Scattering amplitudes in positive Grassmannian: TGD perspective

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Abstract

A generalization of twistor Grassmannian approach defines a very promising vision about the construction of generalized Feynman diagrams. Since particles are replaced with 3-D surfaces and since string like objects emerge naturally in TGD framework, one expects that scattering amplitudes define a generalization of twistor Grassmannian amplitudes with a generalized Yangian symmetry. The realization of this approach has been however plagued by long-standing problems. SUSY in some form seems to be strongly supported by theoretical elegance and TGD indeed suggests a good candidate for a broken SUSY realized in terms of covariantly constant right-handed neutrino and not requiring Majorana spinors. Separate conservation of baryon and lepton number imply that super-generators carry quark or lepton number. This has been the main obstacle in attempts to construct stringly amplitudes. In this article it is found that this obstacles can be overcome and that stringy approach is forced both by the TGD view about physical particles and by the cancellation of UV and IR divergences. Also the planarity restriction emerges automatically in stringy approach. Absolutely essential ingredient is that fundamental fermions can be regarded as massless on-shell fermions having non-physical helicity with propagator replaced with its inverse: this representation follows by perfoming the integration over the virtual four-momentum squared using residue calculus.

1 Introduction

The quite recent but not yet published proposal of Arkani-Hamed and his former student Trnka has gained a lot of attention. There is a popular article in Quanta Magazine about their work at https://www.simonsfoundation.org/quanta/20130917-a-jewel-at-the- There is a video talk by Jaroslav Trnka about positive Grassmannian (the topic is actually touched at the end of the talk but it gives an excellent view about the situation) at http://www.maths.dur.ac.uk/events/Meetings/LMS/2013/PNTPP13/talks/0438trnka.pdf [B4] and a video talk of Nima Arkani-Hamed at http://susy2013.ictp.it/video/05_Friday/2013_08_30_Arkani-Hamed_4-3.html# [B2]. One can also find the slides of Trnka at `http://www.staff.science.uu.nl/~tonge105/igst13/Trnka.pdf` [B4]. For beginners like me the article of Henriette Envang and Yu-tin Huang serves as an enjoyable concretization of the general ideas [B6].

The basic claim is that the construction of Grassmannian amplitudes reduces to the calculation of volumes of positive Grassmannians determined by external particle data and realized as polytopes in Grassmannians such that their facets correspond to logarithmic singularities of a volume form in one-one correspondence with the singularities of the scattering amplitude. Furthermore, the factorization of the scattering amplitude at singularities corresponds to the singularities at facets. Scattering amplitudes would characterize therefore purely geometric objects. The crucial Yangian symmetry would correspond to diffeomorphisms preserving the positivity property. Unitarity and locality would be implied by the volume interpretation. Nima concludes that unitarity and locality, gauge symmetries, space-time, and even quantum mechanics emerge. One can however quite well argue that its the positive Grassmannian property and volume interpretation which emerge. In particular, the existence of twistor structure possible in Minkowskian signature only in M^4 is absolutely crucial for the beautiful outcome, which certainly can mean a revolution as far as calculational techniques are considered and certainly the new view about perturbation theory should be important also in TGD framework. A further nice finding is that CP_2 is the only Euclidian manifold with twistor space which is also Kähler manifold. Therefore the mathematical framework of TGD looks completely unique. Understanding the role of CP_2 twistors is, remains a fascinating challenge.

The talks inspired the consideration of the possible Grassmannian formulation in TGD framework in more detail and to ask whether positivity might have some deeper meaning in TGD framework.

1. The generalization of the BCFW recursion relation using 4-fermion vertex with fermions of internal lines massless in real sense and having unphysical helicity suggests that all loops in Feynman sense vanish and only tree diagrams remain. This would simplify enormously the analog of BCFW construction and would allow to circumvent the restrictions due to the planarity since non-planar diagrams correspond to trees and non-planar diagrams would be obtained by permutations of external particles. Unfortunately this does not work: by the argument implying cancellation of loops involving SUSY also the bosonic wormhole throat propagator should vanish!

   The problem can be circumvented by starting directly from stringy diagrams forced also by the Kähler magnetic charge of wormhole throats and localization of fermions to string world sheets (covariantly constant right handed neutrinos being an exception). BCFW construction generalizes to stringy objects at the formal level at least and cuts are now performed for string world sheets.

2. SUSY in some form seems to be strongly supported by theoretical elegance and TGD indeed suggests a good candidate for a broken SUSY realized in terms of covariantly constant right-handed neutrino and not requiring Majorana spinors. Separate conservation of baryon and lepton number imply that super-generators carry quark or lepton number. This has been the main obstacle in the attempts to construct stringy amplitudes. It will be found that this obstacle can be overcome and that stringy approach is forced both by the TGD view about physical

particles and by the cancellation of UV and IR divergences. Also the planarity restriction emerges automatically in stringy approach. The absolutely essential ingredient is that fundamental fermions can be regarded as massless on-shell fermions having non-physical helicity with propagator replaced with its inverse: this representation follows by performing the integration over the virtual four-momentum squared using residue calculus.

3. The generalization to gravitational sector is not a problem in sub-manifold gravity since M^4 - the only space-time geometry with Minkowski signature allowing twistor structure - appears as a Cartesian factor of the imbedding space. A further finding is that CP_2 and S^4 are the only Euclidian 4-manifolds allowing twistor space with Kähler structure. Since S^4 does not allow Kähler structure, CP_2 is completely unique just like M^4. The analog of twistorial construction in CP_2 degrees of freedom based on the notion of flag manifold and geometric quantization can be considered.

4. In $\mathcal{N} = 4$ theory gauge coupling evolution is trivial and this might be forced also in TGD framework by the analog of $\mathcal{N} = 4$ SUSY. The triviality of coupling constant evolution could be seen as a problem in standard QFT framework but discrete p-adic coupling constant evolution with local RG invariance could resolve the problem: this would give very profound role for the p-adicity. Whether this really happens remains an open question.

5. As both Arkani-Hamed and Trnka state "everything is positive". This is highly interesting since p-adicization involves canonical identification which is well defined only for non-negative reals without further assumptions! This raises the conjecture that positivity is necessary to achieve number theoretical universality.

2 About the definition of positive Grassmannian

The lecture of Trnka [B5, B4] and the earlier article by Arkani-Hamed et al [B7] give an excellent view about positive Grassmannians. The lectures of Postikov (`http://www-math.mit.edu/~ahmorales/18.318lecs/lectures.pdf`) provide a more detailed mathematical summary [B3]. Essentially convex polytopes of Grassmannian are in question.

