Discovering Nature's Hidden Relationships, an Unattainable Goal?

1. INTRODUCTION

No theory explains the secrets of nature - forces and constants are what they are. Why is it difficult to discover hidden relationships that might reveal these secrets? A complete picture of the universe must include both particle physics and cosmology. However, the Standard Model of Particle Physics (SMPP) and the Standard Model of Cosmology (SMC) are disjoint models. Most professional physicists and astronomers would say finding "hidden relationships" is a wild dream, a foolish, almost impossible goal. The noted string theorist, Ed Witten, once said that "he'd given up on ever predicting all constants of nature." [6]

This article, by defining the dimensional and dimensionless constants within the SMPP and the SMC, illustrates the major difficulty. A quote from Paul Davies sets the stage and challenge: "Mathematics and beauty are the foundation stones of the universe. No one who has studied the forces of nature can doubt that the world about us is a manifestation of something very, very clever indeed". [2]

So why is nature disguised so well? Principally because the models of the two worlds, the Standard Model of Particle Physics (**SMPP)** and the Standard Model of Cosmology (**SMC**) have no elements in common, they are completely disjoint. Thus, relationships are hidden and the task of finding them, may prove to be virtually unattainable. However, if secrets are to be found, dimensionless ratios using constants from both models is an appropriate approach.

After explaining the divergent characteristic of the SMPP and the SMC, the disjoint between the two models will be clear. Next, dimensionless numbers are explained, why they are required and how they are calculated. Then, constants and equations from both models are defined. And last, a select group of ratios using Planck and Hubble constants are presented. Is the large number coincidence documented a hidden relationship or speculation?

2. Disjoint Models: Standard Model of Particle Physics (SMPP), Standard Model of Cosmology (SMC)

2.1. Overview

Why are the SMPP and the SMC disjoint models? Table 1 provides an overview of the two models highlighting significant differences. The SMPP involves microscopic sizes (elementary particles, quarks, and bosons); conversely, the SMC deals in the macroscopic world with planets, stars, galaxies and black holes. Differences in sizes are immense, for example, the radius of a hydrogen atom approximately 10^{-13} centimeters is minuscule when compared with cosmological distances measured in light years, each light year about 10^{18} centimeters.

Both models contain successful theories: in SMPP, Quantum Electrodynamics (QED), Quantum Chromodynamics (QCD), and Special Relativity; and in the SMC, the Big Bang Theory, Inflation, and General Relativity. Exact equations are the rule in SMPP, for example in QED, theoretical values match observation to many decimal places; but in the SMC, many calculations are approximate, for example, how exact is the number of baryons (protons and neutrons) in the observable universe which is estimated at 10^{80}?

Both models have numerous unique/mysterious features - the uncertainty principle and quantum weirdness for the SMPP, and dark matter and dark energy for the SMC. Dimensional (Physical) constants applicable in each model have no relation to each other. Elementary particles with their associated mass and charge are quite dissimilar from the concepts of: expanding space, CMB, and critical density. With significant difference in characteristics, sizes, unique features, and constants, the two models earn the right to be labeled disjoint.

The dimensionless constants play integral roles in their respective theories. In the SMPP, the fine structure constant (α) is essential to QED, with the strong (α_S) and weak force (α_W) integral to Electroweak Theory and QCD respectively. In the SMC, the measure of homogeneity (Q) and gravitational force dictate the distribution of galaxies. However, no commonality is identified with dimensionless constants. One attempt to combine constants from both models, defined an equation ($m_{pion} = (\hbar^2 H/Gc)^{1/3}$) using the Planck constant ($\hbar$) and the Hubble constant (H). [9] The resulting value is in units of mass, specifically a very small mass (1.06×10^{-25} gm) - almost, but not exactly, equal to the Pion mass (2.50×10^{-25} gm). Thus, the relationship is probably a coincidence**.** Another attempt used the same constants to calculate a "hypothetical" mass, actually the smallest possible mass (m_H), a minuscule value of 2.53×10^{-66} gm ($m_H = \hbar H/c^2$). [7] This hypothetical particle, possibly the energy of a graviton, can be used to create ratios with other masses as will be demonstrated later. These two equations may be the only good examples of equations that result in units of mass by using both the Planck and Hubble constants. Next, why dimensionless numbers are required when searching for hidden relationships.

