HYPER-FINITE FACTORS, P-ADIC LENGTH SCALE HYPOTHESIS, AND DARK MATTER HIERARCHY

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Preface

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole contact defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy.

The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

Matti Pitkänen

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 32 years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published

result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Matti Pitkänen

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Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. The CMAP files at my homepage provide an overview about ideas and evolution of TGD and make easier to understand what TGD and its applications are about (`http://www.tgdtheory.fi/cmaphtml.html` [L20]).

1.1.1 Basic vision very briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K2].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections ofCP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and

lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogenuity in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.1.2 Two manners to see TGD and their fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A76, A46, A65, A43].

The identification of the space-time as a sub-manifold [A39, A75] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.1.3 Basic objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 p-Adic variants of space-time surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.1.5 The threads in the development of quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name 'TGD as a generalized number theory'. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K93, K71, K61, K110, K81, K109, K108, K78] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K85, K15, K65, K13, K39, K46, K49, K77, K102] are warmly recommended to the interested reader.

Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH ("world of classical worlds",WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A60, A79, A81]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory [1]. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

   M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian

[1] There are four kinds of Dirac operators in TGD. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in propagators

square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The modified gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.

5. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak

form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worldss", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that

clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

The notion of p-adic manifold [K112] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheets as chart maps provides a possible solution of the basic challenge. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, "thought bubbles"). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution. The braids actually co-emerge with string world sheets implied by the condition that em charge is well-defined for spinor modes.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K84]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.

2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K84]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained

by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

3. $M^8 - H$ duality can be generalized to a duality $H \to H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.

4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K84] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the "free" gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.

2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic) sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.

3. For gamma matrix option induced rather than modified gamma matrices must be in question since modified gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially interesting is that p-adic and real regions of the space-time surface might aso emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

1.1.6 Hierarchy of Planck constants and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E25] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K75] .

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K66, K67, K101]) support the view that dark matter might be a key player in living matter.

Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant $\hbar_0$ is assigned with a multiple singular covering of the imbedding space [K32]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invariance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n.

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of biochemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.2 Bird's eye of view about the topics of the book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

1. *Quantum classical correspondence*

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

   The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.

5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

   Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

2. Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

3. Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

4. The organization of the book

The first part of the book is devoted to hyper-finite factors and hierarchy of Planck constants.

1. Configuration space spinors indeed define a canonical example about hyper-finite factor of type II_1. The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of M-matrix. This has turned out to be the case to the extent that a general master formula for M-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

2. The idea about hierarchy of Planck constants emerged from anomalies of biology and the strange finding that planetary orbits could be regarded as Bohr orbits but with a gigantic value of Planck constant. This lead to the vision that dark matter corresponds to ordinary particles but with non-standard value of Planck constant and to a generalization of the 8-D imbedding space to a book like structure with pages partially characterized by the value of Planck constant. Using the intuition provided by the inclusions of hyper-finite factors of type II_1 one ends up to a prediction for the spectrum of Planck constants associated with M^4 and CP_2 degrees of freedom. This inspires the proposal that dark matter could be in quantum Hall like phase localized at light-like 3-surfaces with macroscopic size and behaving in many respects like black hole horizons.

1.3 Sources

The eight online books about TGD [K93, K71, K110, K81, K61, K109, K108, K78] and nine online books about TGD inspired theory of consciousness and quantum biology [K85, K15, K65, K13, K39, K46, K49, K77, K102] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://www.tgdtheory.com/curri.html`) contains a lot of material about TGD. In particular, there is summary about TGD and its applications using CMAP representation serving also as a TGD glossary [L20, L21] (see `http://www.tgdtheory.fi/cmaphtml.html` and `http://www.tgdtheory.fi/tgdglossary.pdf`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (http://journals.sfu.ca/jnonlocality/index.php/jnonlocality founded by Lian Sidorov and in *Prespacetime Journal* (http://prespacetime.com), *Journal of Consciousness Research and Exploration* (https://www.createspace.com/4185546), and *DNA Decipher Journal* (http://dnadecipher.com), all of them founded by Huping Hu. One can find the list about the articles published at http://www.tgdtheory.com/curri.html. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 Part I: Hyper-Finite Factors of Type II and Hierarchy of Planck Constants

What von Neumann Right After All?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology. In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD.

1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [?] Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum

TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the modified gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with M^8-H duality [?]his leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

4. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which modified Dirac action containing measurement interaction term defines stringy propagators. This M-matrix should be consistent with the above proposal.

5. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum

theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

6. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD since 2005. The basic idea was stimulated by the finding of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, where the velocity parameter v_0 has the approximate value $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of

the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

In astrophysics and cosmology the implications are even more dramatic. It was who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Mathematical speculations inspired by the hierarchy of Planck constants

This chapter contains the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) as separate from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first speculative question is about possible relationship between Jones inclusions of hyper-finite factors of type II_1 (hyper-finite factors are von Neuman algebras emerging naturally in TGD framework). The basic idea is that the discrete groups assignable to inclusions could correspond to discrete groups acting in the effective covering spaces of imbedding space assignable to the hierarchy of Planck constants.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

Negentropy Maximization Principle

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The proposal is that the dynamics of consciousness is governed by Negentropy Maximization Principle, which states the information content of conscious experience is maximal. The formulation of NMP is the basic topic of this chapter.

Negentropy Maximization Principle (NMP) codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies at each step a decomposition of the system to unique unentangled subsystems and the process repeats itself for these subsystems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states. The natural assumption is that self loses consciousness when it entangles via bound state entanglement.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement, which need not be bound state entanglement in standard sense. Negentropic entanglement might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of negentropic entanglement the outcome of the reduction is not random: the prediction is that second law is not a universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism. A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

The identification of life as a number theoretically critical phenomenon is also consistent with the idea that the transformation of intention to action corresponds to a U-process inducing leakage between different sectors. This leakage makes sense in the intersection where same mathematical expression defines both real and p-adic partonic 2-surfaces which are the fundamental objects in TGD framework. What these statements really mean requires a construction of number theoretical variant of quantum theory applying in the intersection of real and p-adic worlds.

Besides number theoretic negentropies there are also other new elements as compared to the earlier formulation of NMP. Zero energy ontology modifies dramatically the formulation of NMP since U-matrix acts between zero energy states and can be regarded as a collection of M-matrices, which generalize the ordinary S-matrix and define what might be called a complex square root of density matrix so that kind of a square root of thermodynamics at single particle level justifying also p-adic mass calculations based on p-adic thermodynamics is in question. The hierarchy of Planck constants is a further new element having important implications for conciousness and biology. Hyper-finite factors of type II_1 represent an additional technical complication requiring separate treatment of NMP taking into account finite measurement resolution realized in terms of inclusions of these factors.

NMP has important implications for thermodynamics. In particular, one must give up the standard view about second law and replace it with a formulation taking into accoung the hierarchy of causal diamonds assigned with zero energy ontology and dark matter hierarchy labeled partially by the values of Planck constants, as well as the effects due to negentropic entanglement. In particular, in the case of living matter breaking of second law in standard sense is expected to take place and be crucial for the understanding of evolution. Self hierarchy having the hierarchy of causal diamonds as imbedding space correlate leads naturally to a thermodynamical description of the contents of consciousness and quantum jumps is very much analogous to quantum computation. This leads to a vision about the role of bound state entanglement and negentropic entanglement in the generation of sensory qualia. Negentropic entanglement leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed. A connection with fuzzy qubits and quantum groups with negentropic entanglement is highly suggestive. The implications are highly non-trivial also for quantum computation, which allows three different variants in TGD context. The negentropic variant would correspond to conscious quantum computation like process.

1.4.2 Part II: Applications of p-adic length scale hypothesis and dark matter hierarchy

Recent status of lepto-hadron hypothesis

TGD suggests strongly the existence of lepto-hadron physics. Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of ortopositronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged. Towards the end of 2008 CDF anomaly gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

TGD and Nuclear Physics

This chapter is devoted to the possible implications of TGD for nuclear physics. In the original version of the chapter the focus was in the attempt to resolve the problems caused by the incorrect interpretation of the predicted long ranged weak gauge fields. What seems to be a breakthrough in this respect came only quite recently (2005), more than a decade after the first version of this chapter, and is based on TGD based view about dark matter inspired by the developments in the mathematical understanding of quantum TGD. In this approach condensed matter nuclei can be either ordinary, that is behave essentially like standard model nuclei, or be in dark matter phase in which case they generate long ranged dark weak gauge fields responsible for the large parity breaking effects in living matter. This approach resolves trivially the objections against long range classical weak fields.

The basic criterion for the transition to dark matter phase having by definition large value of $\hbar$ is that the condition $\alpha Q_1 Q_2 \simeq 1$ for appropriate gauge interactions expressing the fact that the perturbation series does not converge. The increase of $\hbar$ makes perturbation series converging since the value of α is reduced but leaves lowest order classical predictions invariant.

This criterion can be applied to color force and inspires the hypothesis that valence quarks inside nucleons correspond to large $\hbar$ phase whereas sea quark space-time sheets correspond to the ordinary value of $\hbar$. This hypothesis is combined with the earlier model of strong nuclear force based on the assumption that long color bonds with p-adically scaled down quarks with mass of order MeV at their ends are responsible for the nuclear strong force.

1. Is strong force due to color bonds between exotic quark pairs?

The basic assumptions are following.

1. Valence quarks correspond to large $\hbar$ phase with p-adic length scale $L(k_{eff} = 129) = L(107)/v_0 \simeq 2^{11}L(107) \simeq 5 \times 10^{-12}$ m whereas sea quarks correspond to ordinary $\hbar$ and define the standard size of nucleons.

2. Color bonds with length of order $L(127) \simeq\simeq 2.5 \times 10^{-12}$ m and having quarks with ordinary $\hbar$ and p-adically scaled down masses $m_q(dark) \simeq v_0 m_q$ at their ends define kind of rubber bands connecting nucleons. The p-adic length scale of exotic quarks differs by a factor 2 from that of dark valence quarks so that the length scales in question can couple naturally. This large length scale as also other p-adic length scales correspond to the size of the topologically quantized field body associated with system, be it quark, nucleon, or nucleus.

3. Valence quarks and even exotic quarks can be dark with respect to both color and weak interactions but not with respect to electromagnetic interactions. The model for binding energies suggests darkness with respect to weak interactions with weak boson masses scaled down by a factor v_0. Weak interactions remain still weak. Quarks and nucleons as defined by their $k = 107$ sea quark portions condense at scaled up weak space-time sheet with $k_{eff} = 111$ having p-adic size 10^{-14} meters. The estimate for the atomic number of the heaviest possible nucleus comes out correctly.

   The wave functions of the nucleons fix the boundary values of the wave functionals of the color magnetic flux tubes idealizable as strings. In the terminology of M-theory nucleons correspond to small branes and color magnetic flux tubes to strings connecting them.

2. General features of strong interactions

This picture allows to understand the general features of strong interactions.

1. Quantum classical correspondence and the assumption that the relevant space-time surfaces have 2-dimensional CP_2 projection implies Abelianization. Strong isospin group can be identified as the SU(2) subgroup of color group acting as isotropies of space-time surfaces. and the U(1) holonomy of color gauge potential defines a preferred direction of strong isospin. Dark color isospin corresponds to strong isospin. The correlation of dark color with weak isospin of the nucleon is strongly suggested by quantum classical correspondence.

2. Both color singlet spin 0 pion type bonds and colored spin 1 bonds are allowed and the color magnetic spin-spin interaction between the exotic quark and anti-quark is negative in this case. p-p and n-n bonds correspond to oppositely colored spin 1 bonds and p-n bonds to colorless spin 0 bonds for which the binding energy is free times higher. The presence of colored bonds forces the presence of neutralizing dark gluon condensate favoring states with $N - P > 0$.

3. Shell model based on harmonic oscillator potential follows naturally from this picture in which the magnetic flux tubes connecting nucleons take the role of springs. Spin-orbit interaction can be understood in terms of the color force in the same way as it is understood in atomic physics.

3. Nuclear binding energies

1. The binding energies per nucleon for $A \leq 4$ nuclei can be understood if they form closed string like structures, nuclear strings, so that only two color bonds per nucleon are possible. This could be understood if ordinary quarks and exotic quarks possessing much smaller mass behave as if they were identical fermions. p-Adic mass calculations support this assumption. Also the average behavior of binding energy for heavier nuclei is predicted correctly.

2. For nuclei with $P = N$ all color bonds can be pion type bonds and have thus largest color magnetic spin-spin interaction energy. The increase of color Coulombic binding energy between colored exotic quark pairs and dark gluons however favors $N > P$ and explains also the formation of neutron halo outside $k = 111$ space-time sheet.

3. Spin-orbit interaction provides the standard explanation for magic numbers. If the maximum of the binding energy per nucleon is taken as a criterion for magic, also Z=N=4,6,12 are magic. The alternative TGD based explanation for magic numbers $Z = N = 4, 6, 8, 12, 20$ would be in terms of regular Platonic solids. Experimentally also other magic numbers are known for neutrons. The linking of nuclear strings provides a possible mechanism producing new magic nuclei from lighter magic nuclei.

4. Stringy description of nuclear reactions

The view about nucleus as a collection of linked nuclear strings suggests stringy description of nuclear reactions. Microscopically the nuclear reactions would correspond to re-distribution of exotic quarks between the nucleons in reacting nuclei.

5. Anomalies and new nuclear physics

The TGD based explanation of neutron halo has been already mentioned. The recently observed tetra-neutron states are difficult to understand in the standard nuclear physics framework since Fermi statistics does not allow this kind of state. The identification of tetra-neutron as an alpha particle containing two negatively charged color bonds allows to circumvent the problem. A large variety of exotic nuclei containing charged color bonds is predicted.

The proposed model explains the anomaly associated with the tritium beta decay. What has been observed is that the spectrum intensity of electrons has a narrow bump near the endpoint energy. Also the maximum energy E_0 of electrons is shifted downwards. I have considered two explanations for the anomaly. The original models are based on TGD variants of original models involving belt of dark neutrinos or antineutrinos along the orbit of Earth. Only recently (towards the end of year 2008) I realized that nuclear string model provides much more elegant explanation of the anomaly and has also the potential to explain much more general anomalies.

Cold fusion has not been taken seriously by the physics community but the situation has begun to change gradually. There is an increasing evidence for the occurrence of nuclear transmutations of heavier elements besides the production of ^4He and ^3H whereas the production rate of ^3He and neutrons is very low. These characteristics are not consistent with the standard nuclear physics predictions. Also Coulomb wall and the absence of gamma rays and the lack of a mechanism transferring nuclear energy to the electrolyte have been used as an argument against cold fusion. TGD based model relying on the notion of charged color bonds explains the anomalous characteristics of cold fusion.

Nuclear String Hypothesis

Nuclear string hypothesis is one of the most dramatic almost-predictions of TGD. The hypothesis in its original form assumes that nucleons inside nucleus form closed nuclear strings with neighboring nuclei of the string connected by exotic meson bonds consisting of color magnetic flux tube with quark and anti-quark at its ends. The lengths of flux tubes correspond to the p-adic length scale of electron and therefore the mass scale of the exotic mesons is around 1 MeV in accordance with the general scale of nuclear binding energies. The long lengths of em flux tubes increase the distance between nucleons and reduce Coulomb repulsion. A fractally scaled up variant of ordinary QCD with respect to p-adic length scale would be in question and the usual wisdom about ordinary pions and other mesons as the origin of nuclear force would be simply wrong in TGD framework as the large mass scale of ordinary pion indeed suggests.

1. $A > 4$ nuclei as nuclear strings consisting of $A \leq 4$ nuclei

In this article a more refined version of nuclear string hypothesis is developed.

1. It is assumed ^4He nuclei and $A < 4$ nuclei and possibly also nucleons appear as basic building blocks of nuclear strings. $A \leq 4$ nuclei in turn can be regarded as strings of nucleons. Large number of stable lightest isotopes of form $A = 4n$ supports the hypothesis that the number of ^4He nuclei is maximal. Even the weak decay characteristics might be reduced to those for $A < 4$ nuclei using this hypothesis.

2. One can understand the behavior of nuclear binding energies surprisingly well from the assumptions that total *strong* binding energy associated with $A \leq 4$ building blocks is *additive* for nuclear strings.

3. In TGD framework tetra-neutron is interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants. For heavier nuclei tetra-neutron is needed as an additional building brick.

2. Bose-Einstein condensation of color bonds as a mechanism of nuclear binding

The attempt to understand the variation of the nuclear binding energy and its maximum for Fe leads to a quantitative model of nuclei lighter than Fe as color bound Bose-Einstein condensates of pion like colored states associated with color flux tubes connecting ^4He nuclei. The color contribution to the total binding energy is proportional to n^2, where n is the number of color bonds. Fermi statistics explains the reduction of E_B for the nuclei heavier than Fe. Detailed estimate favors harmonic oscillator model over free nucleon model with oscillator strength having interpretation in terms of string tension.

Fractal scaling argument allows to understand ^4He and lighter nuclei as strings of nucleons with nucleons bound together by color bonds. Three fractally scaled variants of QCD corresponding $A > 4$, $A = 4$, and $A < 4$ nuclei are involved. The binding energies of also $A \leq 4$ are predicted surprisingly accurately by applying simple p-adic scaling to the model of binding energies of heavier nuclei.

3. Giant dipole resonance as de-coherence of Bose-Einstein condensate of color bonds

Giant resonances and so called pygmy resonances are interpreted in terms of de-coherence of the Bose-Einstein condensates associated with $A \leq 4$ nuclei and with the nuclear string formed from $A \leq 4$ nuclei. The splitting of the Bose-Einstein condensate to pieces costs a precisely defined energy. For ^4He de-coherence the model predicts singlet line at 12.74 MeV and triplet at ~ 27 MeV spanning 4 MeV wide range.

The de-coherence at the level of nuclear string predicts 1 MeV wide bands 1.4 MeV above the basic lines. Bands decompose to lines with precisely predicted energies. Also these contribute to the width. The predictions are in rather good agreement with experimental values. The so called pygmy resonance appearing in neutron rich nuclei can be understood as a de-coherence for $A = 3$ nuclei. A doublet at ~ 8 MeV and MeV spacing is predicted. The prediction for the position is correct.

4. Dark nuclear strings as analogs of as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code

A speculative picture proposing a connection between homeopathy, water memory, and phantom DNA effect is discussed and on basis of this connection a vision about how the tqc hardware represented by the genome is actively developed by subjecting it to evolutionary pressures represented by a virtual world representation of the physical environment. The speculation inspired by this vision is that genetic code as well as DNA-, RNA- and amino-acid sequences should have representation in terms of nuclear strings. The model for dark baryons indeed leads to an identification of these analogs and the basic numbers of genetic code including also the numbers of aminoacids coded by a given number of codons are predicted correctly. Hence it seems that genetic code is universal rather than being an accidental outcome of the biological evolution.

Dark Nuclear Physics and Condensed Matter

In this chapter the possible effects of dark matter in nuclear physics and condensed matter physics are considered. The spirit of the discussion is necessarily rather speculative since the vision about the hierarchy of Planck constants is only 5 years old. The most general form of the hierarchy would involve both singular coverings and factors spaces of CD (causal diamond of M^4) defined as intersection of future and past directed light-cones) and CP_2. There are grave objections against the allowance of factor spaces. In this case Planck constant could be smaller than its standard value and there are very few experimental indications for this. Quite recently came the realization that the hierarchy of Planck constants might emerge from the basic quantum TGD as a consequence of the extreme non-linearity of field equations implying that the correspondence between the derivatives of imbedding space coordinates and canonical momentum is many-to-one. This makes natural to the introduction of covering spaces of CD and CP_2. Planck constant would be effectively replaced with a multiple of ordinary Planck constant defined by the number of the sheets of the covering. The space-like 3-surfaces at the ends of the causal diamond and light-like 3-surfaces defined by wormhole throats carrying elementary particle quantum numbers would be quantum critical in the sense of being unstable against decay to many-sheeted structures. Charge fractionization could be understood in this scenario. Biological evolution would have the increase of the Planck constant as as one aspect. The crucial scaling of the size of CD by Planck constant can be justified by a simple argument. Note that primary p-adic length scales would scale as $\sqrt{\hbar}$ rather than $\hbar$ as assumed in the original model.

1. What darkness means?

Dark matter is identified as matter with non-standard value of Planck constant. The weak form of darkness states that only some field bodies of the particle consisting of flux quanta mediating bound state interactions between particles become dark. One can assign to each interaction a field body (em, Z^0, W, gluonic, gravitational) and p-adic prime and the value of Planck constant characterize the size of the particular field body. One might even think that particle mass can be assigned with its em field body and that Compton length of particle corresponds to the size scale of em field body.

Nuclear string model suggests that the sizes of color flux tubes and weak flux quanta associated with nuclei can become dark in this sense and have size of order atomic radius so that dark nuclear physics would have a direct relevance for condensed matter physics. If this happens, it becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore.

2. What dark nucleons are?

The basic hypothesis is that nuclei can make a phase transition to dark phase in which the size of both quarks and nuclei is measured in Angstroms. For the less radical option this transition could happen only for the color, weak, and em field bodies. Proton connected by dark color bonds super-nuclei with inter-nucleon distance of order atomic radius might be crucial for understanding the properties of water and perhaps even the properties of ordinary condensed matter. Large $\hbar$ phase for weak field body of D and Pd nuclei with size scale of atom would explain selection rules of cold fusion.

3. Anomalous properties of water and dark nuclear physics

A direct support for partial darkness of water comes from the $H_{1.5}O$ chemical formula supported by neutron and electron diffraction in attosecond time scale. The explanation could be that one fourth of protons combine to form super-nuclei with protons connected by color bonds and having distance sufficiently larger than atomic radius.

The crucial property of water is the presence of molecular clusters. Tedrahedral clusters allow an interpretation in terms of magic Z=8 protonic dark nuclei. The icosahedral clusters consisting of 20 tedrahedral clusters in turn have interpretation as magic dark dark nuclei: the presence of the dark dark matter explains large portion of the anomalies associated with water and explains the unique role of water in biology. In living matter also higher levels of dark matter hierarchy are predicted to be present. The observed nuclear transmutation suggest that also light weak bosons are present.

4. Implications of the partial darkness of condensed matter

The model for partially dark condensed matter inspired by nuclear string model and the model of cold fusion inspired by it allows to understand the low compressibility of the condensed matter as being due to the repulsive weak force between exotic quarks, explains large parity breaking effects in living matter, and suggests a profound modification of the notion of chemical bond having most important implications for bio-chemistry and understanding of bio-chemical evolution.

Dark Forces and Living Matter

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether Z^0 charges of elementary particles are screened in electro-weak length scale or not. Same question msut be raised in the case of color charges. For a long time the hypothesis was that the charges are feeded to larger space-time sheets in this length scale rather than screened by vacuum charges so that an effective screening results in electro-weak length scale. This hypothesis turned out to be a failure and was replaced with the idea that the non-linearity of field equations (only topological half of Maxwell's equations holds true) implies the generation of vacuum charge densities responsible for the screening.

The weak form of electric-magnetic duality led to the identification of the long sought for mechanism causing the weak screening in electroweak scales. The basic implication of the duality is that Kähler electric charges of wormhole throats representing particles are proportional to Kähler magnetic charges so that the CP_2 projections of the wormhole throats are homologically non-trivial.

The Kähler magnetic charges do not create long range monopole fields if they are neutralized by wormhole throats carrying opposite monopole charges and weak isospin neutralizing the axial isospin of the particle's wormhole throat. One could speak of confinement of weak isospin. The weak field bodies of elementary fermions would be replaced with string like objects with a length of order W boson Compton length. Electro-magnetic flux would be feeded to electromagnetic field body where it would be feeded to larger space-time sheets. Similar mechanism could apply in the case of color quantum numbers. Weak charges would be therefore screened for ordinary matter in electro-weak length scale but dark electro-weak bosons correspond to much longer symmetry breaking length scale for weak field body. Large values of Planck constant would make it possible to zoop up elementary particles and study their internal structure without any need for gigantic accelerators.

In this chapter possible implications of the dark weak force for the understanding of living matter are discussed. The basic question is how classical Z^0 fields could make itself visible. Large parity breaking effects in living matter suggests which direction one should look for the answer to the question. One possible answer is based on the observation that for vacuum extremals classical electromagnetic and Z^0 fields are proportional to each other and this means that the electromagnetic charges of dark fermions standard are replaced with effective couplings in which the contribution of classical Z^0 force dominates. This modifies dramatically the model for the cell membrane as a Josephson junction and raises the scale of Josephson energies from IR range just above thermal threshold to visible and ultraviolet. The amazing finding is that the Josephson energies for biologically important ions correspond to the energies assigned to the peak frequencies in the biological activity spectrum of photoreceptors in retina suggesting. This suggests that almost vacuum extremals and thus also classical Z^0 fields are in a central role in the understanding of the functioning of the cell membrane and of sensory qualia. This would also explain the large parity breaking effects in living matter.

A further conjecture is that EEG and its predicted fractally scaled variants which same energies in visible and UV range but different scales of Josephson frequencies correspond to Josephson photons with various values of Planck constant. The decay of dark ELF photons with energies of visible photons would give rise to bunches of ordinary ELF photons. Biophotons in turn could correspond to ordinary visible photons resulting in the phase transition of these photons to photons with ordinary value of Planck constant. This leads to a very detailed view about the role of dark electromagnetic radiation in biomatter and also to a model for how sensory qualia are realized. The general conclusion might be that most effects due to the dark weak force are associated with almost vacuum extremals.

Super-Conductivity in Many-Sheeted Space-Time

In this chapter a model for high T_c super-conductivity as quantum critical phenomenon is developed. The relies on the notions of quantum criticality, dynamical quantized Planck constant requiring a generalization of the 8-D imbedding space to a book like structure, and many-sheeted space-time. In particular, the notion of magnetic flux tube as a carrier of supra current of central concept.

With a sufficient amount of twisting and weaving these basic ideas one ends up to concrete model for high T_c superconductors as quantum critical superconductors consistent with the qualitative facts that I am personally aware. The following minimal model looks the most realistic option found hitherto.

1. The general idea is that magnetic flux tubes are carriers of supra currents. In anti-ferromagnetic phases these flux tube structures form small closed loops so that the system behaves as an insulator. Some mechanism leading to a formation of long flux tubes must exist. Doping creates holes located around stripes, which become positively charged and attract electrons to the flux tubes.

2. The higher critical temperature T_{c1} corresponds to a formation local configurations of parallel spins assigned to the holes of stripes giving rise to a local dipole fields with size scale of the order of the length of the stripe. Conducting electrons form Cooper pairs at the magnetic flux tube structures associated with these dipole fields. The elongated structure of the dipoles

favors angular momentum $L = 2$ for the pairs. The presence of magnetic field favors Cooper pairs with spin $S = 1$.

3. Stripes can be seen as 1-D metals with delocalized electrons. The interaction responsible for the energy gap corresponds to the transversal oscillations of the magnetic flux tubes inducing oscillations of the nuclei of the stripe. These transverse phonons have spin and their exchange is a good candidate for the interaction giving rise to a mass gap. This could explain the BCS type aspects of high T_c super-conductivity.

4. Above T_c supra currents are possible only in the length scale of the flux tubes of the dipoles which is of the order of stripe length. The reconnections between neighboring flux tube structures induced by the transverse fluctuations give rise to longer flux tubes structures making possible finite conductivity. These occur with certain temperature dependent probability $p(T, L)$ depending on temperature and distance L between the stripes. By criticality $p(T, L)$ depends on the dimensionless variable $x = TL/\hbar$ only: $p = p(x)$. At critical temperature T_c transverse fluctuations have large amplitude and makes $p(x_c)$ so large that very long flux tubes are created and supra currents can run. The phenomenon is completely analogous to percolation.

5. The critical temperature $T_c = x_c\hbar/L$ is predicted to be proportional to $\hbar$ and inversely proportional to L (, which is indeed to be the case). If flux tubes correspond to a large value of $\hbar$, one can understand the high value of T_c. Both Cooper pairs and magnetic flux tube structures represent dark matter in TGD sense.

6. The model allows to interpret the characteristic spectral lines in terms of the excitation energy of the transversal fluctuations and gap energy of the Cooper pair. The observed 50 meV threshold for the onset of photon absorption suggests that below T_c also $S = 0$ Cooper pairs are possible and have gap energy about 9 meV whereas $S = 1$ Cooper pairs would have gap energy about 27 meV. The flux tube model indeed predicts that $S = 0$ Cooper pairs become stable below T_c since they cannot anymore transform to $S = 1$ pairs. Their presence could explain the BCS type aspects of high T_c super-conductivity. The estimate for $\hbar/\hbar_0 = r$ from critical temperature T_{c1} is about $r = 3$ contrary to the original expectations inspired by the model of of living system as a super-conductor suggesting much higher value. An unexpected prediction is that coherence length is actually r times longer than the coherence length predicted by conventional theory so that type I super-conductor could be in question with stripes serving as duals for the defects of type I super-conductor in nearly critical magnetic field replaced now by ferromagnetic phase.

7. TGD suggests preferred values for $r = \hbar/\hbar_0$. For the most general option the values of $\hbar$ are products and ratios of two integers n_a and n_b. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n_i)$, $i \in \{a, b\}$, in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of r. The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1 + i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four p-adic length scales sith $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of each other leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG.

At qualitative level the model explains various strange features of high T_c superconductors. One can understand the high value of T_c and ambivalent character of high T_c super conductors, the existence of pseudogap and scalings laws for observables above T_c, the role of stripes and doping and the existence of a critical doping, etc...

Quantum Hall effect and Hierarchy of Planck Constants

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

1. The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (Kähler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by delocalized electron at the boundary of the vortex. One can raise two questions.

   Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

   Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

2. Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge fractionalization and fractional braiding statistics. Here the vacuum degeneracy of Kähler action suggests the explanation. Vacuum degeneracy implies that the correspondence between the normal component of the canonical momentum current and normal derivatives of imbedding space coordinates is 1- to-n. These kind of branchings result in multi-furcations induced by variations of the system parameters and the scaling of external magnetic field represents one such variation.

3. At the orbits of wormhole throats, which can have even macroscopic M^4 projections, one has $1 \rightarrow n_a$ correspondence and at the space-like ends of the space-time surface at light-like boundaries of causal diamond one has $1 \rightarrow n_b$ correspondence. This implies that at partonic 2-surfaces defined as the intersections of these two kinds of 3-surfaces one has $1 \rightarrow n_a \times n_b$ correspondence. This correspondence can be described by using a local singular n-fold covering of the imbedding space. Unlike in the original approach, the covering space is only a convenient auxiliary tool rather than fundamental notion.

4. The fractionalization of charge can be understood as follows. A delocalization of electron charge to the n sheets of the multi-furcation takes place and single sheet is analogous to a sheet of Riemann surface of function $z^{1/n}$ and carries fractional charge $q = e/n$, $n = n_a n_b$. Fractionalization applies also to other quantum numbers. One can have also many-electron stats of these states with several delocalized electrons: in this case one obtains more general charge fractionalization: $q = \nu e$.

5. Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the braid rotation by $\Delta\Phi = 2\pi$, where Φ corresponds to M^4 angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate ϕ of the n-branched covering $\Delta\phi = 2/pi$ corresponds to $\Delta\Phi = n \times 2\pi$. If one identifies the sheets of multi-furcation and therefore uses Φ as angle coordinate, single valued angular momentum eigenstates become in general n-valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.

6. How to understand the exceptional values $\nu = 5/2, 7/2$ of the filling fraction? The non-abelian braid group representations can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible.

It seems that the minimum number of braids is $n > 2$ from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives $n > 3$. The minimum value is $n = 4$ consistent with the fractional charge $e/4$.

The model introduces Z_4 valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a Z_4 valued momentum characterizing the four delocalized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would corresponds to that of 4-fold covering space of imbedding space serving as a convenient auxiliary tool. The more standard explanation is that $Z_4 = Z_2 \times Z_2$ such that Z_2:s correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. TGD predicts the possibility of dynamical gauge groups and maybe this kind of gauge group indeed emerges. Dynamical gauge groups emerge also for stacks of N branes and the n sheets of multifurcation are analogous to the N sheets in the stack for many-electron states.

A Possible Explanation of Shnoll Effect

Shnoll and collaborators have discovered strange repeating patterns of random fluctuations of physical observables such as the number n of nuclear decays in a given time interval. Periodically occurring peaks for the distribution of the number $N(n)$ of measurements producing n events in a series of measurements as a function of n is observed instead of a single peak. The positions of the peaks are not random and the patterns depend on position and time varying periodically in time scales possibly assignable to Earth-Sun and Earth-Moon gravitational interaction.

These observations suggest a modification of the expected probability distributions but it is very difficult to imagine any physical mechanism in the standard physics framework. Rather, a universal deformation of predicted probability distributions would be in question requiring something analogous to the transition from classical physics to quantum physics.

The hint about the nature of the modification comes from the TGD inspired quantum measurement theory proposing a description of the notion of finite measurement resolution in terms of inclusions of so called hyper-finite factors of type II_1 (HFFs) and closely related quantum groups. Also p-adic physics -another key element of TGD- is expected to be involved. A modification of a given probability distribution $P(n|\lambda_i)$ for a positive integer valued variable n characterized by rational-valued parameters λ_i is obtained by replacing n and the integers characterizing λ_i with so called quantum integers depending on the quantum phase $q_m = exp(i2\pi/m)$. Quantum integer n_q must be defined as the product of quantum counterparts p_q of the primes p appearing in the prime decomposition of n. One has $p_q = sin(2\pi p/m)/sin(2\pi/m)$ for $p \neq P$ and $p_q = P$ for $p = P$. m must satisfy $m \geq 3$, $m \neq p$, and $m \neq 2p$.

The quantum counterparts of positive integers can be negative. Therefore quantum distribution is defined first as p-adic valued distribution and then mapped by so called canonical identification I to a real distribution by the map taking p-adic -1 to P and powers P^n to P^{-n} and other quantum primes to themselves and requiring that the mean value of n is for distribution and its quantum variant. The map I satisfies $I(\sum P_n) = \sum I(P_n)$. The resulting distribution has peaks located periodically with periods coming as powers of P. Also periodicities with peaks corresponding to $n = n^+n^-$, $n_q^+ > 0$ with fixed $n_q^- < 0$, are predicted. These predictions are universal and easily testable. The prime P and integer m characterizing the quantum variant of distribution can be identified from data. The shapes of the distributions obtained are qualitatively consistent with the findings of Shnoll but detailed tests are required to see whether the number theoretic predictions are correct.

The periodic dependence of the distributions would be most naturally assignable to the gravitational interaction of Earth with Sun and Moon and therefore to the periodic variation of Earth-Sun and Earth-Moon distances. The TGD inspired proposal is that the p-dic prime P and integer m characterizing the quantum distribution are determined by a process analogous to a state function reduction and their most probably values depend on the deviation of the distance R through the formulas $\Delta p/p \simeq k_p \Delta R/R$ and $\Delta m/m \simeq k_m \Delta R/R$. The p-adic primes assignable to elementary particles are very large unlike the primes which could characterize the empirical distributions. The hierarchy of Planck constants allows the gravitational Planck constant assignable to the space-time sheets mediating gravitational interactions to have gigantic values and this allows p-adicity with small values of the p-adic prime P.

Part I

HYPER-FINITE FACTORS AND HIERARCHY OF PLANCK CONSTANTS

Chapter 2

Was von Neumann Right After All?

2.1 Introduction

The work with TGD inspired model [K94] for topological quantum computation [K94] led to the realization that von Neumann algebras [A63, A80, A67, A38] , in particular so called hyper-finite factors of type II_1 [A52] , seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. In this chapter I will discuss various aspects of type II_1 factors and their physical interpretation in TGD framework. The lecture notes of R. Longo [A59] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD.

2.1.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A52] .

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

2.1.2 Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [A37] based on the notion of delta function, plus the emergence of s [A42] , the possibility to formulate the notion of delta function rigorously in terms of distributions [A49, A74] , and the emergence of path integral approach [A68] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A72, A84] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A55] relate closely to type II_1 factors. In topological quantum computation [K94] based on braid groups [A85] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B13] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B26, B6] .

2.1.3 Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K76] . Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products

of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the modified gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality [K20, K25] this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

2.1.4 Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

2.1.5 Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.1.6 Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

This chapter represents a summary about the development of the ideas with last sections representing the recent latest about the realization and role of HFFs in TGD. I have saved the reader from those speculations that have turned out to reflect my own ignorance or are inconsistent with what I regarded established parts of quantum TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hyperfinite factors and TGD [L29]

2.2 Von Neumann algebras

In this section basic facts about von Neumann algebras are summarized using as a background material the concise summary given in the lecture notes of Longo [A59] .

2.2.1 Basic definitions

A formal definition of von Neumann algebra [A80, A67, A38] is as a *-subalgebra of the set of bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space $\mathcal{H}$ closed under weak operator topology, stable under the conjugation $J =^*: x \rightarrow x^*$, and containing identity operator Id. This definition allows also von Neumann algebras for which the trace of the unit operator is not finite.

Identity operator is the only operator commuting with a simple von Neumann algebra. A general von Neumann algebra allows a decomposition as a direct integral of simple algebras, which von Neumann called factors. Classification of von Neumann algebras reduces to that for factors.

$\mathcal{B}(\mathcal{H})$ has involution * and is thus a *-algebra. $\mathcal{B}(\mathcal{H})$ has order order structure $A \geq 0 : (Ax, x) \geq 0$. This is equivalent to $A = BB^*$ so that order structure is determined by algebraic structure. $\mathcal{B}(\mathcal{H})$ has metric structure in the sense that norm defined as supremum of $||Ax||$, $||x|| \leq 1$ defines the notion of continuity. $||A||^2 = inf\{\lambda > 0 : AA^* \leq \lambda I\}$ so that algebraic structure determines metric structure.

There are also other topologies for $\mathcal{B}(\mathcal{H})$ besides norm topology.

1. $A_i \rightarrow A$ strongly if $||Ax - A_ix|| \rightarrow 0$ for all x. This topology defines the topology of C^* algebra. $\mathcal{B}(\mathcal{H})$ is a Banach algebra that is $||AB|| \leq ||A|| \times ||B||$ (inner product is not necessary) and also C^* algebra that is $||AA^*|| = ||A||^2$.

2. $A_i \rightarrow A$ weakly if $(A_ix, y) \rightarrow (Ax, y)$ for all pairs (x, y) (inner product is necessary). This topology defines the topology of von Neumann algebra as a sub-algebra of $\mathcal{B}(\mathcal{H})$.

Denote by M' the commutant of $\mathcal{M}$ which is also algebra. Von Neumann's bicommutant theorem says that $\mathcal{M}$ equals to its own bi-commutant. Depending on whether the identity operator has a finite trace or not, one distinguishes between algebras of type II_1 and type II_∞. II_1 factor allow trace with properties $tr(Id) = 1$, $tr(xy) = tr(yx)$, and $tr(x^*x) > 0$, for all $x \neq 0$. Let $L^2(\mathcal{M})$ be the Hilbert space obtained by completing $\mathcal{M}$ respect to the inner product defined $\langle x|y\rangle = tr(x^*y)$ defines inner product in $\mathcal{M}$ interpreted as Hilbert space. The normalized trace induces a trace in M', natural trace $Tr_{M'}$, which is however not necessarily normalized. JxJ defines an element of M': if $\mathcal{H} = L^2(\mathcal{M})$, the natural trace is given by $Tr_{M'}(JxJ) = tr_M(x)$ for all $x \in M$ and bounded.

2.2.2 Basic classification of von Neumann algebras

Consider first some definitions. First of all, Hermitian operators with positive trace expressible as products xx^* are of special interest since their sums with positive coefficients are also positive.

In quantum mechanics Hermitian operators can be expressed in terms of projectors to the eigen states. There is a natural partial order in the set of isomorphism classes of projectors by inclusion: $E < F$ if the image of $\mathcal{H}$ by E is contained to the image of $\mathcal{H}$ by a suitable isomorph of F. Projectors are said to be metrically equivalent if there exist a partial isometry which maps the images $\mathcal{H}$ by them to each other. In the finite-dimensional case metric equivalence means that isomorphism classes are identical $E = F$.

The algebras possessing a minimal projection E_0 satisfying $E_0 \leq F$ for any F are called type I algebras. Bounded operators of n-dimensional Hilbert space define algebras I_n whereas the bounded operators of infinite-dimensional separable Hilbert space define the algebra I_∞. I_n and I_∞ correspond to the operator algebras of quantum mechanics. The states of harmonic oscillator correspond to a factor of type I.

The projection F is said to be finite if $F < E$ and $F \equiv E$ implies $F = E$. Hence metric equivalence means identity. Simple von Neumann algebras possessing finite projections but no minimal projections so that any projection E can be further decomposed as $E = F + G$, are called factors of type II.

Hyper-finiteness means that any finite set of elements can be approximated arbitrary well with the elements of a finite-dimensional sub-algebra. The hyper-finite II_∞ algebra can be regarded as a tensor product of hyper-finite II_1 and I_∞ algebras. Hyper-finite II_1 algebra can be regarded as a Clifford algebra of an infinite-dimensional separable Hilbert space sub-algebra of I_∞.

Hyper-finite II_1 algebra can be constructed using Clifford algebras $C(2n)$ of $2n$-dimensional spaces and identifying the element x of $2^n \times 2^n$ dimensional $C(n)$ as the element $diag(x, x)/2$ of $2^{n+1} \times 2^{n+1}$-dimensional $C(n + 1)$. The union of algebras $C(n)$ is formed and completed in the weak operator topology to give a hyper-finite II_1 factor. This algebra defines the Clifford algebra of infinite-dimensional separable Hilbert space and is thus a sub-algebra of I_∞ so that hyper-finite II_1 algebra is more regular than I_∞.

von Neumann algebras possessing no finite projections (all traces are infinite or zero) are called algebras of type III. It was later shown by [A33] [A30] that these algebras are labeled by a parameter varying in the range $[0,1]$, and referred to as algebras of type III_x. III_1 category contains a unique hyper-finite algebra. It has been found that the algebras of observables associated with bounded regions of 4-dimensional Minkowski space in quantum field theories correspond to hyper-finite factors of type III_1 [A59] . Also statistical systems at finite temperature correspond to factors of type III and temperature parameterizes one-parameter set of automorphisms of this algebra [B26] . Zero temperature limit correspond to I_∞ factor and infinite temperature limit to II_1 factor.

2.2.3 Non-commutative measure theory and non-commutative topologies and geometries

von Neumann algebras and C^* algebras give rise to non-commutative generalizations of ordinary measure theory (integration), topology, and geometry. It must be emphasized that these structures are completely natural aspects of quantum theory. In particular, for the hyper-finite type II_1 factors quantum groups and Kac Moody algebras [B16] emerge quite naturally without any need for ad hoc modifications such as making space-time coordinates non-commutative. The effective 2-dimensionality of quantum TGD (partonic or stringy 2-surfaces code for states) means that these structures appear completely naturally in TGD framework.

Non-commutative measure theory

von Neumann algebras define what might be a non-commutative generalization of measure theory and probability theory [A59] .

1. Consider first the commutative case. Measure theory is something more general than topology since the existence of measure (integral) does not necessitate topology. Any measurable function f in the space $L^\infty(X,\mu)$ in measure space (X,μ) defines a bounded operator M_f in the space $\mathcal{B}(L^2(X,\mu))$ of bounded operators in the space $L^2(X,\mu)$ of square integrable functions with action of M_f defined as $M_f g = fg$.

2. Integral over $\mathcal{M}$ is very much like trace of an operator $f_{x,y} = f(x)\delta(x,y)$. Thus trace is a natural non-commutative generalization of integral (measure) to the non-commutative case and defined for von Neumann algebras. In particular, generalization of probability measure results if the case $tr(Id) = 1$ and algebras of type I_n and II_1 are thus very natural from the point of view of non-commutative probability theory.

The trace can be expressed in terms of a cyclic vector Ω or vacuum/ground state in physicist's terminology. Ω is said to be cyclic if the completion $\overline{M\Omega} = H$ and separating if $x\Omega$ vanishes only for $x = 0$. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for M'. The expression for the trace given by

$$Tr(ab) = \left(\frac{(ab+ba)}{2}, \Omega\right) \tag{2.2.1}$$

is symmetric and allows to defined also inner product as $(a,b) = Tr(a^*b)$ in $\mathcal{M}$. If Ω has unit norm $(\Omega,\Omega) = 1$, unit operator has unit norm and the algebra is of type II_1. Fermionic oscillator operator algebra with discrete index labeling the oscillators defines II_1 factor. Group algebra is second example of II_1 factor.

The notion of probability measure can be abstracted using the notion of state. State ω on a C^* algebra with unit is a positive linear functional on $\mathcal{U}$, $\omega(1) = 1$. By so called KMS construction [A59] any state ω in C^* algebra $\mathcal{U}$ can be expressed as $\omega(x) = (\pi(x)\Omega,\Omega)$ for some cyclic vector Ω and π is a homomorphism $\mathcal{U} \to \mathcal{B}(\mathcal{H})$.

Non-commutative topology and geometry

C^* algebras generalize in a well-defined sense ordinary topology to non-commutative topology.

1. In the Abelian case Gelfand Naimark theorem [A59] states that there exists a contravariant functor F from the category of unital abelian C^* algebras and category of compact topological spaces. The inverse of this functor assigns to space X the continuous functions f on X with norm defined by the maximum of f. The functor assigns to these functions having interpretation as eigen states of mutually commuting observables defined by the function algebra. These eigen states are delta functions localized at single point of X. The points of X label the eigenfunctions and thus define the spectrum and obviously span X. The connection with topology comes from the fact that continuous map $Y \rightarrow X$ corresponds to homomorphism $C(X) \rightarrow C(Y)$.

2. In non-commutative topology the function algebra $C(X)$ is replaced with a general C^* algebra. Spectrum is identified as labels of simultaneous eigen states of the Cartan algebra of C^* and defines what can be observed about non-commutative space X.

3. Non-commutative geometry can be very roughly said to correspond to *-subalgebras of C^* algebras plus additional structure such as symmetries. The non-commutative geometry of Connes [A31] is a basic example here.

2.2.4 Modular automorphisms

von Neumann algebras allow a canonical unitary evolution associated with any state ω fixed by the selection of the vacuum state Ω [A59] . This unitary evolution is an automorphism fixed apart form unitary automorphisms $A \rightarrow UAU^*$ related with the choice of Ω.

Let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{2.2.2}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$.

2.2.5 Joint modular structure and sectors

Let $\mathcal{N} \subset \mathcal{M}$ be an inclusion. The unitary operator $\gamma = J_N J_M$ defines a canonical endomorphisms $M \rightarrow N$ in the sense that it depends only up to inner automorphism on $\mathcal{N}$, γ defines a sector of $\mathcal{M}$. The sectors of $\mathcal{M}$ are defined as $Sect(\mathcal{M}) = End(\mathcal{M})/Inn(\mathcal{M})$ and form a semi-ring with respected to direct sum and composition by the usual operator product. It allows also conjugation.

$L^2(\mathcal{M})$ is a normal bi-module in the sense that it allows commuting left and right multiplications. For $a, b \in M$ and $x \in L^2(\mathcal{M})$ these multiplications are defined as $axb = aJb^*Jx$ and it is easy to verify the commutativity using the factor $Jy^*J \in \mathcal{M}'$. [A33] [A31] has shown that all normal bi-modules arise in this way up to unitary equivalence so that representation concepts make sense. It is possible to assign to any endomorphism ρ index $Ind(\rho) \equiv M : \rho(\mathcal{M})$. This means that the sectors are in 1-1 correspondence with inclusions. For instance, in the case of hyper-finite II_1 they are labeled by Jones index. Furthermore, the objects with non-integral dimension $\sqrt{[\mathcal{M} : \rho(\mathcal{M})]}$ can be identified as quantum groups, loop groups, infinite-dimensional Lie algebras, etc...

2.2.6 Basic facts about hyper-finite factors of type III

Hyper-finite factors of type II_1, II_∞ and III_1, III_0, III_λ, $\lambda \in (0,1)$, allow by definition hierarchy of finite approximations and are unique as von Neumann algebras. Also hyper-finite factors of type II_∞ and type III could be relevant for the formulation of TGD. HFFs of type II_∞ and III could appear at the level operator algebra but that at the level of quantum states one would obtain HFFs of type II_1. These extended factors inspire highly non-trivial conjectures about quantum TGD. The book of Connes [A31] provides a detailed view about von Neumann algebras in general.

Basic definitions and facts

A highly non-trivial result is that HFFs of type II_∞ are expressible as tensor products $II_\infty = II_1 \otimes I_\infty$, where II_1 is hyper-finite [A31] .

1. The existence of one-parameter family of outer automorphisms

The unique feature of factors of type III is the existence of one-parameter unitary group of outer automorphisms. The automorphism group originates in the following manner.

1. Introduce the notion of linear functional in the algebra as a map $\omega : \mathcal{M} \to C$. ω is said to be hermitian it respects conjugation in $\mathcal{M}$; positive if it is consistent with the notion of positivity for elements of $\mathcal{M}$ in which case it is called weight; state if it is positive and normalized meaning that $\omega(1) = 1$, faithful if $\omega(A) > 0$ for all positive A; a trace if $\omega(AB) = \omega(BA)$, a vector state if $\omega(A)$ is "vacuum expectation" $\omega_\Omega(A) = (\Omega, \omega(A)\Omega)$ for a non-degenerate representation $(\mathcal{H}, \pi)$ of $\mathcal{M}$ and some vector $\Omega \in \mathcal{H}$ with $||\Omega|| = 1$.

2. The existence of trace is essential for hyper-finite factors of type II_1. Trace does not exist for factors of type III and is replaced with the weaker notion of state. State defines inner product via the formula $(x, y) = \phi(y^* x)$ and * is isometry of the inner product. *-operator has property known as pre-closedness implying polar decomposition $S = J\Delta^{1/2}$ of its closure. Δ is positive definite unbounded operator and J is isometry which restores the symmetry between $\mathcal{M}$ and its commutant $\mathcal{M}'$ in the Hilbert space $\mathcal{H}_\phi$, where $\mathcal{M}$ acts via left multiplication: $\mathcal{M}' = J\mathcal{M}J$.

3. The basic result of Tomita-Takesaki theory is that Δ defines a one-parameter group $\sigma_\phi^t(x) = \Delta^{it} x \Delta^{-it}$ of automorphisms of $\mathcal{M}$ since one has $\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}$. This unitary evolution is an automorphism fixed apart from unitary automorphism $A \to UAU^*$ related with the choice of ϕ. For factors of type I and II this automorphism reduces to inner automorphism so that the group of outer automorphisms is trivial as is also the outer automorphism associated with ω. For factors of type III the group of these automorphisms divided by inner automorphisms gives a one-parameter group of $Out(\mathcal{M})$ of outer automorphisms, which does not depend at all on the choice of the state ϕ.

More precisely, let ω be a normal faithful state: $\omega(x^* x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \to x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \to x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{2.2.3}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \to \Delta^{it}\pi\Delta^{-it}$. What makes this result thought provoking is that it might mean a universal quantum dynamics apart from inner automorphisms and thus a realization of general coordinate invariance and gauge invariance at the level of Hilbert space.

2. Classification of HFFs of type III

Connes achieved an almost complete classification of hyper-finite factors of type III completed later by others. He demonstrated that they are labeled by single parameter $0 \leq \lambda \leq 1]$ and that factors of type III_λ, $0 \leq \lambda < 1$ are unique. Haagerup showed the uniqueness for $\lambda = 1$. The idea was that the the group has an invariant, the kernel $T(M)$ of the map from time like R to $Out(M)$, consisting of those values of the parameter t for which σ_ϕ^t reduces to an inner automorphism and to unity as outer automorphism. Connes also discovered also an invariant, which he called spectrum $S(\mathcal{M})$ of $\mathcal{M}$ identified as the intersection of spectra of $\Delta_\phi \backslash \{0\}$, which is closed multiplicative subgroup of R^+.

Connes showed that there are three cases according to whether $S(\mathcal{M})$ is

1. R^+, type III_1
2. $\{\lambda^n, n \in Z\}$, type III_λ.
3. $\{1\}$, type III_0.

   The value range of λ is this by convention. For the reversal of the automorphism it would be that associated with $1/\lambda$.

Connes constructed also an explicit representation of the factors $0 < \lambda < 1$ as crossed product II_∞ factor $\mathcal{N}$ and group Z represented as powers of automorphism of II_∞ factor inducing the scaling of trace by λ. The classification of HFFs of type III reduced thus to the classification of automorphisms of $\mathcal{N} \otimes \mathcal{B}(\mathcal{H}$. In this sense the theory of HFFs of type III was reduced to that for HFFs of type II_∞ or even II_1. The representation of Connes might be also physically interesting.

Probabilistic view about factors of type III

Second very concise representation of HFFs relies on thermodynamical thinking and realizes factors as infinite tensor product of finite-dimensional matrix algebras acting on state spaces of finite state systems with a varying and finite dimension n such that one assigns to each factor a density matrix characterized by its eigen values. Intuitively one can think the finite matrix factors as associated with n-state system characterized by its energies with density matrix ρ defining a thermodynamics. The logarithm of the ρ defines the single particle quantum Hamiltonian as $H = log(\rho)$ and $\Delta = \rho = exp(H)$ defines the automorphism σ_ϕ for each finite tensor factor as $exp(iHt)$. Obviously free field representation is in question.

Depending on the asymptotic behavior of the eigenvalue spectrum one obtains different factors [A31] .

1. Factor of type I corresponds to ordinary thermodynamics for which the density matrix as a function of matrix factor approaches sufficiently fast that for a system for which only ground state has non-vanishing Boltzmann weight.
2. Factor of type II_1 results if the density matrix approaches to identity matrix sufficiently fast. This means that the states are completely degenerate which for ordinary thermodynamics results only at the limit of infinite temperature. Spin glass could be a counterpart for this kind of situation.
3. Factor of type III results if one of the eigenvalues is above some lower bound for all tensor factors in such a manner that neither factor of type I or II_1 results but thermodynamics for systems having infinite number of degrees of freedom could yield this kind of situation.

This construction demonstrates how varied representations factors can have, a fact which might look frustrating for a novice in the field. In particular, the infinite tensor power of $M(2,C)$ with state defined as an infinite tensor power of $M(2,C)$ state assigning to the matrix A the complex number $(\lambda^{1/2}A_{11} + \lambda^{-1/2}\ \phi(A) = A_{22})/(\lambda^{1/2} + \lambda^{-1/2})$ defines HFF III_λ [A31] , [C40] . Formally the same algebra which for $\lambda = 1$ gives ordinary trace and HFF of type II_1, gives III factor only by replacing trace with state. This simple model was discovered by Powers in 1967.

It is indeed the notion of state or thermodynamics is what distinguishes between factors. This looks somewhat weird unless one realizes that the Hilbert space inner product is defined by the "thermodynamical" state ϕ and thus probability distribution for operators and for their thermal expectation values. Inner product in turn defines the notion of norm and thus of continuity and it is this notion which differs dramatically for $\lambda = 1$ and $\lambda < 1$ so that the completions of the algebra differ dramatically.

In particular, there is no sign about I_∞ tensor factor or crossed product with Z represented as automorphisms inducing the scaling of trace by λ. By taking tensor product of I_∞ factor represented as tensor power with induces running from $-\infty$ to 0 and II_1 HFF with indices running from 1 to ∞ one can make explicit the representation of the automorphism of II_∞ factor inducing scaling of trace by λ and transforming matrix factors possessing trace given by square root of index $\mathcal{M} : \mathcal{N}$ to those with trace 2.

2.3 Braid group, von Neumann algebras, quantum TGD, and formation of bound states

The article of Vaughan Jones in [A85] discusses the relation between knot theory, statistical physics, and von Neumann algebras. The intriguing results represented stimulate concrete ideas about how to understand the formation of bound states quantitatively using the notion of join along boundaries bond. All mathematical results represented in the following discussion can be found in [A85] and in the references cited therein so that I will not bother to refer repeatedly to this article in the sequel.

2.3.1 Factors of von Neumann algebras

Von Neumann algebras M are algebras of bounded linear operators acting in Hilbert space. These algebras contain identity, are closed with respect to Hermitian conjugation, and are topologically complete. Finite-dimensional von Neuman algebras decompose into a direct sum of algebras M_n, which act essentially as matrix algebras in Hilbert spaces $\mathcal{H}_{nm}$, which are tensor products $C^n \otimes \mathcal{H}_m$. Here $\mathcal{H}_m$ is an m-dimensional Hilbert space in which M_n acts trivially. m is called the multiplicity of M_n.

A factor of von Neumann algebra is a von Neumann algebra whose center is just the scalar multiples of identity. The algebra of bounded operators in an infinite-dimensional Hilbert space is certainly a factor. This algebra decomposes into "atoms" represented by one-dimensional projection operators. This kind of von Neumann algebras are called type I factors.

The so called type II_1 factors and type III factors came as a surprise even for Murray and von Neumann. II_1 factors are infinite-dimensional and analogs of the matrix algebra factors M_n. They allow a trace making possible to define an inner product in the algebra. The trace defines a generalized dimension for any subspace as the trace of the corresponding projection operator. This dimension is however continuous and in the range $[0, 1]$: the finite-dimensional analog would be the dimension of the sub-space divided by the dimension of $\mathcal{H}_n$ and having values $(0, 1/n, 2/n, ..., 1)$. II_1 factors are isomorphic and there exists a minimal "hyper-finite" II_1 factor is contained by every other II_1 factor.

Just as in the finite-dimensional case, one can to assign a multiplicity to the Hilbert spaces where II_1 factors act on. This multiplicity, call it $dim_M(\mathcal{H})$ is analogous to the dimension of the Hilbert space tensor factor $\mathcal{H}_m$, in which II_1 factor acts trivially. This multiplicity can have all positive real values. Quite generally, von Neumann factors of type I and II_1 are in many respects analogous to the coefficient field of a vector space.

2.3.2 Sub-factors

Sub-factors $N \subset M$, where N and M are of type II_1 and have same identity, can be also defined. The observation that M is analogous to an algebraic extension of N motivates the introduction of index $|M : N|$, which is essentially the dimension of M with respect to N. This dimension is an analog for the complex dimension of CP_2 equal to 2 or for the algebraic dimension of the extension of p-adic numbers.

The following highly non-trivial results about the dimensions of the tensor factors hold true.

1. If $N \subset M$ are II_1 factors and $|M : N| < 4$, there is an integer $n \geq 3$ such $|M : N| = r = 4cos^2(\pi/n)$, $n \geq 3$.

2. For each number $r = 4cos^2(\pi/n)$ and for all $r \geq 4$ there is a sub-factor $R_r \subset R$ with $|R : R_r| = r$.

   One can say that M effectively decomposes to a tensor product of N with a space, whose dimension is quantized to a certain algebraic number r. The values of r corresponding to $n = 3, 4, 5, 6...$ are $r = 1, 2, 1 + \Phi \simeq 2.61, 3, ...$ and approach to the limiting value $r = 4$. For $r \geq 4$ the dimension becomes continuous.

An even more intriguing result is that by starting from $N \subset M$ with a projection e_N: $M \rightarrow N$ one can extend M to a larger II_1 algebra $\langle M, e_N\rangle$ such that one has

$$\begin{aligned} |\langle M, e_N\rangle : M| &= |M : N| \ , \\ tr(xe_N) &= |M : N|^{-1} tr(x) \ , \ \ x \in M \ . \end{aligned} \tag{2.3.1}$$

One can continue this process and the outcome is a tower of II_1 factors $M_i \subset M_{i+1}$ defined by $M_1 = N$, $M_2 = M$, $M_{i+1} = \langle M_i, e_{M_{i-1}}\rangle$. Furthermore, the projection operators $e_{M_i} \equiv e_i$ define a Temperley-Lieb representation of the braid algebra via the formulas

$$\begin{aligned} e_i^2 &= e_i \ , \\ e_i e_{i\pm 1} e_i &= \tau e_i \ , \ \ \tau = 1/|M : N| \\ e_i e_j &= e_j e_i \ , \ \ |i-j| \geq 2 \ . \end{aligned} \tag{2.3.2}$$

Temperley Lieb algebra will be discussed in more detail later. Obviously the addition of a tensor factor of dimension r is analogous with the addition of a strand to a braid.

The hyper-finite algebra R is generated by the set of braid generators $\{e_1, e_2, ...\}$ in the braid representation corresponding to r. Sub-factor R_1 is obtained simply by dropping the lowest generator e_1, R_2 by dropping e_1 and e_2, etc..

2.3.3 II_1 factors and the spinor structure of WCW

The following observations serve as very suggestive guidelines for how one could interpret the above described results in TGD framework.

1. The discrete spectrum of dimensions $1, 2, 1 + \Phi, 3, ..$ below $r < 4$ brings in mind the discrete energy spectrum for bound states whereas the for $r \geq 4$ the spectrum of dimensions is analogous to a continuum of unbound states. The fact that r is an algebraic number for $r < 4$ conforms with the vision that bound state entanglement corresponds to entanglement probabilities in an extension of rationals defining a finite-dimensional extension of p-adic numbers for every prime p.

2. The discrete values of r correspond precisely to the angles ϕ allowed by the unitarity of Temperley-Lieb representations of the braid algebra with $d = -\sqrt{r}$. For $r \geq 4$ Temperley-Lieb representation is not unitary since $cos^2(\pi/n)$ becomes formally larger than one (n would become imaginary and continuous). This could mean that $r \geq 4$, which in the generic case is a transcendental number, represents unbound entanglement, which in TGD Universe is not stable against state preparation and state function reduction processes.

3. The formula $tr(xe_N) = |M : N|^{-1}tr(x)$ is completely analogous to the formula characterizing the normalization of the link invariant induced by the second Markov move in which a new strand is added to a braid such that it braids only with the leftmost strand and therefore does not change the knot resulting as a link closure. Hence the addition of a single strand seems to correspond to an introduction of an r-dimensional sub-factor to II_1 factor.

In TGD framework the generation of bound state has the formation of (possibly braided join along boundaries bonds as a space-time correlate and this encourages a rather concrete interpretation of these findings. Also the I_1 factors themselves have a nice interpretation in terms of the WCW spinor structure.

1. The interpretation of II_1 factors in terms of Clifford algebra of WCW

The Clifford algebra of an infinite-dimensional Hilbert space defines a II_1 factor. The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus to operators of form $(1+\Sigma_{ij})/2$, defined by a subset of sigma matrices. The first guess is that the index pairs are $(i,j) = (1,2),(2,3),(3,4),.....$ The dimension of the Clifford algebra is 2^N for N-dimensional space so that $\Delta N = 1$ would correspond to $r = 2$ in the classical case and to one qubit. The problem with this interpretation is $r > 2$ has no physical interpretation: the formation of bound states is expected to reduce the value of r from its classical value rather than increase it.

One can however consider also the sequence $(i,j) = (1,1+k),(1+k,1+2k),(1+2k,1+3k),....$ For $k = 2$ the reduction of r from $r = 4$ would be due to the loss of degrees of freedom due to the formation of a bound state and $(r = 4, \Delta N = 2)$ would correspond to the classical limit resulting at the limit of weak binding. The effective elimination of the projection operators from the braid algebra would reflect this loss of degrees of freedom. This interpretation could at least be an appropriate starting point in TGD framework.

In TGD Universe physical states correspond to WCW spinor fields, whose gamma matrix algebra is constructed in terms of second quantized free induced spinor fields defined at space-time sheets. The original motivation was the idea that the quantum states of the Universe correspond to the modes of purely classical free spinor fields in the infinite-dimensional configuration space of 3-surfaces (the "world of classical worlds", WCW) possessing general coordinate invariant (in 4-dimensional sense!) Kähler geometry. Quantum information-theoretical motivation could have come from the requirement that these fields must be able to code information about the properties of the point (3-surface, and corresponding space-time sheet). Scalar fields would treat the 3-surfaces as points and are thus not enough. Induced spinor fields allow however an infinite number of modes: according to the naive Fourier analyst's intuition these modes are in one-one correspondence with the points of the 3-surface. Second quantization gives much more. Also non-local information about the induced geometry and topology must be coded, and here quantum entanglement for states generated by the fermionic oscillator operators coding information about the geometry of 3-surface provides enormous information storage capacity.

In algebraic geometry also the algebra of the imbedding space of algebraic variety divided by the ideal formed by functions vanishing on the surface codes information about the surface: for instance, the maximal ideals of this algebra code for the points of the surface (functions of imbedding space vanishing at a particular point). The function algebra of the imbedding space indeed plays a key role in the construction of WCW-geometry besides second quantized fermions.

The Clifford algebra generated by the WCW gamma matrices at a given point (3-surface) of WCW of 3-surfaces could be regarded as a II_1-factor associated with the local tangent space endowed with Hilbert space structure (WCW Kähler metric). The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus operators of form $(G_{\overline{A}B} \times 1 + \Sigma_{\overline{A}B})$ formed as linear combinations of components of the Kähler metric and of the sigma matrices defined by gamma matrices and contracted with the generators of the isometries of WCW. The addition of single complex degree of freedom corresponds to $\Delta N = 2$ and $r = 4$ and the classical limit and would correspond to the addition of single braid. $(r < 4, \Delta N < 2)$ would be due to the binding effects.

$r = 1$ corresponds to $\Delta N = 0$. The first interpretation is in terms of strong binding so that the addition of particle does not increase the number of degrees of freedom. In TGD framework $r = 1$ might also correspond to the addition of zero modes which do not contribute to the WCW metric and spinor structure but have a deep physical significance. $(r = 2, \Delta N = 1)$ would correspond to strong binding reducing the spinor and space-time degrees of freedom by a factor of half. $r = \Phi^2$ $(n = 5)$ *resp.* $r = 3$ $(n = 6)$ corresponds to $\Delta N_r \simeq 1.3885$ *resp.* $\Delta N_r = 1.585$. Using the terminology of quantum field theories, one might say that in the infinite-dimensional context a given complex bound state degree of freedom possesses anomalous real dimension $r < 2$. $r \geq 4$ would correspond to a unbound entanglement and increasingly classical behavior.

2.3.4 About possible space-time correlates for the hierarchy of II_1 subfactors

By quantum classical correspondence the infinite-dimensional physics at WCW level should have definite space-time correlates. In particular, the dimension r should have some fractal dimension as a space-time correlate.

1. Quantum classical correspondence

Join along boundaries bonds serve as correlates for bound state formation. The presence of join along boundaries bonds would lead to a generation of bound states just by reducing the degrees of freedom to those of connected 3-surface. The bonds would constrain the two 3-surfaces to single space-like section of imbedding space.

This picture would allow to understand the difficulties related to Bethe-Salpeter equations for bound states based on the assumption that particles are points moving in M^4. The restriction of particles to time=constant section leads to a successful theory which is however non-relativistic. The basic binding energy would relate to the entanglement of the states associated with the bonded 3-surfaces. Since the classical energy associated with the bonds is positive, the binding energy tends to be reduced as r increases.

By spin glass degeneracy join along boundaries bonds have an infinite number of degrees of freedom in the ordinary sense. Since the system is infinite-dimensional and quantum critical, one expects that the number r of degrees freedom associated with a single join along boundaries bond is universal. Since join along boundaries bonds correspond to the strands of a braid and are correlates for the bound state formation, the natural guess is that $r = 4cos^2(\pi/n)$, $n = 3, 4, 5, ...$ holds true. $r < 4$ should characterize both binding energy and the dimension of the effective tensor factor introduced by a new join along boundaries bond.

The assignment of 2 "bare" and $\Delta N \leq 2$ renormalized real dimensions to single join along boundaries bond is consistent with the effective two-dimensionality of anyon systems and with the very notion of the braid group. The picture conforms also with the fact that the degrees of freedom in question are associated with metrically 2-dimensional light-like boundaries (of say magnetic flux tubes) acting as causal determinants. Also vibrational degrees of freedom described by Kac-Moody algebra are present and the effective 2-dimensionality means that these degrees of freedom are not excited and only topological degrees of freedom coded by the position of the puncture remain.

$(r \geq 4, \Delta N \geq 2)$, if possible at all, would mean that the tensor factor associated with the join along boundaries bond is effectively more than 4-dimensional due to the excitation of the vibrational Kac-Moody degrees of freedom. The finite value of r would mean that most of theme are eliminated also now but that their number is so large that bound state entanglement is not possible anymore.

The introduction of non-integer dimension could be seen as an effective description of an infinite-dimensional system as a finite-dimensional system in the spirit of renormalization group philosophy. The non-unitarity of $r \geq 4$ Temperley-Lieb representations could mean that they correspond to unbound entanglement unstable against state function reduction and preparation processes. Since this kind of entanglement does not survive in quantum jump it is not representable in terms of braid groups.

2. Does r define a fractal dimension of CP_2 projection of partonic 2-surface?

On basis of the quantum classical correspondence one expects that r should define some fractal dimension at the space-time level. Since r varies in the range $1, .., 4$ and corresponds to the fractal dimension of 2-D Clifford algebra the corresponding spinors would have dimension $d = \sqrt{r}$. There are two options.

1. $D = r/2$ is suggested on basis of the construction of quantum version of M^d.

2. $D = log_2(r)$ is natural on basis of the dimension $d = 2^{D/2}$ of spinors in D-dimensional space.

r can be assigned with CP_2 degrees of freedom in the model for the quantization of Planck constant based on the explicit identification of Josephson inclusions in terms of finite subgroups of $SU(2) \subset SU(3)$. Hence D should relate to the CP_2 projection of the partonic 2-surface and one could have $D = D(X^2)$, the latter being the average dimension of the CP_2 projection of the partonic 2-surface for the preferred extremals of Kähler action.

Since a strongly interacting non-perturbative phase should be in question, the dimension for the CP_2 projection of the space-time surface must be at least $D(X^4) = 2$ to guarantee that non-vacuum extremals are in question. This is true for $D(X^2) = r/2 \geq 1$. The logarithmic formula $D(X^2) = log_2(r) \geq 0$ gives $D(X^2) = 0$ for $n = 3$ meaning that partonic 2-surfaces are vacua: space-time surface can still be a non-vacuum extremal.

As n increases, the number of CP_2 points covering a given M^4 point and related by the finite subgroup of $G \subset SU(2) \subset SU(3)$ defining the inclusion increases so that the fractal dimension of the CP_2 projection is expected to increase also. $D(X^2) = 2$ would correspond to the space-time surfaces for which partons have topological magnetic charge forcing them to have a 2-dimensional CP_2 projection. There are reasons to believe that the projection must be homologically non-trivial geodesic sphere of CP_2.

2.3.5 Could binding energy spectra reflect the hierarchy of effective tensor factor dimensions?

If one takes completely seriously the idea that join along boundaries bonds are a correlate of binding then the spectrum of binding energies might reveal the hierarchy of the fractal dimensions $r(n)$. Hydrogen atom and harmonic oscillator have become symbols for bound state systems. Hence it is of interest to find whether the binding energy spectrum of these systems might be expressed in terms of the "binding dimension" $x(n) = 4 - r(n)$ characterizing the deviation of dimension from that at the limit of a vanishing binding energy. The binding energies of hydrogen atom are in a good approximation given by $E(n)/E(1) = 1/n^2$ whereas in the case of harmonic oscillator one has $E(n)/E_0 = 2n + 1$. The constraint $n \geq 3$ implies that the principal quantum number must correspond $n - 2$ in the case of hydrogen atom and to $n - 3$ in the case of harmonic oscillator.

Before continuing one must face an obvious objection. By previous arguments different values of r correspond to different values of $\hbar$. The value of $\hbar$ cannot however differ for the states of hydrogen atom. This is certainly true. The objection however leaves open the possibility that the states of the light-like boundaries of join along boundaries bonds correspond to reflective level and represent some aspects of the physics of, say, hydrogen atom.

In the general case the energy spectrum satisfies the condition

$$\frac{E_B(n)}{E_B(3)} = \frac{f(4 - r(n))}{f(3)} , \tag{2.3.3}$$

where f is some function. The simplest assumption is that the spectrum of binding energies $E_B(n) = E(n) - E(\infty)$ is a linear function of $r(n) - 4$:

$$\frac{E_B(n)}{E_B(3)} = \frac{4 - r(n)}{3} = \frac{4}{3} sin^2(\frac{\pi}{n}) \rightarrow \frac{4\pi^2}{3} \times \frac{1}{n^2} . \tag{2.3.4}$$

In the linear approximation the ratio $E(n + 1)/E(n)$ approaches $(n/n + 1)^2$ as in the case of hydrogen atom but for small values the linear approximation fails badly. An exact correspondence results for

$$\frac{E(n)}{E(1)} = \frac{1}{n^2} ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1 - r(n+2)/4}\right)} - 2 .$$

Also the ionized states with $r \geq 4$ would correspond to bound states in the sense that two particle would be constrained to move in the same space-like section of space-time surface and should be distinguished from genuinely free states when particles correspond to disjoint space-time sheets.

For the harmonic oscillator one express $E(n) - E(0)$ instead of $E(n) - E(\infty)$ as a function of $x = 4 - r$ and one would have

$$\frac{E(n)}{E(0)} = 2n+1 \ ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+3)/4}\right)} - 3 \ .$$

In this case ionized states would not be possible due to the infinite depth of the harmonic oscillator potential well.

2.3.6 Four-color problem, II_1 factors, and anyons

The so called four-color problem can be phrased as a question whether it is possible to color the regions of a plane map using only four colors in such a manner that no adjacent regions have the same color (for an enjoyable discussion of the problem see [A21]). One might call this kind of coloring complete. There is no loss of generality in assuming that the map can be represented as a graph with regions represented as triangle shaped faces of the graph. For the dual graph the coloring of faces becomes coloring of vertices and the question becomes whether the coloring is possible in such a manner that no vertices at the ends of the same edge have same color. The problem can be generalized by replacing planar maps with maps defined on any two-dimensional surface with or without boundary and arbitrary topology. The four-color problem has been solved with an extensive use of computer [A19] but it would be nice to understand why the complete coloring with four colors is indeed possible.

There is a mysterious looking connection between four-color problem and the dimensions $r(n) = 4cos^2(\pi/n)$, which are in fact known as Beraha numbers in honor of the discoverer of this connection [A71] . Consider a more general problem of coloring two-dimensional map using m colors. One can construct a polynomial $P(m)$, so called chromatic polynomial, which tells the number of colorings satisfying the condition that no neighboring vertices have the same color. The vanishing of the chromatic polynomial for an integer value of m tells that the complete coloring using m colors is not possible.

$P(m)$ has also other than integer valued real roots. The strange discovery due to Beraha is that the numbers $B(n)$ appear as approximate roots of the chromatic polynomial in many situations. For instance, the four non-integral real roots of the chromatic polynomial of the truncated icosahedron are very close to $B(5)$, $B(7)$, $B(8)$ and $B(9)$. These findings led Beraha to formulate the following conjecture. Let P_i be a sequence of chromatic polynomials for a graph for which the number of vertices approaches infinity. If r_i is a root of the polynomial approaching a well-defined value at the limit $i \to \infty$, then the limiting value of $r(i)$ is Beraha number.

A physicist's proof for Beraha's conjecture based on quantum groups and conformal theory has been proposed [A71] . It is interesting to look for the a possible physical interpretation of 4-color problem and Beraha's conjecture in TGD framework.

1. In TGD framework $B(n)$ corresponds to a renormalized dimension for a 2-spin system consisting of two qubits, which corresponds to 4 different colors. For $B(n) = 4$ two spin 1/2 fermions obeying Fermi statistics are in question. Since the system is 2-dimensional, the general case corresponds to two anyons with fractional spin $B(n)/4$ giving rise to $B(n) < 4$ colors and obeying fractional statistics instead of Fermi statistics. One can replace coloring problem with the problem whether an ideal antiferro-magnetic lattice using anyons with fractional spin $B(n)/4$ is possible energetically. In other words, does this system form a quantum mechanical bound state even at the limit when the lengths of the edges approach to zero.

2. The failure of coloring means that there are at least two neighboring vertices in the lattice with the property that the spins at the ends of the same edge are in the same direction. Lattice defect would be in question. At the limit of an infinitesimally short edge length the failure of coloring is certainly not an energetically favored option for fermionic spins ($m = 4$) but is allowed by anyonic statistics for $m = B(n) < 4$. Thus one has reasons to expect that when anyonic spin is $B(n)/4$ the formation of a purely 2-anyon bound states becomes possible and they form at the limit of an infinitesimal edge length a kind of topological macroscopic quantum phase with a non-vanishing binding energy. That $B(n)$ are roots of the chromatic polynomial at the continuum limit would have a clear physical interpretation.