1. The starting point is triangle in plane. Its interior points can be defined as center of mass coordinates for a system containing masses at the vertices Z^i of the triangle. As the non-negative masses vary over all possible values one obtains points of the triangle. The generalization to the case of projective plane is obvious. Definition generalizes to n-simplexes defined by $n+1$ points. In three dimensions the construction gives the interior points of tetrahedron. A further generalization of trianges is from projective spaces $G(1,n)$ to projective spaces $G(k,n)$. Now the positivity condition for masses guaranteeing interior point property generalizes to the conditions that all minors defined by the k colmes of the $k \times n$ matrix defining point of $G(k,n)$ are positive. The resulting convex polytope is called $G_+(k,n)$.

2. In the case of projective plane one can also consider convex polygons with $n > 3$ sides. Convexity requires that the minors of the $3 \times n$ matrix (3 is the number of projective coordinates) are positive. Also this construction generalizes to $G(k,n)$.

The positivity makes sense only for real Grassmannian and if the scattering amplitude is volume in strict sense it is real. This cannot make sense in the general case. I have got the impression that the positivity condition generalizes also to the complex case. In the case of $(1,1,-1,-1)$ signature twistors are real: does the positive Grassmannian makes sense in this case and allows to perform calculations and identify scattering amplitude essentially as the volume of a convex polytope and analytically continue the result to the Minkowskian signature.

The scattering amplitude would be the volume for a convex polygon defining a positive Grassmannian. Feynman diagrams and BCFW defines triangulation of this polygon and perturbative calculation of the volume by adding volumes of the parts of the triangulation. The integration measure is defined in the standard representation of scattering amplitudes linearizing the momentum conservation constraint expressed in terms of twistors using the coordinates $C_{\alpha a}$ of $G(k,n)$. Integration measure and the $k \times k$ minors of $k \times n$ matrix representing the point of $G(k,n)$ can be expressed in terms of so called face variables f_i. This allows to express integration measure as product $\prod_i dlog(f_i)$. The integration measure has logarithmic singularities at the facets of the convex polygon defined by the external momenta and helicities. The face variables associated with the loop interiors give a multiplicative factor whereas the integral over the other face variables gives what corresponds to the scattering amplitude as a function of external momenta and helicities. For other than $\mathcal{N} = 4$ theories UV singularities are in the interior of the convex polygon.

3 The recent TGD based view about BCFW construction of scattering amplitudes

What could be the counterpart of BCFW construction in TGD framework? The following view is the latest one and differs from the first guess in that QFT type BCFW is replaced by its stringy variant. Views are still fluctuating wildly.

1. The first task is to define precisely what on-mass-shell and off-mass-shell properties mean. On-mass-shell property for external fermion means that the line is massless and has physical helicity. Internal fermions are also massless but have non-physical helicity. Hence the line containing the inverse of the massless fermion propagator after residue integration over p^2 does not vanish. I have described earlier how pair of fermion and anti-fermion at opposite throats of wormhole contact give rise to effective boson exchanges with space-like momentum (the sign of energy of internal fermion line can be negative). Contrary to the first beliefs, the consideration of the microscopic details of propagation cannot be avoided and their consideration forces stringy variant of BCFW.

2. For 3-vertex of SYM momentum conservation forces the momenta to be parallel. All loop corrections in the sense of Feynman graphs vanish reflecting the fact

that coupling constant renormalization is trivial. 4-fermion vertex can be non-vanishing for non-parallel momenta. Therefore the internal fermion lines can be massless in real sense rather than in only in complex sense as in the case of SYM.

3. Bosonic emergence suggests an additional constraint on diagrams. At least one pair of lines in 4-fermion vertex corresponds to the opposite throats of bosonic wormhole contact. This would reduce the vertex effectively to BFF or BBB vertex. This option looks realistic from QFT point of view. In BCFW construction the cuts would be for bosonic lines. However, Kähler magnetic charges of wormhole throats and localization of induced spinor fields to string world sheets force to accept strings as fundamental objects.

   One might hope that one could at QFT limit neglect the second end of string carrying only neutrino pair neutralizing weak isospin. This hope seems to be unrealistic. It turns out that bosonic wormhole contact propagator diverges in absence SUSY and vanishes if SUSY applies separately to wormhole contacts rather than only to a string like object having wormhole contacts at its "ends". Hence the stringy generalization of twistor Grassmannian approach seems unavoidable unless one is ready to assume that SUSY is broken in long scales and to eliminate the logarithmic divergences appearing already in the emergent gauge boson propagators by using CP_2 mass scale as cutoff scale.

4. The original dream about the cancellation of all loops in QFT sense turned out to be unrealistic and has reincarnated as a dream about the cancellation of stringy loops, and might be equally unrealistic. The idea about discrete p-adic coupling constant evolution with local renormalisation group invariance is however too beautiful to be thrown to the paper basket, and one can hope that stringy BCFW could realise it.

5. Whether SUSY is present or not has been a long standing open question. The argument below relying on the properties of the modified gamma matrices and the special properties of right-handed neutrinos suggests that SUSY emerges from strong gravitation in space-time regions with Euclidian signature - that is inside CP_2 type vacuum extremals defining the lines of generalized Feynman diagrams. SUSY would be broken at very high mass scale - perhaps CP_2 mass scale by a mechanism provided by p-adic thermodynamics.

6. One must consider also CP_2 degrees of freedom. In long length scales one expects that QFT type description of color as spin-like quantum number is a good approximation. In short length scales this cannot be the case. The optimistic guess would be that the construction of the scattering amplitude factorizes so that for a given tree amplitude M^4 and CP_2 degrees can be treated separately and that for a given diagram one obtains just a product of M^4 and CP_2 contributions. The Grassmannian approach following from the momentum conservation constraint is not expected to apply in CP_2 degrees of freedom. If I^3 and Y conservation corresponds to a geometric constraint in F, the question is what happens in vertices.

4 SUSY or no SUSY?

SUSY is the basic poorly understood aspect of TGD. Mathematically SUSY is certainly an extremely attractive idea byt naive physical arguments do not support it.

1. Do covariantly right-handed neutrino and its antiparticle assign to fermions with helicity 1/2 and -1/2 SUSY multiplet with 4 members as mathematical elegance would suggest? Naive physical intuition suggests that the decoupling of right-handed neutrino from standard model interactions implies that fermion and accompanying right-handed neutrinos behave completely independently so that it would not be possible to speak about SUSY multiplets.

   One can of course build SUSY multiplets but SUSY would be badly broken: the spartners of fermions behave just like the fermions with respect to standard model interactions so that it would not be possible to distinguish between fermion and its spartners experimentally? This would lead to contradictions with experimental facts since the number of spartners would appear as degeneracy factor in annihilation rates and in number densities in thermodynamics (say density of photons and photon energy in blackbody radiation). Something clearly goes wrong in this argument.

