist?

267. How one can define neutrosophic maximal N-ideal of a neutrosophic N-LA-semigroup N-ring? Give examples to illustrate it?

268. Does there exist strong neutrosophic minimal N-ideal and strong neutrosophic maximal N-ideal of a neutrosophic N-LA-semigroup?

269. Give some examples in each case of above?

270. How one can define lagrange neutrosophic sub N-LA-semigroup N-ring?

271. Does there exist lagrange neutrosophic N-LA-semigroup N-ring?

272. If yes, give some examples of lagrnage neutrosophic N-LA-semigroup N-ring?

273. What is meant by weakly lagrange neutrosophic N-LA-semigroup N-ring?

274. Does there exist weakly lagrange neutrosophic N-LA-semigroup N-ring?

275. Give some examples of weakly lagrange neutrosophic N-LA-semigroup N-ring?

276. Define lagrange free neutrosophic N-LA-semigroup N-ring?

277. Give examples of lagrange free neutrosophic N-LA-semigroup N-ring?

278. Discuiss about pseudo neutrosophic N-ideals with examples?

279. Give some examples loyal neutrosophic N-ideals?

280. What are quasi loyal neutrosophic N-ideals? Give some examples?

281. Discuiss about bounded quasi neutrosophic N-ideals in detail?

282. Give some examples of bounded quasi neutrosophic N-ideal?

283. Give some different examples of the neutrosophic Loop ring?

284. Does there exist any relation between the neutrosophic Loop ring and a Loop ring? Justify your answer with the help of examples?

285. How one can define zero divisors in a neutrosophic Loop ring?

286. Give some examples of the neutrosophic Loop ring in which zero divisors are exist?

287. How one can define neutrosophic principal ideal of a neutrosophic Loop ring?

288. Can one define strong neutrosophic Loop ring?

289. Discuiss in details different types of neutrosophic subloop rings?

290. Give examples of neutrosophic subloop rings?

291. Discuiss different kinds of strong neutrosophic subloop rings?

292. Can one define neutrosophic minimal ideal of a neutrosophic loop ring?

293. Give 4 examples of neutrosophic minimal ideals of a neutrosophic loop ring, if exist?

294. How one can define neutrosophic maximal ideal of a neutrosophic loop ring? Give examples to illustrate it?

295. Does there exist strong neutrosophic minimal ideal and strong neutrosophic maximal ideal of a neutrosophic loop?

296. Give some examples in each case of above?

297. How one can define lagrange neutrosophic subloop ring?

298. Does there exist lagrange neutrosophic loop ring?

299. If yes, give some examples of lagrnage neutrosophic loop ring?

300. What is meant by weakly lagrange neutrosophic loop ring?

301. Does there exist weakly lagrange neutrosophic loop ring?

302. Give some examples of weakly lagrange neutrosophic loop ring?

303. Define lagrange free neutrosophic loop ring?

304. Give examples of lagrange free neutrosophic loop ring?

305. Discuiss about pseudo neutrosophic ideals with examples?

306. Give some examples loyal neutrosophic ideals?

307. What are quasi loyal neutrosophic ideals? Give some examples?

308. Discuiss about bounded quasi neutrosophic ideals in detail? Give some examples of bounded quasi neutrosophic ideal?

309. Can one define neutrosophic ideal of a loop neutrosophic ring? Give examples of it?

310. Give some different examples of the neutrosophic biloop biring?

311. How one can define neutrosophic principal bi-ideal of a neutrosophic biloop biring?

312. Can one define strong neutrosophic biloop biring?

313. Discuiss in details different types of neutrosophic subbiloop birings?

314. Give examples of neutrosophic subbiloop birings?

315. Discuiss different kinds of strong neutrosophic subbiloop birings?

316. Can one define neutrosophic minimal bi-ideal of a neutrosophic loop biring?

317. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic biloop biring if there exist?

318. How one can define neutrosophic maximal biideal of a neutrosophic biloop biring? Give examples to illustrate it?

319. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic biloop biring?

