An elementary approach to explore possible constraints on the infinite nature of twin primes

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Abstract: Twin prime conjecture states that there are infinite number of twin primes of the form p and p+2. Remarkable progress has recently been achieved by Y. Zhang to show that infinite primes that differ by large gap (~ 70 million) exist and this gap has been further narrowed to ~600 by others. We use an elementary approach to explore any obvious constraint that could limit the infinite nature of twin primes. Using Fermat's little theorem as a surrogate for primality we derive an equation that suggests but not prove that twin primes can be infinite.

Results:

Consider any pair of large twin primes p and p+2.

It follows from Fermat's little theorem that
2^p-2 is divisible by p
$2^{p+2}-2$ is divisible by p+2

Then
$2^p-2=p*a$ …….. (I)
$2^{p+2}-2=(p+2)*b$ ………(II)

where a and b are positive integers.

Subtracting I from II

$2^{p+2}-2-(2^p-2)= (p+2)*b - p*a$

$2^{p+2}-2^p=pb+2b-pa$

$2^p(2^2-1)=p(b-a)+2b$

$3* 2^p= p(b-a)+2b$ …….. (III)

Since p is a large prime it is odd therefore can be written as p=2k+1

Therefore $2^p-2=2^{2k+1}-2= 2(2^{2k}-1)=2(2^k-1)(2^k+1)$

Since $2^{2k}-1,2^k, 2^k+1$ are three consecutive numbers and 2^k cannot be divisible by 3, therefore the product $(2^k-1)(2^k+1)$ must be divisible by 3 and using this we can infer that 2^p-2 is divisible by 6.

Similarly $2^{p+2}-2$ is divisible by 6.

Since p and p+2 are large twin primes therefore the factors a and b can be expressed as 6x and 6y respectively.
a=6x
b=6y

Substituting these values of a and b in Equation III we get,

$3 * 2^p = p(6y-6x)+2(6y)$

$3*2^p = 6[p(y-x)+ 2y]$
$6*2^{p-1}=6[p(y-x)+ 2y]$

$2^{p-1}= p(y-x)+ 2y$

$p=(2^{p-1}-2y)/(y-x)=2(2^{p-2}-y)/(y-x)$……….(IV)

Since p is a large prime therefore (y-x) must be even and can be substituted by 2z in (IV) where z is a positive integer.

Therefore

$p=2(2^{p-2}-y)/(2z)= (2^{p-2}-y)/z$

Therefore

$pz=2^{p-2}-y$ **or** $y=2^{p-2}-pz$

or $2^{p-2}=pz+y$…………..(V)

This is the simple equation (of the form n=pq+r) that must be satisfied if p and p+2 should be twin primes where y and z have unique solutions for each prime pair p and p+2. The simplicity of the equation doesn't reveal any obvious constraints that would make large p, p+2 unlikely.

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