2. Situation is not so gloomy actually. There *is* a coupling between different right- and left handed neutrinos coming from modified gamma matrices which are superpositions of M^4 and CP_2 gamma matrices but the physical interpretation of this coupling has remained open. CP_2 parts of the modified gamma matrices couple the right-handed neutrino to left-handed one and make it possible to talk about massivation of neutrinos.

   This coupling can be classified as gravitational coupling and is extremely small for space-time sheets with Minkowskian signature unless gravitational fields are very strong and the induced metric is very near to Euclidian. For CP_2 type vacuum extremals with Euclidian signature of the induced metric and assigned with the lines of generalized Feynman diagrams the situation is totally different. This would support the idea about separate SUSY multiplets associated with different fermion helicities makes sense in shor enough length scales. The response of spartners to standard model interactions with their entire spin could follow from this coupling. Strong gravitation would generates SUSY dynamically as an ultra-short distance phenomenon. p-adic thermodynamics with different p-adic length scale for members of SUSY multiplet. It would not be terribly surprising if the p-adic length scale for spartners would be rather short so that they would be very massive having mass of order CP_2 mass.

3. The following argument provides additional support for this interpretation. Covariantly constant right-handed neutrinos are associated with entire space-time sheets whereas other fermions are localized at string world sheets. For Minkowskian space-time sheets of macroscopic size right-handed neutrinos are for all practical purposes absent since the macroscopic quantum state has just four SUSY partners having practically no interactions with the state itself! For CP_2 type vacuum externals with strong coupling between left and right-handed neutrinos situation

changes and has important implications already for geneneralized Feynman diagrams identified as stringy diagrams.

5 Bosonic emergence

The Feynman diagrammatics involving only four-fermion vertex with constant value L^2 of the coupling constant strength but no additional assumptions (assumption about bosonic wormhole contacts) looks un-realistic.

1. Dimensional coupling of length squared allows to expect divergences and non-renormalizability. A possible manner to save the situation could be that L^2 corresponds to the square of p-adic length scale L_p determined by the momentum squared assignable to the bosonic wormhole contact.

2. Bosonic emergence requires that standard massless bosonic propagator proportional to $1/p^2$ emerges from fermion loop when combined with vertex factor depending on bosonic line only. If the dimensional coupling L^2 is constant, this is certainly not the case.

3. It is also highly questionable whether it is possible to obtain the analogs of space-like boson exchanges using only four-fermion vertex and tree diagrams even if one allows negative energies. Rather, the theory would look like that of weak interactions with very large weak boson mass.

Bosonic emergence [K2] is one of the basic ideas of TGD approach and means the identification of the basic building blocks of gauge bosons and gravitons as wormhole contacts having fermion and antifermion at their boundaries.

1. Wormhole contacts behave like particles: if the second throat is empty, one has fermion and if the throats carry fermion and antifermion, one has boson. 4-vertices would reduce effectively to 3-vertex with 2 fermionic or bosonic lines and 1 bosonic line and 2-vertex with 2 bosonic vertices. The latter would have interpretation as a mass insertion expected to lead to wave function renormalization of boson propagator.

2. This picture could also resolve the problem created by dimensional coupling constant L^2. BFF coupling would reduce naturally to a product of three factors: Kähler coupling constant, coupling matrix dictated by gauge symmetry and quantum numbers of fermions and boson, and dimensional factor $1/p^2$ replacing L^2: here p is the momentum associated with the wormhole contact corresponding to gauge boson. This identification is indeed possible since wormhole contact property distinguishes bosonic line uniquely for BFF. BBB coupling would involve the product of three bosonic propagators in vertex and BBB cases. Possible BB vertex would have $1/p^2$ factor in vertex.

3. The only vertices would be BFF, BBB, and BB vertices: in BCFW construction these vertices are indeed enough since B^4 vertex of gauge theories is a consequence of off-mass shell gauge invariance and does not appear for on mass shell

amplitudes. In graviton scattering infinite number of higher vertices are consequences of general coordinate invariance and BCFW construction is proposed to yield planar tree diagrams at least.

The basic objection is that bosonic emergence in this form neglects the stringy character of physical particles and cannot work as such. The following arguments show that this anticipation is correct.

6 About BCFW construction of scattering amplitudes

In the fundamental stringy description one can identify string world sheets as loci of the induced spinor fields solving the modified Dirac equation. The condition that electric charge for the spinor modes have a well-defined electric charge - despite the fact that projection of the vielbein connection of CP_2 to space-time surface defines classical electroweak gauge fields having also charged part - forces this [K7].

In the higher level description all fundamental fermions (not the elementary particles) are assumed to be on mass shell fermions in the sense that momenta are light-like. This corresponds to on mass shell property for modified Dirac equation at the microscopic level. In internal lines the fermions must have non-physical helicity since internal line contains the inverse of the Dirac propagator. This gives dimensions correctly when integration is allowed only over light-like momenta. This form can be also interpreted as outcome of residue integration over 4-momenta with massless fermion propagator so that an ad hoc assumption is not in question. Physical fermions and bosons are bound states of massless fundamental fermions and involve pairs of wormhole contacts and a Kähler magnetic monopole flux forming a closed flux loop.

This description leads to either QFT type description or to stringy description at imbedding space level. Both could rely on twistors if both real and virtual fermionic lines have light-like momenta. Hence one would have either QFT type or stringy type generalization of BCFW recursion.

For both options the two 3-vertices of SYM corresponding to $k = 1$ and $k = 2$ negative helicity gauge bosons (black and white) are replaced at microscopic level with fermionic 4-vertex with 2 positive and 2 negative helicities. One cannot assign any color to the vertex since one has 2 positive and 2 negative helicities. For 4-vertex kinematics allows the light-like momenta to be non-parallel and the vertex is not singular. The microscopic description of 4-fermion vertex in terms of the geometry of wormhole contact and its deformations was considered already earlier. For effective 3-vertex the bosonic state represented as wormhole contact is off mass shell and the ordinary and four-momentum conservation forces all four-momenta to be parallel if they are on mass shell and real.

6.1 Is QFT type BCFW construction possible in TGD framework?

Is QFT type BCFW construction neglecting the stringy character of physical particles possible in TGD framework? We have already developed arguments suggesting that

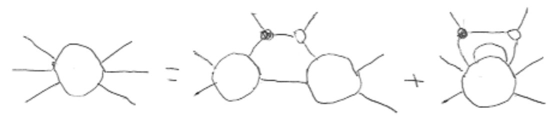

Figure 1: BCFW recursion relation in $\mathcal{N}=4$ Grassmannian construction of scattering amplitudes

this approach fails but the best manner to learn more is to try and see.