320. Give some examples in each case of above?

321. How one can define lagrange neutrosophic sub biloop biring?

322. Does there exist lagrange neutrosophic biloop biring?

323. If yes, give some examples of such lagrnage neutrosophic biloop biring?

324. What is meant by weakly lagrange neutrosophic biloop biring?

325. Does there exist weakly lagrange neutrosophic biloop biring?

326. Give some examples of weakly lagrange neutrosophic biloop biring?

327. Define lagrange free neutrosophic biloop biring?

328. Give examples of lagrange free neutrosophic biloop biring?

329. Discuiss about pseudo neutrosophic bi-ideals with examples?

330. Give some examples loyal neutrosophic bi-ideals?

331. What are quasi loyal neutrosophic bi-ideals? Give some examples?

332. Discuiss about bounded quasi neutrosophic bi-ideals in detail? Give some examples of bounded quasi neutrosophic bi-ideal?

333. Can one define neutrosophic bi-ideal of a biloop neutrosophic biring? Give examples of it?

334. Give some different examples of the neutrosophic N-loop N-ring?

335. How one can define neutrosophic principal N-ideal of a neutrosophic N-loop N-ring?

336. Can one define strong neutrosophic N-loop N-ring?

337. Discuiss in details different types of neutrosophic sub N-loop N-rings?

338. Give examples of neutrosophic sub N-loop N-rings?

339. Discuiss different kinds of strong neutrosophic sub N-loop N-rings?

340. Can one define neutrosophic minimal N-ideal of a neutrosophic N-loop N-ring?

341. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-loop N-ring if there exist?

342. How one can define neutrosophic maximal N-ideal of a neutrosophic N-loop N-ring? Give examples to illustrate it?

343. Does there exist strong neutrosophic minimal N-ideal and strong

neutrosophic maximal N-ideal of a neutrosophic N-loop?

344. Give some examples in each case of above?

345. How one can define lagrange neutrosophic sub N-loop N-ring?

346. Does there exist lagrange neutrosophic N-loop N-ring?

347. If yes, give some examples of lagrnage neutrosophic N-loop N-ring?

348. What is meant by weakly lagrange neutrosophic N-loop N-ring?

349. Does there exist weakly lagrange neutrosophic N-loop N-ring?

350. Give some examples of weakly lagrange neutrosophic N-loop N-ring?

351. Define lagrange free neutrosophic N-loop N-ring?

352. Give examples of lagrange free neutrosophic N-loop N-ring?

353. Discuiss about pseudo neutrosophic N-ideals with examples?

354. Give some examples loyal neutrosophic N-ideals?

355. What are quasi loyal neutrosophic N-ideals? Give some examples?

356. Discuiss about bounded quasi neutrosophic N-ideals in detail? Give some examples of bounded quasi neutrosophic N-ideal?

357. Give some different examples of the neutrosophic groupoid rings?

358. Does there exist any relation between the neutrosophic groupoid ring and a groupoid ring? Justify your answer with the help of examples?

359. How one can define zero divisors in a neutrosophic groupoid ring?

360. Give some examples of the neutrosophic groupoid ring in which zero divisors are exist?

361. How one can define neutrosophic principal ideal of a neutrosophic groupoid ring?

362. Can one define strong neutrosophic groupoid ring?

363. Discuiss in details different types of neutrosophic subgroupoid rings?

364. Give examples of neutrosophic subgroupoid rings?

365. Discuiss different kinds of strong neutrosophic subgroupoid rings?

366. Can one define neutrosophic minimal ideal of a neutrosophic groupoid ring?

367. Give 3 examples of neutrosophic minimal ideals of a neutrosophic groupoid ring if there exist?

368. How one can define neutrosophic maximal ideal of a neutrosophic groupoid ring? Give examples to illustrate it?

369. Does there exist strong neutrosophic minimal ideal and strong neutrosophic maximal ideal of a neutrosophic groupoid ring?

