The Eightfold Way model, the SU(3)-flavour model and the medium-strong interaction

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Abstract

Lack of any baryon number in the Eightfold Way model, and its intrinsic presence in the SU(3)-flavour model, has been a puzzle since the genesis of these models in 1961-1964. In this paper we show that this is linked to the way that the adjoint representation is defined mathematically for a Lie algebra, and how it manifests itself as a physical representation. This forces us to distinguish between the global and the local charges and between the microscopic and the macroscopic models. As a bonus, a consistent understanding of the hitherto mysterious medium-strong interaction is achieved. We also gain a new perspective on how confinement arises in Quantum Chromodynamics.

Keywords: Lie Groups, Lie Algegra, Jacobi Identity, adjoint representation, Eightfold Way model, SU(3)-flavour model, quark model, symmetry breaking, mass formulae

The Eightfold Way model was proposed independently by Gell-Mann and Ne'eman in 1961, but was very quickly transformed into the the SU(3)-flavour model (as known to us at present) in 1964 [1]. We revisit these models and look into the origin of the Eightfold Way model and try to understand as to how it is related to the SU(3)-flavour model. This allows us to have a fresh perspective of the mysterious medium-strong interaction [2], which still remains an unresolved problem in the theory of the strong interaction [1,2,3].

The origin of the Eightfold Way model was the realization that there was a systematic parallelism between the $1/2^+$ baryons and the 0^- mesons when one supplements the isospin number with a new quantum number called the hypercharge Y. This is indicated in the following table:

Table

Parallel structure of $1/2^+$ baryons and 0^- mesons

Y	T	$1/2^+$ baryons	0^- mesons
+1	1/2	p,n	K^+ , K^0
0	1	$\Sigma^+, \Sigma^0, \Sigma^-$	π^+, π^0, π^0
0	0	Λ^0	η^0
-1	1/2	Ξ^0, Ξ^-	K^0, K^-

The Sakata Model of SU(3) with (p, n and Λ) providing its fundamental representation, worked well for the 0^- and 1^+ mesons but failed to account for the baryons. This led Gell-Mann and Ne'eman to propose that the baryons belong to the regular octet representation. Thus the parallelism between the mesons and the baryons was naturally explained. This assignment was called the Eightfold Way [1].

It is important to note a few basic points about the Eightfold Way model [3,p.277; 4]. First, in order to associate the additive quantum numbers of SU(3) with hypercharge and the U-spin, we must choose T_3 and Y as defined in the Appendix here [3,4,5]. This means that there is **no baryon number** appearing in the formula for Y. Thus there is no baryon number in the Eightfold Way model. Only hypercharge, which is elementary and non-composite, arises here in this model.

Note that once one goes to the SU(3) model, then a baryon number gets defined through a new hypercharge given as Y = B + S. Hence in the SU(3) model, hypercharge is a composite of baryon number and strangeness. This is the major difference between the octet baryons in the Eightfold Way model and the same as composite baryons in the SU(3) model. Of course this problem does not arise for the mesons as the baryon number in trivially zero in both the models.

How come the hypercharge number Y, in the Eightfold Way model and the SU(3) models, are so fundamentally different? Clearly these eigenvalues can not be of the same operator. But how can that be, as in SU(3) there is but one more diagonal generator, besides the isospin T_3 ?

There are other significant issues which become apparent on studying the Eightfold Way model and the SU(3) model as given in the Appendix. First let us look at SU(2). Pions as denoted in terms of the cartesian components (π_1, π_2, π_3), can be expressed in the spherical basis as ($\frac{\pi_1+i\pi_2}{2}, \pi_3, \frac{\pi_1-i\pi_2}{2}$) and which are identified with the physical pions, (π^+, π^0, π^-). These can be expressed as 2X2 matrix as given in the Appendix as $\frac{1}{2}\tau_j\pi_j$ with j=1,2,3. All this corresponds to the adjoint representation of SU(2). Next these are also given in tensor expression as $N\bar{N}$ (as given in the Fermi-Yang model) both as (π^+, π^0, π^-) and as a 2X2 matrix. Similarly, one extends to SU(3). In SU(3), both for the pseudoscalar meson and the spin 1/2 baryons, we have similar identifications in terms of the 3X3 adjoint representation matrices. For the mesons and the baryons these are $\frac{1}{2}\lambda_j\pi_j$ and $\frac{1}{2}\lambda_j\mathcal{B}_|$ with j=1,2,..8 respectively, as shown in the Appendix.