2.2 Dimensionless Numbers, Criteria

Since physical constants are defined by a system of units (for example, cgs units - grams, centimeters, and seconds), they do not represent inherent features of the universe - change the units and the comparison changes. Units are arbitrary standards. Thus, ratios where units cancel producing dimensionless numbers are necessary to discover symmetry or fundamental relationships. Using a dimensionless ratio like electron mass divided by proton mass, assures that in any system of units, the ratio would be the same, in this example, β equals 1/1836.12 or 5.44×10^{-4}. Laws of physics are independent of arbitrary units and so are dimensionless ratios. As long as the system of units are the same for the numerator and denominator, the ratio of the two numbers represents an inherent feature of nature. The goal is to search for consistencies among many possible ratios.

But not all ratios reveal fundamental features, for example, one popular comparison is the mass of a typical star divided by the mass of an electron, a ratio of about 10^{60}. This ratio does compare the micro and macro worlds, but since the mass of stars vary by a factor of one thousand, the ratio depends on the star chosen. Ideally, ratios should have some unique physical significance and identify a possible pattern with other ratios. More on this later, but first, a review of the familiar constants in each of the models.

3. Constants and Equations

3.1 Standard Model of Particle Physics

3.1.1 SMPP Constants

The SMPP physical constants include: speed of light, mass of electron and proton, electron charge, Planck constant, and many others (reference Table 2).The dimensional constants are shown with cgs units of measure. These physical constants occur in basic theories of physics (Quantum Mechanics, Newtonian Physics, Relativity, and Electromagnetism) and have universally used symbols. They are a subset of approximately 200 defined fundamental physical constants which are virtually all dimensional.

The SMPP contains 25 dimension**less** (and one dimensional) "input physical parameters" relating to particles and forces. [5] There values cannot be calculated from more fundamental constants because there is nothing more fundamental in SMPP. They are summarized below:

- A. Particle parameters (23)
    1. Coupling constants (G) - twelve (6 quarks, 3 neutrinos , electron, muon and tauon)
    2. Matrix/Phase angles -eight (4 quark and 4 neutrino)
    3. Other - three (2 Higgs coefficients and CP vacuum phase)
- B. Force parameters (3)
    1. Weinberg angle(θ_W)
    2. Weak coupling constant (g)
    3. Strong coupling constant (g_s)

Although no apparent relationships exists among them, it is possible to derive both dimensional "physical parameters", which are primarily masses and forces, and numerous dimensionless constants. For an example derivation, the mass of an electron is: $m_e = vG_e/2^{1/2} = 0.51$ MeV ; where v = Higgs vacuum expectation value, 247 GeV; and, G_e = electron coupling constant, 2.9×10^{-6}. There are also thousands of dimensionless constants (primarily ratios) in physics which can, in principle, be calculated using the laws of physics and the 26 input parameters. [6] The important observation is that the numeric values of the dimensionless input physical parameters, the dimensional physical parameters and the dimensionless constants have no identifiable relationships between them.

Of the four SMPP dimensionless constants shown in Table 2, the two most useful and most mysterious are the fine structure constant (α) and the ratio of the electron to proton mass (β). The fine structure constant (which is a ratio itself) describes the strength of the electromagnetic interaction which determines the structure of atoms/molecules and the behavior of light. The constants α and β are the only two dimensionless constants required for the formulation of Quantum Electrodynamics (QED) and thus, reveal an underlying unity. [1] The values of α and β appear to be unique, no one knows why they are what they are - it is a complete mystery.

In an attempt to predict fundamental particle masses, numerous mathematical analysis have been performed. One clever one, employs the electron mass and the fine structure constant - the muon, pion, and kaon masses are "almost" exactly the electron mass times: $1.5/\alpha$, $2/\alpha$, and $7/\alpha$ respectively. [2] Also, using coupling constant ratios and dimensional analysis, individual masses have been estimated for selective particles (a few fundamental particles). [3] [8] Conversely, plotting the masses of nine fundamental particles (electron, muon, tauon, and six quarks) produces a statistically random distribution rather than a logical pattern. [6] Thus, physicist attempts to define a theory explaining fundamental particle (and subatomic) masses has been, to date, in vain.