1. The obvious manner to proceed is just as in the case of BCFW construction. Unitary cuts would correspond naturally to bosonic wormhole contacts and the two 3-vertices (BFF and BBB) of SYM corresponding to $k=1$ and $k=2$ negative helicity gauge bosons (black and white) represented at microscopic level with fermionic 4-vertex with 2 positive and 2 negative helicities. One cannot assign any color (black white) to the 4-fermion vertex since one has 2 positive and 2 negative helicities. For 4-vertex kinematics allows the light-like momenta to be non-parallel and the vertex is not singular. The microscopic description of 4-fermion vertex in terms of the geometry of wormhole contact and its deformations was considered already earlier.

   Effective 3-vertices co-emerge with bosons identified wormhole contacts formed from fermion-antifermion pairs and one obtains BFF and BBB vertices as in gauge theories. Virtual bosons are in general off mass shell although the fermion and antifermion composing them are massless and on shell but with non-physical helicity. Four-fermion coupling constant is by dimensional considerations proportional to either $1/p^2$ or the p-adic length scale L_p^2 assignable to p^2. For effective 3-vertex the bosonic state represented as wormhole contact is off mass shell and the ordinary four-momentum conservation forces all three four-momenta to be parallel if they are real and on masss shell.

2. The complexification of momenta would be carried exactly as in the case of gauge theories, and would bring in complex number z as a deformation parameter. By expressing the amplitude $A(z=0)$ as a residue integral of the integral $\oint A(z)/z$ one would obtain sum over residues at poles outside origin and identifiable in terms of massless but complex virtual momenta for the bosons at the cut lines. The bosonic propagator in the cut would be the real momentum squared which does not vanish. What is not clear whether the pole at infinity cancels as in $\mathcal{N}=4$ SUSY. One might hope that right-handed neutrino might allow to achieve this. If so, the recursion formula generalizes also for the planar loop diagrams. How to treat the non-planar situation remains a problem unless one assumes the vanishing of loops.

3. For BCFW diagrams the notion of move is essential. There are two basic moves for BCFW diagrams of SYM. Square move replaces in BCFW square diagram black 3-vertices with white and vice versa. In TGD framework square move does

not make sense. Merge expand is second move and replaces BCFW tree diagram analogous to exchange in s-channel with an exchange in t-channel: the colors of the two vertices are same. In TGD framework there BFF and BBB vertices allow the analog of this move. In SYM context moves eliminate a large number of BCFW diagrams.

4. The vision about the reduction of continuous coupling constant evolution to discrete p-adic coupling constant evolution suggests that radiative corrections cold vanish identically due to the SUSY and that the convergence of the theory requires p-adic coupling constant evolution for four-fermion coupling $L^2 \propto L_p^2$. There are some arguments in favour of the vanishing of the loop corrections.

   The breaking of conformal invariance and SUSY takes place *only for the external states* identified as bound states of fermions (via the selection of the p-adic length scale) whereas internal fermion lines remain massless. Therefore the contributions of states and their spartners could cancel each other in self-energy and vertex corrections by the analogy with $calN = 4$ theory. Indeed, if these particular loop corrections are finite they must vanish since their is no scale parameter necessary to construct dimensionless variables from momenta appearing in the correction.

   If this argument generalizes to all loop corrections, BCFW would reduce to that for tree diagrams and non-planar diagrams would not produce any troubles. The objection is that p-adic length scale defines a dimensional parameter. If it appears only in the construction of massive external states as bound states of massless fermions as p-adic thermodynamics suggests, this objection does not bite. Note also that the massivation of external states would resolve the infrared divergences by bringing in natural infrared cutoff as p-adic mass scale.

All this looks good as long as one believes that one can forget the stringy character of the physical particles requiring that the propagation of both wormhole contacts is taken into account and that the fermionic loop defining normalization of the bosonic wormhole contact propagator is finite and non-vanishing. Unfortunately this does not seem to be the case.

6.2 Problem: wormhole fermionic loop diverges in absence of SUSY and vanishes in presence of SUSY!

The fermion loop assignable between two wormhole contacts is essential in the identification of the bosonic wormhole contact propagator. This loop must be by dimensional considerations proportional to p^2, where $p = p_1 + p_2$ is the total momentum of the propagating wormhole contact defined as sum of massless fermion momenta. The questions are following.

Is the resulting number is finite and non-vanishing when stringy character of elementary particles is neglected? Or is finiteness achieved only by integrating simultaneously over both virtual momenta associated with the ends of the string with vertex factor correlating the momenta at the wormhole contacts at the ends of the string?

The original naive expectation was that the light-likeness constraint could make the loop finite: unfortunately SUSY could imply its vanishing! On the other hand, the experience with QFT and string models suggests that strings are necessary and the following arguments support this expectation.

1. The loop integral is defined by performing residue integral over mass squared reducing integration to that over massless momenta for each fermion line restricted by the momentum conservation constraints in various vertices. Since integration measures $d^3p_i/2E_{p_i}$ give massdimension 4, momentum conservation delta function has mass dimension -4, and there are two inverses of massless fermion propagator, the over-all integral has mass dimension 2. A p^2 factor however factors from the fermionic trance and fermionic loop reduces to $p^2 \times \int_{S^2}(E_1/E_2)d\Omega$, where the ratio E_1/E_2 for the energies for fermions depends on angle. Since p^2 can be space-like also negative energies must be allowed. The absolute value of energy must appear in $d^3p_i/2E_{p_i}$ so that the integration measure is positive definite. Singularities of the integrand result as E_1 approaches infinite or E_2 approaches zero and this is possible when p^2 is light-like or space-like. Logarithmic singularities are expected.

2. If the loop is convergent without cutoff, the resulting integral is by dimensional considerations proportional to p^2, and one obtains the standard form of the bosonic propagator if the BFF vertex is proportional to $1/p^2$ (it could be also proportional to the square L_p^2 of the p-adic length scale assignable to p^2). Note that the invariant $p_1 \cdot p_2$ equals to $p^2/2$ so that one cannot and to the vertex dimensionless Lorentz invariants possibly guaranteeing the finiteness of the integral.

Two catastropic events could happen.

1. Formally this integral is just the ordinary diverging fermionic loop encountered in massless gauge theory. Optimistic could argue that just by the divergent character of the loop in the ordinary approach, one could achieve a finite result without posing a cutoff: the residue integral description might be seen as regularization procedure. If the integral divergences, a physical regularization involving the introduction of a p-adic cutoff momentum having interpretation in terms of measurement resolution - lower limit for the size of CDs involved- could give rise to logarithmic factors $log(p^2L_p^2/h_{eff}^2)$. This is very natural expectation in the approach based on QFT. One however want something more elegant than QFT.

