[Manuscript version of the paper published in the Proceedings of Indian National Science Academy]

Some Expressions for Gravity without the big *G* and their Possible Wave-Theoretical-Explanation

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Abstract

This letter presents some new expressions for gravity without the big *G* and proposes their possible wave-theoretical-explanation. This attempt leads to some insight that: (i) We need the proportionality-constant *G* because we measure masses and distances in our arbitrarily-chosen units of kg. and meters; but if we measure 'mass' as a fraction of 'total-mass of the universe' M_0 and measure distances as a fraction of 'radius-of-the-universe' R_0 then there is no need for the proportionality-constant *G*. However, large uncertainties in the M_0 and R_0 limit the general application of this relation presently. (ii) The strength of gravity would be different if the total-mass of the universe were different. Then this possibility is supported with the help of wave-theory. (iii) This understanding of *G* leads to an insight that Planck's-length, Planck-mass and Planck's unit of time are geometric-mean-values of astrophysical quantities like: total-mass of the universe and the smallest-possible-mass $h\, H_0 / c^2$. (iv) There appears a law followed by various systems-of-matter, like: the electron, the proton, the nucleus-of-atom, the globular-clusters, the spiral-galaxies, the galactic-clusters and the whole universe; that their ratio $Mass\ /\ Radius^2$ remains constant. This law seems to be more fundamental than the fundamental-forces because it is obeyed irrespective of the case, whether the system is bound by strong-force, electric-force, or gravitational-force.

1. Introduction

Sir Isaac Newton presented the quantitative description of gravitational attraction between two massive bodies, that the force of attraction is directly proportional to the product of two masses, and inversely proportional to the square of centre-to-centre distance between them; and the value of proportionality-constant *G* was found to remain the same even in the case of planets. But there has been no explanation for why the value of *G* is this much. Einstein also made extensive use of *G* by treating it as a fundamental-physical-constant. Based on my previous works, [1-5] and the works of researchers cited in these papers, this paper presents some alternative expressions for gravity, without the big *G*, and proposes a wave-theoretical-explanation for gravity.

2. New Expressions of gravity without the big *G*

(i) R. K. Adiar, in his book "Concepts in Physics" has given a derivation [6], that the sum of 'gravitational-potential-energy' and 'energy-of-mass' of the whole universe is, strikingly, zero!

i.e. $M_0\, c^2 - G\, M_0\, M_0 / R_0 = 0$

where M_0 and R_0 are total-mass and radius of the universe respectively, and G is Newton's gravitational constant.
i.e. $G M_0^2 / R_0 = M_0 c^2$.

i.e. $G = R_0 c^2 / M_0$

So, by substituting $R_0 c^2 / M_0$ for G in Newton's formula, the gravitational potential energy U_g stored in a system of masses M and m separated by a distance r can be expressed as:

$U_g = (M / M_0)\ m c^2 / (r / R_0)$ …….(1)

Newton's law when expressed as shown in the expression-1, shows that: if we measure masses as a fraction of total-mass of the universe M_0 ; and measure distances as a fraction of radius of the universe R_0 then we do not need the big G .

However, large uncertainties in the M_0 and R_0 limit the general application of this relation presently.

A brief discussion will be in order, how the 'total-mass-of-the-universe' and 'radius-of-the-universe' are derived; and what would be the uncertainties of these?

Total-mass-of-the-universe:

E. P. Hubble's experimental-observations of the 'cosmological-red-shift', when interpreted in terms of 'recession-of-galaxies', gives a linear relation:

$v = H_0 D$

where: v is the 'velocity-of-recession' of a galaxy, H_0 is Hubble's constant and D the luminosity-distance of a galaxy. From this relation we can get an estimate of 'sum-total-of-kinetic-energy-of-the-universe' K_u. This recession-of-galaxies, also known as: 'expansion-of-the-universe', can stop if and when 'kinetic-energy-of-the-universe' K_u becomes equal to 'gravitational-potential-energy-of-the-universe' U_u. By equating $K_u = U_u$, cosmologists have derived the value of 'total-mass-of-the-universe' M_0 .

It has been estimated [7] that the universe would have collapsed to hot-death much sooner than the present-age of the universe if total-mass of the universe were more than M_0; and it would have cooled down to cold-death much earlier than the present-age of the universe if its total-mass were less than M_0. The present-age, of 14 billion years, imply that the total-mass of the universe is indeed M_0 . $M_0 = 10^{82}$ pion-masses.

