Note about "Response to Question #79 Does a plane wave carry spin angular momentum?" (Am. J. Phys. 70, 567)

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It is shown that spin of a light wave is absorbed by all surface of an absorbing plate, while the edge of the plate absorbs orbital angular momentum. Absorption of electrodynamics spin cannot be expressed in terms of the energy-momentum tensor. Spin tensor is needed.

The question is: whether a torque acts on the central part of a plate absorbing a circularly polarized light, or does not act. For the first time, this question was discussed at V.L. Ginzburg Moscow Seminar on spring of 1999, and the problem was formulated in terms of an experiment[1] concerning a two elements absorbing plate comprising a central disc and outer annulus.

I. MOMENT OF MOMENTUM

According to up-to-date paradigm, the question, does *a plane* wave carry spin angular momentum, is incorrect because "plane wave" is an inadmissible concept, and one can ask only, does a circularly polarized *beam* carry spin? The up-to-date answer is: YES, a circularly polarized beam carries spin in the frame of the classical electrodynamics, but this spin is localized in the border of the beam because the Poynting vector is parallel to the wave vector in the central part of the beam. However, such localization provokes the question about torque acting on the central part of an absorbing plate.

L. Allen and M. J. Padgett[2] consider the problem of a circularly polarized plane wave interacting with a round absorbing plate. Firstly, they have represented the wave as the sum of two beams. The first beam has the same diameter as the plate; the rapid falloff in intensity at its edge gives rise to an angular momentum density about the axis, in accordance with the well known formula (see, e.g.[3])

$$j_z = \varepsilon_0[\mathbf{r}\times(\mathbf{E}\times\mathbf{B})]_z = -\frac{\varepsilon_0 r}{2\omega}\frac{\partial E_0^2(r)}{\partial r}. \quad (1)$$

The absorption of this beam with this angular momentum results in a torque τ. So the edge of the plate experiences a torque, and this torque is provided with a moment of the tangent forces acting only near the edge. This is depicted in Fig.1 from[2] (with our additions).