3. Only $B(n) < 4$ defines a sub-factor of von Neumann algebra allowing unitary Temperley-Lieb representations. This is consistent with the fact that for $m = 4$ complete coloring must exists. The physical argument is that otherwise a macroscopic quantum phase with non-vanishing binding energy could result at the continuum limit and the upper bound for r from unitarity would be larger than 4. For $m = 4$ the completely anti-ferromagnetic state would represent the ground state and the absence of anyon-pair condensate would mean a vanishing binding energy.

2.4 Inclusions of II_1 and III_1 factors

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A2] and those of factors of type III by Alain Connes [A30] .

Sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

2.4.1 Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A2] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a) \ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b) \ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{2.4.1}$$

   the numbers at right hand side are known as Beraha numbers [A71] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B16] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A53] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

The interpretation of [A53] is that the subfactors correspond to inclusions $\mathcal{N} \subset \mathcal{M}$ defined in the following manner.

1. Let G be a finite subgroup of SU(2). Denote by R the infinite-dimensional Clifford algebras resulting from infinite-dimensional tensor power of $M_2(C)$ and by R_0 its subalgebra obtained by restricting $M_2(C)$ element of the first factor to be unit matrix. Let G act by automorphisms in each tensor factor. G leaves R_0 invariant. Denote by R_0^G and R^G the sub-algebras which remain element wise invariant under the action of G. The resulting Jones inclusions $R_0^G \subset R^G$ are consistent with the ADE correspondence.

2. The argument suggests the existence of quantum versions of subgroups of SU(2) for which representations are truncations of those for ordinary subgroups. The results have been generalized to other Lie groups.

3. Also $SL(2,C)$ acts as automorphisms of $M_2(C)$. An interesting question is what happens if one allows G to be any discrete subgroups of SL(2,C). Could this give inclusions with $\mathcal{M} : \mathcal{N} > 4$?. The strong analogy of the spectrum of indices with spectrum of energies with hydrogen atom would encourage this interpretation: the subgroup SL(2,C) not reducing to those of SU(2) would correspond to the possibility for the particle to move with respect to each other with constant velocity.

2.4.2 The fundamental construction and Temperley-Lieb algebras

It was shown by Jones [A41] that for a given Jones inclusion with $\beta = \mathcal{M} : \mathcal{N} < \infty$ there exists a tower of finite II_1 factors $\mathcal{M}_k$ for $k = 0, 1, 2, ...$ such that

1. $\mathcal{M}_0 = \mathcal{N}$, $\mathcal{M}_1 = \mathcal{M}$,

2. $\mathcal{M}_{k+1} = End_{\mathcal{M}_{k-1}}\mathcal{M}_k$ is the von Neumann algebra of operators on $L^2(\mathcal{M}_k)$ generated by $\mathcal{M}_k$ and an orthogonal projection $e_k : L^2(\mathcal{M}_k) \to L^2(\mathcal{M}_{k-1})$ for $k \geq 1$, where $\mathcal{M}_k$ is regarded as a subalgebra of $\mathcal{M}_{k+1}$ under right multiplication.

It can be shown that $\mathcal{M}_{k+1}$ is a finite factor. The sequence of projections on $\mathcal{M}_\infty = \cup_{k\geq 0}\mathcal{M}_k$ satisfies the relations

$$
\begin{array}{ll}
e_i^2 = e_i \ , & e_i^{=} e_i \ , \\
e_i = \beta e_i e_j e_i & \text{for } |i-j| = 1 \ , \\
e_i e_j = e_j e_i & \text{for } |i-j| \geq 2 \ .
\end{array} \tag{2.4.2}
$$

The construction of hyper-finite II_1 factor using Clifford algebra $C(2)$ represented by 2×2 matrices allows to understand the theorem in $\beta = 4$ case in a straightforward manner. In particular, the second formula involving β follows from the identification of x at $(k-1)^{th}$ level with $(1/\beta)diag(x,x)$ at k^{th} level.

By replacing 2×2 matrices with $\sqrt{\beta} \times \sqrt{\beta}$ matrices one can understand heuristically what is involved in the more general case. $\mathcal{M}_k$ is $\mathcal{M}_{k-1}$ module with dimension $\sqrt{\beta}$ and $\mathcal{M}_{k+1}$ is the space of $\sqrt{\beta} \times \sqrt{\beta}$ matrices $\mathcal{M}_{k-1}$ valued entries acting in $\mathcal{M}_k$. The transition from $\mathcal{M}_k$ to $\mathcal{M}_{k-1}$ linear maps of $\mathcal{M}_k$ happens in the transition to the next level. x at $(k-1)^{th}$ level is identified as $(x/\beta) \times Id_{\sqrt{\beta}\times\sqrt{\beta}}$ at the next level. The projection e_k picks up the projection of the matrix with $\mathcal{M}_{k-1}$ valued entries in the direction of the $Id_{\sqrt{\beta}\times\sqrt{\beta}}$.

The union of algebras $A_{\beta,k}$ generated by $1, e_1, ..., e_k$ defines Temperley-Lieb algebra A_β [A78] . This algebra is naturally associated with braids. Addition of one strand to a braid adds one generator to this algebra and the representations of the Temperley Lieb algebra provide link, knot, and 3-manifold invariants [A85] . There is also a connection with systems of statistical physics and with Yang-Baxter algebras [A25] .

A further interesting fact about the inclusion hierarchy is that the elements in $\mathcal{M}_i$ belonging to the commutator $\mathcal{N}'$ of $\mathcal{N}$ form finite-dimensional spaces. Presumably the dimension approaches infinity for $n \to \infty$.

2.4.3 Connection with Dynkin diagrams

The possibility to assign Dynkin diagrams ($\beta < 4$) and extended Dynkin diagrams ($\beta = 4$ to Jones inclusions can be understood heuristically by considering a characterization of so called bipartite graphs [A54] , [B16] by the norm of the adjacency matrix of the graph.

Bipartite graphs Γ is a finite, connected graph with multiple edges and black and white vertices such that any edge connects white and black vertex and starts from a white one. Denote by $w(\Gamma)$ ($b(\Gamma)$) the number of white (black) vertices. Define the adjacency matrix $\Lambda = \Lambda(\Gamma)$ of size $b(\Gamma) \times w(\Gamma)$ by

$$w_{b,w} = \begin{cases} m(e) & \text{if there exists } e \text{ such that } \delta e = b - w \ , \\ 0 & \text{otherwise } . \end{cases} \tag{2.4.3}$$

Here $m(e)$ is the multiplicity of the edge e.

Define norm $||\Gamma||$ as

$$\begin{aligned} ||X|| &= max\{||X||; \ ||x|| \leq 1\} \ , \\ ||\Gamma|| &= ||\Lambda(\Gamma)|| = \left|\left| \begin{array}{cc} 0 & \Lambda(\Gamma) \\ \Lambda(\Gamma)^t & 0 \end{array} \right|\right| \ . \end{aligned} \tag{2.4.4}$$

Note that the matrix appearing in the formula is $(m+n) \times (m+n)$ symmetric square matrix so that the norm is the eigenvalue with largest absolute value.

Suppose that Γ is a connected finite graph with multiple edges (sequences of edges are regarded as edges). Then

1. If $||\Gamma|| \leq 2$ and if Γ has a multiple edge, $||\Gamma|| = 2$ and $\Gamma = \tilde{A}_1$, the extended Dynkin diagram for $SU(2)$ Kac Moody algebra.

2. $||\Gamma|| < 2$ if and only Γ is one of the Dynkin diagrams of A,D,E. In this case $||\Gamma|| = 2cos(\pi/h)$, where h is the Coxeter number of Γ.

3. $||\Gamma|| = 2$ if and only if Γ is one of the extended Dynkin diagrams $\tilde{A}, \tilde{D}, \tilde{E}$.

This result suggests that one can indeed assign to the Jones inclusions Dynkin diagrams. To really understand how the inclusions can be characterized in terms bipartite diagrams would require a deeper understanding of von Neumann algebras. The following argument only demonstrates that bipartite graphs naturally describe inclusions of algebras.

1. Consider a bipartite graph. Assign to each white vertex linear space $W(w)$ and to each edge of a linear space $W(b,w)$. Assign to a given black vertex the vector space $\oplus_{\delta e = b-w} W(b,w) \otimes W(w)$ where (b,w) corresponds to an edge ending to b.

2. Define $\mathcal{N}$ as the direct sum of algebras $End(W(w))$ associated with white vertices and $\mathcal{M}$ as direct sum of algebras $\oplus_{\delta e=b-w} End(W(b,w)) \otimes End(W(w))$ associated with black vertices.

3. There is homomorphism $N \rightarrow M$ defined by imbedding direct sum of white endomorphisms x to direct sum of tensor products x with the identity endomorphisms associated with the edges starting from x.

It is possible to show that Jones inclusions correspond to the Dynkin diagrams of A_n, D_{2n}, and E_6, E_8 and extended Dynkin diagrams of ADE type. In particular, the dual of the bi-partite graph associated with $\mathcal{M}_{n-1} \subset \mathcal{M}_n$ obtained by exchanging the roles of white and black vertices describes the inclusion $\mathcal{M}_n \subset \mathcal{M}_{n+1}$ so that two subsequent Jones inclusions might define something fundamental (the corresponding space-time dimension is $2 \times log_2(\mathcal{M} : \mathcal{N}) \leq 4$.

2.4.4 Indices for the inclusions of type III_1 factors

Type III_1 factors appear in relativistic quantum field theory defined in 4-dimensional Minkowski space [B26] . An overall summary of basic results discovered in algebraic quantum field theory is described in the lectures of Longo [A59] . In this case the inclusions for algebras of observables are induced by the inclusions for bounded regions of M^4 in axiomatic quantum field theory. Tomita's theory of modular Hilbert algebras [A77] , [B6] forms the mathematical corner stone of the theory.

The basic notion is Haag-Kastler net [A66] consisting of bounded regions of M^4. Double cone serves as a representative example. The von Neumann algebra $\mathcal{A}(O)$ is generated by observables localized in bounded region O. The net satisfies the conditions implied by local causality:

1. Isotony: $O_1 \subset O_2$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)$.
2. Locality: $O_1 \subset O_2'$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ with O' defined as $\{x : \langle x, y\rangle < 0 \text{ for all } y \in O\}$.
3. Haag duality $\mathcal{A}(O')' = \mathcal{A}(O)$.

   Besides this Poincare covariance, positive energy condition, and the existence of vacuum state is assumed.

DHR (Doplicher-Haag-Roberts) [A70] theory allows to deduce the values of Jones index and they are squares of integers in dimensions $D > 2$ so that the situation is rather trivial. The 2-dimensional case is distinguished from higher dimensional situations in that braid group replaces permutation group since the paths representing the flows permuting identical particles can be linked in $X^2 \times T$ and anyonic statistics [D71, D69] becomes possible. In the case of 2-D Minkowski space M^2 Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ plus a set of discrete values of $\mathcal{M} : \mathcal{N}$ in the range $(4, 6)$ are possible. In [A59] some values are given ($\mathcal{M} : \mathcal{N} = 5, 5.5049..., 5.236...., 5.828...$).

At least intersections of future and past light cones seem to appear naturally in TGD framework such that the boundaries of future/past directed light cones serve as seats for incoming/outgoing states defined as intersections of space-time surface with these light cones. III_1 sectors cannot thus be excluded as factors in TGD framework. On the other hand, the construction of S-matrix at space-time level is reduced to II_1 case by effective 2-dimensionality.

2.5 TGD and hyper-finite factors of type II_1: ideas and questions

By effective 2-dimensionality of the construction of quantum states the hyper-finite factors of type II_1 fit naturally to TGD framework. In particular, infinite dimensional spinors define a canonical representations of this kind of factor. The basic question is whether only hyper-finite factors of type II_1 appear in TGD framework. Affirmative answer would allow to interpret physical M-matrix as time like entanglement coefficients.

2.5.1 What kind of hyper-finite factors one can imagine in TGD?

The working hypothesis has been that only hyper-finite factors of type II_1 appear in TGD. The basic motivation has been that they allow a new view about M-matrix as an operator representable as time-like entanglement coefficients of zero energy states so that physical states would represent laws of physics in their structure. They allow also the introduction of the notion of measurement resolution directly to the definition of reaction probabilities by using Jones inclusion and the replacement of state space with a finite-dimensional state space defined by quantum spinors. This hypothesis is of course just an attractive working hypothesis and deserves to be challenged.

WCW spinors

For WCW spinor s the HFF II_1 property is very natural because of the properties of infinite-dimensional Clifford algebra and the inner product defined by the WCW geometry does not allow other factors than this. A good guess is that the values of conformal weights label the factors appearing in the tensor power defining WCW spinor s. Because of the non-degeneracy and super-symplectic symmetries the density matrix representing metric must be essentially unit matrix for each conformal weight which would be the defining characteristic of hyper-finite factor of type II_1.

Bosonic degrees of freedom

The bosonic part of the super-symplectic algebra consists of Hamiltonians of CH in one-one correspondence with those of $\delta M^4_\pm \times CP_2$. Also the Kac-Moody algebra acting leaving the light-likeness of the partonic 3-surfaces intact contributes to the bosonic degrees of freedom. The commutator of these algebras annihilates physical states and there are also Virasoro conditions associated with ordinary conformal symmetries of partonic 2-surface [K25] . The labels of Hamiltonians of WCW and spin indices contribute to bosonic degrees of freedom.

Hyper-finite factors of type II_1 result naturally if the system is an infinite tensor product finite-dimensional matrix algebra associated with finite dimensional systems [A31] . Unfortunately, neither Virasoro, symplectic nor Kac-Moody algebras do have decomposition into this kind of infinite tensor product. If bosonic degrees for super-symplectic and super-Kac Moody algebra indeed give I_∞ factor one has HFF if type II_∞. This looks the most natural option but threatens to spoil the beautiful idea about M-matrix as time-like entanglement coefficients between positive and negative energy parts of zero energy state.

The resolution of the problem is surprisingly simple and trivial after one has discovered it. The requirement that state is normalizable forces to project M-matrix to a finite-dimensional sub-space in bosonic degrees of freedom so that the reduction $I_\infty \to I_n$ occurs and one has the reduction $II_\infty \to II_1 \times I_n = II_1$ to the desired HFF.

One can consider also the possibility of taking the limit $n \to \infty$. One could indeed say that since I_∞ factor can be mapped to an infinite tensor power of $M(2,C)$ characterized by a state which is not trace, it is possible to map this representation to HFF by replacing state with trace [A31] . The question is whether the forcing the bosonic foot to fermionic shoe is physically natural. One could also regard the II_1 type notion of probability as fundamental and also argue that it is required by full super-symmetry realized also at the level of many-particle states rather than mere single particle states.

How the bosonic cutoff is realized?

Normalizability of state requires that projection to a finite-dimensional bosonic sub-space is carried out for the bosonic part of the M-matrix. This requires a cutoff in quantum numbers of super-conformal algebras. The cutoff for the values of conformal weight could be formulated by replacing integers with Z_n or with some finite field $G(p,1)$. The cutoff for the labels associated with Hamiltonians defined as an upper bound for the dimension of the representation looks also natural.

Number theoretical braids which are discrete and finite structures would define space-time correlate for this cutoff. p-Adic length scale $p \simeq 2^k$ hypothesis could be interpreted as stating the fact that only powers of p up to p^k are significant in p-adic thermodynamics which would correspond to finite field $G(k,1)$ if k is prime. This has no consequences for p-adic mass calculations since already the first two terms give practically exact results for the large primes associated with elementary particles [K55] .

Finite number of strands for the theoretical braids would serve as a correlate for the reduction of the representation of Galois group S_∞ of rationals to an infinite produce of diagonal copies of finite-dimensional Galois group so that same braid would repeat itself like a unit cell of lattice i condensed matter [A9] .

HFF of type III for field operators and HFF of type II_1 for states?

One could also argue that the Hamiltonians with fixed conformal weight are included in fermionic II_1 factor and bosonic factor I_∞ factor, and that the inclusion of conformal weights leads to a factor of type III. Conformal weight could relate to the integer appearing in the crossed product representation $III = Z \times_{cr} II_\infty$ of HFF of type III [A31] .

The value of conformal weight is non-negative for physical states which suggests that Z reduces to semigroup N so that a factor of type III would reduce to a factor of type II_∞ since trace would become finite. If unitary process corresponds to an automorphism for II_∞ factor, the action of automorphisms affecting scaling must be uni-directional. Also thermodynamical irreversibility suggests the same. The assumption that state function reduction for positive energy part of state implies unitary process for negative energy state and vice versa would only mean that the shifts for positive and negative energy parts of state are opposite so that $Z \to N$ reduction would still hold true.

HFF of type II_1 for the maxima of Kähler function?

Probabilistic interpretation allows to gain heuristic insights about whether and how hyper-finite factors of type type II_1 might be associated with WCW degrees of freedom. They can appear both in quantum fluctuating degrees of freedom associated with a given maximum of Kähler function and in the discrete space of maxima of Kähler function.

Spin glass degeneracy is the basic prediction of classical TGD and means that instead of a single maximum of Kähler function analogous to single free energy minimum of a thermodynamical system there is a fractal spin glass energy landscape with valleys inside valleys. The discretization of WCW in terms of the maxima of Kähler function crucial for the p-adicization problem, leads to the analog of spin glass energy landscape and hyper-finite factor of type II_1 might be the appropriate description of the situation.

The presence of the tensor product structure is a powerful additional constraint and something analogous to this should emerge in WCW degrees of freedom. Fractality of the many-sheeted space-time is a natural candidate here since the decomposition of the original geometric structure to parts and replacing them with the scaled down variant of original structure is the geometric analog of forming a tensor power of the original structure.

2.5.2 Direct sum of HFFs of type II_1 as a minimal option

HFF II_1 property for the Clifford algebra of WCW means a definite distinction from the ordinary Clifford algebra defined by the fermionic oscillator operators since the trace of the unit matrix of the Clifford algebra is normalized to one. This does not affect the anti-commutation relations at the basic level and delta functions can appear in them at space-time level. At the level of momentum space I_∞ property requires discrete basis and anti-commutators involve only Kronecker deltas. This conforms with the fact that HFF of type II_1 can be identified as the Clifford algebra associated with a separable Hilbert space.

II_∞ factor or direct sum of HFFs of type II_1?

The expectation is that super-symplectic algebra is a direct sum over HFFs of type II_1 labeled by the radial conformal weight. In the same manner the algebra defined by fermionic anti-commutation relations at partonic 2-surface would decompose to a direct sum of algebras labeled by the conformal weight associated with the light-like coordinate of X^3_l. Super-conformal symmetry suggests that also the configuration space degrees of freedom correspond to a direct sum of HFFs of type II_1.

One can of course ask why not $II_\infty = I_\infty \times II_1$ structures so that one would have single factor rather than a direct sum of factors.

1. The physical motivation is that the direct sum property allow to decompose M-matrix to direct summands associated with various sectors with weights whose moduli squared have an interpretation in terms of the density matrix. This is also consistent with p-adic thermodynamics where conformal weights take the place of energy eigen values.

2. II_∞ property would predict automorphisms scaling the trace by an arbitrary positive real number $\lambda \in R_+$. These automorphisms would require the scaling of the trace of the projectors of Clifford algebra having values in the range $[0, 1]$ and it is difficult to imagine how these automorphisms could be realized geometrically.

How HFF property reflects itself in the construction of geometry of WCW?

The interesting question is what HFF property and finite measurement resolution realizing itself as the use of projection operators means concretely at the level of WCW geometry.

Super-Hamiltonians define the Clifford algebra of the configuration space. Super-conformal symmetry suggests that the unavoidable restriction to projection operators instead of complex rays is realized also WCW degrees of freedom. Of course, infinite precision in the determination of the shape of 3-surface would be physically a completely unrealistic idea.

In the fermionic situation the anti-commutators for the gamma matrices associated with WCW individual Hamiltonians in 3-D sense are replaced with anti-commutators where Hamiltonians are replaced with projectors to subspaces of the space spanned by Hamiltonians. This projection is realized by restricting the anti-commutator to partonic 2-surfaces so that the anti-commutator depends only the restriction of the Hamiltonian to those surfaces.

What is interesting that the measurement resolution has a concrete particle physical meaning since the parton content of the system characterizes the projection. The larger the number of partons, the better the resolution about WCW degrees of freedom is. The degeneracy of WCW metric would be interpreted in terms of finite measurement resolution inherent to HFFs of type II_1, which is not due to Jones inclusions but due to the fact that one can project only to infinite-D subspaces rather than complex rays.

Effective 2-dimensionality in the sense that WCW Hamiltonians reduce to functionals of the partonic 2-surfaces of X^3_l rather than functionals of X^3_l could be interpreted in this manner. For a wide class of Hamiltonians actually effective 1-dimensionality holds true in accordance with conformal invariance.

The generalization of WCW Hamiltonians and super-Hamiltonians by allowing integrals over the 2-D boundaries of the patches of X^3_l would be natural and is suggested by the requirement of discretized 3-dimensionality at the level of WCW.

By quantum classical correspondence the inclusions of HFFs related to the measurement resolution should also have a geometric description. Measurement resolution corresponds to braids in given time scale and as already explained there is a hierarchy of braids in time scales coming as negative powers of two corresponding to the addition of zero energy components to positive/negative energy state. Note however that particle reactions understood as decays and fusions of braid strands could also lead to a notion of measurement resolution.

2.5.3 Bott periodicity, its generalization, and dimension $D = 8$ as an inherent property of the hyper-finite II_1 factor

Hyper-finite II_1 factor can be constructed as infinite-dimensional tensor power of the Clifford algebra $M_2(C) = C(2)$ in dimension $D = 2$. More precisely, one forms the union of the Clifford algebras $C(2n) = C(2)^{\otimes n}$ of $2n$-dimensional spaces by identifying the element $x \in C(2n)$ as block diagonal elements $diag(x, x)$ of $C(2(n + 1))$. The union of these algebras is completed in weak operator topology and can be regarded as a Clifford algebra of real infinite-dimensional separable Hilbert space and thus as sub-algebra of I_∞. Also generalizations obtained by replacing complex numbers by quaternions and octions are possible.

1. The dimension 8 is an inherent property of the hyper-finite II_1 factor since Bott periodicity theorem states $C(n+8) = C_n(16)$. In other words, the Clifford algebra $C(n+8)$ is equivalent with the algebra of 16×16 matrices with entries in $C(n)$. Or articulating it still differently: $C(n+8)$ can be regarded as 16×16 dimensional module with $C(n)$ valued coefficients. Hence the elements in the union defining the canonical representation of hyper-finite II_1 factor are $16^n \times 16^n$ matrices having $C(0)$, $C(2)$, $C(4)$ or $C(6)$ valued valued elements.

2. The idea about a local variant of the infinite-dimensional Clifford algebra defined by power series of space-time coordinate with Taylor coefficients which are Clifford algebra elements fixes the interpretation. The representation as a linear combination of the generators of Clifford algebra of the finite-dimensional space allows quantum generalization only in the case of Minkowski spaces. However, if Clifford algebra generators are representable as gamma *matrices*, the powers of coordinate can be absorbed to the Clifford algebra and the local algebra is lost. Only if the generators are represented as quantum versions of octonions allowing no matrix representation because of their non-associativity, the local algebra makes sense. From this it is easy to deduce both quantum and classical TGD.

2.5.4 The interpretation of Jones inclusions in TGD framework

By the basic self-referential property of von Neumann algebras one can consider several interpretations of Jones inclusions consistent with sub-system-system relationship, and it is better to start by considering the options that one can imagine.

How Jones inclusions relate to the new view about sub-system?

Jones inclusion characterizes the imbedding of sub-system $\mathcal{N}$ to $\mathcal{M}$and $\mathcal{M}$ as a finite-dimensional $\mathcal{N}$-module is the counterpart for the tensor product in finite-dimensional context. The possibility to express $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ states fractality and can be regarded as a kind of self-referential "Brahman=Atman identity" at the level of infinite-dimensional systems.

Also the mysterious looking almost identity $CH^2 = CH$ for the WCW would fit nicely with the identity $M \oplus M = M$. $M \otimes M \subset M$ in WCW Clifford algebra degrees of freedom is also implied and the construction of $\mathcal{M}$ as a union of tensor powers of $C(2)$ suggests that $M \otimes M$ allows $\mathcal{M} : \mathcal{N} = 4$ inclusion to $\mathcal{M}$. This paradoxical result conforms with the strange self-referential property of factors of II_1.

The notion of many-sheeted space-time forces a considerable generalization of the notion of sub-system and simple tensor product description is not enough. Topological picture based on the length scale resolution suggests even the possibility of entanglement between sub-systems of unentangled sub-systems. The possibility that hyper-finite II_1-factors describe the physics of TGD also in bosonic degrees of freedom is suggested by WCW super-symmetry. On the other hand, bosonic degrees could naturally correspond to I_∞ factor so that hyper-finite II_∞ would be the net result.

The most general view is that Jones inclusion describes all kinds of sub-system-system inclusions. The possibility to assign conformal field theory to the inclusion gives hopes of rather detailed view about dynamics of inclusion.

1. The topological condensation of space-time sheet to a larger space-time sheet mediated by wormhole contacts could be regarded as Jones inclusion. $\mathcal{N}$ would correspond to the condensing space-time sheet, $\mathcal{M}$ to the system consisting of both space-time sheets, and $\sqrt{\mathcal{M} : \mathcal{N}}$ would characterize the number of quantum spinorial degrees of freedom associated with the interaction between space-time sheets. Note that by general results $\mathcal{M} : \mathcal{N}$ characterizes the fractal dimension of quantum group ($\mathcal{M} : \mathcal{N} < 4$) or Kac-Moody algebra ($\mathcal{M} : \mathcal{N} = 4$) [B16] .

2. The branchings of space-time sheets (space-time surface is thus homologically like branching like of Feynman diagram) correspond naturally to n-particle vertices in TGD framework. What is nice is that vertices are nice 2-dimensional surfaces rather than surfaces having typically pinch singularities. Jones inclusion would naturally appear as inclusion of operator spaces $\mathcal{N}_i$ (essentially Fock spaces for fermionic oscillator operators) creating states at various lines as sub-spaces $N_i \subset M$ of operators creating states in common von Neumann factor $\mathcal{M}$. This would allow to construct vertices and vertices in natural manner using quantum groups or Kac-Moody algebras.

   The fundamental $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}$ inclusion suggests a concrete representation based on the identification $N_i = M$, where M is the universal Clifford algebra associated with incoming line and $\mathcal{N}$ is defined by the condition that $\mathcal{M}/\mathcal{N}$ is the quantum variant of Clifford algebra of H. N-particle vertices could be defined as traces of Connes products of the operators creating incoming and outgoing states. It will be found that this leads to a master formula for S-matrix if the generalization of the old-fashioned string model duality implying that all generalized Feynman diagrams reduce to diagrams involving only single vertex is accepted.

3. If 4-surfaces can branch as the construction of vertices requires, it is difficult to argue that 3-surfaces and partonic/stringy 2-surfaces could not do the same. As a matter fact, the master formula for S-matrix to be discussed later explains the branching of 4-surfaces as an apparent effect. Despite this one can consider the possibility that this kind of joins are possible so that a new kind of mechanism of topological condensation would become possible. 3-space-sheets and partonic 2-surfaces whose p-adic fractality is characterized by different p-adic primes could be connected by "joins" representing branchings of 2-surfaces. The structures formed by soap film foam provide a very concrete illustration about what would happen. In the TGD based model of hadrons [K58] it has been assumed that join along boundaries bonds (JABs) connect quark space-time space-time sheets to the hadronic space-time sheet. The problem is that, at least for identical primes, the formation of join along boundaries bond fuses two systems to single bound state. If JABs are replaced joins, this objection is circumvented.

4. The space-time correlate for the formation of bound states is the formation of JABs. Standard intuition tells that the number of degrees of freedom associated with the bound state is smaller than the number of degrees of freedom associated with the pair of free systems. Hence the inclusion of the bound state to the tensor product could be regarded as Jones inclusion. On the other hand, one could argue that the JABs carry additional vibrational degrees of freedom so that the idea about reduction of degrees of freedom might be wrong: free system could be regarded as sub-system of bound state by Jones inclusion. The self-referential holographic properties of von Neumann algebras allow both interpretations: any system can be regarded as sub-system of any system in accordance with the bootstrap idea.

5. Maximal deterministic regions inside given space-time sheet bounded by light-like causal determinants define also sub-systems in a natural manner and also their inclusions would naturally correspond to Jones inclusions.

6. The TGD inspired model for topological quantum computation involves the magnetic flux tubes defined by join along boundaries bonds connecting space-time sheets having light-like boundaries. These tubes condensed to background 3-space can become linked and knotted and code for quantum computations in this manner. In this case the addition of new strand to the system corresponds to Jones inclusion in the hierarchy associated with inclusion $\mathcal{N} \subset \mathcal{M}$. The anyon states associated with strands would be represented by a finite tensor product of quantum spinors assignable to $\mathcal{M}/\mathcal{N}$ and representing quantum counterpart of H-spinors.

One can regard $\mathcal{M}:\mathcal{N}$ degrees of freedom correspond to quantum group or Kac-Moody degrees of freedom. Quantum group degrees of freedom relate closely to the conformal and topological degrees of freedom as the connection of II_1 factors with topological quantum field theories and braid matrices suggests itself. For the canonical inclusion this factorization would correspond to factorization of quantum H-spinor from WCW spinor .

A more detailed study of canonical inclusions to be carried out later demonstrates what this factorization corresponds at the space-time level to a formation of space-time sheets which can be regarded as multiple coverings of M^4 and CP_2 with invariance group $G = G_a \times G_b \subset SL(2,C) \times SU(2)$, $SU(2) \subset SU(3)$. The unexpected outcome is that Planck constants assignable to M^4 and CP_2 degrees of freedom depend on the canonical inclusions. The existence of macroscopic quantum phases with arbitrarily large Planck constants is predicted.

It would seem possible to assign the $\mathcal{M}:\mathcal{N}$ degrees quantum spinorial degrees of freedom to the interface between subsystems represented by $\mathcal{N}$ and $\mathcal{M}$. The interface could correspond to the wormhole contacts, joins, JABs, or light-like causal determinants serving as boundary between maximal deterministic regions, etc... In terms of the bipartite diagrams representing the inclusions, joins (say) would correspond to the edges connecting white vertices representing sub-system (the entire system without the joins) to black vertices (entire system).

About the interpretation of $\mathcal{M}:\mathcal{N}$ degrees of freedom

The Clifford algebra $\mathcal{N}$ associated with a system formed by two space-time sheet can be regarded as $1 \leq \mathcal{M}:\mathcal{N} \leq 4$-dimensional module having $\mathcal{N}$ as its coefficients. It is possible to imagine several interpretations the degrees of freedom labeled by β.

1. The $\beta = \mathcal{M}:\mathcal{N}$ degrees of freedom could relate to the interaction of the space-time sheets. Beraha numbers appear in the construction of S-matrices of topological quantum field theories and an interpretation in terms of braids is possible. This would suggest that the interaction between space-time sheets can be described in terms of conformal quantum field theory and the S-matrices associated with braids describe this interaction. Jones inclusions would characterize the effective number of active conformal degrees of freedom. At $n = 3$ limit these degrees of freedom disappear completely since the conformal field theory defined by the Chern-Simons action describing this interaction would become trivial ($c = 0$ as will be found).

2. The interpretation in terms of imbedding space Clifford algebra would suggest that β-dimensional Clifford algebra of $\sqrt{\beta}$-dimensional spinor space is in question. For $\beta = 4$

the algebra would be the Clifford algebra of 2-dimensional space. $\mathcal{M}/\mathcal{N}$ would have interpretation as complex quantum spinors with components satisfying $z_1z_2 = qz_2z_1$ and its conjugate and having fractal complex dimension $\sqrt{\beta}$. This would conform with the effective 2-dimensionality of TGD. For $\beta < 4$ the fractal dimension of partonic quantum spinors defining the basic conformal fields would be reduced and become $d = 1$ for $n = 3$: the interpretation is in terms of strong correlations caused by the non-commutativity of the components of quantum spinor. For number theoretical generalizations of infinite-dimensional Clifford algebras $Cl(C)$ obtained by replacing C with Abelian complexification of quaternions or octonions one would obtain higher-dimensional spinors.

2.5.5 WCW, space-time, and imbedding space and hyper-finite type II_1 factors

The preceding considerations have by-passed the question about the relationship of WCW tangent space to its Clifford algebra. Also the relationship between space-time and imbedding space and their quantum variants could be better. In particular, one should understand how effective 2-dimensionality can be consistent with the 4-dimensionality of space-time.

Super-conformal symmetry and WCW Poisson algebra as hyper-finite type II_1 factor

It would be highly desirable to achieve also a description of the WCW degrees of freedom using von Neumann algebras. Super-conformal symmetry relating fermionic degrees of freedom and WCW degrees of freedom suggests that this might be the case. Super-symplectic algebra has as its generators configuration space Hamiltonians and their super-counterparts identifiable as CH gamma matrices. Super-symmetry requires that the Clifford algebra of CH and the Hamiltonian vector fields of CH with symplectic central extension both define hyper-finite II_1 factors. By super-symmetry Poisson bracket corresponds to an anti-commutator for gamma matrices. The ordinary quantized version of Poisson bracket is obtained as $\{P_i, Q_j\} \rightarrow [P_i, Q_j] = J_{ij}Id$. Finite trace version results by assuming that Id corresponds to the projector CH Clifford algebra having unit norm. The presence of zero modes means direct integral over these factors.

WCW gamma matrices anti-commuting to identity operator with unit norm corresponds to the tangent space $T(CH)$ of CH. Thus it would be not be surprising if $T(CH)$ could be imbedded in the sigma matrix algebra as a sub-space of operators defined by the gamma matrices generating this algebra. At least for $\beta = 4$ construction of hyper-finite II_1 factor this definitely makes sense.

The dimension of WCW defined as the trace of the projection operator to the sub-space spanned by gamma matrices is obviously zero. Thus WCW has in this sense the dimensionality of single space-time point. This sounds perhaps absurd but the generalization of the number concept implied by infinite primes indeed leads to the view that single space-time point is infinitely structured in the number theoretical sense although in the real sense all states of the point are equivalen. The reason is that there is infinitely many numbers expressible as ratios of infinite integers having unit real norm in the real sense but having different p-adic norms.

How to understand the dimensions of space-time and imbedding space?

One should be able to understand the dimensions of 3-space, space-time and imbedding space in a convincing matter in the proposed framework. There is also the question whether space-time and imbedding space emerge uniquely from the mathematics of von Neumann algebras alone.

1. The dimensions of space-time and imbedding space

Two sub-sequent inclusions dual to each other define a special kind of inclusion giving rise to a quantum counterpart of $D = 4$ naturally. This would mean that space-time is something which emerges at the level of cognitive states.

The special role of classical division algebras in the construction of quantum TGD [K84] , $D = 8$ Bott periodicity generalized to quantum context, plus self-referential property of type II_1 factors might explain why 8-dimensional imbedding space is the only possibility.

State space has naturally quantum dimension $D \leq 8$ as the following simple argument shows. The space of quantum states has quark and lepton sectors which both are super-symmetric implying

$D \leq 4$ for each. Since these sectors correspond to different Hamiltonian algebras (triality one for quarks and triality zero for leptonic sector), the state space has quantum dimension $D \leq 8$.

2. How the lacking two space-time dimensions emerge?

3-surface is the basic dynamical unit in TGD framework. This seems to be in conflict with the effective 2-dimensionality [K84] meaning that partonic 2-surface code for quantum states, and with the fact that hyper-finite II_1 factors have intrinsic quantum dimension 2.

A possible resolution of the problem is that the foliation of 3-surface by partonic two-surfaces defines a one-dimensional direct integral of isomorphic hyper-finite type II_1 factors, and the zero mode labeling the 2-surfaces in the foliation serves as the third spatial coordinate. For a given 3-surface the contribution to the WCW metric can come only from 2-D partonic surfaces defined as intersections of 3-D light-like CDs with $X^7_\pm$ [K22] . Hence the direct integral should somehow relate to the classical non-determinism of Kähler action.

1. The one-parameter family of intersections of light-like CD with $X^7_\pm$ inside $X^4 \cap X^7_\pm$ could indeed be basically due to the classical non-determinism of Kähler action. The contribution to the metric from the normal light-like direction to $X^3 = X^4 \cap X^7_\pm$ can cause the vanishing of the metric determinant $\sqrt{g_4}$ of the space-time metric at $X^2 \subset X^3$ under some conditions on X^2. This would mean that the space-time surface $X^4(X^3)$ is not uniquely determined by the minimization principle defining the value of the Kähler action, and the complete dynamical specification of X^3 requires the specification of partonic 2-surfaces X^2_i with $\sqrt{g_4} = 0$.

2. The known solutions of field equations [K12] define a double foliation of the space-time surface defined by Hamilton-Jacobi coordinates consisting of complex transversal coordinate and two light-like coordinates for M^4 (rather than space-time surface). Number theoretical considerations inspire the hypothesis that this foliation exists always [K84] . Hence a natural hypothesis is that the allowed partonic 2-surfaces correspond to the 2-surfaces in the restriction of the double foliation of the space-time surface by partonic 2-surfaces to X^3, and are thus locally parameterized by single parameter defining the third spatial coordinate.

3. There is however also a second light-like coordinate involved and one might ask whether both light-like coordinates appear in the direct sum decomposition of II_1 factors defining $T(CH)$. The presence of two kinds of light-like CDs would provide the lacking two space-time coordinates and quantum dimension $D = 4$ would emerge at the limit of full non-determinism. Note that the duality of space-like partonic and light-like stringy 2-surfaces conforms with this interpretation since it corresponds to a selection of partonic/stringy 2-surface inside given 3-D CD whereas the dual pairs correspond to different CDs.

4. That the quantum dimension would be $2D_q = \beta < 4$ above CP_2 length scale conforms with the fact that non-determinism is only partial and time direction is dynamically frozen to a high degree. For vacuum extremals there is strong non-determinism but in this case there is no real dynamics. For CP_2 type extremals, which are not vacuum extremals as far action and small perturbations are considered, and which correspond to $\beta = 4$ there is a complete non-determinism in time direction since the M^4 projection of the extremal is a light-like random curve and there is full 4-D dynamics. Light-likeness gives rise to conformal symmetry consistent with the emergence of Kac Moody algebra [K12] .

3. Time and cognition

In a completely deterministic physics time dimension is strictly speaking redundant since the information about physical states is coded by the initial values at 3-dimensional slice of space-time. Hence the notion of time should emerge at the level of cognitive representations possible by to the non-determinism of the classical dynamics of TGD.

Since Jones inclusion means the emergence of cognitive representation, the space-time view about physics should correspond to cognitive representations provided by Feynman diagram states with zero energy with entanglement defined by a two-sided projection of the lowest level S-matrix. These states would represent the "laws of quantum physics" cognitively. Also space-time surface serves as a classical correlate for the evolution by quantum jumps with maximal deterministic regions serving as correlates of quantum states. Thus the classical non-determinism making possible cognitive representations would bring in time. The fact that quantum dimension of space-time is smaller than $D = 4$ would reflect the fact that the loss of determinism is not complete.

4. Do space-time and imbedding space emerge from the theory of von Neumann algebras and number theory?

The considerations above force to ask whether the notions of space-time and imbedding space emerge from von Neumann algebras as predictions rather than input. The fact that it seems possible to formulate the S-matrix and its generalization in terms of inherent properties of infinite-dimensional Clifford algebras suggest that this might be the case.

Inner automorphisms as universal gauge symmetries?

The continuous outer automorphisms Δ^{it} of HFFs of type III are not completely unique and one can worry about the interpretation of the inner automorphisms. A possible resolution of the worries is that inner automorphisms act as universal gauge symmetries containing various super-conformal symmetries as a special case. For hyper-finite factors of type II_1 in the representation as an infinite tensor power of $M_2(C)$ this would mean that the transformations non-trivial in a finite number of tensor factors only act as analogs of local gauge symmetries. In the representation as a group algebra of S_∞ all unitary transformations acting on a finite number of braid strands act as gauge transformations whereas the infinite powers $P \times P \times ...$, $P \in S_n$, would act as counterparts of global gauge transformations. In particular, the Galois group of the closure of rationals would act as local gauge transformations but diagonally represented finite Galois groups would act like global gauge transformations and periodicity would make possible to have finite braids as space-time correlates without a loss of information.

Do unitary isomorphisms between tensor powers of II_1 define vertices?

What would be left would be the construction of unitary isomorphisms between the tensor products of the HFFs of type $II_1 \otimes I_n = II_1$ at the partonic 2-surfaces defining the vertices. This would be the only new element added to the construction of braiding M-matrices.

As a matter fact, this element is actually not completely new since it generalizes the fusion rules of conformal field theories, about which standard example is the fusion rule $\phi_i = c_i{}^{jk}\phi_j\phi_k$ for primary fields. These fusion rules would tell how a state of incoming HFF decomposes to the states of tensor product of two outgoing HFFs.

These rules indeed have interpretation in terms of Connes tensor products $\mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$ for which the sub-factor $\mathcal{N}$ takes the role of complex numbers [A40] so that one has $\mathcal{M}$ becomes $\mathcal{N}$ bimodule and "quantum quantum states" have $\mathcal{N}$ as coefficients instead of complex numbers. In TGD framework this has interpretation as quantum measurement resolution characterized by $\mathcal{N}$ (the group G characterizing leaving the elements of $\mathcal{N}$ invariant defines the measured quantum numbers).

2.5.6 Quaternions, octonions, and hyper-finite type II_1 factors

Quaternions and octonions as well as their hyper counterparts obtained by multiplying imaginary units by commuting $\sqrt{-1}$ and forming a sub-space of complexified division algebra, are in in a central role in the number theoretical vision about quantum TGD [K84] . Therefore the question arises whether complexified quaternions and perhaps even octonions could be somehow inherent properties of von Neumann algebras. One can also wonder whether the quantum counterparts of quaternions and octonions could emerge naturally from von Neumann algebras. The following considerations allow to get grasp of the problem.

Quantum quaternions and quantum octonions

Quantum quaternions have been constructed as deformation of quaternions [A64] . The key observation that the Glebsch Gordan coefficients for the tensor product $3 \otimes 3 = 5 \oplus \oplus 3 \oplus 1$ of spin 1 representation of $SU(2)$ with itself gives the anti-commutative part of quaternionic product as spin

1 part in the decomposition whereas the commutative part giving spin 0 representation is identifiable as the scalar product of the imaginary parts. By combining spin 0 and spin 1 representations, quaternionic product can be expressed in terms of Glebsh-Gordan coefficients. By replacing GGC:s by their quantum group versions for group $sl(2)_q$, one obtains quantum quaternions.

There are two different proposals for the construction of quantum octonions [A26, A1] . Also now the idea is to express quaternionic and octonionic multiplication in terms of Glebsch-Gordan coefficients and replace them with their quantum versions.

1. The first proposal [A26] relies on the observation that for the tensor product of $j = 3$ representations of $SU(2)$ the Glebsch-Gordan coefficients for $7 \otimes 7 \to 7$ in $7 \otimes 7 = 9 \oplus 7 \oplus 5 \oplus 3 \oplus 1$ defines a product, which is equivalent with the antisymmetric part of the product of octonionic imaginary units. As a matter fact, the antisymmetry defines 7-dimensional Malcev algebra defined by the anti-commutator of octonion units and satisfying b definition the identity

$$[[x,y,z],x] = [x,y,[x,z]] \quad , \quad [x,y,z] \equiv [x,[y,z]] + [y,[z,x]] + [z,[x,y]] \quad . \tag{2.5.1}$$

   7-element Malcev algebra defining derivations of octonionic algebra is the only complex Malcev algebra not reducing to a Lie algebra. The $j = 0$ part of the product corresponds also now to scalar product for imaginary units. Octonions are constructed as sums of $j = 0$ and $j = 3$ parts and quantum Glebsch-Gordan coefficients define the octonionic product.

2. In the second proposal [A1] the quantum group associated with $SO(8)$ is used. This representation does not allow unit but produces a quantum version of octonionic triality assigning to three octonions a real number.

Quaternionic or octonionic quantum mechanics?

There have been numerous attempts to introduce quaternions and octonions to quantum theory. Quaternionic or octonionic quantum mechanics, which means the replacement of the complex numbers as coefficient field of Hilbert space with quaternions or octonions, is the most obvious approach (for example and references to the literature see for instance [A58] .

In both cases non-commutativity poses serious interpretational problems. In the octonionic case the non-associativity causes even more serious obstacles [B14, A58] , [B14] .

1. Assuming that an orthonormalized state basis with respect to an octonion valued inner product has been found, the multiplication of any basis with octonion spoils the orthonormality. The proposal to circumvent this difficulty discussed in [B14] , [B14] eliminates non-associativity by assuming that octonions multiply states one by one (rather than multiplying each other before multiplying the state). Effectively this means that octonions are replaced with 8×8-matrices.

2. The definition of the tensor product leads also to difficulties since associativity is lost (recall that Yang-Baxter equation codes for associativity in case of braid statistics [A55]).

3. The notion of hermitian conjugation is problematic and forces a selection of a preferred imaginary unit, which does not look nice. Note however that the local selection of a preferred imaginary unit is in a key role in the proposed construction of space-time surfaces as

   hyper-quaternionic or co-hyper-quaternionic surfaces and allows to interpret space-time surfaces either as surfaces in 8-D Minkowski space M^8 of hyper-octonions or in $M^4 \times CP_2$. This selection turns out to have quite different interpretation in the proposed framework.

Hyper-finite factor II_1 has a natural Hyper-Kähler structure

In the case of hyper-finite factors of type II_1 quaternions a more natural approach is based on the generalization of the Hyper-Kähler structure rather than quaternionic quantum mechanics. The reason is that also WCW tangent space should and is expected to have this structure [K22] . The Hilbert space remains a complex Hilbert space but the quaternionic units are represented as operators in Hilbert space. The selection of the preferred unit is necessary and natural. The identity operator representing quaternionic real unit has trace equal to one, is expected to give rise to the series of quantum quaternion algebras in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ having interpretation as N-modules.

The representation of the quaternion units is rather explicit in the structure of hyper-finite II_1 factor. The $\mathcal{M} : \mathcal{N} \equiv \beta = 4$ hierarchical construction can be regarded as Connes tensor product of infinite number of 4-D Clifford algebras of Euclidian plane with Euclidian signature of metric ($diag(-1,-1)$). This algebra is nothing but the quaternionic algebra in the representation of quaternionic imaginary units by Pauli spin matrices multiplied by i.

The imaginary unit of the underlying complex Hilbert space must be chosen and there is whole sphere S^2 of choices and in every point of WCW the choice can be made differently. The space-time correlate for this local choice of preferred hyper-octonionic unit [K84] . At the level of WCW geometry the quaternion structure of the tangent space means the existence of Hyper-Kähler structure guaranteeing that WCW has a vanishing Einstein tensor. It it would not vanish, curvature scalar would be infinite by symmetric space property (as in case of loop spaces) and induce a divergence in the functional integral over 3-surfaces from the expansion of $\sqrt{g}$ [K22] .

The quaternionic units for the II_1 factor, are simply limiting case for the direct sums of 2×2 units normalized to one. Generalizing from $\beta = 4$ to $\beta < 4$, the natural expectation is that the representation of the algebra as $\beta = \mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module gives rise to quantum quaternions with quaternion units defined as infinite sums of $\sqrt{\beta} \times \sqrt{\beta}$ matrices.

At Hilbert space level one has an infinite Connes tensor product of 2-component spinor spaces on which quaternionic matrices have a natural action. The tensor product of Clifford algebras gives the algebra of 2×2 quaternionic matrices acting on 2-component quaternionic spinors (complex 4-component spinors). Thus double inclusion could correspond to (hyper-)quaternionic structure at space-time level. Note however that the correspondence is not complete since hyper-quaternions appear at space-time level and quaternions at Hilbert space level.

Von Neumann algebras and octonions

The octonionic generalization of the Hyper-Kähler manifold does not make sense as such since octonionic units are not representable as linear operators. The allowance of anti-linear operators inherently present in von Neumann algebras could however save the situation. Indeed, the Cayley-Dickson construction for the division algebras (for a nice explanation see [A21]), which allows to extend any * algebra, and thus also any von Neumann algebra, by adding an imaginary unit it and identified as *, comes in rescue.

The basic idea of the Cayley-Dickson construction is following. The * operator, call it J, representing a conjugation defines an *anti-linear* operator in the original algebra A. One can extend A by adding this operator as a new element to the algebra. The conditions satisfied by J are

$$a(Jb) = J(a^*b) \ , \quad (aJ)b = (ab^*)J \ , \quad (Ja)(bJ^{-1}) = (ab)^* \ . \tag{2.5.2}$$

In the associative case the conditions are equivalent to the first condition.

It is intuitively clear that this addition extends the hyper-Kähler structure to an octonionic structure at the level of the operator algebra. The quantum version of the octonionic algebra is fixed by the quantum quaternion algebra uniquely and is consistent with the Cayley-Dickson construction. It is not clear whether the construction is equivalent with either of the earlier proposals [A26, A1] . It would however seem that the proposal is simpler.

Physical interpretation of quantum octonion structure

Without further restrictions the extension by J would mean that vertices contain operators, which are superpositions of linear and anti-linear operators. This would give superpositions of states and their time-reversals and mean that state could be a superposition of states with opposite values of say fermion numbers. The problem disappears if either the linear operators A or anti-linear operators JA can be used to construct physical states from vacuum. The fact, that space-time surfaces are either hyper-quaternionic or co-hyper-quaternionic, is a space-time correlate for this restriction.

The $HQ - coHQ$ duality discussed in [K84] states that the descriptions based on hyper-quaternionic and co-hyper-quaternionic surfaces are dual to each other. The duality can have two meanings.

1. The vacuum is invariant under J so that one can use either complexified quaternionic operators A or their co-counterparts of form JA to create physical states from vacuum.

2. The vacuum is not invariant under J. This could relate to the breaking of CP and T invariance known to occur in meson-antimeson systems. In TGD framework two kinds of vacua are predicted corresponding intuitively to vacua in which either the product of all positive or negative energy fermionic oscillator operators defines the vacuum state, and these two vacua could correspond to a vacuum and its J conjugate, and thus to positive and negative energy states. In this case the two state spaces would not be equivalent although the physics associated with them would be equivalent.

The considerations of [K84] related to the detailed dynamics of $HQ - coHQ$ duality demonstrate that the variational principles defining the dynamics of hyper-quaternionic and co-hyper-quaternionic space-time surfaces are antagonistic and correspond to world as seen by a conscientous book-keeper on one hand and an imaginative artist on the other hand. HQ case is conservative: differences measured by the magnitude of Kähler action tend to be minimized, the dynamics is highly predictive, and minimizes the classical energy of the initial state. $coHQ$ case is radical: differences are maximized (this is what the construction of sensory representations would require). The interpretation proposed in [K84] was that the two space-time dynamics are just different predictions for what would happen (has happened) if no quantum jumps would occur (had occurred). A stronger assumption is that these two views are associated with systems related by time reversal symmetry.

What comes in mind first is that this antagonism follows from the assumption that these dynamics are actually time-reversals of each other with respect to M^4 time (the rapid elimination of differences in the first dynamics would correspond to their rapid enhancement in the second dynamics). This is not the case so that T and CP symmetries are predicted to be broken in accordance with the CP breaking in meson-antimeson systems [K52] and cosmological matter-antimatter asymmetry [K76] .

2.5.7 Does the hierarchy of infinite primes relate to the hierarchy of II_1 factors?

The hierarchy of Feynman diagrams accompanying the hierarchy defined by Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset ...$ gives a concrete representation for the hierarchy of cognitive dynamics providing a representation for the material world at the lowest level of the hierarchy. This hierarchy seems to relate directly to the hierarchy of space-time sheets.

Also the construction of infinite primes [K82] leads to an infinite hierarchy. Infinite primes at the lowest level correspond to polynomials of single variable x_1 with rational coefficients, next level to polynomials x_1 for which coefficients are rational functions of variable x_2, etc... so that a natural ordering of the variables is involved.

If the variables x_i are hyper-octonions (subs-space of complexified octonions for which elements are of form $x + \sqrt{-1}y$, where x is real number and y imaginary octonion and $\sqrt{-1}$ is commuting imaginary unit, this hierarchy of states could provide a realistic representation of physical states as far as quantum numbers related to imbedding space degrees of freedom are considered in M^8 picture dual to $M^4 \times CP_2$ picture [K84] . Infinite primes are mapped to space-time surfaces in a manner analogous to the mapping of polynomials to the loci of their zeros so that infinite primes, integers, and rationals become concrete geometrical objects.

Infinite primes are also obtained by a repeated second quantization of a super-symmetric arithmetic quantum field theory. Infinite rational numbers correspond in this description to pairs of positive energy and negative energy states of opposite energies having interpretation as pairs of initial and final states so that higher level states indeed represent transitions between the states. For these reasons this hierarchy has been interpreted as a correlate for a cognitive hierarchy coding information about quantum dynamics at lower levels. This hierarchy has also been assigned with the hierarchy of space-time sheets. Just as the hierarchy of generalized Feynman diagrams provides self representations of the lowest matter level and is coded by it, finite primes code the hierarchy of infinite primes.

Infinite primes, integers, and rationals have finite p-adic norms equal to 1, and one can wonder whether a Hilbert space like structure with dimension given by an infinite prime or integer makes sense, and whether it has anything to do with the Hilbert space for which dimension is infinite in the sense of the limiting value for a dimension of sub-space. The Hilbert spaces with dimension equal to infinite prime would define primes for the tensor product of these spaces. The dimension of this kind of space defined as any p-adic norm would be equal to one.

One cannot exclude the possibility that infinite primes could express the infinite dimensions of hyper-finite III_1 factors, which cannot be excluded and correspond to that part of quantum TGD which relates to the imbedding space rather than space-time surface. Indeed, infinite primes code naturally for the quantum numbers associated with the imbedding space. Secondly, the appearance of 7-D light-like causal determinants $X^7_\pm = M^4_\pm \times CP_2$ forming nested structures in the construction of S-matrix brings in mind similar nested structures of algebraic quantum field theory [B26] . If this is were the case, the hierarchy of Beraha numbers possibly associated with the phase resolution could correspond to hyper-finite factors of type II_1, and the decomposition of space-time surface to regions labeled by p-adic primes and characterized by infinite primes could correspond to hyper-finite factors of type III_1 and represent imbedding space degrees of freedom.

The state space would in this picture correspond to the tensor products of hyper-finite factors of type II_1 and III_1 (of course, also factors I_n and I_∞ are also possible). III_1 factors could be assigned to the sub-WCWs defined by 3-surfaces in regions of M^4 expressible in terms of unions and intersections of $X^7_\pm = M^4_\pm \times CP_2$. By conservation of four-momentum, bounded regions of this kind are possible only for the states of zero net energy appearing at the higher levels of hierarchy. These sub-WCWs would be characterized by the positions of the tips of light cones $M^4_\pm \subset M^4$ involved. This indeed brings in continuous spectrum of four-momenta forcing to introduce non-separable Hilbert spaces for momentum eigen states and necessitating III_1 factors. Infinities would be avoided since the dynamics proper would occur at the level of space-time surfaces and involve only II_1 factors.

2.6 Could HFFs of type III have application in TGD framework?

One can imagine several manners for how HFFs of type III could emerge in TGD although the proposed view about M-matrix in zero energy ontology suggests that HFFs of type III_1 should be only an auxiliary tool at best. Same is suggested with interpretational problems associated with them. Both TGD inspired quantum measurement theory, the idea about a variant of HFF of type II_1 analogous to a local gauge algebra, and some other arguments, suggest that HFFs of type III could be seen as a useful idealization allowing to make non-trivial conjectures both about quantum TGD and about HFFs of type III. Quantum fields would correspond to HFFs of type III and II_∞ whereas physical states (M-matrix) would correspond to HFF of type II_1. I have summarized first the problems of III_1 factors so that reader can decide whether the further reading is worth of it.

2.6.1 Problems associated with the physical interpretation of III_1 factors

Algebraic quantum field theory approach [B13, B26] has led to a considerable understanding of relativistic quantum field theories in terms of hyper-finite III_1 factors. There are however several reasons to suspect that the resulting picture is in conflict with physical intuition. Also the infinities of non-trivial relativistic QFTs suggest that something goes wrong.

Are the infinities of quantum field theories due the wrong type of von Neumann algebra?

The infinities of quantum field theories involve basically infinite traces and it is now known that the algebras of observables for relativistic quantum field theories for bounded regions of Minkowski space correspond to hyper-finite III_1 algebras, for which non-trivial traces are always infinite. This might be the basic cause of the divergence problems of relativistic quantum field theory.

On basis of this observations there is some temptation to think that the finite traces of hyper-finite II_1 algebras might provide a resolution to the problems but not necessarily in QFT context. One can play with the thought that the subtraction of infinities might be actually a process in which III_1 algebra is transformed to II_1 algebra. A more plausible idea suggested by dimensional regularization is that the elimination of infinities actually gives rise to II_1 inclusion at the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. It is indeed known that the dimensional regularization procedure of quantum field theories can be formulated in terms of bi-algebras assignable to Feynman diagrams and [A32] and the emergence of bi-algebras suggests that a connection with II_1 factors and critical role of dimension $D = 4$ might exist.

Continuum of inequivalent representations of commutation relations

There is also a second difficulty related to type III algebras. There is a continuum of inequivalent representations for canonical commutation relations [A44] . In thermodynamics this is blessing since temperature parameterizes these representations. In quantum field theory context situation is however different and this problem has been usually put under the rug.

Entanglement and von Neumann algebras

In quantum field theories where 4-D regions of space-time are assigned to observables. In this case hyper-finite type III_1 von Neumann factors appear. Also now inclusions make sense and has been studiedin fact, the parameters characterizing Jones inclusions appear also now and this due to the very general properties of the inclusions.

The algebras of type III_1 have rather counter-intuitive properties from the point of view of entanglement. For instance, product states between systems having space-like separation are not possible at all so that one can speak of intrinsic entanglement [A45] . What looks worse is that the decomposition of entangled state to product states is highly non-unique.

Mimicking the steps of von Neumann one could ask what the notion of observables could mean in TGD framework. Effective 2-dimensionality states that quantum states can be constructed using the data given at partonic or stringy 2-surfaces. This data includes also information about normal derivatives so that 3-dimensionality actually lurks in. In any case this would mean that observables are assignable to 2-D surfaces. This would suggest that hyper-finite II_1 factors appear in quantum TGD at least as the contribution of single space-time surface to S-matrix is considered. The contributions for WCW degrees of freedom meaning functional (not path-) integral over 3-surfaces could of course change the situation.

Also in case of II_1 factors, entanglement shows completely new features which need not however be in conflict with TGD inspired view about entanglement. The eigen values of density matrices are infinitely degenerate and quantum measurement can remove this degeneracy only partially. TGD inspired theory of consciousness has led to the identification of rational (more generally algebraic entanglement) as bound state entanglement stable in state function reduction. When an infinite number of states are entangled, the entanglement would correspond to rational (algebraic number) valued traces for the projections to the eigen states of the density matrix. The symplectic transformations of CP_2 are almost $U(1)$ gauge symmetries broken only by classical gravitation. They imply a gigantic spin glass degeneracy which could be behind the infinite degeneracies of eigen states of density matrices in case of II_1 factors.

2.6.2 Quantum measurement theory and HFFs of type III

The attempt to interpret the HFFs of type III in terms of quantum measurement theory based on Jones inclusions leads to highly non-trivial conjectures about these factors.

Could the scalings of trace relate to quantum measurements?

What should be understood is the physical meaning of the automorphism inducing the scaling of trace. In the representation based of factors based on infinite tensor powers the action of g should transform single $n \times n$ matrix factor with density matrix Id/n to a density matrix e_{11} of a pure state.

Obviously the number of degrees of freedom is affected and this can be interpreted in terms of appearance or disappearance of correlations. Quantization and emergence of non-commutativity indeed implies the emergence of correlations and effective reduction of degrees of freedom. In particular, the fundamental quantum Clifford algebra has reduced dimension $\mathcal{M} : \mathcal{N} = r \leq 4$ instead of $r = 4$ since the replacement of complex valued matrix elements with $\mathcal{N}$ valued ones implies non-commutativity and correlations.

The transformation would be induced by the shift of finite-dimensional state to right or left so that the number of matrix factors overlapping with I_∞ part increases or is reduced. Could it have interpretation in terms of quantum measurement for a quantum Clifford factor? Could quantum measurement for $\mathcal{M}/\mathcal{N}$ degrees of freedom reducing the state in these degrees of freedom to a pure state be interpreted as a transformation of single finite-dimensional matrix factor to a type I factor inducing the scaling of the trace and could the scalings associated with automorphisms of HFFs of type III also be interpreted in terms of quantum measurement?

This interpretation does not as such say anything about HFF factors of type III since only a decomposition of II_1 factor to I_2^k factor and II_1 factor with a reduced trace of projector to the latter. However, one can ask whether the scaling of trace for HFFs of type III could correspond to a situation in which infinite number of finite-dimensional factors have been quantum measured. This would correspond to the inclusion $\mathcal{N} \subset \mathcal{M}_\infty = \cup_n \mathcal{M}_n$ where $\mathcal{N} \subset \mathcal{M} \subset ...\mathcal{M}_n...$ defines the canonical inclusion sequence. Physicist can of course ask whether the presence of infinite number of I_2-, or more generally, I_n-factors is at all relevant to quantum measurement and it has already become clear that situation at the level of M-matrix reduces to I_n.

Could the theory of HHFs of type III relate to the theory of Jones inclusions?

The idea about a connection of HFFs of type III and quantum measurement theory seems to be consistent with the basic facts about inclusions and HFFs of type III_1.

1. Quantum measurement would scale the trace by a factor $2^k/\sqrt{\mathcal{M}:\mathcal{N}}$ since the trace would become a product for the trace of the projector to the newly born $M(2,C)^{\otimes k}$ factor and the trace for the projection to $\mathcal{N}$ given by $1/\sqrt{\mathcal{M}:\mathcal{N}}$. The continuous range of values $\mathcal{M}:\mathcal{N} \geq 4$ gives good hopes that all values of λ are realized. The prediction would be that $2^k\sqrt{\mathcal{M}:\mathcal{N}} \geq 1$ holds always true.

2. The values $\mathcal{M} : \mathcal{N} \in \{r_n = 4cos^2(\pi/n)\}$ for which the single $M(2,C)$ factor emerges in state function reduction would define preferred values of the inverse of $\lambda = \sqrt{\mathcal{M}:\mathcal{N}/4}$ parameterizing factors III_λ. These preferred values vary in the range $[1/2, 1]$.

3. $\lambda = 1$ at the end of continuum would correspond to HFF III_1 and to Jones inclusions defined by infinite cyclic subgroups dense in $U(1) \subset SU(2)$ and this group combined with reflection. These groups correspond to the Dynkin diagrams A_∞ and D_∞. Also the classical values of $\mathcal{M} : \mathcal{N} = n^2$ characterizing the dimension of the quantum Clifford $\mathcal{M} : \mathcal{N}$ are possible. In this case the scaling of trace would be trivial since the factor n to the trace would be compensated by the factor $1/n$ due to the disappearance of $\mathcal{M}/\mathcal{N}$ factor III_1 factor.

4. Inclusions with $\mathcal{M} : \mathcal{N} = \infty$ are also possible and they would correspond to $\lambda = 0$ so that also III_0 factor would also have a natural identification in this framework. These factors correspond to ergodic systems and one might perhaps argue that quantum measurement in this case would give infinite amount of information.

5. This picture makes sense also physically. p-Adic thermodynamics for the representations of super-conformal algebra could be formulated in terms of factors of type I_∞ and in excellent approximation using factors I_n. The generation of arbitrary number of type II_1 factors in quantum measurement allow this possibility.

The end points of spectrum of preferred values of λ are physically special

The fact that the end points of the spectrum of preferred values of λ are physically special, supports the hopes that this picture might have something to do with reality.

1. The Jones inclusion with $q = exp(i\pi/n)$, $n = 3$ (with principal diagram reducing to a Dynkin diagram of group SU(3)) corresponds to $\lambda = 1/2$, which corresponds to HFF III_1 differing in essential manner from factors III_λ, $\lambda < 1$. On the other hand, $SU(3)$ corresponds to color group which appears as an isometry group and important subgroup of automorphisms of octonions thus differs physically from the ADE gauge groups predicted to be realized dynamically by the TGD based view about McKay correspondence [A9] .

2. For $r = 4$ SU(2) inclusion parameterized by extended ADE diagrams $M(2,C)^{\otimes 2}$ would be created in the state function reduction and also this would give $\lambda = 1/2$ and scaling by a factor of 2. Hence the end points of the range of discrete spectrum would correspond to the same scaling factor and same HFF of type III. SU(2) could be interpreted either as electro-weak gauge group, group of rotations of th geodesic sphere of $\delta M^4_\pm$, or a subgroup of SU(3). In TGD interpretation for McKay correspondence a phase transition replacing gauge symmetry with Kac-Moody symmetry.

3. The scalings of trace by factor 2 seem to be preferred physically which should be contrasted with the fact that primes near prime powers of 2 and with the fact that quantum phases $q = exp(i\pi/n)$ with n equal to Fermat integer proportional to power of 2 and product of the Fermat primes (the known ones are 5, 17, 257, and $2^{16} + 1$) are in a special role in TGD Universe.

2.6.3 What could one say about II_1 automorphism associated with the II_∞ automorphism defining factor of type III?

An interesting question relates to the interpretation of the automorphisms of II_∞ factor inducing the scaling of trace.

1. If the automorphism for Jones inclusion involves the generator of cyclic automorphism subgroup Z_n of II_1 factor then it would seem that for other values of λ this group cannot be cyclic. SU(2) has discrete subgroups generated by arbitrary phase q and these are dense in $U(1) \subset SU(2)$ sub-group. If the interpretation in terms of Jones inclusion makes sense then the identification $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ makes sense.

2. If HFF of type II_1 is realized as group algebra of infinite symmetric group [A9] , the outer automorphism induced by the diagonally imbedded finite Galois groups can induce only integer values of n and Z_n would correspond to cyclic subgroups. This interpretation conforms with the fact that the automorphisms in the completion of inner automorphisms of HFF of type II_1 induce trivial scalings. Therefore only automorphisms which do not belong to this completion can define HFFs of type III.

2.6.4 What could be the physical interpretation of two kinds of invariants associated with HFFs type III?

TGD predicts two kinds of counterparts for S-matrix: M-matrix and U-matrix. Both are expected to be more or less universal.

There are also *two* kinds of invariants and automorphisms associated with HFFs of type III.

1. The first invariant corresponds to the scaling $\lambda \in]0,1[$ of the trace associated with the automorphism of factor of II_∞. Also the end points of the interval make sense. The inverse of this scaling accompanies the inverse of this automorphism.

2. Second invariant corresponds to the time scales $t = T_0$ for which the outer automorphism σ_t reduces to inner automorphism. It turns out that T_0 and λ are related by the formula $\lambda^{iT_0} = 1$, which gives the allowed values of T_0 as $T_0 = n2\pi/log(\lambda)$ [A31] . This formula can be understood intuitively by realizing that λ corresponds to the eigenvalue of the density matrix $\Delta = e^H$ in the simplest possible realization of the state ϕ.

The presence of two automorphisms and invariants brings in mind U matrix characterizing the unitary process occurring in quantum jump and M-matrix characterizing time like entanglement.

1. If one accepts the vision based on quantum measurement theory then λ corresponds to the scaling of the trace resulting when quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ reduces to a tensor power of $M(2,C)$ factor in the state function reduction. The proposed interpretation for U process would be as the inverse of state function reduction transforming this factor back to $\mathcal{M}/\mathcal{N}$. Thus U process and state function reduction would correspond naturally to the scaling and its inverse. This picture might apply not only in single particle case but also for zero energy states which can be seen as states associated the a tensor power of HFFs of type II_1 associated with partons.
2. The implication is that U process can occur only in the direction in which trace is reduced. This would suggest that the full III_1 factor is not a physical notion and that one must restrict the group Z in the crossed product $Z \times_{cr} II_\infty$ to the group N of non-negative integers. In this kind of situation the trace is well defined since the traces for the terms in the crossed product comes as powers λ^{-n} so that the net result is finite. This would mean a reduction to II_∞ factor.
3. Since time t is a natural parameter in elementary particle physics experiment, one could argue that σ_t could define naturally M-matrix. Time parameter would most naturally correspond to a parameter of scaling affecting all $M^4_\pm$ coordinates rather than linear time. This conforms also with the fundamental role of conformal transformations and scalings in TGD framework.

The identification of the full M-matrix in terms of σ does not seem to make sense generally. It would however make sense for incoming and outgoing number theoretic braids so that σ could define universal braiding M-matrices. Inner automorphisms would bring in the dependence on experimental situation. The reduction of the braiding matrix to an inner automorphism for critical values of t which could be interpreted in terms of scaling by power of p. This trivialization would be a counterpart for the elimination of propagator legs from M-matrix element. Vertex itself could be interpreted as unitary isomorphism between tensor product of incoming and outgoing HFFs of type II_1 would code all what is relevant about the particle reaction.

2.6.5 Does the time parameter t represent time translation or scaling?

The connection $T_n = n2\pi/log(\lambda)$ would give a relationship between the scaling of trace and value of time parameter for which the outer automorphism represented by σ reduces to inner automorphism. It must be emphasized that the time parameter t appearing in σ need not have anything to do with time translation. The alternative interpretation is in terms of $M^4_\pm$ scaling (implying also time scaling) but one cannot exclude even preferred Lorentz boosts in the direction of quantization axis of angular momentum.

Could the time parameter correspond to scaling?

The central role of conformal invariance in quantum TGD suggests that t parameterizes scaling rather than translation. In this case scalings would correspond to powers of $(K\lambda)^n$. The numerical factor K which cannot be excluded a priori, seems to reduce to $K = 1$.

1. The scalings by powers of p have a simple realization in terms of the representation of HFF of type II_∞ as infinite tensor power of $M(p,C)$ with suitably chosen densities matrices in factors to get product of I_∞ and II_1 factor. These matrix algebras have the remarkable property of defining prime tensor power factors of finite matrix algebras. Thus p-adic fractality would reflect directly basic properties of matrix algebras as suggested already earlier. That scalings

by powers of p would correspond to automorphism reducing to inner automorphisms would conform with p-adic fractality.

2. Also scalings by powers $[\sqrt{\mathcal{M}:\mathcal{N}}/2^k]^n$ would be physically preferred if one takes previous arguments about Jones inclusions seriously and if also in this case scalings are involved. For $q = exp(i\pi/n)$, $n = 5$ the minimal value of n allowing universal topological quantum computation would correspond to a scaling by Golden Mean and these fractal scalings indeed play a key role in living matter. In particular, Golden Mean makes it visible in the geometry of DNA.

Could the time parameter correspond to time translation?

One can consider also the interpretation of σ_t as time translation. TGD predicts a hierarchy of Planck constants parameterized by rational numbers such that integer multiples are favored. In particular, integers defining ruler and compass polygons are predicted to be in a very special role physically. Since the geometric time span associated with zero energy state should scale as Planck constant one expects that preferred values of time t associated with σ are quantized as rational multiples of some fundamental time scales, say the basic time scale defined by CP_2 length or p-adic time scales.

1. For $\lambda = 1/p$, p prime, the time scale would be $T_n = nT_1$, $T_1 = T_0 = 2\pi/log(p)$ which is not what p-adic length scale hypothesis would suggest.

2. For Jones inclusions one would have $T_n/T_0 = n2\pi/log(2^{2k}/\mathcal{M} : \mathcal{N})$. In the limit when λ becomes very small (the number k of reduced $M(2,C)$ factors is large one obtains $T_n = (n/k)t_1$, $T_1 = T_0\pi/log(2)$. Approximate rational multiples of the basic length scale would be obtained as also predicted by the general quantization of Planck constant.

p-Adic thermodynamics from first principles

Quantum field theory at non-zero temperature can be formulated in the functional integral formalism by replacing the time parameter associated with the unitary time evolution operator $U(t)$ with a complexified time containing as imaginary part the inverse of the temperature: $t \rightarrow t + i\hbar/T$. In the framework of standard quantum field theory this is a mere computational trick but the time parameter associated with the automorphisms σ_t of HFF of type III is a temperature like parameter from the beginning, and its complexification would naturally lead to the analog of thermal QFT.

Thus thermal equilibrium state would be a genuine quantum state rather than fictive but useful auxiliary notion. Thermal equilibrium is defined separately for each incoming parton braid and perhaps even braid (partons can have arbitrarily large size). At elementary particle level p-adic thermodynamics could be in question so that particle massivation would have first principle description. p-Adic thermodynamics is under relatively mild conditions equivalent with its real counterpart obtained by the replacement of p^{L_0} interpreted as a p-adic number with p^{-L_0} interpreted as a real number.

2.6.6 Could HFFs of type III be associated with the dynamics in $M^4_\pm$ degrees of freedom?

HFFs of type III could be also assigned with the poorly understood dynamics in $M^4_\pm$ degrees of freedom which should have a lot of to do with four-dimensional quantum field theory. Hyper-finite factors of type III_1 might emerge when one extends II_1 to a local algebra by multiplying it with hyper-octonions replaced as analog of matrix factor and considers hyper-quaternionic subalgebra. The resulting algebra would be the analog of local gauge algebra and the elements of algebra would be analogous to conformal fields with complex argument replaced with hyper-octonionic, -quaternionic, or -complex one. Since quantum field theory in M^4 gives rise to hyper-finite III_1 factors one might guess that the hyper-quaternionic restriction indeed gives these factors.

The expansion of the local HFF II_∞ element as $O(m) = \sum_n m^n O_n$, where M^4 coordinate m is interpreted as hyper-quaternion, could have interpretation as expansion in which O_n belongs to

$\mathcal{N}g^n$ in the crossed product $\mathcal{N} \times_{cr} \{g^n, n \in Z\}$. The analogy with conformal fields suggests that the power g^n inducing λ^n fold scaling of trace increases the conformal weight by n.

One can ask whether the scaling of trace by powers of λ defines an inclusion hierarchy of sub-algebras of conformal sub-algebras as suggested by previous arguments. One such hierarchy would be the hierarchy of sub-algebras containing only the generators O_m with conformal weight $m \geq n$, $n \in Z$.

It has been suggested that the automorphism Δ could correspond to scaling inside light-cone. This interpretation would fit nicely with Lorentz invariance and TGD in general. The factors III_λ with λ generating semi-subgroups of integers (in particular powers of primes) could be of special physical importance in TGD framework. The values of t for which automorphism reduces to inner automorphism should be of special physical importance in TGD framework. These automorphisms correspond to scalings identifiable in terms of powers of p-adic prime p so that p-adic fractality would find an explanation at the fundamental level.

If the above mentioned expansion in powers of m^n of $M^4_\pm$ coordinate makes sense then the action of σ^t representing a scaling by p^n would leave the elements O invariant or induce a mere inner automorphism. Conformal weight n corresponds naturally to n-ary p-adic length scale by uncertainty principle in p-adic mass calculations.

The basic question is the physical interpretation of the automorphism inducing the scaling of trace by λ and its detailed action in HFF. This scaling could relate to a scaling in M^4 and to the appearance in the trace of an integral over M^4 or subspace of it defining the trace. Fractal structures suggests itself strongly here. At the level of construction of physical states one always selects some minimum non-positive conformal weight defining the tachyonic ground state and physical states have non-negative conformal weights. The interpretation would be as a reduction to HHF of type II_∞ or even II_1.

2.6.7 Could the continuation of braidings to homotopies involve Δ^{it} automorphisms

The representation of braidings as special case of homotopies might lead from discrete automorphisms for HFFs type II_1 to continuous outer automorphisms for HFFs of type III_1. The question is whether the periodic automorphism of II_1 represented as a discrete sub-group of $U(1)$ would be continued to $U(1)$ in the transition.

The automorphism of II_∞ HFF associated with a given value of the scaling factor λ is unique. If Jones inclusions defined by the preferred values of λ as $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ (see the previous considerations), then this automorphism could involve a periodic automorphism of II_1 factor defined by the generator of cyclic subgroup Z_n for $\mathcal{M} : \mathcal{N} < 4$ besides additional shift transforming II_1 factor to I_∞ factor and inducing the scaling.