370. Give some examples in each case of above?

371. How one can define lagrange neutrosophic subgroupoid ring?

372. Does there exist lagrange neutrosophic groupoid ring?

373. If yes, give some examples of lagrnage neutrosophic groupoid ring?

374. What is meant by weakly lagrange neutrosophic groupoid ring?

375. Does there exist weakly lagrange neutrosophic groupoid ring?

376. Give some examples of weakly lagrange neutrosophic groupoid ring?

377. Define lagrange free neutrosophic groupoid ring?

378. Give examples of lagrange free neutrosophic groupoid ring?

379. Discuiss about pseudo neutrosophic ideals with examples?

380. Give some examples loyal neutrosophic ideals?

381. What are quasi loyal neutrosophic ideals? Give some examples?

382. Discuiss about bounded quasi neutrosophic ideals in detail? Give some examples of bounded quasi neutrosophic ideal?

383. Can one define neutrosophic ideal of a groupoid neutrosophic ring? Give examples of it?

384. Give some different examples of the neutrosophic bigroupoid biring?

385. Does there exist any relation between the neutrosophic bigroupoid biring and a bigroupoid biring? Justify your answer with the help of examples?

386. How one can define neutrosophic principal bi-ideal of a neutrosophic bigroupoid biring?

387. Can one define strong neutrosophic bigroupoid biring?

388. Discuiss in details different types of neutrosophic subbigroupoid birings?

389. Give examples of neutrosophic subbigroupoid birings?

390. Discuiss different kinds of strong neutrosophic subbigroupoid birings?

391. Can one define neutrosophic minimal bi-ideal of a neutrosophic bigroupoid biring?

392. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic bigroupoid biring if there exist?

393. How one can define neutrosophic maximal biideal of a neutrosophic bigroupoid biring? Give examples to illustrate it?

394. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic bigroupoid biring?

395. Give some examples in each case of above?

396. How one can define lagrange neutrosophic subbigroupoid biring?

397. Does there exist lagrange neutrosophic bigroupoid biring?

398. If yes, give some examples of such lagrnage neutrosophic bigroupoid biring?

399. What is meant by weakly lagrange neutrosophic bigroupoid biring?

400. Does there exist weakly lagrange neutrosophic bigroupoid biring?

401. Give some examples of weakly lagrange neutrosophic bigroupoid biring?

402. Define lagrange free neutrosophic bigroupoid biring?

403. Give examples of lagrange free neutrosophic bigroupoid biring?

404. Discuiss about pseudo neutrosophic bi-ideals with examples?

405. Give some examples loyal neutrosophic bi-ideals?

406. What are quasi loyal neutrosophic bi-ideals? Give some examples?

407. Discuiss about bounded quasi neutrosophic bi-ideals in detail?

408. Give some examples of bounded quasi neutrosophic bi-ideal?

409. Can one define neutrosophic bi-ideal of a bigroupoid neutrosophic biring? Give examples of it?

410. Give some different examples of the neutrosophic N-groupoid N-ring?

411. Does there exist any relation between the neutrosophic N-groupoid

N-ring and an N-groupoid N-ring? Justify your answer with the help of examples?

412. How one can define neutrosophic principal N-ideal of a neutrosophic N-groupoid N-ring?

413. Can one define strong neutrosophic N-groupoid N-ring?

414. Discuiss in details different types of neutrosophic sub N-groupoid N-rings?

415. Give examples of neutrosophic sub N-groupoid N-rings?

416. Discuiss different kinds of strong neutrosophic sub N-groupoid N-rings?

417. Can one define neutrosophic minimal N-ideal of a neutrosophic N-groupoid N-ring?

418. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-groupoid N-ring if there exist?

419. How one can define neutrosophic maximal N-ideal of a neutrosophic N-groupoid N-ring? Give examples to illustrate it?

420. Does there exist strong neutrosophic minimal N-ideal and strong neutrosophic maximal N-ideal of a neutrosophic N-groupoid N-ring?

421. Give some examples in each case of above?

422. How one can define lagrange neutrosophic sub N-groupoid N-ring?

423. Does there exist lagrange neutrosophic N-groupoid N-ring?

424. If yes, give some examples of lagrnage neutrosophic N-groupoid N-ring?

425. What is meant by weakly lagrange neutrosophic N-groupoid N-

ring?

