How intelligence is related to matter?

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I can calculate the motion of heavenly bodies, but not the madness of people.

Isaac Newton

In mathematical world, the bridge from matter to intelligence requires extension and modification of quantum physics.

The recent statement about completeness of the physical picture of our Universe made in Geneva raised many questions, and one of them is the ability to create Life and Intelligence out of physical matter without any additional entities. The main difference between living and non-living matter is in directions of their evolution: it has been recently recognized that the evolution of livings is progressive in a sense that it is directed to the highest levels of complexity. Such a property is not consistent with the behavior of *isolated* Newtonian systems that cannot increase their complexity without external forces. That difference created so called Schrödinger paradox: in a world governed by the second law of thermodynamics, all isolated systems are expected to approach a state of maximum *disorder;* since life approaches and maintains a highly *ordered* state – one can argue that this violates the Second Law implicating a paradox,[1]

But livings are not isolated due to such processes as metabolism and reproduction: the increase of order inside an organism is compensated by an increase in disorder outside this organism, and that removes the paradox. Nevertheless it is still tempting to find a mechanism that drives livings from disorder to order. The purpose of this paper is to demonstrate that moving from a disorder to order is not a prerogative of open systems: an isolated system can do it without help from outside. However such system cannot belong to the world of the modern physics: it belongs to the world of living matter, and that lead us to the concept of an intelligent particle – the first step to physics of livings. In order to introduce such a particle, we start with an idealized mathematical model of livings by addressing only one aspect of Life: a ***biosignature,*** i.e. ***mechanical*** invariants of Life, and in particular, the ***geometry and kinematics of intelligent behavior*** disregarding other aspects of Life such as metabolism and reproduction. By narrowing the problem in this way, we are able to extend the mathematical formalism of physics' First Principles to include description of intelligent behavior. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from a disorder to the order.

Starting with quantum mechanics.

The starting point of our approach is the Madelung equation that is a hydrodynamics version of the Schrödinger equation

$$\frac{\partial\rho}{\partial t}+\nabla\bullet\left(\frac{\rho}{m}\nabla S\right)=0 \qquad (1)$$

$$\frac{\partial S}{\partial t}+(\nabla S)^2+F-\frac{\hbar^2\nabla^2\sqrt{\rho}}{2m\sqrt{\rho}}=0 \qquad (2)$$

Here ρ and S are the components of the wave function $\psi=\sqrt{\rho}e^{iS/\hbar}$, and $\hbar$ is the Planck constant divided by 2π. The last term in Eq. (2) is known as quantum potential. From the viewpoint of Newtonian mechanics, Eq. (1) expresses continuity of the flow of probability density, and Eq. (2) is the Hamilton-Jacobi equation for the action S of the particle. Actually the quantum potential in Eq. (2), as a feedback from Eq. (1) to Eq. (2), represents the difference between the Newtonian and quantum mechanics, and therefore, it is solely responsible for fundamental quantum properties.

The Madelung equations (1), and (2) can be converted to the Schrödinger equation using the ansatz

$$\sqrt{\rho}=\Psi\exp(-iS/h) \qquad (3)$$

where ρ and S being real function.

Our approach is based upon a modification of the Madelung equation, and in particular, upon replacing the quantum potential with a different Liouville feedback, Fig.1

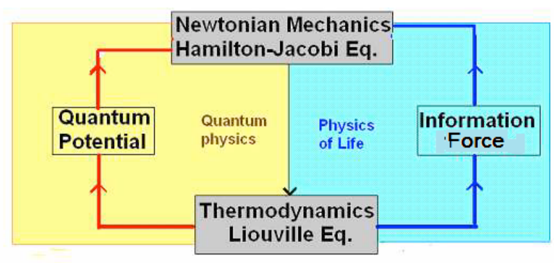

Figure 1. Classic Physics, Quantum Physics and Physics of Life.

In Newtonian physics, the concept of probability ρ is introduced via the Liouville equation

$$\frac{\partial\rho}{\partial t}+\nabla\bullet(\rho\mathbf{F})=0 \qquad (4)$$

generated by the system of ODE

$$\frac{d\mathbf{v}}{dt}=\mathbf{F}[\mathbf{v}_1(t),\ldots\mathbf{v}_n(t),t] \qquad (5)$$

where **v** is velocity vector.