2.6.8 HFFs of type III as super-structures providing additional uniqueness?

If the braiding M-matrices are as such highly unique. One could however consider the possibility that they are induced from the automorphisms σ_t for the HFFs of type III restricted to HFFs of type II_∞. If a reduction to inner automorphism in HFF of type III implies same with respect to HFF of type II_∞ and even II_1, they could be trivial for special values of time scaling t assignable to the partons and identifiable as a power of prime p characterizing the parton. This would allow to eliminate incoming and outgoing legs. This elimination would be the counterpart of the division of propagator legs in quantum field theories. Particle masses would however play no role in this process now although the power of padic prime would fix the mass scale of the particle.

2.7 A vision about the role of HFFs in TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

2.7.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi,\xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A16] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0, 1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X, μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

2. A positive linear functional is weight with $\omega(1)$ finite.

3. A state is a weight with $\omega(1) = 1$.

4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0, 1]$ and for factors of type II_∞ n the range $[0, \infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{2.7.1}$$

   where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in zero energy ontology.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A35, A86] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a manner to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1,h_1)(g_2,h_2) = (g_1h_1(g_2), h_1h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1,g_1)(a_2,g_2) = (a_1g_1(a_2), g_1g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K95] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A2] and those of factors of type III by Alain Connes [A30] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A2] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} & a) \ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ & b) \ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{2.7.2}$$

   the numbers at right hand side are known as Beraha numbers [A71] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B16] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A53] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

Connes tensor product

The inclusions The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} \ = \ \mathcal{M}/\mathcal{N} \otimes \mathcal{N} \ . \tag{2.7.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M} \otimes \mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

2.7.2 Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A59, A35, A86] . There are good arguments showing that in HFFS of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A86] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O} + x)$ where $(\mathcal{O} + x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFS of type III_1s is induced by set theoretic inclusions.

2.7.3 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and zero energy ontology, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the modified Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the modified Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

Zero energy ontology and factors

The first question concerns the identification of the Hilbert space associated with the factors in zero energy ontology. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in zero energy ontology.

1. In zero energy ontology M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFS is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \rightarrow 0$ limit. In quantum TGD the exponent of modified Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Modified Dirac action can therefore be regarded as a "square root" of Kähler action.

3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in zero energy ontology based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the modified Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFS involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of modified Dirac action [K20] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

3. Quantum criticality means that modified Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two superconformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between

tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K32] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

2.7.4 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

The basic action principle

In the following the most recent view about Kähler action and the modified Dirac action (Kähler-Dirac action) is explained in more detail.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action with Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term is chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD).

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

2. By supersymmetry requirement the modified Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain modified gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. At partonic orbits one only assumes that spinors are generalized eigen modes of Chern-Simons Dirac operator with generalized eigenvalues $p^k\gamma_k$ identified as virtual four-momenta so that C-S-D term gives fermionic propagators. At the ends of space-time surface one obtains boundary conditions stating in absence of measurement interaction terms that fundamental fermions are massless on-mass-shell states.

1. Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface if $j \cdot A = 0$ condition holds true as it does for preferred extremals. Note that weak form of electric magnetic duality is not absolutely necessary at space-like ends of the space-time surface but is favored by almost topological QFT property.

2. Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

a) What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi = 0$$

at the space-like ends of space-time surface. The general idea is that the space-time geometry near the fermion line would *define* the on mass shell massless four-momentum propagating along the line and quantum classical correspondence would be realized.

The basic condition is thus that $\sqrt{g_4}\Gamma^n$ is constant at the space-like boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant. It is important to notice that Γ^n brings in the dependence on metric and breaks exact topological QFT property as do also the constraint terms realizing weak form of electric magnetic duality.

Partonic orbits are not boundaries in the usual sense of the word and this condition is not elegant at them since g_4 vanishes at them. The assignment of Chern-Simons Dirac action to partonic orbits required to be continuous at them solves the problems. One can require that the induced spinors are generalized eigenstates of C-S-D operator with eigenvalues with correspond to virtual four-moment. This guarantees that one obtains massless Dirac propagator from C-S-D action. Note that the localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

2. If p^k associated with the partonic orbit is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram since the fermion current in the normal direction vanishes.

The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification of p^k as virtual momentum is correct, continuous mass spectrum suggests itself. Boundary conditions at the ends of CD might imply quantized mass spectrum and the study of C-S-D equation indeed suggets this if periodic boundary conditions are assumed. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation.

b) Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If C-S-D and C-S terms are assigned also with the space-like ends of space-time surface, Kähler action and Kähler function vanish identically if the weak form of em duality holds true. Hence C-S-D and C-S terms can be assigned only with partonic orbits. If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{2.7.4}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = -\gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. The physical interpretation would be as incoming massless fermions.

3. Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Localization of the modes of Kähler-Dirac operator at string world sheets and definition of Dirac determinant

The condition that the modes of Kähler-Dirac operator have well defined electromagnetic charge eigenvalue implies that the modes are restricted to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces [K103]. In the generic case one would have a product of Dirac determinants associated with these 2-surfaces. This obviously simplifies dramatically the definition of Dirac determinant and suggests a reduction to stringy mathematics, where this kind of determinants appear routinely.

The construction of Dirac determinant could proceed in following manner.

1. The spectrum of the Kähler Dirac (KD) operator was originally identified in terms of generalized eigenvalues. The identification coming first in mind would be in terms of conformal weights assignable to the modes of KD operator. The experience with the string models suggests that these conformal weights are integer valued, which would mean that the multiplicative contribution from given string world sheet is constant and cannot depend on 3-surface at all!

2. The boundary conditions at the string curves at the space-like ends of space-time surface however give algebraic form of Dirac equation with the analog of Higgs coupling in algebraic form $(p^k\gamma_k + \Gamma^n)\Psi = 0$, with p^k identifiable as four-momentum of fermionic line emanating from partonic 2-surface. The normal component Γ^n (in time direction) of the vector defined by K-D gamma matrices defines the analog of Higgs vacuum expectation value, and could be covariantly constant along string curve for a suitable choice of string coordinates. $h^2 \equiv (\Gamma^n)^2$ could be interpreted as ground state conformal weight. In p-adic mass calculations ground state conformal weight must be negative half-odd integer and the time-like character of Γ^n could explain this. h^2 could have p-adically small deviation from half-odd integer value and give rise to a Higgs like additional contribution to the conformal weights.

3. The square of the Dirac determinant would be product of eigenvalues mass squared operator assignable to the eigenvalue equation $(p^k\gamma_k+\Gamma^n)^2\Psi = \Lambda_n)\Psi$. If the eigenvalues correspond up to multiplicative factor to integer valued conformal weights, the square of Dirac determinant would be the product of corresponding mass squared values equal to conformal weight with vacuum contribution. The square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + n$, where h is expressed using unit of mass determined by CP_2 radius.

4. One can of course ask whether it might be possible to define even the Dirac determinant itself. Here it seems that the only possible manner to proceed is number theoretic: the factors $p^k\gamma_k+\Gamma^n$ appearing in the formal Dirac determinant should be mapped to complexified octonions and the product of these factors should define Dirac determinant as complex quantity having interpretation as the product of exponents of Kähler for Euclidian and Minkowskian regions metting at wormhole throat. This would be a rather deep connection with the number theoretic approach.

5. Since spinor modes effectively propagate as particles with momentum p^k along braid strands one could argue that one must include h^2 to the integer valued conformal weight so that the square of Dirac determinant would be defined as as the product of conformal weights $h(n) = h^2 + nM_0^2$, M_0 the mass scale determined by CP_2 radius.

   The resulting determinant - if indeed well-defined - would depend on space-time surface and would be obtained as a perturbation from the determinant assignable to Riemann Zeta. Modulus squared for the exponent of vacuum functional would be analogous to the square of Dirac determinant associated with a massless fermion with eigenvalues of m^2 replaced with $h(n)$. The overall determinant would be product over the determinants coming from various strings and possibly also from he partonic 2-surfaces.

One must however be aware about possible objections against the hypothesis that the square of Dirac determinant gives the modulus squared for the vacuum functional.

1. It would be exaggeration to say that Kähler function emerges from K-D action. The reason is that K-D gamma matrices appear in K-D action and internal consistency requires that an extremal of K-D action is in question. Hence it seems that Khler action and K-D action are in completely democratic position and one can wonder whether the possible connection actually gives any profound insights or means anything practical. It could only create technical challenges and one can claim that the definition of exponent of vacuum functional reducing to exponent of Chern-Simons terms looks much more practical and elegant.

2. Kähler function corresponds to Kähler action in Euclidian space-time regions assignable to the lines of generalized Feynman diagrams. It is not clear whether one represent also the Kähler action from Minkowskian regions in this manner.

A proposal for M-matrix

This picture can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW. This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector Chern-Simons Dirac term in the action and the condition that spinors modes localized at string world sheets are eigenstates of C-S-D operator with generalized eigenvalue $p^k\gamma_k$ defining virtual momentum would give effectively rise to massless Dirac action in M^4 and one would obtain massless fermionic propagators. The generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as gauge theory is natural.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts.

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond to the scattering of fermions at opposite wormhole throats. The natural guess is that the propagator is essentially the inverse of the scaling generator L_0 of conformal algebra. Non-locality suggests that one must product for the inverses of the super-generators G and its hermitian conjugate estimated at the two wormhole throats. There the diagrammatics would be combinations of that for QFT with massless fermions and string model diagrammatics. Topologically the vertices would be analogous to Feynman vertices: two 3-surfaces would fuse at vertices to form third. Stringy trouser diagrams would not have interpretation as decays of particle but as particle travelling two different paths.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??**, `http://www.tgdtheory.fi/appfigures/elparticletgd.jpg` or fig. 6, tgdgraphs in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics.

Quantum TGD as square root of thermodynamics

Zero energy ontology (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A59] [K95]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A59], [K95] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the modified Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes. Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K32]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. The inclusions of these conformal algebras would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

Summarizing

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD and that different measurement interactions having interpretation in terms of different kind of quantum measurements causing wave function collapse in zero mode sector of WCW could correspond to various choices of ω. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

2.7.5 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in zero energy ontology

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the

upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

The formulation of quantum TGD in terms of the modified Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

3. Zero energy ontology gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish zero energy ontology allows a symmetry breaking respecting a chosen Cartan algebra.

4. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFS as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) . \tag{2.7.5}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{2.7.6}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^\dagger)|r_2\rangle \ . \tag{2.7.7}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^\dagger) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{2.7.8}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In zero energy ontology the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A8] .

5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S

and guarantee the symmetry. One could speak of a square root of KMS condition [A8] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{2.7.9}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{2.7.10}$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the modified Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain modified gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A20] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

2.7.6 Questions about quantum measurement theory in zero energy ontology

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

2.7.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [K24] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

p-Adic coupling constant evolution

An attractive conjecture is that the coupling constant evolution associated with CDs in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induces p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there seems to be a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end

points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For the weaker condition would be $T_p = pT_0$, p prime, $p \simeq 2^n$ could be seen as an outcome of some kind of "natural selection". In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \ ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

2.7.8 Planar algebras and generalized Feynman diagrams

Planar algebras [A11] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A22] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K18] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

[Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

2.7.9 Miscellaneous

The following considerations are somewhat out-of-date: hence the title 'Miscellaneous'.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A53] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A82] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A28] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K24] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A84] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K94] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K29] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A84] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3 \# S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

2.8 Fresh view about hyper-finite factors in TGD framework

In the following I will discuss the basic ideas about the role of hyper-finite factors in TGD with the background given by a work of more than half decade. First I summarize the input ideas which I combine with the TGD inspired intuitive wisdom about HFFs of type II_1 and their inclusions allowing to represent finite measurement resolution and leading to notion of quantum spaces with algebraic number valued dimension defined by the index of the inclusion.

Also an argument suggesting that the inclusions define "skewed" inclusions of lattices to larger lattices giving rise to quasicrystals is proposed. The core of the argument is that the included HFF of type II_1 algebra is a projection of the including algebra to a subspace with dimension $D \leq 1$. The projection operator defines the analog of a projection of a bigger lattice to the included lattice. Also the fact that the dimension of the tensor product is product of dimensions of factors just like the number of elements in finite group is product of numbers of elements of coset space and subgroup, supports this interpretation.

One also ends up with a detailed identification of the hyper-finite factors in orbital degrees of freedom in terms of symplectic group associated with $\delta M^4_\pm \times CP_2$ and the group algebras of their discrete subgroups define what could be called "orbital degrees of freedom" for WCW spinor fields. By very general argument this group algebra is HFF of type II, maybe even II_1.

2.8.1 Crystals, quasicrystals, non-commutativity and inclusions of hyperfinite factors of type II_1

I list first the basic ideas about non-commutative geometries and give simple argument suggesting that inclusions of HFFs correspond to "skewed" inclusions of lattices as quasicrystals.

1. Quasicrystals (say Penrose tilings) [A13] can be regarded as subsets of real crystals and one can speak about "skewed" inclusion of real lattice to larger lattice as quasicrystal. What this means that included lattice is obtained by projecting the larger lattice to some lower-dimensional subspace of lattice.

2. The argument of Connes concerning definition of non-commutative geometry can be found in the book of Michel Lapidus at page 200. Quantum space is identified as a space of equivalence classes. One assigns to pairs of elements inside equivalence class matrix elements having the element pair as indices (one assumes numerable equivalence class). One considers irreducible representations of the algebra defined by the matrices and identifies the equivalent irreducible representations. If I have understood correctly, the equivalence classes of irreps define a discrete point set representing the equivalence class and it can often happen that there is just single point as one might expect. This I do not quite understand since it requires that all irreps are equivalent.

3. It seems that in the case of linear spaces - von Neumann algebras and accompanying Hilbert spaces - one obtains a connection with the inclusions of HFFs and corresponding quantum factor spaces that should exist as analogs of quantum plane. One replaces matrices with elements labelled by element pairs with linear operators in HFF of type II_1. Index pairs correspond to pairs in linear basis for the HFF or corresponding Hilbert space.

4. Discrete infinite enumerable basis for these operators as a linear space generates a lattice in summation. Inclusion $N \subset M$ defines inclusion of the lattice/crystal for N to the corresponding lattice of M. Physical intuition suggests that if this inclusion is "skewed" one

obtains quasicrystal. The fact the index of the inclusion is algebraic number suggests that the coset space M/N is indeed analogous to quasicrystal.

More precisely, the index of inclusion is defined for hyper-finite factors of type II_1 using the fact that quantum trace of unit matrix equals to unity $Tr(Id(M)) = 1$, and from the tensor product composition $M = (M/N) \times N$ given $Tr(Id(M)) = 1 = Ind(M/N)Tr(P(M \rightarrow N))$, where $P(M \rightarrow N$ is projection operator from M to N. Clearly, $Ind(M/N) = 1/Tr(P(M \rightarrow N))$ defines index as a dimension of quantum space M/N.

For Jones inclusions characterized by quantum phases $q = exp(i2\pi/n)$, $n = 3, 4, ...$ the values of index are given by $Ind(M/N) = 4cos^2(\pi/n)$, $n = 3, 4, ...$ There is also another range inclusions $Ind(M/N) \geq 4$: note that $Tr(P(M \rightarrow N))$ defining the dimension of N as included sub-space is never larger than one for HFFs of type II_1. The projection operator $P(M \rightarrow N)$ is obviously the counterpart of the projector projecting lattice to some lower-dimensional sub-space of the lattice.

5. Jones inclusions are between linear spaces but there is a strong analogy with non-linear coset spaces since for the tensor product the dimension is product of dimensions and for discrete coset spaces G/H one has also the product formula $n(G) = n(H) \times n(G/H)$ for the numbers of elements. Noticing that space of quantum amplitudes in discrete space has dimension equal to the number of elements of the space, one could say that Jones inclusion represents quantized variant for classical inclusion raised from the level of discrete space to the level of space of quantum states with the number of elements of set replaced by dimension. In fact, group algebras of infinite and enumerable groups defined HFFs of type II under rather general conditions (see below).

   Could one generalize Jones inclusions so that they would apply to non-linear coset spaces analogs of the linear spaces involved ? For instance, could one think of infinite-dimensional groups G and H for which Lie-algebras defining their tangent spaces can be regarded as HFFs of type II_1? The dimension of the tangent space is dimension of the non-linear manifold: could this mean that the non-linear infinite-dimensional inclusions reduce to tangent space level and thus to the inclusions for Lie-algebras regarded hyper-finite factors of type II_1 or more generally, type II? This would would rise to quantum spaces which have finite but algebraic valued quantum dimension and in TGD framework take into account the finite measurement resolution.

6. To concretize this analogy one can check what is the number of points map from 5-D space containing aperiodic lattice as a projection to a 2-D irrational plane containing only origin as common point with the 5-D lattice. It is easy to get convinced that the projection is 1-to-1 so that the number of points projected to a given point is 1. By the analogy with Jones inclusions this would mean that the included space has same von Neumann dimension 1 - just like the including one. In this case quantum phase equals $q = exp(i2\pi/n)$, $n = 3$ - the lowest possible value of n. Could one imagine the analogs of $n > 3$ inclusions for which the number of points projected to a given point would be larger than 1? In 1-D case the rational lines $y = (k/l)x$ define 1-D rational analogs of quasi crystals. The points $(x, y) = (m, n)$, $m \ mod \ l = 0$ are projected to the same point. The number of points is now infinite and the ratio of points of 2-D lattice and 1-D crystal like structure equals to l and serves as the analog for the quantum dimension $d_q = 4cos^2(\pi/n)$.

To sum up, this this is just physicist's intuition: it could be wrong or something totally trivial from the point of view of mathematician. The main message is that the inclusions of HFFs might define also inclusions of lattices as quasicrystals.

2.8.2 HFFs and their inclusions in TGD framework

In TGD framework the inclusions of HFFs have interpretation in terms of finite measurement resolution. If the inclusions define quasicrystals then finite measurement resolution would lead to quasicrystals.

1. The automorphic action of N in $M \supset N$ and in associated Hilbert space H_M where N acts generates physical operators and accompanying stas (operator rays and rays) not distinguishable from the original one. States in finite measurement resolution correspond to N-rays rather than complex rays. It might be natural to restrict to unitary elements of N.

   This leads to the need to construct the counterpart of coset space M/N and corresponding linear space H_M/H_N. Physical intuition tells that the indices of inclusions defining the "dimension" of M/N are algebraic numbers given by Jones index formula.

2. Here the above argument would assign to the inclusions also inclusions of lattices as quasicrystals.

Degrees of freedom for WCW spinor field

Consider first the identification of various kinds of degrees of freedom in TGD Universe.

1. Very roughly, WCW ("world of classical worlds") spinor is a state generated by fermionic creation operators from vacuum at given 3-surface. WCW spinor field assigns this kind of spinor to each 3-surface. WCW spinor fields decompose to tensor product of spin part (Fock state) and orbital part ("wave" in WCW) just as ordinary spinor fields.

2. The conjecture motivated by super-symmetry has been that both WCW spinors and their orbital parts (analogs of scalar field) define HFFs of type II_1 in quantum fluctuating degrees of freedom.

3. Besides these there are zero modes, which by definition do not contribute to WCW Kähler metric.

   (a) If the zero zero modes are symplectic invariants, they appear only in conformal factor of WCW metric. Symplectically invariant zero modes represent purely classical degrees of freedom - direction of a pointer of measurement apparatus in quantum measurement - and in given experimental arrangement they entangle with quantum fluctuating degrees of freedom in one-one manner so that state function reduction assigns to the outcome of state function reduction position of pointer. I forget symplectically invariant zero modes and other analogous variables in the following and concentrate to the degrees of freedom contributing WCW line-element.

   (b) There are also zero modes which are not symplectic invariants and are analogous to degrees of freedom generated by the generators of Kac-Moody algebra having vanishing conformal weight. They represent "center of mass degrees of freedom" and this part of symmetric algebra creates the representations representing the ground states of Kac-Moody representations. Restriction to these degrees of freedom gives QFT limit in string theory. In the following I will speak about "cm degrees of freedom".

The general vision about symplectic degrees of freedom (the analog of "orbital degrees of freedom" for ordinary spinor field) is following.

1. WCW (assignable to given CD) is a union over the sub-WCWs labeled by zero modes and each sub-WCW representing quantum fluctuating degrees of freedom and "cm degrees of freedom" is infinite-D symmetric space. If symplectic group assignable to $\delta M^4_+ \times CP_2$ acts as as isometries of WCW then "orbital degrees of freedom" are parametrized by the symplectic group or its coset space (note that light-cone boundary is 3-D but radial dimension is light-like so that symplectic - or rather contact structure - exists).

   Let S^2 be $r_M = constant$ sphere at light-cone boundary (r_M is the radial light-like coordinate fixed apart from Lorentz transformation). The full symplectic group would act as isometries of WCW but does not - nor cannot do so - act as symmetries of Kähler action except in the huge vacuum sector of the theory correspond to vacuum extremals.

2. WCW Hamiltonians can be deduced as "fluxes" of the Hamiltonians of $\delta M^4_+ \times CP_2$ taken over partonic 2-surfaces. These Hamiltonians expressed as products of Hamiltonians of S^2

and CP_2 multiplied by powers r_M^n. Note that r_M plays the role of the complex coordinate z for Kac-Moody algebras and the group G defining KM is replaced with symplectic group of $S^2 \times CP_2$. Hamiltonians can be assumed to have well-defined spin (SO(3)) and color (SU(3)) quantum numbers.

3. The generators with vanishing radial conformal weight ($n = 0$) correspond to the symplectic group of $S^2 \times CP_2$. They are not symplectic invariants but are zero modes. They would correspond to "cm degrees of freedom" characterizing the ground states of representations of the full symplectic group.

Discretization at the level of WCW

The general vision about finite measurement resolution implies discretization at the level of WCW.

1. Finite measurement resolution at the level of WCW means discretization. Therefore the symplectic groups of $\delta M^4_+ \times CP_2$ *resp.* $S^2 \times CP_2$ are replaced by an enumerable discrete subgroup. WCW is discretized in both quantum fluctuating degrees of freedom and "center of mass" degrees of freedom.

2. The elements of the group algebras of these discrete groups define the "orbitals parts" of WCW spinor fields in discretization. I will later develop an argument stating that they are HFFs of type II - maybe even II_1. Note that also function spaces associated with the coset spaces of these discrete subgroups could be considered.

3. Discretization applies also in the spin degrees of freedom. Since fermionic Fock basis generates quantum counterpart of Boolean algebra the interpretation in terms of the physical correlates of Boolean cognition is motivated (fermion number 1/0 and various spins in decomposition to a tensor product of lower-dimensional spinors represent bits). Note that in ZEO fermion number conservation does not pose problems and zero states actually define what might be regarded as quantum counterparts of Boolean rules $A \rightarrow B$.

4. Note that 3-surfaces correspond by the strong form of GCI/holography to collections of partonic 2-surfaces and string world sheets of space-time surface intersecting at discrete set of points carrying fermionic quantum numbers. WCW spinors are constructed from second quantized induced spinor fields and fermionic Fock algebra generates HFF of type II_1.

Does WCW spinor field decompose to a tensor product of two HFFs of type II_1?

The group algebras associated with infinite discrete subgroups of the symplectic group define the discretized analogs of waves in WCW having quantum fluctuating part and cm part. The proposal is that these group algebras are HFFs of type II_1. The spinorial degrees of freedom correspond to fermionic Fock space and this is known to be HFF. Therefore WCW spinor fields would defined tensor product of HFFs of type II_1. The interpretation would be in terms of supersymmetry at the level of WCW. Super-conformal symmetry is indeed the basic symmetry of TGD so that this result is a physical "must". The argument goes as follows.

1. In non-zero modes WCW is symplectic group of $\delta M^4_+ \times CP_2$ (call this group just *Sympl*) reduces to the analog of Kac-Moody group associated with $S^2 \times CP_2$, where S^2 is $r_M =$ constant sphere of light-cone boundary and z is replaced with radial coordinate. The Hamiltonians, which do not depend on r_M would correspond to zero modes and one could not assign metric to them although symplectic structure is possible. In "cm degrees of freedom" one has symplectic group associated with $S^2 \times CP_2$.

2. Finite measurement resolution, which seems to be coded already in the structure of the preferred extremals and of the solutions of the modified Dirac equation, suggests strongly that this symplectic group is replaced by its discrete subgroup or symmetric coset space. What this group is, depends on measurement resolution defined by the cutoffs inherent to the solutions. These subgroups and coset spaces would define the analogs of Platonic solids in WCW!

3. Why the discrete infinite subgroups of *Sympl* would lead naturally to HFFs of type II? There is a very general result stating that group algebra of an enumerable discrete group, which has infinite conjugacy classes, and is amenable so that its regular representation in group algebra decomposes to all unitary irreducibles is HFF of type II. See for examples about HFFs of type II listed in Wikipedia article [A7].

4. Suppose that the group algebras associated the discrete subgroups *Sympl* are indeed HFFs of type II or even type II_1. Their inclusions would define finite measurement resolution the orbital degrees of freedom for WCW spinor fields. Included algebra would create rays of state space not distinguishable experimentally. The inclusion would be characterized by the inclusion of the lattice defined by the generators of included algebra by linearity. One would have inclusion of this lattice to a lattice associated with a larger discrete group. Inclusions of lattices are however known to give rise to quasicrystals (Penrose tilings are basic example), which define basic non-commutative structures. This is indeed what one expects since the dimension of the coset space defined by inclusion is algebraic number rather than integer.

5. Also in fermionic degrees of freedom finite measurement resolution would be realized in terms of inclusions of HFFs- now certainly of type II_1. Therefore one could obtain hierarchies of lattices included as quasicrystals.

What about zero modes which are symplectic invariants and define classical variables? They are certainly discretized too. One might hope that one-one correlation between zero modes (classical variables) and quantum fluctuating degrees of freedom suggested by quantum measurement theory allows to effectively eliminate them. Besides zero modes there are also modular degrees of freedom associated with partonic 2-surfaces defining together with their 4-D tangent space data basis objects by strong form of holography. Also these degrees of freedom are automatically discretized. But could one consider finite measurement resolution also in these degrees of freedom. If the symplectic group of $S^2 \times CP_2$ defines zero modes then one could apply similar argument also in these degrees of freedom to discrete subgroups of $S^2 \times CP_2$.

2.8.3 Little Appendix: Comparison of WCW spinor fields with ordinary second quantized spinor fields

In TGD one identifies states of Hilbert space as WCW spinor fields. The analogy with ordinary spinor field helps to understand what they are. I try to explain by comparison with QFT.

Ordinary second quantized spinor fields

Consider first ordinary fermionic QFT in fixed space-time. Ordinary spinor is attached to an space-time point and there is $2^{D/2}$ dimensional space of spin degrees of freedom. Spinor field attaches spinor to every point of space-time in a continuous/smooth manner. Spinor fields satisfying Dirac equation define in Euclidian metric a Hilbert space with a unitary inner product. In Minkowskian case this does not work and one must introduce second quantization and Fock space to get a unitary inner product. This brings in what is essentially a basic realization of HFF of type II_1 as allowed operators acting in this Fock space. It is operator algebra rather than state space which is HFF of type II_1 but they are of course closely related.

Classical WCW spinor fields as quantum states

What happens TGD where one has quantum superpositions of 4-surface/3-surfaces by GCI/partonic 2-surfaces with 4-D tangent space data by strong form of GCI.

1. First guess: space-time point is replaced with 3-surface. Point like particle becomes 3-surface representing particle. WCW spinors are fermionic Fock states at this surface. WCW spinor fields are Fock state as a functional of 3-surface. Inner product decomposes to Fock space inner product plus functional integral over 3-surfaces (no path integral!). One could speak of quantum multiverse. Not single space-time but quantum superposition of them. This quantum multiverse character is something new as compared to QFT.

2. Second guess: forced by ZEO, by geometrization of Feynman diagrams, etc.

   (a) 3-surfaces are actually not connected 3-surfaces. They are collections of components at both ends of CD and connected to single connected structure by 4-surface. Components of 3-surface are like incoming and outgoing particles in connected Feynman diagrams. Lines are identified as regions of Euclidian signature or equivalently as the 3-D light-like boundaries between Minkowskian and Euclidian signature of the induced metric.

   (b) Spinors(!!) are defined now by the fermionic Fock space of second quantized induced spinor fields at these 3-surfaced and by holography at 4-surface. This fermionic Fock space is assigned to all multicomponent 3-surfaces defined in this manner and WCW spinor fields are defined as in the first guess. This brings integration over WCW to the inner product.

3. Third, even more improved guess: motivated by the solution ansatz for preferred extremals and for modified Dirac equation [K103] giving a connection with string models.

   The general solution ansatz restricts all spinor components but right-handed neutrino to string world sheets and partonic 2-surfaces: this means effective 2-dimensionality. String world sheets and partonic 2-surfaces intersect at the common ends of light-like and space-like braids at ends of CD and at along wormhole throat orbits so that effectively discretization occurs. This fermionic Fock space replaces the Fock space of ordinary second quantization.

2.9 Jones inclusions and cognitive consciousness

WCW spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of WCWs spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

2.9.1 Does one have a hierarchy of U- and M-matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question. The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2, ...$ TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K82] and Jones inclusions are suggestive.

2.9.2 Feynman diagrams as higher level particles and their scattering as dynamics of self consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [K24] .

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K94] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

1. These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

2. States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^\dagger) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

1. Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a manner that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

2. The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

3. After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

4. One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

5. This picture could also relate to the suggested duality between string and parton pictures [K84] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

1. Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.

2. The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.

3. In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

2.9.3 Logic, beliefs, and spinor fields in the world of classical worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

WCW spinors as logic statements

In TGD framework the infinite-dimensional WCW (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [K20] . CH spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [K82] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

1. Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.

2. One can wonder what is the difference between real and p-adic variants of CH spinor fields and whether they could represent reality and beliefs about reality. CH spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real CH spinors as different objects. Real/ p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real CH spinors.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic CH spinor fields could serve as representations of beliefs and real CH spinor fields as representations of reality looks very nice but the fact that the outcomes of p-adic-to-real phase transition and its reversal are highly non-predictable does not support it as such.

Quantum statistical determinism could however come into rescue. Belief could be represented as an ensemble of p-adic mental images resulting in transitions of real mental images representing reality to p-adic states. p-Adic ensemble average would represent the belief.

It is not at all clear whether real-to-padic transitions can occur at high enough rate since p-adic-to-real transition are expected to be highly irreversible. The real initial states much have nearly vanishing quantum numbers emitted in the transition to p-adic state to guarantee conservation laws (p-adic conservation laws hold true only piecewise since conserved quantities are pseudo constants). The system defined by an ensemble of real Boolean mental images representing reality would automatically generate a p-adic variant representing a belief about reality.

p-Adic CH spinors can also represent the cognitive aspects of intention whereas p-adic space-time sheets would represent its geometric aspects reflected in sensory experience.p-Adic space-time sheet could also serve only as a space-time correlate for the fundamental representation of intention in terms of p-adic CH spinor field. This view is consistent with the proposed identification of beliefs since the transitions associated with intentions *resp.* beliefs would be p-adic-to-real *resp.* real-to-padic.

2.9.4 Jones inclusions for hyperfinite factors of type II_1 as a model for symbolic and cognitive representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with CH spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ ($n = \infty$) the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a manner that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since CH spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d < 2$. The maximal reduction occurs to $d = 1$ for $n = 3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n = 3$

corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d < 2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d < 2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \rightarrow p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as $(1/2 + n_1 q^{n_2})r$ and $(1/2 + n_2 q^{n_1})r$. The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \rightarrow \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \rightarrow \infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2 + N_1 n, 1/2 + N_2 n)/[1 + (N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \rightarrow \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

1. Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.
2. Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
   Knowledge and misbelief would involve both the measurement of real and p-adic probabilities
   .
3. Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.

   Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.
4. Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
   In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.
5. Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \to \infty$, which according to the proposal of [K75] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation. A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

2.9.5 Intentional comparison of beliefs by topological quantum computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [K94] . The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

2.9.6 The stability of fuzzy qbits and quantum computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [K94] .

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [K50] . For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

2.9.7 Fuzzy quantum logic and possible anomalies in the experimental data for the EPR-Bohm experiment

The experimental data for EPR-Bohm experiment [J14] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J17] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J14] are not consistent with this prediction [J7] and this is identified as the anomaly.

2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J7] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} &\rightarrow P^2 P_{i,j} + (1-P)^2 P_{i+1,j+1} \\ &+ P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \quad . \end{aligned} \tag{2.9.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned}
P_{0,0} = P_{1,1} &\rightarrow A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\
&= (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\
P_{0,1} = P_{1,0} &\rightarrow A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\
&= (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\
A &= P^2 + (1-P)^2 \ , \ B = 2P(1-P) \ .
\end{aligned} \quad (2.9.2)$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha - \beta$ are scaled by a factor $1 - 4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i (P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \rightarrow [1-4P(1-P)]\, E(\alpha,\beta) \ . \quad (2.9.3)$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha-\beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E = \pm 1$ to $E = \pm(1-4P(1-P))$.

From the figure 1b) of [J7] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P = 1-\epsilon$ one has $A - B \simeq 1 - 4\epsilon$ and $B \simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1 - 4\epsilon$. The graph would give $1 - P \simeq \epsilon \simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [J17, J7] .

A further prediction is that the identities $P(i,i) + P(i+1,i) = 1/2$ should still hold true since one has $P_{i,i} + P_{i,i+1} = (A-B)/2 + B = 1$. This is implied also by probability conservation. The four curves corresponding to these identities do not however co-incide as the figure 6 of [J7] demonstrates. This is regarded as the basic anomaly in [J17, J7] . From the same figure it is also clear that below $\alpha - \beta < 10$ degrees $P_{++} = P_{--}$ $\Delta P_{+-} = -\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying $\Delta P_{++} = -\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij} P_{ij} = 1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii} + P_{ii+1} = 1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i} + P_{ii+1} = 1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-} + P_{--}$ and $P_{+-} + P_{++}$. The constraint $\sum_{ij} P_{ij} = 1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha - \beta > 10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

1. The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative "mundane" explanation.

2. The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.

3. One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P = P(i,j)$ so that one has $(P(-,+), P(+,-)) = (P+\Delta, P-\Delta)$ and $(P(-,-), P(+,+)) = (P+\Delta_1, P-\Delta_1)$. $\Delta \simeq .025$ and $\Delta_1 \simeq \Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i) + P(i,i+1) \geq 1/2$. Only if the procedure extracting the correlations

uses the constraint $\sum_{i,j} P_{ij} = 1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1 \simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of N_in one would have $(N_1 - N_2)/(N_1 + N_2) \simeq .4$ so that one cannot say anything about n in this case. $(N_1, N_2) = (3,1)$ satisfies the condition exactly. For $n = 3$, the smallest possible value of n, this would give $p_1 \simeq .88$ and for $n = 4$ $p_1 = .41$. With high enough precision it might be possible to select between $n = 3$ and $n = 4$ options if small values of N_i are accepted.

2.9.8 Category theoretic formulation for quantum measurement theory with finite measurement resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at http://kea-monad.blogspot.com/) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [A6] . In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [K19]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [A12] . These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [A10] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [A14] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition and intentionality with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

1. Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?

2. How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum subspaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space {0,1} would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [K72] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized imbedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \rightarrow M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

2.10 Appendix: Inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A29] . It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A29] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A29] . For any amenable group G the the inclusion is also unique apart from outer automorphism [A40] .

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [A29] . The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M} .... \otimes_{\mathcal{N}} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A29] .

2.10.1 Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n)|n \geq 3\} \cap [4, \infty)$ [A29] . They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i - j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A29] . It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis [K32] that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$ define orbifold coverings of $H_\pm = M^4_\pm \times CP_2 \to H_\pm/G_a \times G_b$.

2.10.2 Wassermann's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [A29] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2, C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2, C)$ is effectively replaced with the orbifold $M(2, C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \to H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction [A9] . The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

2.10.3 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A83] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n

is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s - r)n/l\right)} \quad . \tag{2.10.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l - 1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \rightarrow M^4/G_a$ or $CP_2 \rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of SU(2) the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3, 1) \times SU(3)$ and $SL(2, C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3, 1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Chapter 3

Does TGD Predict Spectrum of Planck Constants?

3.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [E25] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence. This approach led to the the formula $h_{eff} = n \times h$. Quite recently (2014) it became clear that for microscopic systems the identification $h_{eff} = h_{gr}$ makes sense and predicts universal energy spectrum for cyclotron energies of dark photons identifiable as energy spectrum of bio-photons in TGD inspired quantum biology.

As almost all chapters of the books, also this chapter should be seen as a story about evolution of ideas rather than final summary. I have moved some purely mathematical speculations to second chapter to keep emphasis on TGD inspired physics.

3.1.1 The evolution of mathematical ideas

From the beginning the basic challenge -besides the need to deduce a general formula for the quantized Planck constant- was to understand how the quantization of Planck constant is mathematically possible. From the beginning it was clear that since particles with different values of Planck constant cannot appear in the same vertex, a generalization of space-time concept is needed to achieve this.

During last five years or so many deep ideas -both physical and mathematical- related to the construction of quantum TGD have emerged and this has led to a profound change of perspective in this and also other chapters. The overall view about TGD is described briefly in [L5] .

1. For more than five years ago I realized that von Neumann algebras known as hyperfinite factors of type II_1 (HFFs) are highly relevant for quantum TGD since the Clifford algebra of configuration space ("world of classical worlds", WCW) is direct sum over HFFs. Jones inclusions are particular class of inclusions of HFFs and quantum groups are closely related to them. This led to the conviction that Jones inclusions can provide a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with

M^4 and CP_2 degrees of freedom (later I replaced M^4 by its light cone $M^4_\pm$ and finally with the causal diamond CD defined as intersection of future and past light-cones of M^4). The idea about connection with Jones inclusion can be however questioned and is left another chapter.

2. The notion of zero energy ontology (ZEO) replaces physical states with zero energy states consisting of pairs of positive and negative energy states at the light-like boundaries $\delta M^4_\pm \times CP_2$ of CDs forming a fractal hierarchy containing CDs within CDs. In standard ontology zero energy state corresponds to a physical event, say particle reaction. This led to the generalization of S-matrix to M-matrix - possibly identified as Connes tensor product - characterizing time like entanglement between positive and negative energy states. M-matrix is product of square root of density matrix and unitary S-matrix just like Schrödinger amplitude is product of modulus and phase, which means that thermodynamics becomes part of quantum theory and thermodynamical ensembles are realized as single particle quantum states. This led also to a solution of long standing problem of understanding how geometric time of the physicist is related to the experienced time identified as a sequence of quantum jumps interpreted as moments of consciousness [L3] in TGD inspired theory of consciousness which can be also seen as a generalization of quantum measurement theory [L4] .

3. Another closely related idea was the emergence of measurement resolution as the basic element of quantum theory. Measurement resolution is characterized by inclusion $\mathcal{M} \subset \mathcal{N}$ of HFFs with $\mathcal{M}$ characterizing the measurement resolution in the sense that the action of $\mathcal{M}$ creates states which cannot be distinguished from each other within measurement resolution used. Hence complex rays of state space are replaced with $\mathcal{M}$ rays. One of the basic challenges is to define the nebulous factor space $\mathcal{N}/\mathcal{M}$ having finite fractional dimension $\mathcal{N}:\mathcal{M}$ given by the index of inclusion. It was clear that this space should correspond to quantum counterpart of Clifford algebra of world of classical worlds reduced to a finite-quantum dimensional algebra by the finite measurement resolution [K20] .

4. The realization that light-like 3-surfaces at which the signature of induced metric of space-time surface changes from Minkowskian to Euclidian are ideal candidates for basic dynamical objects besides light-like boundaries of space-time surface was a further decisive step or progress. This led to vision that quantum TGD is almost topological quantum field theory ("almost" because light-likeness brings in induced metric) characterized by Chern-Simons action for induced Kähler gauge potential of CP_2. Together with zero energy ontology this led to the generalization of the notion of Feynman diagram to a light-like 3-surface for which lines correspond to light-like 3-surfaces and vertices to 2-D partonic surface at which these 3-D surface meet. This means a strong departure from string model picture. The interaction vertices should be given by N-point functions of a conformal field theory with second quantized induced spinor fields defining the basic fields in terms of which also the gamma matrices of world of classical worlds could be constructed as super generators of super conformal symmetries [K20] .

5. By quantum classical correspondence finite measurement resolution should have a space-time correlate. The obvious guess was that this correlate is discretization at the level of construction of M-matrix. In almost-TQFT context the effective replacement of light-like 3-surface with braids defining basic objects of TQFTs is the obvious guess. Also number theoretic universality necessary for the p-adicization of quantum TGD by a process analogous to the completion of rationals to reals and various p-adic number fields requires discretization since only rational and possibly some algebraic points of the imbedding space (in suitable preferred coordinates) allow interpretation both as real and p-adic points. It was clear that the construction of M-matrix boils to the precise understanding of number theoretic braids [K20] .

6. The interaction with M-theory dualities [K79] led to a handful of speculations about dualities possible in TGD framework, and one of these dualities- $M^8 - M^4 \times CP_2$ duality - eventually suggests a highly unique identification of number theoretic braids. The dimensions of partonic 2-surface, space-time, and imbedding space strongly suggest that classical number fields, or more precisely their complexifications might help to understand quantum TGD. If the choice

of imbedding space is unique because of uniqueness of infinite-dimensional Kähler geometric existence of world of classical worlds then standard model symmetries coded by $M^4 \times CP_2$ should have some deeper meaning and the most obvious guess is that $M^4 \times CP_2$ can be understood geometrically. $SU(3)$ belongs to the automorphism group of octonions as well as hyper-octonions M^8 identified by subspace of complexified octonions with Minkowskian signature of induced metric. This led to the discovery that hyper-quaternionic 4-surfaces in M^8 can be mapped to $M^4 \times CP_2$ provided their tangent space contains preferred $M^2 \subset M^4 \subset M^4 \times E^4$. Years later I realized that the map generalizes so that M^2 can depend on the point of X^4. The interpretation of $M^2(x)$ is both as a preferred hyper-complex (commutative) sub-space of M^8 and as a local plane of non-physical polarizations so that a purely number theoretic interpretation of gauge conditions emerges in TGD framework. This led to a rapid progress in the construction of the quantum TGD. In particular, the challenge of identifying the preferred extremal of Kähler action associated with a given light-like 3-surface X_l^3 could be solved and the precise relation between M^8 and $M^4 \times CP_2$ descriptions was understood [K20] .

7. Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $h_{eff} = n \times h$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. This reduces the understanding of the effective hierarchy of Planck constants to quantum variant of multi-furcations for the dynamics of preferred extremals of Kähler action. The number of branches of multi-furcation defines the integer n in $\hbar_{eff} = n\hbar$.

3.1.2 The evolution of physical ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.

2. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K64] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E25] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

3. Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K87] , [I14] .

4. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K87] , [L2] .

3.1.3 Brief summary about the generalization of the imbedding space concept

A brief summary of the basic vision in order might help reader to assimilate the more detailed representation about the generalization of imbedding space, which has turned to be only a useful auxiliary tool of the theory rather than basic postulate.

1. The original belief was that the hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time since particles with different values of Planck constant cannot appear in the same interaction vertex. This suggests some kind of book like structure for both M^4 and CP_2 factors of the generalized imbedding space is suggestive. It has turned out that the view about hierarchy of Planck constants as effective hierarchy allows to regard the singular coverings of imbedding space as the natural *auxiliary* tool to describe the quantum view about multi-furcations of preferred extremals.

2. Schrödinger equation suggests that Planck constant corresponds to a scaling factor of M^4 metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in M^4 factor is possible so that the scaling of $\hbar$ corresponds to the scaling of the size of causal diamond CD defined as the intersection of future and past directed light-cones. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states. The infinite-D spaces formed by these 3-surfaces define the fundamental sectors of the configuration space (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces, the coding of radiative corrections to the geometry of space-time sheets becomes possible and Kähler action can be seen as expansion in powers of $\hbar/\hbar_0$.

3. Quantum criticality of the TGD Universe is one of the key postulates of quantum TGD. The most important implication is that Kähler coupling strength is analogous to critical temperature. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality would mean that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes into regions corresponding to different values of Planck constant much like matter decomposes to several phases at thermodynamical criticality.

3.1.4 Basic physical picture as it is now

The basic phenomenological rules are simple and remained roughly the same during years.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through

the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K88].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K64] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. It was Nottale [E25] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

3.1.5 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form.

The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K103] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis. coverings of H are just an auxiliary tool.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

Quantum criticality can be now understood as a direct consequence of the non-determinism of Kähler action and relates directly to h_{eff} hierarchy. There is also a direct relation to generalized conformal symmetries. Critical deformations correspond to Kac-Moody type algebra deforming light-like orbits of partons and respecting their light-likeness and leaving the partonic 2-surfaces at their ends invariant. There are n conformal equivalence classes of these deformations and n defines the value of $h_{eff} = n \times h$.

In this chapter I try to summarize the evolution of the ideas related to Planck constant without systematic attempt to achieve complete internal consistency. I have left the summary about the recent views to the end of the chapter and the reader might find it a good idea to begin from this section.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L27]
- Geometrization of fields [L26]
- Magnetic body [L31]

3.2 Experimental input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

3.2.1 Hints for the existence of large $\hbar$ phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [K75] . This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. With inspiration coming from the finding of Nottale [E25] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [K75] . Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.

2. Living matter could represent a basic example of large $\hbar$ phase [K28, K7] . Even ordinary condensed matter could be "partially dark" in many-sheeted space-time [K30] . In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.

3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [C97]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [K76] . The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [K58, K52] - something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.

4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [K30, K28] .

3.2.2 Quantum coherent dark matter and $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QEd$e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the modified Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in WCW degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

1. The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.

2. The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of WCW geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.

3. In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to WCW functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about imbedding space and forces to generalize the notion of imbedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of imbedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the imbedding space concept.

3.2.3 The phase transition changing the value of Planck constant as a transition to non-perturbative phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulomb contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

3.3 A generalization of the notion of imbedding space as a realization of the hierarchy of Planck constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the imbedding space is given. In [K64] the important delicacies associated with the Kähler structure of generalized imbedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactification leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of modified Dirac action fix to a high degree the vision about generalized imbedding space.

3.3.1 Basic ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of imbedding space geometry provided it is such that it is possible to compare the regions of generalized imbedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a manner that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.

2. The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

3. Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^ik_i$ rather than four-momentum squared $p^2 = p^ip_i$ in CD degrees of freedom and its analog in CP_2 degrees of freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, modified Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the modified Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define WCW spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over WCW defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant WCW metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semi-classical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that WCW Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [K95] . A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} =$

B_n. $\sqrt{b_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{b_n} = 2cos(\pi/n)$ varying thus in the range $[1,2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized imbedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action-finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anti-commutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized imbedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tetrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2,C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two manners for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

3.3.2 The vision

A brief summary of the basic vision behind the generalization of the imbedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of imbedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized imbedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.

2. The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.

3. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of WCW (world of classical worlds).

Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.

4. The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized imbedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

1. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions.

2. The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [K64] .

4. Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$

appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.

5. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.

6. The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

3.3.3 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E25] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K75] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K76] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by

gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K87] .

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K64] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E25] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K87] , [L2] .

The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the

singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

(a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

(b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP_2}\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP_2}/G_b$, $\hat{CD}/G_a \times (\hat{CP_2}\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP_2}/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase

transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K29].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K29] . This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

3.4 Updated view about the hierarchy of Planck constants

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis was that the hierarchy is real. In this formulation the imbedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization has remained somewhat fuzzy [K64]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to N branches is not general enough: the N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

3.4.1 Basic physical ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K88].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K64] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale [E25] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

   It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the modified Dirac equation.

4. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

3.4.2 Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the modified gamma matrices [K103] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book). What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches coincide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

3.4.3 The relationship to the original view about the hierarchy of Planck constants

Originally the hierarchy of Planck constant was assumed to correspond to a book like structure having as pages the n-fold coverings of the imbedding space for various values of n serving therefore as a page number. The pages are glued together along a 4-D "back" at which the branches of n-furcations are degenerate. This leads to a very elegant picture about how the particles belonging to the different pages of the book interact. All vertices are local and involve only particles with the same value of Planck constant: this is enough for darkness in the sense of particle physics. The interactions between particles belonging to different pages involve exchange of a particle travelling from page to another through the back of the book and thus experiencing a phase transition changing the value of Planck constant.

Is this picture consistent with the picture based on n-furcations? This seems to be the case. The conservation of energy in n-furcation in which several sheets are realized simultaneously is consistent with the conservation of classical conserved quantities only if the space-time sheet before n-furcation involves n identical copies of the original space-time sheet or if the Planck constant is $h_{eff} = nh$. This kind of degenerate many-sheetedness is encountered also in the case of branes. The first option means an n-fold covering of imbedding space and h_{eff} is indeed effective Planck constant. Second option means a genuine quantization of Planck constant due to the fact the value of Kähler coupling strength $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is scaled down by $1/n$ factor. The scaling of Planck constant consistent with classical field equations since they involve α_K as an overall multiplicative factor only.

3.4.4 Basic phenomenological rules of thumb in the new framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar_m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible

coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

3.4.5 Charge fractionalization and anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionization and fractionization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$:th branch corresponds to the original one. This suggests that angular momentum fractionization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in imbedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionization however leads to problems with fractionization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

3.4.6 Negentropic entanglement between branches of multi-furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism [K44] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy based on the p-adic norm for the probability appearing as argument of logarithm [K51], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs $a_i \otimes b_i$ in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large $\hbar$ photons.

How the large $\hbar$ photons could carry negentropic entanglement? There are several options to consider and at this stage it is not possible to pinpoint anyone of them as the only possible one. Several of them could also be realized.

1. In zero energy ontology large $\hbar$ photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.

2. The negentropic entanglement of large $\hbar$ photon could be also associated with its positive or energy part or both. Large $\hbar_{eff} = n\hbar$ photon with n-fold energy $E = n \times hf$ is n-sheeted structure consisting of n-photons with energy $E = hf$ de-localized in the discrete space formed by the N space-time sheets. The n single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for N-fold branching the superposition of all $N!/(N-n)!n!$ states obtained by selecting n branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would the quintessence of life.

3. A further very attractive possibility discovered quite recently is that large $h_{eff} = nh$ is closely related to the negentropic entanglement between the states of *two* n-furcated - that is dark - space-time sheets. In the most recent formulation negentropic entanglement corresponds to a state characterized by $n \times n$ identity matrix resulting from the measurement of density matrix. The number theoretic entanglement negentropy is positive for primes dividing p and there is unique prime for which it is maximal.

The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option.

1. One proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.

   The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large $\hbar$ variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW. If each of the n branches carries $1/n$:th part of electron one would have an anyonic state in WCW.

2. One can also make a really crazy question. Could it be that ATP and various bio-molecules form n-particle states at the n-sheet of n-furcation and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [K6] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given CD this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

3.4.7 Dark variants of nuclear and atomic physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [K90].

Before the real understanding what charge fractionization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form n-particle states associated with n-branches of N-furcation with $n = 1, ..., N$. The fractionization for a single particle state de-localized completely to the discrete space of N branches as the analog of plane wave means that single branch carriers charge $1/N$.

The new element is the possibility of n-particle states populating n branches of the N-furcation: note that there is superposition over the states corresponding to different selections of these n branches. $N - k$ and k-nuclei/atoms are in sense conjugates of each other and they can fuse to form N-nuclei/N-atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that N-atoms and even N-molecules could make possible the emergence of symbolic representations with $n \leq N$ serving as a name of atom/molecule and that k- and $N - k$ atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form N-atom/-molecule. For instance, in bio-catalysis k- and $N - k$-atoms/molecules would be paired. The recent picture about n and $N - k$ atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their n-multiples. Synchronized quantum jumps would allow to avoid the disastrous effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

3.4.8 What about the relationship of gravitational Planck constant to ordinary Planck constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the modified Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{3.4.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} \quad . \tag{3.4.2}$$

   For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 \quad . \tag{3.4.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{3.4.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4-momentum squared, then Equivalence Principle implies that h_{gr} can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying h_{gr} can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m^{kl}_{eff} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m^{kl}_{eff}.

3.4.9 Hierarchy of Planck constants and non-determinism of Kähler action

Originally the hierarchy of Planck constant was inspired by empirical inputs from neuroscience, biology, and from Nottale's observations. That it is possible to understand the hierarchy in terms of non-determinism of Kähler action - the fundamental difference between TGD and quantum field theories and string models - is a victory for TGD approach (see fig. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`, which is also in the appendix of this book).

Recall that non-determinism means that all space-time surfaces with CP_2 projection, which is Lagrangian sub-manifold (at most 2-D) of CP_2, carries a vanishing induced Kähler form and is vacuum extremal. The first guess would be that there is a finite number n of space-time sheets connecting given pair of 3-surfaces at the ends of space-time surface at the light-like boundaries of causal diamond (CD). Planck constant would be given as $h_{eff} = n \times h$ in accordance with the earlier interpretation. The degenerate extremals would have same Kähler action and conserved quantities as assumed also in the earlier approach. That the degenerate extremals co-incide at the ends of space-time surface was motivated by mathemtical aesthetics in the earlier approach but finds an interpretation in terms of non-uniqueness of the preferred extremals.

It is essential that these n degrees of freedom are regarded as genuine physical degrees of freedom, which are however discrete. Negentropic entanglement and dark matter would be associated with them naturally. The effective description would be in terms of n-fold singular covering of imbedding space becoming singular at the ends of the space-time surface.

I have also assigned hierarchy of Planck constants with the quantum criticality. Quantum criticality means the existence of an entire continuous family of deformations of space-time sheet with same Kähler action and conserved quantities. The deformations would by definition vanish at the ends of space-time surface. The critical deformations would act as gauge transformations identifiable as conformal symmetries indeed expected to be presents since WCW isometries form a conformal algebra and there is also Kac-Moody type algebra present. The proposal has been that the sub-algebras of conformal algebra for which conformal weights are integer multiples of integer $n = 1, 2, ..$ defined a hierarchy of gauge algebras so that the dynamical algebra reduces to n-dimensional one.

These two identifications seem to be mutually inconsistent. The resolution of the conflict comes from the gauge invariance. For a given Kähler action and conserved quantities there would be n conformal equivalence classes of these 4-surfaces rather than n surfaces, and one would have n-fold degeneracy but for conformal equivalence classes of 4-surfaces rather than 4-surfaces. In Minkowskian regions the degenerate preferred extremals are sheets (graphs of a map from M^4 to CP_2).

3.4.10 Could $h_{gr} = h_{eff}$ hold true?

The obvious question is whether the gravitational Planck constant deduced from the Nottale's considerations and the effective Planck constant $h_{eff} = nh$ deduced from ELF effects on vertebrate brain and explained in terms of non-determinism of Kähler action could be identical. At first this seems to be non-sensical idea since $h_{gr} = GMm/v_0$ has gigantic value.

It is however essential to realize that by Equivalence Principle one describe gravitational interaction by reducing it to elementary particle level. For instance, gravitational Compton lengths do not depend at all on the masses of particles. Also the radii of the planetary orbits are independent of the mass of particle mass in accordance with Equivalence Principle. For elementary particles the values of h_{gr} are in the same range as in quantum biological applications. Typically 10 Hz ELF radiation should correspond to energy $E = h_{eff}f$ of UV photon if one assumes that dark ELF photons have energies of biophotons and transform to them. The order of magnitude for n would be therefore $n \simeq 10^{14}$.

The experiments of M. Tajmar et al [E14, E27] discussed in [K111] provide a support for this picture. The value of gravimagnetic field needed to explain the findings is 28 orders of magnitude higher than theoretical value if one extrapolates the model of Meissner effect to gravimagnetic context. The amazing finding is that if one replaces Planck constant in the formula of gravimagnetic field with h_{gr} associated with Earth-Cooper pair system and assumes that the velocity parameter v_0 appearing in it corresponds to the Earth's rotation velocity around its axis, one obtains correct order of magnitude for the effect requiring $r \simeq 3.6 \times 10^{14}$.

The most important implications are in quantum biology and Penrose's vision about importance of quantum gravitation in biology might be correct.

1. This result allows by Equivalence Principle the identification $h_{gr} = h_{eff}$ at elementary particle level at least so that the two views about hierarchy of Planck constants would be equivalent. If the identification holds true for larger units it requires that space-time sheet identifiable as quantum correlates for physical systems are macroscopically quantum coherent and gravitation causes this. If the values of Planck constant are really additive, the number of parallel space-time sheets corresponding to non-determinism evolution for the flux tube connecting systems with masses M and m is proportional to the masses M and m using Planck mass as unit. Information theoretic interpretation is suggestive since hierarchy of Planck constants is assumed to relate to negentropic entanglement very closely in turn providing physical correlate for the notions of rule and concept.

2. That gravity would be fundamental for macroscopic quantum coherence would not be surprising since by EP all particles experience same acceleration in constant gravitational field, which therefore has tendency to create coherence unlike other basic interactions. This in principle allows to consider hierarchy in which the integers $h_{gr,i}$ are additive but give rise to the same universal dark Compton length.

3. The model for quantum biology relying on the notions of magnetic body and dark matter as hierarchy of phases with $h_{eff} = nh$, and biophotons [K106, K105] identified as decay produces of dark photons. The assumption $h_{gr} \propto m$ becomes highly predictable since cyclotron frequencies would be independent of the mass of the ion.

   (a) If dark photons with cyclotron frequencies decay to biophotons, one can conclude that biophoton spectrum reflects the spectrum of endogenous magnetic field strengths. In the model of EEG [K29] it has been indeed assumed that this kind spectrum is there: the inspiration came from music metaphors suggesting that musical scales are realized in terms of values of magnetic field strength. The new quantum physics associated with gravitation would also become key part of quantum biophysics in TGD Universe.

   (b) For the proposed value of h_{gr} 1 Hz cyclotron frequency associated to DNA sequences would correspond to ordinary photon frequency $f = 3.6 \times 10^{14}$ Hz and energy 1.2 eV just at the lower limit of visible frequencies. For 10 Hz alpha band the energy would be 12 eV in UV. This plus the fact that molecular energies are in eV range suggests very simple realization of biochemical control by magnetic body. Each ion has its own cyclotron frequency but same energy for the corresponding biophoton.

(c) Biophoton with a given energy would activate transitions in specific bio-molecules or atoms: ionization energies for atoms except hydrogen have lower bound about 5 eV (http://en.wikipedia.org/wiki/Ionization_energy). The energies of molecular bonds are in the range 2-10 eV (http://en.wikipedia.org/wiki/Bond-dissociation_energy). If one replaces v_0 with $2v_0$ in the estimate, DNA corresponds to .62 eV photon with energy of order metabolic energy currency and alpha band corresponds to 6 eV energy in the molecular region and also in the region of ionization energies.

Each ion at its specific magnetic flux tubes with characteristic palette of magnetic field strengths would resonantly excite some set of biomolecules.This conforms with the earlier vision about dark photon frequencies as passwords.

It could be also that biologically important ions take care of their ionization self. This would be achieved if the magnetic field strength associated with their flux tubes is such that dark cyclotron energy equals to ionization energy. EEG bands labelled by magnetic field strengths could reflect ionization energies for these ions.

(d) The hypothesis means that the scale of energy spectrum of biophotons depends on the ratio M/v_0 of the planet and on the strength of the endogenous magnetic field, which is .2 Gauss for Earth (2/5 of the nominal value of the Earth's magnetic field). Therefore the astrophysical characteristics of planets should be tuned for molecular life. Taking v_0 to be rotational velocity one obtains for the ratio $M(planet)/v_0(planet)$ using the ratio for Earth as unit the following numbers for the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptune): $M/v_0 = (8.5, 209, 1, .214223, 1613, 6149, 9359)$. If the energy scale of biophotons is required to be the same, the scale of endogenous magnetic field should be divided by this ratio in order to obtain the same situation as in Earth. For instance, in Mars the magnetic field should be roughly 5 times stronger: in reality the magnetic field of Mars is much weaker. Just for fun one can notice that for Sun the ratio is 1.4×10^6 so that magnetic field should be by the inverse of this factor weaker.

4. An interesting question is how large systems can behave as coherent units with $h_{gr} = GMm/v_0$. In living matter one might consider the possibility that entire organism might be this kind of system. Interestingly, for larger masses the gravitational quantum coherence would be easier. For particle with mass m $h_{gr}/h > 1$ requires larger mass to satisfy $M > M_P^2/m_e$. The first guess that life has evolved from long to shorter scales and reached elementary particle last. Planck mass is the critical mass corresponds to the mass of water blog with volume of size scale of 10^{-4} m (big neuron) is the limit.

5. The Universal gravitational Compton wave length of $GM/v_0 \simeq 864$ meters gives an idea about largest possible living matter system if Earth is the second body. Of course, also other large bodies are possible.In the case of solar system this length is 3×10^3 km. The radius of Earth is 6.37×10^3 km - roughly twice the Compton length. The radii of Mercury, Venus, Earth, Mars, Jupiter, Saturnus, Uranus, Neptunus are (.38,.99, .533, 1, 10.6, 8.6, 4.0, 3.9) using Earth radius as unit the value of h_{gr} is by factor 5 larger than for three inner planets so that the values are reasonably near to gravitational Compton length or twice it. Does this mean that dark matter associated with Earth and maybe also other planets is in macroscopic quantum state at some level of the hierarchy of space-time sheets? Does this mean that Mother Gaia as conscious entity might make sense. One can of course make same question in the case of Sun. The universal gravitational Compton length in Sun would be 18 per cent of the radius of Sun if v_0 is taken to be the rotational velocity at the surface of Sun. The radius of solar core, where fusion takes place, is 20-25 per cent of solar radius.

6. There are further interesting numerical co-incidences. One can for a moment forget the standard hostility of scientist towards horoscopes and ask whether Sun and Moon could have somehow affect our life via astrocopic quantum coherence. The gravitational Compton length for particle-Moon or particle-Sun system multiplied by the natural value of magnetic field is the relevant parameter. For Sun the parameters in question are mass of Sun, and rotational velocity of Earth with respect to Sun, plus magnetic fields of Sun at flux tubes associated with solar magnetic field measured to be about 5 nT at the position of Earth and 100 times

stronger than expected from dipole field behavior. This gives that the range of biophoton energies is scaled down with factor of 1/4 in good approximation so that Father Sun might affect terrestrial biology! If one uses for the rotational velocity of particle at surface of Moon as parameter v_0 (particle would be at Moon), biophoton energy scaled scaled up by factor 1.2.

The general proposal discussed above is testable. In particular, a detailed study of molecular energies with those associated with resonances of EEG could be highly rewarding and reveal the speculated spectroscopy of consciousness.

3.4.11 How the effective hierarchy of Planck constants could reveal itself in condensed matter physics

Anderson - one of the gurus of condensed matter physics - has stated that there exists no theory of condensed matter: experiments produce repeatedly surprises and theoreticians do their best to explain them in the framework of existing quantum theory.

This suggests that condensed matter physics might allow room even for new physics. Indeed, the model for fractional quantum Hall effect (FQHE) [K64] strengthened the feeling that the many-sheeted physics of TGD could play a key role in condensed matter physics often thought to be a closed chapter in physics. One implication would be that space-time regions with Euclidian signature of the induced metric would represent the space-time sheet assignable to condensed matter object as a whole as analog of a line of a generalized Feynman diagram. Also the hierarchy of effective Planck constants $\hbar_{eff} = n\hbar$ appears in the model of FQHE.

The recent discussion of possibility of quantum description of psychokinesis [L18] boils down to a model for intentional action based on the notion of magnetic flux tube carrying dark matter and dark photons and inducing macroscopic quantum superpositions of magnetic bubbles of ferromagnet with opposite magnetization. As a by-product the model leads to the proposal that the conduction electrons responsible for ferromagnetism are actually dark (in the sense of having large value of effective Planck constant) and assignable to a multi-sheeted singular covering of space-time sheet assignable to second quantization multi-furcation of the preferred extremal of Kähler action made possible by its huge vacuum degeneracy.

What might be the signatures for $\hbar_{eff} = n\hbar$ states in condensed matter physics and could one interpret some exotic phenomena of condensed matter physics in terms of these states for electrons?

1. The basic signature for the many-electron states associated with multi-sheeted covering is a sharp peak in the density of states due to the presence of new degrees of freedom. In ferromagnets this kind of sharp peak is indeed observed at Fermi energy [D21].

2. In the theory of super-conductivity Cooper pairs are identified as bosons. In TGD framework all bosons - also photons - emerge as wormhole contacts with throats carrying fermion and anti-fermion. I have always felt uneasy with the assumption that two-fermion states obey exact Bose-Einstein statistics at the level of oscillator operators: they are after all two-fermion states. The sheets of multi-sheeted covering resulting in a multi-furcation could however carry both photons identified as fermion-anti-fermion pairs and Cooper pairs and this could naturally give rise to Bose-Einstein statistics in strong sense and also be involved with Bose-Einstein condensates. The maximum number of photons/Cooper pairs in the Bose-Einstein condensate would be given by the number of sheets. Note that in zero energy ontology also the counterparts of coherent states of Cooper pairs are possible: in positive energy ontology they have ill-defined fermion number and also this has made me feel uneasy.

3. Majorana fermions [D12] have become one of the hot topics of condensed matter physics recently.

    (a) Majorana particles are actually quasiparticles which can be said to be half-electrons and half-holes. In the language of anyons would would have charge fractionization $e \rightarrow e/2$. The oscillator operator $a^\dagger(E)$ creating the hole with energy E defined as the difference of real energy and Fermi energy equals to the annihilation operator $a(-E)$ creating a hole: $a^\dagger(E) = a(-E)$. If the energy of excitation is $E = 0$ one obtains $a^\dagger(0) = a(-0)$.

Since oscillator operators generate a Clifford algebra just like gamma matrices do, one can argue that one has Majorana fermions at the level of Fock space rather than at the level of spinors. Note that one cannot define Fock vacuum as a state annihilated by $a(0)$. Since the creation of particle generates a hole equal to particle for $E = 0$, Majorana particles come always in pairs. A fusion of two Majorana particles produces an ordinary fermion.

(b) Purely mathematically Majorana fermion as a quasiparticle would be highly analogous to covariantly constant right-handed neutrino spinor in TGD with vanishing four-momentum. Note that right-handed neutrino allows 4-dimensional modes as a solution of the modified Dirac equation whereas other spinor modes localized to partonic 2-surfaces and string world sheets. The recent view is however that covariantly constant right-handed neutrino cannot not give rise to the TGD counterpart of standard space-time SUSY.

(c) In TGD framework the description that suggests itself is in terms of bifurcation of space-time sheet. Charge -e/2 states would be electrons de-localized to two sheets. Charge fractionization would occur in the sense that both sheets would carry charge -e/2. Bifurcation could also carry two electrons giving charge -e at both sheets. Two-sheeted analog of Cooper pair would be in question. Ordinary Cooper pair would in turn be localized in single sheet of a multi-furcation. The two-sheeted analog of Cooper pair could be regarded as a pair of Majorana particles if the measured charge of electron corresponds to its charge at single sheet of bifurcation (this assumption made also in the case of FQHE is crucial!). Whether this is the case, remains unclear to me.

(d) Fractional Josephson effect in which the current carriers of Josephson current become electrons or quasiparticles with the quantum numbers of electron has been suggested to serve as a signature of Majorana quasiparticles [D14]. An explanation consistent with above assumption is as a two-sheeted analog of Cooper pair associated with a bifurcated space-time sheets.

If the measurements of Josephson current measure the current associated with single branch of bifurcation the unit of Josephson current is indeed halved from -2e to -e. These 2-sheeted Cooper pairs behave like dark matter with respect to ordinary matter so that dissipation free current flow would become possible.

Note that ordinary Cooper pair Bose-Einstein condensate would correspond to N-furcation with N identified as the number of Cooper pairs in the condensate if the above speculation is correct. Fractional Josephson effect generated in external field would correspond to a formation of mini Bose-Einstein condensates in this framework and also smaller fractional charges are expected. In this case the interpretation as Majorana fermion does not seem to make sense.

3.4.12 Summary

The hierarchy of Planck constants reduces to second quantization of multi-furcations in TGD framework and it is somewhat a matter of taste whether one regards the hierarchy as only effective or real. The non-determinism of Kähler action implies the hierarchy. Anyonic physics and effective charge fractionization are consequences of second quantized multi-furcations. This framework also provides quantum version for the transition to chaos via quantum multi-furcations and living matter represents the basic application. The key element of dynamics of TGD is vacuum degeneracy of Kähler action making possible quantum criticality having the hierarchy of multi-furcations as basic aspect. The potential problems relate to the question whether the effective scaling of Planck constant involves scaling of ordinary wavelength or not. For particles confined inside linear structures such as magnetic flux tubes this seems to be the case.

There is also an intriguing connection with the vision about physics as generalized number theory. The conjecture that the preferred extremals of Kähler action consist of quaternionic or co-quaternionic regions led to a construction of them using iteration and also led to the hierarchy of multi-furcations [K103]. Therefore it seems that the dynamics of preferred extremals might indeed reduce to associativity/co-associativity condition at space-time level , to commutativity/co-commutativity condition at the level of string world sheets and partonic 2-surfaces, and to reality at the level of stringy curves (conformal invariance makes stringy curves causal determinants [K97] so that conformal dynamics represents conformal evolution) [K84].

3.5 Vision about dark matter as phases with non-standard value of Planck constant

3.5.1 Dark rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

1. Generalized imbedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.

2. Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [I14] . This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.

3. The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [K64] a detailed model for the Kähler structure of the generalized imbedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full imbedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

3.5.2 Phase transitions changing Planck constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized imbedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.

2. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_{\pm}$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a manner that regions in which induced Kähler form is non-vanishing are contained within single page of imbedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD \times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for appropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

3.5.3 Coupling constant evolution and hierarchy of Planck constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K59] .

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tetrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5 and and thus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

1. In [K5] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.

2. The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0, 1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [K5] .

3.6 Some applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

3.6.1 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [D6] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \quad (3.6.1)$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15...,\ 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9...,\ 1/5, 2/9, 3/13...,\ 2/7, 3/11...,\ 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D6] .

The model of Laughlin [D69] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D65] . Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2\times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [D6, D45] . The fractionized charge is $e/4$ in the latter case [D45, D72] . Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D65] . $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F\int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6\times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature.

The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather ad hoc. Therefore the model can be taken as a warm-up exercise only. In [K64] , where the delicacies of Kähler structure of generalized imbedding space are discussed, also a more detailed of QHE is discussed.

3.6.2 Gravitational Bohr orbitology

The basic question concerns justification for gravitational Bohr orbitology [K75] . The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K75] .

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Schrödinger equation holds true only inside flux quanta for the dark matter.

1. During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).

2. In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.

3. The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

   It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n \ mod \ 5 = 0$ for some reason.

4. A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

   The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

2. Nottale [E25] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [K75]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [K75] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet) n_{F,b}(Earth)/n_{F,a}(Earth) n_{F,b}(planet)$. The deviations are at most 2 per cent.

planet	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
planet	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^4\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 1. The table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k\prod_k F_{n_k}$, where $F_i = 2^{2^i}+1$, $i = 0,1,2,3,4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254}\times 5\times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K29] . For $v_0 = 4.6\times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1+\epsilon)$ of the total mass for Earth gives

$$1+\epsilon = \frac{17}{16} \tag{3.6.2}$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1-\epsilon) \simeq 4.6\times 10^{-4}$ equals with $v_0(eff) = 1/(2^7\times F_2) = 4.5956\times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v\cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface) . Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of imbedding space indeed involve quantum groups central in the modelling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [K64] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized imbedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized imbedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [K89] .

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} =$

$2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

3.6.3 Accelerating periods of cosmic expansion as phase transitions increasing the value of Planck constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E6, E2] . Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be .5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the imbeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D imbedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D imbedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D imbedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to imbedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

3.6.4 Phase transition changing Planck constant and expanding Earth theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.

2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.

3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3,4, and 5 are present.

4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [?] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [?] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [I1] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.

2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.

3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.

4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.

5. I am not sure whether Adams mentions the following objections [?] . Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.

6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [?] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [?] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [?] (orogeny), earth quake zones, and associated zones of volcanic activity [?] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.

2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.

3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [?] , whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.

2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.

3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.

2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.

3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.

4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.

5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...

6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.

7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I9] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.

2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs. The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I15] , which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I13] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [K35] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [E3] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is .131 times that for Earth so that surface gravity would be .532 of that for Earth now and would be reduced to .131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

3.6.5 Allais effect as evidence for large values of gravitational Planck constant?

Allais effect [E1, E9] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E9] .

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5\times 10^{-4}$ [E1, E18] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E24] .

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K10] . If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M,and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

3.6.6 Applications to elementary particle physics, nuclear physics, and condensed matter physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water-might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron [K88] . Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [C125] . Towards the end of 2008 CDF anomaly [C26] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [K88] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [D67] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [L2] , [L2] . In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the cathode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E11]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.

2. The so called $H_{1.5}O$ anomaly of water [D30, D27, D32, D52] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.

3. The mysterious behavior burning salt water [D1] can be also understood in the same framework.

4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [C94, C116] . Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

3.6.7 Applications to biology and neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

1. There is evidence for an icosahedral clustering in [D57] [D31] . Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.

2. It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

3. What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, amino-acids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [K16] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc... [D82, D80] .

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [D82] .

The TGD based model for high T_c super-conductivity [K16] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

1. Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.

2. The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.

3. The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [K16] . p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

1. An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1-5$ μm, the cell nucleus length scale. Hence type I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

2. At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

3. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of .05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [K29] .

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [K29] .

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [K16, K17] .

Dark cyclotron radiation with photon energy above thermal energy could be used for coordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [I12] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [K35] .

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

1. The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

2. The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

3. The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.

4. The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.

5. Each physical system corresponds to an onion-like hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.

6. At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of TQC in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [I7] . Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out

to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to TQC must be weakened appropriately.

2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [C106] generalizes. The ends of the space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical TQC program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that TQC program is automatically written to memory as the braiding of the threads during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

1. Darwinian selection for which standard theory of self-organization [B4] provides a model, should apply also to TQC programs. TQC programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.

2. Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.

3. The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions,...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQC's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.

4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.

5. The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.

6. There are several problems related to the details of the realization. How nucleotides A,T,C,G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High

T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

1. In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

2. The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

3. Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

4. The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

1. The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of TQC. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves

and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

2. During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

3.7 Appendix

3.7.1 About inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A29] . It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A29] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A29] . For any amenable group G the the inclusion is also unique apart from outer automorphism [A40] .

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [A29] . The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_\mathcal{N} \mathcal{M} .... \otimes_\mathcal{N} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A29] .

1. Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n) | n \geq 3\} \cap [4, \infty)$ [A29] . They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$

appears in the relations for the generators of the algebra given by $e_ie_je_i = \lambda e_i$, $|i-j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A29] . It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$ define orbifold coverings of $H_\pm = CD \times CP_2 \rightarrow H_\pm/G_a \times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [A29] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_\mathcal{M}$ it is not invariant under $G_\mathcal{N}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \rightarrow H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

3.7.2 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A83] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M}:\mathcal{N} \; = \; \prod_{1\leq r<s\leq k} \frac{sin^2\left((\lambda_r-\lambda_s+s-r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{3.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1\leq k\leq l-1$ holds true. Only Young diagrams satisfying the condition $l-k=\lambda_1-\lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4\rightarrow M^4/G_a$ or $CP_2\rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1)\times SU(3)$ and $SL(2,C)\times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4\times CP_2$.

1. $n>2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n>3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

Chapter 4

Mathematical Speculations about the Hierarchy of Planck Constants

4.1 Introduction

I decided to separate the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first highly speculative topic discussed in this chapter is about possible connection of the hierarchy of Planck constants with Jones inclusions.

1. The connection with Jones inclusions was originally a purely heuristic guess based on the observation that the finite groups characterizing Jones inclusion characterize also pages of the Big Book. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and that this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized imbedding space could indeed serve as correlates for Jones inclusions. The elements of the included algebra $\mathcal{M}$ are invariant under the action of G and $\mathcal{M}$ takes the role of complex numbers in the resulting non-commutative quantum theory.

2. The understanding of quantum TGD at parton level led to the realization that the dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field. This automatically implies cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups closely associated with Jones inclusions [K11] . The Clifford algebra spanned by the fermionic oscillator operators would provide a realization for the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that its unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 or CP_2 in some sense. The alternative interpretation in terms of two geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom.