For the mesons, there is a direct correspondence with respect to the tensor states of the quark-antiquark in SU(3). But note that in the SU(3) model, the octet meson is a composite of quark-antiquark while in the adjoint representation the mesons arise as a linear combination of the elementary cartesian components, and hence the corresponding pions may be treated as elementary and non-composite. How is this to be understood?

Next, about the charge. We know that quarks are elemenatry entities in the Standard model $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. It has also been shown, that the electric charge, as a microscopic entity, is fully quantized in the Standard Model [6,7]. It is the same elementary quarks which are arising in the SU(3)-flavour model, at least in the current quark form. So the Gell-Mann Nishijima expression for the electric charge, is also the same microscopic quantizd electric charge. So, for the mesons in the SU(3) quark model, the electric chatge is composite and is built up of microscopic quantized entities.

But the charge of the adjoint representation, are produced from elementary cartesian components just by complexification to the spherical component form; e.g. $\frac{\pi^1+i\pi^2}{2}$, has an electric chrage of 1 unit as that of π^+. What is the nature of this charge? Clearly it is not microscopic, but actually global, as the operation of complexification, which generates this charge, is global in nature. Also these charges are elementary. Hence the charges in the adjoint representation here, are completely different from those of the composite meson in the SU(3) model.

Hence we note that the adjoint representation meson is global in nature, it is non-composite and it has global charges. This is completely at variance with the octet meson state in the SU(3) model - where it is microscopic and composite and has local/microscopic charges.

To belabour the point, we identify as in the Appendix that:

$$\pi_3 = \pi^0 \tag{1}$$

Here we are equating two completely different structures, one is global in nature, has global zero charge and is an elementary (non-composite) vector basis state; and the other is composite, local and microscopic and has composite charges. One is arising form a basic adjoint representation of the group while the other is arising as an octet entity built up of more basic fundamental represenation states.

Next for the baryons, in the adjoint represntation, due to the correspondence with the pseudoscalar mesons these arise as 3X3 matrices with $\frac{1}{2}\lambda_j\mathcal{B}_|$, j=1,2,..8. In the Eightdold Way model, as discussed above, these do not have any baryon number. However, the baryons, arisisng as composite of three quark qqq in the SU(3) model, do have a baryon number B=1. This incompatibility can not be ignored.

Hence what we have seen for the octet mesons and baryons, the Eightfold Way model and the SU(3) models, though are describing the same octet entity in isomorphic mathematical Lie algebraic languages, physically are completely different. The difference in the corresponding quantum numbers shows that in the two languages here, these can not be eigenstates of the same operator. The only way to resolve the issue on physical grounds, is to say that these two, the Eightfold way model and the SU(3) models, are different languages, describing the same entity. That is, that there must be a duality in the decription of these octet mesons and baryons. Meaning that these are simultaneous, coexisting and different languages to describe the same entity. The analogy with the well known wave-particle duality, of the quantum mechanical elementary entities, is very striking.

What we have learnt on physical grounds, how can it be justified mathematically? Let us now look at the corresponding Lie algebras. A Lie algebra is defined as [3,4,5]

$$[X_\lambda, X_\mu] = i f_{\lambda\mu\nu} X_\nu \tag{2}$$

where the genetrators X_μ are Hermitian and the structure constants $f_{\lambda\mu\nu}$ are real. In addition to be a Lie algebra. the generators should satisfy another independent condition, the so called Jacobi Identity

$$[X_\lambda, [X_\mu, X_\nu]] + [X_\mu, [X_\nu, X_\lambda]] + [X_\nu, [X_\lambda, X_\mu]] = 0 \tag{3}$$

In terms of the structure constants the Jacobi Identity becomes

$$f_{\lambda\mu\sigma} f_{\sigma\nu\tau} + f_{\mu\nu\sigma} f_{\sigma\lambda\tau} + f_{\nu\lambda\sigma} f_{\sigma\mu\tau} = 0 \tag{4}$$

Now given a matrix represntation of the Lie generators the Lie bracket is defined as

$$[X_\mu, X_\nu] = X_\mu X_\nu - X_\nu X_\mu \tag{5}$$

Now with the above representation in eqn. (5), the jacobi identity in eqn. (3) is automatically satisfied. Thus here, it does look like just an "identity". But in general in Lie algebra, it is not just a trivial identity (as in the above, it apparently appears to be so), but it is truly an independent condition on the generators of the Lie algebra [8, p.52].