It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0=\rho(t=0) \qquad (6)$$

in the initial condition of ODE (6).

Let us rewrite Eq. (2) in the following form

$$\frac{d\mathbf{v}}{dt}=\mathbf{F}[\rho(\mathbf{v})] \qquad (7)$$

where **v** is a velocity of a hypothetical particle.

This is a fundamental step in our approach: in Newtonian dynamics, the probability never explicitly enters the equation of motion, [2,3]. In addition to that, the Liouville equation generated by Eq. (7) is nonlinear with respect to the probability density ρ

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \{\rho \mathbf{F}[\rho(\mathbf{V})]\} = 0 \qquad (8)$$

and therefore, the system (7),(8) departs from Newtonian dynamics. However although it has the same topology as quantum mechanics (since now the equation of motion is coupled with the equation of continuity of probability density), it does not belong to it either. Indeed Eq. (7) is more general than the Hamilton-Jacoby equation (2): it is not necessarily conservative, and **F** is not necessarily the quantum potential although further we will impose some restriction upon it that links **F** to the concept of information, [3]. The relation of the system (7), (8) to Newtonian and quantum physics is illustrated in Fig.1.

Remark. Here and below we make distinction between the random *variable v(t)* and its *values V* in probability space.

Information force instead of quantum potential.

In this section we propose the structure of the force **F** that plays the role of a feedback from the Liouville equation (8) to the equation of motion (7). Turning to one-dimensional case, let us specify this feedback as

$$F = c_0 + \frac{1}{2} c_1 \rho - \frac{c_2}{\rho} \frac{\partial \rho}{\partial v} + \frac{c_3}{\rho} \frac{\partial^2 \rho}{\partial v^2} \qquad (9)$$

$$c_0 > 0, c_1 > 0, c_3 > 0 \qquad (10)$$

Then Eq.(9) can be reduced to the following:

$$\dot{v} = c_0 + \frac{1}{2} c_1 \rho - \frac{c_2}{\rho} \frac{\partial \rho}{\partial v} + \frac{c_3}{\rho} \frac{\partial^2 \rho}{\partial v^2} \qquad (11)$$

and the corresponding Liouville equation will turn into the following PDE

$$\frac{\partial \rho}{\partial t} + (c_0 + c_1 \rho) \frac{\partial \rho}{\partial V} - c_2 \frac{\partial^2 \rho}{\partial v^2} + c_3 \frac{\partial^3 \rho}{\partial V^3} = 0 \qquad (12)$$

This equation is known as the KdV-Bergers' PDE. The mathematical theory behind the KdV equation became rich and interesting, and, in the broad sense, it is a topic of active mathematical research. A homogeneous version of this equation that illustrates its distinguished properties is nonlinear PDE of parabolic type. But a fundamental difference between the standard KdV-Bergers equation and Eq. (12) is that Eq. (12) ***dwells in the probability space***, and therefore, it must satisfy the normalization constraint

$$\int_{-\infty}^{\infty} \rho \, dV = 1 \qquad (13)$$

However as shown in [4], this constraint is satisfied: in physical space it expresses conservation of mass, and it can be easily scale-down to the constraint (13) in probability space. That allows one to apply all the known results directly to Eq. (12). However it should be noticed that all the conservation invariants have different physical meaning: they are not related to conservation of momentum and energy, but rather impose constraints upon the Shannon information.

In physical space, Eq. (12) has many applications from shallow waves to shock waves and solitons. However, application of solutions of the same equations in probability space is fundamentally different. In the last section we present a phenomena that exist neither in Newtonian nor in quantum physics.

Emergence of randomness.

In this section we discuss a fundamentally new phenomenon: transition from determinism to randomness in ODE that coupled with their Liouville PDE.