4. Number theoretic Universality suggests an answer why the hierarchy of Planck constants is necessary. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in p-adic context. All that one can achieve naturally is

the notion of phase defined as root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase if needed. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases geometrically and quantum phases q naturally assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L27]
- Geometrization of fields [L26]
- Magnetic body [L31]

  [L24]

4.2 Jones inclusions and generalization of the imbedding space

The original motivation for the generalization of the imbedding space was the idea that the pages of the Big Book would provide correlates for Jones inclusions. In the following an attempt to formulate this vision more precisely is carried out.

4.2.1 Basic facts about Jones inclusions

Here only basic facts about Jones inclusions are discussed. Appendix contains a more detailed discussion of inclusions of HFFs.

Jones inclusions defined by subgroups of $SL(2,C)\times SU(2)$

Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ have representation as $R_0^G\subset R^G$ with G a discrete subgroup of $SU(2)$. $SO(3)$ or $SU(2)$ can be interpreted as acting in CP_2 as rotations. On quantum spinors the action corresponds to double cover of G.

A more general choice for G would be as a discrete subgroup $G_a\times G_b\subset SL(2,C)\times SU(2)\times SU(2)$. Poincare invariance suggests that the subgroup of $SL(2,C)$ reduces either to a discrete subgroup of $SU(2)$ and in the case that the rotation are genuinely 3-dimensional (E^6,E^8), the only possible interpretation would be as isotropy group of a particle at rest. When the group acts on plane as in case of A_n and D_{2n}, it could be also assigned to a massless particle.

If the group involves boosts it contains an infinite number of elements and it is not clear whether this kind of situation is physically sensible. In this case Jones inclusion could be interpreted as an inclusion for the tensor product of G invariant algebras associated with CD and CP_2 degrees of freedom and one would have $\mathcal{M}:\mathcal{N}=\mathcal{M}:\mathcal{N}(G_a)\times\mathcal{M}:\mathcal{N}(G_b)$. Since the index increases as the order of G increases one has reasons to expect that in the case of $G_a = SL(2,C)$ $N_a = \infty$ implies larger $\mathcal{M} : \mathcal{N}(G_a) > 4$.

A possible interpretation is that the values $\mathcal{M} : \mathcal{N} \leq 4$ are analogous to bound state energies so that a discrete rotation group acting in the relative rotational degrees of freedom can act as a symmetry group whereas the values $\mathcal{M} : \mathcal{N} > 4$ are analogous to ionized states for which particles are almost freely moving with respect to each other with a constant velocity.

When one restricts the coefficients to G-invariant elements of Clifford algebra the Clifford field is G-invariant under the natural action of G. This allows two interpretations. Either the Clifford field is G invariant or that the Clifford field is defined in orbifold $CD/G_a \times CP_2/G_b$. CD/G_a is obtained by replacing hyperboloid H_a ($t^2 - x^2 - y^2 - z^2 = a^2$) with H_a/G_a. These spaces have been considered as cosmological models having 3-space with finite volume [K76] (also a lattice like structure could be in question).

The quantum phases associated with sub-groups of $SU(2)$

It is natural to identify quantum phase as that defined by the maximal cyclic subgroup for finite subgroups of $SU(2)$ and infinite subgroups of $SL(2,C)$. Before continuing a brief summary about quantum phases associated with finite subgroups of $SU(2)$ is in order. E_6 corresponds to $N = 24$ and $n = 3$ and E_8 to icosahedron with $N = 120$, $n = 5$ and Golden mean and the minimal value of n making possible universal topological quantum computer [K94] .

D_n and A_n have orders $2n$ and $n + 1$ and act as symmetry groups of n-polygon and have n-element cyclic group as a maximal cyclic subgroup. For double covers the orders are twice this. Thus A_n *resp.* D_{2n} correspond to $q = exp(i\pi/n)$ *resp.* $q = exp(i\pi/2n)$. Note that the restriction $n \geq 3$ means geometrically that only non-trivial polygons are allowed.

4.2.2 Jones inclusions and the hierarchy of Planck constants

The anyonic arguments for the quantization of Planck constant suggest that one can assign separate scalings of Planck constant to CD and CP_2 degrees of freedom and that these scalings in turn reflect as scalings of $M^4\pm$ and CP_2 metrics. This is definitely not in accordance with the original TGD vision based on uniqueness of imbedding space but makes sense if space-time and imbedding space are emergent concepts as the hierarchy of number theoretical von Neumann algebra inclusions indeed suggests. Indeed, the scaling factors of CD and CP_2 metric remain non-fixed by the general uniqueness arguments since Cartesian product is in question.

Hierarchy of Planck constants and choice of quantization axis

Jones inclusions seem to relate in a natural manner to the selection of quantization axis.

1. In the case of CD the orbifold singularity is for all groups G_a except E_6 and E_8 the time-like plane M^2 corresponding to a radial ray through origin defining the quantization axis of angular momentum and intersecting light-cone boundary along a preferred light-like ray. For E_6 and E_8 (tetrahedral and icosahedral symmetries) the singularity consists of planes M^2 related by symmetries of G sharing time-like line M^1 and in this case there are several alternative identifications of the quantization axes as axis around which the maximal cyclic subgroup acts as rotations.

2. From this it should be obvious that Jones inclusions represented in this manner would relate very closely to the selection of quantization axes and provide a geometric representation for this selection at the level of space-time and WCW. The existence of the preferred direction of quantization at a given level of dark matter level should have observable consequences. For instance, in cosmology this could mean a breaking of perfect rotational symmetry at dark matter space-time sheets. The interpretation would be as a quantum effect in cosmological length scales. An interesting question is whether the observed asymmetry of cosmic microwave background could have interpretation as a quantum effect in cosmological length and time scales.

Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of the singular coverings and and factor spaces? If both geodesic spheres of CP_2 are allowed $\mathcal{M}:\mathcal{N}=4$ could correspond to the allowance of cosmic strings and other analogous objects. This option is however asymmetric with respect to CD and CP_2 and the more plausible option is that the two kinds of Jones inclusions correspond to singular factor spaces and coverings.

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M}:\mathcal{N}<4$ and it naturally corresponds to the coset spaces. For $\mathcal{M}:\mathcal{N}=4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$. For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane. This picture is also consistent with the G singlets of the quantum states despite the fact that fermionic oscillator operators belong to non-trivial irreps of G.

Coverings and factors spaces form an algebra like structure

It is easy to see that coverings and factor spaces defining the pages of the Big Book form an algebra like structure.

1. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labeled by a subset of discrete subgroups of $SU(2)$.

2. The discrete subgroups of $SU(2)$ with fixed quantization axis possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tetrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2 generated by reflection, and tetrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in WCW labeled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

Connection between Jones inclusions, hierarchy of Planck constants, and finite number of spinor modes

The original generalization of the imbedding space to accommodate the hierarchy of Planck constants was based on the idea that the singular coverings and factor spaces associated with the causal diamond CD and CP_2, which appears as factors of $CD \times CP_2$ correspond somehow to Jones inclusions, and that the integers n_a and n_b characterizing the orders of maximal cyclic groups of groups G_a and G_b associated with the two Cartesian factors correspond to quantum phases $q = exp(i2\pi/n_i)$ in such a manner that singular factor spaces correspond to Jones inclusions with index $\mathcal{M} : \mathcal{N} < 4$ and coverings to those with index $\mathcal{M} : \mathcal{N} = 4$.

Since Jones inclusions are interpreted in terms of finite measurement resolution, the mathematical realization of this heuristic picture should rely on the same concept realized also by the fact that the number of non-zero modes for induced spinor fields is finite. This allows to consider two possible interpretations.

1. The finite number of modes defines an approximation to the hyper-finite factor of type II_1 defined by WCW Clifford algebra.

2. The Clifford algebra spanned by fermionic oscillator operators is quantum Clifford algebra and corresponds to the somewhat nebulous object $\mathcal{N}/\mathcal{M}$ associated with the inclusion $\mathcal{M} \subset \mathcal{N}$ and coding the finite measurement resolution to a finite quantum dimension of the Clifford algebra. The fact that quantum dimension is smaller than the actual dimension would reflect correlations between spinor components so that they are not completely independent.

If the latter interpretation is correct then second quantized induced spinor fields should obey quantum variant of anti-commutation relations reducing to ordinary anti-commutation relations only for $n_a = n_b = 0$ (no singular coverings nor factor spaces). This would give the desired connection between inclusions and hierarchy of Planck constants. It is possible to have infinite number of quantum group like structure for $\hbar = \hbar_0$.

There are two quantum phases q and one should understand what is the phase that appears in the quantum variant of anti-commutation relations. A possible resolution of the problem relies on the observation that there are two kinds of number theoretic braids. The first kind of number theoretic braid is defined as the intersection of M_+ (or light-like curve of δM^4_+ in more general case) and of δM^4_+ projection of X^2. Second of braid is defined as the intersection of CP_2 projection of X^2 of homologically non-trivial sphere S^2_{II} of CP_2. The intuitive expectation is that these dual descriptions apply for light-like 3-surfaces associated *resp.* co-associative regions of space-time surface and that both descriptions apply at wormhole throats. The duality of these descriptions is guaranteed also at wormhole throats if physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations. This would suggest that it is $q = exp(i2\pi/r)$, which appears in quantum variant of anti-commutation relations of the induced spinor fields.

The action of $G_a \times G_b$ on WCW spinors and spinor fields

The first question is what kind of measurement resolution is in question. In zero energy ontology the included states would typically correspond to insertion of zero energy states to the positive or negative part of the physical state in time scale below the time resolution defined by the time scale assignable to the smallest CD present in the zero energy state. Does the description in terms of G invariance apply in this case or does it relate only to time and length scale resolution whereas hierarchy of Planck constants would relate to angle resolution? Assume that this is the case.

The second question is how the idea about $\mathcal{M}$ as an included algebra defining finite measurement resolution and G invariance as a symmetry defining $\mathcal{M}$ as the included algebra relate to each other.

1. One cannot say that G creates states, which cannot be distinguished from each other. Rather G-invariant elements of $\mathcal{M}$ create states whose presence in the state cannot be detected.

2. For covering space option $\mathcal{M}$ represents states which are invariant under discrete subgroup of $SU(2)$ acting in the covering. States with integer spin would be below measurement resolution and only factional spins of form j/n would be observable. For factor space option $\mathcal{M}$ would

represents states which are invariant under discrete subgroup of $SU(2)$ acting in H-say states with spin. States with spin which is multiple of n would be below measurement resolution. The situation would be very similar to each other. Number theoretic considerations and the fact that the number of fermionic oscillator operators is finite suggest that that for coverings the condition $L_z < 1$ and for factor spaces the condition $L_z < n$ is satisfied by the generators of Clifford algebra regarded as irreducible representation of G. For factor spaces the interpretation could be in terms of finite angular resolution $\Delta\phi \leq 2\pi/n$ excluding angular momenta $L_z \geq n$. For coverings the resolution would be related to rotations (or rather, braidings) as multiples of 2π: multiples $m2\pi$ $m \geq n$ cannot be distinguished from $m \ mod \ n$ multiples.

3. The minimal assumption is that integer orbital angular momenta are excluded for coverings and n-multiples are excluded for factors spaces. The stronger assumption would be that there is angular momentum cutoff. This point is however very delicate. The states with $j > n$ can be obtained as tensor products of representations with $j = m$. If entanglement is present one cannot anymore express the state as a product of $\mathcal{M}$ element and $\mathcal{N}$ element so that the states $j > n$ created in this manner would not be equivalent with those with $j \ mod \ n$. The replacement of the ordinary tensor product with Connes tensor product would indeed generate automatically entangled states and one could interpret Connes tensor product as a manner to create only the allowed states.

4. For quantum groups allow only finite number of representations up to some maximum spin determined by the integer n characterizing quantum phase q. This would mean angular momentum cutoff leaving only a finite number of representations of quantum group [K11] . This fits nicely with what one obtains in the case of factor spaces. For coverings the new element is that the unit of spin becomes $1/n$: otherwise the situation seems to be similar. Quantum group like structure is obtained if the fermionic oscillator operators satisfy the quantum version of anti-commutation relations. The algebra would be very similar except that the orbital angular momentum labeling oscillator operators has different unit. Oscillator operators are naturally in irreducible representations of G and only the non-trivial representations of G are allowed.

5. Besides Jones inclusions corresponding to $\mathcal{M} : \mathcal{N} < 4$ there are inclusions with $\mathcal{M} : \mathcal{N} = 4$ to which one can also assign quantum phases. It would be natural to assign covering spaces and factor spaces to these two kinds of inclusions. For the minimal option excluding only the orbital angular momentum which are integers or multiples of n the fraction of excluded states is very small for coverings so that $\mathcal{M} : \mathcal{N} = 4$ is natural for this option. $\mathcal{M} : \mathcal{N} < 4$ would in turn correspond naturally to factor spaces.

6. Since the two kinds of number theoretic braids correspond to points which belong to M^2 or S^2, one might argue that several quantum anti-commutation relations must be satisfied simultaneously. This is not the case since the eigen modes of D_{C-S} and hence also oscillator operators code information about partonic surface X^2 itself and also about $X^4(X^3_l)$ rather than being purely local objects. In the case of covering space the oscillator operators can be arranged to irreducible representations of G and in the case of factor space the oscillator operators are G-invariant.

One must distinguish between G invariance for WCW spinors and spinor fields.

1. In the case of factor spaces 3-surface are G invariant so that there is no difference between spinors and spinor fields as far as G is considered. Irreducible representations of G would correspond to the superpositions of G-transforms of oscillator operators for a fixed G-invariant X^3_l.

2. For covering space option G-invariance would mean that 3-surface is a mere G-fold copy of single 3-surface. There is no obvious reason to assume this. Hence one cannot separate spinorial degrees of freedom from WCW degrees of freedom since G affects both the spin degrees of freedom and the 3-surface. Irreducible representations of G would correspond to genuine WCW spinor fields involving a superposition of G-transforms of also X^3_l. The

presence of both orbital and spin degrees of freedom could provide alternative explanation for why $\mathcal{M} : \mathcal{N} = 4$ holds true for covering space option.

If the fermionic oscillator algebra is interpreted as a representation for $\mathcal{N}/\mathcal{M}$, allowed fermionic oscillator operators belong to non-trivial irreps of G. One can however ask whether the many-fermion states created by these operators are G-invariant for some physical reason so that one would have kind of G-confinement forcing the states to be many-fermion states with standard unit of quantum numbers for coverings and integer multiples of n for factor spaces. This would conform with the ideas that anyonicity is a microscopic property not visible at the level of entire state and that many-fermion systems in the anyonic state resulting in strong coupling limit for ordinary value of $\hbar$ are in question. The processes changing the value of Planck constant would be phase transitions involving all fermions of the G-invariant state and would be slow for this reason. This would also contribute to the invisibility of dark matter.

4.2.3 Questions

What is the role of dimensions?

Could the dimensions of CD and CP_2 and the dimensions of spaces defined by the choice of the quantization axes play a fundamental role in the construction from the constraint that the fundamental group is non-trivial?

1. Suppose that the sub-manifold in question is geodesic sub-manifold containing the orbits of its points under Cartan subgroup defining quantization axes. A stronger assumption would be that the orbit of maximal compact subgroup is in question.

2. For M^{2n} Cartan group contains translations in time direction with orbit M^1 and Cartan subgroup of $SO(2n-1)$ and would be M^n so that $\hat{M^{2n}}$ would have a trivial fundamental group for $n > 2$. Same result applies in massless case for which one has $SO(1,1) \times SO(2n-2)$ acts as Cartan subgroup. The orbit under maximal compact subgroup would not be in question.

3. For CP_2 homologically non-trivial geodesic sphere CP_1 contains orbits of the Cartan subgroup. For $CP_n = SU(n+1)/SU(n) \times U(1)$ having real dimension $2n$ the sub-manifold CP_{n-1} contains orbits of the Cartan subgroup and defines a sub-manifold with codimension 2 so that the dimensional restriction does not appear.

4. For spheres $S^{n-1} = SO(n)/SO(n-1)$ the dimension is $n-1$ and orbit of $SO(n-1)$ of point left fixed by Cartan subgroup $SO(2) \times ..$ would for $n = 2$ consist of two points and S_{n-2} in more general case. Again co-dimension 2 condition would be satisfied.

What about holes of WCW?

One can raise analogous questions at the level of WCW geometry. Vacuum extremals correspond to Lagrangian sub-manifolds $Y^2 \subset CP_2$ with vanishing induced Kähler form. They correspond to singularities of WCW ("world of classical worlds") and WCW spinor fields should vanish for the vacuum extremals. Effectively this would mean a hole in configuration space, and the question is whether this hole could also naturally lead to the introduction of covering spaces and factor spaces of the WCWs. How much information about the general structure of the theory just this kind of decomposition might allow to deduce? This kind of singularities are infinite-dimensional variants of those discussed in catastrophe theory and this suggests that their understanding might be crucial.

Are more general inclusions of HFFs possible?

The proposed scenario could be criticized because discrete subgroups of $SU(2)$ are in a preferred position. The Jones inclusions considered correspond to quantum spinor representations of various quantum groups $SU(2)_q$, $q = exp(i2\pi/n)$. This explains the result $\mathcal{M} : \mathcal{N} \leq 4$. These representations are certainly in preferred role as far as WCW spinor fields are considered but it is possible to assign a hierarchy of inclusions of HFFs labeled by quantum phase q with arbitrary representation of an arbitrary compact Lie group. These inclusions would be analogous to discrete states in the continuum $\mathcal{M} : \mathcal{N} > 4$.

Since the inclusions are characterized by single quantum phase $q = exp(i2\pi/n)$ in the case of compact Lie groups (Appendix), one can ask whether more general discrete groups than subgroups of $SU(2)$ should be allowed. The inclusions of HFFs associated with higher dimensional Lie groups have $\mathcal{M} : \mathcal{N} > 4$ and are analogous to bound states in continuum (Appendix). In the case of CP_2 this would allow to consider much more general sub-groups.

The question is therefore whether some principle selects subgroups of $SU(2)$. There are indeed good arguments supporting the hypothesis that only discrete Abelian subgroups of $SU(2)$ are possible.

1. The notion of number theoretic braid allows only the only subgroups of rotation group leaving M^2 invariant and sub-groups of $SU(3)$ leaving geodesic sphere S^2_i invariant. This would drop groups having genuinely 3-D action. In the case of $SU(3)$ discrete subgroups of $SO(3)$ or $U(2)$ remain under consideration. The geodesic sphere of type II is however analogous to North/South pole of S^2 and second phase factor associated with the coordinates (ξ^1, ξ^2) becomes redundant since $(|\xi^1|^2 + |\xi^2|^2)^{1/2}$ becomes infinite at S^2_{II} so that ξ^1/ξ^2 becomes appropriate coordinate. Hence action of $U(2)$ reduces to that of $SU(2)$ since ξ^1 and ξ^2 correspond to same value of color hyper charge associated with $U(1)$.

2. A physically attractive possibility is that $G_a \times G_b$ leaves the choice of quantization axes invariant. This condition makes sense also for coverings. This would leave only Abelian groups into consideration and drop D_{2n}, E_6, and E_8. It is quite possible that only these groups define sectors of the generalized imbedding space. This means that $G_b = Z_{n_1} \times Z_{n_2} \subset U(1)_I \times U(1)_Y \subset SU(2) \times U(1)_Y$ and even more general subgroups of $SU(3)$ (if non-commutativity is allowed) are a priori possible. Again the first argument reduces the list to cyclic subgroups of $SU(2)$.

3. The products of groups Z_n are also number theoretically in a very special position since they relate naturally to the finite cyclic extensions and also to the maximal Abelian extension of rationals. With this restriction on $G_a \times G_b$ one can consider the hypothesis that elementary particles correspond are maximally quantum critical systems left invariant by all groups $G_a \times G_b$ respecting a given choice of quantization axis and implying that darkness is associated only to field bodies and Planck constant becomes characterizer of interactions rather than elementary particles themselves.

4.3 Some mathematical speculations

4.3.1 The content of McKay correspondence in TGD framework

The possibility to assign Dynkin diagrams with the inclusions of II_1 algebras is highly suggestive concerning possible physical interpretations. The basic findings are following.

1. For $\beta = \mathcal{M} : \mathcal{N} < 4$ Dynkin diagrams code for the inclusions and correspond to simply laced Lie algebras. $SU(2)$, D_{2n+1}, and E_7 are excluded.

2. Extended ADE Dynkin diagrams coding for simply laced ADE Kac Moody algebras appear at $\beta = 4$. Also $SU(2)$ Kac Moody algebra appears.

Does TGD give rise to ADE hierarchy of gauge theories

The first question is whether any finite subgroup $G \subset SU(2)$ acting in CP_2 degrees of freedom could somehow give rise to multiplets of the corresponding gauge group having interactions described by a gauge theory. Orbifold picture suggests that might be the case.

1. The "sheets" for the space-time sheet forming an $N(G)$-fold cover of CD are in one-one correspondence with group G. This degeneracy gives rise to additional states and these states

correspond to the group algebra having basis given by group characters $\chi(g)$. One obtains irreducible representations of G with degeneracies given by their dimensions. Altogether one obtains $N(G)$ states in this manner. In the case of $A(n)$ the number of these states is $n+1$, the number of the states of the fundamental representation of $SU(n+1)$. In the same manner, for D_{2n} the number of these states equals to the number of states in the fundamental representation of D_{2n}. It seems that the rule is quite general. Thus these representations would in the case of fermions give the states of the fundamental representation of the corresponding gauge group.

2. From fermion and anti-fermion states one can construct in a similar manner pairs giving $N(G)^2$ states defining in the case of $A(n)$ n^2-1-dimensional gauge boson multiplet plus singlet. Also other groups must give boson multiplet plus possible other multiplets. For instance, for $D(4)$ the number of states is 64 and boson multiplet is 8-dimensional so that many other spin 1 states result.

3. These findings give hopes that the orbifold multiplets could be modelled by a gauge theory based on corresponding gauge group. What is nice that this huge hierarchy of gauge theories is associated with dark matter so that the predictivity and falsifiability are not lost unlike in M-theory.

Does one obtain also a hierarchy of conformal theories with ADE Kac Moody symmetry?

Consider next the question Kac Moody interactions correspond to extended ADE diagrams are possible.

1. In this case the notion of orbifold seems to break down since the symmetry related points form a continuum $SU(2)$ and space-time surface would become 6-dimensional if the CD projection is 4-dimensional. If one takes space-time as something which emerges, one could take this possibility half seriously. A more natural natural possibility is that CD projection is 2-dimensional geodesic sphere in which case one would have string like objects so that conformal field theory with Kac-Moody algebra would emerge naturally.

2. The new degrees of freedom would define 2-dimensional continuum and it would not be completely surprising if conformal field theory based on ADE Kac Moody algebra could describe the situation. One possibility is that these continua for different inclusions correspond to $SU(2)$ decompose to an $N(G)$-fold covers of S^2/G orbifold so that also now groups G would be involved with the Jones inclusions, which might provide a hint about how to construct them. S^2/G would play the role of stringy world sheet for the conformal field theory in question. This effective re-arrangement of the topology S^2 might be due to the fact that conformal fields possess G symmetry which effectively groups points of S^2 to $n(G)$-multiplets. The localized representations of the Lie group corresponding to G would correspond to the multiplets obtained from the representations of group algebra of G as in previous case.

3. The formula for the scaling factor of CD metric would give infinite scaling factor if one identifies the scaling factor as maximal order of cyclic subgroup of $SU(2)$. As a matter fact there is no finite cyclic subgroup of this kind. The solution to the problem would be identification of the scaling factor as the order of the maximal cyclic subgroup of G so that the scaling factors would be same for the two situations related by McKay correspondence.

Generalization to CD degrees of freedom

One can ask whether the proposed picture generalizes formally also the case of CD.

1. In this case quantum groups would correspond to discrete subgroups $G \subset SL(2,C)$. Kac Moody group would correspond to G-Kac Moody algebra made local with respect to $SL(2,C)$ orbit in CD divided by G. These orbits are 3-dimensional hyperboloids H_a with a constant value of light cone proper time a so that the division by G gives fundamental domain H_a/G with a finite 3-volume.

2. The 4-dimensionality of space-time would require 1-dimensional CP_2 projection. Vacuum extremals of Kähler action would be in question. Robertson-Walker metric have 1-dimensional CP_2 projection and carry non-vanishing density of gravitational mass so that in this sense the theory would be non-trivial. G would label different lattice like cosmologies defined by tesselations with fundamental domain H_a/G.

3. The multiplets of G would correspond to collections of points, one from each cells of the lattice like structure. Macroscopic quantum coherence would be realized in cosmological scales. If one takes seriously the vision about the role of short distance p-adic physics as a generator of long range correlations of the real physics reflected as p-adic fractality, this idea does not look so weird anymore.

   Complexified modular group $SL(2, Z + iZ)$ and its subgroups are interesting as far as p-adicization is considered. The principal congruence subgroups $\Gamma(N)$ of $SL(2, Z + iZ)$ which are unit matrices modulo N define normal subgroups of the complex modular group and are especially interesting candidates for groups $G \subset SL(2, C)$. The group $\Gamma(N = p^k)$ labeling fundamental domains of the tesselation $H_a/\Gamma(N = p^k)$ defines a mathematically attractive candidate for a point set associated with the intersections of p-adic space-time sheets with real space-time sheets. Also analogous groups for algebraic extensions of Z are interesting.

   The simplest discrete subgroup of SL(2,C) with infinite number of elements would corresponds to powers of boost to single direction and correspond at the non-relativistic limit to multiples of basic velocity. This could also give rise to quantization of cosmic recession velocities. There is evidence for the quantization of cosmic recession velocities (for a model in which single object produces quantized redshifts see [K26]) and it is interesting to see whether they could be interpreted in terms of the lattice like periodicity in cosmological length scales implied by the effective reduction of physics to M^4_+/G_n. In [E19] the values $z = 2.63, 3.45, 4.47$ of cosmic red shift are listed. These correspond to recession velocities $v = (z^2 - 1)/(z^2 + 1)$ are (0.75,0.85,0.90). The corresponding hyperbolic angles are given by $\eta = acosh(1/(1 - v^2))$ and the values of η are $(1.46, 1.92, 2.39)$. The differences $\eta(2) - \eta(1) = .466$ and $\eta(3) - \eta(2) = .467$ are same within experimental uncertainties. One has however $\eta(n)/(\eta(2) - \eta(1)) = (3.13, 4.13, 5.13)$ instead of $(3, 4, 5)$. A possible interpretation is in terms of the velocity of the observer with respect to the frame in which quantization of η happens.

Quantitative support for the interpretation

A more detailed analysis of the situation gives support for the proposed vision.

1. A given value of quantum group deformation parameter $q = exp(i\pi/n)$ makes sense for any Lie algebra but now a preferred Lie-algebra is assigned to a given value of quantum deformation parameter. At the limit $\beta = 4$ when quantum deformation parameter becomes trivial, the gauge symmetry is replaced by Kac Moody symmetry.

2. The prediction is that Kac-Moody central extension parameter should vanish for $\beta < 4$. There is an intriguing relationship to formula for the quantum phase q_{KM} associated with (possibly trivial) Kac-Moody central extension and the phase defined by ADE diagram

$$
\begin{array}{ll}
q_{KM} = exp(i\phi) \ , & \phi_1 = \frac{\pi}{k+h^v} \ , \\
q_{Jones} = exp(i\phi) \ , & \phi = \frac{\pi}{h}
\end{array}
$$

   In the first formula sum of Kac-Moody central extension parameter k and dual Coxeter number h^v appears whereas Coxeter number h appears in the second formula. Internal consistency requires

$$
k + h^v = h \ . \tag{4.3.1}
$$

It is easy see that the dual Coxeter number h^v and Coxeter number h given by $h = (dim(g) - r)/r$, where r is the dimension of Cartan algebra of g, are identical for ADE algebras so that the Kac-Moody central extension parameter k must indeed vanish. For $SO(2n+1)$, $Sp(n)$, G_2, and F_4 the condition $h = h^v$ does not hold true but one has $h(n) = 2n = h^v + 1$ for $SO(2n+1)$, $h(n) = 2n = 2(h^v - 1)$ for $Sp(n)$, $h = 6 = h^v + 2$ for G_2, and $h = 12 = h^v + 3$ for F_4.

What is intriguing that G_2, which seems to play a fundamental role in the dual formulation of quantum TGD based on the identification of space-times as surfaces in hyper-octonionic space M^8 [K84] is not allowed. As a matter fact, $G_2 \rightarrow SU(3)$ reduction occurs also in the dual formulation based on $G_2/SU(3)$ coset model and is required by the separate conservation of quark and lepton numbers predicted by TGD. ADE groups would be associated with the interaction between space-time sheets rather than entire dynamics and need not have anything to do with the Kac-Moody algebra associated with color and electro-weak interactions appearing in the construction of physical states [K48] .

3. There seems to be a concrete connection with conformal field theories. This connection would allow to understand the emergence of quantum groups appearing naturally in these theories. Quite generally, the conformal central extension parameter for unitary Virasoro representations resulting by Sugawara construction from Kac Moody representations satisfies either of the conditions

$$\begin{aligned} c &\geq \frac{kdim(g)}{k+h^v} + 1 \ , \\ c &= \frac{kdim(g)}{k+h^v} + 1 - \frac{6}{(h-1)h} \ . \end{aligned} \tag{4.3.2}$$

For $k = 0$, which should be interesting for $\beta < 4$, the second formula reduces to

$$c = 1 - \frac{6}{(h-1)h} \ . \tag{4.3.3}$$

The formula gives the values of c for minimal conformal field theories with finite number of conformal fields and real conformal weights. Indeed, h in this formula seems to correspond to the same h as appearing in the expression $\beta \equiv \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h)$.

$\beta = 3, h = 6$ corresponds to three-state Potts model with $c = 4/5$ which should thus have a gauge group for which Coxeter number is 6: the group should be either $SU(6)$ or $SO(8)$. Two-state Potts model, that is Ising model with $\beta = 2, h = 4$ would correspond to $c = 1/2$ and to a gauge group $SU(4)$ or $SO(4)$. For $h = 3$ ("one-state Potts model") with group $SU(3)$ one would have $c = 0$ and vanishing conformal anomaly so that conformal degrees of freedom would become pure gauge degrees of freedom.

These observations give support for the following picture.

1. Quite generally, the number of states of the generalized β-state Potts model has an interpretation as the dimension $\beta = \mathcal{M} : \mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module. Besides the models with integer number of states there is an infinite number of models for which the number of states is not an integer. The conditions $c \leq 1$ guaranteeing real conformal weights and $\beta \leq 4$ correspond to each other for these models.

2. $\beta > 4$ Potts models would be formally obtained by allowing h to be imaginary in the defining formula for $\mathcal{M} : \mathcal{N}$. In this case c would be however complex so that the theory would not be unitary.

3. For minimal models with ($\beta < 4, c < 1$) Kac-Moody central extension parameter is vanishing so that Kac Moody algebra indeed acts like gauge symmetries and gauge symmetries would be in question. ($\beta = 4, c = 1$) would define a "four-state Potts model" with infinite-dimensional unitary group acting as a gauge group. On the other hand, the appearance of extended ADE Dynkin diagrams suggests strongly that this limit is not realized but that $\beta = \mathcal{M} : \mathcal{N} = 4$ corresponds to $k = 1$ conformal field theory allowing Kac Moody symmetries for any ADE group, which as simply-laced groups allows vertex operator construction. The appearance of $kdim(g)/(k + g)$ in the more general formula would thus code the Kac Moody group whereas for $\beta < 4$ ADE diagram codes for the preferred gauge group characterizing the minimal CFT.

4. The possibility that any ADE gauge group or Kac-Moody group can characterize the interaction between space-time sheets conforms with the idea about Universe as a Topological Quantum Computer able to simulate any conceivable quantum dynamics. Of course, one cannot exclude the possibility that only electro-weak and color symmetries are realized in this manner.

G_a as a symmetry group of magnetic body and McKay correspondence

The group $G_a \subset SU(2) \subset SL(2, C)$ means exact rotational symmetry realized in terms of CD coverings of CP_2. The 5 and 6-cycles in biochemistry (sugars, DNA,....) are excellent candidates for these symmetries. For very large values of Planck constant, say for the values $\hbar(CD)/\hbar(CP_2) = GMm/v_0 = (n_a/n_b)\hbar_0$, $v_0 = 2^{-11}$, required by the model for planetary orbits as Bohr orbits [K75] , G_a is huge and corresponds to either Z_{n_a} or in the case of even value of n_a to the group generated by Z_n and reflection acting on plane and containing $2n_a$ elements.

The notion of magnetic body seems to provide the only conceivable candidate for a geometric object possessing G_a as symmetries. In the first approximation the magnetic field associated with a dark matter system is expected to be modellable as a dipole field having rotational symmetry around the dipole axis. Topological quantization means that this field decomposes into flux tube like structures related by the rotations of Z_n or D_{2n}. Dark particles would have wave functions de-localized to this set of these flux quanta and span group algebra of G_a. Magnetic flux quanta are indeed assumed to mediate gravitational interactions in the TGD based model for the quantization of radii of planetary orbits and this explains the dependence of $\hbar_{gr}$ on the masses of planet and central object [K75] .

For the model of dark matter hierarchy appearing in the model of living matter one has $n_a = 2^{11k}$, $k = 1, 2, 3, .., 7$ for cyclotron time scales below life cycle for a magnetic field $B_d = .2$ Gauss at $k = 4$ level of hierarchy (the field strength is fixed by the model for the effects of ELF em fields on vertebrate brain at harmonics of cyclotron frequencies of biologically important ions [K29]). Note that B_d scales as 2^{-11k} from the requirement that cyclotron energy is constant.

ADE correspondence between subgroups of $SU(2)$ and Lie groups in ADE hierarchy encourages to consider the possibility that TGD could mimic ADE hierarchy of gauge theories. In the case of G_a this would mean that many fermion states constructed from single fermion states, which are in one-one correspondence with the elements of G_a group algebra, would define multiplets of the gauge group corresponding to the Dynkin diagram characterizing G_a: for instance, $SU(n_a)$ in the case of Z_{n_a}. Fermion multiplet would contain n_a states and gauge boson multiplet $n_a^2 - 1$ states. This would provide enormous information processing capacity since for $n_a = 2^{11k}$ fermion multiplet would code exactly $11k$ bits of information. Magnetic body could represent binary information using the many-particle states belonging to the representations of say $SU(n_a)$ at its flux tubes.

4.3.2 Jones inclusions, the large N limit of $SU(N)$ gauge theories and AdS/CFT correspondence

The framework based on Jones inclusions has an obvious resemblance with larger N limit of $SU(N)$ gauge theories and also with the celebrated AdS/CFT correspondence [B20] so that a more detailed comparison is in order.

Large N limit of gauge theories and series of Jones inclusions

The large N limit of $SU(N)$ gauge field theories has as definite resemblance with the series of Jones inclusions with the integer $n \geq 3$ characterizing the quantum phase $q = exp(i\pi/n)$ and the order of the maximal cyclic subgroup of the subgroup of $SU(2)$ defining the inclusion. Recall that all ADE groups except D_{2n+1} and E_7 are allowed (SU(2) is excluded since it would correspond to $n = 2$).

The limiting procedure keeps the value of g^2N fixed. Rather remarkably, this is equivalent with keeping αN constant but assuming $\hbar$ to scale as $n = N$. Thus the quantization of Planck constants would provide a physical laboratory for the testing of large N limit.

The observation suggesting a description of YM theories in terms of closed strings is that Feynman diagrams can be interpreted as being imbedded at closed 2-surfaces of minimal genus guaranteeing that the internal lines meet except in vertices. The contribution of genus g diagrams is proportional to N^{g-1} at the large N limit. The interpretation in terms of closed partonic 2-surfaces is highly suggestive and the N^{g-1} should come from the multiple covering property of CP_2 by N CD-points (or vice versa) with the finite subgroup of $G \subset SU(2)$ defining the Jones inclusion and acting as symmetries of the surface.

Analogy between stacks of branes and multiple coverings of CD and CP_2

An important aspect of AdS/CFT dualities is a prediction of an infinite hierarchy of gauge groups, which as such is as interesting as the claimed dualities. The prediction relies on the notion Dp-branes. Dp-branes are $p+1$-dimensional surfaces of the target space at which the ends of open strings can end. In the simplest situation one considers N parallel p-branes at the limit when the distances between branes characterized by an expectation value of Higgs fields approach zero to obtain what is called N-stack of branes. There are N^2 different strings connecting the branes and the heuristic idea is that they correspond to gauge bosons of $U(N)$ gauge theory. Note that the requirement that AdS/CFT dualities exist forces the introduction of branes and the optimistic interpretation is that a non-perturbative effect of still unknown M-theory is in question. In the limit of an ideal stack one assumes that $U(N)$ gauge theory at the brane representing the stack is obtained. The branes must also carry a p-form defining gauge potential for a closed $p+1$-form. This Ramond charge is quantized and its value equals to N.

Consider now the group $G_a \times G_b \subset SL(2,C) \times SU(2) \subset SU(3)$ defining double Jones inclusion and implying the scalings $\hbar(M^4) \rightarrow n(G_b)\hbar(M^4)$ and $\hbar(CP_2) \rightarrow n(G_a)\hbar(CP_2)$. These space-time surfaces define $n(G_a)$-fold multiple coverings of CP_2 and $n(G_b)$-fold multiple coverings of CD. In CP_2 degrees of freedom the collection of G_b-related partonic 2-surfaces (/3-surfaces/4-surfaces) is highly analogous to the stack of branes. In CD degrees of freedom the stack of copies of surface typically correspond to along a circle (A_n, D_{2n} or at vertices of tetrahedron or isosahedron.

In TGD framework the interpretation strings are not needed to define gauge fields. The group algebra of G realized as discrete plane waves at G-orbit gives rise to representations of G. The hypothesis supported by few examples is that these additional degrees of freedom allow to construct multiplets of the gauge group assignable to the ADE diagram characterizing the inclusion.

AdS/CFT duality

AdS/CFT duality is a further aspect of the brane construction. The dual description of the situation is in terms of a string theory in a background in which N-brane acts as a macroscopic object giving rise to a black-hole like object in (say) 10-dimensional target space. This background has the form $AdS_5 \times X_5$, where AdS_5 is 5-dimensional hyperboloid of M^6 and thus allows $SO(4,2)$ as isometries. X_5 is compact constant curvature space. S_5 gives rise to $N=4$ SUSY in M^4 with M^4 interpreted as a brane. The first support for the dualities comes from the symmetries: for instance, the $N=4$ super-symmetrized isometries of $AdS_5 \times S^5$ are same as the symmetries of 4-dimensional $N=4$ SUSY for $p=3$ branes. N-branes can be used as models for black holes in target space and black-hole entropy can be calculated using either target space picture or conformal field theory at brane and the results turn out be the same.

Does the TGD equivalent of this duality exists in some sense?

1. As far as partonic 2-surfaces identified as 1-branes are considered, conformal field theory description is trivially true. In TGD framework the analog of Ramond charges are the integers n_a and n_b characterizing the multiplicities of the maximal Abelian subgroups having clear topological meaning. This conforms with the observation that large N limit of the gauge field theories can be formulated in terms of closed surfaces at which the Feynman diagrams are imbedded without self crossings. It seems that the integers n_a and n_b characterizing the Jones inclusion naturally take the role of Ramond charge: this does not of course exclude the possibility they can be expressed as fluxes at space-time level as will be indeed found.

2. Conformal field theory description can be generalized in the sense that one replaces the $n(G_a) \times n(G_b)$ partonic surfaces with single one and describes the new states as primary fields arranged into representations of the ADE group in question. This would mean that the standard model gauge group extends by additional factor which is however non-trivially related to it.

3. If one can accept the idea that the conformal field theory description for partons gives rise to M^4 gauge theory as an approximate description, it is not too difficult to imagine that also ADE hierarchy of gauge theories results as a description of the exotic states. One can say that CFT in p-brane is replaced now with CFT on partonic 2-surface (1-brane) analogous to a closed string.

4. In the minimal interpretation there is no need to add strings connecting the branches of the double covering of the partonic 2-surface whose function is essentially that of making possible gauge bosons as fermion anti-fermion pairs. One could of course imagine gauge fluxes as counterparts of strings but just the fact that G-invariance dictates the configurations completely forces to question this kind of dynamics.

5. There is no reason to expect the emergence of $N = 4$ super-symmetric field theory in M^4 as in the case of super-string models. The reasons should be already obvious: super-conformal generators G anti-commute to L_0 proportional to mass squared rather than four-momentum and the spectrum extended by $G_a \times G_b$ degeneracy contains more states.

One can of course ask whether higher values of p could make sense in TGD framework.

1. It seems that the light-like orbits of the partonic 2-surfaces defining 2-branes do not bring in anything new since the generalized conformal invariance makes it possible the restriction to a 2-dimensional cross section of the light like causal determinant.

2. The idea of regarding space-time surface X^4 as a 3-brane in H in which some kind of conformal field theory is defined is in conflict with the basis ideas of TGD. The role of X^4 interior is to provide classical correlates for quantum dynamics to make possible quantum measurement theory and also introduce correlations between partonic 2-surfaces even in the case that partonic conformal dynamics reduces to a topological string theory. It is quantum classical correspondence which corresponds to this duality.

What is the counterpart of the Ramond charge in TGD?

The condition that there exist a p-form defining $p+1$-gauge field with p-charge equal to n_a or n_b is a rather stringent additional condition also in TGD framework. For $n < \infty$ this kind of charge is defined by Jones inclusion and represented topologically so that Ramond charge is not needed in $n < \infty$ case. By the earlier arguments one must however be able to assign integers n_a and n_b also to $G = SU(2)$ inclusions with Kac-Moody algebra characterized by an extended ADE diagram with the phases $q_i = exp(i\pi/n_i)$ relating to the monodromy of the theory. Since Jones inclusion does not define in this case the value of $n < \infty$ in any obvious manner, the counterpart of the Ramond charge is needed.

1. For partonic 2-surfaces ordinary gauge potential would define this form and the condition would state that magnetic flux equals to n so that the anyonic partonic two-surfaces would be homologically non-trivial in CP_2 degrees of freedom. String ends would define basic example

of this situation. This would be the case also in M^4_+ degrees of freedom: the partonic 2-surface would essentially wind n_a times around the tip of δCD and the gauge field in question would be monopole magnetic field in δCD. This kind of situation need not correspond to anything cosmological since future and past light-cones appear in the basic definition of the scattering amplitudes.

2. For $p = 3$ Chern-Simons action for the induced CP_2 Kähler form associated with the partonic 2-surface indeed defines this kind of charge. Ramond charge should be simply N. CP_2 type extremals or their small deformations satisfy this constraint and are indeed very natural in elementary particle physics context but too restrictive in a more general context.

Note that the light-like orbits of non-deformed CP_2 extremals have light-like random curve as an M^4 projection and the conformal symmetries of M^4 obviously respect light-likeness property. Hence $SO(4,2)$ symmetry characterizing AdS_5/CFT is not excluded but would be broken by p-adic thermodynamics and by TGD based Higgs mechanism involving the identification of inertial momentum as average value of non-conserved gravitational momentum parallel to the light-like zitterbewegung orbit.

Can one speak about black hole like structures in TGD framework?

For AdS/CFT correspondence there is also a dynamical coupling to the target space metric. The coupling to H-metric is present also now since the overall scalings of the CD *resp.* CP_2 metrics by n_b *resp.* by n_a are involved. This applies to when multiple covering is used explicitly. In the description in which one replaces the multiple covering by ordinary $M^4 \times CP_2$, the metric suffers a genuine change and something analogous to the black-hole type metrics encountered in AsS/CFT correspondence might be encountered.

Consider as an example an n_a-fold covering of CP_2 points by M^4 points (ADE diagram A_{n_a-1}). The n-fold covering means only $n2\pi$ rotation for the phase angle ψ of CP_2 complex coordinate leads to the original point. The replacement $\psi \rightarrow \psi/n_a$ gives rise to what would look like ordinary $M^4 \times CP_2$ but with a modified CP_2 metric. The metric components containing ψ as index are scaled down by $1/n_a$ or $1/n_a^2$. Notice that Ψ effectively disappears from the dynamics at the large n_a limit.

If one uses an effective description in which covering is eliminated the metric is indeed affected at the level of imbedding space black hole like structures at the level of dynamic space might make emerge also in TGD framework at large N limit since the masses of the objects in question become large and CP_2 metric is scaled by N so that CP_2 has very large size at this limit. This need not lead to any inconsistencies if these phases are interpreted as dark matter. At the elementary particle level p-adic thermodynamics predicts that p-adic entropy is proportional to thermal mass squared which implies elementary particle black-hole analogy.

Other dualities

Also quantum classical correspondence defines in a loose sense a duality justifying the basic assumptions of quantum measurement theory. The light-like orbits of 2-D partons are characterized by a generalization of ordinary 2-D conformal invariance so that CFT part of the duality would be very natural. The dynamical target space would be replaced with the space-time surface X^4 with a dynamical metric providing classical correlates for the quantum dynamics at partonic 2-surfaces. The duality in this sense cannot be however exact since classical dynamics cannot fully represent quantum dynamics.

Classical description is not expected to be unique. The basic condition on space-time surfaces assignable to a given configuration of partonic 2-surfaces associated with the surface X^3_V defining S-matrix element are posed by quantum classical correspondence. Both hyper-quaternionic and co-hyper-quaternionic space-time surfaces are acceptable and this would define a fundamental duality.

A concrete example about this HQ-coHQ duality would be the equivalence of space-time descriptions using 4-D CP_2 type extremals and 4-D string like objects connecting them. If one restricts to CP_2 type extremals and string like objects of from $X^2 \times Y^2$, the target space reduces effectively to M^4 and the dynamical degrees of freedom correspond in both cases to transversal M^4

degrees of freedom. Note that for CP_2 type extremals the conditions stating that random light-likeness of the M^4 projection of the CP_2 type extremal are equivalent to Virasoro conditions. CP_2 type extremals could be identified as co-HQ surfaces whereas stringlike objects would correspond to HQ aspect of the duality.

HQ-coHQ provides dual classical descriptions of same phenomena. Particle massivation would be a basic example. Higgs mechanism in a gauge theory description based on CP_2 type extremals would rely on zitterbewegung implying that the average value of gravitational mass identified as inertial mass is non-vanishing and is discussed already. Higgs field would be assigned to the wormhole contacts. The dual description for the massivation would be in terms of string tension and mass squared would be proportional to the distance between G-related points of CP_2.

These observations would suggest that also a super-conformal algebra containing $SL(2,R) \times SU(2)_L \times U(1)$ or its compact version exists and corresponds to a trivial inclusion. This is indeed the case [A18] . The so called large $N = 4$ super-conformal algebra contains energy momentum current, 2+2 super generators G, $SU(2) \times SU(2) \times U(1)$ Kac-Moody algebra (both $SU(2)$ and SL(2,R) could be interpreted as acting on M^4 spin degrees of freedom, and 2 spin 1/2 fermionic currents having interpretation in terms of right handed neutrinos corresponding to two H-chiralities. Interestingly, the scalar generator is now missing.

4.3.3 Could McKay correspondence and Jones inclusions relate to each other?

The understanding of Langlands correspondence for general reductive Lie groups in TGD framework seems to require some physical mechanism allowing the emergence of these groups in TGD based physics. The physical idea would be that quantum dynamics of TGD is able to emulate the dynamics of any gauge theory or even stringy dynamics of conformal field theory having Kac-Moody type symmetry and that this emulation relies on quantum deformations induced by finite measurement resolution described in terms of Jones inclusions of sub-factors characterized by group G leaving elements of sub-factor invariant. Finite measurement resolution would would result simply from the fact that only quantum numbers defined by the Cartan algebra of G are measured.

There are good reasons to expect that infinite Clifford algebra has the capacity needed to realize representations of an arbitrary Lie group. It is indeed known that that any quantum group characterized by quantum parameter which is root of unity or positive real number can be assigned to Jones inclusion [A40] . For $q = 1$ this would gives ordinary Lie groups. In fact, all amenable groups define unique sub-factor and compact Lie groups are amenable ones.

It was so called McKay correspondence [A53] which originally stimulated the idea about TGD as an analog of Universal Turing machine able to mimic both ADE type gauge theories and theories with ADE type Kac-Moody symmetry algebra. This correspondence and its generalization might also provide understanding about how general reductive groups emerge. In the following I try to cheat the reader to believe that the tensor product of representations of SU(2) Lie algebras for Connes tensor powers of $\mathcal{M}$ could induce ADE type Lie algebras as quantum deformations for the direct sum of n copies of $SU(2)$ algebras This argument generalizes also to the case of other compact Lie groups.

About McKay correspondence

McKay correspondence [A53] relates discrete finite subgroups of $SU(2)$ ADE groups. A simple description of the correspondences is as follows [A53].

1. Consider the irreps of a discrete subgroup $G \subset SU(2)$ which correspond to irreps of G and can be obtained by restricting irreducible representations of $SU(2)$ to those of G. The irreducible representations of $SU(2)$ define the nodes of the graph.

2. Define the lines of graph by forming a tensor product of any of the representations appearing in the diagram with a doublet representation which is always present unless the subgroup is 2-element group. The tensor product regarded as that for $SU(2)$ representations gives representations $j - 1/2$, and $j + 1/2$ which one can decompose to irreducibles of G so that a branching of the graph can occur. Only branching to two branches occurs for subgroups

yielding extended ADE diagrams. For the linear portions of the diagram the spins of corresponding SU(2) representations increase linearly as $.., j, j+1/2, j+1, ...$

One obtains extended Dynkin diagrams of ADE series representing also Kac-Moody algebras giving A_n, D_n, E_6, E_7, E_8. Also A_∞ and $A_{-\infty,\infty}$ are obtained in case that subgroups are infinite. The Dynkin diagrams of non-simply laced groups B_n ($SO(2n+1)$), C_n (symplectic group $Sp(2n)$ and quaternionic group $Sp(n)$), and exceptional groups G_2 and F_4 are not obtained.

ADE Dynkin diagrams labeling Lie groups instead of Kac-Moody algebras and having one node less, do not appear in this context but appear in the classification of Jones inclusions for $\mathcal{M} : \mathcal{N} < 4$. As a matter fact, ADE type Dynkin diagrams appear in very many contexts as one can learn from John Baez's This Week's Finds [A15] .

1. The classification of integral lattices in Rn having a basis of vectors whose length squared equals 2
2. The classification of simply laced semisimple Lie groups.
3. The classification of finite sub-groups of the 3-dimensional rotation group.
4. The classification of simple singularities . In TGD framework these singularities could be assigned to origin for orbifold CP_2/G, $G \subset SU(2)$.
5. The classification of tame quivers.

Principal graphs for Connes tensor powers $\mathcal{M}$

The thought provoking findings are following.

1. The so called principal graphs characterizing $\mathcal{M} : \mathcal{N} = 4$ Jones inclusions for $G = SU(2)$ are extended Dynkin diagrams characterizing ADE type affine (Kac-Moody) algebras. D_n is possible only for $n \geq 4$.
2. $\mathcal{M} : \mathcal{N} < 4$ Jones inclusions correspond to ordinary ADE type diagrams for a subset of simply laced Lie groups (all roots have same length) A_n ($SU(n)$), D_{2n} ($SO(2n)$), and E_6 and E_8. Thus D_{2n+1} ($SO(2n+2)$) and E_7 are not allowed. For instance, for $G = S_3$ the principal graph is not D_3 Dynkin diagram.

The conceptual background behind principal diagram is necessary if one wants to understand the relationship with McKay correspondence.

1. The hierarchy of higher commutations defines an invariant of Jones inclusion $\mathcal{N} \subset \mathcal{M}$. Denoting by $\mathcal{N}'$ the commutant of $\mathcal{N}$ one has sequences of horizontal inclusions defined as $C = \mathcal{N}' \cap \mathcal{N} \subset \mathcal{N}' \cap \mathcal{M} \subset \mathcal{N}' \cap \mathcal{M}^1 \subset ...$ and $C = \mathcal{M}' \cap \mathcal{M} \subset \mathcal{M}' \cap \mathcal{M}^1 \subset ....$ There is also a sequence of vertical inclusions $\mathcal{M}' \cap \mathcal{M}^k \subset \mathcal{N}' \cap \mathcal{M}^k$. This hierarchy defines a hierarchy of Temperley-Lieb algebras [A78] assignable to a finite hierarchy of braids. The commutants in the hierarchy are direct sums of finite-dimensional matrix algebras (irreducible representations) and the inclusion hierarchy can be described in terms of decomposition of irreps of k^{th} level to irreps of $(k-1)^{th}$ level irreps. These decomposition can be described in terms of Bratteli diagrams [A25] .
2. The information provided by infinite Bratteli diagram can be coded by a much simpler bipartite diagram having a preferred vertex. For instance, the number of $2k$-loops starting from it tells the dimension of k^{th} level algebra. This diagram is known as principal graph.

Principal graph emerges also as a concise description of the fusion rules for Connes tensor powers of $\mathcal{M}$.

1. It is natural to decompose the Connes tensor powers [A53] $\mathcal{M}_k = \mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$ to irreducible $\mathcal{M} - \mathcal{M}$, $\mathcal{N} - \mathcal{M}$, $\mathcal{M} - \mathcal{N}$, or $\mathcal{N} - \mathcal{N}$ bi-modules. If $\mathcal{M} : \mathcal{N}$ is finite this decomposition involves only finite number of terms. The graphical representation of these decompositions gives rise to Bratteli diagram.

2. If $\mathcal{N}$ has finite depth the information provided by Bratteli diagram can be represented in nutshell using principal graph. The edges of this bipartite graph connect $\mathcal{M}-\mathcal{N}$ vertices to vertices describing irreducible $\mathcal{N}-\mathcal{N}$ representations resulting in the decomposition of $\mathcal{M}-\mathcal{N}$ irreducibles. If this graph is finite, $\mathcal{N}$ is said to have finite depth.

A mechanism assigning to tensor powers Jones inclusions ADE type gauge groups and Kac-Moody algebras

The earliest proposals inspired by the hierarchy of Jones inclusions is that in $\mathcal{M}:\mathcal{N}<4$ case it might be possible to construct ADE representations of gauge groups or quantum groups and in $\mathcal{M}:\mathcal{N}=4$ using the additional degeneracy of states implied by the multiple-sheeted cover $H\rightarrow H/G_a\times G_b$ associated with space-time correlates of Jones inclusions. Either G_a or G_b would correspond to G. In the following this mechanism is articulated in a more refined manner by utilizing the general properties of generators of Lie-algebras understood now as a minimal set of elements of algebra from which the entire algebra can be obtained by repeated commutation operator (I have often used " Lie algebra generator" as an synonym for "Lie algebra element"). This set is finite also for Kac-Moody algebras.

1. Two observations

The explanation to be discussed relies on two observations.

1. McKay correspondence for subgroups of G ($\mathcal{M}:\mathcal{N}=4$) *resp.* its variants ($\mathcal{M}:\mathcal{N}<4$) and its counterpart for Jones inclusions means that finite-dimensional irreducible representations of allowed $G\subset SU(2)$ label both the Cartan algebra generators and the Lie (Kac-Moody) algebra generators of t_+ and t_- in the decomposition $g=h\oplus t_+\oplus t_-$, where h is the Lie algebra of maximal compact subgroup.

2. Second observation is related to the generators of Lie-algebras and their quantum counterparts (see Appendix for the explicit formulas for the generators of various algebras considered). The observation is that each Cartan algebra generator of Lie- and quantum group algebras, corresponds to a triplet of generators defining an SU(2) sub-algebra. The Cartan algebra of affine algebra contains besides Lie group Cartan algebra also a derivation d identifiable as an infinitesimal scaling operator L_0 measuring the conformal weight of the Kac-Moody generators. d is exceptional in that it does not give rise to a triplet. It corresponds to the preferred node added to the Dynkin diagram to get the extended Dynkin diagram.

2. Is ADE algebra generated as a quantum deformation of tensor powers of $SU(2)$ Lie algebras representations?

The ADE type symmetry groups could result as an effect of finite quantum resolution described by inclusions of HFFs in TGD inspired quantum measurement theory.

1. The description of finite resolution typically leads to quantization since complex rays of state space are replaced as $\mathcal{N}$ rays. Hence operators, which would commute for an ideal resolution cease to do so. Therefore the algebra $SU(2)\otimes ...\otimes SU(2)$ characterized by n mutually commuting triplets, where n is the number of copies of $SU(2)$ algebra in the original situation and identifiable as quantum algebra appearing in $\mathcal{M}$ tensor powers with $\mathcal{M}$ interpreted as $\mathcal{N}$ module, could suffer quantum deformation to a simple Lie algebra with $3n$ Cartan algebra generators. Also a deformation to a quantum group could occur as a consequence.

2. This argument makes sense also for discrete groups $G\subset SU(2)$ since the representations of G realized in terms of WCW spinor s extend to the representations of SU(2) naturally.

3. Arbitrarily high tensor powers of $\mathcal{M}$ are possible and one can wonder why only finite-dimensional Lie algebra results. The fact that $\mathcal{N}$ has finite depth as a sub-factor means that the tensor products in tensor powers of $\mathcal{N}$ are representable by a finite Dynkin diagram. Finite depth could thus mean that there is a periodicity involved the kn tensor powers decomposes to representations of a Lie algebra with $3n$ Cartan algebra generators. Thus the additional requirement would be that the number of tensor powers of $\mathcal{M}$ is multiple of n.

3. Space-time correlate for the tensor powers $\mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$

By quantum classical correspondence there should exist space-time correlate for the formation of tensor powers of $\mathcal{M}$ regarded as $\mathcal{N}$ module. A concrete space-time realization for this kind of situation in TGD would be based on n-fold cyclic covering of H implied by the $H \to H/G_a \times G_b$ bundle structure in the case of say G_b. The sheets of the cyclic covering would correspond to various factors in the n-fold tensor power of $SU(2)$ and one would obtain a Lie algebra, affine algebra or its quantum counterpart with n Cartan algebra generators in the process naturally. The number n for space-time sheets would be also a space-time correlate for the finite depth of $\mathcal{N}$ as a factor.

WCW spinors could provide fermionic representations of $G \subset SU(2)$. The Dynkin diagram characterizing tensor products of representations of $G \subset SU(2)$ with doublet representation suggests that tensor products of doublet representations associated with n sheets of the covering could realize the Dynkin diagram.

Singlet representation in the Dynkin diagram associated with irreps of G would not give rise to an SU(2) sub-algebra in ADE Lie algebra and would correspond to the scaling generator. For ordinary Dynkin diagram representing gauge group algebra scaling operator would be absent and therefore also the exceptional node. Thus the difference between $(\mathcal{M} : \mathcal{N} = 4)$ and $(\mathcal{M} : \mathcal{N} < 4)$ cases would be that in the Kac-Moody group would reduce to gauge group $\mathcal{M} : \mathcal{N} < 4$ because Kac-Moody central charge k and therefore also Virasoro central charge resulting in Sugawara construction would vanish.

4. Do finite subgroups of $SU(2)$ play some role also in $\mathcal{M} : \mathcal{N} = 4$ case?

One can ask wonder the possible interpretation for the appearance of extended Dynkin diagrams in $(\mathcal{M} : \mathcal{N} = 4)$ case. Do finite subgroups $G \subset SU(2)$ associated with extended Dynkin diagrams appear also in this case. The formal analog for $H \to G_a \times G_b$ bundle structure would be $H \to H/G_a \times SU(2)$. This would mean that the geodesic sphere of CP_2 would define the fiber. The notion of number theoretic braid meaning a selection of a discrete subset of algebraic points of the geodesic sphere of CP_2 suggests that $SU(2)$ actually reduces to its subgroup G also in this case.

5. Why Kac-Moody central charge can be non-vanishing only for $\mathcal{M} : \mathcal{N} = 4$?

From the physical point of view the vanishing of Kac-Moody central charge for $\mathcal{M} : \mathcal{N} < 4$ is easy to understand. If parton corresponds to a homologically non-trivial geodesic sphere, space-time surface typically represents a string like object so that the generation of Kac-Moody central extension would relate directly to the homological non-triviality of partons. For instance, cosmic strings are string like objects of form $X^2 \times Y^2$, where X^2 is minimal surface of M^2 and Y^2 is a holomorphic sub-manifold of CP_2 reducing to a homologically non-trivial geodesic sphere in the simplest situation. A conjecture that deserves to be shown wrong is that central charge k is proportional/equal to the absolute value of the homology (Kähler magnetic) charge h.

6. More general situation

McKay correspondence generalizes also to the case of subgroups of higher-dimensional Lie groups [A53]. The argument above makes sense also for discrete subgroups of more general compact Lie groups H since also they define unique sub-factors. In this case, algebras having Cartan algebra with nk generators, where n is the dimension of Cartan algebra of H, would emerge in the process. Thus there are reasons to believe that TGD could emulate practically any dynamics having gauge group or Kac-Moody type symmetry. An interesting question concerns the interpretation of non-ADE type principal graphs associated with subgroups of SU(2).

7. Flavor groups of hadron physics as a support for HFF?

The deformation assigning to an n-fold tensor power of representations of Lie group G with k-dimensional Cartan algebra a representation of a Lie group with nk-dimensional Cartan algebra could be also seen as a dynamically generated symmetry. If quantum measurement is characterized by the choice of Lie group G defining measured quantum numbers and defining Jones inclusion characterizing the measurement resolution, the measurement process itself would generate these dynamical symmetries. Interestingly, the flavor symmetry groups of hadron physics cannot be justified from the structure of the standard model having only electro-weak and color group as fundamental symmetries. In TGD framework flavor group $SU(n)$ could emerge naturally as a fusion of n quark doublets to form a representation of $SU(n)$.

4.3.4 Farey sequences, Riemann hypothesis, tangles, and TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labeled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

Farey sequences

Some basic facts about Farey sequences [A4] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.

2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.

3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| \quad = \quad |F(N-1)| + \phi(N-1) \ . \tag{4.3.4}$$

   Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p + 1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p + 1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the imbedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labeled by integers N and in direct correspondence with the hierarchy of quantum critical phases [K25] would naturally relate to the Farey sequence.

Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n:th term of the N:th Farey sequence, and the n:th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\begin{aligned} &\sum_{n=1,\ldots,|F_N|} |d_{n,N}| = O(N^r) \ \text{ for any } \ r > 1/2 \ , \\ &\sum_{n=1,\ldots,|F_N|} d_{n,N}^2 = O(N^r) \ \text{ for any } \ r > 1 \ . \end{aligned} \tag{4.3.5}$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.

2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labeled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [K25, K23] .

3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors. At least the S-matrix associated with p-adic-to-real transitions and more generally $p_1 \rightarrow p_2$ transitions between states for which the partonic space-time sheets are p_1- resp. p_2-adic can involve only this kind of algebraic phases. One can also say that cognitive representations can involve only algebraic phases and algebraic numbers in general. For real-to-real transitions and real-to-padic transitions U-matrix might be non-algebraic or obtained by analytic continuation of algebraic U-matrix. S-matrix is by definition diagonal with respect to number field and similar continuation principle might apply also in this case.

4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labeled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labeled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [K25] .

Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$:th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [A56] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad - bc = \pm 1$ so that the pair defines element of the modular group SL(2,Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.
2. In the general case the sum of $M-$ and $N-$tangles is $M+N-2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M+N$-tangle looks to me physically more natural than the sum.
3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N>2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N > 2$. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.
2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N:th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.
3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a,b]^T \rightarrow a/b$ from a rational 2-spinor $[a,b]^T$ to which SL(2(N-1),Z) acts. Equivalence means that the columns $[a,b]^T$ and $[c,d]^T$ combine to form element of SL(2,Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?

4. Could N-tangles be characterized by $N-1$ $2(N-1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N-1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N-1$ spinors combine to form $N-1+N-1$ columns of $SL(2(N-1), Z)$ matrix. Could N-tangles quite generally correspond to collections of projective $N-1$ spinors having as components algebraic integers and could $ad - bc = \pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g,Z) so that $N-1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.

5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2, Q)$ labeled by N (the generator $\tau \rightarrow \tau + 2$ of modular group is replaced with $\tau \rightarrow \tau + 2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N-1), Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N:th root of unity instead of phase U satisfying $U^2 = 1$?

How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N:th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g > 0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).

2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like "written language representations" of genetic programs represented as number theoretic braids.

4.3.5 Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2, F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \ , \tag{4.3.6}$$

read as

$$\begin{array}{lll} ab = qba \ , & ac = qac \ , & bd = qdb \ , cd = qdc \ , \\ [ad, da] = (q - q^{-1})bc \ , & bc = cb \ . & \end{array} \tag{4.3.7}$$

These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a, a^\dagger] = [b, b^\dagger] = [c, c^\dagger] = [d, d^\dagger] = 1 \ . \tag{4.3.8}$$

The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a - a^\dagger, d - d^\dagger, b - c^\dagger, c - b^\dagger\} \ . \end{array} \tag{4.3.9}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. This is the case and means also that the operators representing M^4 coordinates commute and it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

$M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})bc$ does not vanish. The observation that M^4 coordinates can be regarded as eigenvalues of commuting observables proves that quantum CD and its orbifold description are equivalent.

What about M^8: does it have analogous description? The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

Introduce the coefficients of E^4 gamma matrices having interpretation as quaterionic units as

$$\begin{array}{ll} a_0 = ix(a + d) \ , & a_3 = x(a - d) \ , \\ a_1 = x(b + c) \ , & a_2 = x(ib - c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

The selections of commutative and quaternionic sub-algebras of octonion space are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of octonions the selection of quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned}
[a_0,a_3] &= -i(q-q^{-1})(a_1^2+a_2^2) \ , \\
[a_i,a_j] &= 0 \ , \ i,j \neq 0,3 \ , \\
a_0a_i &= qa_ia_0 \ , \ i\neq 0,3 \ , \\
a_3a_i &= qa_ia_3 \ , \ i\neq 0,3 \ .
\end{aligned} \tag{4.3.10}$$

Checking that M^8 indeed corresponds to commutative subspace defined by the eigenvalues of operators is straightforward.

The argument generalizes easily to other dimensions $D\geq 4$ but now quaternionic and octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Thus the special role of classical number fields and uniqueness of space-time and imbedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. It is possible to construct the quantal variants of the coset spaces $M^4\times E^4/G_a\times G_b$ by simply posing restrictions on the of eigen states of the commuting coordinate operators. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible.

Chapter 5

Negentropy Maximization Principle

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5.1 Introduction

Quantum TGD involves 'holy trinity' of time developments. There is the geometric time development dictated by the preferred extremal of Kähler action crucial for the realization of General Coordinate Invariance and analogous to Bohr orbit. There is the unitary "time development" U: $\Psi_i \to U\Psi_i \to \Psi_f$, associated with each quantum jump, which is the counterpart of the Schrödinger time evolution $U(-t, t \to \infty)$. There is however no actual Schrödinger equation involved: situation is in practice same also in quantum field theories. Quantum jump sequence itself defines what might be called subjective time development.

Some dynamical principle governing subjective time evolution should exist and explain state function reduction with the characteristic one-one correlation between macroscopic measurement variables and quantum degrees of freedom and state preparation process. Negentropy Maximization Principle is the candidate for this principle, which I have been developing during last fifteen years.

The evolution of ideas related to NMP has been slow and tortuous process characterized by misinterpretations, over-generalizations, and unnecessarily strong assumptions, and has been basically evolution of ideas related to the anatomy of quantum jump and of quantum TGD itself.

5.1.1 The notion of entanglement entropy

1. The first form of NMP was rather naive. There was no idea about the anatomy of quantum jump and NMP only stated that the allowed quantum jumps are such that the information gain of conscious experience measured by the reduction of entanglement entropy resulting in the reduction of entanglement between the subsystem of system and its complement is maximal. Later it became clear that quantum jump has a complex anatomy consisting of unitary process U followed by the TGD counterpart of state function reduction serving as a state preparation for the next quantum jump.

2. The attempts to formulate NMP in p-adic physics led to the realization that one can distinguish between three kinds of information measures.

   (a) In real physics the negative of the entanglement entropy defined by the standard Shannon formula defines a natural information measure, which is always non-positive.

   (b) In p-adic physics one can generalize this information measure to p-adic valued information measure by replacing the logarithms of p-adic valued probabilities with the p-based logarithms $log_p(|P|_p)$ which are integer valued and can be interpreted as p-adic numbers. This p-adic valued entanglement entropy can be mapped to a non-negative real number by the so called canonical identification $x = \sum_n x_n p^n \to \sum_n x_n p^{-n}$. In both cases a non-positive information measure results.

(c) When the entanglement probabilities are rational numbers or at most finitely algebraically extended rational numbers one can still define logarithms of probabilities as p-based logarithms $log_p(|P|_p)$ and interpret the entropy as a rational or algebraic number. In this case the entropy can be however negative and positive definite information measure is possible. Irrespective of number field one can in this case define entanglement entropy as a maximum of number theoretic entropies S_p over the set of primes. The first proposal was that the algebraic entanglement corresponds to bound state entanglement turned out to be wrong.

3. At some stage the importance of the almost trivial fact that bound state entanglement must be kinematically stable against NMP became obvious. One can imagine that the state function reduction proceeds step by step by reducing the state to two parts in such a manner that the reduction of entanglement entropy is maximal.

   (a) If a resulting subsystem corresponds to a bound state having no decomposition to free subsystems the process stops for this subsystem. The natural assumption is that subsystems lose their consciousness when U process leads to bound state entanglement whereas bound state itself can be conscious.

   (b) If the entanglement is negentropic (and thus rational or algebraic) a more natural interpretation consistent with the teaching of spiritual practices is that subsystems experience a fusion to a larger conscious entity. The negentropic entanglement between free states is stabilized by NMP and negentropically entangled states need not reside at the bottom of potential well forbidding the reduction of entanglement. This makes possible new kinds of correlated states for which binding energy can be negative. Bound state entanglement would be like the jail of organized marriage and negentropic entanglement like a love marriage in which companions are free to leave but do not what it. The existence of this kind of negentropic entanglement is especially interesting in living matter, where metabolism (high energy phosphate bond in particular) and the stability of DNA and other highly charged polymers is poorly understood physically: negentropic entanglement could be responsible for stabilization making possible the transfer of metabolic energy [K35] .

4. For the negentropic entanglement the outcome of the state function reduction ceases to be random as it is for the standard definition of entanglement entropy. Note however that U process as a creative act yielding superposition of possibilities from which state function reduction selects leaves means non-determinism. This has far reaching consequences. Ordinary state function reductions for an ensemble of systems lead to a generation of thermodynamical entropy and this explains the second law of thermodynamics. In the case of negentropic entanglement situation changes and the predicted breaking of second law of thermodynamics provides a new view to understand self-organization [K73] , and living matter could be identified as something residing in the intersection of real and p-adic worlds where p-adic intentions can be transformed to real actions.

5. One particular choice involved with state function reduction process could be the choice between generic entanglement and number theoretic entanglement possible only in the intersection of p-adic and real WCWs. If the choice is the generic entanglement, system ends up either to an unentangled state with maximal conscious freedom or to a bound state with a loss of consciousness. If the choice is algebraic entanglement, system ends up to negentropic entanglement and correlations with external world and experiences an expansion of consciousness. Maybe ethical choices are basically choices between these two options. Also positive emotions like love and experience of understanding could directly relate to various aspects of the negentropic entanglement.

5.1.2 Zero energy ontology

Zero energy ontology changes considerably the interpretation of the unitary process. In zero energy ontology quantum states are replaced with zero energy states defined as a superpositions of pairs of positive and negative energy states identified as counterparts of initial and final states of a physical event such as particle scattering. The matrix defining entanglement between positive and negative - christened as M-matrix- is the counterpart of the ordinary S-matrix but need not be unitary. It can be identified as a "complex square root" of density matrix expressible as a product of positive square root of diagonal density matrix and unitary S-matrix. Quantum TGD can be seen as defining a "square root" of thermodynamics, which thus becomes an essential part of quantum theory.

U-matrix is defined between zero energy states and cannot therefore be equated with the S-matrix used to describe particle scattering events. Unitary conditions however imply that U-matrix can be seen as a collection of M-matrices labelled by zero energy states so that the knowledge of U-matrix implies the knowledge of M-matrices. The unitarity conditions will be discussed later. A natural guess is that U is directly related to consciousness and the description of intentional actions. For positive energy ontology state function reduction would serve as a state preparation for the next quantum jump. In zero energy ontology state function preparation and reduction can be assigned to the positive and negative energy states defining the initial and final states of the physical event. The reduction of the time-like entanglement during the state function reduction process corresponds to the measurement of the scattering matrix. In the case of negentropic time-like entanglement the reduction process is not random anymore and the resulting dynamics is analogous to that of cellular automata providing a natural description of the dynamics of self-organization in living matter. This self-organization is also 4-dimensional in ZEO: this is of utmost importance in attempts to understand living matter.

According to standard quantum measurement theory state function reductions can take place repeatedly without any change in the state. In ZEO state function reduction to a given boundary of CD can occur repeatedly without changing the corresponding part of zero energy state but affecting the part at the opposite boundary. Superposition of CDs with different sizes is possible and one can assign to the second boundary a wave function in the space of moduli, which includes the proper time distance between the tips of CD. This distance must increase in average sense and this gives rise to the arrow of experienced time. Self can be identified as a sequence of quantum jumps reducing to same boundary of CD.

Zero energy ontology leads to a precise identification of the subsystem at space-time level. General coordinate invariance in 4-D sense means that 3-surfaces related by 4-D diffeomorphisms are physically equivalent. It is convenient to perform a gauge fixing by a introducing a natural choice for the representatives of the equivalence classes formed by diffeo-related 3-surfaces.

1. Light-like 3-surfaces identified as surfaces at which the Minkowskian signature of the induced space-time metric changes to Euclidian one - wormhole contacts- are excellent candidates in this respect. The intersections of these surfaces with the light-like boundaries of CD defined 2-D partonic surfaces. Also the 3-D space-like ends of space-time sheets at the light-like boundaries of CDs are very natural candidates for preferred 3-surfaces.

2. The condition that the choices are mutually consistent implies effective 2-dimensionality. The intersections of these surfaces defining partonic 2-surface plus the distribution of 4-D tangent spaces at its points define the basic dynamical objects with 4-D general coordinate invariance reduced to 2-dimensional one. This effective 2-dimensionality was clear from the very beginning but is only apparent since also the data about 4-D tangent space distribution is necessary to characterize the geometry of WCW and quantum states. The descriptions in terms of 3-D light-like or space-like surfaces and even in terms of 4-D surfaces are equivalent but redundant descriptions.

As far as consciousness is considered effective 2-dimensionality means holography and could relate to the fact that at least our visual experience is at least effectively 2-dimensional.

5.1.3 Connection with standard quantum measurement theory

TGD allows to deduce the standard quantum measurement theory involving the notion of classical variables and their correlation with quantum numbers in an essential manner. WCW (or "world of classical worlds", briefly WCW) is a union over zero modes labelling infinite-dimensional symmetric spaces having interpretation as classical non-quantum fluctuating classical variables such as the pointer of a measurement apparatus essential for the standard quantum measurement theory [K22]

. Quantum holography states that partonic 2-surfaces at the light-like boundaries of CDs plus the corresponding distributions of 4-D tangent spaces of space-time surfaces at carry the information about quantum state and space-time sheet. The distribution of values of induced Kähler form of CP_2 at these surfaces defines zero modes whereas quantum fluctuating degrees of freedom correspond to the deformations of space-time surface by the flows induced by Hamiltonians associated with the degenerate symplectic structure of $\delta M^4_\pm \times CP_2$.

There exists no well-defined metric integration measure in the infinite-dimensional space of zero modes, which by definition do not contribute to the line element of WCW. This does not lead to difficulties if one assumes that a complete localization in zero modes occurs in each quantum jump. A weaker condition is that wave functions are localized to discrete subsets in the space of zero modes. An even weaker and perhaps the most realistic condition is that a localization to a finite-dimensional 2n-dimensional manifold with induced symplectic form defining a positive definite integration volume takes place.

The fundamental formulation of quantum TGD in terms of the modified Dirac action [K20, K33] containing a measurement interaction term guarantees quantum classical correspondence in the sense that the geometry of the space-time surface correlates with the values of conserved quantum numbers. The resulting correlation of zero modes with the values of quantum numbers can be interpreted as an abstract form of quantum entanglement reduced in quantum jump for the standard definition of the entanglement entropy. This reproduces standard quantum measurement theory.