The three SMPP forces - electromagnetic, weak, and strong - are indeed strange and thus difficult to relate. They operate over distance in totally dissimilar ways, for example, the strong force increases with distance but only acts over a extremely short range, the weak force also acts over a short range but decreases with distance, the electromagnetic force decreases with distance squared but has unlimited range. How could they be more different?

3.1.2 SMPP Equations

The equations to calculate values for the electromagnetic force, the fine structure constant, classical electron radius, and Planck entities are also shown in Table 2. These values can be used to formulate dimensionless ratios. The Planck mass is calculated from just three "universal" physical constants: speed of light, gravitation constant, and Planck constant. By forming equations with these three constants, natural or Planck units are also created for length, time, temperature, energy, and density. These values are primarily used in theoretical calculations. The Planck length is extremely small even when compared to nuclear sizes, but the Planck mass is relatively gigantic, entering the macro world with the mass of a grain of sand.

3.2 Standard Model of Cosmology

3.2.1 SMC Constants

The SMC constants address aspects of space (reference Table 3); they have virtually nothing in common and are not derived from the SMPP constants. [4] The SMC dimensional constants shown are: Hubble constant (H), CMB temperature (T_γ), energy density (ρ_C, ρ_Λ, ρ_B, ρ_{CDM}, ρ_γ), and number density of photons and baryons (η_γ,η_B). All except the dark energy density (ρ_Λ) actually vary over time but are considered constants because they change so slowly. The density of dark energy is predicted to remain constant per volume as the universe ages. Assuming the amount of baryon and cold dark matter is constant over time, the expanding universe dictates their decreasing density.

The SMC also has few dimensionless constants. One well known is the measure of homogeneity in the universe, denoted by "Q". It dictates how galaxies and clusters of galaxies form and is actually a ratio - the energy required to disperse cosmic structures (stars, galaxies, and clusters of galaxies) divided by their rest-mass energy. Q is validated by the difference in the CMB radiation intensity, about two in 100,000. [5] Other familiar dimensionless constants are: density ratios (matter, cold dark matter, and dark energy), the gravitation force strength, and ratios based on number of things, like number density of photons divided by number density of baryons. The dimensionless density ratios (Ω_{Λ}, Ω_B, Ω_{CDM}) reflect current matter/energy density - as we know at very early times radiation dominated, then matter, and now in our current epoch and in the future, dark energy dominates. The total density divided by the critical density equals one (within 1-2 percent), assuring a flat universe. The gravitational force plays an essential role in cosmology but not in the SMPP.

3.2.2 SMC Equations

The three equations for radius, mass/energy and critical density, are derived using three constants: speed of light (c), gravitational constant (G), and Hubble constant (H), reference Table 3. The mass and radius define a Hubble sphere, a finite space which encompasses the gravitationally connected universe. The radius of the Hubble sphere is 13.8 billion light years. A flat universe, requires the observed density to equal the critical density which amazingly it does. The strength of the gravitational force is calculated by the equation shown using the mass of a proton and electron.

4. Comparing Dimensionless Constants and Ratios

4.1 Dimensionless Constants

In both the SMPP and SMC, there are few dimensionless constants to compare, reference Table 2 and Table 3. Three of the SMC dimensionless constants reflect force strength ranging in value from about one to 7×10^{-6}. These forces act in totally different ways as previously noted. The commonality with SMC, is that both the gravitational force and electromagnetic force decrease exactly the same, directly proportional to distance squared. This commonality is offset by the huge disparity of their magnitudes.

The SMC density ratios vary from 00.5 (Ω_B - baryon density/critical density) to 0.68 (Ω_{Λ} - dark energy density/critical density), a relatively small range which reflects the epoch of our time. However, there is no apparent relationship to SMPP constants. Two relatively close values are β (mass electron/mass proton) and Q (measure of homogeneity), 5×10^{-4} and 2×10^{-5} respectively. Is there a possible relationship between them? If so, it is not obvious. Thus, if there is any chance of finding hidden relationships, dimensionless ratios derived with dimensional values from both the SMPP and SMC must be employed.