In order to complete the solution of the system (11), (12), one has to substitute the solution of Eq. (12):

$$\rho = \rho(V,t) \quad at \quad V = v \qquad (14)$$

into Eq.(11). Since the transition from determinism to randomness occurs at $t \to 0$, let us turn to Eq. (12) with sharp initial condition

$$\rho_0(V) = \delta(V)\, at \qquad t = 0, \qquad (15)$$

Then applying one of the standard analytical approximations of the delta-function, one obtains the asymptotic solution

$$\rho = \frac{1}{t\sqrt{\pi}} e^{-\frac{V^2}{t^2}} \quad at \quad t \to 0 \qquad (16)$$

Substitution this solution into Eq. (14) shows that

$$O(c_0 + \frac{1}{2}c_1\rho) = \frac{1}{t}, \quad O(\frac{c_2}{\rho}\frac{\partial \rho}{\partial v}) = \frac{1}{t^2},$$
$$and \quad O(\frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2}) = \frac{1}{t^4} \quad at \quad t \to 0, \quad v \neq 0 \qquad (17)$$

i.e.

$$c_0 + \frac{1}{2}c_1\rho << \frac{c_2}{\rho}\frac{\partial \rho}{\partial v} << \frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2} \quad at \quad t \to 0, \quad v \neq 0 \qquad (18)$$

and therefore, the first three terms in Eq. (11) can be ignored

$$\dot{v} = \frac{c_3}{\rho}\frac{\partial^2 \rho}{\partial v^2} \quad at \quad t \to 0, \quad v \neq 0 \qquad (19)$$

or after substitution of Eq. (16)

$$\dot{v} = \frac{4c_3 v^2}{t^4} \quad at \quad t \to 0, \quad v \neq 0 \qquad (20)$$

Eq. (20) has the following solution (see Fig. 2)

$$v = \frac{t^3}{4c_3 + Ct^3} \quad at \quad t \to 0, \quad v \neq 0 \qquad (21)$$

where C is an arbitrary constant.

This solution has the following property: the Lipchitz condition at $t \to 0$ fails

$$\frac{\partial \dot{v}}{\partial v} = \frac{8c_3 v}{t^4} = \frac{8c_3 t^3}{t^4(4c_3 + Ct^3)} \to \infty \quad at \quad t \to 0, \quad v \neq 0 \qquad (22)$$

and as a result of that, the uniqueness of the solution is lost. Indeed, as follows from Eq. (21), for any value of the arbitrary constant C, the solutions are different, but they satisfy the same initial condition

$$v \to 0 \quad at \quad t \to 0 \qquad (23)$$

Due to violation of the Lipchitz condition (22), the solution becomes unstable. That kind of instability when infinitesimal errors lead to finite deviations from basic motion (the Lipchitz instability) has been discussed in [5,6]. This instability leads to unpredictable shift of solution from one value of *C* to another. It means that appearance of any specified solution out of the whole family is random, and that randomness is controlled by the feedback (9) from the Liouville equation (12). Indeed if the solution (21) runs independently many times with the same initial conditions, and the statistics is collected, the probability density will satisfy the Liouville equation (12), Fig.3.

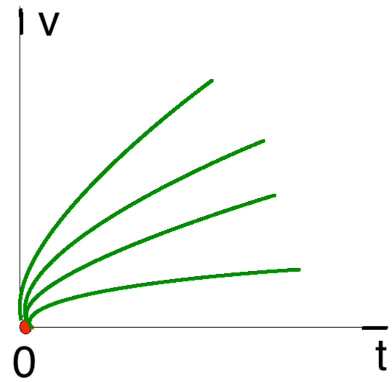

Figure 2. Family of random solutions describing transition from determinism to stochastisity.

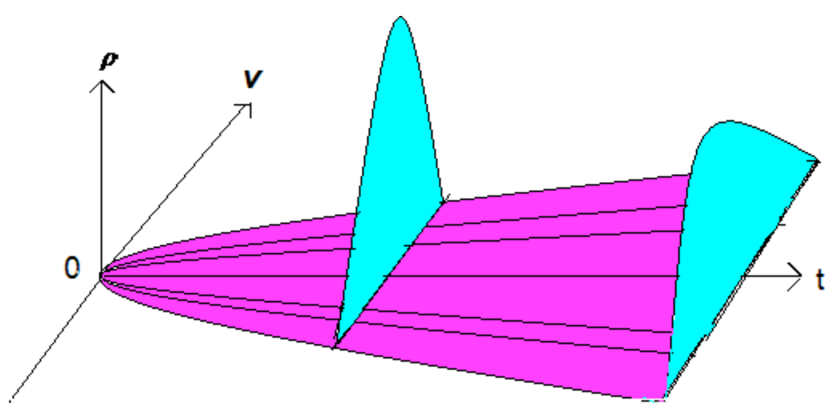

Figure 3. Stochastic process and probability density.