An indpendent representation of basic significance is the adjoint representation which is provided by the structure constants themselves. Define

$$[F_\lambda]_{\mu\nu} = i f_{\lambda\mu\nu} \tag{6}$$

from the Jacobi Identity eqn. (3) above we see immediately that

$$[F_\lambda, F_\mu] = i f_{\lambda\mu\nu} F_\nu \tag{7}$$

This is a Lie algebra and is isomorphic to the original Lie algebra eqn. (2) above. And this isomorphism is what everyone, has been using and thus identifying the 3X3 adjoint representation of the octet in terms of the Eightfold Way model and the SU(3) model. But this isomorphism for the adjoint representation leads to a group homomorphism. In fact it should be clear that the above lie algebra given in eqn. (7) is a different algebra and completely independnt of the other Lie algebra in eqn.(2). However, this independenece and basic difference between the two algebra is well known in mathematics [9, p.374]. We may say that the Jacobi Identity, in terms of the structure constants eqn. (4), **is actually an independent Lie algebra in disguise, giving the adjoint representation**.

We treat the above independence and differene as being physically significant. So, there are two independent and coexsiting algebras for the octet representation. The second one treats the adjoint representation as sacred and independent. While the other one, starting with the defining and fundamental represenation, reproduces the same octet represenation, but in a physically different manner. Clearly being independent, the second diagonal generator in SU(3) is allowed to have different eigenvalues, e,g, the baryon-number-less hypecharge in the Eightfold Way model, in contrast to the composite hypercharge with baryon number in it, as in the SU(3)-flavour model.

Thus the duality for the octet representation for the mesons and the baryons that we had identified earlier, is actually being mapped by the two dual representations as provided by the two independent Lie algebras above. Clearly this duality must be as fundamental as the well known wave-particle duality.

What are some obvious conclusions that we can make based on this duality that we have discovered?

(1) There has been a commonly-felt feeling amongst physicists that any global symmetry or any global structure in physics are at best only true as some sort of approximation of some more basic putative micrsocopic symmetry or structure. We quote Gross [10] here, "Today we believe that global symmetries are unnatural. They smell of action at a distance." But in this paper we have found a global/macroscopic adjoint represenation, which is as basic and as primitive as the local/microscopic fundamental representtion in hadron physics.

(2). Thus due to the duality for the octet spin-half baryons, we have a microscopic model and we have another coexsiting macroscopic model. We may propose that for these baryons, inner structure should be well described by the microscopic model, and towards the outer regions and on the surface and outside the baryons, it would be best to use the macroscopic coexisting structure. This will be the way that this duality may manifest itself. There is thus color degree of freedom inside and colour plays no role for the adjoint represenation, and that these octet baryons would be trivially colour singlet outside. This would therefore explain the confinement problem in QCD.

(3). One conclusion here is that the electric charges are of two kind - one global and the other local. We know that in the Standard model, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ the manner in which the electric charge of W^+ and W^- is defined, it is global in nature. But the electric charge of the matter particles is local in nature [6,7]. However clearly charges which are global, these necessarily exist everywhere, including any local point also. And hence these gauge mediators can couple to the locally charged microscopic matter fields without any problem.

(4). However, the gluon in QCD is microscopic and thus local in character. It therefore interacts as per QCD rules with the local and microscopic quarks. But its colour charge being local, can not interact with any global fields. Thus it does not interact with any global adjoint representation states. This will expalin how gluon field may be confined within the colour singlet outer adjoint represenattion states in hadrons. This also explains why there cannot be bound states of pure gluons as glueballs, as these hadrons do not have the necessary dual structure as do the baryons have.

(5) The medium-strong interaction which is invoked to get the successful Gell-Mann-Okubo mass formula [1-5] has been a big puzzle since it was invoked in the Eightfold Way model. First, why one can use perturbation theory to get it and secondly why the symmetry breaking is through the the eighth member of the octet. These have been contentious issues in these models. Within our new description presented here, these questions find natural explanations. The eighth member is of course due to the independent adjoint representation Lie algebra. First, these are two co-existing models here and each may have its own Hamiltonin associated with it. In adition these two are quite independent of each other in the lowest order. Thus if these do interact through the next lowest order, then it is natural that this interaction would be "weak", thus allowing a perturbative treatment. Also because of the fact that in one model there is no baryon number, and in the

other model, there does exist a baryon number, there is a narrow window through the T=0 and Y=0 term, through which this putative pertubative interaction may take place. Thus the mysterious medium-strong interaction finds a natural explanation in our model.

Appendix

For details, one may refer to [3,4,5].

