Reconstruction of Quantum Field Theory as Extension of Wave Mechanics

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Abstract

The task to be carried out should be clear from the title. One motivation for this endeavour is coming from the fact that the usual version of quantum field theory is not acceptable. In the paper, above all, three intentions are pursued (a) an adequate consideration of the interaction (b) a proof that the means of classical field theory are sufficient (c) a new attempt to describe particles by stable wave packets.

1. What is the problem?

Quantum mechanics is a theory, which is well suited to describe the 'behaviour' of a quantum object under the influence of an external potential, as immediately can be seen by a glance at the Schrödinger equation or at the equation for the harmonic oscillator. However interaction between such objects is lacking. Hence Einstein was right with his claim that quantum mechanics is incomplete. But meanwhile quantum mechanics has been completed by quantum field theory. In that theory interaction is not only adequately respected, but also playing a central role, for instance, in the analysis of scattering processes.

So far all is quite right.

But now a problem arises. The usual version of quantum field theory, as can be found in the textbooks, is not acceptable. It is true that its results are supported by the experiments. But the theoretical foundation is complicated, faulty, and partially dispensable. The reasons for such a far reaching thesis are given in the next section. In the present paper the ansatz is pursued, not to carry out the extension to quantum field theory by starting from the common version of quantum mechanics, but to choose wave mechanics as the base for such an enlargement, although this early variant of quantum mechanics is not much appreciated nowadays. With the planned endeavour three aims, above all, are intended.

(a) an adequate consideration of the interaction, which is not yet inherent in wave mechanics by itself,

(b) a proof that the means of classical field theory are sufficient, and

(c) a new attempt to describe elementary particles by stable wave packets.

In section 3 first of all scattering processes are analyzed in the frame of perturbation theory. Section 4 contains a short historical review of wave mechanics, including the ansatz of Schrödinger to represent particles by wave packets. A renewed attempt to pertain this is made using all means available today and, above all, including interaction. The question, whether extended wave mechanics is a field theory or a quantum theory, shall be discussed in section 5. The summary in section 6 is striking the balance of the extension of wave mechanics to a version of quantum field theory. In the list of literature also some older textbooks are quoted. They have the advantage of being thorough and explicit.

2. Why is the usual version of quantum field theory not acceptable?

A closer inspection of quantum field theory, as it is presented in the textbooks, would reveal that quantum field theory is blown up, sometimes even faulty, and in this version superfluous. This is the case especially for the following points

(a) the LSZ-reduction

(b) perturbation theory

(c) the concept of particle in the Copenhagen interpretation

(d) quantum statistics for the ingoing particles of scattering processes

(e) quantum statistics for the outgoing particles of scattering processes.

In detail:

2.1 The LSZ-reduction: analysis

The aim of the LSZ-reduction is to transform the matrix elements of the S-matrix into a vacuum expectation value. Here only a small part of the whole procedure shall be referred. It already will contain the essentials and be independent of special models of quantum field theory. For this purpose the following assumptions are needed:

(a) The asymptotic condition

$$\lim_{t \to -\infty} < \phi|\varphi(t,x)|\psi > = < \phi|\varphi_{in}(t,x)|\psi > \tag{1}$$

$$\lim_{t \to -+\infty} < \phi|\varphi(t,x)|\psi > = < \phi|\varphi_{out}(t,x)|\psi > \tag{2}$$

is granting the existence of the two limits for arbitrary states ϕ and ψ and for the operator φ of the field leaving away all normating factors.

(b) The function f^p is a solution of a homogeneous field equation, which is the Euler-Lagrange equation of the classical Lagrangian density of the chosen model.