2. There is also another catastrophe lurking there. The supersymmetry induced by the possibility to have spartners of fermions in the loop corresponding to 4 states constructed from covariantly constant ν_R and its charge conjugate $\overline{\nu}_R$ would most naturally imply that the loops sum up to zero! This result holds completely generally if virtual fermions are massless.

We are clearly sailing between Scylla and Charybdis!

6.3 Stringy variant of BCFW construction

Suppose that the doomsday scenario for QFT type BCFW is realized: the basic fermionic loop diverges without SUSY and vanishes for SUSY. How to proceed? First of all, one must remember that one is basically constructing zero energy states rather than scattering matrix between positive energy states. Hence the only rule to be obeyed is that the zero energy state is well-defined mathematically and therefore free of divergences.

1. It is physically completely natural and in harmony with the vision about finite measurement resolution that CP_2 length scale or some p-adic length scale would define a momentum cutoff. The challenge is to formulate the cutoff in an elegant manner as a restriction on the momentum squared of the wormhole contact propagator emerging spontaneously rather than being put in by hand. The natural assumption is that this cutoff applies only to positive and negative energy parts of the zero energy states and not on propagators and vertices.

2. In string theory one can avoid infinities and this suggests the introduction of the stringy description from the beginning as required also by the fact wormhole contacts carry Kähler magnetic charges.

    (a) The standard stringy approach would be stringy perturbation theory based on super-conformal algebra. This would bring in CP_2 scale and perhaps also the hierarchy of p-adic length scales defining the mass scale of conformal excitations. In TGD framework however the fact that the fermionic generators of the super-conformal algebra carry fermion number seems to produce insurmountable difficulties in this approach.

    (b) The natural constraint is that p-adic length scale is associated only with the positive and negative energy parts of the states and does not affect at all the online massless propagation of fermions. This suggests a fresh approach to strings based on twistors and Grassmannians. Virtual fundamental fermions remain massless but form only basic building bricks of real and virtual particles identified as pair sof wormhole contacts. For physical particles both fermionic and bosonic propagator lines are replaced by pairs of string world sheets with wormhole contacts at their ends. Also hadronic strings result in this manner. Stringy structure implies the breaking of the generalization of the 2-D conformal invariance and the fact that covariantly constant right-handed neutrinos are associated with entire space-time sheet implies SUSY breaking. Hence stringy propagators for elementary particles can be finite and non-vanishing.

The rough vision about stringy diagrammatics and its BCFW variant would be following.

1. To avoid confusion, it should be made clear that one has three kinds of lines to consider.

    (a) Fundamental fermion lines. These are assigned to wormhole throats and accompanied by massless fermion propagators. After residue integration over p^2 they give rise to inverses $p^k\gamma_k$ of the massless Dirac propagator estimated on shell and non-vanishing only when the fundamental fermion has non-physical helicity. This micro-anatomy is not present in string models and expresses the idea that all particle states emerge from massless on mass shell fermions making in turn possible to express the momenta of wormhole contacts and of string like object itself in terms of twistors. This gives hope about BCFW recursion with cuts defined for fermionic and bosonic strings. Also Grassmannian formulation might make sense since it results from momentum conservation for states decomposing into many particle states carrying massless momenta.

(b) Wormhole contact lines. In the bosonic case these contain inverses $p^k\gamma_k$ of massless fermion propagators at the two fundamental fermion lines. The ends of the wormhole contact line contain the generalization of bosonic propagator $1/p_i^2$ to $1/L_{0,i}$ as vertex factors at the four-fermion vertices at its ends. Fermionic wormhole contact line involves the super generator G and its hermitian conjugate $G^\dagger$ at the 4-fermion vertices at the ends of the line. This boils down to the general assumption that each fermion line in the 4-fermion vertex contains $1/G$ or $1/G^\dagger$. For bosonic wormhole contacts this gives $1/GG^\dagger = 1/L_0$ as vertex factor. These replacements bring in the dependence on CP_2 length scale defining physical UV cufoff. Note that p-adic thermodynamics is associated with external lines only. The earlier proposal [K1] is that Neveu-Schwartz *resp.* Ramond representation of Super Virasoro algebra occurs for quarks *resp.* leptons.

The problem in understanding stringy diagrammatics in TGD framework has been that G and $G^\dagger$ carry fermion number rather than being hermitian operators as in superstring model relying on the Majorana property of spinors. The solution of the problem emerges from the that that in the recent approach the ends of fermion number carrying wormhole contact contains $1/G$ and $1/G^\dagger$ respectively so that at the low energy limit one obtains just ordinary Dirac propagator.

(c) Stringy lines. Stringy propagator for a physical particle is obtained by integrating over light-like momenta of the fermionic lines. Correlations between the momentum integrations follow only from the momentum conserving delta function.

The first difference with respect to string models is that massless fermions are fundamental and strings are emergent, and also physical particles are string like objects. Second difference is that the super generator G carries fermion number. Third difference is on mass shell light-likeness of fundamental fermions giving hopes about the applicability of twistor Grassmannian formalism.

2. The momentum conservation constraint for the string like object makes vertices non-local in the scale of string. Stringy emergence allows only this kind of non-locality. One could of course consider also a more general non-locality. Stringy vertex could contain a dependence on the invariants constructed from the light-like fermionic momenta p_i at the ends of the string. These invariants correspond to dimensionless invariants X_{ij}/X_{kl}, $X_{ij} = p_i \cdot p_j$ and $p^2 = (\sum p_i)^2$. If the first wormhole contact carries only fermion and second wormhole contact a neutrino-antineutrino pair neutralizing weak isospin, one obtains 3 inner products and

three dimensionless invariants. If both ends correspond to bosons one obtains 4 inner products affecting the stringy loop integral.

3. As explained, bosonic vertex factors $1/p_i^2$ are replaced with the Virasoro generators $1/L_{0,i}$. In the case of of fermionic lines single particle super Virasoro generator $1/G_i$ defines the analog of the inverse of the Dirac operator $p_i^k \gamma_k$ at the level of "world of classical worlds" (WCW). There is however a problem here.

   (a) If fermionic wormhole contacts carry momentum only at the second throat, they are massless and the dependence of G on four-momentum disappears completely since it reduces to the sum of CP_2 part and "vibrational" part. p-Adic mass calculations however suggest that also the second throat must carry massless four-momentum.