Departure from Newtonian and quantum physics.

In this section we will derive a distinguished property of the system (16),(17) that is associated with violation of the second law of thermodynamics i.e. with the capability of moving from disorder to order without help from outside. That property can be predicted qualitatively even prior to analytical proof: due to the nonlinear term in Eq. (17), the solution form shock waves and solitons in probability space, and that can be interpreted as "concentrations" of probability density, i.e. departure from disorder. In order to demonstrate it analytically, let us turn to Eq. (17) at

$$c_1 >> |c_2|, c_3 \tag{24}$$

and find the change of entropy H

$$\frac{\partial H}{\partial t}=-\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\rho\ln\rho\, dV=-\int_{-\infty}^{\infty}\frac{1}{c_1}\dot{\rho}(\ln\rho+1)\,dV=\int_{-\infty}^{\infty}\frac{1}{c_1}\frac{\partial}{\partial V}(\rho^2)\ln(\rho+1)dV$$
$$=\frac{1}{c_1}[\,\Big|_{-\infty}^{\infty}\rho^2(\ln\rho+1)-\int_{-\infty}^{\infty}\rho\, dV]=-\frac{1}{c_1}<0 \tag{25}$$

At the same time, the original system (11), (12) is isolated: it has no external interactions. Indeed the information force Eq. (9) is generated by the Liouville equation that, in turn, is generated by the equation of motion (11). Therefore the solution of Eqs. (11), and (12) can violate the second law of thermodynamics, and that means that this class of dynamical systems does not belong to physics as we know it. This conclusion triggers the following question: are there any phenomena in Nature that can be linked to dynamical systems (11), (12)? The answer will be discussed bellow.

Thus despite the mathematical similarity between Eq.(12) and the KdV-Bergers equation, the physical interpretation of Eq.(12) is fundamentally different: it is a part of the dynamical system (11),(12) in which Eq. (12) plays the role of the Liouville equation generated by Eq. (11). As follows from Eq. (25), this system being isolated has a capability to decrease entropy, i.e. to move from disorder to order without external resources. In addition to that, the system displays transition from deterministic state to randomness (see Eq. (22)).

This property represents departure from classical and quantum physics, and, as shown in [2,3], provides a link to behavior of livings. That suggests that this kind of dynamics requires extension of modern physics to include physics of life.

The process of violation of the second law of thermodynamics is illustrated in Fig. 4: the higher values of ρ propagate faster than lower ones. As a result, the moving front becomes steeper and steeper, and that leads to formation of solitons (c_3>0), or shock waves (c_3=0) ***in probability space.*** This process is accompanied by decrease of entropy.

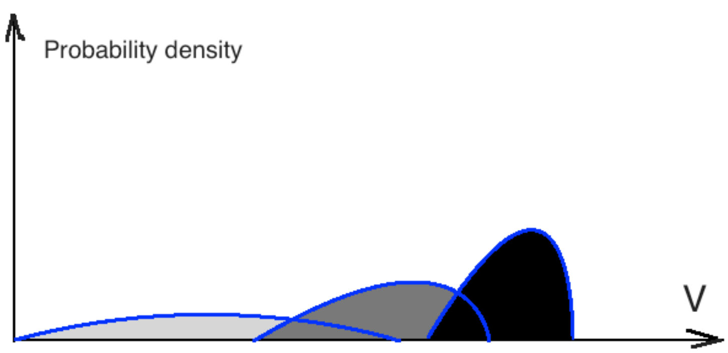

Figure 4. Formation of shock waves in probability space.