It is surprising [8] that cosmologists are so far able to experimentally detect only the baryonic-matter, which is hardly 4% of the total-mass M_0 ! At least 70% of the total-mass M_0 is believed to be in the form of 'dark-energy', and remaining 26% in the form of 'dark-matter'. 'Dark-matter' is needed to explain the 'flattening-of-galaxies-rotation-curves'. That is, the estimates of total-mass of the universe depend on 26% share from 'dark-matter', and 70% share from 'dark-energy' which are yet to be detected.

Radius-of-the-universe:

The distance at which a galaxy can attain the velocity-of-light, that is, when Hubble's expression becomes: $H_0 R_0 = c$, where c is the speed-of-light, this distance R_0 is called: 'the-radius-of-the-universe'. Even if universe-tip may be moving with speed higher than light-speed, the 'visible' horizon will be limited by the equation $c = H_0 R_0$ [8]. So, the value of radius of the universe is taken as 10^{26} meters, i.e. = 10^{40} classical-radius of the electron. Here H_0 is Hubble's constant.

As far as accuracy of the values of M_0 and R_0 are concerned, there must be large amount of uncertainties. We can not expect to improve current value of G form them. Our expression of gravity without G can only help us to gain an insight, that the strength of gravitational-force seems to depend on total-mass and radius of the universe. Similarly, we can gain some insight in to Planck's natural units, and Milgrom's new constant of nature a_0, termed as the 'critical-acceleration' of Modified Newtonian Dynamics [MOND].

Now, let us move to some more expressions without the big *G*

(ii) Milgrom's expression for the constant velocity v of the stars at the out-skirts of a spiral-galaxy of mass M is conventionally expressed as [6-7]:

$\mathrm{v} = [\, G\, M\, a_0]^{1/4}$ …………………...(2)

Since: $G = R_0\, c^2/M_0$, and $a_0 = c^2 / R_0$, as discussed in [8], the expression-2 can be re-expressed without G as:

$\mathrm{v} = [\, (R_0\, c^2/M_0)\, M\; (\, c^2/ R_0\,)\,]^{1/4}$

i.e. $\mathrm{v} = [\, M / M_0\,]^{1/4}\; c$ …..………(3)

In the expression-3, c is the speed of light in vacuum, and M_0 and R_0 are total-mass and radius of the universe respectively.

(iii) We can express the radii of the globular-clusters, the spiral-galaxies and the galactic-clusters as:

$R_{\mathrm{globu}} = [M_{\mathrm{globu}}/ M_0]^{1/2}\, R_0 = [\, r_{\mathrm{Gglo}}\;\; R_0\,]^{1/2}$ Where r_{Gglo} is gravitational-radius of the globular-cluster….(4)

$R_{\mathrm{galaxy}} = [\, M_{\mathrm{galaxy}} / M_0\,]^{1/2}\, R_0 = [\, r_{\mathrm{Ggal}}\;\; R_0\,]^{1/2}$ Where r_{Ggal} is gravitational-radius of the galaxy……..(5)

$R_{\mathrm{gal\text{-}clust}} = [\, M_{\mathrm{gal\text{-}clust}} / M_0\,]^{1/2}\;\; R_0 = [\, r_{\mathrm{Ggal\text{-}clust}}\;\; R_0\,]^{1/2}$ Where $r_{\mathrm{Ggal\text{-}clust}}$ is gravitational-radius of the galactic-cluster……………………………………………………………………………………………....(6)

Even the classical-radius of the electron $r_e = e^2/m_e\, c^2$ can also be expressed as:

$r_{\mathrm{e}} = [\, m_{\mathrm{e}} / M_0\,]^{1/2}\, R_0 = [\, r_{\mathrm{G\text{-}e}}\;\; R_0\,]^{1/2}$ Where $r_{\mathrm{G\text{-}e}}$ is gravitational-radius of the electron .... (7)

Radius of the pi-meson $r_{\mathrm{pi}} = N\, g^2/ m_{\mathrm{pi}}\, c^2$ can also be expressed as:

$r_{\mathrm{pi}} = [\, m_{\mathrm{pi}} / M_0\,]^{1/2}\;\; R_0 = [\, r_{\mathrm{G\text{-}pi}}\;\; R_0\,]^{1/2}$ Where $r_{\mathrm{G\text{-}pi}}$ is gravitational-radius of the pi-meson ....(8)

And the radius of nucleus of an atom r_{n} can also be expressed as:

$r_{\mathrm{n}} = [\, m_{\mathrm{n}} / M_0\,]^{1/2}\, R_0 = [\, r_{\mathrm{G\text{-}n}}\;\; R_0\,]^{1/2}$ Where $r_{\mathrm{G\text{-}n}}$ is gravitational-radius of the nucleus-of-atom…....(9)

The expressions 4 to 9 can be jointly expressed as [8]:

$M_0 / R_0^{\,2} = m_{\mathrm{P}} / r_{\mathrm{P}}^{\,2} = m_{\mathrm{e}} / r_{\mathrm{e}}^{\,2} = m_{\mathrm{n}} / r_{\mathrm{n}}^{\,2} = M_{\mathrm{gc}} / R_{\mathrm{gc}}^{\,2} = M_{\mathrm{gal}} / R_{\mathrm{gal}}^{\,2} = M_{\mathrm{cg}} / R_{\mathrm{cg}}^{\,2}$
$= H_0\, c/ G$ ……………………………..(4-9)

We shall consider a possible 'wave-theoretical-explanation' for the expressions 4 to 9 in the section-4.

Since the classical-radius of the electron $r_e = e^2/m_e\, c^2$, radius of the pi-meson $r_{\mathrm{pi}} = N\, g^2/ m_{\mathrm{pi}}\, c^2$ and the radius of nucleus of an atom r_{n} can also be expressed in the similar manner by inserting the masses of the electron, the pi-meson and the nucleus in the right-hand-sides of the above expressions, though they are bound by electric-force, strong-force and the nuclear-force respectively, it suggests a possibility that the currently-believed fundamental-forces may not be truly fundamental; rather, the law followed by them, as expressed in the expressions 4 to 9, may be more fundamental than the ‘fundamental-forces’; and the strengths of forces may be getting decided by these expressions 4 to 9. It also suggest a possibility that when a ‘black-hole’ has some ‘mass’ then it has to have a ‘radius’.

(iv) We can express the cosmological red-shift z_c smaller than unity as:

$z_c = [\, D / R_0 \,]$ ……………………….(10)

And we can express the accelerated-expansion of the universe, the deceleration of the cosmologically red-shifted photon, the deceleration of the Pioneer-10 , 11 , Galileo and Ulysses space-probes and the ‘critical-acceleration’ of MOND as [9]:

$a_0 = c^2 / R_0$ ……………………...(11)

3. Some insight into Planck’s units:

From the law of equality of gravitational-potential-energy and energy-of-mass of the universe we found that: $G = R_0\, c^2/M_0$. Now let us make use of this expression to get some insight into Planck’s units of length, mass and time:

Planck’s-length $L^* = [h\, G / c^3\,]^{1/2}$

Substituting $R_0\, c^2/M_0$ for G in the above expression,

Planck’s-length $L^* = [\, h\, R_0\, c^2 / M_0\, c^3]^{1/2}$

i.e. Planck’s-length $L^* = [(h / M_0\, c)\,(\, R_0\,)]^{1/2}$ …….(12)

i.e. Planck’s-length L^* is a geometric-mean of: Compton-wavelength and Gravitational radius of total-mass of the universe, because $R_0 = G\, M_0 / c^2$.

Planck-mass $M^* = [\, h\, c / G\,]^{1/2}$

Substituting $R_0\, c^2/M_0$ for G in the above expression,

Planck-mass $M^* = [\, h\, c\, M_0 / R_0\, c^2\,]^{1/2}$

i.e. Planck-mass $M^* = [(\, h / R_0\, c\,)\ (M_0\,)]^{1/2}$

i.e. Planck-mass $M^* = [(\, h\, H_0 / c^2\,)\ (M_0\,)]^{1/2}$ …….(13)

That is Planck’s unit of mass is a geometric-mean of: total-mass of the universe and smallest-possible-mass, corresponding to Hubble’s constant ($h\, H_0 / c^2$).

Similarly, Planck’s unit of time T^* is a geometric-mean of: age-of-the-universe T_0 and the period

($h / M_0\, c^2$):

i.e. $T^* = [\, T_0\ (\, h / M_0\, c^2\,)\,]^{1/2}$ …………………(14)

4. Possible wave-theoretical explanation for gravity:

Let us assume that there are some most-fundamental-particles, and a long-range fundamental-force. We can take the mass of the 'most-fundamental-particle' as a unity, and think that all the massive objects are collections of the 'most-fundamental-particles'.

Now, by a 'particle' we mean an entity which is localized in an extremely small space; so, a 'particle' can be mathematically represented in the space-domain as an impulse-function. This impulse-function can be Fourier-transformed into the 'wave-number-domain'. Then assuming a constant velocity of transmission of these waves, at the velocity of light, we can represent these waves in the 'frequency-domain' as a wide band of frequencies. A particle of matter has a wide band of frequency-spectrum and a definite phase-spectrum. When this wide band of waves travels in space, then a 'particle' becomes manifest only at a place and time when-and-where all the spectral-components add constructively, and have a particular, definite phase-relation, otherwise the particle remains dissolved in the un-manifest-state.