   (b) The most obvious manner to overcome the problem relates to the electroweak symmetry breaking requiring a pair of left- and right handed neutrinos to cancel the net weak charge in the length scale of string. One could also assume that the right-handed anti-neutrino and fermion reside at the first wormhole contact so that this state can develop mass squared by p-adic thermodynamics whereas left handed neutrino would reside at the second wormhole throat but could not develop any mass squared in this manner. The roles of ν_R and ν_L could be of course changed. Note also that the existence of ν_R modes delocalised at the entire space-time sheet of string like object does not mean the non-existence of modes localised at wormhole throats and the mixing of left- and right-handed neutrinos implied by modified gamma matrices indeed suggests this.

   (c) One can of course wonder whether this problem might be a connected with Higgs mechanism and vacuum expectation of Higgs: could fermionic wormhole contact contain Higgs or analog of coherent Higgs state? This does not seem plausible. TGD predicts Higgs like scalar particles but no Higgs vacuum expectation since p-adic thermodynamics explains massivation. The mass-proportionality of the couplings of Higgs to fermions follows form gradient coupling with same universal scalar coupling so that no problems with naturally are encountered.

4. Super conformal generators contain the dependence on CP_2 length scale so that the cutoff mass scale emerges naturally and without any ad hoc procedures. It is essential that wormhole contact propagators are correlated by momentum conservation constraint as parts of stringy propagator: this expresses non-locality in the scale of string. At low energy limit one can replace strings with points and stringy propagators with ordinary propagators. This forces to pose artificial UV cutoff in order to obtain a finite boson propagator.

5. The expressions for the stringy propagators should remain non-vanishing as one performs sums over spartners. Here one must notice that covariantly constant right-handed neutrinos are associated with the interiors of space-time sheets, and one is led to double counting if one assumes independent super-symmetries at the ends of the stringy propagator might lead to the breaking of SUSY in the

p-adic length scale in question. This is completely analogous to the reduction of rotational symmetry in two particle system to that for cm degrees of freedom. The summation over spartner combinations is possible only for stringy propagators and approximate SUSY can guarantee that they are finite and non-vanishing.

To get a more concrete view about stringy propagators it is good to look at two examples.

1. Consider a stringy diagram with bosonic wormhole contact propagator $1/L_{0,i}$ at both ends of wormhole contact orbit and reducing to $1/p_i^2$ at low energy limit. There are 4 fermionic momentum integrations d^3p_i/E_{p_i}, 4-D delta function for momentum conservation, and 4 inverse fermionic propagators: this gives contribution $\Delta D = 4 \times 2 - 4 + 4 = 8$ to the mass dimension of the integral. Bosonic emergence suggests $1/p_i^2$ factor from both ends of each bosonic propagator identifiable as low energy limit of the Virasoro generator $1/L_0$: p_i^2 *cannot* be taken outside the integral sign now so that one obtains the contribution $\Delta D = -4 \times 2 = -8$ to the overall mass dimension of the integrand. Bosonic propagator must have mass dimension -2 so that there must be an additional overall factor with mass dimension $D = -2$. This gives hopes about convergence of the integral. This additional factor could correspond to string tension identified in terms of CP_2 scale or p-adic scale. The presence of Super Virasoro generators bringing in dependence on CP_2 mass scale is expected to be crucial for the cancellation of UV divergences.

2. Similar consideration applies to the propagator of physical fermion with fermionic propagator at the first wormhole contact (idealization only) and bosonic propagator at second wormhole contact. There are 3 fermionic momentum integrations d^3p_i/E_{p_i}, 4-D delta function for momentum conservation, and 3 inverse fermionic propagators: this gives contribution $\Delta D = 3 \times 2 - 4 + 3 = 5$ to the mass dimension of the integral. $1/p_i^2$ factor from both ends of each bosonic propagator gives a contribution $\Delta D = -4$ to the overall mass dimension of the integrand. The overall contribution to the mass dimension is $\Delta D = 1$ from these sources. Fermionic propagator must have mass dimension -1 so that there must be an additional overall factor with mass dimension $D = -2$ identifiable in terms of string tension.

The overall conclusion would be that although fundamental fermions propagate as massless particles, physical particles can propagate only as string like objects as forced also by the Kähler magnetic charges of wormhole throats. Stringy propagators are finite and non-vanishing by general dimensional arguments. The QFT type BCFW works also assuming that SUSY is broken at some very high mass scale but one must introduce CP_2 scale as cutoff scale in order to obtain finite and non-vanishing bosonic propagators.

6.4 Twistorialization of gravitation by twistorial string diagrams

The twistorialization of gravitation is problem of the standard twistorialization approach since curved space-times do not allow twistor structure. In TGD framework this is not a problem. The above approach giving QFT type picture treats particles as wormhole contacts neglecting the fact that second wormhole contact must be present by the conservation of magnetic flux and absence of Dirac monopoles (magnetic flux lines are closed). The other wormhole contact carries weak quantum numbers neutralizing the weak quantum numbers of particles in the case of leptons. In the case of quarks the cancellation of Kähler (color-) magnetic charge might take place only at the level of the entire hadron. For gravitons second wormhole contact is necessary in order to obtain spin two states and this forces stringy picture.

The generalization of twistorial diagrams to twistorial string diagrams is forced by the replacement of wormhole contacts with pairs of them and connected by closed fermionic string (having pieces at separate space-time sheets) and also by the failure of the QFT type BCFW approach. Localization to string world sheets is implied by the modified Dirac equation and the requirement of well-defined em charge for spinor modes either than right-handed neutrino. The lines of Feynman diagrams are replaced by closed strings connecting two wormhole contacts along first space-time sheet and returning along the second one. Elementary particles correspond to pairs of string world sheets: for fermions second string world sheet is empty and for bosons the two string world sheets carry fermion and antifermion quantum numbers.

6.5 Could positivity be a prerequisite for number theoretical universality?

Physics as infinite-dimensional geometry of WCW ("world of classical worlds") [K3]and physics as generalized number theory [K4] are the two complementary visions about TGD. For the latter vision number theoretical universality has served as the basic guide line. It states that scattering amplitudes should make sense in both real and p-adic number fields and their algebraic extensions (and perhaps even non-algebraic but finite-dimensional extensions, say the extension obtained by adding Neper number e). This principle suggests an interpretation for the positivity of Grassmannian as a prerequisite for p-adicization [K5].

Already p-adic mass calculations [K1] forced to consider the question how to map real and p-adic numbers to each other. One can imagine two quite different manners to achieve this.

1. Direct correspondence via rationals would respect algebra and symmetries realized in terms of matrices with rational elements. It is however extremely discontinuous and not complete since p-adic integers for which the pinary expansion is infinite and not periodic do not correspond to any rational number.