Remark. The system (11), (12) displays transition from deterministic state to randomness (see Eq. (22)), and this property can be linked to the similar property of the Madelung equation, although strictly speaking, Eq.(1) is a "truncated" version of the Liouvile equation: it does not include the contribution of the quantum potential. Nevertheless the origin of randomness in quantum mechanics is the same as in the system (11), (12) as demonstrated in [3,6].

Comparison with quantum mechanics.

a. Mathematical Viewpoint. The model of intelligent particle is represented by a nonlinear ODE (7) and a nonlinear parabolic PDE (8) coupled in a master-slave fashion: Eq. (8) is to be solved independently, prior to solving Eq. ((7). The coupling is implemented by a feedback that includes the probability density and its space derivatives, and that converts the first order PDE (the Liouville equation) to the second or higher order nonlinear PDE. As a result of the nonlinearity, the solutions to PDE can have attractors (static, periodic, or chaotic) in probability space. The solution of ODE (7) represents another major departure from classical ODE: due to violation of Lipchitz conditions at states where the probability density has a sharp value, the solution loses its uniqueness and becomes random. However, this randomness is controlled by the PDE (8) in such a way that each random sample occurs with the corresponding probability, Fig.3.

b. Physical Viewpoint. The model of intelligent particle represents a fundamental departure from both Newtonian and quantum mechanics. The fundamental departure of all the modern physics is the violation of the second laws of thermodynamics,(see Eq.(25), and Fig. 4). However a more detailed analysis, [3], shows that due to similar dynamics topology to quantum mechanics,(see Fig.1) the model preserves some quantum properties such as entanglement and interference of probabilities.

Origin of intelligence.

a. Relevance to model of intelligent particle. The proposed model illuminates the "border line" between living and non-living systems. The model introduces an intelligent particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the intelligent particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (such as Eq (11)) can be identified with a motor dynamics, while the evolution of density (see Eq. (12)) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information potential and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of intelligent systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density ρ from Equation (12), creates functions of this density, converts them into the information force and applies this force to the equation of motion (11). As already mentioned above, because of that property of the model, the evolution of the probability density can become nonlinear, and the entropy may decrease "against the second law of thermodynamics". Actually the proposed model represents governing equations for interactions of intelligent agents. In order to emphasize the autonomy of the agents' decision-making process, we will associate the proposed models with ***self-supervised (SS) active systems.*** By an active system we will understand here a set of interacting intelligent agents capable of processing information, while an intelligent agent is an autonomous entity, which observes and acts upon an environment and directs its activity towards achieving goals. The active system is not derivable from the Lagrange or Hamilton principles, but it is rather created for information processing. One of specific differences between active and physical systems is that the former are supposed to act in uncertainties originated from incompleteness of information. Indeed, an intelligent agent almost never has access to the whole truth of its environment. Uncertainty can also arise because of incompleteness and incorrectness in the agent's understanding of the properties of the environment. That is why *quantum-inspired SS* systems represented by the particles under consideration are well suited for representation of active systems, and the hypothetical particle introduced above can be associated with the term "intelligent" particle. It is important to emphasize that self-supervision is implemented by the feedback from mental dynamics, i.e. by internal force, since the mental dynamics is generated by intelligent particle itself.

b. Comparison with control systems. In this sub-section we will establish a link between the concepts of intelligent control and phenomenology of behavior of intelligent particle.

*Example**.*** One of the limitations of classical dynamics, and in particular, neural networks, is inability to change their structure without an external input. As will be shown below, an intelligent particle can change the locations and even the type of the attractors being triggered only by information forces i.e. by an internal effort. We will start with a simple dynamical system

$$\dot{v} = 0, \quad v = 0 \quad at\ t = 0 \tag{26}$$

and than apply the following control

$$F = -k\overline{v} + a\overline{\overline{v}} - \sigma \frac{\partial}{\partial v} \ln \rho, \tag{27}$$

where $\overline{\overline{V}} = \int_{-\infty}^{\infty} \rho (V - \overline{V})^2 dV, \qquad \overline{V} = \int_{-\infty}^{\infty} \rho V dV,$ (28)

and k, a, σ are constant coefficients.

Then the controlled version of the motor dynamics (26) is changed to

$$\dot{v} = -k\overline{v} + a\overline{\overline{v}} - \sigma \frac{\partial}{\partial v} \ln \rho \tag{29}$$

while F represents the information forces that play the role of ***internal*** actuator.