That state function can occur at both boundaries of CD localizing the boundary in question reducing the part of zero energy state associated with it is the new element of TGD inspired quantum measurement theory and allows to understand how the arrow of experienced time emerges and precisely define self - observer - as a part of system interacting with it. Also the possibility that the arrow of time changes at some level of the self hierarchy is predicted. In living matter this is expected to occur routinely as already Fantappie speculated [J15]: the first state function reduction in the sequence of them and changing the arrow of time is indeed naturally identified as a correlate for the volitional act.

5.1.4 Quantum classical correspondence

Quantum classical correspondence has served as a guideline in the evolution of the ideas and the identification of the geometric correlates of various quantum notions at the level of imbedding space and space-time surfaces has been an important driving force in the progress of ideas.

1. In zero energy ontology causal diamonds (CDs) identified roughly as intersections of future and past directed light-cones are in key role. At imbedding space level CD is a natural correlate for self and sub-CDs serve as correlates of sub-selves identified as mental images. At space-time level the space-time sheets having their ends at the light-like boundaries of CD serve as correlates for self. For a system characterized by a primary p-adic length scale $L_p \propto 2^{k/2}$ the size scale of CD is secondary p-adic scale $L_{p,2} = \sqrt{p}L_p \propto 2^k$. p-Adic length scale hypothesis follows if the proper time distance between the tips of CDs is quantized in powers of 2. This quantization should relate directly to almost equivalence of octaves associated with music experience.

2. At the level of space-time the identification of join along boundaries bonds between space-time sheets (more precisely, between partonic 2-surfaces) as a correlate for bound state entanglement suggests itself. Join along boundaries bonds correspond typically to magnetic flux tubes in the TGD inspired quantum model of living matter. The size scale of the magnetic body of system is given by the size scale of CD and much larger than the size of the system itself.

3. The space-time sheets in the intersection of the real and p-adic WCWs characterized by the property that the mathematical representation of the partonic 2-surfaces at the ends representing holograpically the state allows interpretation in both real and p-adic sense would correspond to the correlates for negentropic entanglement. Rational and algebraic 2-surfaces (in preferred coordinates) would be the common points of realities and p-adicities.

Quantum classical correspondence allows also to generate new views about quantum theory itself. Many-sheeted space-time and p-adic length scale hierarchy force to generalize the notion of sub-system. The space-time correlate for the negentropic and bound state entanglement is the formation of join along boundaries bonds connecting two space-time sheets. The basic realization is that two disjoint space-time sheets can contain smaller space-time sheets topologically condensed at them and connected by join along boundaries bonds. Thus systems un-entangled at a given level of p-adic hierarchy -that is in the measurement resolution defined by the level considered - can contain entanglement subsystems at lower level not visible in the resolution used.

In TGD inspired theory of consciousness this makes possible sharing and fusion of mental images by entanglement. The resolution dependence for the notions of sub-system and entanglement means that the entanglement between sub-systems is not "seen" in the length scale resolution of unentangled systems. This phenomenon does not result as an idealization of theoretician but is a genuine physical phenomenon. Obviously this generalized view about sub-system poses further challenges to the detailed formulation of NMP. Note that the resulting mental image should depend on whether sub-selves are entangled by bound state entanglement or negentropic entanglement.

5.1.5 Fusion of real and p-adic physics

The fusion of real and p-adic physics to a larger structure has been a long standing challenge for TGD. The motivations come both from elementary particle physics and TGD inspired theory of consciousness, in particular from the attempt to model how intentions proposed to have p-adic space-time sheets as space-time correlates are transformed to actions having real space-time sheets as correlates. The basic idea is that various number fields are fused to a larger structure by gluing them along rationals and common algebraic numbers. The challenge is to imagine what quantum jump and NMP could mean in this framework. The first question is how the unitary process acts.

1. U-process acts in spinorial degrees of freedom of WCW (fermionic Fock space for a given 3-surface) and in WCW degrees of freedom (the space of partonic 2-surfaces roughly). The transformation of intention to action would correspond to a leakage from p-adic to real sector of WCW.

2. At the level of WCW one can only speak about classical spinor fields and the idea about tensor product of states corresponding to different sectors of WCW does not look reasonable at the first glance. Rather, a quantum superposition of WCW spinor fields localized at various sectors would look more appropriate. Therefore the WCW spinor field would be in fixed number field after state function reduction if it involves localization in this sense. This does not look sensible. The tensor product for fermionic Fock spaces is indeed very natural and strongly suggested also by the interpretation of the 3-surfaces as particles. One can indeed consider CDs and their unions and it would seem reasonable to assign to the unions of CDs tensor products of the corresponding WCW spinor fields. Let us assume this.

3. Let us assume that the initial zero energy state state represents an un-entangled tensor product of states in various number fields. The simplest assumption is that U process can induce a leakage between different sectors only in the intersection of real and p-adic worlds. This would also hold true as far as entanglement between different number fields is considered. This would allow to realize intentional action geometrically as a p-adic-to-real transition. The p-adic and real variants of a state quantum entangled with a third (say real) state would define the entangled system and state function reduction would select either p-adic or real variant of the state. The selection would be whether to transform action to its cognitive representation or intention to action. Also a transformation of a real zero energy state to its cognitive representation in p-adic sense is possible as also transformations between p-adic cognitive representations characterized by different primes.

4. For partonic 2-surfaces the quantum superposition of quantum states belonging to different number fields in the intersection would mean a quantum superposition of real and various p-adic variants of the surface with given mathematical representation forming tensor products with the states of second system, which could be real for instance. U-matrix could lead to this kind of quantum superposition. U-matrix between different number fields should be expressible using only the geometric data from the intersection of the real and p-adic variants of the partonic surface- that is rational points and common algebraic points, whose number

is expected to be finite. Some kind of number theoretic quantum field theory should describe the U-matrix. State function reduction would involve the selection of whether the outcome is action or intention (or cognitive representation). Note that if the real-real entanglement is non-algebraic the NMP leads to a final state with algebraic entanglement between real system and p-adic cognitive representation of the other system. If real-real entanglement is algebraic, the reduction can lead from intention to action as a more negentropic final state.

5. It has been assumed that entanglement and matrix elements of U between different number fields are possible only in the intersection of the real and p-adic worlds. This is natural if entanglement coefficients between different number fields are represented in terms of the data provided by the intersection of the real and p-adic variants of partonic 2-surfaces involved and consisting of rational points and some algebraic points. Outside the intersection real and p-adic worlds would evolve independently. One could criticize this picture as raising the intersection of real and p-adic worlds to a singular position. Life is however something very special and the interpretation in terms of number theoretical criticality justifies this singular character.

5.1.6 Dark matter hierarchy

The identification of dark matter as phases having large value of Planck constant [K75, K32, K28] led to a vigorous evolution of ideas. Entire dark matter hierarchy with levels labelled by increasing values of Planck constant is predicted, and in principle TGD predicts the values of Planck constant if physics as a generalized number theory vision is accepted [K32] .

The hierarchy of Planck constants is realized in terms of a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. $CD \times CP_2$ is generalized by gluing singular coverings and factor spaces of both CD and CP_2 together like pages of book along common back, which is 2-D sub-manifold which is M^2 for CD and homologically trivial geodesic sphere S^2 for CP_2 [K32] . The value of the Planck constant characterizes partially the given page and arbitrary large values of $\hbar$ are predicted so that macroscopic quantum phases are possible since the fundamental quantum scales scale like $\hbar$. The most general spectrum comes in rational multiples of standard value of Planck constant which corresponds to the unit of rationals. For CDs the scaling of Planck constants means scaling of the size of CD. This could explain why the rational multiples of the fundamental frequency are so special for music experience.

All particles in the vertices of Feynman diagrams have the same value of Planck constant so that particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to $\hbar$ changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory. These observations motivate the tentative identification of the macroscopic quantum phases in terms of dark matter and also of dark energy with gigantic "gravitational" Planck constant.

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by the following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

At least dark matter could be a key player in quantum biology.

1. Dark matter hierarchy and p-adic length scale hierarchy would provide a quantitative formulation for the self hierarchy. To a given p-adic length scale one can assign a secondary p-adic time scale as the temporal distance between the tips of the CD. For electron this time scale is .1 second, the fundamental bio-rhythm. For a given p-adic length scale dark matter hierarchy gives rise to additional time scales coming as $\hbar/\hbar_0$ multiples of this time scale.

2. The predicted breaking of second law of thermodynamics chacterizing living matter - if identified as something in the intersection of real and p-adic words - would be always below the time scale of CD considered but would take place in arbitrary long time scales at appropriate levels of the hierarchy. The scaling up of $\hbar$ also scales up the time scale for the breaking of the second law.

3. The hypothesis that magnetic body is the carrier of dark matter in large $\hbar$ phase has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of [J6] [K29] . Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K47, K29] . A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K29] .

5.1.7 Hyper-finite factors of type II_1 and quantum measurement theory with a finite measurement resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the imbedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the imbedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in zero energy ontology states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.

2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.

3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.

As already explained, the topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a manner that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

Also the notions of resolution and monitoring pop up naturally in this framework. p-Adic probabilities relate very naturally to hyper-finite factors of type II_1 and extend the expressive power of the ordinary probability theory. p-Adic thermodynamics with conformal cutoff is very natural for hyper-finite factors of type II_1 and explains p-adic length scale hypothesis $p \simeq 2^k$, k prime characterizing exponentially smaller p-adic length scale

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- TGD inspired theory of consciousness [L42]
- Negentropy Maximization Principle [L32]
- Zero Energy Ontology (ZEO) [L45]

5.2 Basic view about NMP

The following represents a brief overall view about the notions of quantum jump, self, and NMP.

5.2.1 The general structure of quantum jump

It has gradually become clear that TGD involves 'holy trinity' of dynamics.

1. The dynamics defined by the preferred extremals of Kähler action identifiable as counterparts of Bohr orbits corresponds to the dynamics of material existence, with matter defined as 'res extensa', three-surfaces.

2. The dynamics defined by the action of the unitary "time development" operator U can be regarded as informational "time development" occurring at the level of objective existence. U brings in mind the time evolution operator $U(-t,t)$, $t \rightarrow \infty$ associated with the scattering solutions of Schrödinger equation. It seems however un-necessary and also impossible to assign Schrödinger equation with U. Furthermore, U acts between zero energy states in zero energy ontology and is more naturally assigned with intentional action rather than to the description of particle scattering.

3. The dynamics of quantum jumps governed by U and by NMP corresponds to the dynamics of subjective existence.

In accordance with this, quantum jump decomposes into informational time development

$$\Psi_i \rightarrow U\Psi_i \ ,$$

followed by a sequence of self measurements (generalization of state function reduction)

$$\Psi_{f_0} \rightarrow \Psi_{f_1} ..... \rightarrow \Psi_f$$

governed by NMP. At given step subsystem the decomposition to two un-entangled systems is such that maximum reduction of entanglement entropy is achieved. This means that the reduction process proceeds as a binary tree. If subsystem does not allow a decomposition to a pair of free subsystems with entropic entanglement the process stops.

Zero energy ontology means that one must distinguish between M-matrix and U-matrix. M-matrix characterizes the time like entanglement between positive and negative energy parts of zero energy state and is measured in particle scattering experiments. M-matrix need not be unitary and can be identified as a "complex" square root of density matrix representable as a product of its real and positive square root and of unitary S-matrix so that thermodynamics becomes part of quantum theory with thermodynamical ensemble being replaced with a zero energy state. The unitary U-matrix describes quantum transitions between zero energy states and is therefore something genuinely new. It is natural to assign the statistical description of intentional action with U-matrix since quantum jump occurs between zero energy states.

U process is in zero energy ontology something totally new and can be seen as representing an act of genuine re-creation of the Universe. The following metaphors might help to understand what is involved.

1. A good metaphor for the quantum jump is as Djinn leaving the bottle (U) fulfilling the wish realized as a choice between various option that is state function reduction. In the case that final state has negentropic entanglement wish is realized in different manner.

2. A second useful metaphor is as generation of infinite number of quantum parallel potentialities in which entire universe is in a totally entangled holistic state of oneness followed by state function reduction and self measurement cascade analyzing the state into maximally unentangled subsystems. NMP states that the analysis produces maximum amount of conscious information. For irreducible selves analysis process do not continue and the sequences of quantum jumps effectively take the role of single quantum jump. A further element is the expansion of consciousness when negentropic entanglement is generated. Therefore this structure characterizes also conscious experience in macro-temporal time scales. Clearly, quantum measurement theory has fascinating parallels with Krishnamurti's philosophy of consciousness which underlines the competing holistic and reductionistic aspects of consciousness.

3. A third metaphor comes from particle physics. Moment of consciousness can be seen as elementary particle of consciousness and selves as the atoms, molecules, ...galaxies,... of consciousness. Fractality hypothesis allows to get general vision about structure of consciousness even in the time scale of human life.

If quantum jump occurs between two different time evolutions of Schrödinger equation (understood here in very metaphorical sense) rather than interfering with single deterministic Schrödinger evolution, the basic problem of quantum measurement theory finds a resolution. The interpretation of quantum jump as a moment of consciousness means that volition and conscious experience are outside space-time and state space and that quantum states and space-time surfaces are "zombies".

5.2.2 NMP and the notion of self

Negentropy Maximization Principle (NMP) codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies decomposition of the system to unique unentangled systems and the process repeats itself for these systems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states.

Intuitively self corresponds to a sequence of quantum jumps which somehow integrates to a larger unit much like many-particle bound state is formed from more elementary building blocks. It also seems natural to assume that self stays conscious as long as it can avoid bound state entanglement with the environment in which case the reduction of entanglement is energetically impossible. One could say that everything is conscious and consciousness can be only lost when the system forms bound state entanglement with environment.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement, which need not be bound state entanglement in standard sense. Negentropic entanglement might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of negentropic entanglement the outcome of the reduction is not random: the prediction is that second law is not universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism [K35] . A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

If one accepts the hierarchy of Planck constants [K32] , it might be un-necessary to distinguish between self and quantum jump. The hierarchy of Planck constants interpreted in terms of dark matter hierarchy predicts a hierarchy of quantum jumps such that the size of space-time region contributing to the contents of conscious experience scales like $\hbar$. Also the hierarchy of space-time sheets labeled by p-adic primes suggests the same. That sequence of sub-selves/sub-quantum jumps are experienced as separate mental images explains why we can distinguish between digits of phone number. The irreducible component of self (pure awareness) would correspond to the highest level in the "personal" hierarchy of quantum jumps and the sequence of lower level quantum jumps would be responsible for the experience of time flow. Entire life cycle would correspond to single quantum jump at the highest(?) level of the personal self hierarchy and pure awareness would prevail during sleep: this would make it possible to experience directly that I existed yesterday. Whether these two definitions of self are in some sense equivalent will be discussed later.

How the contents of consciousness of self are determined

The hypothesis that the experiences of self associated with the quantum jumps occurred after the last 'wake-up' sum up to single experience, implies that self can have memories about earlier moments of consciousness. Therefore self becomes an extended object with respect to subjective time and has a well defined 'personal history'. If temporal binding of experiences involves kind of averaging, quantum statistical determinism makes the total experience defined by the heap of the experiences associated with individual quantum jumps reliable. Subjective memory has natural identification as a short term memory.

A given self S behaves essentially as a separate sub-Universe with respect to NMP. If one postulates that the conscious experiences of sub-selves S_i of an self S integrate with the self experience of S to single experience, one obtains a filtered hierarchy of conscious experiences with increasingly richer contents and at the top of the hierarchy is entire universe, God, enjoying eternal self-consciousness since it cannot get entangled with any larger system.

An attractive hypothesis is that the experience of self is abstraction in the sense that the experiences of sub-selves S_{ij} of S_i are abstracted to average experience $\langle S_{ij}\rangle$. This implies that the experiences of sub-sub-...selves of S are effectively unconscious to S. This hierarchy obviously has extremely far-reaching consequences. Temporal binding implies that experiences of individual selves are reliable and abstraction brings in the possibility of quantum statistical determinism at the level of ensembles.

The binding of *experiencers* is also possible. The binding of selves by quantum entanglement however destroys the component selves (note however the comment about situation in which the p-adic primes are different for real entangling selves). This process could correspond to the formation as wholes from their parts, say the formation of the mental image representing word from the mental images representing letters, which are all represented as sub-selves. Associative learning might correspond to the generation of entanglement between selves representing objects of the sensory experience and conscious association would correspond to the reduction of this entanglement generating associated sub-selves. The entanglement of sub-selves of two selves is possible if one accepts the length scale dependent notion of subsystem and means sharing and fusion of mental images, binding of experiences. Entanglement might make possible communication between selves belonging to different levels of the self hierarchy and to different number fields: this entanglement would be reduced always in state function reduction step.

Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy as a hierarchy defined by quantized Planck constants leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K28, K29] .

The hierarchy of dark matter levels is labeled by the values of Planck constant having quantized but arbitrarily large values. For the most general option the values of $\hbar$ are products and ratios of two integers. The products of distinct Fermat primes and power of two are number theoretically favored values for these integers. p-Adic length scale hypothesis favors powers of two. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T \propto \hbar$ of the quantum jump.

Dark matter hierarchy suggests a modification of the notion of self, in fact a reduction of the notion of self to that of quantum jump alone. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. This indeed looks extremely natural and the hypothesis that self remains un-entangled for a longer duration than single quantum jump unnecessary. It is perhaps un-necessary to emphasize that the reduction of the notion of self to that of quantum jump means conceptual economy and somewhat ironically, would also a return to the original hypothesis but with a quantized Planck constant.

The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

5.2.3 NMP, self measurements, cognition, state preparation, qualia

NMP can be seen as the variational principle governing the dynamics of *self measurements* giving rise to state preparation and reduction finding a unified description as state function reduction in zero energy ontology.

1. NMP applies to any unentangled subsystem resulting in this cascade of self measurements and tells that self measurement is performed for the subsystem (or equivalently, its complement) which gives rise to maximum entanglement negentropy gain in the self measurement.

2. This self measurement process continues until the system decomposes into unentangled subsystems consisting of subsystems for which the entanglement is bound state entanglement or negentropic entanglement.

NMP dictates the anatomy of a single quantum so that there is actually no need to mention the notion of self at all in the context of NMP (note however the possibility that the notions of self and quantum are one and same). Despite this it is useful to briefly introduce the basic concepts related to the notion of self. Self is a subsystem able to remain unentangled in sequential quantum jumps and preserving its identity in some sense: presumably the p-adic prime characterizing self (and also the real space-time sheet associated with self) is what characterizes the self identity. One can define irreducible self as a self which does not decompose to further sub-selves in state preparation process. A second reason for introducing the notion of self is that for a self in a state of macro-temporal quantum coherence the sequence of quantum jumps effectively fuses to single quantum jump representing single long lasting moment of consciousness. With this definition self ceases to exist as it fuses to another self by bound state entanglement of negentropic entanglement. In the latter case self however experiences expansion of consciousness rather than losing it.

Some further comments about NMP are in order.

1. Standard quantum measurement theory does not allow a spontaneous reduction of entanglement between quantum fluctuating degrees of freedom of two subsystems associated with

a 3-surface. Only the entanglement between quantum fluctuating and zero mode degrees of freedom, that is between quantum system and observer can be reduced. The question is therefore whether one should restrict NMP to the entanglement between zero modes and quantum fluctuating degrees of freedom or allow also the reduction of entanglement between quantum fluctuating degrees of freedom. Self measurements affecting entanglement between quantum fluctuating degrees of freedom are distinguishable from standard quantum measurements. The working hypothesis is that state function reduction applies to any kind of entanglement.

2. Self measurement involves the division of unentangled subsystem (possibly self, mental image) into two unentangled subsystems. Analytical thought creates separations and comparisons so that this division could be identified as the basic mechanism of cognition. Also sensory experience generates separations and distinctions so that NMP should be identified as the variational principle governing the dynamics of cognition and perception. State reduction process makes the world of conscious experience to look completely classical since only bound state entanglement and negentropic entanglement are stable against self measurement. One can thus say that state function reduction leads from a maximally entangled multiverse state $U\Psi_i$ to a maximally analyzed state: from quantum holism to classical reductionism. At the level of standard quantum measurement theory this process is equivalent with state preparation process yielding totally unentangled product state as incoming state of particle physics experiment.

3. The fact that self measurement reduces entanglement entropy allows the system to remain conscious (unless it generates bound state entanglement) but leads to a generation of thermodynamical entropy at the level of ensemble. Thermodynamical ensemble of sub-sub-selves means fuzzy mental images at the level of self. Thermodynamical ensemble of sub-selves could give rise to statistical determinism and be essential for sensory representations.

4. Irreducible self *effectively* obeys in quantum fluctuating degrees of freedom a unitary time development defined by n:th power of U for a sequence of n quantum jumps, at least in reasonable approximation. This means fractality of consciousness: one can approximate sequences of quantum jumps with single quantum jump such as one can approximate molecules consisting of elementary particles with a point like particle. This observation is of crucial importance for understanding how quantum computing is possible in TGD universe despite that single quantum jump to an increment of psychological time equal to CP_2 time. Also Penrose-Hameroff hypothesis generalizes to TGD framework and one can understand the purely phenomenological notion of quantum de-coherence at fundamental level and also how the quantum spin glass nature of TGD Universe allows to circumvent the objections against Penrose-Hameroff hypothesis.

5. The fact that state preparation is not a deterministic process, forces a statistical modelling of the state of self using the ensemble formed by the prepared states defined by the sequence of quantum jumps in turn defining the contribution to the contents of consciousness of self as a statistical average. The simplest description is in terms of thermodynamics. Thermodynamical density matrix gives the probabilities for various states of a subsystem in the sequence of quantum jumps occurred after the last 'wake-up'. What is of paramount importance is that the contents of consciousness of self can be modelled using statistical thermodynamics. Non-geometric sensory qualia indeed have a close relationship with conjugate pairs of thermodynamical variables such as temperature-entropy, pressure-volume, chemical potential-particle number,... The sequence of quantum jumps also defines a sequence of quantum jumps in zero modes. Statistical averaging is not so natural for the values of zero modes characterizing the outcomes of the quantum measurements, which suggests that they could be experienced as separate ones by self and would correspond to geometric qualia experienced as being sharp and dynamical.

5.3 Physics as fusion of real and p-adic physics and NMP

In this section the vision about state function reduction and preparation processes as number theoretic necessities is developed: also the chapter "Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory" contains related topics. The proposal raises NMP to fundamental principle applying also to the state function reduction step.

5.3.1 Basic definitions related to density matrix and entanglement entropy

In this sequel the detailed definitions of density matrix and entropy are discussed. It has become clear that one must distinguish between three kinds of systems systems.

1. Genuinely real systems for which entanglement probabilities are not rational numbers or finitely extended rational numbers. In this case one can regard the probabilities as limiting values of frequencies for outcomes of measurement defined by a time series. This is also the case when the entanglement coefficients are rational or algebraic numbers but the number of entangled state pairs is infinite so that the entanglement probabilities need not be algebraic numbers anymore.

2. A genuinely p-adic system is a p-adic system in which entanglement probabilities are not positive rational numbers so that one cannot interpret the entanglement probabilities as a limit for frequencies defined by any ensemble.

3. Finitely extended rational entanglement probabilities allow an interpretation as ordinary probabilities. In this case one can regard the probabilities as belonging to an extension of rationals or to any p-adic number field. What is essential is that the number field is now discrete whereas it is continuous in above mentioned cases.

   One must use different definition for the real counterpart of the entanglement entropy in these two cases. In the first case standard Shannon's entropy works. In the second case p-adic counterpart of the Shannon entropy mapped to a real number by the canonical identification is the only possibility. In the third case the number theoretic entropies S_p based on p-adic norm can be regarded as extended rational numbers as such. In this case S_p can be negative, and one can fix the value of p used to define the entropy by requiring that entropy is maximally negative and thus identifiable as a genuine information measure.

Density matrix

The density matrix of subsystem, call it A, can be defined using the standard formulas of QM: essentially trace over the degrees of freedom associated with the complement of A, call it B, is performed. B could effectively reduce to a sub-system of the complement. Density matrix is hermitian matrix and can be diagonalized in the real context. Eigenvalues are real and give the weights for various eigen states in the superposition. There is important *duality* present: in the basis of A in which the density matrix for A is diagonal also the density matrix of B is diagonal.

Density matrix actually determines one-one-correspondence between certain states of the system A and system B. The state in eigen state basis can be written as

$$|A,B\rangle = \sum_m c_m|m\rangle \times |M(m)\rangle \ , \tag{5.3.1}$$

where the map $m \rightarrow M(m)$ defines identification of certain states of A with certain states of B.

Quantum measurement of density matrix means that subsystem goes to an eigen state of density matrix. In the p-adic context the diagonalization of the density matrix requires special assumptions about the form of the state since the p-adic number fields are not closed with respect to algebraic operations. There is an algebraic extension obtained by requiring that each 'real' p-adic number has square root [K56] . The extension is 4-dimensional for $p \geq 3$ and 8-dimensional for $p = 2$. It can quite well happen that density matrix can be diagonalized only partially in this extension since the eigenvalues of the density matrix are in general algebraic numbers determined as a solution of polynomial eigenvalue equation.

One can however allow the extension of the p-adic number field to allow eigenvalues in an algebraic extension. Unless this is allowed the concepts of density matrix and entropy are not well defined for a generic subsystem. Physically this would mean that quantum state can have irreducible number theoretic entanglement besides the entanglement related to the quantum statistics. The vision about TGD as a generalized number theory encourages the allowance of the algebraic extension. This means that quantum subsystems can be classified using as criterion the dimension of the p-adic algebraic extension needed to define the eigen states and eigenvalues of the density matrix. In well defined sense physical systems generate increasingly complicated number fields as algebraic extensions of the p-adic numbers.

An interesting possibility is that hermiticity in the p-adic context must be defined so that the eigenvalues of the density matrix are *ordinary p-adic numbers*: if this is the case then the algebraic extension is needed only for the diagonalization of the density matrix but the diagonalized density matrix itself is 'p-adically real'. This option seems however un-necessarily restrictive and will not be considered in the sequel.

If entanglement coefficients are algebraic numbers then also entanglement probabilities are algebraic numbers in the case that the number of entanglement state pairs is finite. Even finite-dimensional extensions of p-adic number numbers involving transcendentals such as $e, e^2, ..., e^{p-1}$) can be allowed. If the number of entangled state pairs is infinite, entanglement probabilities need not belong to a finite extension of rationals and it seems that entanglement cannot be regarded as bound state entanglement in this case.

p-Adic entanglement negentropy

In the real context negentropy is defined using the standard formula for Shannon entropy:

$$N = \sum_k p_k \cdot log(p_k) \quad . \tag{5.3.2}$$

In the real context one could equally well replace the e-based logarithm $log(x)$ by a-based logarithm (a could be any positive real) since this introduces only multiplicative factor ($log_a(x) = \frac{log(x)}{log(a)}$).

p-Adic thermodynamics has turned out to be surprisingly successful for the calculation of elementary particle masses. p-Adic thermodynamics is however naturally based on p-based logarithm log_p rather than the ordinary e-based logarithm since Boltzmann weights are powers of p rather than exponents. This would suggest the following definition

$$N = \sum_k p_k \cdot log_p(p_k) \quad . \tag{5.3.3}$$

There are however two problems:

1. p-based logarithm exists only for $p_k = p^r$, that is power of p. One should somehow modify the definition of the logarithm so that it is defined for all p-adic numbers.

2. Since the probabilities p_k correspond to eigenvalues of density matrix, they in general belong to some algebraic extension of p-adic numbers. Thus the modified logarithm should also exist for any algebraic extension of p-adic numbers.

The definition of the modified p-based logarithm $Log_p(x)$ should satisfy following constraints.

1. If argument is power of p then modified logarithm must be equal to p-based logarithm:

$$Log_p(p^n) = log_p(p^n) \quad .$$

2. Modified logarithm must be additive in order to make negentropy additive for systems having no interactions:

$$Log_p(xy) = Log_p(x) + Log_p(y) \ .$$

These requirements fix the definition of logarithm uniquely. The modified logarithm can depend on the p-adic norm of the argument only. Or in terms of canonical identification

$$I : \sum x_n p^n \rightarrow \sum x_n p^{-n} \ ,$$

mapping p-adics to reals and p-adic norm $N_p(x)$ one must have

$$\begin{aligned} Log_p(x) &= log_p([x]) \ , \\ [x] &= I^{-1}(N_p(x)) \ , \\ &= \left[\sum_{n\geq n_0} x_n p^n\right] = p^{n_0} \ . \end{aligned} \tag{5.3.4}$$

This definition works also for the algebraic extensions, for which p-adic norm is defined as the p-adic norm for the determinant of the linear map induced by a multiplication with z in algebraic extension: it is easy to see that the determinant of this map is indeed a power of p always (note that this norm is multiplicative, which implies the additivity of modified logarithm and entropy).

For the algebraic extensions of p-adic numbers one must define how the units e_k of algebraic extension $z = x + \sum_k y^k e_k$ are mapped to the reals in the canonical identification map. e_k are typically roots of integers in the range $-1, ..., p$. The rule is following: if e_k is not a root of p then it is mapped to e_k interpreted as a real number: for instance, $2^{1/3}$ is mapped to $2^{1/3}$ for $p \neq 2$ in case that $2^{1/3}$ does not exist as p-adic number. If e_k is root of p it is mapped to its inverse: for instance, $\sqrt{p}$ is mapped to $\frac{1}{\sqrt{p}}$.

Note that p-adic entanglement entropy can be also expressed as a sum over the derivatives of the p-adic entanglement probabilities with respect to p:

$$S = \sum_i \frac{d}{dp} p_i \ . \tag{5.3.5}$$

The real counterpart of the p-adic entanglement entropy is obtained by canonical identification $x = \sum x_n p^n \rightarrow \sum x_n p^{-n} = x_R$

$$S_r = S_R \times log(p) \ . \tag{5.3.6}$$

$log(p)$ factor must be included in order to make possible the comparison of entropies associated with different values of p.

The value of the p-adic entanglement entropy is always non-negative. It vanishes if the p-adic entanglement entropies have unit p-adic norm. Thus $S = 0$ p-adic entanglement is possible. This entanglement need not be stable since a direct sum of eigen spaces of density matrix with finitely extended rational entanglement probabilities has negative entanglement entropy.

Unless some p-adic probabilities do not have p-adic norm larger than one, p-adic entanglement entropy is of order $O(p)$ for genuinely p-adic systems so that negentropy gain is below $log(p)$ irrespective of the size of the system. This situation is realized in p-adic thermodynamics. There is a nice connection with p-adic mass calculations: p-adic thermal mass squared expectation value is essentially the p-adic entropy. This connection was noticed already [L9] [K60] and it was suggested that p-adic primes associated with elementary particles could correspond to entropy maxima as function of p. This connection suggests that the proper definition of p-adic entropy is based on the canonical identification.

Remark: Statistics does not give rise to entanglement entropy as one might erratically conclude by considering the symbolic representation of tensor product suggesting the identification of 'left' and 'right' members of the tensor product as subsystems A and B: the concrete representation of the states using oscillator operators associated with Y^3 and its complement shows that there is no statistical entanglement entropy between the subsystem and its complement: if this were the case the entire universe should behave like a single conscious being and this would be a catastrophe as far as NMP is considered.

Systems with finitely extended rational entanglement

In the case of an finitely extended rational entanglement one can map the p-adic entropy to its real counterpart using the identification by common rationals instead of the canonical identification. This gives the formula

$$\begin{aligned} S_R &= S_p log(p) \ , \\ S_p &= \sum_n p_k Log_p(p_k) log(p) \ , \\ Log_p(x) &= log_p(|x|_p) \ . \end{aligned} \tag{5.3.7}$$

where the p-adic entropy which can be regarded as a rational number is re-interpreted as a real number. Note that the probabilities p_k are positive numbers. What is remarkable is that in this case entanglement entropy can be a negative rational number or a number in a finite extension of rational numbers. This observation encourages the definition of the number theoretic entanglement negentropy as maximum information in the set of all p-adic number fields and their extensions:

$$I \equiv Max\{-S_p, \ p \ \text{prime}\} \ . \tag{5.3.8}$$

Since the numbers $log(p)$ are independent transcendentals there exists a unique prime for which the maximum is achieved.

The original identification of negentropic entanglement as bound state entanglement is unnecessary and the observation that negentropic entanglement is possible withing binding energy might have far reaching consequences concerning the understanding of metabolism and stability of fundamental bio-polymers.

The consistency with the standard quantum measurement theory requires that the process corresponds to a measurement of the density matrix so that a projection must occur to an eigen space or sub-space of eigen space of the density matrix if this maximizes negentropy gain. The density matrix of the system would become

$$\rho \ \rightarrow \ \frac{1}{D_i} P_i \ . \tag{5.3.9}$$

Here D_i and P_i denote the dimension of the eigen space associated with p_i and corresponding projection operator. Assuming that D_i has the decomposition

$$D_i = \prod_{i \in I} q_i^{n_i}$$

to a product of powers of primes, the negentropy of the final state can be written as

$$N_R = Max\{n_i log(q_i) | i \in I\} \ . \tag{5.3.10}$$

The maximization of the increment of entanglement entropy gives a criterion selecting the final eigen space or its sub-space. Quantum classical correspondence suggests that one can assign similar inherent negentropy to the space-time sheet consisting of D strictly deterministic regions.

For the negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) the state function reduction process is far from being random. It is quite possible that the reduction takes to unique final state for which the common denominator of entanglement probabilities is power of prime. This is achieved if the reduction occurs to a sub-space for which the denominator measuring roughly the number of states is reduced to a number having very large p-adic norm for some prime. This suggests that the quantum behavior of negentropic states resembles more that of cellular automata than of ordinary quantum states.

The eigen spaces of the density matrix with dimensions $D = p^N$ are of special interest. The entanglement negentropy for $D = p^N n_0$, n_0 integer not divisible by p, is $N_R = Nlog(p)$. The reduction to a sub-space of the eigen space can yield higher negentropy gain than the reduction to the entire eigen space and powers of prime are favored as dimensions of these sub-spaces.

The entanglement negentropy per single dimension of eigen space is $N_R/D = Nlog(p)p^{-N}/n_0$. For $D = p^N$ the entanglement negentropy per dimension of eigen space is $N_R/D = Nlog(p)/p^N = log(D)/D$ and maximum as a function of n_0. N_R/D as a function of D has a maximum $N_R/D = .3662$ for $D = 3$ rather than $D = 2$ as one might expect. For $D = 2$ and $D = 4$ one has $N_R/D = .3466$ (note that there are 4 DNA nucleotides). For other values of D N_R/D is smaller.

For extended rational entanglement the measurement of the density matrix can occur only in special cases. For instance, when the probabilities p_k belong to a finite extension of rational numbers and are different, the measurement of the density matrix would reduce the negentropy to zero and NMP does not therefore allow the measurement of density matrix to occur. Degenerate eigen spaces do not correspond to the maximum entanglement negentropy per dimension. $p_k = n_k/p^N$, n_k not divisible by p, gives $N_R = Nlog(p)$ irrespective of dimension D, and $N_R/D = Nlog(p)/2$ for $D = 2$ ($p_1 = m/p^N$ and $p_2 = (p^N - m)/p^N$, m not divisible by p) is the best one can achieve. Since there is no upper bound for N nor p even in the case of a 2-state system, the negentropy gain can be arbitrarily high. One could criticize this result as counter intuitive.

5.3.2 Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements and state preparation for next quantum jump is state reduction for the previous quantum jump. In zero energy ontology one can interpret the state preparation for positive and negative energy parts of the state as reduction and preparation in the sense of standard physics. Each self measurement means a decomposition of the sub-system involved to two unentangled parts unless the system is bound state. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

Bound state entanglement is stable against self measurement simply because energy conservation prevents the decay to a pair of free (uncorrelated) subsystems. The generalized definition of entanglement entropy allows to assign a negative value of entanglement entropy to rational and algebraic entanglement, so that this kind of entanglement would actually carry information, in fact conscious information (experience of understanding). This kind of entanglement cannot be reduced in state function reduction. Macro-temporal quantum coherence could correspond to a generation of either bound state entanglement or negentropic entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images. Generation of negentropic entanglement would involve experience about expansion of consciousness and that of bound states entanglement a loss of consciousness.

The mathematical models for quantum computers typically operate with systems for which entanglement probabilities are identical. Also rational numbers are involved. Does this mean that negentropic entanglement makes possible quantum computation? This does not seem to be the case. State function reduction with random outcomes is a central element of quantum computation which suggests that quantum computation must be based on entropic entanglement with large enough value of $\hbar$ to overcome the restrictions caused by the interactions with the external world. The negentropic entanglement in turn would relate to conscious information processing involving experience of understanding represented by negentropic entanglement. Negentropic entanglement would make possible conscious cellular automaton type information processing much closer to that carried out by ordinary computers and this information processing might be equally important in living systems.

5.3.3 Number theoretic information measures at the space-time level

Quantum classical correspondence suggests that the notion of entropy should have also space-time counterpart. Entropy requires ensemble and both the p-adic non-determinism and the non-determinism of Kähler action allow to define the required ensemble as the ensemble of strictly deterministic regions of the space-time sheet. One can measure various observables at these space-time regions, and the frequencies for the outcomes are rational numbers of form $p_k = n(k)/N$, where N is the number of strictly deterministic regions of the space-time sheet. The number theoretic entropies are well defined and negative if p divides the integer N. Maximum is expected to result for the largest prime power factor of N. This would mean the possibility to assign a unique prime to a given real space-time sheet.

The classical non-determinism resembles p-adic non-determinism in the sense that the space-time sheet obeys effective p-adic topology in some length and time scale range is consistent with this idea since p-adic fractality suggests that N is power of p.

5.3.4 Number theoretical Quantum Mechanics

The vision about life as something in the intersection of the p-adic and real worlds requires a generalization of quantum theory to describe the U-process properly. One must answer several questions. What it means mathematically to be in this intersection? What the leakage between different sectors does mean? Is it really possible to formally extend quantum theory so that direct sums of Hilbert spaces in different number fields make sense? Or should one consider the possibility of using only complex, algebraic, or rational Hilbert spaces also in p-adic sectors so that p-adicization would take place only at the level of geometry?

What it means to be in the intersection of real and p-adic worlds?

The first question is what one really means when one speaks about a partonic 2-surface in the intersection of real and p-adic worlds or in the intersection of two p-adic worlds.

1. Many algebraic numbers can be regarded also as ordinary p-adic numbers: square roots of roughly one half of integers provide a simple example about this. Should one assume that all algebraic numbers representable as ordinary p-adic numbers belong to the intersection of the real and p-adic variants of partonic 2-surface (or to the intersection of two different p-adic number fields)? Is there any hope that the listing of the points in the intersection is possible without a complete knowledge of the number theoretic anatomy of p-adic number fields in this kind of situation? And is the set of common algebraic points for real and p-adic variants of the partonic 2-surface X^2 quite too large- say a dense sub-set of X^2?

   This hopeless looking complexity is simplified considerably if one reduces the considerations to algebraic extensions of rationals since these induce the algebraic extensions of p-adic numbers. For instance, if the p-adic number field contains some n:th roots of integers in the range $(1, p-1)$ as ordinary p-adic numbers they are identified with their real counterparts. In principle one should be able to characterize the -probably infinite-dimensional- algebraic extension of rationals which is representable by a given p-adic number field as p-adic numbers of unit norm. This does not look very practical.

2. At the level WCW one must direct the attention to the function spaces used to define partonic 2-surfaces. That is the spaces of rational functions or even algebraic functions with coefficients of polynomials in algebraic extensions of rational numbers making sense with arguments in all number fields so that algebraic extensions of rationals provide a neat hierarchy defining also the points of partonic 2-surfaces to be considered. If one considers only the algebraic points of X^2 belonging to the extension appearing in the definition the function space as common to various number fields one has good hopes that the number of common points is finite.

3. Already the ratios of polynomials with rational coefficients lead to algebraic extensions of rationals via their roots. One can replace the coefficients of polynomials with numbers in algebraic extensions of rationals. Also algebraic functions involving roots of rational functions can be considered and force to introduce the algebraic extensions of p-adic numbers.

For instance, an n:th root of a polynomial with rational coefficients is well defined if n:th roots of p-adic integers in the range $(1, p-1)$ are well well-defined. One clearly obtains an infinite hierarchy of function spaces. This would give rise to a natural hierarchy in which one introduces n:th roots for a minimum number of p-adic integers in the range $(1, p-1)$ in the range $1 \leq n \leq N$. Note that also the roots of unity would be introduced in a natural manner.

The situation is made more complex because the partonic 2-surface is in general defined by the vanishing of six rational functions so that algebraic extensions are needed. An exception occurs when six preferred imbedding space coordinates are expressible as rational functions of the remaining two preferred coordinates. In this case the number of common rational points consists of all rational points associated with the remaining two coordinates. This situation is clearly non-generic. Usually the number of common points is much smaller (the set of rational points satisfying $x^n + y^n = z^n$ for $n > 2$ is a good example). This however suggests that these surfaces are of special importance since the naive expectation is that the amplitude for transformation of intention to action or its reversal is especially large in this case. This might also explain why these surfaces are easy to understand mathematically.

4. These considerations suggest that the numbers common to reals and p-adics must be defined as rationals and algebraic numbers appearing explicitly in the algebraic extension or rationals associated with the function spaces used to define partonic 2-surfaces. This would make the deduction of the common points of partonic 2-surface a task possible at least in principle. Algebraic extensions of rationals rather than those of p-adic numbers would be in the fundamental role and induce the extensions of p-adic numbers.

Let us next try to summarize the geometrical picture at the level of WCW and WCW spinor fields.

1. WCW decomposes into WCWs associated with CDs and there unions. For the unions one has Cartesian product of WCWs associated with CDs. At the level of WCW spinor fields one has tensor product.

2. The WCW for a given CD decomposes into a union of sectors corresponding to various number fields and their algebraic extensions. The sub-WCW corresponding to the intersection consists of partonic 2-surfaces X^2 (plus distribution of 4-D tangent spaces $T(X^4)$ at X^2 - a complication which will not be considered in the sequel), whose mathematical representation makes sense in real number field and in some algebraic extensions of p-adic number fields. The extension of p-adic number fields needed for algebraic extension of rationals depends on p and is in general sub-extension of the extension of rationals. This sub-WCW is a sub-manifold of WCW itself. It has also a filtering by sub-manifolds of QCW. For instance, partonic 2-surfaces representable using ratios of polynomials with degree below fixed number N defines an inclusion hierarchy with levels labelled by N.

3. The spaces of WCW spinors associated with these sectors are dictated by the second quantization of induced spinor fields with dynamics dictated by the modified Dirac action in more or less one-one correspondence. The dimension for the modes of induced spinor field (solutions of the modified Dirac equation at the space-time surface holographically assigned with X^2 plus the 4-D tangent space-space distribution) in general depends on the partonic 2-surface and the classical criticality of space-time surface suggests an inclusion hierarchy of super-conformal algebras corresponding to a hierarchy of criticalities. For instance, the partonic 2-surfaces X^2 having polynomial representations in referred coordinates could correspond to simplest possible surfaces nearest to the vacuum extremals and having in a well define sense smallest (but possibly infinite) dimension for the space of spinor modes.

4. For each CD one can decompose the Hilbert space to a formal direct sum of orthogonal state spaces associated with various number fields

$$H = \oplus_F H_F \ . \tag{5.3.11}$$

Here F serves as a label for number fields. For the sake of simplicity and to get idea about what is involved, all complications due to algebraic extensions are neglected in the sequel so that only rational surfaces are regarded as being common to various sectors of WCW.

5. The states in the direct sum make sense only formally since the formal inner product of these states would be a sum of numbers in different number fields unless one assigns complex Hilbert space with each sector or restricts the coefficients to be rational which is of course also possible. This problem is avoided if the state function reduction process induces inside each CD a choice of the number field. One could say that state function is a number theoretical necessity at least in this sense.

   (a) Should the state function reduction in this sense involve a reduction of entanglement between distinct CDs is not clear. One could indeed consider the possibility of a purely number theoretical reduction not induced by NMP and taking place in the absence of entanglement with reduction probabilities determined by the probabilities assignable to various number fields which should be rational or at most algebraic. Hard experience however suggests that one should not make exceptions from principles.

   (b) The alternative is to allow the Hilbert spaces in question to have rational or at most algebraic coefficients in the intersection of real and various p-adic worlds. This means that the entanglement is algebraic and NMP need not lead to a pure state: the superposition of pairs of entangled states is however mathematically well defined since inner products give algebraic numbers. Cognitive entanglement stable under NMP would become possible. The experience of understanding could be a correlate for it. The pairs in the sum defining the entangled state defined the instances of a concept as a mapping of real world state to its symbol structurally analogous to a Boolean rule. The entangled states between different p-adic number fields would define maps between symbolic representations.

6. Assume that each H_F allows a decomposition to a direct sum of two orthogonal parts corresponding to WCW spinor fields localized to the intersection of number fields and to the complements of the intersection:

$$
\begin{aligned}
H &= H_{nm} \oplus H_m \ , \\
H_{nm} &= \oplus_F H_{nm,F} \ , \ H_m = \oplus_F H_{m,F} \ .
\end{aligned}
\tag{5.3.12}
$$

   Here nm stands for 'no mixing' (no mixing between different number fields and localization to the complement of the intersection) and m for 'mixing' (mixing between different number fields in the intersection). F labels the number fields. Orthogonal direct sum might be mathematically rather singular and un-necessarily strong assumption but the notion of number theoretical criticality favors it.

The general structure of U-matrix neglecting the complexities due to algebraic extensions

M-matrix is diagonal with respect to the number field for obvious reasons. U-matrix can however induce a leakage between different number fields as well as entanglement between different number fields when unions of CDs are considered. The simplest assumption is that this entanglement is induced by the leakage between different number fields for single CD but not directly. For instance, the members of entangled pair of real states associated with two CDs leak to various p-adic sectors and induce in this manner entanglement beween different number fields. One must however notice that the part of U-matrix acting in the tensor product of Hilbert spaces assignable to separate CDs must be considered separately: it seems that the entanglement inducing part of U is diagonal with respect to number field except in the intersection.

To simplify the rather complex situation consider first the U matrix for a given CD by neglecting the possibility of algebraic extensions of the p-adic number fields. Restrict also the consideration to single CD.

1. The unitarity conditions do not make sense in a completely general sense since one cannot add numbers belonging to different number fields. The problem can be circumvented if the U-matrix decomposes into a product of U-matrices, which both are such that unitarity conditions make sense for them. Here an essential assumption is that unit matrix and projection operators are number theoretically universal. In this spirit assume that for a given CD U decomposes to a product of two U-matrices U_{nm} inducing no mixing between different number fields and U_m inducing the mixing in the intersection:

$$U = U_{nm}U_m \ . \tag{5.3.13}$$

Here the subscript 'nm' (no mixing) having nothing to do with the induces of U as a matrix means that the action is restricted to a dispersion in a sector of WCW characterized by particular number field. The subscript 'm' (mixing) in turn means that the action corresponds to a leakage between different number fields possible in the intersection of worlds corresponding to different number fields and that U_m acts non-trivially in this intersection.

2. Assume that U_{nm} decomposes into a formal direct sum of U-matrices associated with various number fields F:

$$U_{nm} = \oplus_F U_{nm,F} \ . \tag{5.3.14}$$

$U_{nm,F}$ acts inside H_F in both WCW and spin degrees of freedom, does not mix states belonging to different number fields, and creates a state which is always mathematically completely well defined in particular number field although the direct sum over number fields is only formally defined. Unitarity condition gives a direct sum of projection operators to Hilbert spaces associated with various number fields. One can assume that this object is number theoretically universal.

3. U_m acts in the intersection of the real and p-adic worlds identified in the simplified picture in terms of surfaces representable using ratios of polynomials with rational coefficients. The resulting superposition of WCW spinor fields in different number fields is as such not mathematical sensible although the expression of U_m is mathematically well-defined. If the leakage takes place with same probability amplitude irrespective of the quantum state, U_m is a unitary operator, not affecting at all the spinor indices of WCW spinor fields characterizing quantum numbers of the state and whose action is analogous to unitary mixing of the identical copies of the state in various number fields.

   The probability with which the intention is realized as action would not therefore depend at all on the quantum number fields, but only on the data at points common to the variants of the partonic 2-surface in various number fields. Intention would reduce completely to the algebraic geometry of partonic 2-surfaces. This assumption allows to write U in the form

$$U = U_{nm}U_m \ , \tag{5.3.15}$$

where U_m acts as an identity operator in H_{nm}.

The general structure of U-matrix when algebraic extensions of rationals are allowed

Consider now the generalization of the previous argument allowing also algebraic extensions.

1. For each algebraic extension of rationals one can express WCW as a union of two parts. The first one corresponds to to 2-surfaces, which belong to the intersection of real and p-adic worlds. The second one corresponds to 2-surfaces in the algebraic extension of genuine p-adic numbers and having necessarily infinite size in real sense. Therefore the decomposition of U to a product $U = U_{nm}U_m$ makes sense also now.

2. It is natural to assume that U_m decomposes to a product of two operators: $U_m = U_H U_Q$. The strictly horizontal operator U_H connects only same algebraic extensions of rationals assigned to different number fields. Here one must think that p-adic number fields represent a large number of algebraic extensions of rationals without need for an algebraic extension in the p-adic sense. The second unitary operator U_Q describes the leakage between different algebraic extensions of rationals. Number theoretical universality encourages the assumption that this unitary operator reduces to an operator U_Q acting on algebraic extensions of rationals regarded effectively as quantum states so that it would be same for all number fields. One can even consider the possibility that U_Q depends on the extensions of rationals only and not at all on partonic 2-surfaces. One cannot assume that U_Q corresponds just to an inclusion to a larger state space since this would give an infinite number of identical copies of same state and imply a non-normalizable state. Physically U_Q would define dispersion in the space of algebraic extension of rationals defining the rational function space giving rise to the sub-WCW. The simplest possibility is that U_Q between different algebraic extensions is just the projection operator to their intersection multiplied by a numerical constant determined number theoretical in terms of ratios of dimensions of the algebraic extensions so that the diffusion between extensions products unit norm states.

One must take into account the consistency conditions from the web of inclusions for the algebraic extensions of rationals inducing extensions of p-adic numbers.

1. There is an infinite inverted pyramide-like web of natural inclusions of WCWs associated with algebraic extensions of rational numbers and one can assign a copy of this web to all number fields if a given p-adic number field is characterized by a web defined by algebraic extensions of rationals numbers, which it is able to represent without explicit introduction of the algebraic extension, so that the pyramide is same for all number fields. For instance, the WCW corresponding to p-adic numbers proper is included to the WCWs associated with any of its genuine algebraic extensions and defines the lower tip of the inverted pyramide. From this tip an arrow emerges connecting it to every algebraic extension defining a node of this web. Besides these arrows there are arrows from a given extension to all extensions containing it.

2. These geometric inclusions induce inclusions of the corresponding Hilbert spaces defined by rational functions and possibly by algebraic functions in which case sub-web must be considered (all n:th roots of integers in the range $(1, p{-}1)$ must be introduced simultaneously). Leakage can occur between different extensions only through WCW spinor fields located in the common intersection of these spaces containing always the rational surfaces. The intersections of WCWs associated with various extensions of p-adic number fields correspond to WCWs assignable to rational functions with coefficients in various algebraic extensions of rationals using preferred coordinates of CD and CP_2.

Together with unitarity conditions this web poses strong constraints on the unitary matrices U_m and U_Q expressible conveniently in terms of commuting diagrams. There are two kinds of webs. The vertical webs are defined by the algebraic extensions of rationals. These form a larger web in which lines connect the nodes of identical webs associated with various p-adic number fields and represent algebraic extensions of rationals.

1. One has the general product decomposition $U = U_{nm} U_Q U_m$, where U_{nm} does not induce mixing between number fields, and U_m does it purely horizontally but without affecting quantum states in WCW spin degrees of freedom, and $P(H_{nm})$ projects to the complement of the intersection of number fields holds true also now.

2. Each algebraic extension of rationals gives unitary conditions for the corresponding $U_{nm,F}$ for each p-adic number field with extensions included. These conditions are relatively simple and no commuting diagrams are needed.

3. In the horizontal web U_m mixes the states in the intersections of two number fields but connects only same algebraic extensions so that the lines are strictly horizontal. U_Q acts strictly vertically in the web formed by algebraic extension of rationals and its action is

unitary. One has infinite number of commuting diagrams involving U_m and U_Q since the actions along all routes connecting given points between p_1 and p_2 must be identical.

4. If algebraic universality holds in the sense that U_m is expressible using only the data about the common points of 2-surfaces in the intersection defined by particular extensions using some universal functions, and U_Q is purely number theoretical unitary matrix having no dependence on partonic 2-surfaces, one can hope that the constraints due to commuting diagrams in the web of horizontal inclusions can be satisfied automatically and only the unitarity constraints remain. This web of inclusions brings strongly in mind the web of inclusions of hyper-finite factors.

5.4 Anatomy of quantum jump in zero energy ontology

Consider now the anatomy of quantum jump identified as a moment of consciousness in the framework of ZEO [K51].

1. Quantum jump begins with unitary process U described by unitary matrix assigning to a given zero energy state a quantum superposition of zero energy states. This would represent the creative aspect of quantum jump - generation of superposition of alternatives.

2. The next step is a cascade of state function reductions proceeding from long to short scales. It starts from some CD and proceeds downwards to sub-CDs to their sub-CDs to ...... At a given step it induces a measurement of the quantum numbers of either positive or negative energy part of the quantum state. This step would represent the measurement aspect of quantum jump - selection among alternatives.

3. The basic variational principle is Negentropy Maximization Principle (NMP) [K51] stating that the reduction of entanglement entropy in given quantum jump between two subsystems of CD assigned to sub-CDs is maximal. Mathematically NMP is very similar to the second law although states just the opposite but for individual quantum system rather than ensemble. NMP actually implies second law at the level of ensembles as a trivial consequence of the fact that the outcome of quantum jump is not deterministic.

   For ordinary definition of entanglement entropy this leads to a pure state resulting in the measurement of the density matrix assignable to the pair of CDs. For hyper-finite factors of type II_1 (HFFs) state function reduction cannot give rise to a pure state and in this case one can speak about quantum states defined modulo finite measurement resolution and the notion of quantum spinor emerges naturally. One can assign a number theoretic entanglement entropy to entanglement characterized by rational (or even algebraic) entanglement probabilities and this entropy can be negative. Negentropic entanglement can be stable and even more negentropic entanglement can be generated in the state function reduction cascade.

The irreversibility is realized as a property of zero energy states (for ordinary positive energy ontology it is realized at the level of dynamics) and is necessary in order to obtain non-trivial U-matrix. State function reduction should involve several parts. First of all it should select the density matrix or rather its Hermitian square root. After this choice it should lead to a state which prepared either at the upper or lower boundary of CD but not both since this would be in conflict with the counterpart for the determinism of quantum time evolution.

5.4.1 Generalization of S-matrix

ZEO forces the generalization of S-matrix with a triplet formed by U-matrix, M-matrix, and S-matrix. The basic vision is that quantum theory is at mathematical level a complex square roots of thermodynamics. What happens in quantum jump was already discussed.

1. U-matrix as has its rows M-matrices , which are matrices between positive and negative energy parts of the zero energy state and correspond to the ordinary S-matrix. M-matrix is a product of a hermitian square root - call it H - of density matrix ρ and universal S-matrix S commuting with H: $[S, H] = 0$. There is infinite number of different Hermitian square

roots H_i of density matrices which are assumed to define orthogonal matrices with respect to the inner product defined by the trace: $Tr(H_iH_j) = 0$. Also the columns of U-matrix are orthogonal. One can interpret square roots of the density matrices as a Lie algebra acting as symmetries of the S-matrix.

2. One can consider generalization of M-matrices so that they would be analogous to the elements of Kac-Moody algebra. These M-matrices would involve all powers of S.

   (a) The orthogonality with respect to the inner product defined by $\langle A|B\rangle = Tr(AB)$ requires the conditions $Tr(H_1H_2S^n) = 0$ for $n \neq 0$ and H_i are Hermitian matrices appearing as square root of density matrix. H_1H_2 is hermitian if the commutator $[H_1, H_2]$ vanishes. It would be natural to assign n:th power of S to the CD for which the scale is n times the CP_2 scale.

   (b) Trace - possibly quantum trace for hyper-finite factors of type II_1) is the analog of integration and the formula would be a non-commutative analog of the identity $\int_{S^1} exp(in\phi)d\phi = 0$ and pose an additional condition to the algebra of M-matrices. Since $H = H_1H_2$ commutes with S-matrix the trace can be expressed as sum $\sum_{i,j} h_is_j(i) = \sum_{i,j} h_i(j)s_j$ of products of correspondence eigenvalues and the simplest condition is that one has either $\sum_j s_j(i) = 0$ for each i or $\sum_i h_i(j) = 0$ for each j.

   (c) It might be that one must restrict M-matrices to a Cartan algebra for a given U-matrix and also this choice would be a process analogous to state function reduction. Since density matrix becomes an observable in TGD Universe, this choice could be seen as a direct counterpart for the choice of a maximal number of commuting observables which would be now hermitian square roots of density matrices. Therefore ZEO gives good hopes of reducing basic quantum measurement theory to infinite-dimensional Lie-algebra.

5.4.2 A concise description of quantum jump

In the following a minimalistic view about quantum jump is described. Both U-process and state preparation reduce to state function reductions to two basis for zero energy states characterized by opposite arrows of geometric time.

Unitary process and choice of the density matrix

The basic question concerning U process is which of the following two options U-process corresponds to.

1. U-process occurs for zero energy states. U-matrix would be defined in the space of zero energy states and would represent kind of higher order scattering whereas M-matrix and S-matrix as time-like entanglement coefficients would describe what happens in a scattering experiment. This kind of possibility can be certainly considered since one can form zero energy states using zero energy states as building bricks. Entire hierarchy of zero energy states could be constructed in this manner.

2. U-process can be said to occur for either positive or negative energy parts of zero energy states. This option is definitely minimal and in this case U-process for positive (negative) energy part of the state is dual to state function reduction for the negative (positive) energy part of the state. Furthermore, state function reduction is dual to state preparation. For this reason this option deserves to be called minimalistic.

During years I have considered both options without clearly distinguishing between them. The notion of time is very difficult concept: we do not have brain for time. Below I will consider only the minimalistic option in the hope that Nature would prefer minimalism also at this time. There is no need to emphasize how speculative these considerations are.

Consider first unitary process followed by the choice of the density matrix for the minimalistic option.

1. There are two natural state basis for zero energy states. The states of these state basis are prepared at the upper or lower boundary of CD respectively and correspond to various M-matrices M_K^+ and M_L^-. U-process is simply a change of state basis meaning a representation of the zero energy state $M_K^{\pm}$ in zero energy basis $M_K^{\mp}$ followed by a state preparation to zero energy state $M_K^{\pm}$ with the state at second end fixed in turn followed by a reduction to $M_L^{\mp}$ to its time reverse, which is of same type as the initial zero energy state.

   The state function reduction to a given M-matrix $M_K^{\pm}$ produces a state for the state is superposition of states which are prepared at either lower or upper boundary of CD. It does not yet produce a prepared state on the ordinary sense since it only selects the density matrix.

2. The matrix elements of U-matrix are obtained by acting with the representation of identity matrix in the space of zero energy states as

$$I = \sum_K |K^+\rangle\langle K^+|$$

   on the zero energy state $|K^-\rangle$ (the action on $|K^+\rangle$ is trivial!) and gives

$$U^+_{KL} = Tr(M_K^+ M_L^+) \ .$$

   In the similar manner one has

$$U^-_{KL} = (U^{+\dagger})_{KL} = Tr(M_L^- M_K^-) = \overline{U^+_{LK}} \ .$$

   These matrices are Hermitian conjugates of each other as matrices between states labelled by positive or negative energy states. The interpretation is that two unitary processes are possible and are time reversals of each other. The unitary process produces a new state only if its time arrow is different from that for the initial state. The probabilities for transitions $|K_+\rangle \to |K_-\rangle$ are given by $p_{mn} = |Tr(M_K^+ M_L^+)|^2$.

State function preparation

Consider next the counterparts of the ordinary state preparation process.

1. The ordinary state function process can act either at the upper or lower boundary of CD and its action is thus on positive or negative energy part of the zero energy state. At the lower boundary of CD this process selects one particular prepared states. At the upper boundary it selects one particular final state of the scattering process.

2. Restrict for definiteness the consideration to the lower boundary of CD. Denote also M_K by M. At the lower boundary of CD the selection of prepared state - that is preparation process- means the reduction

$$\sum_{m^+n^-} M^{\pm}_{m^+n^-} |m^+\rangle|n^-\rangle \to \sum_{n^-} M^{\pm}_{m^+n^-} |m^+\rangle|n^-\rangle \ .$$

   The reduction probability is given by

$$p_m = \sum_{n^-} |M_{m^+n^-}|^2 = \rho_{m^+m^+} \ .$$

   For this state the lower boundary carries a prepared state with the quantum numbers of state $|m_+\rangle$. For density matrix which is unit matrix (this option giving pure state might not be possible) one has $p_m = 1$.

State function reduction process

The process which is the analog of measuring the final state of the scattering process is also needed and would mean state function reduction at the upper end of CD - to state $|n^-\rangle$ now.

1. It is impossible to reduce to arbitrary state $|m_+\rangle|n_-\rangle$ and the reduction must at the upper end of CD must mean a loss of preparation at the lower end of CD so that one would have kind of time flip-flop!

2. The reduction probability for the process

$$|m_+ \equiv \sum_{n^-} M_{m^+n^-}|m^+\rangle|n^-\rangle \to n_- = \sum_{m^+} M_{m^+n^-}|m^+\rangle|n^-\rangle$$

would be

$$p_{mn} = |M_{mn}^2| \ .$$

This is just what one would expect. The final outcome would be therefore a state of type $|n^-\rangle$ and - this is very important- of the same type as the state from which the process began so that the next process is also of type U^+ and one can say that a definite arrow of time prevails.

3. Both the preparation and reduction process involves also a cascade of state function reductions leading to a choice of state basis corresponding to eigenstates of density matrices between subsystems.

5.4.3 Questions and answers

Answering to question is the best possible manner to develop ideas in more comprehensible form. In this respect the questions of Hamed at my blog have been especially useful. Many questions below are originally made by him and inspired the objections, many of them discussed also in previous discussions. The answers to these questions have changed during latest years as the views about self and the relation between experienced time and geometric time have developed. The following answers are the most recent ones.

Question: The minimalistic option suggests very strongly that our sensory perception can be identified as quantum measurement assignable to state function reductions for upper or lower boundaries of our personal CD. Our sensory perception does not however jump between future and past boundaries of our personal CD (containing sub-CDS in turn containing)! Why?

Possible answer: The answer to this question comes from the realization that in ordinary quantum theory state function reductions leaving the reduced state invariant are possible. This must have counterpart in ZEO. In ZEO reduces zero energy states are superpositions of zero energy states associated with CDs with second boundary fixes inside light-cone boundary and the position of the second boundary of CD varying: one can speak about wave function in the moduli space of CDs. The temporal distance between the tips of CD and discrete lattice of the 3-D hyperbolic space defined by the Lorentz boosts leaving second tip invariant corresponds to the basic moduli.

The repeated state function reductions leave both the fixed boundary and parts of zero energy states associated with this boundary invariant. They however induce dispersion in the moduli space and the average temporal distance between the tips of CDs increases. This gives rise to the flow of psychological time and to the arrow of time. Self as counterpart of observer can be identified as a sequence of quantum jumps leaving the fixed boundary of CD invariant. Sensory perception gives information about varying boundary and the fixed boundary creates the experience about self as invariant not changed during quantum jumps.

Self hierarchy corresponds to the hierarchy of CDs. For instance, we perceive from day to day the - say- positive energy part of a state assignable to this very big CD. Hence the world looks rather stable.

Question: This suggests that our sensory perception actually corresponds to sequences of state function reductions to the two fixed boundaries of CDs of superposition of CDs so that our sensory inputs would alternately be about upper and lower boundaries of personal CDs. Sleep-awake cycle could correspond to a flip flop in which self falls asleep at boundary and wakes up at opposite boundary. Doesn't this lead to problems with the arrow of time?

Possible answer: If we measure time relative to the fixed boundary then the geometric time defined as the average distance between tips in superposition of CDs would increase steadily and we get older also during sleep. Hence we would experience subjective time to increase. Larger CDs than our personal CD for which the arrow of time remains fixed in the time scale of life cycle would provide the objective measure of geometric time.

Question: What is the time scale assignable to my personal CD: the typical wake-up cycle: 24 hours? Or of the order of life span. Or perhaps shorter?

Possible answer: The durations of wake-up periods for self is determined by NMP: death means that NMP favors the next state function to take place at the opposite boundary. The first naive guess is that the duration of the wake up period is of the same order of magnitude as the geometric time scale of our personal CD. In wake-up state we we would be performing state function reduction repeatedly to say "lower" boundary of our personal CD and sensory mental images as sub-CDs would be concentrated near opposite boundary. During sleep same would happen at lower boundary of CD and sensory mental images would be at opposite boundary (dreams,).

Question: Are dreams sensory perceptions with opposite arrow of time or is some sub-self in wake-up state and experiences same arrow of time as we during wake-up state? If the arrow is different in dreams, is the "now" of dreams in past and "past" in the recent of wake-upe state

Possible answer: Here I can suggest an answer based on my own subjective experiences and it would be cautious "yes".

Question: Why we do remember practically nothing about sensory perceptions during sleep period? (Note that we forget actively dream experiences).

Possible answer: That we do not have many memories about sleep and dream time existence and that these memories are unstable should relate to the change of the arrow of personal time as we wake up. Wake-up state should somehow rapidly destroy the ability to recall memories about dreams and sleep state. Wake-up memory recall means communications to geometric past, that is to the boundary of CD which remains fixed during wake-up state. In memory recall for dreams in wake-up state these communications should take place to geometric future. Memory recall of dreams would be seeing to future and much more difficult since the future is changing in each state function reduction so that dream memories are erased automatically during wake-up.

Question: Does the return to childhood at old age relate with this time flip-flop of arrow of time in the scale of life span: do we re-incarnate in biologically death at opposite end of CD with scale of life span?

Possible answer: Maybe this is the case. If this boundary corresponds to time scale of life cycle, the memories would be about childhood. Dreams are often located to the past and childhood.

5.4.4 More about the anatomy of state function reduction

In a comment to previous posting Ulla gave a link to an interesting article by George Svetlichny [J20] describing an attempt to understand free will in terms of quantum measurement. After reading of the article I found myself explaining once again to myself what state function reduction in TGD framework really means.

The proposal of Svetlichny

The basic objection against assigning free will to state function reduction in the sense of wave mechanics is that state function reduction from the point of view of outsider is like playing dice. One can of course argue that for an outsider *any* form of free will looks like throwing a dice since causally effective experience of free will is accompanied by non-determinism. We simply do cannot know what is the experience possibly associated with the state function reduction. The lesson is that we must carefully distinguish between two levels: the single particle level and ensemble level - subjective and objective. When we can say that something is random, we are talking about ensembles, not about single member of ensemble.

The author takes the objection seriously and notices that quantum measurement means a division of system to three parts: measured system, measuring system and external world and argues that in some cases this division might not be unique. The choice of this division would have interpretation as an act of free will. I leave it to the reader can decide whether this proposal is plausible or not.

TGD view about state function reduction

What can one say about the situation in TGD framework? There are several differences as compared to the standard measurement "theory", which is just certain ad hoc rules combined with Born rule, which applies naturally also in TGD framework and which I do not regard as ad hoc in infinite-D context.

In the sequel I will discuss the possible anatomy of the state function reduction part of the quantum jump.

1. TGD ontology differs from the standard one. Space-time surfaces and quantum states as such are zombies in TGD Universe: consciousness is in the quantum jump. Conscious experience is in the change of the state of the brain, brain state as such is not conscious. Self means integration of quantum jumps to higher level quantum jumps and the hierarchy of quantum jumps and hierarchy of selves can be identified in ZEO . It has the hierarchy of CDs and space-time sheets as geometrical correlates. In TGD Universe brain and body are not conscious: rather, conscious experience is about brain and body and this leads to the illusion caused by the assimilation with the target of sensory input: I am what I perceive.

2. In TGD framework one does not assume the division of the system to a product of measured system, measuring system, and external world before the measurement. Rather, this kind of divisions are outcomes of state function reduction which is part of quantum jump involving also the unitary process. Note that standard measurement theory is not able to say anything about the dynamics giving rise to this kind of divisions.

3. State function reduction cascade as a part of quantum jump - this holistic view is one new element - proceeds in zero energy ontology (ZEO) from long to short length scales $CD \rightarrow sub-CDs \rightarrow ...$, and stops when Negentropy Maximization Principle (NMP [K51] defining the variational principle of consciousness is also something new) does not allow to reduce entanglement entropy for any subsystem pair of subsystem un-entangled with the external world. This is the case if the sub-system in question is such that all divisions to two parts are negentropically entangled or form an entangled bound state.

   An interesting possibility is that negentropic entanglement does not correspond to bound state entanglement. The negentropically entangled particles would remain correlated by NMP rather than being in the jail defined by the interaction potential. I have proposed that this analog of love marriage could be fundamental for understanding living matter and that high energy phosphate bond central for ADP-ATP process could involve negentropic entanglement [K44].

   For a given subsystem occurring in the cascade the splitting into an unentangled pair of measured and measuring system can take place if the entanglement between these subsystems is entropic. The splitting takes place for a pair with largest entanglement entropy and defines measuring and measured system.

   Who measures whom? This seems to be a matter of taste and one should not talk about measuring system as conscious entity in TGD Universe, where consciousness is in quantum jump.

4. The factorization of integer to primes is a rather precise number theoretical analogy for what happens, and the analogy might actually have a deeper mathematical meaning since Hilbert spaces with prime dimension cannot be decomposed into tensor products. Any factorization

of integer to a product of primes corresponds to a cascade of state function reductions. At the first step division takes place to two integers and several alternative divisions are possible. The pair for which the reduction of entanglement entropy is largest, is preferred. The resulting two integers can be further factorized to two integers, and the process continues and eventually stops when all factors are primes and no further factorization is possible.

One could even assign to any decomposition $n = rs$ the analogs of entanglement probabilities as $p_1 = log(r)/log(n)$ and $p_2 = log(s)/log(n)$. NMP would favor the divisions to factors r and s which are as near as possible to $n/2$.

Negentropically entangled system is like prime. Note however that these systems can still make an analog of state function reduction which does not split them but increases the negentropy for all splittings of system to two parts. This would be possible only in the intersection of real and p-adic worlds, that is for living matter. My cautious proposal is that just this kind of systems - living systems - can experience free will: either in the analog of state function reduction process increasing their negentropy or in state function process reducing their entanglement with environment.

5. In standard measurement theory observer chooses the measured observables and the theory says nothing about this process. In TGD the measured observable is the density matrix for a pair formed by any two entangled parts of sub-system division for which negentropy gain is maximal in quantum measurement defines the pair. Therefore both the measurement axis and the pair representing the target of measurement and measurer are selected in quantum jump.

6. Quantum measurement theory assumes that measurement correlates classical long range degrees of freedom with quantal degrees of freedom. One could say that the direction of the pointer of the measurement apparatus correlates faithfully with the value of the measured microscopic observable. This requires that the entanglement is reduced between microscopic and macroscopic systems .

   I have identified the "classical" degrees of freedom in TGD framework as zero modes which by definition do not contribute to the line-element of WCW although the WCW metric depends on zero modes as external parameters. The induced Kähler field represents an infinite number of zero modes whereas the Hamiltonians of the boundaries of CD define quantum fluctuating degrees of freedom.

   The reduction of the entanglement between zero modes and quantum fluctuating degrees of freedom is an essential part of quantum measurement process. Also state function reductions between microscopic degrees of freedom are predicted to occur and this kind of reductions lead to de-coherence so that one can apply quantum statistical description and derive Boltzmann equations. Also state function reductions between different values of zero modes are possible are possible and one could perhaps assign "telepathic" effects with them.

   The differences with respect to the standard quantum measurement theory are that several kinds of state function reductions are possible and that the division to classical and quantum fluctuating degrees of freedom has a purely geometric meaning in TGD framework.

7. One can even imagine quantum parallel state function reduction cascades. This would make possible quantum parallel dissipation, which would be something new. My original proposal was that in hadronic physics this could make possible a state function reduction cascade proceeding in quark scales while hadronic scales would remain entangled so that one could apply statistical description to quarks as parts of a system, which is quantum coherent in hadronic length scale.

   This looks nice but...! It is a pity that eventually an objection pops up against every idea irrespective how cute it looks like. The p-adic primes associated with light quarks are larger than that associated with hadron so that quarks - or rather, their magnetic bodies are larger than that hadron's magnetic body. This looks strange at first but actually conforms with Uncertainty Principle and the observation that the charge radius of proton is slightly smaller than predicted (see this, [K53]), gives support for this picture. Geometrically the situation might change if quarks are highly relativistic and color magnetic fields of quarks are dipole fields compressed to cigar like shape: Lorentz contraction could reduce the size scale of their magnetic bodies in the direction of their motion. [Note that p-adic length scale hypothesis applies in the rest system of the particle so that Lorentz contraction is in conflict with it]. Situation remains unsettled.

Further questions

There are many other interesting issues about which my understanding could be much better.

1. In ZEO the choice of the quantization axes and would fix the moduli of the causal diamond CD: the preferred time direction defined by the line connecting the tips of CD, the spin quantization axis, etc.. This choice certainly occurs. Does it reduce to the measurement of a density matrix for some decomposition of some subsystem to a pair? Or should one simply assume state function reductions also at this level meaning localization to a sector of WCW corresponding to given CD. This would involve localization in the moduli space of CDs selecting some boost of a CD with fixed quantized proper time distance between it tips, fixed spin directions for positive and negative energy parts of zero energy states defined by light-like geodesics at its light-like boundary. Preferred complex coordinates for CP_2, etc...

2. Zero energy states are characterized by arrow of geometric time in the sense that either positive or negative energy parts of states have well defined particles numbers and single particle numbers but not both. State function reduction is possible only for positive or negative energy part of the state but not both. This should relate very closely to the fact that our sensory percepts defined by state function reductions are mostly about the upper or lower boundary of CD, or to the fact that we do not remember the percepts made from the other boundary during sleeping period.

3. In ZEO also quantum jumps can also lead to generation of new sub-Universes, sub-CDs carrying zero energy states. Quantum jumps can also involve phase transitions changing p-adic space-time sheets to real ones and these could serve as quantum correlates for intentional actions. Also the reverse process changing matter to thoughts is possible. These possibilities are totally unimaginable in the quantum measurement theory for systems describable by wave mechanics.

4. There is also the notion of finite measurement resolution described in terms of inclusions of hyperfinite factors at quantum level and in terms of braids at space-time level.

To summarize, a lot of theory building is needed in order to fuse all new elements to a coherent framework. In this framework standard quantum measurement theory is only a collection of ad hoc rules and can catch only a small part of what really happens. Certainly, standard quantum measurement theory is is far from being enough for the purposes of consciousness theorist.

5.5 Generalization of NMP to the case of hyper-finite type II_1 factors

The intuitive notions about entanglement do not generalize trivially to the context of relativistic quantum field theories as the rigorous algebraic approach of [C40] based on von Neumann algebras demonstrates. von Neumann algebras can be written as direct integrals of basic building blocks referred to as factors [A38] . Factors can be classified to three basic types labelled as type I, II, and III. Factors of type I appear in non-relativistic quantum theory whereas factors of type III_1 in relativistic QFT [C40] . Factors of type II_1 [A52] , believed by von Neumann to be fundamental, appear naturally in TGD framework [K95] .

5.5.1 Factors of type I

The von Neuman factors of type I correspond to the algebras of bounded operators in finite or infinite-dimensional separable Hilbert spaces. In the finite-dimensional case the algebra reduces to the ordinary matrix algebra in the finite-dimensional case and to the algebra of bounded operators of a separable Hilbert space in the infinite-dimensional case. Trace is the ordinary matrix trace. The algebra of projection operators has one-dimensional projectors as basic building blocks (atoms), the notion of pure state is well-defined, and the decomposition of entangled state to a superposition of products of pure states is unique. This case corresponds to the ordinary non-relativistic quantum theory. Ordinary quantum measurement theory and also the theory of quantum computation has been formulated in terms of type I factors. Also the discussion of NMP has been formulated solely in terms of factors of type I.