2. Canonical identification- call it I - maps the pinary expansion for a positive real number to p-adic pinary expansion by just inverting the powers of p: $\sum x_n p^n \rightarrow x_n p^{-n}$. It is continuous map in both directions but maps two p-adic numbers to single real number as the p-adic generalization of $1 = .99999...$ implies. Therefore the inverse is two valued for real numbers with a finite number of pinary digits. Canonical identification respects continuity but not algebra and breaks algebraic symmetries. There is clearly a tension between symmetries and continuity.

As discussed in [K8], one can define a variant of canonical identification which is a kind of compromise between algebra and topology.

1. This variant maps positive real rationals smaller than some power p^N to itself so that symmetries are realized algebraically in finite measurement resolution. Reals with larger number of pinary digits are mapped by canonical identifying based on expansion in powers of p^N by mapping coeffiecients $0 \leq x_n < p^N$ to itself and inverting powers of p^N: $\sum x_n p^{nN} \rightarrow \sum x_n p^{-nN}$. Now continuity is respected.

2. Canonical identification in this generalized sense is used to define the notion of p-adic manifold as what might be called cognitive representation of real manifold. The inverse map define the space-time correlate for intention. The basic ideas is that chart leafs for p-adic manifold are not p-adic but real and canonical identification defines them. This allows to transfer basic notions of real topology to p-adic context. One can also define p-adic chart leafs for real manifold and they have interpretation as space-time correlates cognitive representations.

3. The condition that preferred externals of Käbler action appear at both real and p-adic sides brings in additional pinary cutoff. The preferred extremal property of Kähler action forces canonical identification since the canonical image ofa real (p-adic) space-time surface would not be differentiable in p-adic (real) sense. This requires finite pinary cutoff $M > N$ for the canonical identification. The cutoff has an interpretation as finite measurement resolution in the sense that chart maps involve only discrete set of points on both sides. The completion of discrete point set to a preferred extremal can be performed on both sides. Note that the completion need not be unique but this is of course consistent with the finite measurement resolution.

There is a problematic feature related to the canonical identification and possibly closely related to the positivity. What to do with *negative* real numbers. p-Adic -1 has representation $(p-1)(1+p+p^2+...)$ and maps by the inverse of the canonical identification to a positive real number p. Hence one cannot map real -1 to p-adic -1. Canonical identification makes sense only for non-negative reals.

1. One cannot introduce a p-adic counterpart of real -1 via algebraic extension of p-adic numbers as $-1 \equiv exp(i\pi)$ interpreted as a phase factor defining angle π and map it to real -1 since $(exp(i\pi)-1)(exp(i\pi)+1)$ would vanish and one would not have a number field anymore. Note that in p-adic context all angles $2\pi/N$ for prime values of N must be introduced via algebraic extensions of p-adics since the obvious candidates for the p-adic trigonometric functions are not periodic. This forces finite angular - or rather phase - - resolution.

2. A possible manner to cope with the situation would be to divide real axis to positive and negative half-axes and interpret reals as a 1-dimensional manifold with two coordinate charts and use positive coordinate for both so that p-adic counterpart could be defined by canonical identification. This construction generalizes to n-dimensional case in an obvious manner.

What makes this so interesting is that everything in the positive Grassmannian approach is positive as Nima and Jaroslav Trnka state it. The positivity of Grassmannian means positivity of all elements of $k \times n$ matrix and of all minors associated with the rows labelled by integers $< i < j < ....$ Also the scattering amplitude itself is positive as a volume as are also external data - at least in the signature (1,1,-1,-1). Could this be interpreted as guaranteeing number theoretical universality allowing to algebraically continue from real to p-adic context using some variant of canonical identification with a cutoff. Of course, an interesting question is what happens as one continues to other signatures.

6.6 What about CP_2 twistorialization?

CP_2 allows twistorialization in terms of 6-D flag manifold $F = SU(3)/U(1) \times U(1)$ having interpretation as a space for the choices of all possible quantization axes for color isospin and hypercharge defined by the Cartan algebra $u(1) \oplus u(1)$. Since CP_2 is the only Euclidian compact four-manifold allowing twistor space which is also Kähler manifold, one might expect that CP_2 twistorialization involves deep physics and could make TGD Universe unique. The coordinatization for the choices of quantization axes corresponds to the complex coordinates assignable to π^+, K^+, K^0 and their complex conjugates assignable to $\pi^-, K^-, \overline{K}^0$ in the octet representation of tangent space of $SU(3)$ in terms of generators with quantum numbers of mesons.

It is of course far from obvious whether twistorialization in CP_2 degrees of freedom is useful. The original argument was that twistorialization is necessary for color symmetry but this argument need not be quite correct. One might quite well consider the possibility that one has just color conservation in vertices. If this is the case the color would be present rather passively. Hence it makes sense to ask what twistorialization in CP_2 degrees of freedom could make sense and what it could mean. In particular, one can ask whether the crucial vanishing of total momentum as a constraint generalizes to the case of color quantum numbers.

1. Should one introduce the choices of color quantization axes as a moduli space assignable to the external particles and over which it might be necessary to integrate? I_3 and Y define the counterparts of momentum components and correspond to the complement of twistorial tangent space in SU(3) Lie-algebra. One might hope that incidence relations make sense for the Hamiltonians representing I_3 and Y in F as bilinears of holomorphic coordinates and their conjugates.

2. If Lie group G acts in symplectic manifold M, the so called moment map assigns to the Lie-algebra generators of G their Hamiltonians in M as inner product of Killing vector field and 1-form defining the momentum map. F is symplectic manifold because it is Kähler manifold. Rather remarkably, only the twistor spaces associated with CP_2 and S^4 are Kähler manifolds in 4-D Eucdlian case [B1] (`http://www.math.ucla.edu/~greene/YauTwister(8-9).pdf`). Furthermore, S^4 does not allow Kähler structure so that CP_2 and M^4 are completely unique! F is known to allow two non-equivalent Einstein metrics (Einstein tensor proportional to the metric tensor).

3. The vanishing of the total momentum for the diagram should have CP_2 analog and one might hope that the linearization of this constraint could lead to Grassmannian formulation. The vanishing of the sums $\sum_i Y_i$ and $\sum_i I_{3,i}$ of hyper-charge isospin Hamiltonians represent the vanishing of total quantum numbers and would select a co-dimension 2 sub-manifold in the Cartesian product of twistor spaces associated with the external particles and in this manner correlate CP_2 twistorial degrees of freedom. If the Hamiltonians Y and I_3 are bilinear in holomorphic twistor coordinates and their conjugates and therefore analogous to harmonic oscillator Hamiltonians, the constraint is quadratic and there are hopes about the analog of Grassmannian formulation obtained by linearizing the constraints. The exterior product $\prod_k^{\wedge} \omega_k \wedge J_n^2$, $\omega_i = J_i \wedge J_i \wedge J_i \equiv \wedge J_i^3$ or the symmetrization of this form would define the symplectic volume form to be used in the integration.