(c) The operator $a_{in}(p)$ is created out of the operator $\varphi_{in}(t,x)$ of the ingoing field. Its definition is

$$a_{in}(p) = \int dx f^p(t,x)\varphi_{in}(t,x) \tag{3}$$

(d) The eigen state $|p>$ of an asymptotic free particle with the momentum p is created out of the vacuum state according to

$$|p_{in}>= \varphi_{in}(t,x)|0> \tag{4}$$

In the sequel the transformation of $< k|p'_{ein}p_{ein} >$ into a vacuum expectation value shall be carried out under the condition that in a first step $< k|p'_{ein}p_{ein} >$ has already been transformed into $< k|\varphi(t',x')|p_{ein} >$. The notation $|k>$ is standing for the multiparticle state of a system of asymptotic free particles and $|p'_{ein}p_{ein} >$ for the state of a system of two asymptotic free particles with the momenta p_{in} and p'_{in}. The first seven steps of the reduction of $< k|\varphi(t',x')|p_{ein} >$ are

1. The state $|p'_{in} >$ is created out of $|0>$.

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad < k|\varphi(t',x')a_{in}(p)|0 >$$

2. enlarging according to the scheme $a = b - (b-a)$

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad < k|a_{out}(p)\varphi(t',x')|0 > - < k|(a_{out}(p)\varphi(t',x') - \varphi(t',x')a_{in}(p))|0 >$$

3. The first term is vanishing

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad - < k|(a_{out}(p)\varphi(t',x') - \varphi(t',x')a_{in}(p))|0 >$$

4. smearing out by a test function

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad - < k|\int_{-\infty}^{+\infty} dx(a_{out}(p)\varphi(t',x') - \varphi(t',x')a_{in}(p))f^p(t,x)|0 >$$

5. application of the asymptotic condition

   $$\begin{aligned} < k|\varphi(t',x')|p_{in} > \quad = \quad & - < k|\int_{-\infty}^{+\infty} dx \lim_{t\to+\infty} \varphi(t,x)\varphi(t',x')f^p(t,x)|0 > \\ & + < k|\int_{-\infty}^{+\infty} dx' \lim_{t\to-\infty} \varphi(t',x')\varphi(t,x)f^{p'}(t,x)|0 > \end{aligned}$$

6. combining the two terms by using the definition of the time ordered product

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad - < k|\int_{-\infty}^{+\infty} dx(\lim_{t\to+\infty} - \lim_{t\to-\infty})T(\varphi(t,x)\varphi(t',x'))f^p(t,x)|0 >$$

7. transformation according to the scheme $f(b) - f(a) = \int_a^b f(x)dx$

   $$< k|\varphi(t',x')|p_{in} > \quad = \quad - < k|\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dt(\frac{\partial}{\partial t}(T(\varphi(t,x)\varphi(t',x'))f^p(t,x)|0 >$$

2.2. The LSZ-reduction: criticism

So far the deduction is independent of special examples. At any rate the sequence of factors in the time ordered product is reflecting the sequence of the steps in the reduction. Hence it doesn't matter, because the asymptotic states are independent of one another. Hence the LSZ-formalism is delivering the correct propagator for bosons. This is also true for the original paper, because Lehman, Symanzik and Zimmermann deliberately have confined themselves to the case of bosons.

A mistake will occur, if one is trying to apply the formalism to fermions. Then a wrong propagator will occur. In the usual representation of quantum field theory this mistake is cured by changing the definition of the time ordered product into

$$T(\psi(t,x),\psi(t',x')) = \left\{ \begin{array}{ll} +\,\psi(t,x)\psi(t',x') & x^0 > y^0 \\ -\,\psi(t',x')\psi(t,x) & x^0 < y^0 \end{array} \right\} \tag{5}$$

and at the same time submitting the multiparticle system of free particles to a correlation by the condition of antisymmetry. But this attempt of repair cannot be judged to be a correct deduction, because a definition is no proof, and a correlation between asymptotic free particles already was refuted earlier.

2.3 Perturbation theory

Even after the LSZ-reduction has been done, there is still a long distance to be covered, before one will arrive at perturbation theory. First of all the Theorem of Wick must be applied. Then a detour over the interaction picture is following. It is necessary in order to get explicitly the Hamilton operator of the interaction term. When after all this effort finally the result

$$< 0|T\{\phi(x_1)...\phi(x_n)\}|0 >= \frac{< 0|T\{\phi_I(x_1)...\phi_I(x_n)\exp[-i\int d^4x\mathcal{H}_I]\}|0 >}{< 0|T\{\exp[-i\int d^4x\mathcal{H}_I]\}|0 >} \tag{6}$$

appears, it is not yet the turn to begin. Then the exponential function must be expanded into a Taylor series and each term of it into a series according to the coupling constant. Not until in the coefficients

$$P_{r,n} = \phi_I(x_1)...\phi_I(x_r)\int dy_1...dy_n\mathcal{H}_I(y_1)...\mathcal{H}_I(y_n) \tag{7}$$

contractions for all pairs of factors ϕ_I have been introduced, the perturbation series for the S-matrix can be deduced.