5.5.2 Factors of type II_1

The so called hyper-finite type II_1 factors, which are especially natural in TGD framework, can be identified in terms of the Clifford algebra of an infinite-dimensional separable Hilbert space such that the unit operator has unit trace. Essentially the fermionic oscillator operator algebra associated with a separable state basis is in question. The theory of hyper-finite type II_1 factors is rich and has direct connections with conformal field theories [A53] , quantum groups [A55] , knot and 3-manifold invariants [A72, A84, A85] , and topological quantum computation [K94] , [K94] .

The origin of hyper-finite factors of type II_1 in TGD

Infinite-dimensional Clifford algebra corresponds in TGD framework to the super-algebra generated by complexified WCW gamma matrices creating WCW spinor s from vacuum spinor which is the counterpart of Fock vacuum [K95] . By super-conformal symmetry also WCW degrees of freedom correspond to a similar factor. For type hyper-finite II_1 factors the trace is by definition finite and normalized such that the unit operator has unit trace. As a consequence, the traces of projection operators have interpretation as probabilities.

Finite-dimensional projectors have vanishing traces so that the notion of pure state must be generalized. The natural generalization is obvious. Generalized pure states correspond to states for which density matrix reduces to a projector with a finite norm. The physical interpretation is that physical measurements are never able to resolve completely the infinite state degeneracy identifiable in TGD framework as spin glass degeneracy basically caused by the vacuum degeneracy implying non-determinism of Kähler action. An equivalent interpretation is in terms of state space resolution, which can never be complete.

In TGD framework the relevant algebra can also involve finite-dimensional type I factors as tensor factors. For instance, the entanglement between different space-time sheets could be of this kind and thus completely reducible whereas the entanglement in configuration space spin and "vibrational" degrees of freedom (essentially fermionic Fock space) would be of type II_1. The finite state-space resolution seems to effectively replace hyper-finite type II_1 factors with finite-dimensional factors of type I.

The new view about quantum measurement theory

This mathematical framework leads to a new kind of quantum measurement theory. The basic assumption is that only a finite number of degrees of freedom can be quantum measured in a given measurement and the rest remain untouched. What is known as Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras allow to realize mathematically this idea [K95] . $\mathcal{N}$ characterizes measurement resolution and quantum measurement reduces the entanglement in the non-commutative quantum space $\mathcal{M}/\mathcal{N}$. The outcome of the quantum measurement would still represented by a unitary S-matrix but in the space characterized by $\mathcal{N}$. It is not possible to end up with a pure state with a finite sequence of quantum measurements.

The measurement of components of quantum spinors does not make sense since it due to the non-commutativity it is not possible to talk about quantum spinor with single non-vanishing component. Therefore the measurements must be thought of as occurring in the state space associated with quantum spinors. The possible consequences of non-commutativity are considered from the point of view of cognition in [K95] by starting from the observation that the moduli squared of quantum spinor components are commuting hermitian operators possessing a universal rational valued spectrum which suggests interpretation in terms of quantum version of fuzzy belief.

The obvious objection is that the replacement of a universal S-matrix coding entire physics with a state dependent unitary entanglement matrix is too heavy a price to be paid for the resolution of the above mentioned paradoxes. Situation could be saved if the S-matrices have fractal structure. The quantum criticality of TGD Universe indeed implies fractality. The possibility of an infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras expresses this fractal character algebraically. Thus one can hope that the S-matrix appearing as entanglement coefficients is more or less universal in the same manner as Mandelbrot fractal looks more or less the same in all length scales and for all resolutions. Whether this kind of universality must be posed as an additional condition on entanglement coefficients or is an automatic consequence of unitarity in type II_1 sense is an open question.

What happens in repeated measurements?

The assumption of the standard quantum measurement theory is that the outcome of state function reduction does not change in further measurements if the combined system consisting of measured system and performer of measurement is isolated. This hypothesis generalizes to the case of hyper-finite type II_1 factors. Suppose that the outcome of a quantum jump represented by a projection operator P. If the combined system is not isolated, P can replaced by an arbitrary projection operator in the next unitary process. If the combined system is isolated, the next unitary process leads to a state in which P is replaced by a state expressible in terms of projection operators P_i projecting to the sub-space defined by P, and one of them is selected in the next state function reduction or state preparation. A never-ending series of quantum jumps forcing the state to a smaller and smaller but always infinite-dimensional corner of the state-space would result in absence of the unitary process regenerating the entanglement. This process could be seen as a counterpart for the process in which state function reduction and state preparation processes propagate from long to short length scales.

The notion of rational entanglement has a natural type II_1 counterpart and corresponds to rational valued traces for the projection operators involved and rational valued coefficients for these projection operators in the expression of the density matrix. The idea about rational entanglement (or algebraic entanglement in algebraic extension of p-adics in question) as bound state entanglement carrying negative entanglement entropy generalizes.

Rational density matrices are in a special role since they can be thought of as being common to the real and p-adic variants of the state space. The information measures based on p-adic norm and allowing negative entanglement entropy make sense also now. The question whether there might be some deeper justification for the stability of the generalized rational (algebraic) entanglement against state function reduction/preparation reducing entanglement negentropy in the context of hyper-finite type II_1 factors, remains to be answered.

Consider a rationally entangled state characterized by projection operators P_i such that the probabilities p_i are rational and remain stable in the unitary process. For factor of type I, a situation in which P_i are replaced by 1-dimensional projectors $Q_i < P_i$ is achieved sooner or later. In the infinite-dimensional case this situation can be approached but never reached.

p-Adic thermodynamics with conformal cutoff and hyper-finite factors of type II_1

For hyper-finite factors of type II_1 the unit matrix has unit trace. Hence real probabilities assignable to finite-dimensional projectors vanish so that the eigenvalues of the density matrix are always infinitely degenerate in the real context. p-Adic probabilities however make sense as finite p-adic numbers even if they vanish as real numbers. This raises the idea that p-adic probabilities are more natural for hyper-finite factors of type II_1 than real ones. Indeed, in p-adic context one could have finite probabilities for even one-dimensional sub-spaces, which would definitely mean an enhanced expressive power of the formalism. Thus hyper-finite factors of II_1 would give the reason why for p-adic thermodynamics [K55] .

The interpretation of p-adic probabilities is of overall importance from the point of view of physics. When probabilities are rational, the number field does not matter. If not, it seems necessary to map the p-adic probabilities to real ones. One can ask whether this mapping should respect probability conservation without normalization by hand. The variants of canonical identification with some additional conditions on probabilities satisfied for instance in p-adic thermodynamics provide a possible manner to perform this map (see [K55]). In [K83, K56] it is found that so called canonical identification seems to provide a tool to achieve this.

Canonical identification in its basic form is defined as $I : \sum_{k=0}^{\infty} \alpha_k p^k \mapsto \sum_{k=0}^{\infty} \alpha_k p^{-k}$.

Canonical identification for rational numbers is defined using the unique representation $q = r/s$ as

$$I(\frac{r}{s}) = \frac{I(r)}{I(s)} \quad . \tag{5.5.1}$$

Canonical identification allows a further generalization to the case of p-adic thermodynamics where Boltzmann weights b_n are fundamental and their sum defines partition function as $Z = \sum_{n=0}^{\infty} g_n b_n$, where g_n is the degeneracy of the state with a given "energy" (or any conserved quantity whose thermal average is fixed). In real thermodynamics Boltzmann weights are given by

$$b(E_n) = g(E_n) exp(-E_n/T) \quad , \tag{5.5.2}$$

where E_n is "energy" and $g(E_n)$ the integer valued degeneracy of states with energy E_n. In p-Adic thermodynamics the partition function would not converges for this form of Boltzmann weights, which are therefore replaced by $b(E_n) = g(E_n) p^{E_n/T}$ and E_n/T is integer valued to guarantee the p-adic existence of the conformal weight. The quantization of E_n/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance in applications of topological geometrodynamics (see [K55, K84]), which is indeed a basic symmetry of the theory [K25] . Thus the mere number theoretical existence fixes the physics to a high degree and indeed leads to the understanding of elementary particle mass scales. For applications to the calculations of elementary particle masses see [K55] .

In p-adic thermodynamics the probabilities would be given by $p_n = b_n/Z$ and N_{max} would be replaced by Z. When b_n are integers it is natural to define the canonical identification as

$$I(p_n) = I(\frac{b_n}{Z}) \equiv \frac{I(b_n)}{I(Z)} \quad . \tag{5.5.3}$$

A physically very powerful additional constraint is that the additivity of probabilities for independent events holds true also for the *real* counterparts of the p-adic probabilities obtained by canonical identification so that one would obtain also a real probability theory without ad hoc normalization of the real images of p-adic probabilities. This condition is satisfied only if the Boltzman weights b_{n_1} and b_{n_2} for any pair (n_1, n_2) are p-adic integers having no common pinary digits so that no "interference" in the sum of the p-adic probabilities occurs.

The selection of a basis for independent events would correspond to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory such that quantum measurement can give only one of these states as an outcome. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product $\sum_n \alpha_n \beta_n p^n$ of integers $x = \alpha_n p^n$ and $y = \beta_n p^n$ defining analogs of wave functions in the space of pinary digits. Or putting it somewhat differently: Boltzmann weights b_n for orthogonal quantum states represent them as orthogonal states in the space of binary digits with orthogonality realized as vanishing of the overlap for non-negative "wave functions". This map puts strong constraints on the probabilities of elementary independent events and is therefore highly interesting from the point of view of physics.

p-Adic thermodynamics satisfies the constraint that p-adic probabilities have no common pinary digits provided the degeneracies satisfy the condition $g(E_n) < p$ (later a somewhat more general conditions is deduced). For p-adic mass calculations (see [K48]) the degeneracies $g(n)$ of states with conformal weight $L_0 = n$ (taking the role of "energy") however increase exponentially so that the condition is not satisfied for very large values of n. Since $g(n)$ increases exponentially (say as

2^{nx}, where x is some parameter), probability conservation requires a cutoff of order $n_{max} \sim log_2(p)$ to the number of terms in the sum defining the partition function. In practice this cutoff has no implications since already the two lowest terms give excellent approximation to the elementary particle masses.

For instance, the value of p is $M_{127} = 2^{127} - 1 \sim 10^{38}$ in the case of electron so that higher terms in partition function Z are extremely small. The physical interpretation for the cutoff n_{max} would be in terms of p-adic length scale hypothesis (see [K83, K56] stating that the length scales $L_p \propto \sqrt{p}$ with primes $p \simeq 2^k$, k prime, are physically favored and the exponentially smaller p-adic length scale $L_k \propto \sqrt{k}$ defines the size scale of the elementary particle [K48] .

For the ordinary thermodynamics of strings the exponential increase gives rise to Hagedorn temperature T_H as the maximal temperature possible for strings (see [B8]). The interpretation is that the heat capacity of system grows without bound since the number of excited degrees of freedom increases without bound as T_H is approached. Clearly Hagedorn temperature is somewhat analogous to the pinary cutoff in p-adic thermodynamics.

The interpretation of the conformal cutoff in terms of factors of type II_1 factor would be that all conformal weights $n > n_{cr}$ correspond to the same p-adic probability so that it is not possible to distinguish experimentally between these states. This interpretation fits nicely with the notions of resolution and monitoring.

5.5.3 Factors of type III

For algebras of type III associated with non-separable Hilbert spaces all projectors have infinite trace so that the very notion of trace becomes obsolete. The factors of type III_1 are associated with quantum field theories in Minkowski space.

The highly counter-intuitive features of entanglement for type III factors are discussed in [C40] .

1. The von Neumann algebra defined by the observables restricted to an arbitrary small region of Minkowski space in principle generates the whole algebra. Expressed in a more technical jargon, any field state with a bound energy is cyclic for each local algebra of observables so that the field could be obtained in entire space-time from measurements in an arbitrary small region of space-time. This kind of quantum holography looks too strong an idealization.

   In TGD framework the replacement of Minkowski space-time with space-time sheet seems to restrict the quantum holography to the boundaries of the space-time sheet. Furthermore, in TGD framework the situation is nearer to the non-relativistic one since Poincare transformations are not symmetries of space-time and because 3-surface is the fundamental unit of dynamics. Also in TGD framework M^4 cm degrees of 3-surfaces are present but it would seem that they appear as labels of type II_1 factors in direct integral decomposition rather than as arguments of field operators.

2. The notion of pure state does not make sense in this case since the algebra lacks atoms and projector traces do not define probabilities. The generalization of the notion of pure state as in II_1 case does not make sense since projectors have infinite trace.

3. Entanglement makes sense but has very counter-intuitive properties. First of all, there is no decomposition of density matrix in terms of projectors to pure states nor any obvious generalization of pure states. There exists no measure for the degree of entanglement, which is easy to understand since one cannot assign probabilities to the projectors as their traces.

4. For any pair of space-like separated systems, a dense set of states violates Bell inequalities so that correlations cannot be regarded as classical. This is in a sharp contrast with elementary quantum mechanics, where "de-coherence effects" are believed to drive the states into a classically correlated states.

5. No local measurement can remove the entanglement between a local system and its environment. In TGD framework local operations would correspond to operations associated with a given space-time sheet. Irreducible type II_1 entanglement between different space-time sheets, if indeed present, might have an interpretation in terms of a finite resolution at state space level due to spin glass degeneracy.

On basis of these findings, one might well claim that the axiomatics of relativistic quantum field theories is not consistent with the basic physical intuitions.

5.6 Some consequences of NMP

In the sequel the most obvious consequences of self measurement and NMP are discussed from the point of view of physics, biology, cognition, and quantum computing. The recent discussion differs considerably from the earlier one since several new elements are involved. Zero energy ontology and the hierarchy of CDs, the hierarchy of Planck constants and dark matter, and -perhaps most importantly- the better understanding negentropic entanglement as something genuinely new and making sense in the interaction of real and various p-adic worlds at which living matter is assumed to reside.

5.6.1 NMP and thermodynamics

The physical status of the second law has been a longstanding open issue in physics- in particular biophysics. In positive energy ontology the understanding of the origin of second law is simple. Quantum jumps involve state function reduction (or more generally, self measurement) with a random outcome and in the case of ensemble of identical system this leads to to a probability distribution for the states of the members of the ensemble. This implies Boltzmann equations implying the second law. In TGD framework there are many elements which force to question this simple picture: zero energy ontology and CDs, effective four-dimensionality of the ensemble defined by states assignable to sub-CDs, hierarchy of Planck constants, and the possibility of negentropic entanglement.

Zero energy ontology and thermodynamical ensembles

Zero energy ontology means that the thermodynamics appears both at the level of quantum states and at the level of ensembles. At the level of quantum states this means that M-matrix can be seen as a complex square root of the density matrix: $\rho = MM^\dagger$, where M is expressible as a product of a positive and diagonal square root of density matrix and unitary S-matrix identifiable as the S-matrix used in quantum physics. U matrix can be seen as a collection of M-matrices as will be found later so that U-matrix fixes M-matrices contrary to what was believed originally. One can say that thermodynamics -at least in some sense- is represented at the level of single particle states. It is natural to assume that this density matrix is measured in particle physics experiment, and that this measurement corresponds to a state function reduction, which in standard physics picture corresponds to a preparation for the initial states and state function reduction for the final states.

The p-adic thermodynamics, which applies to conformal weights rather than energy, predicts successfully elementary particle masses [K55] and should reduce to this thermodynamics. That p-adic thermodynamics can be applied at all suggests that even elementary particles reside in the intersection of the real and p-adic worlds so that either p-adic thermodynamics or real thermodynamics with additional constraints on temperature implied by number theory applies.

Thermodynamical ensembles are 4-dimensional

The hierarchy of CDs within CDs defines a hierarchy of sub-systems and sub-CDs define in a natural manner 4-dimensional ensemble. If the state function reduction leads to unentangled states, the outcome is an ensemble describable by the density matrix assignable to the single particle states. The sequence of quantum jumps is expected to lead to a 4-D counterpart of thermodynamical ensemble and thermodynamics results when one labels the states by the quantum numbers assignable to their positive energy part. Entropy is assigned with entire 4-D CD rather than to its 3-dimensional time=constant snapshots. The thermodynamical time is basically the subjective time and measured in terms of quantum jumps but has a correlation with geometric time as explained in [K6] and explained briefly below.

This picture differs from the standard views, and this might explain the paradoxical situation in cosmology resulting from the fact that the initial state of the universe in the standard sense of the word looks highly entropic whereas second law would suggest the opposite [K76] . The cosmological entropy is assigned with a CD of size scale defined by the value of the age of the universe. In this kind of situation each quantum jump replaces the zero energy state with a new one and also induces a drift in the space of CDs to the direction of larger CDs with size defined by the proper time distance between the tips of CD coming as power of 2. Entropy as a function of cosmic time corresponds in TGD framework to the increase of the 4-D entropy as a function of the quantized proper time distance between the tips of the CD.

In this framework it is possible to understand second law in cosmic time scales apart from the possible effects related to the negentropic entanglement responsible for the evolution and breaking of second law in arbitrarily long time scales. For instance, the number of sub-CDs increases meaning the increase of the size of the ensemble and the emergence of new p-adic length scales as the size of cosmic CD increases. What is fascinating is that the TGD counterpart of cosmic time is quantized in powers of two. This might have predictable effects such as the occurrence of the cosmic expansion in a jump-wise manner. I have discussed an explanation of the accelerated cosmic expansion in terms of quantum jumps of this kind but starting from somewhat different picture [K76] .

How second law must be modified?

Second law as such does not certainly apply in TGD framework.

1. The hierarchy of CDs forces to introduce a fractal version of the second law taking into account the p-adic length scale hypothesis and dark matter hierarchy. This means that the idea about quantum parallel Universes generalizes to that of quantum parallel dissipating Universes. For instance, the parton model of hadrons based on quarks and gluons relies on kinetic equations and is basically thermodynamical whereas the model for hadron applied at low energies is quantum mechanical. These two views are consistent if quantum parallel dissipation realized in terms of a hierarchy of CDs is accepted. p-Adic length scale hierarchy with p-adic length scale hypothesis stating that primes near powers of two are preferred corresponds to this dissipative quantum parallellism. Dark matter hierarchy brings in a further dissipative quantum parallelism.

2. Second law should always be applied only at a given level of p-adic and dark matter hierarchy and one must always take into account two time scales involved corresponding to the time scale assignable to the system identifiable as the time scale characterizing corresponding CD and the time scale in which the system is observed. Only if the latter time scale is considerably longer than the CD time scale, second law is expected to make sense in TGD framework - this provided one restricts the consideration to the entropic entanglement. The reason is that the Boltzmann equations implying the second law require that the geometric time scale assignable to quantum jump is considerably shorter than the time scale of observation: this guarantees that the random nature of quantum jump allows to use statistical approach.

3. The possibility of negentropic entanglement in time scale of CD brings a further new element strongly suggesting that the mechanical application of second law does to living matter does not make sense. The basic time scales for CDs come as powers of two and the hierarchy of Planck constants in the most general case allows rational multiples of these. If a restriction is made to singular covering spaces of CD and CP_2 (this might well be consistent with experimental inputs), only integer multiples of these time scales are predicted at the level of dark matter. The increase of Planck constant allows to scale up the time scale of quantum coherence associated with the negentropic entanglement and this provides a further good reason for why large values of Planck constant should be favored in living matter.

4. The reduction of entanglement entropy at single particle level implies the increase of thermodynamical entropy at the level of ensemble in the case of entropic non-binding entanglement. This applies also to bound state entanglement leading to a generation of entropy at the level of binding systems and a reduction of the contribution of the bound systems to the entropy of the entire system. Note however the emission of binding energy -say in form of photons-could take care of the compensation so that entropy would be never reduced for ensemble. In the case of negentropic entanglement the situation is different.

The entropy of the negentropically entangled system is negative and the synergic aspect of negentropic entanglement means that the system does not contribute to thermodynamical entropy. This means that second law could be broken in the geometric time scale considered. One must of course be careful in distinguishing between geometric and subjective time. In the case of subjective time the negentropic situation could continue forever unless the CD disappears in some quantum jump (highly non-probable for large enough CDs). If not, then endless evolution at the level of conscious experience is possible in the intersection of real and p-adic worlds and heat death is not the fate of the Universe as in ordinary thermodynamics.

5. The breaking of second law must correspond to the breaking of ergodicity. Spin glasses are non-ergodic systems and TGD Universe is analogous to a 4-D quantum spin glass by the failure of strict non-determinism of Kähler action reflecting itself as vacuum degeneracy. Does the quantum spin glass property of the TGD universe imply the breaking of the second law? Gravitation has been seen as one possible candidate for the breaking second law because of its long range nature. It is indeed classical gravitational energy which distinguishes between almost degenerate spin glass states. The huge value of gravitational Planck constant associated with space-time sheets mediating gravitational interaction and making possible perturbative quantum treatment of gravitational interaction would indeed suggest the breaking of second law in cosmological time scales. For instance, black hole entropy which is inversely proportional to $GM^2/\hbar_{gr}$ would be for the values of gravitational Planck constant involved of the order of unity.

What do experiments say about second law?

That the status of the second law is far from settled is demonstrated by an experiment performed by a research group in Australian National University [D39] . The group studied a system consisting of 100 small beads in water. One bead was shot by a laser beam so that it became charged and was trapped. The container holding the beads was then moved from side to side 1000 times per second so that the trapped bead dragged first one way and then another. The system was monitored and for monitoring times not longer than .1 seconds second law did not hold always: entropy could also decrease.

1. What is remarkable that .1 seconds defines the duration τ of the memetic code word and corresponds to the secondary p-adic time scale $T_p(2) = \sqrt{p}L_p/c$ associated with Mersenne prime $p = M_{127}$ characterizing electron. This correspondence follows solely from the model of genetic code predicting hierarchy of codes associated with $p = 3, 7, 127$ (genetic code), $p = M_{127}$,... τ should be the fundamental time scale of consciousness. For instance, average alpha frequency 10 Hz corresponds to this time scale and 'features' inside cortex representing sensory percepts have average duration of .1 seconds.

   For electrons the CDs would have spatial size $L = 3 \times 10^7$ meters, which is slightly smaller than the circumference of Earth ($L = cT$, $T = .1$ s, the duration of sensory moment) so that they would have a strong overlap. One can of course ask whether this is an accident. For instance, the lowest Schumann frequency is around 7.8 Hz and not far from 10 Hz. What is interesting that Bohr orbit model [K75] predicts that Universe might be populated by Earth like systems having same distance from their Sun (stars with mass near that of Sun are very frequent). Bohr orbitology applied to Earth itself could also lead to the quantization of the radius of Earth.

2. The first observation was made for more than 15 years ago. Even more remarkable is the recent observation that the time scale of CD associated with electron is .1 seconds. Can one assign the breaking of the second law with the field bodies of electrons?

3. The experiment involves also a millisecond time scale. I do not know whether it is essential that the time scale is just this but one can play with the though that it is. Millisecond time scale is roughly the duration of seventh bit of the genetic codeword if its bits correspond to CDs with sizes coming as subsequence octaves of the basic time scale. Millisecond defines also the time scale for the duration of the nerve pulse and the frequency of kHz cortical synchrony.

At the level of CDs millisecond time scale would correspond to a secondary p-adic time scale assignable to $k = 120$. Only u and d quarks, which appear with several p-adic mass scales in hadron physics and are predicted to be present as light variants also in nuclear physics as predicted by TGD, could correspond to this p-adic length scale: the prediction for their mass scale would be 5 MeV. Does this mean that the basic time scales of living matter correspond directly to the basic time scales of elementary particle physics?

4. A further interesting point is that neutrinos correspond to .1 eV mass scale. This means that the p-adic length scale is around $k = 167$ which means that the corresponding CD has time scale which is roughly 2^{40} times that for electron and corresponds to the primary p-adic length scale of 2.5 μm (size of cellular nucleus) and tothe time scale of 10^4 years. I have proposed that so called cognitive neutrino pairs consisting of neutrino and antineutrino assignable to the opposite throats of wormhole contact could play key a role in the formation of cognitive representations [K68] . This assumption looks now un-necessarily restrictive but one could quite well consider the possibility that neutrinos are responsible for the longest time scales assignable to consciousness for ordinary value of $\hbar$ (not necessarily our consciousness!). Large value of $\hbar$ could make also possible the situation in which intermediate gauge bosons are effectively massless in cell length scale so that electro-weak symmetry breaking would be absent. This would require $\hbar \simeq 2^{33}$. For this value of $\hbar$ the time scale of electronic CD is of the order of the duration of human of human life cycle. This would scale up the Compton length of neutrino to about 10 kilometers and the temporal size of neutrino CD to a super-cosmological time scale.

5.6.2 NMP and self-organization

NMP leads to new vision about self-organization about which adetailed vision is discussed in [K73] . Here only some key points are emphasized.

1. Dissipation selects the asymptotic self-organization patterns in the standard theory of self-organization and the outcomes are interesting in the presence of energy feed. The feed of energy can be generalized to feed of any kind of quantum numbers: for instance, feed of quantum numbers characterizing qualia. In fact, energy increment in quantum jump defines one particular kind of quale [K38] .

2. The notion of self relates very closely to self-organization in TGD framework [K73] . Self is a dissipative structure because it has sub-selves which dissipate quantum parallelly with it. Self as a perceiver maps the dissipation at the level of quantities in the external world to dissipation at the level of qualia in the internal world.

3. Dissipation leads to self-organization patterns and in the absence of external energy feed to thermal equilibrium. Thus thermodynamics emerges as a description for an ensemble of selves or for the time average behavior or single self when external energy feed to system is absent. One can also understand how the dissipative universe characterized by the presence of parameters like diffusion constants, conductivities, viscosities, etc.. in the otherwise reversible equations of motion, emerges. Dissipative dynamics is in a well defined sense the envelope for the sequence of reversible dynamical evolutions modelling the sequence of final state quantum histories defined by quantum jumps.

4. Quantum self-organization can be seen as iteration of the unitary process followed by state function reduction and leads to fixed point self-organization patterns analogous to the patterns emerging in Benard flow. Since selves approach 'asymptotic selves', dissipation can be regarded as a Darwinian selector of both genes and memes. Thus not only surviving physical systems but also stable conscious experiences of selves, habits, skills, behaviors, etc... are a result of Darwinian selection.

5. In TGD one must distinguish between two kinds of self organizations corresponding to the entropic bound state entanglement and negentropic entanglement. Biological self-organization could be therefore fundamentally different from the non-biological one. The succes of the p-adic mass calculations suggest that even elementary particles live in the intersection of real

and p-adic worlds so that one should be very cautious in making strong conclusions. Certainly the intentional, goal-directed behavior of the system in some time scale is a signature of negentropic self-organization but it is difficult to apply this criterion in time scales vastly different from human time scales. It is the field bodies (or magnetic bodies) , which can be assigned naturally to CDs which suggests that the negentropic self organization occurs at this level. TGD based vision about living matter actually assumes this implicitly.

6. What is new that even quantum jump itself can be seen as a self-organization process analogous to Darwinian selection, which eliminates all unbound entanglement and yields a state containing only bound state state entanglement or negentropic entanglement and representing analog of the self-organization patterns. By macro-temporal quantum coherence effectively gluing quantum jumps sequences to single quantum jump this pattern replicates itself fractally in various time scales. Thus self-organization patterns can be identified as bound states and states paired by a negentropic entanglement and the development of the self-organization pattern as a fractally scaled up version of single quantum jump. Second new element is that dissipation is not mere destruction of order but producer of jewels. A further new element is that dissipation can occur in quantum parallel manner in various scales.

7. The failure of the determinism in standard sense for Kähler action is consistent with the classical description of dissipation. In particular, the emergence of sub-selves inside self looks like dissipation from outside but corresponds to self-organization from the point of view of self. 4-dimensional spin glass degeneracy meaning breaking of ergodicity crucial for self-organization is highly suggestive on basis of the vacuum degeneracy of Kähler action, and this alone predicts ultra-metric topology for the landscape of the maxima of Kähler function defined in terms of Kähler action so that p-adicity emerges naturally also in this manner.

One particularly interesting concrete prediction is that the time scales assignable to CDs come as powers of two. This predicts fundamental frequencies coming as powers of two, and the hierarchy of Planck constants predicts rational or at least integer multiples of these frequencies. Could these powers of two relate to frequency doubling rather generally observed in hydrodynamical self-organizing systems?

5.6.3 NMP and p-adic length scale hypothesis

The original form of the p-adic length scale hypothesis stated that physically most interesting p-adic primes satisfy $p \simeq 2^k$, k prime or power of prime. It has however turned out that all positive integers k are possible. Surprisingly few new length scales are predicted by this generalization in physically interesting length scales. p-Adic length scale hypothesis leads to excellent predictions for elementary particle masses (note that the mass prediction is exponentially sensitive to the value of k) and explains also some interesting length scales of biology: for instance, the thicknesses of the cell membrane and of single lipid layer of cell membrane correspond to $k = 151$ and $k = 149$ respectively.

The big problem of p-adic TGD is to derive this hypothesis from the basic structure of the theory.

1. One argument is based on black hole-elementary particle analogy [K60] leading to the generalization of the Hawking-Bekenstein formula: the requirement leading to the p-adic length scale hypothesis is that the radius of the so called elementary particle horizon is itself a p-adic length scale. This argument involves p-adic entropy essentially and it seems that information processing is somehow involved.

2. Zero energy ontology predicts p-adic length scale hypothesis if one accepts the assumption that the proper time distances between the tips of CDs come as powers of 2 [K60] . A more general highly suggestive proposal is that the relative position between tips forms a lattice at proper time constant hyperboloid having as a symmetry group discrete subgroup of Lorentz group (which could reduce to a subgroup of the group $SO(3)$ acting as isotropy group for the time-like direction defined by the relative coordinate between the tips of CD [K76] .

p-Adic length scale hypothesis could be understood as a resonance in frequency domain -most naturally for massless particles like photons. The secondary p-adic time scale for favored p-adic primes must be as near as possible to the proper time distance between the tips of CD. Mersenne primes $M_n = 2^n - 1$ (n is prime) satisfy this condition. Also $log(p)$ is in this case as near as possible to $log(2^n)$ and in the sense that the unit of negentropy defined as $log(2^n - m(n))/log(2^n)$ is maximized. This argument might work also for Gaussian Mersennes $G_n = (1+i)^n - 1$ (n is prime also now) if one restricts the consideration to Gaussian primes.

A more general and more realistic looking hypothesis is that a given CD can have partonic light-like 3-surfaces ending at its boundaries for all p-adic length scales up to that associated with CD: powers of 2 would be favored by the condition of commeasurability very much analogous to frequency doubling.

3. An exciting possibility, suggested already earlier half seriously, is that evolution is present already at elementary particle level. This is the case if elementary particles reside in the intersection of real and p-adic worlds. The success of p-adic mass calculations and the identification of p-adic physics as physics of cognition indeed forces this interpretation. In particular, one can understand p-adic length scale hypothesis as reflecting the survival of the cognitively fittest p-adic topologies.

I have discussed also other explanations.

1. A possible physical reason for the primes near prime powers of 2 is that survival necessitates the ability to co-operate, to act in resonance: this requirement might force commeasurability of the length scales for p-adic space-time sheet (p_1) glued to larger space-time sheet ($p_2 > p_1$). The hierarchy would state from 2-adic level having characteristic fractal length scales coming as powers of $\sqrt{2}$. When $p > 2$ space-time sheet is generated during cosmological evolution $L(p)$ for it must correspond to power of $\sqrt{2}$ so that one must have $p \simeq 2^n$.

2. A model for learning [K21] as a transformation of the reflective level of consciousness to proto level supports the view that evolution and learning occur already at elementary particle level as indeed suggested by NMP: the p-adic primes near power of prime powers of two are the fittest ones. The core of the argument is the characterization of learning as a map from 2^N many-fermion states to M association sequences. The number of association sequences should be as near as possible equal to 2^N. If M is power of prime: $M = p^K$, association sequences can be given formally the structure of a finite field $G(p,K)$ and p-adic length scale hypothesis follows as a consequence of $K = 1$. NMP provides the reason for why $M = p^K$ is favored: in this case one can construct realization of quantum computer with entanglement probabilities $p_k = 1/M = 1/p^K$ and the negentropy gain in quantum jump is $Klog(p)$ while for M not divisible by p the negentropy gain is zero.

5.6.4 NMP and biology

The notion of self is crucial for the understanding of bio-systems and consciousness. It seems that the negentropic entanglement is the decisive element of life and that one can say that in metaphorical sense life resides in the intersection of real and p-adic worlds.

Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that life and conscious intelligence reside in the intersection of the real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

The identification of intentionality as the basic aspect of life seems to be consistent with this idea.

1. The proposed realization of the intentional action has been as a transformation of p-adic space-time sheet to a real one. Also transformations of real space-time sheets to p-adic space-time sheets identifiable as cognitions are possible. Algebraic entanglement is a prerequisite for the realization of intentions in this manner. Essentially a leakage between p-adic and real worlds is in question and makes sense only in zero energy ontology. The reason is that various quantum numbers in real and p-adic sectors are not in general comparable in positive energy ontology so that conservation laws would be broken or even cease to make sense.

2. The transformation of intention to action can occur if the partonic 2-surfaces and their 4-D tangent space-distributions are representable using rational functions with rational (or even algebraic) coefficients in preferred coordinates for the imbedding space dictated by symmetry considerations. Intentional systems must live in the intersection of real and p-adic worlds.

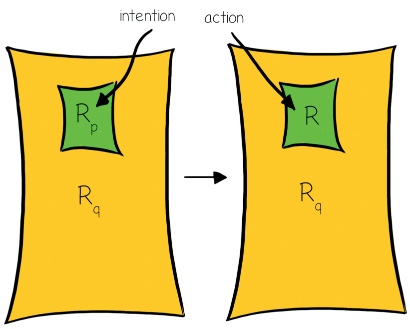

Figure 5.1: Transformation of intention to action as real-to-p-adc transformation and realization of intention as action as its reversal.

3. For the minimal option life would be also effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic partonic two-surfaces appears in U-matrix so that only the data from rational and some algebraic points of the partonic 2-surface dictate U-matrix. This means discretization at parton level and something which might be called number theoretic quantum field theory should emerge as a description of intentional action.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K31] . Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K73] .

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

That only algebraic extensions are possible is of course only a working hypothesis. Also finite-dimensional extensions of p-adic numbers involving transcendentals are possible and might in fact be necessary. Consider for instance the extension containing $e, e^2, .., e^{p-1}$ as units (e^p is ordinary p-adic number. Infinite number of analogous finite-dimensional extensions can be constructed by taking a function of integer variable such that $f(p)$ exists both p-adically and as a real transcendental number. The powers of $f(p)^{1/n}$ for a fixed value of n define a finite-dimensional transcendental extension of p-adic numbers if the roots do not exist p-adically.

Numbers like $log(p)$ and π cannot belong to a finite-dimensional extension of p-adic numbers [K36] . One cannot of course take any strong attitude concerning the possibility of infinite-dimensional extensions of p-adic numbers but the working hypothesis has been that they are absent. The phases $exp(i2\pi/n)$ define finite dimensional extensions allowing to replace the notion of angle in finite measurement resolution with the corresponding phase factors in finite measurement. The functions $exp(i2\pi q/n)$, where q is arbitrary p-adic integers define in a natural manner the physical counterparts of plane waves and angular momentum eigenstates not allowing an identification as ordinary p-adic exponential functions. They are clearly strictly periodic functions of q with a finite value set. If n is divisible by a power of p, these functions are continuous since the values of the function for q and $q + kp^n$ are identical for large enough values of n. This condition is essential and means in the case of plane waves that the size scale of a system (say one-dimensional box) is multiple of a power of p.

Evolution and second law

Evolution has many facets in TGD framework.

1. A natural characterization of evolution is in terms of p-adic topology relating naturally to cognition. p-Adic primes near powers of two are favored if CDs have the proposed discrete size spectrum. From the point of view of self this would be essentially cosmic expansion in discrete jumps. CDs and can be characterized by powers of 2 and if partonic 2-surfaces correspond to effective p-adic p-adic topology characterized by a power of two, one obtains the commeasurability of the secondary p-adic time scale of particle and that of CD in good approximation.

2. The notion of infinite primes motivates the hypothesis that the many-sheeted structure of space-time can be coded by infinite primes [K82] . The number of primes larger than given infinite prime P is infinitely larger than the number of primes than P. The infinite prime P characterizing the entire universe decomposes in a well defined manner to finite primes and p-adic evolution at the level of entire universe is implied by local p-adic evolution at the level of selves. Therefore maximum entanglement negentropy gain for p-adic self increases at least as $log(p)$ with p in the long run. This kind of relationship might hold true for real selves of p-adic physics is physics of cognitive representations of real physics as suggested by the success of p-adic mass calculations. Thus it should be possible to assign definite p-adic prime to each partonic 2-surface.

3. A further aspect of evolution relates to the hierarchy of Planck constants implying that at dark matter levels rational or at least integer multiples of the favored p-adic time scales are realized. The latter option is favored by the idea that the book like structure with pages consisting of many-sheeted coverings of CD and CP_2, and correlates with the emergence of algebraic extensions of p-adic numbers defined by the roots $exp(i2\pi/n)$ of unity. For the latter option evolution by quantum jumps would automatically imply the drifting of the partonic 2-surfaces to the pages of books labelled by increasing values of Planck constant. For more general option one might argue that drifting to pages with small values of Planck constant is also possible. This would give kind of antizooms of long length scale physics to short scales. Both kind of temporal zooms could be crucial for conscious intelligence building scaled models about time evolution in various scales.

4. The generation of negentropic entanglement between different number fields would of course be the fundamental aspect of evolution. It would give rise to increasingly complex and negentropic sensory perceptions and cognitive representations based on conscious rules coded by negentropic entanglement. This would justify the association concept as it used in neuro-science. Negentropic entanglement could be also crucial for the basic mechanism of metabolism and make possible conscious co-operation even in nano-scales.

Just for fun one can play also with numbers.

1. The highest dark matter level associated with self corresponds to its geometric duration which can be arbitrarily long: the typical duration of the memory span gives an idea about the level of dark matter hierarchy involved if one assumes that the time scale .1 seconds assignable to electrons is the fundamental time scale. If the time scale T of human life cycle corresponds to a secondary p-adic time scale then $T = 100$ years gives the rough estimate $r \equiv \hbar/\hbar_0 = 2^{33}$ if this time scale corresponds to that for dark electron. The corresponding primary p-adic time length scale corresponds to $k = 160$ and is 2.2×10^{-7} meters.

2. If human time scale -taken to be $T = 100$ years- corresponds to primary p-adic time scale of electron, one must have roughly $r = 2^{97}$.

I have already discussed the second law in TGD framework and it seems that its applies only when the time scale of perception is longer than the time scale characterizing the level of the p-adic and dark matter hierarchy. Second law as it is usually stated can be seen as an unavoidable implication of the materialistic ontology.

Stable entanglement and quantum metabolism as different sides of the same coin

The notion of binding has two meanings. Binding as a formation of bound state and binding as a fusion of mental images to larger ones essential for the functioning of brain and regarded as one the big problems of consciousness theory.

Only bound state entanglement and negentropic entanglement are stable against the state reduction process. Hence the fusion of the mental images implies the formation of a bound entropic state- in this case the two interpretations of binding are equivalent- or a negentropic state, which need not be bound state.

1. In the case of negentropic entanglement bound state need not be formed and the interesting possibility is that the negentropic entanglement could give rise to stable states without binding energy. This could allow to understand the mysterious high energy phosphate bond to which metabolic energy is assigned in ATP molecule containing three phosphates and liberated as ATP decays to ADP and phosphate molecule. Negentropic entanglement could also explain the stability of DNA and other highly charged biopolymers. In this framework the liberation of metabolic (negentropic) energy would involve dropping of electrons to a larger space-time sheets accompanying the process $ATP \rightarrow ADP + P_i$. A detailed model of this process is discussed in [K35] .

   In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

2. The formation of bound state entanglement is expected to involve a liberation of the binding energy and this energy might be a usable energy. This process could perhaps be coined as quantum metabolism and one could say that quantum metabolism and formation of bound states are different sides of the same coin. It is known that an intense neural activity, although it is accompanied by an enhanced blood flow to the region surrounding the neural activity, does not involve an enhanced oxidative metabolism [J9] (that is $ATP \rightarrow ADP$ process and

its reversal). A possible explanation is that quantum metabolism accompanying the binding is involved. Note that the bound state is sooner or later destroyed by the thermal noise so that this mechanism would in a rather clever manner utilize thermal energy by applying what might be called buy now–pay later principle.

If these interpretations are correct, there would be two modes of metabolism corresponding to two different kinds of fusion of mental images.

5.6.5 NMP, consciousness, and cognition

As already found NMP dictates the subjective time development of self and is therefore the basic law of consciousness. If p-adic physics is the physics of cognition, the most exotic implications of NMP relate to cognition rather than standard physics.

Thermodynamics for qualia

If only entropic entanglement is assumed, second law seems to hold also at the level of conscious experience of self, which can be seen as an ensemble of its sub-selves assignable to sub-CDs. The randomness of the state function reduction process implies that conscious experience involves statistical aspects in the sense that the experienced qualia correspond to the averages of quantum number and zero mode increments over the sub-selves assignable to sub-CDs. When the number of quantum jumps in the ensemble defining self increases, qualia get more entropic and fuzzy unless macro-temporal quantum coherence changes the situation.

Negentropic entanglement means departure from this picture if sub-CDs can generate negentropic entanglement. This is expected to be true if they overlap if one believes on standard argument for the formation of macroscopic quantum phases. In this case the flux tubes connecting space-time sheets assignable to the sub-CDs would serve as a space-time correlate for the negentropic entanglement.

The basic questions are whether sensory qualia can really correspond to the increments of quantum numbers in quantum jump and whether these quantum jumps are assignable to entropic or negentropic qualia. What is clear that the sensory qualia such as colors are assigned to an object of external world rather predictably. This is not obvious if this process is based on quantum jump.

1. Qualia are determined basically as increments of quantum numbers [K38] whereas in ordinary statistical physics measured quantities would correspond to quantum numbers basically. The basic function of sensory organs is to map quantum numbers to quantum number increments so that our sensory perception is in reasonable approximation about world rather than changes of the world.

2. In zero energy ontology the increments must correspond to increments of quantum numbers for (say) positive energy part of the state. A sensation of (say) given color requires a continual feed of corresponding quantum number increment to the positive energy part of the system. Some kind of far from equilibrium thermodynamics seems to be necessary with external feed of quantum numbers generalizing the external feed of energy. The capacitor model of a sensory receptor [K38] realizes this idea in terms of generalized di-electric breakdown implying opposite charging of the capacitor plates in question. Note that in zero energy ontology also the positive and negative energy parts of the zero energy state assignable to capacitor plates would be also analogous to a pair of oppositely charged capacitor plates and one can speak about capacitor also in time direction.

3. If entropic entanglement is reduced to zero in quantum jump for individual sensory recepto, the outcome involves all possible values of quale, say different fundamental colors for which I have proposed a model in terms of QCD color [K38] . If the probability of particular value of quale is much larger than others, one can have statistical ensemble giving rise to predictable quale as ensemble average.

4. If negentropic entanglement is in question, similar situation is encountered but the perception is a mixture of qualia. For large values of p-adic prime one could have almost complete dominance of a particular instance of quale also now. One could argue that the perception

represents also the definition of the concept of a particular quale as a superposition of pairs of consisting of the state inducing the instance of the quale and the state representing it. The fact that there are very many negentropic superpositions however suggests that the superposition represents both the definition of quale and average value of quale. For instance, the fusion of various colors could rely on negentropic entanglement.

5. Both these representations of qualia could realized and one can ask whether the entropic representation could be aesthetically less pleasing than the negentropic representation involving also the notion of quale.

Questions about various kinds of entropies

There are three kinds of entropies and the basic question is how these entropies relate.

1. Does the entropy characterizing the experience of self relate to the thermodynamical entropy of some system? The fact that non-geometric sensory qualia have a statistical interpretation, suggests that the entropy associated with the qualia of the mental image corresponds to the thermodynamical entropy for a system giving rise to the qualia via the sensory mapping. The thermodynamics of quantities in the external world would thus be mapped to the thermodynamics of qualia, increments of quantities, in the inner world. Selves could also represent the fundamental thermodynamical ensembles since they define also statistical averages of quantum numbers and zero modes although these are not directly experienced.

2. Could one interpret the entropies of the space-time sheets as entropies associated with the symbolic representations of conscious experiences of selves? Could one see the entire classical reality as a symbolic representation? Does the entropy of conscious experience correspond to the thermodynamical entropy of the perceived system, which in turn would correspond to the classical space-time entropy of the system representing the perceived system symbolically? Does this conclusion generalize to the case of p-adic entropy? Quantum-classical correspondence would encourage to cautiously think that the common answer to these questions might be yes.

The arrow of psychological time and second law

The arrow of psychological time is closely related to the second law and I have considered several alternative identifications for the arrow of psychological time. These identifications are discussed in [K91, K6, K92] . The latest option favored by zero energy ontology is discussed in [K6] and involves two aspects: the one related to the arrow of time coordinate assignable to the space-time sheet and the other one to the relative proper time coordinate between the tips of CD.

A simple argument show that this distance quantized in powers of 2 should increase gradually in statistical sense since the size of CD can also change in quantum jump. This would have have interpretation in terms of a flow of "cosmic time" (CD is analogous to big bang followed by big crunch). Interestingly, CD with time scale of order 10^{11} years (age of the universe) corresponds primary p-adic length scale of only only 10^{-4} meters, the size of a large neuron, and also the length scale in which the blob of water has Planck mass so that the quantization of gravitational Planck constant should become important [K75] . Could this mean that the CDs assignable to large neurons make possible to develop the idea about the cosmology and cosmology itself? Could it really be that that our cognitive representations about Universe quite concretely have the size of the Universe itself as p-adic view about cognition requires?

Quantum jump and cognition

The fusion of sub-selves can take place in two manners: by real bound state entanglement and by negentropic entanglement. The resulting mental images must differ somehow, and the proposal is that the entanglement associated with the negentropic mental defines a conscious cognitive representation: kind of rule. Schrödinger cat negentropically entangled with the bottle of poison knows that it is not a good idea to open the bottle: open bottle-dead cat, closed bottle-living cat. Negentropic entanglement would generate rules and counterparts of conscious associations fundamental in brain functioning. For the mental image associated with bound state entanglement the information about bound systems would be lost. Bound state entanglement could however give rise to stereo-consciousness essential for (say) stereo vision.

One can imagine several kinds of negentropic entanglements of this kind. Between two real systems, between real and p-adic systems, and between two p-adic systems possibly characterized by different values of p: all these systems assigned with distinct but overlapping CDs. These entanglements would correspond to different aspects of conscious experience. Maybe the real-real entanglement could correspond to a positive emotion- perhaps love-, and the remaining to experiences of understanding generating a connection between two different things: between real world even and its cognitive representation or between two cognitive representations. Note that the entanglement probabilities can vary considerably and one can obtain identical a spectrum of entanglement probabilities by permuting them. This should relate to the character of the experience of understanding. Schrödinger cat which is almost dead has strong conviction that it is better to not open the bottle. The optimal situation concerning understanding would be identical probabilities.

Analysis and conceptualization (synthesis) - formation of rules- could be seen as the reductionistic and holistic aspects of consciousness. The interpretation of quantum jump as a creation of a totally entangled holistic state, which is then analyzed to stable entangled pieces allows to interpret self measurement cascade as a conscious analysis. The resulting stable negentropic pieces give rise to experience of understanding and conceptualization - rules and abstractions. Perhaps the holistic character assigned ot right brain hemisphere could be interpreted in terms of specialization to conceptualization and reductionist character of left brain to entropic analysis to smallest possible pieces.

There are rather interesting connections with altered states of consciousness and states of macro-temporal quantum coherence.

1. Making mind empty of mental images could perhaps be interpreted as a mechanism of achieving irreducible self state. If self entangles negentropically with larger conscious entity this would lead to experiences characterized as expansion of consciousness, even cosmic consciousness. One could also consider the possibility the sub-selves representing mental images fuse to single long-lasting negentropic mental image. The absence of dissipation could relate to the reports of meditators about lowered metabolic needs.

2. The ordinary wake-up consciousness is identifiable as the analytical mode in which entropic entanglement dominates so that each U process is followed by a rather complete state function reduction. The reason for this could be sensory input and motor activities, which would create effective heat bath destroying holistic mental images.

3. Krishnamurti has talked a lot about states of consciousness in which no separations and discriminations occur and timelessness prevails. These states could correspond to long-lived negentropic entanglement with large $\hbar$ with larger conscious entities giving rise to very long effective moments of consciousness. In this kind of situation NMP does not force cognitive self measurements to occur and analysis and separations can thus be avoided.

4. Sharing and fusion of mental images by entanglement of sub-selves of separate selves makes possible quantum realization of telepathy and could be a universal element of altered states of consciousness. Also this entanglement could be bound state entanglement or negentropic entanglement.

Cognitive codes

p-Adic length scale hypothesis leads to the idea that each $p \simeq 2^k$, k integer, defines a hierarchy of cognitive codes with code word having duration given by the n-ary p-adic time scale $T(n, k)$ and number of bits given by any factor of k. Especially interesting codes are those for which the number of bits is prime factor or power of prime factor of k. $n = 2$ seems to be in special position in zero energy ontology. This is a strong quantitative prediction since the duration of both the code word and bit correspond to definite frequencies serving as signatures for the occurrence of commutations utilizing these codes.

If k is prime, the amount of information carried by the codon is maximal but there is no obvious manner to detect errors. If k is not prime there are several codes with various numbers of bits:

information content is not maximal but it is possible to detect errors. For instance, $k = 252$ gives rise to code words for which the number of bits is $k_1 = 252, 126, 63,\ 84, 42, 21_2, 9,\ 7, 6_2, 4, 3_2, 2$: the subscript $_2$ tells that there are two non-equivalent manners to get this number of bits. For instance, $126 = 42 \times 3$-bit codon can have 42 -bit parity codon: the bits of this codon would be products of three subsequent bits of 126-bit codon. This allows error detection by comparing the error codon for communicated codon and communicated error codon.

Abstraction hierarchy and genetic code

Mersenne primes $M_n = 2^n - 1$, which seem to play fundamental role in elementary particle physics and it has been already found that their emergence is natural consequence of NMP. This would put primes 3, 7, 31, 127, etc. in a special position. Primes appear frequently in various bio-structures and this might reflect the underlying p-adicity for the association sequences providing 'plan' for the development of bio-system. For instance, we have actually 7 (!) fingers: two of them have degenerated during evolution but can be seen in the developing embryo. There are 31 subunits in our spinal chord, etc...

In the model of genetic code based on a simple model of abstraction process [K40] the so called Combinatorial Hierarchy $2, 3, 7, 127, 2^{127} - 1, ...$ of Mersenne primes emerges naturally. The construction for a model of abstraction process proceeds as follows.

1. At lowest level there are two digits. The statements Yes and No.

2. At the next level one considers all Boolean statements about these two statements which can be regarded as maps from 2-element set to 2-element set. There are 4 of them. Throw one away and you get 3 statements.

3. At the next level one considers all Boolean statements about these 3 statements and the total number of them is 2^3. Throw one away and you get 7 statements. And so on.

The mystery is why one statement must be thrown away at each level of the construction. The answer might relate to a concrete model of quantum computation.

1. A possible neurolevel realization of a quantum computation is following. Entangle in the proposed manner two memetic codewords represented as temporal sequences of 127 cognitive Z^0 magnetized antineutrino ensembles with bit represented as the magnetization direction. The phase transitions changing the direction of magnetization are assumed to involve classical non-determinism.

2. Nerve pulse (or pulse like membrane oscillation) results from each flip of the direction of the Z^0 magnetization. The temporal sequence for which all Z^0 magnetization are in the the direction of the external Z^0 magnetic field is excluded because this state does not give rise to a nerve pulse pattern (or membrane oscillation pattern). In this manner a quantum computer with $N = 1$ and $p = 2^{127} - 1$ results. Incoming nerve pulse patterns could be taken to be identical memetic codewords and out would go a a pair of memetic codewords representing the initial memetic codeword and the result of the quantum computation. The duration of the computation is .1 seconds and involves $2^{127} - 1$ quantum jumps effectively glued to single quantum jump by macro-temporal quantum coherence.

The concepts of resolution and monitoring

The following considerations represent a rather early idea related to p-adic physics, and I am not sure whether to take it seriously or not. The basic observation is that genuinely p-adic probabilities can sum up to zero, and this might make possible some rather exotic looking effects in genuinely p-adic sectors of state space.

When the fundamental observable (density matrix or entropy operator) has degenerate eigenvalues, one can only speak about probability for quantum jump to a particular eigen space of the the observable since there is no preferred basis in this eigen space. This leads to the concept of cognitive resolution: one cannot distinguish between states belonging to a given eigen space of density matrix and one can make predictions for the probabilities for quantum jumps to given eigen space only.

1. Resolution and monitoring

p-Adic probability concept allows to consider an additional exotic effect.

1. The total real probability for quantum jump to degenerate subspace is the real counterpart for sum of p-adic probabilities rather than sum of the real counterparts of the p-adic probabilities. This can lead to rather dramatic effects: for instance, the sum of p-adic probabilities can be very small even when the sum of the real probabilities is large.

2. The notion of resolution is closely related to the notion of monitoring: resolution can be defined as a decomposition of the p-adic state space to a direct sum of subspaces such that the p-adic density matrix is degenerate inside each subspace. If p-adic probabilities are defined modulo $O(p)$ pinary cutoff this kind of degeneracy is bound to occur if the dimension of the state space is larger than p.

An interesting possibility is that the notions of resolution and monitoring could be important in the physics of cognition. Perhaps the well-known fact that the behavior of cognitive systems is sensitive to monitoring, might have something to do with the density matrix characterizing the entanglement between the monitoring and monitored systems. The behavior of monitored system would depend on the resolution of the monitoring, that is on how interested monitorer is about behavior of monitored system. In the limit that monitorer is not interested at all on the behavior, entanglement probabilities would in general be identical and unless the number of states is power of p, $S = 0$ state would result.

The total probability for a set of independent events to occur depends on the resolution of monitoring: not only the behavior of individual quantum system in ensemble but also the *statistical* behavior of the ensemble of systems characterized by same p-adic prime depends on the resolution of the monitoring.

Standard probability theory, which also lies at the root of the standard quantum theory, predicts that the probability for a certain outcome of experiment does not depend on how the system is monitored. For instance, if system has N outcomes $o_1, o_2, ...o_N$ with probabilities $p_1, ..., p_N$ then the probability that o_1 or o_2 occurs does not depend on whether common signature is used for o_1 and o_2 or whether observer also detects which of these outcomes occurs. The crucial signature of p-adic probability theory is that monitoring affects the behavior of the system. NMP provides precise definition for the concept of monitoring. There are two forms of monitoring depending on whether the fundamental observable, denote it by O, is density matrix or entropy operator.

Consider first the situation in which all entanglement probabilities have p-adic norm different from unity. Physically monitoring is represented by quantum entanglement and differentiates between two eigen states of O (density matrix or entropy operator) only provided the eigenvalues of O are different. If there are several degenerate eigenvalues, quantum jump occurs to any state in the eigen space and one can predict only the total probability for the quantum jump into this eigen space. Hence the p-adic probability for a quantum jump to a given eigen space of density matrix is p-adic sum of probabilities over the eigen states belonging to this eigen space:

$$P_i = \frac{(n(i)P(i))_R}{\sum_j (n(j)P(j))_R} \quad .$$

Here n_i are dimensions of various eigen spaces.

If the degeneracy of the eigenvalues is removed by an arbitrary small perturbation, the total probability for the transition to the same subspace of states becomes the sum for the real counterparts of probabilities and one has in good approximation:

$$P^R = \frac{n(i)P(i)_R}{[\sum_{j\neq i}\sum_j (n(j)P(j))_R + n(i)P(i)_R]} \quad .$$

Rather dramatic effects could occur. Suppose that that the entanglement probability $P(i)$ is of form $P(i) = np$, $n \in \{0, p-1\}$ and that n is large so that $(np)_R = n/p$ is a considerable fraction of unity. Suppose that this state becomes degenerate with a degeneracy m and $mn > p$ as integer.

In this kind of situation modular arithmetics comes into play and $(mnp)_R$ appearing in the real probability $P(1\ or\ 2)$ can become very small. The simplest example is $n = (p+1)/2$: if two states i and j have *very nearly equal but not identical* entanglement probabilities $P(i) = (p+1)p/2 + \epsilon$, $P(j) = (p+1)p/2 - \epsilon$, monitoring distinguishes between them for arbitrary small values of ϵ and the total probability for the quantum jump to this subspace is in a good approximation given by

$$\begin{aligned} P(1\ or\ 2) &\simeq \frac{x}{\left[\sum_{k\neq i,j}(P_k)_R + x\right]} \ , \\ x &= 2\left[(p+1)p/2\right]_R \ . \end{aligned} \tag{5.6.1}$$

and is rather large. For instance, for Mersenne primes $x \simeq 1/2$ holds true. If the two states become degenerate then one has for the total probability

$$\begin{aligned} P(1\ or\ 2) &\simeq \frac{x}{\left[\sum_{k\neq i,j}(P_k)_R + x\right]} \ , \\ x &= \frac{1}{p} \ . \end{aligned} \tag{5.6.2}$$

The order of magnitude for $P(1\ or\ 2)$ is reduced by a factor of order $1/p$!

A test for the notion of p-adic quantum cognition would be provided by the study of the dependence of the transition rates of quantum systems on the resolution of monitoring defined by the dimensions of the degenerate eigen spaces of the subsystem density matrix (or entropy operator). One could even consider the possibility of measuring the value of the p-adic prime in this manner. The behavior of living systems is known to be sensitive to monitoring and an exciting possibility is that this sensitivity, if it really can be shown to have statistical nature, could be regarded as a direct evidence for TGD inspired theory of consciousness. Note that the mapping of the physical quantities to entanglement probabilities could provide an ideal manner to compare physical quantities with huge accuracy! Perhaps bio-systems have invented this possibility before physicists and this could explain the miraculous accuracy of biochemistry in realizing genetic code.

If some entanglement probabilities have unit norm so that their contributions to the p-adic entanglement entropy vanish, quantum jump to an entangled final state can occur: this is genuinely p-adic effect and serves as a second test for p-adic cognition. If density matrix is the fundamental observable, quantum jump can occur to an entangled final state, which corresponds to any $S = 0$ subspace of $S = 0$ eigen space of the entropy operator with is eigen space of the density matrix. If entropy operator is the fundamental observable, quantum jump can occur to any $S = 0$ subspace of entropy operator. Again the total probability for the transition is determined by the p-adic sum of the probabilities and dramatic 'interference' effects at the level of probabilities are possible.

An alternative interpretation for the degenerate eigenvalues has emerged years after writing this. The sub-spaces corresponding to given eigenvalue of density matrix represent entangled states resulting in state function reduction interpreted as measurement of density matrix. This entanglement would be negentropic and represent a rule/concept, whose instances the superposed state pairs are. The information measure would Shannon entropy based on the replacement of the probability appearing as argument of logarithm with its p-adic norm. This entropy would be negative and therefore measure the information associated with the entanglement. This number theoretic entropy characterizes two particle state rather than single particle state and has nothing to do with the ordinary Shannon entropy.

Maybe one could say that finite measurement resolution implies automatically conceptualization and rule building. Abstractions are indeed obtained by dropping out the details.

Resolution and monitoring and hyperfinite factors of type II_1

The notion of resolution emerges naturally for the hyper-finite factors of type II_1. The trace of the unit operator is unit for the infinite-dimensional space in question so that any projector with a finite trace must project to an infinite dimensional space so that there would always an infinite-dimensional degeneracy involved with the eigenvalues of the measured observables.

One could however consider the formulation of the theory in terms of p-adic probabilities and for this formulation resolution and monitoring emerge naturally. One could go even further. For instance, if one can specify the infinite number of degrees of freedom as a p-adic integer, say $N = -1 = (p-1)\sum_{k=0}^{\infty} p^k$, which in a well-defined sense represents the largest p-adic integer, one can say that the p-adic probability for a given state is $1/N$ and finite as a p-adic number. It is finite also as a real number and equal to $1/p$ if canonical identification is used to map N to a real number. For a given finite-dimensional density matrix with finite number of distinct eigenvalues it would be possible to have projections to one-dimensional subspace but there would always infinitely degenerate eigenvalue present in accordance with the notion of finite resolution.

A natural question concerns the implications of the assumption that the map of p-adic probabilities to real ones conserves probabilities without additional normalization.

5.6.6 NMP and quantum computer type systems

TGD Universe can be regarded as an infinite quantum computer. Unitarity process U is analogous to a quantum computation. The state function reduction process represents a stepwise halting of the computation proceeding until the resulting states are eith bound states or negentropically entangled states. U matrix is between zero energy states and can be regarded as a collection of M-matrices labelled by zero energy states. The possibility of two kinds of entropic and negentropic entanglement makes possible two kinds of quantum computations and negentropic quantum computations based on states which are longlived by the properties of the negentropic entanglement could be the one realized in living matter.

The relationship between U-matrix and M-matrix

Before proceeding it is a good idea to clarify the relationship between the notions of U-matrix and M-matrix. If state function reduction associated with time-like entanglement leads always to a product of positive and negative energy states (so that there is no counterpart of bound state entanglement and negentropic entanglement possible for zero energy states) U-matrix and can be regarded as a collection of M-matrices

$$U_{m_+n_-,r_+,s_-} = M(m_+,n_-)_{r_+,s_-} \tag{5.6.3}$$

labeled by the pairs (m_+,n_-) labelling zero energy states assumed to reduced to pairs of positive and negative energy states. M-matrix element is the counterpart of S-matrix element $S_{r,s}$ in positive energy ontology. Unitarity conditions for U-matrix read as

$$\begin{aligned}
(UU^\dagger)_{m_+n_-,r_+s_-} &= \sum_{k_+,l_-} M(m_+,n_-)_{k_+,l_-}\overline{M}(r_+,s_-)_{k_+,l_-} = \delta_{m+r_+,n_-s_-} \quad , \\
(U^\dagger U)_{m_+n_-,r_+s_-} &= \sum_{k_+,l_-} \overline{M}(k_+,l_-)_{m_+,n_-}M(k_+,l_-)_{r_+,s_-} = \delta_{m+r_+,n_-s_-} \quad .
\end{aligned} \tag{5.6.4}$$

The conditions state that the zero energy states associated with different labels are orthogonal as zero energy states and also that the zero energy states defined by the dual M-matrix

$$M^\dagger(m_+,n_-)_{k_+,l_-} \equiv \overline{M}(k_+l_-)_{m_+,n_-} \tag{5.6.5}$$

-perhaps identifiable as phase conjugate states- define an orthonormal basis of zero energy states.

When time-like binding and negentropic entanglement are allowed also zero energy states with a label not implying a decomposition to a product state are involved with the unitarity condition but this does not affect the situation dramatically. As a matter fact, the situation is mathematically the same as for ordinary S-matrix in the presence of bound states.

How quantum computation in zero energy ontology differs from ordinary quantum computation

Quantum computation in zero energy ontology differs in several respects from ordinary quantum computation.

1. The time parameter defining quantum computation as a unitary time evolution in standard quantum physics disappears and corresponds to the U-matrix for single quantum jump. Quantum computation corresponds to the U-matrix assignable to single quantum jump if one restricts to sub-CDs with given time scale inside larger CD. The quantum jump for given sub-CD would represent single quantum computation and the outcome of the quantum computation would be determined statistically from the distribution of the outcomes of state function reductions for over sub-CDs.

   Quantum classical correspondence encourages to assign to the quantum computation an interval of psychological time equal to the proper time distance between the tips of CD. For instance, .1 seconds would the be time scale assignable to quantum computations possibly assignable to electrons.

   The hierarchies of CDs and Planck constants make possible zoomed up variants of quantum computations. This kind of zooming might be essential for intelligent behavior since it is useful to simulate dynamics of the external world in the time scales natural for brain and shorter than the time scale during which it is necessary to react in order to survive. The geometric duration of the shortest possible quantum computation with respect to the psychological time of self is of order CP_2 time about 10^4 Planck times, if the simplest estimate is correct.

2. The classical space-time correlates for the quantum computation are four-dimensional unlike in the case of ordinary quantum compitation. In living matter nerve pulses and EEG frequencies would be very natural correlates of this kind. The model for DNA as topological quantum computer [K31] has as its space-time correlates magnetic flux tubes connecting DNA nucleotides and lipids of nuclear and cell membranes defining the braiding coding for the topological quantum computation. Dynamical flow of lipids defines the braiding in time direction and the memory representation is in terms of the braiding of the flux tubes induced by this flow. A good metaphor is in terms of dancers connected to a wall by threads. Dancing is the correlate for the running quantum computer program and the geometric entanglement of threads the correlate for the storage of the program to computer memory.

3. The outcome of quantum computation is described statistically in terms of a large set of quantum computations. The statistical description of the conscious experience of ensemble of sub-selves implies that mathematically the situation is very much analogous with that encountered in the standard quantum computation and it is attractive to assume that conscious experience codes for the outcome of quantum computation via the average quantities assignable to the distribution of zero energy quantum states assignable to sub-CDs.

4. A further new element is macro-temporal quantum coherence involving several aspects. One of these aspects is that the time scale of CD defines macrotemporal quantum coherence at least at the level of the field body assignable to the physical system such as electron. It is not quite clear whether electrons correspond to distinct overlapping CDs of size scale defined by .1 second time scale and of the order of Earth circumference and thus satisfying the basic criterion of quantum coherence or whether one should speak about anyonic many particle states assignable to single CD or whether both interpretations can make sense depending on situation. In living matter also millisecond time scale is important and would correspond naturally to the CDs assignable to u and d quarks in nuclei and perhaps also with the ends of magnetic flux tubes in the model of DNA as topological quantum computer. In the proposed model quarks and antiquarks at the ends of flux tubes represent genetic codons and their entanglement is responsible for the realization of the program at quantum level. The millisecond time scale of synchronous cortical firing and of nerve pulse could correspond to the time scale of CDs associated with u and d quarks at the ends of the flux tube. Note that

larger value of $\hbar$ would scale up this time scale. Quantum parallel dissipation taking place at various size scales for CD is a further new element.

5. One must generalize the standard quantum computer paradigm since ordinary quantum computers represent only the lowest, 2-adic level of the p-adic intelligence. Qubits must be replaced by qupits since for algebraic entanglement two-state systems are naturally replaced with p-state systems. For primes of order say $p \simeq 2^{167}$ (the size of small bacterium) this means about 167 bits, which would mean gigantic quantum computational resources. The secondary p-adic time scale $T_2(127) \simeq .1$ seconds basic bit-like unit corresponds to $M_{127} = 2^{127} - 1$ M_{127}-qupits making about 254 bits. The size of neuron corresponds to CD with time scale equal to the age of the universe and in this case the maximum the number of pinary digits is 171.

   The finite measurement resolution for qubits of course poses strong limitations to the actual number of bits since the negentropic zero energy qubits must be in reasonable approximation pure qubits distinguishable from each other and could correspond CDs with time scales coming as powers of two from $n = k_{min}$ to k so that the effective number of qubits would go like 2-based logarithm of the p-adic prime. For instance, electron could correspond to six bits assignable to genetic code plus parity bit corresponding to time scale range from 1 ms to 100 ms. In any case the idea about neuron as a classical bit might be completely wrong!

6. Spin glass degeneracy also provides the needed huge number of degrees of freedom making quantum computations very effective. These degrees of freedom are associated with the join along boundaries bonds -say magnetic flux tubes- and are essentially gravitational so that a connection with Penrose-Hameroff hypothesis suggests itself. The space-time sheets mediating gravitational interaction are predicted to have a huge gravitational Planck constant $\hbar_{gr} = GMm/v_0$, $v_0/c < 1$, particles at these space-time sheets are predicted to have huge Compton wavelengths and the plausible looking identification is in terms of dark energy [K75, K62] . This would make quantum computation like activities possible in super-astronomical time scales.

Three kinds of quantum computations are possible in TGD Universe

In TGD Universe one must distinguish between three kinds of quantum computational modes. Ordinary quantum computation utilizes only the part of U-matrix for which zero energy states involved are unentangled products of positive and negative energy states. In this case quantum coherence is extremely fragile and lasts for single quantum jump only but even in this case one might hope that coherence time correspondences to the time scale CD. U-matrix can also correspond to the analogous of bound states for real time-like entanglement. If the proposed interpretation makes sense these state pairs would not correspond to conscious rules. Negentropic entanglement in time direction is the third option. For living quantum computers entanglement could correspond to bound state entanglement or negentropic entanglement and NMP takes care that the character of both these states is preserved. Thus bio-systems would be especially attractive candidates for performers of quantum computation like processes.

Negentropic quantum computations, fuzzy qubits, and quantum groups

1. The possibility of negentropic entanglement is certainly the basic distinction making in the intersection of real and p-adic worlds possible conscious process at least analogous to a quantum computation and accompanied by a conscious understanding. What makes this possible is the fact that the negentropically entangled states of N basic states have permutation of the basis states as a symmetry. For instance, states for which bit 1 appears with almost unit probability gives by permutation a state for which bit 0 appears with almost unit probability. This suggests that the outcome of quantum computation is expressed in terms of almost bits with a small mixing implying that the outcome has interpretation both as a rule and as almost bit in the ordinary sense. The conscious quantum computation would utilize states with negentropic entanglement in time direction. Also the analogies of bound states for time-like engtanglement are possible and might make possible the counterpart of ordinary

quantum computation without the higher level conscious experience about rules defined by the entangled states.

2. Negentropic entanglement for positive and negative energy parts of bits stable and pinary digits stable under NMP means that the logic is always fuzzy. I have proposed the mathematical description of this in terms of quantum spinors for which the components do not commute anymore implying that only the probability for either spin state is is an observable [K95] . This suggests that negentropic entanglement might be describable in terms of quantum spinors and that it would be the unavoidable fuzziness which would make possible the representation conscious rules. What is interesting that for quantum spinors the spectrum of the probabilities for given spin is universal and depends only on the integers characterizing the quantum phase $q = exp(i2\pi/n)$. An alternative interpretation is that fuzzy logic relates to a finite measurement resolution. These interpretation need not be in conflict with each other. Since quantum groups are associated with anyonic systems, this suggests that negentropic quantum computations take place in anyonic systems assignable to phases with large value of $\hbar$. This encourages to consider the possibility that quantum phases define algebraic extensions of p-adic numbers.

3. In living systems it might be more appropriate to talk about conscious problem solving instead of quantum computation. In this framework the periods of macro-temporal quantum coherence replace the unitary time evolutions at the gates of the quantum computer as the basic information processing units and entanglement bridges between selves act as basic quantum communication units with the sharing of mental images providing a communication mode not possible in standard quantum mechanics.

5.7 Generalization of thermodynamics allowing negentropic entanglement and a model for conscious information processing

Costa de Beauregard considers a model for information processing by a computer based on an analogy with Carnot's heat engine [J11] , [J11] . I am grateful for Stephen Paul King for bringing this article to my attention in Time discussion group and also for inspiring discussions which also led to the birth of this section. As such the model Beauregard for computer does not look convincing as a model for what happens in biological information processing.

Combined with TGD based vision about living matter, the model however inspires a model for how conscious information is generated and how the second law of thermodynamics must be modified in TGD framework. The basic formulas of thermodynamics remain as such since the modification means only the replacement $S \rightarrow S - N$, where S is thermodynamical entropy and N the negentropy associated with negentropic entanglement. This allows to circumvent the basic objections against the application of Beauregard's model to living systems. One can also understand why living matter is so effective entropy producer as compared to inanimate matter and also the characteristic decomposition of living systems to highly negentropic and entropic parts as a consequence of generalized second law.

5.7.1 Beauregard's model for computer

Beauregard's model describes computer as information processor analogous to heat engine. The work done by a heat engine is replaced with information generated by the computer and printing makes this information manifest.

1. In Carnot cycle thermal energy is transformed to work and one gets the well known upper bound for the efficiency from second law as $\eta = W/Q_{in} \leq \Delta T/T_{in}$.

2. Beauregard a model for an ideal computer is as a system which performs no work but prints instead. One studies information flow instead of energy flow. Negentropy is identified as a negative of thermodynamical entropy. Incoming negative negentropy flow means coding of program metaphorically at least and outgoing negentropy flow to what results, when this

coding is erased in computer memory. The printed text carries the negentropy which in the optimal situation is the difference between incoming and outgoing negentropies. This negentropy is sucked from the incoming negative negentropy flow so that second law holds true.

3. In terms of formulas one has $dW = dQ_{out} - dQ_{in} = 0$ and $dS = dQ_{out}/T_{out} - dQ_{in}/T_{in} = dQ_{in}(1/T_{out} - 1/T_{in}) \geq 0$. In the ideal case that the total entropy does not increase, this entropy growth must be compensated by the reduction of the entropy of the printer by amount dS interpreted as negentropy of the output.

4. This vision about computing is based on second law and identifies information gain as difference between two entropies. System can gain information by feeding disorder to the environment. The best possible situation is that one has no information at all.

Criticism of the model

This model seems consistent with thermodynamics and skeptic would argue that what we see around us could be seen as a support for this view about information processing in living systems. One can however argue that the view about information as absence of entropy does not really make sense in living matter.

1. p-Adic physics encourages the belief in genuine information. If living matter is identified as something in the intersection of real and p-adic worlds it is possible to have a genuine information represented as a negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book). The number theoretic variant of Shannon entropy gives a natural measure for this information since it can be negative and there is a unique p-adic prime minimizing it. Conscious information is a rule $A \leftrightarrow B$ in which the pairs $a \otimes b$ in the quantum superposition represent the instances of the rule. Schrödinger cat knows that it should not open the bottle by being a little bit dead but negentropically so.

2. Second point is that Boltzmann's kinetic theory leading to the second law is based on the assumption that quantum coherence is not present in the time scales considered. If this assumption fails one cannot treat the system as a thermodynamical system (atoms represent standard example of this). In zero energy ontology and accepting the hierarchy of Planck constants, there are always levels of hierarchy for which second law does not make sense in a given time scale.

3. There is also a direct experimental evidence for the reversal of thermodynamical time and therefore breaking of second law in time scales below .1 seconds, which happens to correspond to the time scale assignable to the CD of electron and to a fundamental biorhythm. The evidence comes from a system consisting of beads on necklace [D39] .

   (a) Standard physics explanation would be in terms of fluctuation in the value of entropy. Fluctuation theorem [B2] allows to deduce a precise expression for the ratio of probabilities of entropy fluctuations of same magnitude but opposite sign as $exp(A)$ where A represents the magnitude of the fluctuation. The appearance of .1 second time scale however forces to challenge this interpretation.

   (b) In TGD framework one possibility is that the spontaneous local reversal of the arrow of geometric time induced from that of experienced time implies that second law with reversed arrow of geometric time is operating. Second possibility is that genuine increase of negentropy is in question.

Problems of Beauregard's model if interpreted as a model for information processing in living systems

Beauregard's model for what he calls "printer" looks problematic for several reasons.

1. Living matter and computers are in good approximation at the same temperature as environment and temperature T and volume V are not changed during the process so that free energy F is minimized rather than thermodynamical negentropy. This kind of systems are not analogous to steam engines for which one has has incoming steam at higher temperature. Beauregard's analog of Carnot engine satisfies $dW = dQ_{out} - dQ_{in} = 0$ and indeed gives for $T_{in} = T_{out}$ the trivial result $dN = 0$. No information is generated. Even worse, living systems are typically at higher temperature than environment so that the heat engine analogy does not seem to work well.

2. In the analog of steam engine one actually assumes that the entropy difference for outgoing and incoming beams corresponds to a positive negentropy assignable to the printing. One can however treat the printer and computer as a single system in which case one can draw only one conclusion from standard thermodynamics: this negentropy corresponds to work done by the combined system and one has just the ideal steam engine but the work interpreted as printout. Something however distinguishes between printer and steam engine.

5.7.2 TGD based variant of Beauregard's model and generalization of thermodynamics

The TGD inspired variant of Beauregard's model leads naturally to a generalization of the second law of thermodynamics taking into account the possibility of negentropic entanglement.

Questions

Something distinguishes between printer and steam engine and standard thermodynamics is not able to express this difference. What this something is? The proposal to be discussed is that the positive entanglement negentropy assignable to rational (or even algebraic) entanglement generated in the process in which conscious information is created. It is best to proceed by making questions.