4.2 Combined SMPP and SMC Ratios

4.2.1 Ratios with no significance

Although not comprehensive, the data referenced in Table 2 and Table 3 provide an extensive source for creating ratios between the SCM and SMPP. Numerous ratios are possible but finding comparisons that result in more than a coincidence is challenging, for example, compare the ratio of the electromagnetic force divided by gravitational force (2.3×10^{39}) to the ratio of the universe radius (assuming a Hubble sphere) divided by classical electron radius (4.9×10^{40}). Although the values differ by more than ten times, the similar size of the exponents, 39 and 40, encourages an interpretation of a possible relationship - something more than a coincidence. However, this type of comparison and similar attempts have evolved into numerology, the unscientific manipulation of numbers to substantiate theories. Examples of these two and six other similar "large number" ratios are shown in Table 4. All have no apparent significance.

4.2.2 One Possibility

4.2.2.1 Hubble and Planck constants

One novel approach is based on only three fundamental constants (speed of light, gravitational constant, Planck constant) and the Hubble Constant. Six dimensionless ratios and their supporting equations are listed in Table 5. The numerator and denominator, each from their respective models, have values with the same units (in each ratio, the numerator is the larger number eliminating negative exponents or small fractions). The first four ratios based on the Hubble constant and the Planck constant produce an interesting result - an exact value of 6.05×10^{60} - a number labeled "N". Also, critical density divided by Planck density (ρ_C/ρ_{PL}) equals 36.6×10^{120} or N^2. Using the hypothetical mass with both the Planck and universe masses produce the same two values (N and N^2). Do the six ratios identify a hidden relationship or is there an explanation? How are the exact values of N and N^2 calculated? The next section addresses these questions.

4.2.2.2 Black Holes

How do black holes play a part in this coincidence game? The mass of a static non-rotating black hole is proportional to the Schwarzschild radius ($m = r_s c^2/2G$). The equations to compute both the universe mass/radius and the Planck mass/radius satisfy the Schwarzschild relationship (although expanding space is quite different than a black hole). Thus, as the chart in Fig. 1 shows, based on algebra, N equals: universe mass divided by Planck mass (M_U/m_{PL}), radius divided by length (R_U/l_{PL}), age divided by time (T_U/ t_{PL}), and the square root of Planck Density divided by Critical Density (ρ_{pl}/ρ_C). If the mass to radius proportionality is the same for any two masses, this relationship holds although the value of N is determined by the specific mass. Thus, since both mass/radius relationships are equal, algebraic correlation guarantees the other three relationships as shown in Fig. 1 - they are not unique coincidences, but rather a direct result of the Schwarzschild relationship.

4.2.2.3 Equations

Now for an explanation of why the ratios equal an exact value of N (6.05×10^{60}) or N^2 (36.6×10^{120}). It is because the ratios are derived from equations, reference Fig. 2 which defines relationships between three basic ratios: mass of the universe divided by Planck mass (M_U/m_{PL}); Planck mass divided by the hypothetical mass (m_{PL}/m_H); and, mass of the universe divided by the hypothetical mass (M_U/ m_H). When we divide the numerator and denominator for each ratio, the resulting equation is: $N = (c^5/(2\hbar G H^2))^{1/2}$ for the first two and $N^2 = c^5/(2\hbar G H^2)$ for the third ratio. The value of the first equation is a dimensionless number, 6.05×10^{60}, the value of the second equation is 36.6×10^{120}.

Other comparisons of large numbers, are not exact; but, these ratios, calculated from four constants of nature, are exact. Although we are now dealing with only three "basic" ratios, it is remarkable that they produce the same exact equation and equation squared for a value of N and N^2. [7] Is this one unique calculation or does N have additional ramifications, does it reveal a hidden relationship in nature? The reader can decide.

5. CONCLUSION

In our quest to discover hidden relationships, we have documented how the SMPP and SMC are disjoint models with significant differences in general characteristics, objects, unique features, dimensional constants and dimensionless constants. The SMPP has numerous exact dimensional constants. The SMC has few dimensional constants mostly with approximate values and some varying with time like the Hubble constant. Both models contain only a few dimensionless constants. Thus, the search for hidden relationships is based on dimensionless ratios. Comparisons are difficult because of the diversity in the characteristics, objects, features and constants between the micro and macro environments. They are also suspect because the time dependence of key SMC constants. A number of familiar ratios, derived from micro and macro dimensionless constants have no significance. However, ratios created with four "constants" and supported by equations provide a possible inherent relationship based on a large number (N).