2.4 The concept of particle in the Copenhagen interpretation

According to the ideas of the Copenhagen interpretation we don't know what a particle is 'an sich', but only how it appears to us in an experiment: either as wave or as a corpusculum. The latter one is nothing else than the mass point of classical mechanics, while the waves are amplitudes belonging to the probability interpretation of Born. This curious idea must be estimated as an improper attempt to introduce the Transzendentalphilosophie of Kant into theoretical physics, or said a bit impolite: There is no place in theoretical physics for schizophrenic objects like the particles of the Copenhagen interpretation. On the other hand a concept of particle is lacking that describes those really existing particles, with which experimentally oriented physicists are working.

2.5 The quantum statistics for the ingoing particles of scattering processes

For particles that are free in the sense that they satisfy the homogeneous part of a field equation, and moreover never have been in a common interaction, any correlation is a contradiction in itself. This especially is true for the ingoing particles of scattering processes, for they are generated by independent sources, and hence are independent themselves.

There is still another proof of the fact that that problems of quantum statistics are not valid for the ingoing particles. It is contained in the following quotation from [1] p. 149. "The relative minus sign between the direct and exchange terms is due to the Fermi statistics, which requires the amplitude to be antisymmetric under interchange of the two final electrons. It is also antisymmetric under interchange of the two initial electrons as required by the statistics." Assumed that this statement is true. Then one could in the electron-electron-scattering to second order interchange the two vertices of the exchange term, which would leave the photon propagator unchanged. By this procedure the exchange of the ingoing particles could be transmitted to the exchange of the outgoing particles and vice versa. Hence at most the outgoing particles are relevant for topics of quantum statistics.

2.6 The quantum statistics for the outgoing particles of scattering processes

In section 2.1 it was shown that the correct propagator and the correct statistics for fermions only can be achieved, if the systems of ingoing particles are submitted to a binding and if the time ordered product is redefined. Both manipulations already in section 2.2 were refuted as an incorrect deduction.

Another justification of quantum statistics is arguing that from a violation of quantum statistics one could infer a violation of micro causality. But micro causality is one of the fundamental principles of quantum field theory. Nevertheless the violation of micro causality can, according to the following quotation, occur without being noticed by anybody. "It is worthwhile observing that, if one quantizes, say, a Bose field with anticommutators, the violation of microscopic causality is sizable only at distances comparable to the Compton wavelength of the particle involved, generally $\sim 10^{-13}$cm." (see [2] p.172). This remark is to be supplemented by the statement that a location below the Compton wavelength is needing such high energy that it would destroy the system. No measurement within this system is possible (cf. Thirring [7]). Hence the violation of micro causality may be true, but it cannot be realized by a measurement.

3. Perturbation theory

3.1 Preliminary remark and survey of the intended procedure

The aim of perturbation theory is to determine the coefficients S_n in the Taylor series

$$S = \sum_{n=0}^{\infty} S_n \cdot g^n \tag{8}$$

of the S-Matrix. The usual derivation of the S-Matrix concededly is complicated and faulty. But the result is supported by the experiments. Hence in the sequel first of all the Feynman diagrams shall be decomposed step by step and are afterwards rebuilt in the same order. But, before doing this, the propagator function is to be derived from the field equation. At the end of the section a remark is made concerning the exchange terms in case of identical particles.

This decomposition and rebuilding shall serve as a guiding principle for a correct deduction of the Feynman diagrams and the corresponding Feynman integrals only by means of classical field theory.