4. Does the notion of positive/negative helicity have any meaning in CP_2 degrees of freedom? For M^4 spinors helicity corresponds to the eigenvalues of γ_5. The eigenvalue of γ_5 in CP_2 part of the imbedding space spinor would define the notion of helicity in CP_2 degrees of freedom. These eigenvalues are correlated since their product tells whether the imbedding space chirality of spinor corresponds to quark or lepton. They are of same sign for quarks and of opposite sign for leptons (this is of course a convention only). For antiparticles the signs are opposite. Anomalous hyper-charge could play the role of helicity since it has opposite sign for fermions and anti-fermions.

7 About emergence

Nima's dream is that not only gauge symmetry (that is gauge redundancy), unitary, locality, space-time and even quantum theory emerge from their approach and claim that positivity and interpretation as scattering amplitude as volume is the fundamental principle implying even quantum theory.

I cannot agree with this. For me it is much more natural to interpret the representation of scattering amplitude as a volume as emergence forced by fundamental physical principles. Even a new fundamental principle would be involved and would be number theoretical universality involving p-adicization using canonical identification: this requires positivity unless additional assumptions are made. In any case, it is interesting to consider the emergence from TGD point of view.

1. Consider first the emergence of space-time. Twistors are present and represent four-momentum. For Minkowskian signature twistors are possible only in Minkowski space so that not only space-time but also M^4 seem to be necessary. This means a severe problem for the twistorial approach to gravitation ("googly" problem). Space-time as 4-surface in $M^4 \times CP_2$ is the elegant solution allowing twistorialization also in CP_2 degrees of freedom. Also half-odd integer spin and SUSY are involved and require M^4.

2. One can say that in TGD electroweak gauge symmetries emerge from the geometrical gauge symmetry related to the freedom to choose vielbein. Electroweak gauge group corresponds to the holonomy group of CP_2 having concrete

geometric interpretation. Global gauge transformations do not mean mere gauge redundancy. Color symmetries correspond to isometries of CP_2 and color gauge symmetry is approximate and emergent at long length scales.

3. Gauge bosons and graviton emerge in TGD as bound states of massless fundamental fermions defining the fundamental particle like excitations. Even the representations of infinite-dimensional super-conformal symmetry algebras emerge and their states are expressible as bound states of massless fermions. There are also the WCW degrees of freedom represented as Super-Kac-Moody and super-symplectic algebras in WCW and one can assign color degrees of freedom to these as well as stringy geometric degrees of freedom relevant for hadron like objects. Fermionization allows to have non-singular fundamental vertices and allows real light-likeness for internal lines.

4. In zero energy ontology (ZEO) one must introduce what I have called U-matrix having as rows M-matrices, which are products of hermitian square roots of density matrices with unitary S-matrix. Each M-matrix corresponds to an analog of S-matrix in thermal QFT and S-matrix should have the standard interpretation. Therefore the notion of unitary is generalized. Locality is definitely lost since point-like particle is replaced with 3-surface - or by strong form of holography with particle 2-surface together with its 4-D tangent space data defining the basic dynamical unit. Locality emerges at the point-like limit of the theory.

5. Yangian symmetry in $\mathcal{N} = 4$ SYM extends the conformal symmetries of M^4 and should be present also in TGD framework. Besides this there is a generalization of the Yangian symmetry with super-conformal algebras associated with partonic 2-surfaces and the integer $n = 1, 2, ..$ defining the characteristic "n-point" property of the generators of Yangian corresponds concretely to the number partonic 2-surfaces to which the Yangian generator acts. Hence the finite-dimensional conformal Lie-algebra is replaced with infinite-dimensional conformal algebras assignable with the collections of partonic 2-surfaces associated with the space-time surface.

   In the case of $N = 4$ SYM conformal Yangian corresponds to diffeomorphisms preserving the positive Grassmannian property of the polytope (intuitively clear since conformal invariance respects light-likeness). Whether also the huge Yangian associated with super-conformal symmetries acts as a symmetry of the polytope possibly associated with the scattering amplitudes in TGD framework is an open question. Certainly these scattering amplitudes must have additional symmetries if all loop corrections in Feynman sense vanish.

8 Possible problems

Consider next the possible problems of $\mathcal{N} = 4$ SYM and TGD approach assuming that the proposed conjecture makes sense.

1. Ordinary Grassmannian approach applies only to planar Feynman graphs. Stringy twistorialization and BCFW recursion is free of this problem.

2. Gravitation is problem in standard QFT approach since twistors make sense only for M^4 if Minkowskian signature is assumed. Sub-manifold gravity of TGD would resolve the problem. Twistor diagrams have a natural stringy generalization forced by internal consitsency and allowing the description of all elementary particles in similar footing.

3. The basic problem of $\mathcal{N}=4$ SYM is that there is no coupling constant evolution. For stringy BCFW one has SUSY breaking and non-vanishing loops so that the problem is probably not encountered. p-Adic coupling constant evolution is however a highly attractive notion in TGD framework. Coupling constant evolution would discretize and mass squared scales would given by inverses of primes with primes near certain powers of two favored.

   Discretization would mean that each interval between two subsequent prime corresponds to a fixed point of renormalization group. Primes or preferred primes would label the fixed points of coupling constant evolution. Also the scales of CDs could define mass scale hierarchy. No breaking of conformal symmetry and SUSY would take for internal fermion lines and these symmetries would be broken only for the external states and characterized by p-adic mass scale defining also natural IR cutoff.

4. Nima notices in his lecture that BCF equations have exactly the same form as renormalization group equations. In TGD framework the equations would indeed state the triviality of the renormalization group flow and different solutions for the condition satisfied by 4-vertex could correspond to the hierarchy of CDs, to different p-adic primes, or subset of them allowed by p-adic length scale hypothesis.

5. The connection with the notion of finite measurement resolution is interesting. Intuitively finite length scale resolution corresponds to a minimum size scale for the causal diamonds (CDs) taken into account in the generalized Feynman diagrams. In the similar manner upper size scale for CD corresponds to IR cutoff. Does the proposed description make sense only for single CD? Or should one combine different CDs somehow in the general situation? Hyperfinite factors [K6] have been proposed to describe the finite measurement resolution and the question is whether there is a hierarchy of polytopes corresponding to the hierarchy of CDs/p-adic length scales. Does the inclusion for HFFs correspond to inclusion of corresponding CDs with sub-CD defining measurement resolution?

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