The framework defined provides perspective; but, have we identified how nature's secrets are so ingeniously disguised? Unfortunately, no. Why things "are the way they are" is still a mystery - discovery possibly unattainable via analysis of dimensionless numbers. However, from a mathematical point of view, if a Theory of Everything (TOE) is realized via String Theory or another theory, the answer may not be complex, quoting John Wheeler:

"Behind it all is surely an idea so simple, so beautiful, that when we grasp it - in a decade, a century, or a millennium - we will all say to each other, how could it have been otherwise?"

Table 1. Two Disjoint Models - Particle Physics and Cosmology

	SMPP	SMC
General Characteristics	Exact calculations	Approximate calculations
	Microscopic size	Macroscopic size
	QED Theory	Big Bang Theory
	QCD Theory	Inflation
	Special Relativity	General Relativity
Objects	Elementary particles	Planets, comets, etc.
	Short lived particles	Stars
	Quarks	Black holes
	Bosons (force carriers)	Galaxies
Unique Features	Uncertainty principle	Dark matter
	Quantum weirdness	Dark energy
Dimensional Constants	$\hbar$ (Planck)	H (Hubble constant)
	c (speed of light)	T_γ (CMB temperature)
	m_e (mass electron)	ρ_C, ρ_Λ, ρ_B, ρ_{CDM}, ρ_γ (densities)
	m_p (mass proton)	Few others
	e (electron charge)	
	Many others	
Dimensionless Constants	α (fine structure constant)	Q (homogeneous universe)
	β (mass electron/mass proton)	Ω, Ω_Λ, Ω_B, Ω_{CDM}, (density ratios)
	α_S (strong force strength)	η (baryon-to-photon ratio)
	α_W (weak force strength)	α_G (gravitational force)

Table 2. SMPP Selected Constants and Equations

Dimensional Constants	
Speed of light	$c = 3.00 \times 10^{10}$ cm/sec
Gravitational constant	$G = 6.67 \times 10^{-8}$ cm^3/(gm sec^2)
Planck's constant reduced	$\hbar = 1.06 \times 10^{-27}$ gm cm^2/sec
Charge of electron	$e = 4.80 \times 10^{-10}$ e.s.u. (gm$^{(1/2)}$ cm$^{(3/2)}$)/sec
Mass of electron	$m_e = 9.11 \times 10^{-28}$ gm
Mass of proton (neutron)	$m_p = 1.67 \times 10^{-24}$ gm
Dimensionless Constants	
Electromagnetic force	$\alpha = 7.29 \times 10^{-3}$
Strong force[1]	$\alpha_S \approx 0.83$
Weak force[2]	$\alpha_W \approx 7.3 \times 10^{-6}$
Mass electron/mass proton	$\beta = 5.45 \times 10^{-4}$
SMPP input parameters	Particle masses/force parameters
Equations using c, ħ, G, e, m_e	
Electromagnetic force [3]	$\alpha = e^2/(\hbar c) = 7.29 \times 10^{-3}$
Classical electron radius	$r_e = e^2/(m_e c^2) = 2.81 \times 10^{-13}$ cm
Bohr radius	$\alpha_O = \hbar^2/(m_e e^2) = 5.26 \times 10^{-9}$ cm
Atomic time	$T = e^2/(m_e c^3) = 9.36 \times 10^{-24}$ sec
Planck time	$t_{PL} = l_{PL}/c = (\hbar G/c^5)^{1/2} = 7.60 \times 10^{-44}$ sec
Planck mass[4]	$m_{PL} = (\hbar c/2G)^{1/2} = 1.53 \times 10^{-5}$ gm
Planck length[4]	$l_{PL} = (2\hbar G/c^3)^{1/2} = 2.28 \times 10^{-33}$ cm
Planck temperature	$T_{PL} = m_{PL}c^2/k_B = 1.42 \times 10^{32}$ K
Planck energy	$E_{PL} = (\hbar c^5/2G)^{1/2} = 1.38 \times 10^{16}$ erg
Planck density	$\rho_{PL} = 3c^5/(16\pi\hbar G^2) = 3.11 \times 10^{92}$ gm/cm^3

