NEGATIVE TEMPERATURES REVISITED.

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Abstract.
In view of some fairly recent comments in the literature, well-known facts concerning systems capable of achieving both positive and negative absolute temperatures are reviewed. These considerations are shown to raise serious unanswered questions concerning the widely accepted expression for the entropy of a Schwarzschild black hole.

Several recent articles on the two subjects of negative temperatures and black holes have provoked this short follow-up to earlier comments [1] on recent incorrect statements concerning negative absolute temperatures. Here attention turns to the link between the theories surrounding negative temperatures and the thermodynamics of black holes, although the motivation is provided by the apparent first chink in the belief in the existence of black holes [2].

Various aspects of, and approaches to, thermodynamics make it seem an extremely abstract subject. Nevertheless, it is a branch of physics with roots firmly embedded in physical reality and whose purpose is to help in the explanation of physical phenomena. Nowhere is this link with reality better revealed than in the everyday notions of "hotter" and "colder". Here the everyday linguistic meaning of the terms is used in the physical theory. As Weinreich [3] points out, when two systems are placed in contact via a diathermic wall, the one which gives up heat is called the hotter and that which absorbs heat is the colder. The property of being hotter or colder is found to be transitive and this may be used to order all states of systems so that any state will give up heat only to states which are in lower positions on the list. The property determining position on this list is temperature and the hotter state is said to possess the higher temperature.

Again, each thermodynamic system must be capable of coming to thermal equilibrium with another system; that is, it must possess the property of thermal stability. This means that, if two systems at different temperatures exchange heat, the result must be to reduce the temperature difference between them. It follows from the First Law that if, in a process during which no work is done, heat flows from a hotter to a cooler system, the internal energy of the cooler system will increase while that of the hotter system will decrease. These changes must correspond to a warming up of the cooler system and a cooling down of the hotter system. This in turn implies that the temperature of each system must be a monotonically increasing function of the internal energy; that is

$$\left(\frac{\partial T}{\partial U}\right)_{W=0} > 0, \tag{1}$$

where T and U represent temperature and internal energy respectively and $W = 0$ means that no work is done during the process.

The entropy S of a system may be written as a function $S(U, X_1, X_2, \ldots)$ of the internal energy U and the deformation (or work) variables $X_1, X_2, \ldots$ Now, since

$$\left(\frac{\partial^2 S}{\partial U^2}\right)_{X_i} = -\frac{1}{T^2}\left(\frac{\partial T}{\partial U}\right)_{X_i} \tag{2}$$

where X_i indicates that all the X_i are held constant for these partial differentiations, the above criterion for thermal stability [3] implies that the curve of S against U is concave. Hence, if a system is capable of achieving both positive and negative temperatures, the equilibrium curve of S as a function of U will possess a maximum and, for values of the internal energy less than that for which the maximum occurs, the temperature, given by

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{X_i} \tag{3}$$

is seen to be positive; while for those greater than that for which the maximum occurs, the temperature is negative. It is seen immediately that an equilibrium state of negative temperature has a higher internal energy than an equilibrium state of positive temperature at the same value of the entropy and work variables. Hence, in order to preserve the property of temperature being a monotonically increasing function of the internal energy, negative temperatures must be higher than positive temperatures.

This latter point was emphasised first by Ramsey [4] who pointed out that, due to the form of the entropy curve discussed above for systems which exhibit both positive and negative temperatures, it follows that, in cooling from negative to positive temperatures, such a system passes through infinite temperature and *not* through absolute zero. He also drew attention to the fact that the negative temperature cooling curves produced experimentally by Purcell and Pound [5] support this view. All this adds undeniable support to the views expressed in the earlier communication [1] pointing out the grave and misleading errors in a relatively recent article and textbook.

Hence, as has been known for some years now, the fundamental theory covering systems which can exhibit negative temperatures is well-established and, most importantly, is supported by actual experiment. However, this does raise at least one big question:

Considering the foregoing discussion, where does that leave entropies of the form

$$S = kU^2,$$

which is supposedly the form of the entropy for a Schwarzschild black hole?

For a system having such an entropy, the thermal stability criterion (1) is violated, (it might be noted that two such systems could only come to thermal equilibrium with one another if heat flowed from the cooler to the hotter system, in direct violation of the Second Law of Thermodynamics; the curve of S plotted against U is not concave but is convex; and, most importantly in the present context, negative temperatures for such a system would appear to be lower than positive temperatures on a scale running from -0^{o}K to ∞^{o}K to +0^{o}K. This latter point is extremely important since the temperature scale to which attention was drawn initially by the work of Ramsey, Purcell and Pound in the 1950's is well-established both theoretically *and* experimentally and obviously it is impossible to have two totally different temperature scales existing in nature simultaneously. Therefore, it would appear that something is radically wrong with the presently accepted expression for the entropy of a Schwarzschild black hole.

Of course, it should be remembered that this note draws attention once again to one major objection to the popularly accepted Bekenstein-Hawking expression for the entropy of a black hole; there are many other objections which have been voiced by such as Stephen Crothers, Bernard Lavenda and myself both on this site and in several prestigious academic publications to both this expression and, possibly more importantly, to the whole idea of a black hole both as both a theoretical and physical entity. In view of the recent publications alluded to earlier [2] as well as these comments, it is to be hoped that this whole question will be examined afresh with scientifically open minds as a matter of genuine urgency. It might be noted also that the existence of black holes is central to many explanations of observations which rely on knowledge of what lies at the centres of galaxies. The notion that black holes are at the centres of galaxies has become accepted as almost a fact by many, including the general public. However, other feasible explanations for the relevant observations do exist as may be seen by any who check up on ideas of a plasma universe or, alternatively, an electric universe. Bearing in mind the reliance on experiment to support ideas of negative temperatures, it is worth realising that much of the work covered by these latter two theories is backed up by a wealth of experimental and observational evidence going back at least to the work of Kristian Birkeland at the beginning of the last century.

References.

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