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Roughness of Neutrosophic Sets

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ABSTRACT.
Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. In this paper we define rough neutrosophic sets and study their properties. Some propositions in this notion are proved. Possible application to computer sciences is touched upon.

Introduction

The fuzzy set was introduced by Zadeh [31] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. After the introduction of the neutrosophic set concept [5-20]. The fundamental concepts of neutrosophic set, introduced by Smarandache in [18, 19, 20], Salama et al. in [5-17], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [21-29], such as a neutrosophic set theory. There are several non-classical and higher order fuzzy sets ([1], [6,7] ,[9-11]) all having very good application potential in the area of Computer Science. One of the interesting generalizations of the theory of fuzzy sets is the theory of intuitionstic fuzzy sets introduced by Atanassov [1]. Intuitionstic fuzzy sets are fuzzy sets described by two functions: a membership function and a non-membership function that are loosely related. While the fuzzy set is a powerful tool to deal with vagueness, the theory of rough sets introduced by Pawlak [30] is a powerful mathematical tool to deal with incompleteness. Fuzzy sets and rough sets are two different topics none conflicts the other. In [23], Dubois and Prade defined rough fuzzy sets and fuzzy rough sets providing hints on some research directions on them. Nanda [28], Nakamura [27] also defined fuzzy rough sets independently in different ways. Fuzzy rough sets and rough fuzzy sets are concerned with both of vagueness and incompleteness. In the present paper we define rough neutrosophic sets and study their properties.

Preliminaries

In this section we present some preliminaries which will be useful to our work in the next section .We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [18, 19, 20] and Salama et al. [15-17]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]{}^{-}0, 1^{+}\right[$ is non-standard unit interval.

Definition 2.1. [18, 19, 20]

Let T, I, F be real standard or nonstandard subsets of $\left]{}^{-}0, 1^{+}\right[$, with

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Sup_T=t_sup, inf_T=t_inf

Sup_I=i_sup, inf_I=i_inf

Sup_F=f_sup, inf_F=f_inf

n-sup=t_sup+i_sup+f_sup

n-inf=t_inf+i_inf+f_inf,

T, I, F are called neutrosophic components

Definition 2.2. [15, 16, 17]

Let X be a non-empty fixed set. A neutrosophic set (NS for short or $((A \in N^X))A$ is an object having the form $A = \{<\mu_A(x), \nu_A(x), \gamma_A(x)>, x \in X\}$ Where $\mu_A(x), \nu_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\nu_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A, where $0 \le \mu_A(x) + \nu_A(x) + \gamma_A(x) \le 3$

Definition 2.1 [30]

Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets $A_1(X) = \{x : [x]_R \subseteq X\}$ and $A_2(X) = \{x : [x]_R \cap X \ne \Phi\}$ are called the lower approximation and upper approximation, respectively of X, where the pair $S = (U, R)$ is called an approximation space. This equivalent relation R is called indiscernibility relation. The pair $A(X) = (A_1(X), A_2(X))$ is called the rough set of X in S. Here [x] R denotes the equivalence class of R containing x. For more details on the algebra and operations on rough sets [29, 30] may be seen.

Rough Neutrosophic Sets

In this section we define rough neutrosophic sets and some operations viz. union, intersection, inclusion and equalities over them.

Definition 3.1

Let X be a non-null set and R be an equivalence relation on X. Let $A = \{<\mu_A(x), \nu_A(x), \gamma_A(x)>, x \in X\}$ be a neutrosophic set in X with the membership function μ_A, non-membership function γ_A and indeterminacy ν_A. The lower and the upper approximations $R_1(A)$ and $R_2(A)$ respectively of the neutrosophic set A are neutrosophic sets of the quotient set XlR with

(i) Membership function defined by

$$\mu_{R_1(A)}(X_i) = \inf\{\mu_A(x) : x \in X_i\}$$

$$\mu_{R_2(A)}(X_i) = \sup\{\mu_A(x) : x \in X_i\}$$

(ii) and indeterminacy may be defined as two types

Type 1:

$$\nu_{R_1(A)}(X_i) = \inf\{\nu_A(x) : x \in X_i\}$$

$$\nu_{R_2(A)}(X_i) = \sup\{\nu_A(x) : x \in X_i\}$$

Type 2:

$$\nu_{R_1(A)}(X_i) = \sup\{\nu_A(x) : x \in X_i\}$$

$$\nu_{R_2(A)}(X_i) = \inf\{\nu_A(x) : x \in X_i\}$$

(iii) and non-membership function defined by

$$\gamma_{R_1(A)}(X_i) = \sup\{\mu_A(x) : x \in X_i\}$$

$$\mu_{R_2(A)}(X_i) = \inf\{\mu_A(x) : x \in X_i\}$$

We prove that $R_1(A)$ and $R_2(A)$ defined in this way are NS.