1. The work done by steam engine is "useful" work. What does this mean? Something which does not have meaning for us but is a prerequisite for having meaning. Perhaps metabolic energy at the basic level. This work can be eventually transformed to metabolic energy needed to build mental images generated by the text.

2. What metabolic energy is? In TGD Universe there are two kinds of entanglements: the entropic bound state entanglement and negentropic entanglement which is rational or even algebraic and possible in the intersection of real and p-adic worlds. Bound state entanglement is stable under NMP by binding energy. This kind of entanglement is like a marriage based on social conventions, a jail.

   Negentropic entanglement does not involve binding energy and can be compared to a marriage based on freedom and love. The positive energy associated with the negentropic entanglement has wrong sign to be interpreted as binding energy and is identifiable as metabolic energy. This identification could explain the long standing mystery of the high energy phosphate bond central for the functioning of ATP and ADP. ATP-ADP process would be basically a transfer of negentropic entanglement and thus information to the living system and at work at all levels in living matter.

3. What is the process giving meaning to the text? This process must generate negentropic entanglement. The corresponding entanglement negentropy is something independent of thermodynamic entropy and the safest assumption is that the generation of negentropic entanglement is accompanied by the generation of thermodynamical entropy at least compensating it so that second law in a generalized form continues to hold true.

What happens in quantum jump?

Quantum jump involves U process and state function reduction cascade. Negentropy Maximization Principle implies second law for the standard view about state function reduction: second law states that the ensemble entropy increases by the randomness of the outcome of the state function reduction process. When negentropic entanglement is present the situation is not so clear. Before proceeding to consider the modification of the second law one must define more precisely what U process is.

The simplest view about quantum jump is as a unitary U-process followed by as a cascade of state function reductions proceeding from top to bottom. But what is the top?

1. In positive energy ontology it would be entire Universe. Quantum classical correspondence suggests that one should be able to assign to quantum jump a duration of geometric time. For this proposal this time is most naturally infinite.

2. The vision about fractal hierarchy of selves and quantum jumps together with ZEO suggests a more refined view about quantum jump in which. U-process and subsequence state function reduction cascade could occur independently for disjoint CDs. For a given CD the new sub-CDs (representing mental images of the corresponding self) can be created and old destroyed so that the only constraint would be that only disjoint CDs can perform quantum jumps independently. For this option the duration of geometric time assignable to the quantum jump would naturally correspond to the temporal distance between the tips of CD: p-adic length scale hypothesis and number theoretical vision suggest that this distance comes as an octave of CP_2 time scale (prime or integer multiple is the more general option). For infinitely large CD this would mean infinite duration. This picture is consistent with the TGD view about how the arrow of subjective time induces the arrow of geometric time [K6] .

Modification of thermodynamics to take into account negentropic entanglement

What does the presence of this negentropic entanglement mean from the point of view of thermodynamics? There are two obvious options to consider. The optimistic option is just the standard thermodynamics saying nothing about negentropy generation. The pessimistic option is that the generation of negentropy must be accompanied by a generation of at least the same amount of entropy: the good news is that this entropy can be carried by different system and it is possible to have genuinely negentropic systems. The following consideration is restricted to the pessimistic option which seems to be more realistic view about the world we live in.

1. One must generalize the basic expression for energy differential

$$dE = TdS - dW \rightarrow T(dS - dN) - dW \ . \tag{5.7.1}$$

This means that there are two kinds of energies given out by the system. The useful work dW and negentropic energy TdN. For steam engine only dW is present. For ideal system only negentropic energy would be present.

2. What happens to the second law? The pessimistic guess is that generation of negentropy requires a generation of at least same amount of entropy so that one would have

$$\Delta S - \Delta N \geq 0 \ . \tag{5.7.2}$$

Here S can be interpreted as a sum of two terms. The first part corresponds to the ensemble entropy generated by the randomness of ordinary quantum jumps, and second part to the entropy assignable as maximal entanglement entropy assignable to the decompositions of bound state to two parts. N corresponds to maximal negentropy for the decompositions of negentropic sub-system to pairs. One can criticize these definitions and a possible modification of could be as as the average for the entanglement entropies over this kind of decompositions.

3. Quite generally, Clausius inequality allowing to deduce extremization conditions for various thermodynamical potentials generalizes to

$$T_0(\Delta S - \Delta N) - \Delta E - P_0 \Delta V \quad \geq \quad 0 \ . \tag{5.7.3}$$

where T_0 and P_0 and temperature and pressure of heat bath. Living systems would be entropy producers and this seems to conform with what we see around us.

For instance, for a system in constant volume one would have

$$\Delta S - \Delta N - \frac{\Delta E}{T} \quad \geq \quad 0 \ . \tag{5.7.4}$$

so that systems developing negentropy would also generate thermodynamics entropy. For a system in heat bath one has $T = T_0$ and Clausius inequality gives

$$\Delta F \quad = \quad -\Delta W \tag{5.7.5}$$

stating that increase of free energy at constant temperature requires work done on the system ($dW < 0$): otherwise $\Delta F \leq 0$ holds true.

By using the variable $S-N$ instead of S all formulas reduce formally to standard thermodynamics except that S can be negative. This is absolutely crucial for distinguishing TGD counterpart of Beauregard's printer -identifiable as conscious reader rather than printer - from Carnot engine.

The analog of Carnot cycle for information processing in living matter

Consider now Carnot heat engine and its information theoretic analog in this framework.

1. The basic equation for Carnot engine is

$$dW \quad = \quad dQ_{in} - dQ_{out} \geq 0 \ . \tag{5.7.6}$$

Optimal efficiency corresponds to $dS_{out} = dS_{in}$.

2. For the information theoretic analog one would have

$$dW \quad = \quad 0 \ , \tag{5.7.7}$$

and

$$dN \quad = \quad dS_{out} - dS_{in} \geq 0 \ . \tag{5.7.8}$$

The interpretation would be that incoming entropy flow leaves the computer in a state of higher entropy and the difference corresponds to information dN feeded to say printer. The increase of entropy would have interpretation in terms of erasing of data from computer memory.

The problematic aspect of the model is that it requires $T_{in} > T_{out}$ in order to have $dN > 0$. For living systems one has however typically $T_{in} < T_{out}$. Already for $T_{in} = T_{out}$ the situation trivializes since one has

$$dN = 0 \quad (5.7.9)$$

by $dW = 0$ and $dS = dQ/T$.

3. Now however a more general condition

$$T_{in}d(S_{in} - N_{in}) - T_{out}d(S_{out} - N_{out}) \geq 0 \quad (5.7.10)$$

holds true and allows to generate conscious information provided it is compensated by thermodynamical entropy. Note that the temperature of the environment can be even lower than the temperatures of the system.

It is also possible to transform information to work as the expression for the differential $dF = -SdT - TdN - dW$ of the generalized free energy $E = E - TS$ shows. The increase of dW for the work done by the system is compensated by the reduction of information dN so that system loses negentropy in the process keeping dF constant. The loss of negentropy couild be interpreted in terms of a loss of metabolic energy which corresponds to negentropic entanglement for AMP, ADP, and ATP molecules.

4. Beauregard calls the information engine printer. What does this "printing" correspond from the point of view of negentropic entanglement? Is the negentropic entanglement is generated during physical printing or during the reading? If the negentropic entanglement is generated before reading, there must be some other conscious entity for which the text has meaning. This seems un-necessary assumption so that ordinary computers would not generate negentropic entanglement. For the second and much more reasonable looking option the above process takes place during the reading and the "printing" as a name for the above process is misleading: conscious reading is in question.

Some clarifying comments

Some clarifying comments about biological implications are in order. Many of them are inspired by the questions of Stephen Paul King in Time discussion group.

1. There is no need to restrict the consideration to equilibrium systems. First of all, the environment and living system are in general at different temperatures and temperature difference is typically of wrong sign for the model of Beauregard to work in this context. Beauregard's model is of course a model for computation, not for the generation of negentropic mental images. Maybe cognitive machine might be proper term for what the modified model could describe.

2. Quite generally, self-organization requires a feed of energy to the system so that one has flow equilibrium. In the case of living system this feed of energy is metabolic energy associated with the negentropic entanglement transferred to the system in the ATP-ADP process. Self-organization driven by negentropic entanglement leads to standardized negentropic mental images automatically as asymptotic self-organization patterns in 4-D sense (CDs within CDs within ...).

3. No explicit assumptions about computational aspects of the process has been made. Just a generation of conscious information identified in terms of negentropic entanglement is assumed. The basic character quantum jump as U-process followed by the cascade of state function reductions represents a fractal hierarchy of what can be seen as quantum computations and are distinguished from classical computations in that the process proceeds from top

to bottom rather than being a local process. The result of computation is represented using statistical ensembles defined by sub-CDs at various levels of the hierarchy and is in principle communicable by classical fields (say EEG patterns in the case of brain) to higher levels of self hierarchy which in turn can induces the same distributions so that communication of the objective aspects of the experience with the mediation of "medium" is possible. The presence of the "medium" seems unavoidable. Magnetic body would be this medium in TGD inspired biology.

5.7.3 About implications of generalized second law

Generalized second law allows to sharpen the basic picture about implications of the second law.

Biological implications

Living matter involves also another aspect made possible by the generalized second law obtained by the replacement $S \rightarrow S - N$. Subsystem can have also negative net entropy and split to two highly negentropic and entropic pieces. In the extreme situation this is nothing but excretion, which is absolutely essential element of being alive but sometimes forgotten from the lists of properties distinguishing living matter from inanimate matter. It is not at all clear whether this is possible for standard non-equilibrium systems defining information as a reduction of disorder. At all levels of the fractal hierarchy division into negentropic and entropic subsystems is expected.

This picture seems to be in accordance with basic chemistry of energy metabolism.

1. The process creating both negentropy and entropy would be standardized in living matter and mean a generation of high energy phosphate bonds assignable to AMP, ADP, and ATP containing 1, 2, and 3 phosphates respectively besides the sugar residue. Sugar residue is basic nutrient and would provide the stored metabolic energy transformed to the negentropic energy of the high energy phosphate bonds if the proposed view is correct. Also other DNA nucleotides such as G can appear besides A but in metabolism A has a preferred role.

2. The basic metabolic cycle provides ADP with an additional phosphate energizing it to ATP and the reverse process transfers the metabolic energy and also negentropic entanglement to the acceptor molecule. Also ADP can provide metabolic energy by transforming to AMP when ATP is not available in sufficient amounts. That the catabolism of AMP creates urea excreted out of the system fits with the general picture. The catabolism for nutrients would create the entropy compensating for the negentropy of the high energy phosphate bonds.

3. The backbone of DNA is made of sugar and phosphate residues and corresponds to a sequence of XMP, $X = A, T, C, G$ with each XMP presumably containing single high energy phosphate bond serving as a storage or potential source of negentropy. This conforms with the view that DNA carries conscious information.

Negentropic and entropic entanglement are assumed to generate mental images with opposite emotional colors. This connects information processing with emotions. From neuroscience point of view this is not a news: peptides are molecules of emotions on one hand and molecules of information on the other hand [J19] . The well-known specialization of the left and right hand sides of the amygdala to experience positive and negatively colored emotions could be seen as one instance of this connection and representing also an example about fractal negentropic-entropic differentiation.

The interpretation of generalized second law in a wider context

Leaving the narrow confines of thermodynamics one could try to interpret the generalized second law in a wider context.

1. The generalized second law unavoidably brings in mind the Good-Evil dichotomy. Good deeds seem to induce evil deeds. Maybe this kind of polarization effect is indeed unavoidable in the situations for which thermodynamics applies. The crucification of a man whose sole crime was to suggest that we should love also our enemies expresses this paradoxical truth

in very deep manner. Thermodynamical approximation can however fail and the hierarchy of Planck constants and zero energy ontology predict that this occurs. Maybe the Eastern teachings promising a way out from the cycle of endless suffering are inspired by experiences in which no Good-Evil polarization takes place. The ATP-ADP cycle generating negentropy and at least same amount of entropy has more than obvious analogy with the Karma's cycle.

2. One cannot avoid associations with the basic teachings of Christianity. U process would correspond to Genesis creating the paradise. Eating the fruits from the tree of Good and Bad Knowledge would correspond to the emergence of cognition producing islands of negentropy and entropy and meaning a banishment from paradise. "With hard work of you hands must you will get your bread" would correspond to endless fight for getting metabolic energy transformed to energy associated with the negentropic entanglement.

   Heaven and hell would be the islands of negentropy and entropy resulting during the state function reduction process. The next U-process re-creating the heaven and and Earth would be the new Genesis and the moment of mercy meaning a new possibility to be used or lost for both saints and sinners. If U-process is local in the sense that it can occur independently for disjoint CDs, the situation is rather comforting since salvation possibly brought by the next moment of recreation requires only a finite time of waiting.

5.8 Further progress in the understanding of NMP

I have collected in this section the updates motivated by the progress in TGD and TGD inspired theory of consciousness since 2012. NMP [K51] implies that negentropic entanglement is approximately invariant under quantum jumps. This allows to build a direct connection with the basic idea of quantum biology about the braiding of magnetic flux tubes as a correlate for the negentropic entanglement and identify braidings as kind of "Akashic records" giving rise to various representations (sensory - , memory - , cognitive -) defining reflective level of consciousness as opposed to phenomenal consciousness defined by sensory qualia. NMP in the rational intersection of realities and p-adicities in turn fixes the p-adic prime associated with the criticality at the intersection. Also a close connection between quantum criticality, life as something in the intersection of realities and p-adicities, hierarchy of effective vales of Planck constant, negentropic entanglement, and p-adic view about cognition emerges. The reader interested in details can consult a more detailed representation about the recent vision about TGD inspired theory of consciousness [K107].

5.8.1 The anatomy of quantum jump in zero energy ontology (ZEO)

Zero energy ontology emerged around 2005 and has had profound consequences for the understanding of quantum TGD. The basic implication is that state function reductions occur at the opposite light-like boundaries of causal diamonds (CDs) forming a hierarchy, and produce zero energy states with opposite arrows of imbedding space time. Also concerning the identification of quantum jump as moment of consciousness ZEO encourages rather far reaching conclusions. In ZEO the only difference between motor action and sensory representations on one hand, and intention and cognitive representation on the other hand , is that the arrows of imbedding space time are opposite for them. Furthermore, sensory perception followed by motor action corresponds to a basic structure in the sequence of state function reductions and it seems that these processes occur fractally for CDs of various size scales.

1. State function reduction can be performed to either boundary of CD but not both simultaneously. State function reduction at either boundary is equivalent to state preparation giving rise to a state with well defined quantum numbers (particle numbers, charges, four-momentum, etc...) at this boundary of CD. At the other boundary single particle quantum numbers are not well defined although total conserved quantum numbers at boundaries are opposite by the zero energy property for every pair of positive and negative energy states in the superposition. State pairs with different total energy, fermion number, etc.. for other boundary are possible: for instance, t coherent states of super-conductor for which fermion number is ill defined are possible in zero energy ontology and do not break the super-selection rules.

2. The basic objects coding for physics are U-matrix, M-matrices and S-matrix. M-matrices correspond to a orthogonal rows of unitary U-matrix between zero energy states, and are expressible as products of a hermitian square root of density matrix and of unitary S-matrix which more or less corresponds to ordinary S-matrix. One can say that quantum theory is formally a square root of thermodynamics. The thermodynamics in question would however relate more naturally to NMP rather than second law, which at ensemble level and for ordinary entanglement can be seen as a consequence of NMP.

   The non-triviality of M-matrix requires that for given state reduced at say the "lower" boundary of CD there is entire distribution of states at "upper boundary" (given initial state can lead to a continuum of final states). Even more, all size scales of CDs are possible since the position of only the "lower" boundary of CD is localized in quantum jump whereas the location of upper boundary of CD can vary so that one has distribution over CDs with different size scales and over their Lorentz boots and translates.

3. The quantum arrow of time follows from the asymmetry between positive and negative energy parts of the state: the other is prepared and the other corresponds to the superposition of the final states resulting when interactions are turned on. What is remarkable that the arrow of time at imbedding space level at least changes direction when quantum jump occurs to opposite boundary.

   This brings strongly in mind the old proposal of Fantappie [J15] that in living matter the arrow of time is not fixed and that entropy and its diametric opposite syntropy apply to the two arrows of the imbedding space time. The arrow of subjective time assignable to second law would hold true but the increase of syntropy would be basically a reflection of second law since only the arrow of the geometric time at imbedding space level has changed sign. The arrow of geometric at space-time level which conscious observer experiences directly could be always the same if quantum classical correspondence holds true in the sense that the arrow of time for zero energy states corresponds to arrow of time for preferred extremals. The failure of strict non-determinism making possible phenomena analogous to multi-furcations makes this possible.

4. This picture differs radically from the standard view and if quantum jump represents a fundamental algorithm, this variation of the arrow of geometric time from quantum jump to quantum jump should manifest itself in the functioning of brain and living organisms. The basic building brick in the functioning of brain is the formation of sensory representation followed by motor action. These processes look very much like temporal mirror images of each other such as the state function reductions to opposite boundaries of CD look like. The fundamental process could correspond to a sequences of these two kinds of state function reductions for opposite boundaries of CDs and maybe independently for CDs of different size scales in a "many-particle" state defined by a union of CDs.

How the formation of cognitive and sensory representations could relate to quantum jump?

1. ZEO allows quantum jumps between different number fields so that p-adic cognitive representations can be formed and intentional actions realized. How these quantum jumps are realized at the level of generalized Feynman diagrams is non-trivial question: one possibility suggested by the notion of adele combining reals and various p-adic number fields to a larger structure is that the lines and vertices of generalized Feynman diagrams can correspond to different number fields [K100].

   The formation of cognitive representation could correspond to a quantum jump in which real space-time sheet identified as a preferred extremal is mapped to its p-adic counterpart or superposition of them with the property that the discretized versions of all p-adic counterparts are identical. In the latter case the chart map of real preferred extremal would be quantal and correspond to de-localized state in WCW. The p-adic chart mappings are not expected to take place but with some probabilities determined by the number theoretically universal U-matrix.

2. Similar consideration applies to intentional actions realized as real chart maps for p-adically realized intention. The natural interpretation of the process is as a time reversal of cognitive

map. Cognitive map would be generated from real sensory representation and intentional action would transform time reversed cognitive map to real "motor" action identifiable as time reversal of sensory perception. This would occur in various length scales in fractal manner.

3. The formation of superpositions of preferred extremals associated with discrete p-adic chart maps from real preferred extremals could be interpreted as an abstraction process. Similar abstraction could take place also in the mapping of p-adic space-time surface to a superposition of real preferred extremals representing intentional action. U-matrix should give also the probability amplitudes for these processes, and the intuitive idea is that the larger then number of common rational and algebraic points of real and p-adic surfaces is, the higher the probability for this is: the first guess is that the amplitude is proportional the number of common points. On the other hand, large number of common points means high measurement resolution so that the number of different surfaces in superposition tends to be smaller.

4. One should not make any un-necessary assumptions about the order of various kinds of quantum jumps. For the most general option real-to-padic and p-adic-to-real quantum jumps can follow any quantum jumps and state function reductions to opposite boundaries of CD can also occur any time in any length scale. Also the length scale of resolution scale assignable to the cognitive representation should be determined probabilistically. Quantal probabilities for quantum jumps should therefore apply to all aspect of quantum jump and now ad hoc assumptions should be made. Very probably internal consistency allows only very few alternative scenarios. The assumption that the cascade beginning from given CD continues downwards until stops due to the emergence of negentropic entanglement looks rather natural constraint.

5.8.2 About NMP and quantum jump

NMP is assumed to be the variational principle telling what can happen in quantum jump and says that the information content of conscious experience for the entire system is maximized. In zero energy ontology (ZEO) the definition of NMP is far from trivial and the recent progress - as I believe - in the understanding of structure of quantum jump forces to check carefully the details related to NMP. A very intimate connection between quantum criticality, life as something in the intersection of realities and p-adicities, hierarchy of effective values of Planck constant, negentropic entanglement (NE), and p-adic view about cognition emerges. One ends up also with an argument why p-adic sector is necessary if one wants to speak about conscious information. I will proceed by making questions.

What happens in single state function reduction?

State function reduction is a measurement of density matrix. The condition that a measurement of density matrix takes place implies standard measurement theory on both real and p-adic sectors: system ends to an *eigen-space* of density matrix. This is true in both real and p-adic sectors. NMP is stronger principle at the real side and implies state function reduction to 1-D subspace - its eigenstate.

The resulting N-dimensional space has however rational entanglement probabilities $p = 1/N$ so that one can say that it is the intersection of realities and p-adicities. If the number theoretic variant of entanglement entropy is used as a measure for the amount of entropy carried by entanglement rather than either entangled system, the state carries genuine information and is stable with respect to NMP if the p-adic prime p divides N. NMP allows only single p-adic prime for real $\rightarrow$ p-adic transition: the power of this prime appears is the largest power of prime appearing in the prime decomposition of N. Degeneracy means also criticality so that that ordinary quantum measurement theory for the density matrix favors criticality and NMP fixes the p-adic prime uniquely.

If one - contrary to the above conclusion - assumes that NMP holds true in the entire p-adic sector, NMP gives in p-adic sector rise to a *reduction* of the negentropy in state function reduction if the original situation is negentropic and the eigen-spaces of the density matrix are 1-dimensional. This situation is avoided if one assumes that state function reduction cascade in real or genuinely p-adic sector occurs first (without NMP) and gives therefore rise to N-dimensional eigen spaces. The state is negentropic and stable if the p-adic prime p divides N. Negentropy is generated.

The real state can be transformed to a p-adic one in quantum jump (defining cognitive map) if the entanglement coefficients are rational or belong to an algebraic extension of p-adic numbers in the case that algebraic extension of p-adic numbers is allowed (number theoretic evolution gradually generates them). The density matrix can be expressed as sum of projection operators multiplied by probabilities for the projection to the corresponding sub-spaces. After state function reduction cascade the probabilities are rational numbers of form $p = 1/N$.

Number theoretic entanglement entropy also allows to avoid some objections related to fermionic and bosonic statistics. Fermionic and bosonic statistics require complete anti-symmetrization/symmetrization. This implies entanglement which cannot be reduced away. By looking for symmetrized or antisymmetrized 2-particle state consisting of spin 1/2 fermions as the simplest example one finds that the density matrix for either particle is the simply unit 2×2 matrix. This is stable under NMP based on number theoretic negentropy. One expects that the same result holds true in the general case. The interpretation would be that particle symmetrization/antisymmetrization carries negentropy.

The degeneracy of the density matrix is of course not a generic phenomenon and one can argue that it corresponds to some very special kind of physics. The identification of space-time correlates for the hierarchy for the effective values $\hbar_{eff} = n\hbar$ of Planck constant as n-furcations of space-time sheet suggests strongly the identification of this physics in terms of this hierarchy. Hence quantum criticality, the essence of life as something in the rational intersection of realities and p-adicities, the hierarchy of effective values of $\hbar$, negentropic quantum entanglement, and the possibility to make real-p-adic transitions and thus cognition and intentionality would be very intimately related. This is a highly satisfactory outcome, since these ideas have been rather loosely related hitherto.

What happens in quantum jump?

Suppose that everything can be reduced to what happens for a given CD characterized by a scale. There are at least two questions to be answered.

1. There are two processes involved. State function reduction and quantum jump transforming real state to p-adic state (matter to cognition) and vice versa (intention to action). Do these transitions occur independently or not? Does the ordering of the processes matter? The proposed view about state function reduction strongly suggests that the p-adic $\leftrightarrow$real transition (if possible at all) can occur any time without affecting the outcome of the state function reduction.

2. State function reduction cascade in turn consists of two different kinds of state function reductions. The M-matrix characterizing the zero energy state is product of square root of density matrix and of unitary S-matrix and the first step means the measurement of the projection operator. It defines a density matrix for both upper and lower boundary of CD and these density matrices are essentially same.

   (a) At the first step a measurement of the density matrix between positive and negative energy parts of the quantum state takes place for CD. One can regard both the lower and upper boundary as an eigenstate of density matrix in absence of NE. The measurement is thus completely symmetric with respect to the boundaries of CDs. At the real sector this leads to a 1-D eigen-space of density matrix if NMP holds true. In the intersection of real and p-adic sectors this need not be the case if the eigenvalues of the density matrix have degeneracy. Zero energy state becomes stable against further state function reductions! The interactions with the external world can of course destroy the stability sooner or later. An interesting question is whether so called higher states of consciousness relate to this kind of states.

   (b) If the first step gave rise to 1-D eigen-space of the density matrix, a state function reduction cascade at either upper of lower boundary of CD proceeding from long to short scales. At given step divides the sub-system into two systems and the sub-system-complement pair which produces maximum negentropy gain is subject to quantum measurement maximizing negentropy gain. The process stops at given subsystem resulting in the process if the resulting eigen-space is 1-D or has NE (p-adic prime p divides the dimension N of eigenspace in the intersection of reality and p-adicity).

Negentropic entanglement, NMP, braiding and TQC

NMP and evolution of intelligence

Alexander Wissner-Gross, a physicist at Harvard University and the Massachusetts Institute of Technology, and Cameron Freer, a mathematician at the University of Hawaii at Manoa, have developed a theory that they say describes many intelligent or cognitive behaviors, such as upright walking and tool use [J16, J5]. The basic idea of the theory is that intelligent system collects information about large number of histories and preserves it. Thermodynamically this means large entropy so that the evolution of intelligence would be rather paradoxically evolution of highly entropic systems. According to standard view about Shannon entropy transformation of entropy to information (or the reduction of entropy to zero) requires a process selecting one of instances of thermal ensemble with a large number of degenerate states and one can wonder what is this selection process. This sounds almost like a paradox unless one accepts the existence of this process. I have considered the core of this almost paradox in TGD framework already earlier.

According to the popular article (`http://www.insidescience.org/content/physicist-proposes-new-way-think-987`) the model does not require explicit specification of intelligent behavior and the intelligent behavior relies on "causal entropic forces" (here one can counter argue that the selection process is necessary if one wants information gain). The theory requires that the system is able to collect information and predict future histories very quickly.

The prediction of future histories is one of the basic characters of life in TGD Universe made possible by zero energy ontology (ZEO) predicting that the thermodynamical arrow of geometric time is opposite for the quantum jumps reducing the zero energy state at upper and lower boundaries of causal diamond (CD) respectively. This prediction means quite a dramatic deviation from standard thermodynamics but is consistent with the notion of syntropy introduced by Italian theoretical physicist Fantappie already for more than half a century ago as well as with the reversed time arrow of dissipation appearing often in living matter.

The hierarchy of Planck constants makes possible negentropic entanglement and genuine information represented as negentropic entanglement in which superposed state pairs have interpretation as incidences $a_i \leftrightarrow b_i$ of a rule $A \leftrightarrow B$: apart from possible phase the entanglement coefficients have same value $1/\sqrt{n}$, where $n = h_{eff}/h$ define the value of effective Planck constant and dimension for the effective covering of imbedding space. This picture generalizes also to the case of multipartite entanglement but predicts similar entanglement for all divisions of the system to two parts. There are however still some questions which are not completely settled and leave some room for imagination. Therefore a small digression from the main topic is perhaps allowed.

1. Negentropic entanglement is possible in the discrete degrees of freedom assignable to the n-fold covering of imbedding space allowing to describe situation formally. For $h_{eff}/h = n$ one can introduce $SU(n)$ as dynamical symmetry group and require that n-particle states are singlets under $SU(n)$. $SU(n)$ brings in mind the dynamical gauge symmetry group introduced earlier for inclusions of hyper-finite factors of type II_1 to which one can assign simply laced Lie groups such as $SU(n)$. These groups were proposed to make possible emulation of all possible gauge group dynamics. They would also characterize the finite measurement resolution (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book).

   This gives rise to n-particle states constructed by contracting product of some number k of n-dimensional permutation symbols contracted with many particle states assignable to the m factors. These states would generalize k-particle states. For $k = 1$ and $m > 1$ one would have single particle state in "schizophrenic state" consisting of m particles with fractional quantum numbers n_i/n times the usual quantum numbers. Spin-statistics connection might produce problems - at least it is non-trivial - since one possible interpretation is that the states carry fractional quantum numbers - in particular fractional fermion number and charges.

   These states generalize the notion of N-atom proposed earlier as emergence of symbols and "sex" at molecular level [K28]. "Molecular sex" means that all states can be seen as composites of two states with opposite fractional $SU(n)$ quantum numbers (this decomposition need not be unique!). This brings in mind the monogamy theorem for ordinary entanglement stating that maximal entanglement means this kind of decomposition to two parts.

2. Is negentropic entanglement possible only in the new covering degrees of freedom or is it possible in more familiar angular momentum, electroweak, and color degrees of freedom?

   (a) One can imagine that also states that are singlets with respect to rotation group SO(3) and its covering SU(2) (2-particle singlet states constructed from two spin 1 states and spin singlet constructed from two fermions) could carry negentropic entanglement. The latter states are especially interesting biologically and from the point of view of photosynthesis and navigation of birds: long-lived negentropically entangled spin singlet electron-hole pairs and electron pairs are proposed as explanation of the experimental findings.

   (b) In TGD framework all space-time surfaces can be seen at least 2-fold coverings of M^4 locally since boundary conditions do not seem to allow 3-surfaces with spatial boundaries so that finiteness of the space-time sheet requires covering structure in M^4. This forces to ask whether this double covering could provide a geometric correlate for fermionic spin 1/2 suggested by quantum classical correspondence taken to extreme. Fermions are indeed fundamental particles in TGD framework and it would be nice if also 2-sheeted coverings would define fundamental building bricks of space-time.

   (c) Color group SU(3) for which color triplets defines singlets can be also considered. I have been even wondering whether quark color could actually correspond to 3-fold or 6-fold (color isospin corresponds to SU(2)) covering so that quarks would be dark leptons, which correspond $n = 3$ coverings of CP_2 and to fractionization of color hypercharge and electromagnetic charge. The motivation came from the inclusions of hyper-finite factors of type II_1 labelled by integer $n \geq 3$. If this were the case then only second H-chirality would be realized and leptonic spinors would be enough. What this would mean from the point of view of separate B and L conservation remains an open and interesting question. This kind of picture would allow to consider extremely simple genesis of matter from right-handed neutrinos only [?].

   There are two objections against this naive picture. The fractionization associated with h_{eff} should be same for all quantum numbers so that different fractionizations for color isospin and color hyper charge does not seem to be possible. One can of course ask whether the different quantum numbers are fractionized independently and what this could mean geometrically. Second, lethal looking objection is that fractional quark charges involve also shift of em charge so that neutrino does not remain neutral it becomes counterpart of u quark.

Negentropy Maximization Principle (NMP) resolves also the above mentioned almost paradox related to entropy contra intelligence. I have proposed analogous principle but relying on generation of negentropic entanglement and replacing entropy with number theoretic negentropy obeying modification of Shannon formula involving p-adic norm in the logarithm $log(|p|_p)$ of probability. The formula makes sense for probabilities which are rational or in algebraic extension of rational numbers and requires that the system is in the intersection of real and p-adic worlds. The dark matter matter with integer value of Planck constant and $h_{eff} = nh$ predicts rational entanglement probabilities: their values are simply $p_i = 1/n$ since the entanglement coefficients define a diagonal matrix proportional to unit matrix. Negentropic entanglement makes sense also for n-particle systems.

Negentropic entanglement corresponds therefore always to $n \times n$ density matrix proportional to unit matrix: this means maximal entanglement and maximal number theoretic entanglement negentropy for two entangled systems with number n of entangled states. n corresponds to Planck constant $h_{eff} = n \times h$ so that a connection with hierarchy of Planck constants is also obtained. Power of the p-adic prime by definition defines largest prime power divisor of n. Individually negentropically entangled systems would be very entropic since there would be n energy-degenerate states with the same Boltzmann weight. Negentropic entanglement changes the situation: thermodynamics of course does not apply anymore. Hence TGD produces same prediction as thermodynamical model but avoids the almost paradox.

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Part II

P-ADIC LENGTH SCALE HYPOTHESIS AND DARK MATTER HIERARCHY

Chapter 6

Recent Status of Lepto-Hadron Hypothesis

6.1 Introduction

TGD suggest strongly ('predicts' is perhaps too strong expression) the existence of color excited leptons. The mass calculations based on p-adic thermodynamics and p-adic conformal invariance lead to a rather detailed picture about color excited leptons.

1. The simplest color excited neutrinos and charged leptons belong to the color octets ν_8 and L_{10} and $L_{\overline{10}}$ decouplet representations respectively and lepto-hadrons are formed as the color singlet bound states of these and possible other representations. Electro-weak symmetry suggests strongly that the minimal representation content is octet and decouplets for both neutrinos and charged leptons.

2. The basic mass scale for lepto-hadron physics is completely fixed by p-adic length scale hypothesis. The first guess is that color excited leptons have the levels $k = 127, 113, 107, ...$ ($p \simeq 2^k$, k prime or power of prime) associated with charged leptons as primary condensation levels. p-Adic length scale hypothesis allows however also the level $k = 11^2 = 121$ in case of electronic lepto-hadrons. Thus both $k = 127$ and $k = 121$ must be considered as a candidate for the level associated with the observed lepto-hadrons. If also lepto-hadrons correspond non-perturbatively to exotic Super Virasoro representations, lepto-pion mass relates to pion mass by the scaling factor $L(107)/L(k) = k^{(107-k)/2}$. For $k = 121$ one has $m_{\pi_L} \simeq 1.057$ MeV which compares favorably with the mass $m_{\pi_L} \simeq 1.062$ MeV of the lowest observed state: thus $k = 121$ is the best candidate contrary to the earlier beliefs. The mass spectrum of lepto-hadrons is expected to have same general characteristics as hadronic mass spectrum and a satisfactory description should be based on string tension concept. Regge slope is predicted to be of order $\alpha' \simeq 1.02/MeV^2$ for $k = 121$. The masses of ground state lepto-hadrons are calculable once primary condensation levels for colored leptons and the CKM matrix describing the mixing of color excited lepton families is known.

The strongest counter arguments against color excited leptons are the following ones.

1. The decay widths of Z^0 and W boson allow only $N = 3$ light particles with neutrino quantum numbers. The introduction of new light elementary particles seems to make the decay widths of Z^0 and W intolerably large.

2. Lepto-hadrons should have been seen in e^+e^- scattering at energies above few MeV. In particular, lepto-hadronic counterparts of hadron jets should have been observed.

A possible resolution of these problems is provided by the loss of asymptotic freedom in lepto-hadron physics. Lepto-hadron physics would effectively exist in a rather limited energy range about one MeV.

The development of the ideas about dark matter hierarchy [K37, K80, K30, K28] led however to a much more elegant solution of the problem.

1. TGD predicts an infinite hierarchy of various kinds of dark matters which in particular means a hierarchy of color and electro-weak physics with weak mass scales labelled by appropriate p-adic primes different from M_{89}: the simplest option is that also ordinary photons and gluons are labelled by M_{89}.

2. There are number theoretical selection rules telling which particles can interact with each other. The assignment of a collection of primes to elementary particle as characterizer of p-adic primes characterizing the particles coupling directly to it, is inspired by the notion of infinite primes [K82] , and discussed in [K37] . Only particles characterized by integers having common prime factors can interact by the exchange of elementary bosons: the p-adic length scale of boson corresponds to a common primes.

3. Also the physics characterized by different values of $\hbar$ are dark with respect to each other as far quantum coherent gauge interactions are considered. Laser beams might well correspond to photons characterized by p-adic prime different from M_{89} and de-coherence for the beam would mean decay to ordinary photons. De-coherence interaction involves scaling down of the Compton length characterizing the size of the space-time of particle implying that particles do not anymore overlap so that macroscopic quantum coherence is lost.

4. Those dark physics which are dark relative to each other can interact only via graviton exchange. If lepto-hadrons correspond to a physics for which weak bosons correspond to a p-adic prime different from M_{89}, intermediate gauge bosons cannot have direct decays to colored excitations of leptons irrespective of whether the QCD in question is asymptotically free or not. Neither are there direct interactions between the QED:s and QCD:s in question if M_{89} characterizes also ordinary photons and gluons. These ideas are discussed and applied in detail in [K37, K80, K30] .

Skeptic reader might stop the reading after these counter arguments unless there were definite experimental evidence supporting the lepto-hadron hypothesis.

1. The production of anomalous e^+e^- pairs in heavy ion collisions (energies just above the Coulomb barrier) suggests the existence of pseudo-scalar particles decaying to e^+e^- pairs. A natural identification is as lepto-pions that is bound states of color octet excitations of e^+ and e^-.

2. The second puzzle, Karmen anomaly, is quite recent [C92] . It has been found that in charge pion decay the distribution for the number of neutrinos accompanying muon in decay $\pi \rightarrow \mu + \nu_\mu$ as a function of time seems to have a small shoulder at $t_0 \sim ms$. A possible explanation is the decay of charged pion to muon plus some new weakly interacting particle with mass of order 30 MeV [C118] : the production and decay of this particle would proceed via mixing with muon neutrino. TGD suggests the identification of this state as color singlet leptobaryon of, say type $L_B = f_{abc}L_8^a L_8^b \bar{L}_8^c$, having electro-weak quantum numbers of neutrino.

3. The third puzzle is the anomalously high decay rate of orto-positronium. [C44] . e^+e^- annihilation to virtual photon followed by the decay to real photon plus virtual lepto-pion followed by the decay of the virtual lepto-pion to real photon pair, $\pi_L\gamma\gamma$ coupling being determined by axial anomaly, provides a possible explanation of the puzzle.

4. There exists also evidence for anomalously large production of low energy e^+e^- pairs [C70, C41, C65, C20] in hadronic collisions, which might be basically due to the production of lepto-hadrons via the decay of virtual photons to colored leptons.

In this chapter a revised form of lepto-hadron hypothesis is described.

1. Sigma model realization of PCAC hypothesis allows to determine the decay widths of lepto-pion and lepto-sigma to photon pairs and e^+e^- pairs. Ortopositronium anomaly determines the value of $f(\pi_L)$ and therefore the value of lepto-pion-lepto-nucleon coupling and the decay rate of lepto-pion to two photons. Various decay widths are in accordance with the experimental data and corrections to electro-weak decay rates of neutron and muon are small.

2. One can consider several alternative interpretations for the resonances.

   Option 1: For the minimal color representation content, three lepto-pions are predicted corresponding to $8, 10, \overline{10}$ representations of the color group. If the lightest lepto-nucleons e_{ex} have masses only slightly larger than electron mass, the anomalous e^+e^- could be actually $e_{ex}^+ + e_{ex}^-$ pairs produced in the decays of lepto-pions. One could identify 1.062, 1.63 and 1.77 MeV states as the three lepto-pions corresponding to $8, 10, \overline{10}$ representations and also understand why the latter two resonances have nearly degenerate masses. Since d and s quarks have same primary condensation level and same weak quantum numbers as colored e and μ, one might argue that also colored e and μ correspond to $k = 121$. From the mass ratio of the colored e and μ, as predicted by TGD, the mass of the muonic lepto-pion should be about 1.8 MeV in the absence of topological mixing. This suggests that 1.83 MeV state corresponds to the lightest $g = 1$ lepto-pion.

   Option 2: If one believes sigma model (in ordinary hadron physics the existence of sigma meson is not established and its width is certainly very large if it exists), then lepto-pions are accompanied by sigma scalars. If lepto-sigmas decay dominantly to e^+e^- pairs (this might be forced by kinematics) then one could adopt the previous scenario and could identify 1.062 state as lepto-pion and 1.63, 1.77 and 1.83 MeV states as lepto-sigmas rather than lepto-pions. The fact that muonic lepto-pion should have mass about 1.8 MeV in the absence of topological mixing, suggests that the masses of lepto-sigma and lepto-pion should be rather close to each other.

   Option 3: One could also interpret the resonances as string model 'satellite states' having interpretation as radial excitations of the ground state lepto-pion and lepto-sigma. This identification is not however so plausible as the genuinely TGD based identification and will not be discussed in the sequel.

3. PCAC hypothesis and sigma model leads to a general model for lepto-hadron production in the electromagnetic fields of the colliding nuclei and production rates for lepto-pion and other lepto-hadrons are closely related to the Fourier transform of the instanton density $\bar{E} \cdot \bar{B}$ of the electromagnetic field created by nuclei. The first source of anomalous e^+e^- pairs is the production of $\sigma_L \pi_L$ pairs from vacuum followed by $\sigma_L \rightarrow e^+e^-$ decay. If $e_{ex}^+ e_{ex}^-$ pairs rather than genuine e^+e^- pairs are in question, the production is production of lepto-pions from vacuum followed by lepto-pion decay to lepto-nucleon pair.

   Option 1: For the production of lepto-nucleon pairs the cross section is only slightly below the experimental upper bound for the production of the anomalous e^+e^- pairs and the decay rate of lepto-pion to lepto-nucleon pair is of correct order of magnitude.

   Option 2: The rough order of magnitude estimate for the production cross section of anomalous e^+e^- pairs via $\sigma_l \pi_l$ pair creation followed by $\sigma_L \rightarrow e^+e^-$ decay, is by a factor of order $1/\sum N_c^2$ (N_c is the total number of states for a given colour representation and sum over the representations contributing to the ortopositronium anomaly appears) smaller than the reported cross section in case of 1.8 MeV resonance. The discrepancy could be due to the neglect of the large radiative corrections (the coupling $g(\pi_L \pi_L \sigma_L) = g(\sigma_L \sigma_L \sigma_L)$ is very large) and also due to the uncertainties in the value of the measured cross section.

   Given the unclear status of sigma in hadron physics, one has a temptation to conclude that anomalous e^+e^- pairs actually correspond to lepto-nucleon pairs.

4. The vision about dark matter suggests that direct couplings between leptons and lepto-hadrons are absent in which case no new effects in the direct interactions of ordinary leptons are predicted. If colored leptons couple directly to ordinary leptons, several new physics effects such as resonances in photon-photon scattering at cm energy equal to lepto-pion masses and the production of $e_{ex}\bar{e}_{ex}$ (e_{ex} is leptobaryon with quantum numbers of electron) and $e_{ex}\bar{e}$ pairs in heavy ion collisions, are possible. Lepto-pion exchange would give dominating contribution to $\nu - e$ and $\bar{\nu} - e$ scattering at low energies. Lepto-hadron jets should be observed in e^+e^- annihilation at energies above few MeV:s unless the loss of asymptotic freedom restricts lepto-hadronic physics to a very narrow energy range and perhaps to entirely non-perturbative regime of lepto-hadronic QCD.

This chapter is a revised version of the earlier chapter [K3] and still a work in progress. I apologize for the reader for possible inconvenience. The motivation for the re-writing came from the evidence for the production of τ-pions in high energy proton-antiproton collisions [C26] . Since the kinematics of these collisions differs dramatically from that for heavy ion collisions, a critical re-examination of the earlier model - which had admittedly somewhat ad hoc character- became necessary. As a consequence the earlier model simplified dramatically. As far as basic calculations are considered, the modification makes itself visible only at the level of coefficients. Even more remarkably, it turned out possible to calculate exactly the lepto-pion production amplitude under a very natural approximation, which can be also generalized so that the calculation of production amplitude can be made analytically in high accuracy and only the integration over lepto-pion momentum must be carried out numerically. As a consequence, a rough analytic estimate for the production cross section follows and turns out to be of correct order of magnitude. It must be however stressed that the cross section is highly sensitive to the value of the cutoff parameter (at least in this naive estimate) and only a precise calculation can settle the situation.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- TGD view about elementary particles [L43]
- p-Adic mass calculations [L35]
- Leptohadron hypothesis [L30]

6.2 Lepto-hadron hypothesis

6.3 Lepto-hadron hypothesis

6.3.1 Anomalous e^+e^- pairs in heavy ion collisions

Heavy ion collision experiments carried out at the Gesellschaft fur Schwerionenforschung in Darmstadt, West Germany [C58, C61, C72, C56] have yielded a rather puzzling set of results. The expectation was that in heavy ion collisions in which the combined charge of the two colliding ions exceeds 173, a composite nucleus with $Z > Z_{cr}$ would form and the probability for spontaneous positron emission would become appreciable.

Indeed, narrow peaks of widths of roughly 50-70 keV and energies about 350$\pm$ 50 keV were observed in the positron spectra but it turned out that the position of the peaks seems to be a constant function of Z rather that vary as Z^{20} as expected and that peaks are generated also for Z smaller than the critical Z. The collision energies at which peaks occur lie in the neighborhood of 5.7-6 MeV/nucleon. Also it was found that positrons are accompanied by e^-- emission. Data are consistent with the assumption that some structure at rest in cm is formed and decays subsequently to e^+e^- pair.

Various theoretical explanations for these peaks have been suggested [C24, C96] . For example, lines might be created by pair conversion in the presence of heavy nuclei. In nuclear physics explanations the lines are due to some nuclear transition that occurs in the compound nucleus formed in the collision or in the fragmets formed. The Z-independence of the peaks seems however to exclude both atomic and nuclear physics explanations [C24] . Elementary particle physics explanations [C24, C96] seem to be excluded already by the fact that several peaks have been observed in the range $1.6 - 1.8\ MeV$ with widths of order $10^1 - 10^2\ keV$. These states decay to e^+e^- pairs. There is evidence for one narrow peak with width of order one keV at 1.062 Mev [C24] : this state decays to photon-photon pairs.

Thus it seems that the structures produced might be composite, perhaps resonances in e^+e^- system. The difficulty of this explanation is that conventional QED seems to offer no natural explanation for the strong force needed to explain the energy scale of the states. One idea is that the strong electromagnetic fields create a new phase of QED [C24] and that the resonances are analogous to pseudo-scalar mesons appearing as resonances in strongly interacting systems.

TGD based explanation relies on the following hypothesis motivated by Topological Geometrodynamics.

1. Ordinary leptons are not point like particles and can have colored excitations, which form color singlet bound states. A natural identification for the primary condensate level is $k = 121$ so that the mass scale is of order one MeV for the states containing lowest generation colored leptons. The fact that d and s quarks, having the same weak quantum numbers as charged leptons, have same primary condensation level, suggests that both colored electron and muon condense to the same level. The expectation that lepto-hadron physics exists in a narrow energy interval only, suggests that also colored τ should condense on the same level.

2. The states in question are lepto-hadrons, that is color confined states formed from the colored excitations of e^+ and e^-. The decay rate to lepto-nucleon pairs $e^+_{ex}e^-_{ex}$ is large and turns out to give rise to correct order of magnitude for the decay width. Hence two options emerge.

   Option 1: Lepto-nucleons e_{ex} have masses only slightly above the electron mass and since they behave like electrons, anomalous e^+e^- pairs could actually correspond to lepto-nucleon pairs created in the decays of lepto-pions. 1.062, 1.63 and 1.77 MeV states can be identified as lowest generation lepto-pions correspond to octet and two decouplets. 1.83 MeV state could be identified as the second generation lepto-pion corresponding to colored muon. The small branching fraction to gamma pairs explains why the decays of the higher mass lepto-pions to gamma pairs has not been observed. $g = 0$ lepto-pion decays to lepto-nucleon pairs can be visualized as occurring via dual diagrams obeying Zweig's rule (annihilation is not allowed inside incoming or outgoing particle states). The decay of $g = 1$ colored muon pair occurs via Zweig rule violating annihilation to two gluon intermediate state, which transforms back to virtual $g = 0$ colored electron pair decaying via dual diagram: the violation of Zweig's rule suggests that the decay rate for 1.8 MeV state is smaller than for the lighter states. Quantitive model shows that this scenario is the most plausible one.

   Option 2: Lepto-sigmas, which are the scalar partners of lepto-pions predicted by sigma model, are the source of anomalous (and genuine) e^+e^- pairs. In this case 1.062 state must correspond to lepto-pion whereas higher states must be identified as lepto-sigmas. Also now new lepto-pion states decaying to gamma pairs are predicted and one could hence argue that this prediction is not consistent with what has been observed. A crucial assumption is that lepto-sigmas are light and cannot decay to other lepto-mesons. Ordinary hadronic physics suggests that this need not be the case: the hadronic decay width of the ordinary sigma, if it exists, is very large.

The program of the section is following:

1. PCAC hypothesis, successful in low energy pion physics, is generalized to the case of lepto-pion. Hypothesis allows to deduce the coupling of lepto-pion to leptons and lepto-baryons in terms of leptobaryon-lepton mixing angles. Ortopositronium anomaly allows to deduce precise value of $f(\pi_L)$ characterizing the decay rate of lepto-pion so that the crucial parameters of the model are completely fixed. The decay rates of lepto-pion to photon pair and of lepto-sigma to ordinary e^+e^- pairs are within experimental bounds and corrections to muon and beta decay rates are small. New calculable resonance contributions to photon-photon scattering at cm energy equal to lepto-pion masses are predicted.

2. If anomalous e^+e^- pairs are actually lepto-nucleon pairs, only a model for the creation of lepto-pions from vacuum is needed. In an external electromagnetic field lepto-pion develops a vacuum expectation value proportional to electromagnetic anomaly term [B17] so that the production amplitude for the lepto-pion is essentially the Fourier transform of the scalar product of the electric field of the stationary target nucleus with the magnetic field of the colliding nucleus.

3. If anomalous e^+e^- pairs are produced in the decays of lepto-sigmas, the starting point is sigma model providing a realization of PCAC hypothesis. Sigma model makes it possible to

relate the production amplitude for $\sigma_L\pi_L$ pairs to the lepto-pion production amplitude: the key element of the model is the large value of the $\sigma\pi_L\pi_L$ coupling constant.

4. Lepto-hadron production amplitudes are proportional to lepto-pion production amplitude and this motivates a detailed study of lepto-pion production. Two models for lepto-pion production are developed: in classical model colliding nucleus is treated classically whereas in quantum model the colliding nucleus is described quantum mechanically. It turns out that classical model explains the peculiar production characteristics of lepto-pion but that production cross section is too small by several orders of magnitude. Quantum mechanical model predicts also diffractive effects: production cross section varies rapidly as a function of the scattering angle and for a fixed value of scattering angle there is a rapid variation with the collision velocity. The estimate for the total lepto-pion production cross section increases by several orders of magnitude due to the coherent summation of the contributions to the amplitude from different values of the impact parameter at the peak.

5. The production rate for lepto-nucleon pairs is only slightly smaller than the experimental upper bound but the e^+e- production rate predicted by sigma model approach is still by a factor of order $1/\sum N_c^2$ smaller than the reported maximum cross section. A possible explanation for this discrepancy is the huge value of the coupling $g(\pi_L,\pi_L,\sigma_L) = g(\sigma_L,\sigma_L,\sigma_L)$ implying that the diagram involving the exchange of virtual sigma can give the dominant contribution to the production cross section of $\sigma_L\pi_L$ pair.

6.3.2 Lepto-pions and generalized PCAC hypothesis

One can say that the PCAC hypothesis predicts the existence of pions and a connection between the pion nucleon coupling strength and the pion decay rate to leptons. In the following we give the PCAC argument and its generalization and consider various consequences.

PCAC for ordinary pions

The PCAC argument for ordinary pions goes as follows [B24] :

1. Consider the contribution of the hadronic axial current to the matrix element describing lepton nucleon scattering (say $N+\nu \to P+e^-$) by weak interactions. The contribution in question reduces to the well-known current-current form

$$\begin{aligned} M &= \frac{G_F}{\sqrt{2}} g_A L_\alpha \langle P|A^\alpha|P\rangle \ , \\ L_\alpha &= \bar{e}\gamma_\alpha(1+\gamma_5)\nu \ , \\ \langle P|A^\alpha|P\rangle &= \bar{P}\gamma^\alpha N \ , \end{aligned} \tag{6.3.1}$$

where $G_F = \frac{\pi\alpha}{2m_W^2 sin^2(\theta_W)} \simeq 10^{-5}/m_p^2$ denotes the dimensional weak interaction coupling strength and g_A is the nucleon axial form factor:$g_A \simeq 1.253$.

2. The matrix element of the hadronic axial current is not divergenceless, due to the non-vanishing nucleon mass,

$$a_\alpha\langle P|A^\alpha|P\rangle \simeq 2m_p\bar{P}\gamma_5 N \ . \tag{6.3.2}$$

Here q^α denotes the momentum transfer vector. In order to obtain divergenceless current, one can modify the expression for the matrix element of the axial current

$$\langle P|A^\alpha|N\rangle \rightarrow \langle P|A^\alpha|N\rangle - q^\alpha 2m_p\bar{P}\gamma_5 N\frac{1}{q^2} \ . \tag{6.3.3}$$

3. The modification introduces a new term to the lepton-hadron scattering amplitude identifiable as an exchange of a massless pseudo-scalar particle

$$\delta T = \frac{G_F g_A}{\sqrt{2}} L_\alpha \frac{2m_p q^\alpha}{q^2} \bar{P}\gamma_5 N \ . \tag{6.3.4}$$

   The amplitude is identifiable as the amplitude describing the exchange of the pion, which gets its mass via the breaking of chiral invariance and one obtains by the straightforward replacement $q^2 \rightarrow q^2 - m_\pi^2$ the correct form of the amplitude.

4. The nontrivial point is that the interpretations as pion exchange is indeed possible since the amplitude obtained is to a good approximation identical to that obtained from the Feynman diagram describing pion exchange, where the pion nucleon coupling constant and pion decay amplitude appear

$$T_2 = \frac{G}{\sqrt{2}} f_\pi q^\alpha L_\alpha \frac{1}{q^2 - m_\pi^2} g\sqrt{2}\bar{P}\gamma_5 N \ . \tag{6.3.5}$$

   The condition $\delta T \sim T_2$ gives from Goldberger-Treiman [B24]

$$g_A(\simeq 1.25) = \sqrt{2}\frac{f_\pi g}{2m_p}(\simeq 1.3) \ , \tag{6.3.6}$$

   satisfied in a good accuracy experimentally.

PCAC in leptonic sector

A natural question is why not generalize the previous argument to the leptonic sector and look at what one obtains. The generalization is based on following general picture.

1. There are two levels to be considered: the level of ordinary leptons and the level of leptobaryons of, say type $f_{ABC}\nu_8^A\nu_8^B\bar{L}_{10}^C$, possessing same quantum numbers as leptons. The interaction transforming these states to each other causes in mass eigenstates mixing of leptobaryons with ordinary leptons described by mixing angles. The masses of lepton and corresponding leptobaryon could be quite near to each other and in case of electron this should be the case as it turns out.

2. A counterargument against the applications of PCAC hypothesis at level of ordinary leptons is that baryons and mesons are both bound states of quarks whereas ordinary leptons are not bound states of colored leptons. The divergence of the axial current is however completely independent of the possible internal structure of leptons and microscopic emission mechanism. Ordinary lepton cannot emit lepto-pion directly but must first transform to leptobaryon with same quantum numbers: phenomenologically this process can be described using mixing angle $sin(\theta_B)$. The emission of lepto-pion proceeds as $L \rightarrow B_L$: $B_L \rightarrow B_L + \pi_L$: $B_L \rightarrow L$, where B_L denotes leptobaryon of type structure $f_{ABC}L_8^A L_8^B \bar{L}_8^C$. The transformation amplitude $L \rightarrow B_L$ is proportional to the mixing angle $sin(\theta_L)$.

Three different PCAC type identities are assumed to hold true:
PCAC1) The vertex for the emission of lepto-pion by ordinary lepton is equivalent with the graph in which lepton L transforms to leptobaryon L^{ex} with same quantum numbers, emits lepto-pion and transforms back to ordinary lepton. The assumption relates the couplings $g(L_1, L_2)$ and $g(L_1^{ex}, L_2^{ex})$ (analogous to strong coupling) and mixing angles to each other

$$g(L_1,L_2) \quad = \quad g(L_1^{ex},L_2^{ex})sin(\theta_1)sin(\theta_2) \ . \tag{6.3.7}$$

The condition implies that in electro-weak interactions ordinary leptons do not transform to their exotic counterparts.
PCAC2) The generalization of the ordinary Goldberger-Treiman argument holds true, when ordinary baryons are replaced with leptobaryons. This gives the condition expressing the coupling $f(\pi_L)$ of the lepto-pion state to axial current defined as

$$\langle vac|A_\alpha|\pi_L\rangle = ip_\alpha f(\pi_L) \ , \tag{6.3.8}$$

in terms of the masses of leptobaryons and strong coupling g.

$$f(\pi_L) \quad = \quad \sqrt{2}g_A\frac{(m_{ex}(1)+m_{ex}(2))sin(\theta_1)sin(\theta_2)}{g(L_1,L_2)} \ , \tag{6.3.9}$$

where g_A is parameter characterizing the deviation of weak coupling strength associated with leptobaryon from ideal value: $g_A \sim 1$ holds true in good approximation.
PCAC3) The elimination of leptonic axial anomaly from leptonic current fixes the values of $g(L_i,L_j)$.

1. The standard contribution to the scattering of leptons by weak interactions given by the expression

$$\begin{aligned} T &= \frac{G_F}{\sqrt{2}}\langle L_1|A^\alpha|L_2\rangle\langle L_3|A_\alpha|L_4\rangle \ , \\ \langle L_i|A^\alpha|L_j\rangle &= \bar{L}_i\gamma^\alpha\gamma_5 L_j \ . \end{aligned} \tag{6.3.10}$$

2. The elimination of the leptonic axial anomaly

$$q_\alpha\langle L_i|A^\alpha|L_j\rangle \quad = \quad (m(L_i)+m(L_j))\bar{L}_i\gamma_5 L_j \ , \tag{6.3.11}$$

by modifying the axial current by the anomaly term

$$\langle L_i|A^\alpha|L_j\rangle \quad \rightarrow \quad \langle L_i|A^\alpha|L_j\rangle - (m(L_i)+m(L_j))\frac{q^\alpha}{q^2}\bar{L}_i\gamma_5 L_j \ , \tag{6.3.12}$$

induces a new interaction term in the scattering of ordinary leptons.

3. It is assumed that this term is equivalent with the exchange of lepto-pion. This fixes the value of the coupling constant $g(L_1,L_2)$ to

$$\begin{aligned} g(L_1,L_2) &= 2^{1/4}\sqrt{G_F}(m(L_1)+m(L_2))\xi \ , \\ \xi(charged) &= 1 \ , \\ \xi(neutral) &= cos(\theta_W) \ . \end{aligned} \tag{6.3.13}$$

Here the coefficient ξ is related to different values of masses for gauge bosons W and Z appearing in charged and neutral current interactions. An important factor 2 comes from the modification of the axial current in both matrix elements of the axial current.

Lepto-pion exchange interaction couples right and left handed leptons to each other and its strength is of the same order of magnitude as the strength of the ordinary weak interaction at energies not considerably large than the mass of the lepto-pion. At high energies this interaction is negligible and the existence of the lepto-pion predicts no corrections to the parameters of the standard model since these are determined from weak interactions at much higher energies. If lepto-pion mass is sufficiently small (as found, $m(\pi_L) < 2m_e$ is allowed by the experimental data), the interaction mediated by lepto-pion exchange can become quite strong due to the presence of the lepto-pion progator. The value of the lepton-lepto-pion coupling is $g(e,e) \equiv g \sim 5.6 \cdot 10^{-6}$. It is perhaps worth noticing that the value of the coupling constant is of the same order as lepton-Higgs coupling constant and also proportional to the mass of the lepton.

PCAC identities fix the values of coupling constants apart from the values of mixing angles. If one assumes that the strong interaction mediated by lepto-pions is really strong and the coupling strength $g(L_{ex}, L_{ex})$ is of same order of magnitude as the ordinary pion nucleon coupling strength $g(\pi NN) \simeq 13.5$ one obtains an estimate for the value of the mixing angle $sin(\theta_e)$ $sin^2(\theta_e) \sim \frac{g(\pi NN)}{g(L,L)} \sim 2.4 \cdot 10^{-6}$. This implies the order of magnitude $f(\pi_L) \sim 10^{-6} m_W \sim 10^2\ keV$ for $f(\pi_L)$. The order of magnitude is correct as will be found. Ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma \sim 10^{-3}$ and the assumption $m_{ex} \geq 1.3\ MeV$ (so that $e_{ex}\bar{e}$ decay is not possible) gives the upper bound $sin(\theta_e) \leq x \cdot \sqrt{N_c} \cdot 10^{-4}$, where the value of $x \sim 1$ depends on the number of the lepto-pion type states and on the precise value of the Op anomaly.

6.3.3 Lepto-pion decays and PCAC hypothesis

The PCAC argument makes it possible to predict the lepto-pion coupling and decay rates of the lepto-pion to various channels. Actually the orders of magnitude for the decay rates of the lepto-sigma and other lepto-mesons can be deduced also. The comparison with the experimental data is made difficult by the uncertainty of the identifications. The lightest candidate has mass 1.062 MeV and decay width of order 1 keV [C24] : only photon photon decay has been observed for this state. The next lepto-meson candidates are in the mass range $1.6 - 1.8\ MeV$. Perhaps the best status is possessed by 'Darmstadtium' with mass 1.8 MeV. For these states decays to final states identified as e^+e^- pairs dominate: if indeed e^+e^- pairs, these states probably correspond to the decay products of lepto-sigma. Another possibility is that pairs are actually lepto-nucleon pairs with the mass of the lepto-nucleon only slightly larger than electron mass. Hadron physics experience suggests that the decay widths of the lepto-hadrons (lepto-pion forming a possible exception) should be about 1-10 per cent of particle mass as in hadron physics. The upper bounds for the widths are indeed in the range $50 - 70\ keV$ [C24] .

$\Gamma(\pi_L \rightarrow \gamma\gamma)$

As in the case of the ordinary pion, anomaly considerations give the following approximate expression for the decay rate of the lepto-pion to two-photon final states [B17])

$$\Gamma(\pi_L \rightarrow \gamma\gamma) = \frac{N_c^2 \alpha^2 m^3(\pi_L)}{64 f(\pi_L)^2 \pi^3} \ . \qquad (6.3.14)$$

$N_c = 8, 10$ is the number of the colored lepton states coming from the axial anomaly loop. For $m(\pi_L) = 1.062\ MeV$ and $f(\pi_L) = N_c \cdot 7.9\ keV$ implied by the ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma = 10^{-3}$ one has $\Gamma(\gamma\gamma) = .52\ keV$, which is consistent with the experimental estimate of order 1 keV [C24] .

In fact, several lepto-pion states could exist (4 at least corresponding to the resonances at 1.062, 1.63, 1.77 and 1.83 MeV). Since all these lepto-pion states contribute to Op decay rate, the actual value of $f(\pi_L)$ assumed to scale as $m(\pi_L)$, is actually larger in this case: it turns out that $f(\pi_L)$ for the lightest lepto-pion increases to $f(\pi_L)(lightest) = N_c \cdot 15\ keV$ and gives $\Gamma(\gamma\gamma) \simeq .13\ keV$ in case of the lightest lepto-pion if lepto-pions are assumed to correspond the resonances. Note that the order of magnitude for $f(\pi_L)$ is same as deduced from the assumption that lepto-hadronic counterpart of $g(\pi NN)$ equals to the ordinary $g(\pi NN)$. The increase of the ortopositronium anomaly by a factor of, say 4, implies corresponding decrease in $f(\pi_L)^2$. The value of $f(\pi_L)$ is also sensitive to the precise value of the mass of the lightest lepto-pion.

Lepto-pion-lepton coupling

The value of the lepto-pion-lepton coupling can be used to predict the decay rate of lepto-pion to leptons. One obtains for the decay rate $\pi_L^0 \rightarrow e^+e^-$ the estimate

$$\begin{aligned} \Gamma(\pi_L \rightarrow e^+e^-) &= 4\frac{g(e,e)^2\pi}{2(2\pi)^2}(1-4x^2)m(\pi_L) \\ &= 16Gm_e^2cos^2(\theta_W)\frac{\sqrt{2}}{4\pi}(1-4x^2)m(\pi_L) \ , \\ x &= \frac{m_e}{m(\pi_L)} \ . \end{aligned} \tag{6.3.15}$$

for the decay rate of the lepto-pion: for lepto-pion mass $m(\pi_L) \simeq 1.062\ MeV$ one obtains for the decay rate the estimate $\Gamma \sim 1/(1.3 \cdot 10^{-8}\ sec)$: the low decay rate is partly due to the phase space suppression and implies that e^+e^- decay products cannot be observed in the measurement volume. The low decay rate is in accordance with the identification of the lepto-pion as the $m = 1.062\ MeV$ lepto-pion candidate. In sigma model lepto-pion and lepto-sigma have identical lifetimes and for lepto-sigma mass of order $1.8\ MeV$ one obtains $\Gamma(\sigma_L \rightarrow e^+e^-) \simeq 1/(8.2 \cdot 10^{-10}\ sec)$: the prediction is larger than the lower limit $\sim 1/(10^{-9}\ sec)$ for the decay rate implied by the requirement that σ_L decays inside the measurement volume. The estimates of the lifetime obtained from heavy ion collisions [C78] give the estimate $\tau \geq 10^{-10}\ sec$. The large value of the lifetime is in accordance with the limits for the lifetime obtained from Babbha scattering [C69] , which indicate that the lifetime must be longer than 10^{-12} sec.

For lepto-meson candidates with mass above $1.6\ MeV$ no experimental evidence for other decay modes than $X \rightarrow e^+e^-$ has been found and the empirical upper limit for $\gamma\gamma/e^+e^-$ branching ratio [C59] is $\Gamma(\gamma\gamma)/\Gamma(e^+e^-) \leq 10^{-3}$. If the identification of the decay products as e^+e^- pairs is correct then the only possible conclusion is that these states cannot correspond to lepto-pion since lepto-pion should decay dominantly into photon photon pairs. Situation changes if pairs of lepton-ucleons $e_{ex}\overline{e}_{ex}$ of type $e_{ex} = e_8\nu_8\overline{\nu}_8$ pair are in question.

I realized that this conclusion might be questioned for more than decade after writing the above text as I developed a model for CDF anomaly suggesting the existence of τ-pions. Since colored leptons are color octets, anomalous magnetic moment type coupling of form $\overline{L}Tr(F^{\mu\nu}\Sigma_{\mu\nu}L_8)$ (the trace is over the Lie-algebra generators of $SU(3)$ and $F^{\mu\nu}$ denotes color gauge field) between ordinary lepton, colored lepton and lepto-gluon is possible. The exchange of a virtual lepto-gluon allows lepto-pion to decay by lepto-strong interactions to electron-positron pairs. The decay rate is limited by the kinematics for the lightest state very near to the final state mass and might make decay rate to in this case very small. If the rate for the decay to electron-positron pair is comparable to that for the decay to two photons the production rate for electron-positron pairs could be of the same order of magnitude as lepto-pion production rate. The anomalous magnetic moment of electron however poses strong limitations on this coupling and it might be that the coupling is too small. This coupling could however induce the mixing of e_{ex} with e.

$\Gamma(\pi_L \rightarrow e + \bar{\nu_e})$

The expression for the decay rate $\pi_L \rightarrow e + \bar{\nu_e}$ reads as

$$\begin{aligned} \Gamma(\pi_L^- \rightarrow e\nu_e) &= 8Gm_e^2\frac{(1-x^2)^2}{2(1+x^2)}\frac{\sqrt{2}}{(2\pi)^5}m(\pi_L) \ , \\ &= \frac{4}{cos^2(\theta_W)}\frac{(1-x^2)}{(1+x^2)(1-4x^2)}\Gamma(\pi_L^0 \rightarrow e^+e^-) \ , \end{aligned} \tag{6.3.16}$$

and gives $\Gamma(\pi_L^- \rightarrow e\nu_e) \simeq 1/(3.6 \cdot 10^{-10}\ sec)$ for $m(\pi_L) = 1.062\ MeV$.

$\Gamma(\pi_L/\sigma_L \rightarrow e_{ex}\overline{e}_{ex})$ **and** $\Gamma(\pi_L/\sigma_L \rightarrow e_{ex}\overline{e})$

Sigma model predicts lepto-pion and lepto-sigma to have same coupling to lepto-nucleon e_{ex} pair so that in the sequel only lepto-pion decay rates are considered. One must consider also the possibility that lepto-pion decay products are either $e_{ex}\bar{e}_{ex}$ or $e_{ex}\bar{e}$ pairs with e_{ex} having mass of near the mass of electron so that it could be misidentified as electron. If the mass of lepto-nucleon e_{ex} with quantum numbers of electron is smaller than $m(\pi_L)/2$ it can be produced in lepto-pion annihilation. One must also assume $m(e_{ex}) > m_e$: otherwise electrons would spontaneously decay to lepto-nucleons via photon emission. The production rate to lepto-nucleon pair can be written as

$$\begin{aligned} \Gamma(\pi_L \rightarrow e^+_{ex} e^-_{ex}) &= \frac{1}{sin^4(\theta_e)} \frac{(1-4y^2)}{(1-4x^2)} \Gamma(\pi_L \rightarrow e^+ e^-) \ , \\ y &= \frac{m(e_{ex})}{m(\pi_L)} \ . \end{aligned} \tag{6.3.17}$$

If $e - e_{ex}$ mass difference is sufficiently small the kinematic suppression does not differ significantly from that for e^+e^- pair. The limits from Babbha scattering give no bounds on the rate of $\pi_L \rightarrow e^+_{ex} e^-_{ex}$ decay. The decay rate $\Gamma \sim 10^{20}/sec$ implied by $sin(\theta_e) \sim 10^{-4}$ implies decay width of order .1 MeV: the order of magnitude is the naively expected one and means that the decay to $e^+_{ex} e^-_{ex}$ pairs dominates over the decay to gamma pairs except in the case of the lightest lepto-pion state for which the decay is kinematically forbidden.

The decay rate of the lepto-pion to $\bar{e}e_{ex}$ pair has sensible order of magnitude: for $sin(\theta_e) = 1.2 \cdot 10^{-3}$, $m_{\sigma_L} = 1.8\ MeV$ and $m_{e_{ex}} = 1.3\ MeV$ one has $\Gamma \simeq 60\ eV$ allowed by the experimental limits. This decay is kinematically possible only provided the mass of e_{ex} is in below $1.3\ MeV$. These decays should dominate by a factor $1/sin^2(\theta_e)$ over e^+e^- decays if kinematically allowed.

A signature of these events, if identified erratically as electron positron pairs, is the non-vanishing value of the energy difference in the cm frame of the pair: $E(e^-) - E(e^+) \simeq (m^2(e_{ex}) - m_e^2)/2E > 160\ keV$ for $E = 1.8\ MeV$. If the decay $e_{ex} \rightarrow e + \gamma$ takes place before the detection the energy asymmetry changes its sign. Energy asymmetry [C64] increasing with the rest energy of the decaying object has indeed been observed: the proposed interpretation has been that electron forms a bound state with the second nucleus so that its energy is lowered. Also a deviation from the momentum distribution implied by the decay of neutral particle to e^+e^- pair (momenta are opposite in the rest frame) results from the emission of photon. This kind of deviation has also been observed [C64] : the proposed explanation is that third object is involved in the decay. A possible alternative explanation for the asymmetries is the production mechanism ($\sigma_L \pi_L$ pairs instead of single particle states).

$\Gamma(e_{ex} \rightarrow e + \gamma)$

The decay to electron and photon would be a unique signature of e_{ex}. The general feature of of fermion family mixing is that mixing takes place in charged currents. In present case mixing is of different type so that $e_{ex} \rightarrow e + \gamma$ might be allowed. If this is not the case then the decay takes place as weak decay via the emission of virtual W boson: $e_{ex} \rightarrow e + \nu_e + \bar{\nu}_e$ and is very slow due to the presence of mixing angle and kinematical suppression. The energy of the emitted photon is $E_\gamma = (m^2_{ex} - m^2_e)/2m_e$. The decay rate $\Gamma(e_{ex} \rightarrow e + \gamma)$ is given by

$$\begin{aligned} \Gamma(e_{ex} \rightarrow e + \gamma) &= \alpha_{em} sin^2(\theta_e) X m_e \ , \\ X &= \frac{(m_1 - m_e)^3 (m_1 + m_e) m_e}{(m_1^2 + m_e^2)^2 m_1} \ . \end{aligned} \tag{6.3.18}$$

For $m(e_{ex}) = 1.3\ MeV$ the decay of order $1/(1.4 \cdot 10^{-12}\ sec)$ for $sin(\theta_e) = 1.2 \cdot 10^{-3}$ so that lepto-nucleons would decay to electrons in the measurement volume. In the experiments positrons are identified via pair annihilation and since pair annihilation rate for $\bar{e}_{ex}$ is by a factor $sin^2(\theta_e)$ slower than for e^+ the particles identified as positrons must indeed be positrons. For sufficiently small mass difference $m(e_{ex}) - m_e$ the particles identified as electron could actually be e_{ex}. The decay of e_{ex} to electron plus photon before its detection seems however more reasonable alternative since it could explain the observed energy asymmetry [C64] .

Some implications

The results have several implications as far as the decays of on mass shell states are considered:

1. For $m(e_{ex}) > 1.3\ MeV$ the only kinematically possible decay mode is the decay to e^+e^- pair. Production mechanism might explain the asymmetries [C64] . The decay rate of on mass shell π_L and σ_L (or $\eta_L, \rho_L, ..$) is above the lower limit allowed by the detection in the measurement volume.

2. If the mass of e_{ex} is larger than .9 MeV but smaller than 1.3 MeV $e_{ex}\bar{e}$ decays dominate over e^+e^- decays. The decay $e_{ex} \rightarrow e + \gamma$ before detection could explain the observed energy asymmetry.

3. It will be found that the direct production of $e_{ex}\bar{e}$ pairs is also possible in the heavy ion collision but the rate is much smaller due to the smaller phase space volume in two-particle case. The annihilation rate of $\bar{e}_{ex}$ in matter is by a factor $sin^2(\theta_e)$ smaller than the annihilation rate of positron. This provides a unique signature of e_{ex} if e^+ annihilation rate in matter is larger than the decay rate of $\bar{e}_{ex}$. In lead the lifetime of positron is $\tau \sim 10^{-10}$ sec and indeed larger that e_{ex} lifetime.

Karmen anomaly

A brief comment on the Karmen anomaly [C92] observed in the decays of π^+ is in order. The anomaly suggests the existence [C118] of new weakly interacting neutral particle x, which mixes with muon neutrino. Since $g = 1$ neutrino corresponds to charmed quark in hadron physics context having $k = 103$ rather than $k = 107$ as primary condensation level, a natural guess for its primary condensation level is $k = 113$, which would mean that the mass scale would be of order muon mass: the particle candidate indeed has mass of order 30 MeV. One class of solutions to laboratory constraints, which might evade also cosmological and astrophysical constraints, corresponds to object x mixing with muon type neutrino and decaying radiatively to $\gamma + \nu_\mu$ via the emission of virtual W boson. The value of the mixing parameter $U(\mu, x)$ describing $\nu_{mu} - x$ mixing satisfies $|U_{\mu,x}|^4 \simeq .8 \cdot 10^{-10}$.

The following naive PCAC argument gives order of magnitude estimate for $|U(\mu, x)| \sim sin(\theta_\mu)$. The value of $g(\mu, \mu)$ is by a factor $m(\mu)/m_e$ larger than $g(e, e)$. If the lepto-hadronic couplings $g(\mu_{ex}, \mu_{ex})$ and $g(e_{ex}, e_{ex})$ are of same order of magnitude then one has $sin(\theta_\mu) \leq .02$ (3 lepto-pion states and Op anomaly equal to $Op = 5 \cdot 10^{-3}$): the lower bound is 6.5 times larger than the value .003 deduced in [C118] . The actual value could be considerably smaller since e_{ex} mass could be larger than 1.3 MeV by a factor of order 10.

6.3.4 Lepto-pions and weak decays

The couplings of lepto-meson to electro-weak gauge bosons can be estimated using PCAC and CVC hypothesis [B17] . The effective $m_{\pi_L} - W$ vertex is the matrix element of electro-weak axial current between vacuum and charged lepto-meson state and can be deduced using same arguments as in the case of ordinary charged pion

$$\langle 0|J_A^\alpha|\pi_L^-\rangle = Km(\pi_l)p^\alpha \quad , \tag{6.3.19}$$

where K is some numerical factor and p^α denotes the momentum of lepto-pion. For neutral lepto-pion the same argument gives vanishing coupling to photon by the conservation of vector current. This has the important consequence that lepto-pion cannot be observed as resonance in e^+e^- annihilation in single photon channel. In two photon channel lepto-pion should appear as resonance. The effective interaction Lagrangian is the 'instanton' density of electromagnetic field giving additional contribution to the divergence of the axial current and was used to derive a model for lepto-pion production in heavy ion collisions.