3.2 Field - field equation - propagator function

A field shall be understood as a function with complex values and the coordinates of the four dimensional space as its arguments. Finite systems of such functions also shall be considered as fields. The physical meaning of a field is to describe the state of some object.

Field theories usually are characterized by Lagrangian densities. The corresponding field equations are derived from them as the Euler-Lagrangian equations. This procedure can be reverted according to a theorem of Darboux, quoted and reported reported in Bolza [13 p.37 f.]. According to this theorem any common or partial differential equation of at most second order may be considered as the Euler-Lagrangian equation of a suited Lagrangian function or Lagrangian density.

In this paper the application of field equations or of systems of equations as, for instance, the Dirac-Maxwell system

$$\begin{aligned} i\gamma^\mu \partial_\mu \psi(x) - m\psi(x) &= e\gamma^\mu A_\mu(x)\psi(x) \\ -i\partial_\mu \bar{\psi}(x)\gamma^\mu - m\bar{\psi}(x) &= e\bar{\psi}(x)\gamma^\mu A_\mu(x) \\ \partial_\mu \partial^\mu A^\nu(x) &= e\bar{\psi}(x)\gamma^\nu \psi(x) \end{aligned}$$

of differential equations is preferred to the use of Lagrange densities. As far as only one single field equation is given, it shall be of the form

$$D\varphi(x) = gP(\varphi(x)) \qquad x = x^\mu \qquad 0 \le \mu \le 3$$

with an at most quadratic function D of differential operators and a polynomial P of the functional values $\varphi(x)$.

All, what is to do else, shall be demonstrated by the example of a model with the equation

$$(\partial_\mu \partial^\mu + m^2)\varphi(x) = g\varphi(x)^2 \tag{9}$$

This model is unrealistic, because it has no minimal energy. But it seems to be suited to demonstrate the applied methods.

The corresponding propagator function Δ_F is satisfying the equation

$$(\partial_\mu \partial^\mu + m^2)\Delta_F(x) = \delta^{(4)}(x) \tag{10}$$

The method to get an explicit representation of the propagator function by Fourier representation, decompositon of partial fractions and the residue theorem of Cauchy is generally known and can be taken for granted.

The physical meaning of the propagator function $\Delta_F(x-y)$ is consisting in the description of a spreading wave, which is emerging at x and absorbed at y , provided $x_0 < y_0$. For the sake of linguistic simplicity the temporal sequence shall be left away, and the reverse case of a wave spreading from y to x, if $x_0 > y_0$, also be subsumed under the same expression 'spreading of a wave from x to y'.

3.3 Decomposition and rebuilding of Feynman diagrams

A Feynman diagram with n vertices, r external and l inner lines may be given. Since three lines are hanging on every vertex, one has

$$3n = 2l + r$$

For any external line there are three possibilities according to the two cases, whether (a) it is hanging together with another external line on the same vertex or (b) with two inner lines. The decomposition of the diagram is done by removing step for step either the two external lines together with the common vertex in case (a) or the single external line inclusive the vertex in case (b). By this procedure the number of vertices is decreasing step by step, such that finally a simple vertex part will remain as the only diagram of order 1.

In the same way, as it is decomposed, the diagram can be rebuilt. This procedure will be the guiding line for the analysis of a scattering process and at the same time serve as a proof that every Feynman diagram will be reached.

3.4 Analysis of scattering processes

In a scattering process particles are generated and prepared in suited sources. They are meeting in a small region and interacting there. Afterwards other particles leave the place of their creation. They are registered and measured in detectors.

Now such a scattering process shall be analyzed within the frame of perturbation theory and only by means of classical field theory.

The region Z of four dimensional space may be the region, where the interaction is concentrated. Moreover r points x_1 to x_r may be given, all of them far away from Z, and n points y_1 to y_n located within Z. There may be a non negative integer l such that

$$3n = 2l + r \qquad r, n \geq 1$$

Now it may be assumed that from a point x_i, far away from Z, a scattering wave is spreading and participating in the interaction at a vertex y_j. Then there are two possibilities: Either (a) it is meeting at y_j another wave coming from far away, or (b) it produces two scattering waves.