426. Does there exist weakly lagrange neutrosophic N-groupoid N-ring?

427. Give some examples of weakly lagrange neutrosophic N-groupoid N-ring?

428. Define lagrange free neutrosophic N-groupoid N-ring?

429. Give examples of lagrange free neutrosophic N-groupoid N-ring?

430. Discuiss about pseudo neutrosophic N-ideals with examples?

431. Give some examples loyal neutrosophic N-ideals?

432. What are quasi loyal neutrosophic N-ideals? Give some examples?

433. Discuiss about bounded quasi neutrosophic N-ideals in detail?

434. Give some examples of bounded quasi neutrosophic N-ideal of a

neutrosophic groupoid ringt?

435. Give some different examples of the neutrosophic biring?

436. Does there exist any relation between the neutrosophic biring and a biring? Justify your answer with the help of examples?

437. How one can define zero divisors in a neutrosophic biring?

438. Give some examples of the neutrosophic biring in which zero divisors are exist?

439. How one can define neutrosophic principal bi-ideal of a neutrosophic biring?

440. Can one define strong neutrosophic biring?

441. Discuiss in details different types of neutrosophic subbirings?

442. Give examples of neutrosophic sub bibirings?

443. Discuiss different kinds of strong neutrosophic subbirings?

444. Can one define neutrosophic minimal bi-ideal of a neutrosophic biring?

445. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic biring if there exist?

446. How one can define neutrosophic maximal biideal of a neutrosophic biring? Give examples to illustrate it?

447. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic biring?

448. Give some examples in each case of above?

449. How one can define lagrange neutrosophic subbiring?

450. Does there exist lagrange neutrosophic biring?

451. If yes, give some examples of such lagrnage neutrosophic biring?

452. What is meant by weakly lagrange neutrosophic biring?

453. Does there exist weakly lagrange neutrosophic biring?

454. Give some examples of weakly lagrange neutrosophic biring?

455. Define lagrange free neutrosophic biring?

456. Give examples of lagrange free neutrosophic biring?

457. Discuiss about pseudo neutrosophic bi-ideals with examples?

458. Give some examples loyal neutrosophic bi-ideals?

459. What are quasi loyal neutrosophic bi-ideals? Give some examples?

460. Discuiss about bounded quasi neutrosophic bi-ideals in detail? Give some examples of bounded quasi neutrosophic bi-ideal?

461. Give some different examples of the neutrosophic N-ring?

462. How one can define neutrosophic principal N-ideal of a neutrosophic N-ring?

463. Can one define strong neutrosophic N-ring?

464. Discuis in details different types of neutrosophic sub N-rings?

465. Give examples of neutrosophic sub N-rings?

466. Discuiss different kinds of strong neutrosophic sub N-rings?

467. Can one define neutrosophic minimal N-ideal of a neutrosophic N-ring?

468. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-ring if there exist?

469. How one can define neutrosophic maximal N-ideal of a neutrosophic N-ring? Give examples to illustrate it?

470. Does there exist strong neutrosophic minimal N-ideal and strong neutrosophic maximal N-ideal of a neutrosophic N-ring?

471. Give some examples in each case of above?

472. How one can define lagrange neutrosophic sub N-ring?

473. Does there exist lagrange neutrosophic N-ring?

474. If yes, give some examples of lagrnage neutrosophic N-ring?

475. What is meant by weakly lagrange neutrosophic N-ring?

476. Does there exist weakly lagrange neutrosophic N-ring?

477. Give some examples of weakly lagrange neutrosophic N-ring?

478. Define lagrange free neutrosophic -ring?

479. Give examples of lagrange free neutrosophic N-ring?

480. Discuiss about pseudo neutrosophic N-ideals with examples?

481. Give some different examples of the neutrosophic bigroup biring?

482. Does there exist any relation between the neutrosophic bigroup ring and a bigroup biring? Justify your answer with the help of examples?

483. How one can define zero divisors in a neutrosophic bigroup biring?

484. Give some examples of the neutrosophic group biring in which zero divisors are exist?

485. How one can define neutrosophic principal bi-ideal of a neutrosophic bigroup biring?

486. Can one define strong neutrosophic bigroup biring?

487. Discuis in details different types of neutrosophic sub bigroup birings?

488. Give examples of neutrosophic sub bigroup birings?

489. Discuiss different kinds of strong neutrosophic sub bigroup birings?

490. Can one define neutrosophic minimal bi-ideal of a neutrosophic bigroup biring?

491. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic bigroup biring if there exist?