In SU(2), the mesons may be represented either as an isovector or as a traceles matrix:

$$\begin{pmatrix} \frac{p\bar{p}-n\bar{n}}{2} & p\bar{n} \\ n\bar{p} & \frac{-p\bar{p}-n\bar{n}}{2} \end{pmatrix} = \begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix} \tag{8}$$

We can apply this to the pion triplet, which can be written as an isovector

$$\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} OR \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \tag{9}$$

or as traceless isovector

$$\pi_\alpha{}^\beta = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{\pi_3}{\sqrt{2}} & \frac{\pi_1-i\pi_2}{\sqrt{2}} \\ \frac{\pi_1+i\pi_2}{\sqrt{2}} & \frac{\pi_3}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\tau_j\pi_j \tag{10}$$

In Sakata model [3,4,5] in terms of the standard definition of the generators in SU(3), the hypercharge and the electric charge are defined as

$$Y = \frac{2}{\sqrt{3}}F_8 + \frac{2}{3}B \tag{11}$$

$$Q = (F_3 + \frac{1}{\sqrt{3}}F_8) + \frac{B}{3} \tag{12}$$

Where B is the baryon number.

In the Eightfold Way model, there is no baryon number and from the above equations one has

$$Y = \frac{2}{\sqrt{3}}F_8 = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{-2}{3} \end{pmatrix} \tag{13}$$

$$Q = F_3 + \frac{1}{\sqrt{3}} F_8 = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 \\ 0 & 0 & \frac{-1}{3} \end{pmatrix} \tag{14}$$

From SU(2) one simply extrapolates to SU(3) as

$$M_\alpha{}^\beta = \begin{pmatrix} 2p\bar{p} - n\bar{n} - \lambda\bar{\lambda} & p\bar{n} & p\bar{\lambda} \\ n\bar{p} & \frac{1}{3}(2n\bar{n} - p\bar{p} - \lambda\bar{\lambda}) & n\bar{\lambda} \\ \lambda\bar{p} & \lambda\bar{n} & \frac{1}{3}(-p\bar{p} - n\bar{n} + 2\lambda\bar{\lambda}) \end{pmatrix} \tag{15}$$

This is equated to the octet mesons

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix} \tag{16}$$

In terms of the cartesian coordinates $P_j, j = 1, ..8$ the above states can be written as

$$P = \begin{pmatrix} \frac{P_8}{\sqrt{6}} + \frac{P_3}{\sqrt{2}} & \frac{P_1 - iP_2}{\sqrt{2}} & \frac{P_4 - iP_5}{\sqrt{2}} \\ \frac{P_1 + iP_2}{\sqrt{2}} & \frac{P_8}{\sqrt{6}} - \frac{P_3}{\sqrt{2}} & \frac{P_6 - iP_7}{\sqrt{2}} \\ \frac{P_4 + iP_5}{\sqrt{2}} & \frac{P_6 + iP_7}{\sqrt{2}} & -\frac{2}{\sqrt{6}} P_8 \end{pmatrix} = \frac{1}{\sqrt{2}} \lambda_j P_j \tag{17}$$

In the same manner, the $1/2^+$ baryons octet can be described either by an 8-dimensional vector or by a traceless matrix $\frac{1}{\sqrt{2}}\lambda_j \mathcal{B}_|$. Here the components of $\mathcal{B}_|$ are related to the physical baryons N, Λ, σ, Ξ in the same manner as for the above mesons as:

$$B = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \tag{18}$$

References

(1). M. Gell-Mann and Y. Ne'eman, "The Eightfold Way", W. A. Benjamin Inc., New York, 1964

(2). Y. Ne'eman. "The fifth interaction - origin of the mass breaking symmetry", Phys. Rev. 134 (1964) B1355

(3). S. Gasiorowicz, "Elementary Particle Physics", John Wiley and Sons, New York, 1966

(4). P. A. Carruthers, "Introduction to Unitary Symmetry", Interscience Pub. New York, 1966

(5). N. W. Dean, "Introduction to the Strong Interaction", Gordon and Breach Science Pub. New York, 1976

(6). A. Abbas, "Anomalies and charge quantization in the Standard Model with arbitrary number of colours", Phys. Lett **B 238**, 344 (1990)

(7). A. Abbas, "Spontaneous symmetry breaking, quantization of the electric charge and the anomalies", J. Phys. **G 16**, L163 (1990)

(8). J. Fuchs and S. Schweigert, "Symmetries, Lie Algebras and Representations", Cambridge University Press, Cambridge, 1997

(9). R. Mirman, "Group Theory: An Intuitive Approach", World Scientific Press, Singapore, 1995

(10). D. J. Gross, "Triumphs and limitations of quantum field theory", in Ed. T. Y. Cao, "Conceptual foundations of quantum field theory", Cambridge University Press, Cambridge (1999), p. 56-67