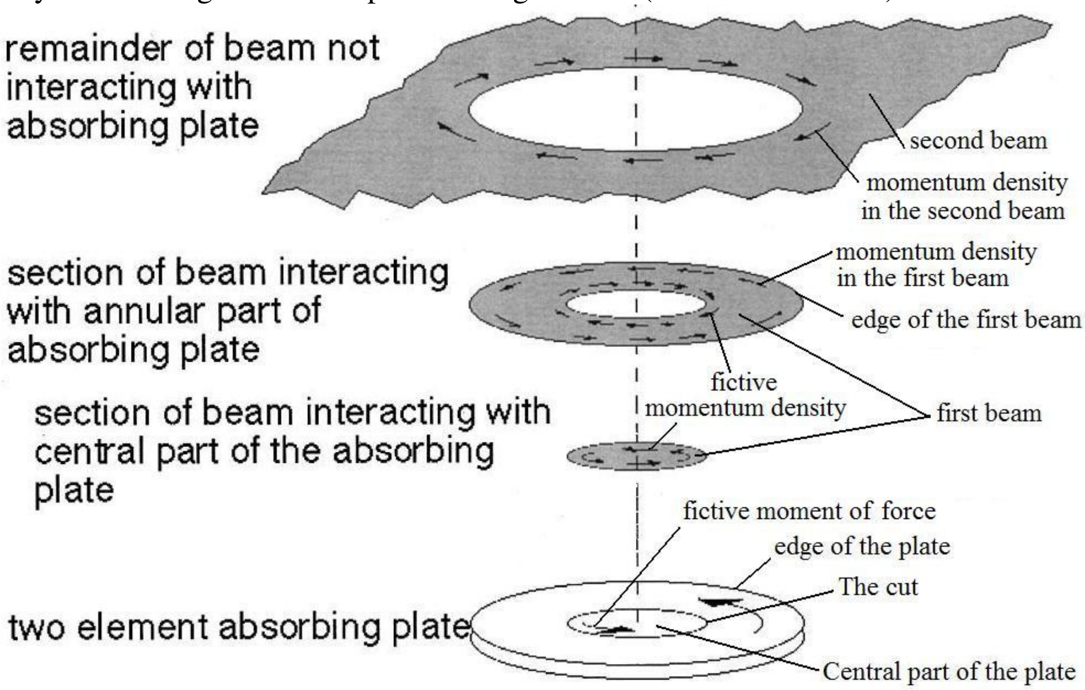


Fig. 1. When suspended in a circularly polarized plane wave, a two element absorbing plate comprising a central disc and outer annulus experiences a torque on both components. The torques arise from the effective aperturing of the light beam, such that the large intensity gradient at the perimeter of the plates results in azimuthal components to the momentum density.

We must note here that, since this angular momentum and this torque come into existence as a moment of momentum and as a moment of force, this angular momentum is an orbital angular momentum rather than spin, because spin is armless.

Integrating of the angular momentum density (1) across the first beam yields this torque as

$$\tau = P/\omega, \tag{2}$$

where P is power of the first beam, and ω is the friquence.[3]

The second beam corresponds to the rest of the plane wave and has an equal but opposite angular momentum near its inner edge. However, the second beam plays no role as it does not overlap with the plate, is not absorbed. This second beam is depicted by Simmonds and Gutmann[4]; they consider a wide beam of radius R_0 instead of the plane wave, but it is of no importance. In Fig. 9.4 from,[4] r_0 is the radius of the absorbing plate.

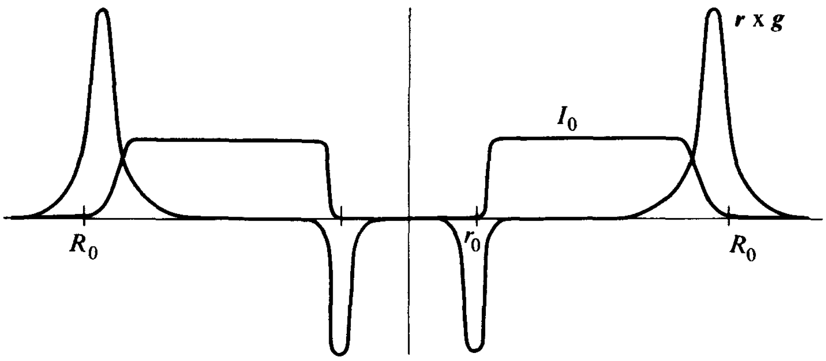


Fig. 9.4. The intensity and the angular momentum density across a cylindrical beam after an absorbing disk has been placed in the center of the beam.

Formula (1) shows that the moment of the local momentum density is proportional to the radial intensity gradient of a light beam. So, since there is no such a gradient in the central part of the first beam, there is no moment of momentum density in the central part. So, no forces and no moment of forces act on the central part of the plate.

However, authors of[2] assert that a cut in the plate induces momentum density in the first light beam, although the cut does not change the light beam! The beam obviously is not apertured by the cut.

Authors depicted this induced momentum density in Fig. 1 by arrows. But this momentum density is a fiction. So, no moment of force acts on the central part of the plate according to up-to-date paradigm.

Note that a cut is a singular formation; a cut has no width. So there is no area where the asserted momentum density exists.

We criticized this assertion long ago.[5] We wrote: “An intensity gradient near a wall of a beam results in the azimuthal component of momentum density only in the case of a real beam satisfying the Maxwell equations. There are no azimuthal components in a piece of a wave that is simply cut off from a whole wave. Such a piece cannot be considered at all because it does not satisfy the Maxwell equations”.

II. SPIN

Meanwhile, as we wrote in,[1] R. Feynman[6] clearly showed how a circularly polarized wave transfers a spin torque to an absorbing medium (see Fig. 17-5). Beth[7] wrote: “The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant is a tensor. Consequently the electric intensity $\mathbf{E}$ is not parallel to the electric polarization $\mathbf{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is $\tau / V = \mathbf{P} \times \mathbf{E}$" (Fig. 2).