In case (a) a scattering wave is spreading from y_j and absorbed at y_k, while in case (b) two scattering waves are spreading from y_j and absorbed at y_k and y_l. This process can be continued such that the decomposition and rebuilding of an arbitrary Feynman diagram may serve as a guiding principle.

When the whole procedure is done, a propagator $\Delta_F(x - y)$ is attached to every inner line connecting x with y. Every vertex will get a factor g, and to every point far away the value $\varphi(x)$ of the field φ is attached. The product of these different kinds of expressions is dependent on x_1 to x_r and y_1 to y_n. When finally the integral over the variables y_1 to y_n is done, a not truncated Feynman integral is emerging as the result, up to numerical and combinatorial factors.

3.5 Exchange terms in case of outgoing particles being of the same kind

The whole afford that is done in the usual deduction of the Feynman rules has essentially the aim to justify the quantum statistics for identical particles. This is to say that an exchange term is necessary, if two outgoing particles are of the same kind, as for instance two electrons or to pions, and that the relative phase must be $+1$ for bosons and -1 for fermions. Since the justification of this rule is not admissible, one should drop it, and instead add a merely empirical rule to the Feynman rules.

Also in the present ansatz no possibility of a theoretical justification can be seen.

4. Wave mechanics

4.1 A historical remark

After Einstein in 1905 had inferred from experimental results and theoretical reasons that there must be a particle in correspondence to the electromagnetic field, which nowadays is called photon, de Broglie had the idea to invert this relation and to assign a wave to all material objects (cf. e. g. [9]). That was the beginning of wave mechanics. It was further elaborated by Schrödinger with the equation now bearing his name. Since then the harmonic oscillator is estimated as a classical example for the transition from classical mechanics to quantum mechanics.

The next step consisted in Schrödinger's [10] attempt to describe elementary particles by wave packets. For this purpose he developed a model, in which the wave packet is constructed by superposition of eigen functions of the harmonic oscillator. The decisive point of the construction is that such a wave packet is stable. But the hope that in a similar way the electrons in the orbit of an atom can be described, too, by packets of matter waves was in vain. For Heisenberg showed in the same paper [11], in which he published the uncertainty relations, that the model of Schrödinger has an equidistant energy spectrum and hence is the only example of a stable wave packet (cf. also [12]). The usual argument against such attempts nowadays is that the wave packets are dispersing, comparable with the dispersion of a heat pole on a heat leading material.

4.2 A new attempt to describe elementary particles by stable

The ansatz of Schrödinger could not succeed, because it was undertaken in the frame of quantum mechanics. But this theorie is linear und thus not suited to describe interaction. Hence the attempt has failed because of the insufficient means of the year 1926. Here a new ansatz is made in order to describe really existing elementary particles with explicit reference to interaction.

Provided a wave packet is given built up out of solutions of the Dirac equation with the parameters of the electron and concentrated in a region of about a Compton wave length of the electron. Then this assumption is only an illusion. In reality the electron is indivisibly coupled to its own electromagnetic field. Hence interaction takes place, which may be considered, too, as the self interaction of the whole system. Within perturbation theory the electron is surrounding itself with a cloud of virtual photons, electrons and positrons, the expression 'virtual particle' to be read as 'scattering wave'. In a further step one can consider the originally given electron as one of the virtual particles. Under the assumption that the whole object proves to be stable even beyond perturbation theory, one has a model for a really existing electron. It is free in the sense that it does not interact with other particles. But it is not free in so far, as it does not yield the homogeneous part of a field equation.

More cannot be given here, because theoretical physics is far away from being able to manage interaction. At a later occasion it shall be discussed, why the possibilities of experimental investigation are bounded, too. At any rate the presented ansatz, in spite of all speculative elements, seems to be better suited to describe particles than the ideas of the Copenhagen interpretation.