For $x \in X_i$, we obtain successively:

$$\mu_A(x)+\gamma(\mathrm{x})+\nu(\mathrm{x}) \le 3\ ,\mu_A(x) \le 3-(\gamma(\mathrm{x})+\nu(\mathrm{x})), \sup\{\mu_A(x): x \in X_i\} \le \sup\{3-(\gamma(\mathrm{x})+\nu(\mathrm{x}))\},$$

$$\sup\{\mu_A(x): x \in X_i\} \le 3-\mathrm{Inf}\{(\gamma(\mathrm{x})+\nu(\mathrm{x})): \mathrm{x} \in \mathrm{X_i}\},$$

$$\sup\{\mu_A(x): x \in X_i\}+\mathrm{Inf}\{(\gamma(\mathrm{x})+\nu(\mathrm{x})): \mathrm{x} \in \mathrm{X_i}\} \le 3,$$

$$\sup\{\mu_A(x): x \in X_i\}+\mathrm{Inf}\{(\gamma(\mathrm{x}): \mathrm{x} \in \mathrm{X_i}\}+Inf\{\nu(\mathrm{x})): \mathrm{x} \in \mathrm{X_i}\} \le 3$$

Hence $R_1(A)$ is a NS. Similarly we can prove that $R_2(A)$ is a NS. The rough neutrosophic set of A is $R(A)$ given by the pair $R(A)=\langle R_1(A), R_2(A)\rangle$.

Definition 3.2

If $R(A)=\langle R_1(A), R_2(A)\rangle$ is a rough neutrosophic set A in (X,R), the rough complement of $R(A)$ is the rough neutrosophic set denoted by $R^c(A)$ and is defined by $R^c(A)=\left\langle R_1{}^c(A), R_2{}^c(A)\right\rangle$ where $R_1{}^c(A)$, $R_2(A)$ are the complements of the neutrosophic sets $R_1(A)$ and $R_2(A)$ respectively.

Definition 3.3

If $R(A_1)$ and $R(A_2)$ are two rough neutrosophic sets of the neutrosophic sets A_1 and A_2 respectively in X, then we define the following:

(i) $R(A_1) = R(A_2)$ iff $R_1(A_1) = R_1(A_2)$ and $R_2(A_1) = R_2(A_2)$.

(ii) $R(A_1) \subseteq R(A_2)$ iff R1 $R_1(A_1) \subseteq R_1(A_2)$ and $R_2(A_1) \subseteq R_2(A_2)$.

(iii) $R(A_1) \cup R(A_2) = \langle R_1(A_1) \cup R_1(A_2), R_2(A_1) \cup R_2(A_2)\rangle$.

(iv) $R(A_1) \cap R(A_2) = \langle R_1(A_1) \cap R_1(A_2), R_2(A_1) \cap R_2(A_2)\rangle$.

(v) $R(A_1) + R(A_2) = \langle R_1(A_1) + R_1(A_2), R_2(A_1) + R_2(A_2)\rangle$

(vi) $R(A_1) \bullet R(A_2) = \langle R_1(A_1) \bullet R_1(A_2), R_2(A_1) \bullet R_2(A_2)\rangle$.

(vii) $R(A_1) = \langle [\ ]R_1(A_1), [\ ]R_2(A_1)\rangle$.

(viii) $R(A_1) = \langle \langle\ \rangle R_1(A_1), \langle\ \rangle R_2(A_1)\rangle$

If R, S, T are rough neutrosophic sets in (X,R) ,then the results in the following proposition are straightforward from definitions.

Proposition 3.1

(i) $R^{cc} = R$

(ii) $R \cup S = S \cup R,\ R \cap S = S \cap R,$

(iii) $(R \cup S) \cup T = R \cup (S \cup T),\ (R \cap S) \cap T = R \cap (S \cap T),$

(vi) $(R \cup S) \cap T = (R \cup S) \cap (R \cup T),$

(v) $(R \cap S) \cup T = (R \cap S) \cup (R \cap T),$

De Morgan's laws are satisfied for rough neutrosophic sets:

Proposition 3.2

(i) $\left(R(A_1) \cup R(A_2)\right)^c = R^c(A_1) \cap R^c(A_2)$

(ii) $\left(R(A_1) \cap R(A_2)\right)^c = R^c(A_1) \cap R^c(A_2)$

Proof

$$\left(R(A_1) \cup R(A_2)\right)^c = \left(\{R_1(A_1) \cup R_1(A_2)\}, \{R_2(A_1) \cup R_2(A_2)\}\right)^c$$

$$= \left(\{R_1(A_1) \cup R_1(A_2)\}^c, \{R_2(A_1) \cup R_2(A_2)\}^c\right)$$

$$= \left(\{R_1{}^c(A_1) \cap R_1{}^c(A_2)\}, \{R_2{}^c(A_1) \cap R_2{}^c(A_2)\}\right)$$

Hence $\left(R(A_1) \cup R(A_2)\right)^c = R^c(A_1) \cap R^c(A_2)$. Hence Proved.

(ii) Similar to the proof of (i).

Proposition 3.3

If A_1 and A_2 are two neutrosophic sets in X such that $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$ in (X, R).

Proof: Straightforward

Proposition 3.4

$$R(A_1 \cup A_2) \supseteq R(A_2) \cup R(A_2)$$

$$R(A_1 \cap A_2) \subseteq R(A_2) \cap R(A_2)$$

Proof: Clear

The following proposition relates the rough neutrosophic set of a neutrosophic set with the rough neutrosophic set of its complement.

Proposition 3.5

Rough complement of the rough neutrosophic set of an neutrosophic set is the rough neutrosophic set of its complement.

Proof: clear

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Conclusion

In this paper we have defined the notion of rough neutrosophic sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and neutrosophic set theory. While neutrosophic set theory is mainly concerned with vagueness, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently, by the way they are defined; it is clear that rough neutrosophic sets can be utilized for dealing with both of vagueness and incompleteness.

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