Notes

1. Sixty times α at a separation of 3×10^{-15} cm at low energy levels
2. α times 10^{-4} at a separation of 3×10^{-15} cm at low energy levels
3. Ratio of electrostatic energy of repulsion between two elementary charges, e, separated by one Compton wavelength, to the rest energy of a single charge: $e^2/(\hbar/(m_e c))/(m_e c^2) = e^2/(\hbar c)$
4. Planck length/mass calculated by setting the Compton wavelength, λ, equal to the gravitational radius (Schwarzschild): $\lambda = \hbar/(m_{PL} c) = r_s = 2Gm_{PL}/c^2$

Table 3. SMC Selected Constants and Equations

Dimensional Constants	
Hubble constant based on 67.15 km/sec/Mpc	$H = 2.18 \times 10^{-18}$ sec
CMB temperature	$T_\gamma = 2.726$ K $= 3.76 \times 10^{-16}$ erg
Critical density	$\rho_C = 0.85 \times 10^{-29}$ gm/cm^3
Dark energy density, cosmological constant[1]	$\rho_\Lambda = 5.8 \times 10^{-30}$ gm/cm^3
Baryon energy density[1]	$\rho_B = 4.1 \times 10^{-31}$ gm/cm^3
Cold dark matter energy density[1]	$\rho_{CDM} = 2.3 \times 10^{-30}$ gm/cm^3
Photon energy density[1]	$\rho_\gamma = 4.4 \times 10^{-34}$ gm/cm^3
Number density of photons	$\eta_\gamma = 413/$ cm^3
Number density of baryons	$\eta_B = 2.5 \times 10^{-7}/$ cm^3
Dimensionless Constants	
Measure of homogeneity	$Q = 2.0 \times 10^{-5}$
Actual density/critical density	$\Omega = \rho/\rho_C \approx 1.0$
Dark energy density/critical density	$\Omega_\Lambda = \rho_\Lambda/\rho_C \approx 0.68$
Baryon density/critical density	$\Omega_B = \rho_B/\rho_C \approx 0.05$
Cold dark matter density/critical density	$\Omega_{CDM} = \rho_{CDM}/\rho_C \approx 0.27$
Baryon-to-photon ratio	$\eta = \eta_B/\eta_\gamma = 6.05 \times 10^{-10}$
Gravitational force (proton and electron)[2]	$\alpha_G = 3.19 \times 10^{-42}$
Equations using c, H, G	
Hubble radius	$R_U = c/H = 1.38 \times 10^{28}$ cm
Hubble time, age of universe	$T_U = 1/H = 4.60 \times 10^{17}$ sec
Mass/energy of Hubble sphere	$M_U = c^3/2GH = 9.25 \times 10^{55}$ gm
Critical density	$\rho_C = 3H^2/(8\pi G) = 8.50 \times 10^{-30}$ gm/cm^3

Notes

1. Symbols/abbreviations: Λ = cosmological constant; B = baryons; and CDM = cold dark matter; and, ɣ = photons
2. Gravitational force = $\alpha_G = Gm_pm_e/(\hbar c) = 3.19 \times 10^{-42}$

Table 4. Ratios Having no Significance

Description	Sym	Value	Sym	Value	EQ	Ratio
Sun/atom radius (Bohr)	R_Θ	7.00E+10	r_B	5.26E-09	R_Θ/r_B	1.33E+19
Electromagnetic/gravitational force[1]	α	7.29E-03	α_G	3.19E-42	α/α_G	2.29E+39
Universe/atomic time	T_U	4.60E+17	Ƭ	9.36E-24	T_U/Ƭ	4.91E+40
Universe/classical electron radius	R_U	1.38E+28	r_e	2.81E-13	R_U/r_e	4.91E+40
Neutron/critical density	ρ_N	4.00E+14	ρ_{CR}	8.50E-30	ρ_N/ρ_{CR}	4.71E+43
Sun/electron mass	M_Θ	2.00E+33	m_e	9.11E-28	M_Θ/m_e	2.20E+60
Universe/proton mass	M_U	9.25E+55	m_P	1.67E-24	M_U/m_P	5.54E+79
Universe/electron mass	M_U	9.25E+55	m_E	9.11E-28	M_U/m_E	1.02E+83