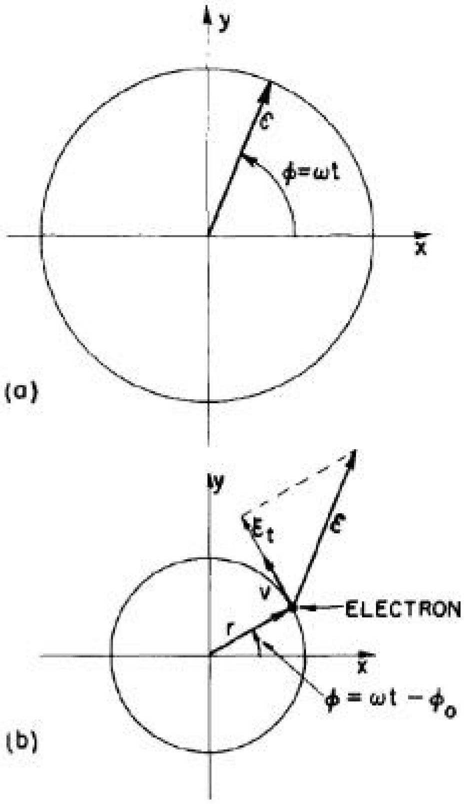
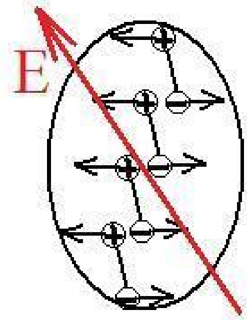


Fig. 17–5. (a) The electric field ε in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.



The absorption of spin angular momentum can be thought of as a couple acting on a small area of the absorbing plate from the light.

Fig. 2.

So, any area of the plate experiences a torque from the light. The concept of forces *acting from light* on the central part is wrong (except light pressure). If the plate is not suspended but is anchored by a support and is not split, the equilibrium of the area requires tangential forces acting along the perimeter of the area. (By the way, if the area is a disk of radius r, and the flux density of spin is Υ, then the linear density of the force, $f = \Upsilon / 2$, is independent of r and can be found from $\Upsilon \pi r^2 = f 2\pi r r$). Light, which illuminates adjacent area elements, cannot provide such force density. This light does not touch the area under consideration. So, a mechanical shear stress of the plate is the only possibility to provide perimeter of the area with the need force density.[5] And this shear stress causes a (spin) torque acting on the support of the plate in addition to the moment of the edge forces. The famous Haumblet identity,[8]

$$\varepsilon_0 \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV = \varepsilon_0 \int \mathbf{E} \times \mathbf{A} \, dV , \tag{3}$$

between the total moment of the boundary momentum and the total spin shows that the total torque is

$$\tau_{tot} = 2P / \omega, \tag{4}$$

III. CONCLUSION

Circularly polarized beam carries the double angular momentum.

ACKNOWLEDGMENTS

I am eternally grateful to Professor Robert H. Romer for valiant publishing of my question [1] (submitted on 7 October 1999). That issue was the last, which Robert Romer edited.

APPENDIX: HISTORY OF SUBMITTING

First submission

Subject : MS# **15263** answer to #79
From : AJP
Date : Mon, 10 Sep 2001
Dear Prof. Khrapko, We have received your manuscript:

MS# 15263. Answer to question # 79: Plane wave carries a spin

The classical electrodynamics is not complete. Electrodynamics spin tensor is not zero (physics/0102084, physics/0105031). So, "ponderomotive forces" acting on an surface element da_j consist of both, the force itself, $dF^i = T^{ij} da_j$,

and of a torque, $d\tau^{ik} = \Upsilon^{ikj} da_j$.

Here T^{ij} is the stress tensor, i.e. the space part of the energy-momentum tensor, and Υ^{ikj} is the space part of a 4-spin tensor (they are rather tensor densities).

So, the central part of the round flat target which absorbs a circularly polarized wave will perceive a torque. The central part will become twisted.