492. How one can define neutrosophic maximal biideal of a neutrosophic bigroup biring? Give examples to illustrate it?

493. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic bigroup biring?

494. Give some examples in each case of above?

495. How one can define lagrange neutrosophic sub bigroup biring?

496. Does there exist lagrange neutrosophic bigroup biring?

497. If yes, give some examples of such lagrnage neutrosophic bigroup biring?

498. What is meant by weakly lagrange neutrosophic bigroup biring?

499. Does there exist weakly lagrange neutrosophic bigroup biring?

500. Give some examples of weakly lagrange neutrosophic bigroup biring?

501. Define lagrange free neutrosophic bigroup biring?

502. Give examples of lagrange free neutrosophic bigroup biring?

503. Discuiss about pseudo neutrosophic bi-ideals with examples?

504. Give some examples loyal neutrosophic bi-ideals?

505. What are quasi loyal neutrosophic bi-ideals? Give some examples?

506. Discuiss about bounded quasi neutrosophic bi-ideals in detail? Give some examples of bounded quasi neutrosophic bi-ideal?

507. Can one define neutrosophic bi-ideal of a bigroup neutrosophic biring? Give examples of it?

508. Give some different examples of the neutrosophic N-group N-ring?

509. How one can define neutrosophic principal N-ideal of a neutrosophic N-group N-ring?

510. Can one define strong neutrosophic N-group N-ring?

511. Discuis in details different types of neutrosophic sub N-group N-rings?

512. Give examples of neutrosophic sub N-group N-rings?

513. Discuiss different kinds of strong neutrosophic sub N-group N-rings?

514. Can one define neutrosophic minimal N-ideal of a neutrosophic N-group N-ring?

515. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-group N-ring if there exist?

516. How one can define neutrosophic maximal N-ideal of a neutrosophic N-group N-ring? Give examples to illustrate it?

517. Does there exist strong neutrosophic minimal N-ideal and strong neutrosophic maximal N-ideal of a neutrosophic N-group?

518. Give some examples in each case of above?

519. How one can define lagrange neutrosophic sub N-group N-ring?

520. Does there exist lagrange neutrosophic N-group N-ring?

521. If yes, give some examples of lagrnage neutrosophic N-group N-ring?

522. What is meant by weakly lagrange neutrosophic N-group N-ring?

523. Does there exist weakly lagrange neutrosophic N-group N-ring?

524. Give some examples of weakly lagrange neutrosophic N-group N-ring?

525. Define lagrange free neutrosophic N-group N-ring?

526. Give examples of lagrange free neutrosophic N-group N-ring?

527. Discuiss about pseudo neutrosophic N-ideals with examples?

528. Give some examples loyal neutrosophic N-ideals?

529. What are quasi loyal neutrosophic N-ideals? Give some examples?

530. Discuiss about bounded quasi neutrosophic N-ideals in detail?

Give some examples of bounded quasi neutrosophic N-ideal?

531. Give some different examples of the neutrosophic bisemigroup biring?

532. Does there exist any relation between the neutrosophic bisemigroup ring and a bisemigroup biring? Justify your answer with the help of examples?

533. How one can define zero divisors in a neutrosophic bisemigroup biring?

534. Give some examples of the neutrosophic bisemigroup biring in which zero divisors are exist?

535. How one can define neutrosophic principal bi-ideal of a neutrosophic bisemigroup biring?

536. Can one define strong neutrosophic bisemigroup biring?

537. Discuis in details different types of neutrosophic sub bisemigroup birings?

538. Give examples of neutrosophic sub bisemigroup birings?

539. Discuiss different kinds of strong neutrosophic subbisemigroup birings?

540. Can one define neutrosophic minimal bi-ideal of a neutrosophic bisemigroup biring?

541. Give 2 examples of neutrosophic minimal bi-ideals of a neutrosophic bisemigroup biring if there exist?

542. How one can define neutrosophic maximal biideal of a neutrosophic bisemigroup biring? Give examples to illustrate it?