[1] Proton and electron mass

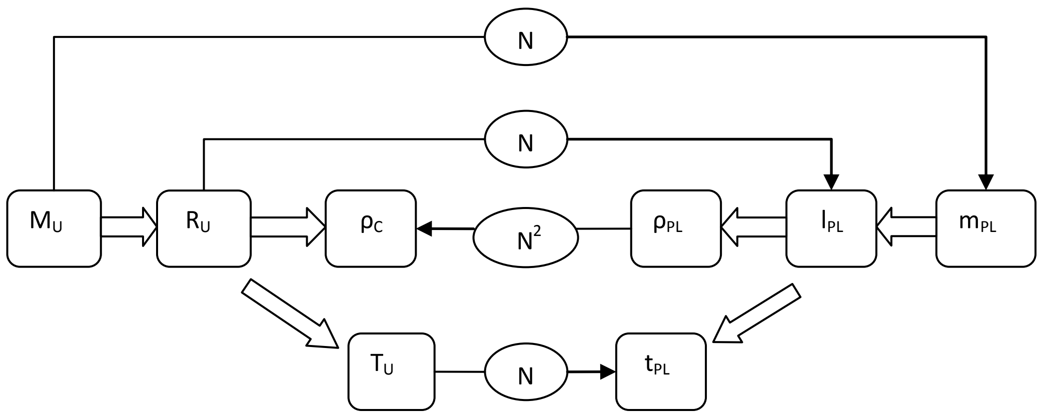
Table 5. Ratios Using c, G, ħ, and H

SMPP	Sym	Value	SMC	Sym	Value	EQ	Ratio
Planck mass	m_{PL}	1.53E-05	Hypoth. mass	m_H	2.53E-66	m_{PL}/m_H	6.05E+60
Planck mass	m_{PL}	1.53E-05	Univ. mass	M_U	9.25E+55	M_U/m_{PL}	6.05E+60
Planck length	l_{PL}	2.28E-33	Univ. radius	R_U	1.38E+28	R_U/l_{PL}	6.05E+60
Planck time	t_{PL}	7.60E-44	Univ. age	T_U	4.60E+17	T_U/t_{PL}	6.05E+60
Planck density	ρ_{PL}	3.11E+92	Critical density	ρ_C	8.50E-30	ρ_{PL}/ρ_C	3.66E+121
Hypoth. mass	m_H	2.53E-66	Univ. mass	M_U	9.25E+55	M_U/m_H	3.66E+121

Equations with Hubble and Planck constants

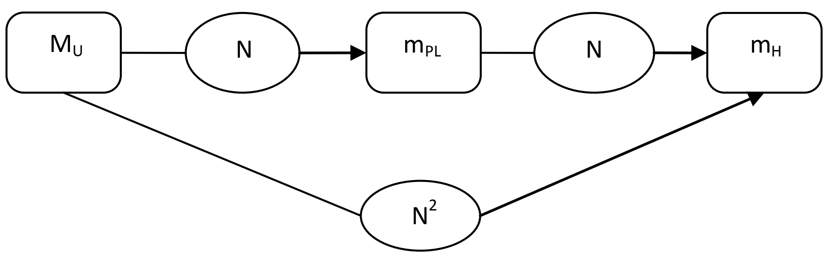
Hubble	Mass/energy of Hubble sphere	$M_U = c^3/2GH$
	Hubble radius	$R_U = c/H$
	Hubble time, age of universe	$T_U = 1/H$
	Critical density	$\rho_C = 3H^2/(8\pi G)$
Planck	Planck mass	$m_{PL} = (\hbar c/2G)^{1/2}$
	Planck length	$l_{PL} = (2\hbar G/c^3)^{1/2}$
	Planck time	$t_{PL} = l_{PL}/c = (\hbar G/c^5)^{1/2}$
	Planck density	$\rho_{PL} = 3c^5/(16\pi\hbar G^2)$
Both	Hypothetical mass	$m_H = \hbar/Hc^2$

Figure 1. Black Hole Relationships



Notes
N = constant for any two black holes
Line points to ratio denominator; Bold arrow points to dependent equation
$M_U/m_{PL} = R_U/l_{PL} = T_U/t_{PL} = N$; $\rho_{PL}/\ \rho_C = N^2$

Figure 2. Relationships for Three Basic Ratios



Notes - Ratios for exact value of N and N^2
$M_U/m_{PL} = m_{PL}/m_H = (c^5/(2\hbar GH^2))^{1/2} = 6.05 \times 10^{60} = N$
$M_U/m_H = c^5/(2\hbar GH^2) = 36.60 \times 10^{120} = N^2$

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