I suggested that the 4-spin tensor of electromagnetic wave, $\Upsilon^{\alpha\gamma\beta}$, is expressed symmetrically in terms of the magnetic A^α and electric Π^α vector potentials

$(\alpha, \gamma, ... = 0,1,2,3)$: $\Upsilon^{\alpha\gamma\beta} = A^{[\alpha}\partial^{|\beta|}A^{\gamma]} + \Pi^{[\alpha}\partial^{|\beta|}\Pi^{\gamma]}$, $\partial_\alpha A^\alpha = \partial_\alpha \Pi^\alpha = 0$,

where $2\partial_{[\zeta}(A^\alpha g_{|\alpha|\rho]}/\sqrt{g}) = F_{\zeta\rho}$, $\partial_\zeta(\Pi^\alpha \varepsilon_{\alpha\mu\nu\sigma}\sqrt{g}\, g\, g^{\mu\rho} g^{\nu\xi} g^{\sigma\zeta}) = F^{\rho\xi}$,

and $F_{\varsigma\rho}$, $F^{\rho\xi}$ are the covariant and contravariant electromagnetic tensors, respectively ($F^{\rho\xi}$ is rather a tensor density).

R. I. Khrapko

Subject : MS# **15263**
From : AJP
Date : 24 Oct 2001
Dear Professor Khrapko,
Below you will see comments from two reviewers of your answer to question #79. Neither reviewer recommended publication. Because we have two other answers that both reviewers recommend publishing with revisions, we are not planning on publishing your answer.
Sincerely, Jan Tobochnik

Referee A
Question #79 is an intriguing question - on a problem I had never heard of. I was tempted to send it back and say I could not comment, but somehow held off, talked to various people, and finally have come to an opinion: Manuscripts # 12470 (Allen and Padgett[2]) and #15056 (Yurchenko[9]) should be published, with some minor changes as described below, but not #15263 (Khrapko) in its present form. (Please see about Yurchenko in[10] – *R. Khrapko*).

Referee B
Comment on Manuscript **#15263** (Khrapko, answering his own question): I did not understand a word of it. Very few readers of the AJP would get anything out of it. It needs a lot more discussion and explanation.

Second submission

Subject : MS# 15916
From : AJP
Date : 03 Jun 2002
Dear Prof. Khrapko,
We have received your manuscript entitled,

"Answer to Question #79: Classical Electrodynamics' Spin"[11]

Sincerely yours, Julie Wenzel

Subject : MS# 15916
From : AJP
Date : 19 Jun 2002
Dear Prof. Khrapko,

Below you will find a copy of the reviewer's report on your manuscript MS# 15916, "Answer to Question #79: Classical Electrodynamics' Spin"

As you can see, this reviewer does not recommend publication, and our own reading of the manuscript concurs with the reviewer's opinion. Hence, we have decided not to publish your manuscript.

Sincerely, Jan Tobochnik

REVIEW

I believe that this paper should not be published. I think it attempts to raise issues with, and modify, the standard formulation of electrodynamics that already does an adequate job of describing physical systems. The editorial policy of American Journal of Physics, as stated in the January issue each year, states that "manuscripts questioning well-established and successful theories are more appropriately submitted to one of the archival research mournals for evaluation by specialists." That policy clearly applies here and provides in itself a basis of rejection of the manuscript.

The present manuscript presents the usual derivation, by variation of the action, of the canonical energy-momentum tensor following Landau and Lifshitz (LL) and shows that there are two deficiencies in this: It is not symmetric and it does not include the effects of particle currents. LL correct this by adding two terms. The author introduces a so-called spin tensor, which he claims should be modified in an analogous way. The author claims that the usual approach leads to a vanishing spin tensor; this result corresponds to the usual derivation in many references that a circularly polarized plane wave that is infinite in extent carries no angular momentum along its direction z of travel.