Secondly, we can not expect any coherence between the spectral-components of one and the other 'particle'. That means, that when two or more such fundamental-particles come close to each-other, the wide bands of their waves add like the incoherent superimposition of wideband-noise.

We know that the superimposition of n number of wide-band noise-sources of unit-amplitude is square-root-of n; like the vector-sum of n mutually orthogonal unit-vectors. That is:

$$N(t) = [\ (N_1(t))^2 + (N_2(t))^2 + (N_3(t))^2 \ \ldots\ldots + (N_n(t))^2\ ]^{1/2}$$

Now, if the strength of 'coupling-constant' of a fundamental-force is, say, e^2, which is the strength of electric-force of the proton, then the strength of 'coupling-constant' of a new "fundamental-force", which is actually due to 'incoherent-superimposition', within the system of n fundamental-particles will be: $[(n^{1/2}\, e^2) \,/\, n]$. Since the total-mass of the universe M_0 is 10^{80} proton-masses, the strength of gravitational-force between the two protons is expected to be:

$$G\, M_0\, m_p = (\text{Total-number of protons in the universe})^{1/2}\, e^2$$

i.e. $G\, m_p^2 = (10^{80})^{1/2}\ e^2 / 10^{80}$

i.e. $G\, m_p^2 = 10^{-40}\, e^2$ .... (14)

[Note: This is just an order-of-magnitude-estimate]

Now, if the force within a system is stronger than gravity by a multiplication-factor, say, k-times, then the density of matter within that system is also logically expected to be k- times higher. That is, in our example of proton and the universe:

$$e^2 / G\, m_p^2 = [\ M_0 / m_p\ ]^{1/2}$$

$$= [\ m_p / (4/3)\ \pi\ r_p^3\ ] / [\ M_0 / (4/3)\ \pi\ R_0^3\ ]$$

i.e. $e^2 / G\, m_p^2 = [\ M_0 / m_p\ ]^{1/2} = [\ m_p\, R_0^3\ / M_0\ r_p^3\ ]$

i.e. $[\ M_0 / m_p\ ]^{3/2} = [\ R_0^3\ /\ r_p^3\ ]$

i.e. $[\ M_0 / m_p\ ]^{1/2} = [\ R_0 / r_p\ ] = e^2 / G\, m_p^2$ ....(15)

i.e. $M_0 / R_0^2 = m_p /\ r_p^2$ ....(16)

The expression-15 was noticed as the 'large-number-coincidence [LNC], whereas here we derived it with the help of wave-theory.

Sivaram [10] had noticed a relation between masses and radii of the electron, the proton the nucleus-of-atoms, the globular-clusters, the spiral-galaxies, the galactic-clusters and the universe as shown in the expression-17 below. The expression-17 is similar to the expression-16 derived by us using wave-theory. So our derivation based on wave-theory matches with the observations presented by Sivaram.

$$M_0 /R_0^2 = M_{gal\text{-}clust} /R_{gal\text{-}clust}{}^2 = M_{gal} /R_{gal}{}^2 = M_{globu} /R_{globu}{}^2 = m_n /r_n{}^2 = m_{pi} /R_{pi}{}^2 = m_e /r_e{}^2$$

$$= H_0\, c/G .... (17)$$

Even the mysterious-looking Weinberg-formula can be re-written, and explained, as follows: Weinberg's formula is: $m_{pi}{}^3 = h^2 H_0 / c\, G$, which can be re-written as: $m_{pi} / (h / m_{pi}\, c)^2 = H_0\, c / G$. Weinberg's formula has an imbalance of one order of magnitude which can be corrected by replacing Compton-wavelength of the pion by radius of the pion, i.e. $m_{pi} /R_{pi}{}^2 = H_0\, c/ G$. So the mysterious-looking Weinberg-formula is also a part of the expression-17.

4. Conclusion

Now we have an explanation for why we need the gravitational constant G. The strength of gravity seems to depend on the total-mass M_0 and radius R_0 of the universe. However, large uncertainties in the M_0 and R_0 limit the general application of this relation presently. Secondly, gravity may not be an independent 'fundamental-force'; it may be arising due to 'in-coherent super-imposition' of wave-amplitudes of very wide-band of waves of total number of fundamental-particles contained in the universe. The theory also explained the large-number-coincidence, and the mysterious-looking Weinberg formula. We also gained some insight into Planck's units that: Planck-length, Planck-mass and Planck's unit of time are geometric-mean-values of astrophysical quantities like: total-mass of the universe and the smallest-possible-mass hH_0 / c^2.

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