543. Does there exist strong neutrosophic minimal bi-ideal and strong neutrosophic maximal bi-ideal of a neutrosophic bisemigroup biring?

544. Give some examples in each case of above?

545. How one can define lagrange neutrosophic sub bisemigroup biring?

546. Does there exist lagrange neutrosophic bisemigroup biring?

547. If yes, give some examples of such lagrnage neutrosophic bisemigroup biring?

548. What is meant by weakly lagrange neutrosophic bisemigroup biring?

549. Does there exist weakly lagrange neutrosophic bisemigroup biring?

550. Give some examples of weakly lagrange neutrosophic bisemigroup biring?

551. Define lagrange free neutrosophic bisemigroup biring?

552. Give examples of lagrange free neutrosophic bisemigroup biring?

553. Discuiss about pseudo neutrosophic bi-ideals with examples?

554. Give some examples loyal neutrosophic bi-ideals?

555. What are quasi loyal neutrosophic bi-ideals? Give some examples?

556. Discuiss about bounded quasi neutrosophic bi-ideals in detail? Give some examples of bounded quasi neutrosophic bi-ideal?

557. Can one define neutrosophic bi-ideal of a bisemigroup neutrosophic biring? Give examples of it?

558. Give some different examples of the neutrosophic N-semigroup N-ring?

559. How one can define neutrosophic principal N-ideal of a neutrosophic N-semigroup N-ring?

560. Can one define strong neutrosophic N-semigroup N-ring?

561. Discuis in details different types of neutrosophic sub N-semigroup N-rings?

562. Give examples of neutrosophic sub N-semigroup N-rings?

563. Discuiss different kinds of strong neutrosophic sub N-semigroup N-rings?

564. Can one define neutrosophic minimal N-ideal of a neutrosophic N-semigroup N-ring?

565. Give 3 examples of neutrosophic minimal N-ideals of a neutrosophic N-semigroup N-ring if there exist?

566. How one can define neutrosophic maximal N-ideal of a neutrosophic N-semigroup N-ring? Give examples to illustrate it?

567. Does there exist strong neutrosophic minimal N-ideal and strong neutrosophic maximal N-ideal of a neutrosophic N-semigroup?

568. Give some examples in each case of above?

569. How one can define lagrange neutrosophic sub N-semigroup N-ring?

570. Does there exist lagrange neutrosophic N-semigroup N-ring?

571. If yes, give some examples of lagrnage neutrosophic N-semigroup N-ring?

572. What is meant by weakly lagrange neutrosophic N-semigroup N-ring?

573. Does there exist weakly lagrange neutrosophic N-semigroup N-ring?

574. Give some examples of weakly lagrange neutrosophic N-semigroup N-ring?

575. Define lagrange free neutrosophic N-semigroup N-ring?

576. Give examples of lagrange free neutrosophic N-semigroup N-ring?

577. Discuiss about pseudo neutrosophic N-ideals with examples?

578. Give some examples loyal neutrosophic N-ideals?

579. What are quasi loyal neutrosophic N-ideals? Give some examples?

580. Discuiss about bounded quasi neutrosophic N-ideals in detail? Give some examples of bounded quasi neutrosophic N-ideal?

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An elaborative list of references is given for the readers to study the notions of neutrosophic set theory in innovative directions that several researchers carried out.

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In this book, we define several new neutrosophic algebraic structures and their related properties. The main focus of this book is to study the important class of neutrosophic rings such as neutrosophic LA-semigroup ring, neutrosophic loop ring, neutrosophic groupoid ring and so on. We also construct their generalization in each case to study these neutrosophic algebraic structures ina broader sense. The indeterminacy element " I " give rise to a more bigger algebraic structure than the classical algebraic structures. It mainly classifies the algebraic structures in three catagories such as: neutrosophic algebraic structures, strong neutrosophic algebraic structures, and classical algebraic structures respectively. This reveals the fact that a classical algebraic structure is a part of the neutrosophic algebraic structures. This open a new way for the researchers to think in a broader way to visualize these vast neutrosophic algebraic structures.

ISBN 978-1-59973-313-5

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