As several references quoted by the author show, one can explain the seeming paradox that a beam can impart z angular momentum to absorbers by considering the finite extent of a real beam which then has E and B not perpendicular to z everywhere. An answer (Ref 13) to the authors question in AJP concerning the spin of a plane wave (Ref 12) does a nice job in explaining this by considering the beam as made up of components that are "aperatured." The author questions the validity of this explanation and proposes that the correct approach is by introducing his non-gauge invariant spin tensor, which has been corrected for the presence of currents in a way analogous to the LL correction of the energy-momentum tensor.

To me the analysis of angular momentum carried by EM waves presents no unresolved paradoxes. We know how to treat the problem in QED; so in fact we don't really need a classical solution to the problem. However, I think that Ref 13, for example, gives a simple valid explanation of the author's AJP question. The author's introduction of a new formulation of the spin tensor solves a problem that apparently does not exist and does so only by breaking gauge invariance. This leads me to strongly doubt the validity of the author's arguments. However, to a large extent the validity or not of the paper is irrelevant, because accepting the paper would violate the editorial policy concerning manuscripts questioning well-established theories.

The analysis of the problem is carried out in very formal tensor language, which would severely limit the number of readers of AJP who could follow the argument. Ref 9, which appeared in AJP also dealt with an analogous problem, but with a much more transparent and less formal approach. The esoteric presentation of the present paper would in itself merit rejection in my opinion.

Subject :Re: MS# 15916

From : Prof. Khrapko

Dear Jan Tobochnik,

Allen & Padgett's answer to the question # 79 is obviously wrong. They write, ``Any form of aperture introduces an intensity gradient, and a field component is induced in the propagation direction and so the dilemma is potentially resolved."

But a small clearance between the inner disc and outer annulus does not aperture a wave and does not induce a field component in the propagation direction. The imaginary decomposition of the

plane wave into three beams, the inner beam, the annular beam, and the remainder, is not capable to create longitudinal field components and, correspondingly, transverse momentum and torque acting on the disc. Maxwell stress tensor cannot supply the disc with torque.
So, the question # 79 has no answer. The only answer is contained in my unpublished papers. Please, publish them.

I shall try to eliminate the esotericancy of my paper's presentation.

Sincerely, Radi Khrapko

Subject : Re: MS# 15916
From : AJP
Date : 21 Jun 2002
Dear Prof. Khrapko,

We have been through this many times. Perhaps AJP in not the appropriate journal for your answers.

Thank you, Jan Tobochnik

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[a] E-mail: khrapko_ri@hotmail.com, site: http://khrapkori.wmsite.ru

[1] Khrapko R.I.,"Does plane wave not carry a spin?" Amer. J. Phys. 69, 405 (2001)

[2] Allen L., M. J. Padgett, "Response to Question #79. Does a plane wave carry spin angular momentum?" Am. J. Phys. 70, 567 (2002) (Manuscript #12470).

[3] Allen L., M. J. Padgett, M. Babiker, "The orbital angular momentum of light" in Progress in Optics XXXIX (Elsevier, Amsterdam, 1999)

[4] Simmonds J. W., M. J. Guttmann, States, Waves and Photons (Addison-Wesley, Reading, MA, 1970)

[5] Khrapko R.I., "Mechanical stresses produced by a light beam". J. Modern Optics, 55, 1487-1500 (2008)

[6] Feynman R. P., R. B. Leighton, M. Sands, The Feynman Lectures on Physics (Addison–Wesley, London, 1965) Vol. 3, p. 17–10.

[7] Beth R. A., "Mechanical detection and measurement of the angular momentum of light" Phys. Rev. 50, 115 (1936).

[8] Humblet J., "Sur le moment d'impulsion d'une onde electromagnetique". *Physica (Utrecht)* **10** (7): 585 (1943)

[9] Yurchenko V. B., "Answer to Question #79. Does plane wave not carry a spin?" *Am. J. Phys*. 70, 568 (2002) (Manuscript #15056).

[10] Khrapko R.I. "Comments on the rejection" http://khrapkori.wmsite.ru/ftpgetfile.php?id=103&module=files

[11] Khrapko R.I. "Answer to Question #79: Classical Electrodynamics' Spin" http://khrapkori.wmsite.ru/ftpgetfile.php?id=127&module=files