Let us notice that the internal actuator (27) is a particular case of the information force (9) at

$$c_0 = -k\overline{v} + a\overline{\overline{v}}, \quad c_1 = 0, \quad c_2 = \sigma, \quad c_3 = 0 \tag{30}$$

For a closure, Eq. (29) is complemented by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} = k\overline{V} \frac{\partial \rho}{\partial V} - a\overline{\overline{V}} \frac{\partial \rho}{\partial V} + \sigma \frac{\partial^2 \rho}{\partial V^2}, \tag{31}$$

to be solved subject to sharp initial condition

$$\rho_0(V) = \delta(V)\, at \qquad t = 0, \tag{32}$$

As shown above, the solution of Eq.(29) is random, (see Eq. (21) and Fig. 2) while this randomness is controlled by Eq. (31). Therefore in order to describe it, we have to transfer to the mean values $\overline{v}$ and $\overline{\overline{v}}$. For that purpose, let us multiply Eq.(1) by V .Then integrating it with respect to V over the whole space, one arrives at ODE for the expectation $\overline{v}(t)$

$$\dot{\overline{v}} = -k\overline{v} + a\overline{\overline{v}} \tag{33}$$

Multiplying Eq.(31) by V^2, then integrating it with respect to V over the whole space, one arrives at ODE for the variance $\overline{\overline{v}}(t)$

$$\dot{\overline{\overline{v}}} = -2k\overline{\overline{v}} + 2a\overline{v}\,\overline{\overline{v}} + 2\sigma \tag{34}$$

Let us find fixed points of the system (33) and (34) by solving the system of algebraic equations:

$$0 = -k\overline{v} + a\overline{\overline{v}} \tag{35}$$

$$0 = -2k\overline{\overline{v}} + 2a\overline{v}\,\overline{\overline{v}} + 2\sigma \tag{36}$$

By selecting

$$\sigma = \frac{k^3}{2a^2} \tag{37}$$

we arrive at the following single fixed point

$$\overline{v}^* = \frac{k}{2a}, \quad \overline{\overline{v}}^* = \frac{k^2}{2a^2} \tag{38}$$

In order to establish whether this fixed point is an attractor or a repeller, we have to analyze stability of the homogeneous version of the system (33), (34) linearized with respect to the fixed point (38)

$$\dot{\overline{v}} = -k\overline{v} + a\overline{\overline{v}} \tag{39}$$

$$\dot{\overline{\overline{v}}} = -k\overline{\overline{v}} + \frac{k^2}{a}\overline{v} \tag{40}$$

Analysis of its characteristic equation shows that it has non-positive roots:

$$\lambda_1 = 0, \quad \lambda_2 = -2k < 0 \tag{41}$$

and therefore, the fixed point (38) is a stochastic attractor with stationary mean and variance. However the higher moments of the probability density are not necessarily stationary: they can be found from the original PDE (31).

Thus as a result of a ***mental*** control, an ***isolated*** dynamical system (26) that prior to control was at rest, moves to the stochastic attractor (38) having the expectation $\overline{v}*$ and the variance $\overline{\overline{v}}*$.

The distinguished property of the particle introduced above definitely fits into the concept of intelligence. Indeed, the evolution of intelligent living systems is directed toward the highest levels of complexity if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. At the same time, the solutions to the models based upon dissipative Newtonian dynamics eventually approach attractors where the evolution stops while these attractors dwell on the subspaces of lower dimensionality, and therefore, of the lower complexity (until a “master” reprograms the model). Therefore, such models fail to provide an autonomous progressive evolution of intelligent systems (i.e. evolution leading to increase of complexity). At the same time, a self-controlled particle can create its own complexity based only upon an *internal* effort.

Thus the actual source of intelligent behavior of the particle introduced above is a new type of force - the information force - that contributes its work into the Law of conservation of energy. However this force is internal: it is generated by the particle itself with help of the Liouvile equation. The machinery of the intelligence is similar to that of control system with the only difference that control systems are driven by external actuators while the intelligent particle is driven by a feedback from the Liouvile equation without any external resources.

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