4.3. Inferences

The draft of the last section is speculative insofar, as it, like in almost all other cases with interaction, cannot be assured by a concrete calculation. Nevertheless it seems to be a plausible alternative to the concept of particle in the Copenhagen interpretation of quantum mechanics. If the ansatz is true, it describes a stable wave packet being a model for a particle with finite extension. Its magnitude might be of the order of a Compton wave length. According to Thirring [7] this length is marking the region, in which further location is impossible. The attempt to confine an electron even more would afford such high energies that the particle would be destroyed. "...the Compton wave length of the electron ... is the smallest size within which the electron can be compressed." In a region free from external potentials such a particle can move uniform and rectilinear without dissipation. It therefore would be the 'really existing particle' the experimentally oriented physicists are working with.

Interaction between two particles is taking place by penetration of the clouds of virtual particles. But the detail of this process is not observable on principle, for otherwise the impact of a measuring device would imply that a process between three participants would take place instead with the two partners of the process to be measured. As one can see, here, too, the important idea of quantum theory appears, stating that the influence of a measuring process on a quantum object must not be neglected. In a field theory as, for instance wave mechanics, it is particularly impressive, for in a world described by it all is consisting of waves, and waves cannot be grasped by waves with arbitrary precision.

Hence scattering processes are dependent on chance. Thus for the directions, into which the outgoing particles are moving, provided the ingoing particles are given, only a probability density can be expected. Since the process is an interaction between waves, the Feynman integrals deliver probability amplitudes, the norms of which are the probabilities. This statement is not in contradiction to the fact that there is a correlation between the outgoing particles caused by the validity of the laws of conservation.

Diffraction at an obstacle is possible, if the extent of the obstacle is of the same order as the size of the particles. For this reason diffraction of electrons is only possible at the lattices of crystals. At this order of magnitude the crystals themselves are composed of particles, and that is to say of waves. Hence the diffraction of particles at the lattices of crystals may be considered as a case of interaction.

The uncertainty relations don't need any further discussion, because they already are valid in classical physics: The latitude of a wave packet is reciprocal proportional to the extension of the corresponding frequency range.

5. Is quantum wave theory a field theory or a quantum theory?

A field had been defined as an array of complex functions having the time and the three spatial coordinates as its arguments. Since all concepts of the theory, including that of an elementary particle, are reduced to the concept of field, the answer is clear.

The extended wave mechanics is a field theory.

But that is not all.

The extended wave mechanics is also a quantum theory.

In order to justify this assertion first of all the concept of quantum must be cleared. There are two different kinds of definition.

On the one hand the eigenvalues of differential equations having discrete spectra of eigenvalues are considered to be quanta, so for instance the eigenvalues for the equation of the harmonic oscillator. But that shall not be adopted here for two reasons. First of all the Schrödinger equation, like similar equations, meanwhile is judged to be classical field equation. Secondly already in classical electrodynamics the expansion of the potentials into multipoles contains 'quantized' angular momenta. On the other hand stable elementary particles are considered to be quanta. That seems to be a reasonable concept and shall be adopted here.

Extended quantum wave theory is a quantum theory for two reasons. First of all the stable wave packets as the elementary particles of extended wave mechanics are clearly separated objects of the micro world and hence quanta. Moreover, for them an essential insight of quantum mechanics is valid: The interaction between quantum objects and a measuring device must not be neglected.

Any measurement is disturbing the object to be measured and by this impact it can destroy the results of other measurements.

By the way: in a recent publication (Scientific American, August 2014, 29-33, p. 32) the existence of gravitational waves is celebrated as a proof that the general theory of gravitation is a quantum theory. According to this sentence already classical electrodynamics would be a quantum theory, because there are electromagnetic waves in it.

6. Summary

The task was to reconstruct the essential parts of quantum field theory and for this purpose to take wave mechanics as a starting point. The reason for this endeavour was the estimation that the present state of quantum field theory is not acceptable. One essential point of the extension consists in the effort of adequately respecting the role of interaction, another one in a new attack to the old problem to describe elementary particles by stable wave packets. But this time the attempt was made with all means nowadays being available. Since the propagator function already is given by the homogeneous part of the field equation, classical field theory reveals to be sufficient for justifying the Feynman rules. But the rule for treating exchange terms cannot be deduced in either version of quantum field theory. The result of the present construction reveals to be both, a field theory and a quantum theory.

7. Literature

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