Lepto-hadrons and lepton decays

The lifetime of charged lepto-pion is from PCAC estimates larger than 10^{-10} seconds by the previous PCAC estimates. Therefore lepto-pions are practically stable particles and can appear in the final states of particle reactions. In particular, lepto-pion atoms are possible and by Bose statistics have the peculiar property that ground state can contain many lepto-pions.

Lepton decays $L \rightarrow \nu_\mu + H_L$, $L = e, \mu, \tau$ via emission of virtual W are kinematically allowed and an anomalous resonance peak in the neutrino energy spectrum at energy

$$E(\nu_L) = \frac{m(L)}{2} - \frac{m_H^2}{2m(L)} \ , \tag{6.3.20}$$

provides a unique test for the lepto-hadron hypothesis. If lepto-pion is too light electrons would decay to charged lepto-pions and neutrinos unless the condition $m(\pi_L) > m_e$ holds true.

The existence of a new decay channel for muon is an obvious danger to the lepto-hadron scenario: large changes in muon decay rate are not allowed.

Consider first the decay $\mu \rightarrow \nu_\mu + \pi_L$ where π_L is on mass shell lepto-pion. Lepto-pion has energy $\sim m(\mu)/2$ in muon rest system and is highly relativistic so that in the muon rest system the lifetime of lepto-pion is of order $\frac{m(\mu)}{2m(\pi_L)}\tau(\pi_L)$ and the average length traveled by lepto-pion before decay is of order 10^8 meters! This means that lepto-pion can be treated as stable particle. The presence of a new decay channel changes the lifetime of muon although the rate for events using $e\nu_e$ pair as signature is not changed. The effective $H_L - W$ vertex was deduced above. The rate for the decay via lepto-pion emission and its ratio to ordinary rate for muon decay are given by

$$\begin{aligned}
\Gamma(\mu \rightarrow \nu_\mu + H_L) &= \frac{G^2K^2}{2^5\pi} m^4(\mu)m^2(H_L)(1 - \frac{m^2(H_L)}{m^2(\mu)})\frac{(m^2(\mu) - m^2(H_L))}{(m^2(\mu) + m^2(H_L))} \ , \\
\frac{\Gamma(\mu \rightarrow \nu_\mu + H_L)}{\Gamma(\mu \rightarrow \nu_\mu + e + \bar{\nu}_e)} &= 6 \cdot (2\pi^4)K^2\frac{m^2(H_L)}{m^2(\mu)}\frac{(m^2(\mu) - m^2(H_L))}{(m^2(\mu) + m^2(H_L))} \ ,
\end{aligned} \tag{6.3.21}$$

and is of order $.93K^2$ in case of lepto-pion. As far as the determination of G_F or equivalently m_W^2 from muon decay rate is considered the situation seems to be good since the change introduced to G_F is of order $\Delta G_F/G_F \simeq 0.93K^2$ so that K must be considerably smaller than one. For the physical value of K: $K \leq 10^{-2}$ the contribution to the muon decay rate is negligible.

Lepto-hadrons can appear also as virtual particles in the decay amplitude $\mu \rightarrow \nu_\mu + e\nu_e$ and this changes the value of muon decay rate. The correction is however extremely small since the decay vertex of intermediate off mass shell lepto-pion is proportional to its decay rate.

Lepto-pions and beta decay

If lepto-pions are allowed as final state particles lepto-pion emission provides a new channel $n \rightarrow p + \pi_L$ for beta decay of nuclei since the invariant mass of virtual W boson varies within the range $(m_e = 0.511\ MeV, m_n - m_p = 1.293 MeV$. The resonance peak for $m(\pi_L) \simeq 1\ MeV$ is extremely sharp due to the long lifetime of the charged lepto-pion. The energy of the lepto-pion at resonance is

$$E(\pi_L) = (m_n - m_p)\frac{(m_n + m_p)}{2m_n} + \frac{m(\pi_L)^2}{2m_n} \simeq m_n - m_p \ . \tag{6.3.22}$$

Together with long lifetime this lepto-pions escape the detector volume without decaying (the exact knowledge of the energy of charged lepto-pion might make possible its direct detection).

The contribution of lepto-pion to neutron decay rate is not negligible. Decay amplitude is proportional to superposition of divergences of axial and vector currents between proton and neutron states.

$$M = \frac{G}{\sqrt{2}} K m(\pi_L)(q^\alpha V_\alpha + q^\alpha A_\alpha) \ . \tag{6.3.23}$$

For exactly conserved vector current the contribution of vector current vanishes identically. The matrix element of the divergence of axial vector current at small momentum transfer (approximately zero) is in good approximation given by

$$\begin{aligned} \langle p | q^\alpha A_\alpha | n \rangle &= g_A (m_p + m_n) \bar{u}_p \gamma_5 u_n \ , \\ g_A &\simeq 1.253 \ . \end{aligned} \tag{6.3.24}$$

Straightforward calculation shows that the ratio for the decay rate via lepto-pion emission and ordinary beta decay rate is in good approximation given by

$$\begin{aligned} \frac{\Gamma(n \rightarrow p + \pi_L)}{\Gamma(n \rightarrow p + e + \bar{\nu}_e)} &= \frac{30\pi^2 g_A^2 K^2}{0.47 \cdot (1 + 3g_A^2)} \frac{m_{\pi_L}^2 (\Delta^2 - m_{\pi_L}^2)^2}{\Delta^6} \ , \\ \Delta &= m(n) - m(p) \ . \end{aligned} \tag{6.3.25}$$

Lepto-pion contribution is smaller than ordinary contribution if the condition

$$K < \left[\frac{.47 \cdot (1 + 3g_A^2)}{30\pi^2 g_A^2} \frac{\Delta^6}{(\Delta^2 - m_{\pi_L}^2)^2 m_{\pi_L}^2} \right]^{1/2} \simeq .28 \ , \tag{6.3.26}$$

is satisfied. The upper bound $K \leq 10^{-2}$ coming from the lepto-pion decay width and Op anomaly implies that the contribution of the lepto-pion to beta decay rate is very small.

6.3.5 Ortopositronium puzzle and lepto-pion in photon photon scattering

The decay rate of ortopositronium (Op) has been found to be slightly larger than the rate predicted by QED [C44, C103] : the discrepancy is of order $\Delta\Gamma/\Gamma \sim 10^{-3}$. For parapositronium no anomaly has been observed. Most of the proposed explanations [C103] are based on the decay mode $Op \rightarrow X + \gamma$, where X is some exotic particle. The experimental limits on the branching ratio $\Gamma(Op \rightarrow X + \gamma)$ are below the required value of order 10^{-3}. This explanation is excluded also by the standard cosmology [C103] .

Lepto-pion hypothesis suggests an obvious solution of the Op-puzzle. The increase in annihilation rate is due to the additional contribution to $Op \rightarrow 3\gamma$ decay coming from the decay $Op \rightarrow \gamma_V$ (V denotes 'virtual') followed by the decay $\gamma_V \rightarrow \gamma + \pi_L^V$ followed by the decay $\pi_L^V \rightarrow \gamma + \gamma$ of the virtual lepto-pion to two photon state. $\gamma\gamma\pi_L$ vertices are induced by the axial current anomaly $\propto E \cdot B$. Also a modification of parapositronium decay rate is predicted. The first contribution comes from the decay $Op \rightarrow \pi_L^V \rightarrow \gamma + \gamma$ but the contribution is very small due the smallness of the coupling $g(e,e)$. The second contribution obtained from ortopositronium contribution by replacing one outgoing photon with a loop photon is also small. Since the production of a real lepto-pion is impossible, the mechanism is consistent with the experimental constraints.

The modification to the Op annihilation amplitude comes in a good approximation from the interference term between the ordinary e^+e^- annihilation amplitude F_{st} and lepto-pion induced annihilation amplitude F_{new}:

$$\Delta\Gamma \propto 2Re(F_{st}\bar{F}_{new}) \ , \tag{6.3.27}$$

and rough order of magnitude estimate suggests $\Delta\Gamma/\Gamma \sim K^2/e^2 = \alpha^2/4\pi \sim 10^{-3}$. It turns out that the sign and the order of magnitude of the new contribution are correct for $f(\pi_L) \sim 2\ keV$ deduced also from the anomalous e^+e^- production rate.

The new contribution to $e^+e^- \rightarrow 3\gamma$ decay amplitude is most easily derivable using for lepto-pion-photon interaction the effective action

$$\begin{aligned} L_1 &= K\pi_L F \wedge F \ , \\ K &= \frac{\alpha_{em} N_c}{8\pi f(\pi_L)} \ , \end{aligned} \tag{6.3.28}$$

where F is quantized electromagnetic field. The calculation of the lepto-pion contribution proceeds in manner described in [B17] , where the expression for the standard contribution and an elegant method for treating the average over e^+e^- spin triplet states and sum over photon polarizations, can be found. The contribution to the decay rate can be written as

$$\begin{aligned} \frac{\Delta\Gamma}{\Gamma} &\simeq K_1 I_0 \ , \\ K_1 &= \frac{3\alpha N_c^2}{(\pi^2-9)2^9(2\pi)^3}(\frac{m_e}{f(\pi_L)})^2 \ , \\ I_0 &= \int_0^1 \int_{-1}^{umax} \frac{f}{v+f-1-x^2} v^2(2(f-v)u+2-v-f)dvdu \ , \\ f &\equiv f(v,u) = 1-\frac{v}{2}-\sqrt{(1-\frac{v}{2})^2-\frac{1-v}{1-u}} \ , \\ u &= \bar{n}_1 \cdot \bar{n}_2 \ , \ \bar{n}_i = \frac{\bar{k}_i}{\omega_i} \ , \ umax = \frac{(\frac{v}{2})^2}{(1-\frac{v}{2})^2} \ , \\ v &= \frac{\omega_3}{m_e} \ , \ x = \frac{m_{\pi_L}}{2m_e} \ . \end{aligned} \tag{6.3.29}$$

ω_i and $\bar{k}_i$ denote the energies of photons, u denotes the cosine of the angle between first and second photon and v is the energy of the third photon using electron mass as unit. The condition $\Delta\Gamma/\Gamma = 10^{-3}$ gives for the parameter $f(\pi_L)$ the value $f(\pi_L)(1.062\ MeV) \simeq N_c \cdot 7.9\ keV$. If there are several lepto-pion states, they contribute to the decay anomaly additively. If the four known resonances correspond directly to lepto-pions decaying to lepto-nucleon pairs and $f(\pi_L)$ is assumed to scale as $N_c m_{\pi_L}$, one obtains $f(\pi_L)(1.062\ MeV) \simeq N_c \cdot 14.7\ keV$. From the PCAC relation one obtains for $sin(\theta_e)$ the upper bound $sin(\theta_e) \leq x \cdot \sqrt{N_c}10^{-4}$ assuming $m_{ex} \geq 1.3\ MeV$ (so that $e_{ex}\bar{e}$ decay is not possible), where $x = 1.2$ for single lepto-pion state and $x = 1.36$ for four lepto-pion states identified as the observed resonances.

Lepto-pion photon interaction implies also a new contribution to photon-photon scattering. Just at the threshold $E = m_{\pi_L}/2$ the creation of lepto-pion in photon photon scattering is possible and the appearance of lepto-pion as virtual particle gives resonance type behaviour to photon photon scattering near $s = m_{\pi_L}^2$. The total photon-photon cross section in zero decay width approximation is given by

$$\sigma = \frac{\alpha^4 N_c^2}{2^{14}(2\pi)^6} \frac{E^6}{f_{\pi_L}^4 (E^2 - \frac{m_{\pi_L}^2}{4})^2} \ . \tag{6.3.30}$$

N	$Op/10^{-3}$	$f(\pi_L)/(N_c keV)$	$sin(\theta_e)(m_{ex}/1.3\ MeV)^{1/2}$	$\Gamma(\pi_L)/keV$
1	1	7.9	$1.2 \cdot 10^{-4}\sqrt{N_c}$	.51
3	1	14.7	$1.7 \cdot 10^{-4}\sqrt{N_c}$	.13
3	5	6.5	$3.6 \cdot 10^{-4}\sqrt{N_c}$	.73

Table 1: The dependence of various quantities on the number of lepto-pion type states and Op anomaly, whose value is varied assuming the proportionality $f(\pi_L) \propto N_c m_{\pi_L}$. N_c refers to the number of lepto-pion states in given representation and Op denotes lepto-pion anomaly.

6.3.6 Spontaneous vacuum expectation of lepto-pion field as source of lepto-pions

The basic assumption in the model of lepto-pion and lepto-hadron production is the spontaneous generation of lepto-pion vacuum expectation value in strong nonorthogonal electric and magnetic fields. This assumption is in fact very natural in TGD [1].

1. The well known relation [B17] expressing pion field as a sum of the divergence of axial vector current and anomaly term generalizes to the case of lepto-pion

$$\pi_L = \frac{1}{f(\pi_L)m^2(\pi_L)}(\nabla \cdot j^A + \frac{\alpha_{em}N_c}{2\pi}E\cdot B) \ . \tag{6.3.31}$$

   In the case of lepto-pion case the value of $f(\pi_L)$ has been already deduced from PCAC argument. Anomaly term gives rise to pion decay to two photons so that one obtains an estimate for the lifetime of the lepto-pion.

   This relation is taken as the basis for the model describing also the production of lepto-pion in external electromagnetic field. The idea is that the presence of external electromagnetic field gives rise to a vacuum expectation value of lepto-pion field. Vacuum expectation is obtained by assuming that the vacuum expectation value of axial vector current vanishes.

$$\begin{aligned} \langle vac \mid \pi \mid vac\rangle &= KE\cdot B \ , \\ K &= \frac{\alpha_{em}N_c}{2\pi f(\pi_L)m^2(\pi_L)} \ . \end{aligned} \tag{6.3.32}$$

   Some comments concerning this hypothesis are in order here:

   (a) The basic hypothesis making possible to avoid large parity breaking effects in atomic and molecular physics is that p-adic condensation levels with length scale $L(n) < 10^{-6}\ m$ are purely electromagnetic in the sense that nuclei feed their Z^0 charges on condensate levels with $L(n) \geq 10^{-6}\ m$. The absence of Z^0 charges does not however exclude the possibility of the classical Z^0 fields induced by the nonorthogonality of the ordinary electric and magnetic fields (if Z^0 fields vanish E and B are orthogonal in TGD.

   (b) The non-vanishing vacuum expectation value of the lepto-pion field implies parity breaking in atomic length scales. This is understandable from basic principles of TGD since classical Z^0 field has parity breaking axial coupling to electrons and protons. The non-vanishing classical lepto-pion field is in fact more or less equivalent with the presence of classical Z^0 field.

2. The amplitude for the production of lepto-pion with four momentum $p = (p_0, \bar{p})$ in an external electromagnetic field can be deduced by writing lepto-pion field as sum of classical and quantum parts: $\pi_L = \pi_L(class) + \pi_L(quant)$ and by decomposing the mass term into interaction term plus c-number term and standard mass term:

$$\begin{aligned} \frac{m^2(\pi_L)\pi_L^2}{2} &= L_{int} + L_0 \ , \\ L_0 &= \frac{m^2(\pi_L)}{2}(\pi_L^2(class) + \pi_L^2(quant)) \ , \\ L_{int} &= m^2(\pi_L)\pi_L(class)\pi_L(quant) \ . \end{aligned} \tag{6.3.33}$$

[1] 'Instanton density' generates coherent state of lepto-pions just like classical em current generates coherent state of photons

Interaction Lagrangian corresponds to L_{int} linear in lepto-pion oscillator operators. Using standard LSZ reduction formula and normalization conventions of [B17] one obtains for the probability amplitude for creating lepto-pion of momentum p from vacuum the expression

$$\begin{aligned} A(p) &\equiv \langle a(p)\pi_L\rangle = (2\pi)^3 m^2(\pi_L)\int f_p(x)\langle vac \mid \pi \mid vac\rangle d^4x \ , \\ f_p &= e^{ip\cdot x} \ . \end{aligned} \quad (6.3.34)$$

The probability for the production of lepto-pion in phase space volume element $d3p$ is obtained by multiplying with the density of states factor $d^3n = V\frac{d^3p}{(2\pi)^3}$:

$$\begin{aligned} dP &= A|U|^2 V d^3p \ , \\ A &= (\frac{\alpha_{em}N_c^2 m^2(\pi_L)}{2\pi f(\pi_L)})^2 \ , \\ U &= \int e^{ip\cdot x} E\cdot B d^4x \ . \end{aligned} \quad (6.3.35)$$

The first conclusion that one can draw is that nonstatic electromagnetic fields are required for lepto-pion creation since in static fields energy conservation forces lepto-pion to have zero energy and thus prohibits real lepto-pion production. In particular, the spontaneous creation lepto-pion in static Coulombic and magnetic dipole fields of nucleus is impossible.

6.3.7 Sigma model and creation of lepto-hadrons in electromagnetic fields

Why sigma model approach?

For several reasons it is necessary to generalize the model for lepto-pion production to a model for lepto-hadron production.

1. Sigma model approach is necessary if one assumes that anomalous e^+e^- pairs are genuine e^+e^- pairs rather lepto-nucleon pairs produced in the decays of lepto-sigmas.
2. A model for the production of lepto-hadrons is obtained from an effective action describing the strong and electromagnetic interactions between lepto-hadrons. The simplest model is sigma model describing the interaction between lepto-nucleons, lepto-pion and a hypothetical scalar particle σ_L [B17] . This model realizes lepto-pion field as a divergence of the axial current and gives the standard relation between $f(\pi_L), g$ and m_{ex}. All couplings of the model are related to the masses of e_{ex}, π_L and σ_L. The generation of lepto-pion vacuum expectation value in the proposed manner takes place via triangle anomaly diagrams in the external electromagnetic field.
3. If needed the model can be generalized to contain terms describing also other lepto-hadrons. The generalized model should contain also vector bosons ρ_L and ω_L as well as pseudo-scalars η_L and η'_L and radial excitations of π_L and σ_L. An open question is whether also η and η' generate vacuum expectation value proportional to $E \cdot B$. Actually all these states appear as 3-fold degenerate for the minimal color representation content of the theory.
4. The following observations are useful for what follows.
   (a) Ortopositronium decay width anomaly gives the estimate $f(\pi_L) \sim N_c\cdot 7.9\ keV$ and from this one can deduce an upper bound for lepto-pion production cross section in an external electromagnetic field. The calculation of lepto-pion production cross section shows that lepto-pion production cross section is somewhat smaller than the upper bound for the observed anomalous e^+e^- production cross section, even when one tunes the values of the various parameters. This is consistent with the idea that lepto-nucleon pairs, with lepto-nucleon mass being only slightly larger than electron mass, are in question.

(b) Also the direct production of the lepto-nucleon pairs via the interaction term $gcos(\theta_e)\bar{e}_{ex}\gamma_5 e_{ex}\pi_L(cl)$ is possible but gives rise to continuum mass squared spectrum rather than resonant structures. The direct production of the pairs via the interaction term
$gsin(\theta_e)\bar{e}\gamma_5 e_{ex}\pi_L(cl)$ from is much slower process than the production via the meson decays and does not give rise to resonant structures since Also the production via the $\bar{e}e_{ex}$ decay of virtual lepto-pion created from classical field is slow process since it involves $sin^2(\theta_e)$.

(c) e^+e^- production can also proceed also via the creation of many particle states. The simplest candidates are $V_L + \pi_L$ states created via $\partial_\alpha \pi_L V^\alpha \pi_L(class)$ term in action and $\sigma_L + \pi_L$ states created via the the $k\sigma_L\pi_L\pi_L(class)$ term in the sigma model action. The production cross section via the decays of vector mesons is certainly very small since the production vertex involves the inner product of vector boson 3 momentum with its polarization vector and the situation is non-relativistic.

(d) If the strong decay of σ_L to lepto-mesons is kinematically forbidden (this is not suggested by the experience with the ordinary hadron physics), the production rate for σ_L meson is large since the coupling k turns out to be given by $k = (m^2_{\sigma_L} - m^2_{\pi_L})/2f(\pi_L)$ and is anomalously large for the value of $f(\pi_L) \geq 7.9 \cdot N_c\ keV$ derived from ortopositronium anomaly: $k \sim 336m(\pi_L)/N_c$ for $f(\pi_L) \sim N_c \cdot 7.9\ keV$. The resulting additional factor in the production cross section compensates the reduction factor coming from two-particle phase space volume. Despite this the estimate for the production cross section of anomalous e^+e^- pairs is roughly by a factor $1/N_c^2$ smaller than the maximum experimental cross section. The radiative corrections are huge and should give the dominant contribution to the cross section. It is however questionable very the assumed small lepto-hadronic decay width and mass of σ_L is consistent with the extremely strong interactions of σ_L.

Simplest sigma model

A detailed description of the sigma model can be found in [B17] and it suffices to outline only the crucial features here.

1. The action of lepto-hadronic sigma model reads as

$$\begin{aligned} L &= L_S + c\sigma_L \ , \\ L_S &= \bar{\psi}_L(i\gamma^k\partial_k + g(\sigma_L + i\pi_L\cdot\tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma_L)^2) \\ &- \frac{\mu^2}{2}(\sigma_L^2 + \pi_L^2) - \frac{\lambda}{4}(\sigma_L^2+\pi_L^2)^2 \ . \end{aligned} \tag{6.3.36}$$

π_L is isospin triplet and σ_L isospin singlet. ψ_L is isospin doublet with electro-weak quantum numbers of electron and neutrino (e_{ex} and ν_{ex}). The model allows $so(4)$ symmetry. Vector current is conserved but for $c \neq 0$ axial current generates divergence, which is proportional to pion field: $\partial^\alpha A_\alpha = -c\pi_L$.

2. The presence of the linear term implies that σ_L field generates vacuum expectation value $\langle 0|\sigma_L|0\rangle = v$. When the action is written in terms of new quantum field $\sigma'_L = \sigma_L - v$ one has

$$\begin{aligned} L &= \bar{\psi}_L(i\gamma^k\partial_k + m + g(\sigma'_L + i\pi_L\cdot\tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma'_L)^2) \\ &- \frac{1}{2}m^2_{\sigma_L}(\sigma'_L)^2 - \frac{m^2_{\pi_L}}{2}\pi_L^2 \\ &- \lambda v\sigma'_L((\sigma'_L)^2 + \pi_L^2) - \frac{\lambda}{4}((\sigma'_L)^2+\pi_L^2)^2 \ , \end{aligned} \tag{6.3.37}$$

The masses are given by

$$
\begin{aligned}
m^2_{\pi_L} &= \mu^2 + \lambda v^2 \ , \\
m^2_{\sigma_L} &= \mu^2 + 3\lambda v^2 \ , \\
m &= -gv \ .
\end{aligned}
\tag{6.3.38}
$$

These formulas relate the parameters μ, v, g to lepto-hadrons masses.

3. The requirement that σ'_L has vanishing vacuum expectation implies in Born approximation

$$
c - \mu^2 v - \lambda v^3 = 0 \ , \tag{6.3.39}
$$

which implies

$$
\begin{aligned}
f(\pi_L) &= -v = -\frac{c}{m^2(\pi_L)} \ , \\
m_{ex} &= g f(\pi_L) \ .
\end{aligned}
\tag{6.3.40}
$$

Note that e_{ex} and ν_{ex} are predicted to have identical masses in this approximation. The value of the strong coupling constant g of lepto-hadronic physics is indeed strong from $m_{ex} > m_e$ and $f(\pi_L) < N_c \cdot 10$ keV.

4. A new feature is the generation of the lepto-pion vacuum expectation value in an external electromagnetic field (of course, this is possible for the ordinary pion field, too!). The vacuum expectation is generated via the triangle anomaly diagram in a manner identical to the generation of a non-vanishing photon-photon decay amplitude and is proportional to the instanton density of the electromagnetic field. By redefining the pion field as a sum $\pi_L = \pi_L(cl) + \pi'_L$ one obtains effective action describing the creation of the lepto-hadrons in strong electromagnetic fields.

5. As far as the production of $\sigma_L \pi_L$ pairs is considered, the interaction term $\lambda v \sigma'_L \pi_L^2$ is especially interesting since it leads to the creation of $\sigma_L \pi_L$ pairs via the interaction term $k\lambda v \sigma'_L \pi_L(qu)\pi_L(cl)$.

   The coefficient of this term can be expressed in terms of the lepto-meson masses and $f(\pi_L)$:

$$
\begin{aligned}
k &\equiv 2\lambda v = \frac{m^2_{\sigma_L} - m^2_{\pi_L}}{2f(\pi_L)} = x m_{\pi_L} \ , \\
x &= \frac{1}{2}\left(\frac{m^2_{\sigma_L}}{m^2_{\pi_L}} - 1\right)\frac{m_{\pi_L}}{f(\pi_L)} \ .
\end{aligned}
\tag{6.3.41}
$$

   The large value of the coupling deriving from $f(\pi_L) = N_c \cdot 7.9\ keV)$ compensates the reduction of the production rate coming from the smallness of two-particle phase space volume as compared with single particle-phase space volume but fails to produce large enough production cross section. The large value of $g(\sigma_L, \sigma_L, \sigma_L) = g(\sigma_L, \pi_L, \pi_L)$ however implies that the radiative contribution to the production cross section coming from the emission of a virtual sigma in the production vertex is much larger than the lowest order production cross section and with a rather small value of the relative $\sigma_L - \pi_L$ mass difference correct order of magnitude of cross section should be possible.

6.3.8 Classical model for lepto-pion production

The nice feature of both quantum and classical model is that the production amplitudes associated with all lepto-hadron production reactions in external electromagnetic field are proportional to the lepto-pion production amplitude and apart from phase space volume factors production cross sections are expected to be given by lepto-pion production cross section. Therefore it makes sense to construct a detailed model for lepto-pion production despite the fact that lepto-pion decays probably contribute only a very small fraction to the observed e^+e^- pairs.

General considerations

Angular momentum barrier makes the production of lepto-mesons with orbital angular momentum $L > 0$ improbable. Therefore the observed resonances are expected to be $L = 0$ pseudo-scalar states. Lepto-pion production has two signatures which any realistic model should reproduce.

1. Data are consistent with the assumption that states are produced at rest in cm frame.

2. The production probability has a peak in a narrow region of velocities of colliding nucleus around the velocity needed to overcome Coulomb barrier in head on collision. The relative width of the velocity peak is of order $\Delta\beta/\beta \simeq \cdot 10^{-2}$ [C72] . In Th-Th system [J10] two peaks at projectile energies 5.70 MeV and 5.75 MeV per nucleon have been observed. This suggests that some kind of diffraction mechanism based on the finite size of nuclei is at work.
   In this section a model treating nuclei as point like charges and nucleus-nucleus collision purely classically is developed. This model yields qualitative predictions in agreement with the signature 1) but fails to reproduce the possible diffraction behavior although one can develop argument for understanding the behavior above Coulomb wall.

The general expression for the amplitude for creation of lepto-pion in external electric and magnetic fields has been derived in Appendix. Let us now specialize to the case of heavy ion collision. We consider the situation, where the scattering angle of the colliding nucleus is measured. Treating the collision completely classically we can assume that collision occurs with a well defined value of the impact parameter in a fixed scattering plane. The coordinates are chosen so that target nucleus is at rest at the origin of the coordinates and colliding nucleus moves in z-direction in y=0 plane with velocity β. The scattering angle of the scattered nucleus is denoted by α, the velocity of the lepto- pion by v and the direction angles of lepto-pion velocity by (θ, ϕ).

The minimum value of the impact parameter for the Coulomb collision of point like charges is given by the expression

$$\begin{aligned} b &= \frac{b_0 cot(\alpha/2)}{2} \ , \\ b_0 &= \frac{2Z_1Z_2\alpha_{em}}{M_R\beta^2} \ , \end{aligned} \tag{6.3.42}$$

where b_0 is the expression for the distance of the closest approach in head on collision. M_R denotes the reduced mass of the nucleus-nucleus system.

To estimate the amplitude for lepto-pion production the following simplifying assumptions are made.

1. Nuclei can be treated as point like charges. This assumption is well motivated, when the impact parameter of the collision is larger than the critical impact parameter given by the sum of radii of the colliding nuclei:

$$b_{cr} = R_1 + R_2 \ . \tag{6.3.43}$$

   For scattering angles that are sufficiently large the values of the impact parameter do not satisfy the above condition in the region of the velocity peak. p-Adic considerations lead to the conclusion that nuclear condensation level corresponds to prime $p \sim 2^k$, $k = 113$ (k is prime). This suggest that nuclear radius should be replaced by the size $L(113)$ of the p-adic convergence cube associated with nucleus (see the chapter "TGD and Nuclear Physics": $L(113) \sim 1.7 \cdot 10^{-14}\ m$ implies that cutoff radius is $b_{cr} \sim 2L(113) \sim 3.4 \cdot 10^{-14}\ m$.

2. Since the velocities are non-relativistic (about $0.12c$) one can treat the motion of the nuclei non-relativistically and the non-retarded electromagnetic fields associated with the exactly known classical orbits can be used. The use of classical orbit doesn't take into account recoil effect caused by lepto-pion production. Since the mass ratio of lepto-pion and the reduced mass of heavy nucleus system is of order 10^{-5} the recoil effect is however negligible.

3. The model simplifies considerably, when the orbit is idealized with a straight line with impact parameter determined from the condition expressing scattering angle in terms of the impact parameter. This approximation is certainly well founded for large values of impact parameter. For small values of impact parameter the situation is quite different and an interesting problem is whether the contributions of long range radiation fields created by accelerating nuclei in head-on collision could give large contribution to lepto-pion production rate. On the line connecting the nuclei the electric part of the radiation field created by first nucleus is indeed parallel to the magnetic part of the radiation field created by second nucleus. In this approximation the instanton density in the rest frame of the target nucleus is just the scalar product of the Coulombic electric field E of the target nucleus and of the magnetic field B of the colliding nucleus obtained by boosting it from the Coulomb field of nucleus at rest.

Expression of the classical cross section

First some kinematical notations. lepto-pion four-momentum in the rest system of target nucleus is given by the following expression

$$
\begin{aligned}
p &= (p_0, \bar{p}) = m\gamma_1(1, vsin(\theta)cos(\phi), vsin(\theta)sin(\phi), vcos(\theta)) \ , \\
\gamma_1 &= 1/(1-v^2)^{1/2} \ .
\end{aligned}
\tag{6.3.44}
$$

The velocity and Lorentz boost factor of the projectile nucleus are denoted by β and $\gamma = 1/\sqrt{1-\beta^2}$.

The double differential cross section in the classical model can be written as

$$
\begin{aligned}
d\sigma &= dP 2\pi b db \ , \\
dP &= K|A(b,p)|^2 d^3n \ , per d^3n = V\frac{d^3p}{(2\pi)^3} \ , \\
K &= (Z_1Z_2)^2(\alpha_{em})^4 \times N_c^2(\frac{m(\pi_L}{f(\pi_L)})^2\frac{1}{2\pi^{13}} \ , \\
A(b,p) &= N_0\frac{4\pi}{Z_1Z_2\alpha_{em}} \times U(b,p) \ , \\
U(b,p) &= \int e^{ip\cdot x} E\cdot B d^4x \ , \\
N_0 &= \frac{(2\pi)^7}{i} \ .
\end{aligned}
\tag{6.3.45}
$$

where b denotes impact parameter. The formula generalizes the classical formula for the cross section of Coulomb scattering. In the calculation of the total cross section one must introduce some cutoff radii and the presence of the volume factor V brings in the cutoff volume explicitly (particle in the box description for lepto-pions). Obviously the cutoff length must be longer than lepto-pion Compton length. Normalization factor N_0 has been introduced in order to extract out large powers of 2π.

From this one obtains differential cross section as

$$
\begin{aligned}
d\sigma &= P2\pi bdb \ , \\
P &= \int K|A(b,p)|^2 V\frac{d^3p}{(2\pi)^3} \ , \ .
\end{aligned}
\tag{6.3.46}
$$

The first objection is the need to explicitly introduce the reaction volume: this obviously breaks manifest Lorentz invariance. The cross section was estimated in the earlier version of the model [K3] and turned to be too small by several orders of magnitude. This inspired the idea that constructive interference for the production amplitudes for different values of impact parameter could increase the cross section.

6.3.9 Quantum model for lepto-pion production

There are good reasons for considering the quantum model. First, the lepto-pion production cross section is by several orders of magnitude too small in classical model. Secondly, in Th-Th collisions there are indications about the presence of two velocity peaks with separation $\delta\beta/\beta \sim 10^{-2}$ [C72] and this suggests that quantum mechanical diffraction effects might be in question. These effects could come from the upper and/or lower length scale cutoff and from the de-localization of the wave function of incoming nucleus.

The question is what quantum model means. The most natural thing is to start from Coulomb scattering and multiply Coulomb scattering amplitude for a given impact parameter value b with the amplitude for lepto-pion production. This because the classical differential cross section given by $2\pi bdb$ in Coulomb scattering equals to the quantum cross section. One might however argue that on basis of $S = 1 + T$ decomposition of S-matrix the lowest order contribution to lepto-pion production in quantum situation corresponds to the absence of any scattering. The lepto-pion production amplitude is indeed non-vanishing also for the free motion of nuclei. The resolution of what looks like a paradox could come from many-sheeted space-time concept (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book): if no scattering occurs, the space-time sheets representing colliding nuclei do not touch and all and there is no interference of em fields so that there is no lepto-pion production. It turns however that lowest order contribution indeed corresponds to the absence of scattering in the model that works.

Two possible approaches

One can imagine two approaches to the construction of the model for production amplitude in quantum case.

The first approach is based on eikonal approximation [B27] . Eikonal approximation applies at high energy limit when the scattering angle is small and one can approximate the orbit of the projectile with a straight orbit.

The expression for the scattering amplitude in eikonal approximation reads as

$$
\begin{aligned}
f(\theta,\phi) &= \frac{k}{2\pi i}\int d^2b exp(-ik\cdot b)exp(i\xi(b))-1) \ , \\
\xi(b) &= \frac{-m}{k\hbar^2}\int_{z=-\infty}^{z=\infty} dzV(z,b) \ , \\
\frac{d\sigma}{d\Omega} &= |f^2| \ .
\end{aligned}
\tag{6.3.47}
$$

as one expands the exponential in lowest in spherically symmetric potential order one obtains the

$$
f(\theta,\phi) \simeq -\frac{m}{2\pi\hbar^2}\int J_0(k_Tb)\xi(b)bdb \ .
\tag{6.3.48}
$$

The challenge is to find whether it is possible to generalize this expression so that it applies to the production of lepto-pions.

1. The simplest guess is that one should multiply the eikonal amplitude with the dimensionless amplitude $A(b)$:

$$\begin{aligned} f(\theta,\phi) & \rightarrow & f(\theta,\phi,p) = \frac{k}{2\pi i}\int d^2b exp(-ik\cdot b)exp(i\xi(b))-1)A(b,p) \\ & \simeq & -\frac{m}{2\pi\hbar^2}\int J_0(k_Tb)\xi(b)A(b,p)bdb \ . \end{aligned} \quad (6.3.49)$$

2. Amplitude squared must give differential cross section for lepto-pion production and scattering

$$\begin{aligned} d\sigma & = & |f(\theta,\phi,p)|^2 d\Omega d^3n \ , \\ d^3n & = & Vd^3p \ . \end{aligned} \quad (6.3.50)$$

   This requires an explicit introduction of a volume factor V via a spatial cutoff. This cutoff is necessary for the coordinate z in the case of Coulomb potential, and would have interpretation in terms of a finite spatio-temporal volume in which the space-time sheets of the colliding particles are in contact and fields interfere.

3. There are several objections against this approach. The loss of a manifest relativistic invariance in the density of states factor for lepto-pion does not look nice. One must keep count about the scattering of the projectile which means a considerable complication from the point of view of numerical calculations. In classical picture for orbits the scattering angle in principle is fixed once impact parameter is known so that the introduction of scattering angles does not look logical.

Second approach starts from the classical picture in which each impact parameter corresponds to a definite scattering angle so that the resulting amplitude describes lepto-pion production amplitude and says nothing about the scattering of the projectile. This approach is more in spirit with TGD since classical physics is exact part of quantum TGD and classical orbit is absolutely real from the point of view of lepto-pion production amplitude.

1. The counterpart of the eikonal exponent has interpretation as the exponent of classical action associated with the Coulomb interaction

$$S(b) = \int_{\gamma} Vds \quad (6.3.51)$$

   along the orbit γ of the particle, which can be taken also as a real classical orbit but will be approximated with rectilinear orbit in sequel.

2. The first guess for the production amplitude is

$$\begin{aligned} f(p) & = & \int d^2b exp(-i\Delta k(b)\cdot b)exp[\frac{i}{\hbar}S(b)]A(b,p) \\ & = & \int J_0(k_T(b)b)(1+\frac{i}{\hbar}\int_{z=-a}^{z=a} dzV(z,b)+..)A(b,p) \ . \end{aligned} \quad (6.3.52)$$

   Δk is the change of the momentum in the classical scattering and in the scattering plane. The cutoff $|z|\leq a$ in the longitudinal direction corresponds to a finite imbedding space volume inside which the space-time sheets of target and projectile are in contact.

3. The production amplitude is non-trivial even if the interaction potential vanishes being given by

$$f(p) = \int d^2bexp(-ik\cdot b))A(b,p) = 2\pi int J_0(k_T(b)b)\times A(b,p)bdb \ . \quad (6.3.53)$$

This formula can be seen as a generalization of quantum formula in the sense that incoherent integral over production probabilities at various values of b is replaced by an integral over production amplitude over b so that interference effects become possible.

4. This result could be seen as a problem. On basis of $S = 1 + iT$ decomposition corresponding to free motion and genuine interaction, one could argue that since the exponent of action corresponds to S, $A(p,b)$ vanishes when the space-time sheets are not in contact. The improved guess for the amplitude is

$$\begin{aligned} f(p) &= \int d^2bexp(-ik\cdot b)exp(\frac{i}{\hbar}S(b))A(b,p) \\ &= \int J_0(k_T(b)b)(\frac{i}{\hbar}\int_{z=-a}^{z=a} V(z,b)+..)A(b,p) \ . \end{aligned} \quad (6.3.54)$$

This would mean that there would be no classical limit when coherence is assumed to be lost. At this stage one must keep mind open for both options.

5. The dimension of $f(p)$ is $L^2/\hbar$

$$d\sigma = |f(p)|^2\frac{d^3p}{2E_p(2\pi)^3} \ . \quad (6.3.55)$$

has correct dimension. This model will be considered in sequel. The earlier work in [K3] was however based on the first option.

Production amplitude

The Fourier transform of $E\cdot B$ can be expressed as a convolution of Fourier transforms of E and B and the resulting expression for the amplitude reduces by residue calculus (see APPENDIX) to the following general form

$$\begin{aligned} A(b,p) &\equiv N_0\times\frac{4\pi}{Z_1Z_2\alpha_{em}}\times U(b,p) = 2\pi i(CUT_1+CUT_2) \ , \\ N_0 &= \frac{(2\pi)^7}{i} \ . \end{aligned} \quad (6.3.56)$$

where nuclear charges are such that Coulomb potential is $1/r$. The motivation for the strange looking notation is to extract all powers of 2π so that the resulting amplitudes contain only factors of order unity.

The contribution of the first cut for $\phi \in [0,\pi/2]$ is given by the expression

$$\begin{aligned}
CUT_1 &= D_1 \times \int_0^{\pi/2} exp(-\frac{b}{b_0}cos(\psi))A_1 d\psi \ , \\
D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ b_0 = \frac{\hbar}{m}\frac{\beta\gamma}{\gamma_1} \ , \\
A_1 &= \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\
A &= sin(\theta)cos(\phi) \ , \ B = K \ , \\
C &= K\frac{cos(\phi)}{sin(\theta)} \ , \ D = -sin^2(\phi) - \frac{K^2}{sin^2(\theta)} \ , \\
K &= \beta\gamma(1 - \frac{v_{cm}}{\beta}cos(\theta)) \ , \ v_{cm} = \frac{2v}{1+v^2} \ .
\end{aligned} \quad (6.3.57)$$

The definitions of the various kinematical variables are given in previous formulas. The notation is tailored to express the facts that A_1 is rational function of $cos(\psi)$ and that integrand depends exponentially on the impact parameter.

The expression for CUT_2 reads as

$$\begin{aligned}
CUT_2 &= D_2 \times \int_0^{\pi/2} exp(i\frac{b}{b_1}cos(\psi))A_2 d\psi \ , \\
D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \times exp(-\frac{b}{b_2}) \ , \\
b_1 &= \frac{\hbar}{m}\frac{\beta}{\gamma_1} \ , \ b_2 = \frac{\hbar}{mb}\frac{1}{\gamma_1 \times sin(\theta)cos(\phi)} \\
A_2 &= \frac{Acos(\psi) + B}{cos^2(\psi) + Ccos(\psi) + D} \ , \\
A &= sin(\theta)cos(\phi)u \ , \ B = \frac{w}{v_{cm}} + \frac{v}{\beta}sin^2(\theta)[sin^2(\phi) - cos^2(\phi)] \ , \\
C &= 2i\frac{\beta w}{uv_{cm}}\frac{cos(\phi)}{sin(\theta)} \ , \ D = -\frac{1}{u^2}(\frac{sin^2(\phi)}{\gamma^2} + \beta^2(v^2 sin^2(\theta) - \frac{2vw}{v_{cm}})cos^2(\phi)) \\
&+ \frac{w^2}{v_{cm}^2 u^2 sin^2(\theta)} + 2i\frac{\beta v}{u}sin(\theta)cos(\phi) \ , \\
u &= 1 - \beta v cos(\theta) \ , \ w = 1 - \frac{v_{cm}}{\beta}cos(\theta) \ .
\end{aligned} \quad (6.3.58)$$

(6.3.59)

The denominator X_2 has no poles and the contribution of the second cut is therefore always finite. Again the expression is tailored to make clear the functional dependence of the integrand on $cos(\psi)$ and on impact parameter. Besides this the exponential damping makes in non-relativistic situation the integrand small everywhere expect in the vicinity of $cos(\Psi) = 0$ and for small values of the impact parameter.

Using the symmetries

$$\begin{aligned}
U(b,p_x,-p_y) &= -U(b,p_x,p_y) \ , \\
U(b,-p_x,-p_y) &= \bar{U}(b,p_x,p_y) \ ,
\end{aligned} \quad (6.3.60)$$

of the amplitude one can calculate the amplitude for other values of ϕ.

CUT_1 gives the singular contribution to the amplitude. The reason is that the factor X_1 appearing in denominator of cut term vanishes, when the conditions

$$
\begin{aligned}
cos(\theta) &= \frac{\beta}{v_{cm}} \ , \\
sin(\phi) &= cos(\psi) \ ,
\end{aligned}
\tag{6.3.61}
$$

are satisfied. In forward direction this condition tells that z- component of the lepto-pion momentum in velocity center of mass coordinate system vanishes. In laboratory this condition means that the lepto-pion moves in certain cone defined by the value of its velocity. The condition is possible to satisfy only above the threshold $v_{cm} \geq \beta$.
For $K = 0$ the integral reduces to the form

$$
CUT_1 = \frac{1}{2}cos(\phi)sin(\phi)\lim_{\varepsilon\rightarrow 0}\frac{\int_0^{\pi/2} exp(-\frac{cos(\psi)}{sin(\phi_0)})d\psi}{(sin^2(\phi) - cos^2\psi + i\varepsilon)} \ .
\tag{6.3.62}
$$

One can estimate the singular part of the integral by replacing the exponent term with its value at the pole. The integral contains two parts: the first part is principal value integral and second part can be regarded as integral over a small semicircle going around the pole of integrand in upper half plane. The remaining integrations can be performed using elementary calculus and one obtains for the singular cut contribution the approximate expression

$$
\begin{aligned}
CUT_1 &\simeq e^{-(b/a)(sin(\phi)/sin(\phi_0))}(\frac{ln(X)}{2} + \frac{i\pi}{2}) \ , \\
X &= \frac{((1+s)^{1/2} + (1-s)^{1/2})}{((1+s)^{1/2} - (1-s)^{1/2})} \ , \\
s &= sin(\phi) \ , \\
sin(\phi_0) &= \frac{\beta\gamma}{\gamma_1 m(\pi_L)a} \ .
\end{aligned}
\tag{6.3.63}
$$

The principal value contribution to the amplitude diverges logarithmically for $\phi = 0$ and dominates over 'pole' contribution for small values of ϕ. For finite values of impact parameter the amplitude decreases exponentially as a function of ϕ.

If the singular term appearing in CUT_1 indeed gives the dominant contribution to the lepto-pion production one can make some conclusions concerning the properties of the production amplitude. For given lepto-pion cm velocity v_{cm} the production associated with the singular peak is predicted to occur mainly in the cone $cos(\theta) = \beta/v_{cm}$: in forward direction this corresponds to the vanishing of the z-component of the lepto-pion momentum in velocity center of mass frame. Since the values of $sin(\theta)$ are of order .1 the transversal momentum is small and production occurs almost at rest in cm frame as observed. In addition, the singular production cross section is concentrated in the production plane ($\phi = 0$) due to the exponential dependence of the singular production amplitude on the angle ϕ and impact parameter and the presence of the logarithmic singularity. The observed lepto-pion velocities are in the range $\Delta v/v \simeq 0.2$ [C72] and this corresponds to the angular width $\Delta\theta \simeq 34$ degrees.

Differential cross section in the quantum model

There are two options to consider depending on whether one uses $exp(iS)$ or $exp(iS) - 1$ to define the production amplitude.

1. For the $exp(iS)$ option the expression for the differential cross section reads in the lowest order as

$$\begin{aligned} d\sigma &= K|f_B|^2\frac{d^3p}{2E_p} \ , \\ f_B &\simeq i\int exp(-i\Delta k\cdot r)(CUT_1+CUT_2)bdbdzd\phi \ , \\ K &= (Z_1Z_2)^2\alpha_{em}^4N_c^2(\frac{m(\pi_L)}{f(\pi_L)})^2\frac{1}{(2\pi)^{15}} \ . \end{aligned} \quad (6.3.64)$$

Here Δk is the momentum exchange in Coulomb scattering and a vector in the scattering plane so that the above described formula is obtained for the linear orbits.

2. For the $exp(iS)-1$ option the differential production cross section for lepto-pion is in the lowest non-trivial approximation for the exponent of action S given by the expression

$$\begin{aligned} d\sigma &= K|f_B|^2\frac{d^3p}{2E_p} \ , \\ f_B &\simeq \int exp(-i\Delta k\cdot r)V(z,b)(CUT_1+CUT_2)bdbdzd\phi \ , \\ V(z,b) &= \frac{1}{r} \ , \\ K &= (Z_1Z_2)^4\alpha_{em}^6N_c^2(\frac{m(\pi_L)}{f(\pi_L)})^2\frac{1}{(2\pi)^{15}} \ . \end{aligned} \quad (6.3.65)$$

Effectively the Coulomb potential is replaced with the product of the Coulomb potential and lepto-pion production amplitude $A(b,p)$. Since α_{em} is assumed to correspond to relate to its standard value by a scaling $\hbar_0/\hbar$ factor.

3. Coulomb potential brings in an additional $(Z_1Z_2\alpha_{em})^2$ factor to the differential cross section, which in the case of heavy ion scattering increases the contribution to the cross section by a factor of order 3×10^3 but reduces it by a factor of order 5×10^{-5} in the case of proton-antiproton scattering. The increase of $\hbar$ expected to be forced by the requirement that perturbation theory is not lost however reduces the contribution from higher orders in V. It should be possible to distinguish between the two options on basis of these differences.

The scattering amplitude can be reduced to a simpler form by using the defining integral representation

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} exp(-ixsin(\phi))d\phi$$

of Bessel functions.

1. For $exp(iS)$ option this gives

$$\begin{aligned} f_B &= 2\pi i\int J_0(\Delta kb)(CUT_1+CUT_2)bdb \ , \\ \Delta k &= 2ksin(\frac{\alpha}{2}) \ , \ k=M_R\beta \ , \\ M_R &\simeq A_Rm_p \ , \quad A_R=\frac{A_1A_2}{A_1+A_2} \ , \end{aligned} \quad (6.3.66)$$

where the length scale cutoffs in various integrations are not written explicitly. The value of α can be deduced once the value of impact parameter is known in the case of the classical Coulomb scattering.

2. For $exp(iS) - 1$ option one has

$$
\begin{aligned}
f_B &= 2\pi i \int F(b) J_0(\Delta k b)(CUT_1 + CUT_2) bdb \ , \\
F(b \geq b_{cr}) &= 2 \int dz \frac{1}{\sqrt{z^2+b^2}} = ln(\frac{\sqrt{a^2-b^2}+a}{b}) \ , \\
&.
\end{aligned}
\tag{6.3.67}
$$

Note that the factors K appearing in the different cross section are different in these to cases.

Calculation of the lepto-pion production amplitude in the quantum model

The details related to the calculation of the production amplitude can be found in appendix and it suffices to describe only the general treatment here. The production amplitude of the quantum model contains integrations over the impact parameter and angle parameter ψ associated with the cut. The integrands appearing in the definition of the contributions CUT_1 and CUT_2 to the scattering amplitude have simple exponential dependence on impact parameter. The function F appearing in the definition of the scattering amplitude is a rather slow varying function as compared to the Bessel function, which allows trigonometric approximation and for small values of scattering angle equals to its value at origin. This motivates the division of the impact parameter range into pieces so that F can approximated with its mean value inside each piece so that integration over cutoff parameters can be performed exactly inside each piece.

In Appendix the explicit expansion in power series with respect to impact parameter is derived by assuming $J_0(k_T b) \simeq 1$ and $F(b) = F = constant$. These formulas can be easily generalized by assuming a piecewise constancy of these two functions. This means that the only the integration over the lepto-pion phase space must be carried out numerically.

CUT_1 becomes also singular at $cos(\theta) = \beta/v_{cm}$, $cos(\psi) = \sin(\phi)$. The singular contribution of the production amplitude can be extracted by putting $cos(\psi) = sin(\phi)$ in the arguments of the exponent functions appearing in the amplitude so that one obtains a rational function of $cos(\psi)$ and $sin(\psi)$ integrable analytically. The remaining nonsingular contribution can be integrated numerically.

Formula for the production cross section

In the case of heavy ion collisions the rectilinear motion is not an excellent approximation since the anomalous events are observed near Coulomb wall and $\beta \simeq .1$ holds true. Despite this this can be taken as a first approximation.

The expression for the differential cross section for lepto-pion production in heavy ion collisions is given by

$$
d\sigma = KF^2 |\int (CUT_1 + CUT_2) bdb|^2 \frac{d^3p}{2E} \ ,
\tag{6.3.68}
$$

This expression and also the expressions of the integrals of CUT_1 and CUT_2 are calculated explicitly as powers series of the impact parameter in the Appendix.

1. For $exp(iS)$ option one has

$$
\begin{aligned}
K &= (Z_1 Z_2)^2 \alpha_{em}^4 N_c^2 [\frac{m(\pi_L)}{f(\pi_L)}]^2 \frac{1}{(2\pi)^{13}} \ , \\
F &= 1 \ .
\end{aligned}
\tag{6.3.69}
$$

2. For $exp(iS)-1$ option one has

$$
\begin{aligned}
K &= (Z_1Z_2)^4\alpha_{em}^6 N_c^2[\frac{m(\pi_L)}{f(\pi_L)}]^2\frac{1}{(2\pi)^{13}} \ , \\
F &= 2\langle\langle ln(\frac{\sqrt{a^2-b^2}+a}{b})\rangle\rangle \ .
\end{aligned}
\quad (6.3.70)
$$

In the approximation that F is constant the two lowest order predictions are related by a scaling factor

$$
R = (Z_1Z_2\alpha_{em})^2F^2 \ . \quad (6.3.71)
$$

It is interesting to get a rough order of magnitude feeling about the situation assuming that the contributions of CUT_1 and CUT_2 are of order unity. For $Z_1 = Z_2 = 92$ and $m(\pi_L)/f(\pi_L) \simeq 1.5$ -as in the case of ordinary pion- one obtains following results. It must be emphasized that these estimates are extremely sensitive to the over all scaling of f_B and to the choice of the cutoff parameter a and cannot be taken too seriously.

1. From $\beta \simeq .1$ one has $b_0 \simeq .1/m(\pi_L)$. One can argue that the impact parameter cutoff $a = xb_0$ should satisfy $a \geq 1/m_{\pi_L}$ so that $x \geq 10$ should hold true.

2. For $expi(S)-1$ option one has $K = 4.7\times 10^{-6}$. From the classical model the allowed phase space volume is of order $\frac{1}{3}\Delta v^3 \sim 10^{-4}$. By using $a = m(\pi_L)$ as a cutoff and $m(\pi_L) \simeq 2m_e$ one obtains $\sigma \sim 4$ μb, which is of same order of magnitude as the experimental estimate 5 μb.

3. For $exp(iS)$ option one has $K = 1.2\times 10^{-9}$ and the estimate for cross section is 1.1 nb for $a = 1/m(\pi_L)$. A correct order of magnitude is obtained by assuming $a = 5.5/m(\pi_L)$ and that a^4 scaling holds true. At larger values of impact parameter a^2 scaling sets on and would require $a \sim 30/m(\pi_L)$ which would correspond to .36 A and to atomic length scale. It is not possible to distinguish between the two options.

4. The singular contribution near to production plane at the cone $v_{cm}cos(\theta) = \beta$ is expected to enhance the total cross section. The strong sensitivity of the cross section to the choice of the cutoff parameter allows to reproduce the experimental findings easily and it would be important to establish strong bounds on the value of the impact parameter.

Dominating contribution to production cross section and diffractive effects

Consider now the behavior of the dominating singular contribution to the production amplitude at the cone $cos(\theta) = \beta/v_{cm}$ depending on b via the exponent factor . This amplitude factorizes into a product

$$
\begin{aligned}
f_{B,sing} &= K_0a^2B(\Delta k)A_{sing}(b,p) \ , \\
B(\Delta k) &= \int F(ax)J_0(\Delta kax)exp(-\frac{sin(\phi)}{sin(\phi_0)}x)xdx \ , \\
&\sim \sqrt{\frac{2}{\pi\Delta ka}}\int F(ax)cos(\Delta kax-\frac{\pi}{4})exp(-\frac{sin(\phi)}{sin(\phi_0)}x)\sqrt{x}dx \ , \\
x &= \frac{b}{a} \ .
\end{aligned}
\quad (6.3.72)
$$

The factor $A_{sing}(b,p) \equiv (4\pi/(Z_1Z_2\alpha_{em})U_{sing}(b,p)$ is the analytically calculable singular and dominating part of the lepto-pion production amplitude (see appendix) with the exponential factor excluded. The factor B is responsible for diffractive effects. The contribution of the peak to the total production cross section is of same order of magnitude as the classical production cross section.

At the peak $\phi \sim 0$ the contribution the exponent of the production amplitude is constant at this limit one obtains product of the Fourier transform of Coulomb potential with cutoffs with the production amplitude. One can calculate the Fourier transform of the Coulomb potential analytically to obtain

$$\begin{aligned} f_{B,sing} &\simeq 4\pi K_0 \frac{(cos(\Delta ka) - cos(\Delta kb_{cr}))}{\Delta k^2} CUT_1 \\ \Delta k &= 2\beta sin(\frac{\alpha}{2}) \ . \end{aligned} \quad (6.3.73)$$

One obtains oscillatory behavior as a function of the collision velocity in fixed angle scattering and the period of oscillation depends on scattering angle and varies in wide limits.

The relationship between scattering angle α and impact parameter in Coulomb scattering translates the impact parameter cutoffs to the scattering angle cutoffs

$$\begin{aligned} a &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(min)/2) \ , \\ b_{cr} &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(max)/2) \ . \end{aligned} \quad (6.3.74)$$

This gives for the argument Δkb of the Bessel function at lower and upper cutoffs the approximate expressions

$$\begin{aligned} \Delta ka &\simeq \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124}{\beta} \ , \\ \Delta kb_{cr} &\simeq x_0 \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124 x_0}{\beta} \ . \end{aligned} \quad (6.3.75)$$

The numerical values are for $Z_1 = Z_2 = 92$ (U-U collision). What is remarkable that the argument Δka at upper momentum cutoff does not depend at all on the value of the cutoff length. The resulting oscillation at minimum scattering angle is more rapid than allowed by the width of the observed peak: $\Delta\beta/\beta \sim 3 \cdot 10^{-3}$ instead of $\Delta\beta/\beta \sim 10^{-2}$: of course, the measured value need not correspond to minimum scattering angle. The oscillation associated with the lower cutoff comes from $cos(2M_R b_{cr} \beta sin(\alpha/2))$ and is slow for small scattering angles $\alpha < 1/A_R \sim 10^{-2}$. For $\alpha(max)$ the oscillation is rapid: $\delta\beta/\beta \sim 10^{-3}$.

In the total production cross section integrated over all scattering angles (or finite angular range) diffractive effects disappear. This might explain why the peak has not been observed in some experiments [C72] .

Cutoff length scales

Consider next the constraints on the upper cutoff length scale.

1. The production amplitude turns out to decrease exponentially as a function of impact parameter b unless lepto-pion is produced in scattering plane. The contribution of lepto-pions produced in scattering plane however gives divergent contribution to the total cross section integrated over all impact parameter values and upper cutoff length scale a is necessary. If one considers scattering with scattering angle between specified limits this is of course not a problem of classical model.

2. Upper cutoff length scale must be longer than the Compton length of lepto-pion.

3. Upper cutoff length scale a should be certainly smaller than the interatomic distance. For partially ionized atoms a more stringent upper bound for a is the size r of atom defined as the distance above which atom looks essentially neutral: a rough extrapolation from hydrogen

atom gives $r \sim a_0/Z^{1/3} \sim 1.5 \cdot 10^{-11}\ m$ (a_0 is Bohr radius of hydrogen atom). Therefore cutoff scale would be between Bohr radius $a_0/Z \sim .5 \cdot 10^{-12}\ m$ and r. In the recent case however atoms are completely ionized so that cutoff length scale can be longer. It turns out that 10 A reproduces the empirical estimate for the cross section correctly.

Numerical estimate for the electro-pion production cross section

The numerical estimate for the electro-pion production cross section is carried out for thorium with $(Z = 90, A = 232)$. The value of the collision velocity of the incoming nucleus in the rest frame of the second nucleus is taken as $\beta = .1$. From the width $\delta v/v = .2$ of velocity distribution in the same frame the upper bound $\gamma \leq 1 + \delta$, $\delta \simeq 2 \times 10^{-3}$ for the Lorentz boost factor of electro-pion in cm system is deduced. The cutoff is necessary because energy conservation is not coded to the structure of the model.

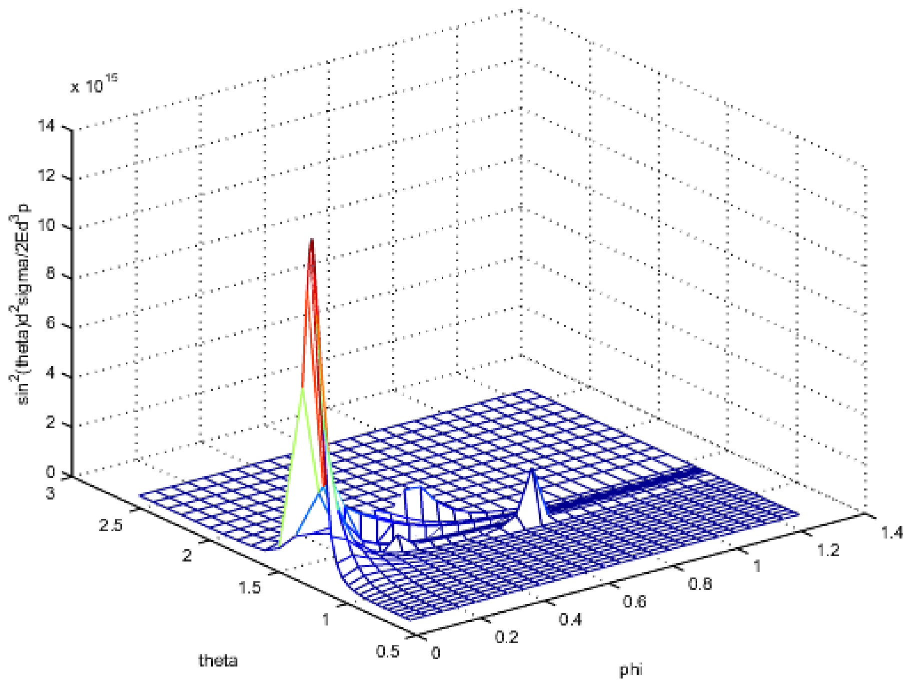

Figure 6.1: Differential cross section $sin^2(\theta) \times \frac{d^2\sigma}{2Ed^3p}$ for τ-pion production for $\gamma_1 = 1.0319 \times 10^3$ in the rest system of antiproton for $\delta = 1.5$. $m(\pi_\tau)$ defines the unit of energy and nb is the unit for cross section. The ranges of θ and ϕ are $(0, \pi)$ and $(0, \pi/2)$.

As expected, the singular contribution from the cone $v_{cm}cos(\theta) = \beta$, $v_{cm} = 2v/(1 + v^2)$ gives the dominating contribution to the cross section. This contribution is proportional to the value of b_{max}^2 at the limit $\phi = 0$. Cutoff radius is taken to be $b_{max} = 150 \times \gamma_{cm}\hbar/m(\pi_e) = 1.04$ A. The numerical estimate for the cross section using the parameter values listed comes out as $\sigma = 5.6\ \mu$b to be compared with the rough experimental estimate of about 5 μb. The interpretation would be that the space-time sheet associated with colliding nuclei during the collision has this transversal size in cm system. At this space-time sheet the electric and magnetic fields of the nuclei interfere.

From this one can cautiously conclude that lepto-pion model is consistent with both electro-pion production and τ-pion production in proton antiproton collisions. One can of course criticize the large value of impact parameter and a good justification for 1 Angstrom should be found. One could also worry about the singular character of the amplitude making the integration of total cross section somewhat risky business using the rather meager numerical facilities available. The rigorous method to calculate the contribution near the singularity relies on stepwise halving of the increment $\Delta\theta$ as one approaches the singularity. The calculation gives essentially the same result as that with constant value of $\Delta\theta$. Hence it seems that one can trust on the result of calculation.

Figure 2. gives the differential production cross section for $\gamma_1 = 1.0319$. Obviously the differential cross section is strongly concentrated at the cone due to singularity of the production amplitude for fixed b.

The important conclusion is that the same model can reproduce the value of production cross section for both electro-pions explaining the old electron-positron anomaly of heavy ion collisions and τ-pions explaining the CDF anomaly of proton-antiproton collisions at cm energy $\sqrt{s} = 1.96$ TeV (to be discussed later) with essentially same and rather reasonable assumptions (do not however forget the large maximal value of the impact parameter!).

In the case of electro-pions one must notice that depending on situation the final states are gamma pairs for the electron-pion with mass very nearly equal to electron mass. In the case of neutral tau-pion the strong decay to three p-adically scaled down versions of τ-pion proceeds faster or at least rate comparable to that for the decay to gamma pair. For higher mass variants of electro-pion for which there is evidence (for instance, one with mass 1.6 MeV) the final states are dominated by electron-positron pairs. This is true if the primary decay products are electro-baryons of form (say) $e_{ex} = e_8\nu_8\nu_{c,8}$ resulting via electro-strong decays instead of electrons and having slightly larger mass than electron. Otherwise the decay to gamma pair would dominate also the decays of higher mass states. A small magnetic moment type coupling between e, e_{ex} and electro-gluon field made possible by the color octet character of colored leptons induces the mixing of e and e_{ex} so that e_{ex} can transform to e by the emission of photon. The anomalous magnetic moment of electron poses restrictions on the color magnetic coupling.

$e^+_{ex}e^-_{ex}$ pairs from lepto-pions or e^+e^- pairs from lepto-sigmas?

If one assumes that anomalous e^+e^- pairs correspond to lepto-nucleon pairs, then lepto-pion production cross section gives a direct estimate for the production rate of e^+e^- pairs. The results of the table 3 show that in case of 1.8 MeV state, the predicted cross section is roughly by a factor 5 smaller than the experimental upper bound for the cross section. Since this lepto-pion state is rather massive, positron decay width allows smaller $f(\pi_L)$ in this case and the production cross section could be larger than the estimate used by the $1/f(\pi_L)^2$ proportionality of the cross section. Both the simplicity and predictive power of this option and the satisfactory agreement with the experimental data suggest that this option provides the most plausible explanation of the anomalous e^+e^- pairs.

N	$Op/10^{-3}$	$\Gamma(\pi_L)/keV$	$\sigma(\pi_L)/\mu b$	$\sigma(\pi_L)/\mu b$
			$a = .01$	$a = .1$
1	1	.51	.13	1.4
3	1	.13	.04	.41
3	5	.73	.19	2.1

Table 2. The table summarizes lepto-pion lifetime and the upper bounds for lepto-pion (and lepto-nucleon pair) production cross sections for the lightest lepto-pion. N refers to the number of lepto-pion states and $Op = \Delta\Gamma/\Gamma$ refers to ortopositronium decay anomaly. The values of upper cutoff length a are in units of 10^{-10} m.

If one assumes that anomalous e^+e^- pairs result from the decays of lepto-sigmas, the value of e^+e^- production cross section can be estimated as follows. e^+e^- pairs are produced from via the creation of $\sigma_L\pi_L$ pairs from vacuum and subsequent decay σ_L to e^+e^- pairs. The estimate for (or rather for the upper bound of) $\pi_L\sigma_L$ production cross section is obtained as

$$\begin{aligned}
\sigma(e^+e^-) &\simeq X\sigma(\pi_L) \ , \\
X &= \frac{V_2}{V_1}(\frac{km_{\sigma_L}}{m_{\pi_L}^2})^2 \ , \\
\frac{V_2}{V_1} &= V_{rel} = \frac{v_{12}^3}{3(2\pi)^2} \sim 1.1 \cdot 10^{-5} \ , \\
\frac{k}{m_{pi_L}} &= \frac{(m_\sigma^2 - m_{\pi_L}^2)}{2m_{\pi_L} f(\pi_L)} \ .
\end{aligned} \quad (6.3.76)$$

Here V_2/V_1 of two-particle and single particle phase space volumes. V_2 is in good approximation the product $V_1(cm)V_1(rel)$ of single particle phase space volumes associated with cm coordinate and relative coordinate and one has $V_2/V_1 \sim V_{rel} = \frac{v_{12}^3}{3(2\pi)^2)} \simeq 1.1 \cdot 10^{-5}$ if the maximum value of the relative velocity is $v_{12} \sim .1$.

Situation is partially saved by the anomalously large value of $\sigma_L\pi_L\pi_L$ coupling constant k appearing in the production vertex $k\sigma_L\pi_L\pi_L(class)$. Production cross section is very sensitive to the value of $f(\pi_L)$ and Op anomaly $\Delta\Gamma/\Gamma = 5 \cdot 10^{-3}$ gives upper bound 2 $\mu b/N_c^2$ for $a = 10^{-11}$ m, which is considerably smaller than the experimental upper bound 5 μb. The huge value of the $g(\pi_L, \pi_L, \sigma_L)$ and $g(\sigma_L, \sigma_L, \sigma_L)$, however implies that radiative corrections to the cross section given by σ exchange are much larger than the lowest order contribution to the cross section! If this is the case then lepto-sigma option might survive but perturbative approach probably would not make sense. On the other hand, one could argue that sigma model action should be regarded as an effective action giving only tree diagrams so that radiative corrections cannot save the situation. There are also purely physical counter arguments against lepto-sigma option: hadronic physics experience suggests that the mass of lepto-sigma is much larger than lepto-pion mass so that lepto-sigma becomes very wide resonance decaying strongly and having negligibly small branching ratio to e^+e^- pairs.

It must be emphasized that the estimates are very rough (the replacement of the integral over the angle α with rough upper bound, estimate for the phase space volume, the values of cutoff radii, the neglect of the velocity dependence of the production cross section, the estimate for the minimum scattering angle, ...). Also the measured production cross section is subject to considerable uncertainties (even the issue whether or not anomalous pairs are produced is not yet completely settled!).

Summary

The usefulness of the modelling lepto-pion production is that the knowledge of lepto-pion production rate makes it possible to estimate also the production rates for other lepto-hadrons and even for many particle states consisting of lepto-hadrons using some effective action describing the strong interactions between lepto-hadrons. One can consider two basic models for lepto-pion production. The models contain no free parameters unless one regards cutoff length scales as such. Classical model predicts the singular production characteristics of lepto-pion. Quantum model predicts several velocity peaks at fixed scattering angle and the distance between the peaks of the production cross section depends sensitively on the value of the scattering angle. Production cross section depends sensitively on the value of the scattering angle for a fixed collision velocity. In both models the reduction of the lepto-pion production rate above Coulomb wall could be understood as a threshold effect: for the collisions with impact parameter smaller than two times nuclear radius, the production amplitude becomes very small since $E \cdot B$ is more or less random for these collisions in the interaction region. The effect is visible for fixed sufficiently large scattering angle only. The value of the anomalous e^+e^- production cross section is of nearly the observed order of magnitude provided that e^+e^- pairs are actually lepto-nucleon pairs originating from the decays of the lepto-pions. Alternative mechanism, in which anomalous pairs originate from the creation of $\sigma_L\pi_L$ pairs from vacuum followed by the decay $\sigma_L \to e^+e^-$ gives too small production cross section by a factor of order $1/N_c^2$ in lowest order calculation. This alternative works only provided that radiative corrections give the dominant contribution to the production rate of $\pi_L\sigma_L$ pairs as is the case if $\pi_L\sigma_L$ mass difference is of order ten per cent. The existence of at least three colored leptons and family replication provide the most plausible explanation the appearance of several peaks.

The proposed models are certainly over idealizations: in particular the approximation that nuclear motion is free motion fails for those values of the impact parameter, which are most important in the classical model. To improve the models one should calculate the Fourier transform of $E \cdot B$ using the fields of nuclei for classical orbits in Coulomb field rather than free motion. The second improvement is related to the more precise modelling of the situation at length scales below b_{cr}, where nuclei do not behave like point like charges. A peculiar feature of the model from the point of view of standard physics is the appearance of the classical electromagnetic fields associated with the classical orbits of the colliding nuclei in the definition of the quantum model. This is in spirit with Quantum TGD: Quantum TGD associates a unique space-time surface (classical history) to a given 3-surface (counterpart of quantum state).

6.4 Further developments

This section represents further developments of lepto-hadron model which have emerged during years after the first version of the model published in International Journal of Theoretical Physics.

6.4.1 How to observe leptonic color?

The most obvious argument against lepto-hadrons is that their production via the decay of virtual photons to lepto-mesons has not been observed in hadronic collisions. The argument is wrong. Anomalously large production of low energy e^+e^- pairs [C70, C41, C65, C20] in hadronic collisions has been actually observed. The most natural source for photons and e^+e^- pairs are lepto-hadrons. There are two possibilities for the basic production mechanism.

1. Colored leptons result directly from the decay of hadronic gluons. Internal consistency excludes this alternative.

2. Colored leptons result from the decay of virtual photons. This hypothesis is in accordance with the general idea that the QCD:s associated with different condensate levels of p-adic topological condensate do not communicate. More precisely, in TGD framework leptons and quarks correspond to different chiralities of WCW spinor s: this implies that baryon and lepton numbers are conserved exactly and therefore the stability of proton. In particular, leptons and quarks correspond to different Kac Moody representations: important difference as compared with typical unified theory, where leptons and quarks share common multiplets of the unifying group. The special feature of TGD is that there are several gluons since it is possible to associate to each Kac-Moody representation gluons, which are "irreducible" in the sense that they couple only to a single Kac Moody representation. It is clear that if the physical gluons are "irreducible" the world separates into different Kac Moody representations having their own color interactions and communicating only via electro-weak and gravitational interactions. In particular, no strong interactions between leptons and hadrons occur. Since colored lepton corresponds to colored ground state of Kac-Moody representations the gluonic color coupling between ordinary lepton and colored lepton vanishes.

If this picture is correct then lepto-hadrons are produced only via the ordinary electro-weak interactions: at higher energies via the decay of virtual photon to colored lepton pair and at low energies via the emission of lepto-pion by photon. Consider next various manners to observe the effects of lepton color.

1. Resonance structure in the photon-photon scattering and energy near lepto-pion mass is a unique signature of lepto-pion.

2. The production of lepto-mesons in strong classical electromagetic fields (of nuclei, for example) is one possibility. There are several important constraints for the production of lepto-pions in this kind of situation.

   i) The scalar product $E \cdot B$ must be large. Faraway from the source region this scalar product tends to vanish: consider only Coulomb field.

ii) The region, where $E \cdot B$ has considerable size cannot be too small as compared with lepto-pion de Broglie wavelength (large when compared with the size of nuclei for example). If this condition doesn't hold true the plane wave appearing in Fourier amplitude is essentially constant spatially and since the fields are approximately static the Fourier component of $E \cdot B$ is expressible as a spatial divergence, which reduces to a surface integral over a surface faraway from the source region. Resulting amplitude is small since fields in faraway region have essentially vanishing $E \cdot B$.

iii) If fields are exactly static, then energy conservation prohibits lepto-hadron production.

3. Also the production of $e^+_{ex}e^-_{ex}$ and $e^+e^-_{ex}$ pairs in nuclear electromagnetic fields with non-vanishing $E \cdot B$ is possible either directly or as decay products of lepto-pions. In the direct production, the predicted cross section is small due to the presence of two-particle phase space factor. One signature of e^-_{ex} is emission line accompanying the decay $e^-_{ex} \rightarrow e^- + \gamma$. The collisions of nuclei in highly ionized (perhaps astrophysical) plasmas provide a possible source of leptobaryons.

4. The interaction of quantized em field with classical electromagnetic fields is one experimental arrangement to come into mind. The simplest arrangement consisting of linearly polarized photons with energy near lepto-pion mass plus constant classical em field does not however work. The direct production of $\pi_L - \gamma$ pairs in rapidly varying classical electromagnetic field with frequency near lepto-pion mass is perhaps a more realistic possibility . An interesting possibility is that violent collisions inside astrophysical objects could lead to gamma ray bursts via the production of pions and lepto-pions in rapidly varying classical E and B fields.

5. In the collisions of hadrons, virtual photon produced in collision can decay to lepto-hadrons, which in turn produce lepto-pions decaying to leptonucleon pairs. As already noticed, anomalous production of low energy e^+e^- pairs (actually leptonucleon pairs!) [C70] in hadronic collisions has been observed.

6. $e - \nu_e$ and $e - \bar{\nu}_e$ scattering at energies below one MeV provide a unique signature of lepto-pion. In $e - \bar{\nu}_e$ scattering π_L appears as resonance.

7. If leptonic color coupling strength has sufficiently small value in the energy range at which lepto-hadronic QCD exists, e^+e^- annihilation at energies above few MeV should produce colored pairs and lepto-hadronic counterparts of the hadron jets should be observed. The fact that nothing like this has been observed, suggests that lepto-hadronic coupling constant evolution does not allow the perturbative QCD phase.

6.4.2 New experimental evidence

After writing this chapter astrophysical support for the notion of lepto-pions has appeared. There is also experimental evidence for the existence of colored muons

Could lepto-hadrons correspond to dark matter?

The proposed identification of cosmic strings (in TGD sense) as the ultimate source of both visible and dark matter discussed in [K26] does not exclude the possibility that a considerable portion of topologically condensed cosmic strings have decayed to some light particles. In particular, this could be the situation in the galactic nuclei.

The idea that lepto-hadrons might have something to do with the dark matter has popped up now and then during the last decade but for some reason I have not taken it seriously. Situation changed towards the end of the year 2003. There exist now detailed maps of the dark matter in the center of galaxy and it has been found that the density of dark matter correlates strongly with the intensity of monochromatic photons with energy equal to the rest mass of electron [E12] .

The only explanation for the radiation is that some yet unidentified particle of mass very nearly equal to $2m_e$ decays to an electron positron pair. Electron and positron are almost at rest and this implies a high rate for the annihilation to a pair of gamma rays. A natural identification for the particle in question would be as a lepto-pion (or rather, electro-pion). By their low mass lepto-pions, just like ordinary pions, would be produced in high abundance, in lepto-hadronic strong reactions and therefore the intensity of the monochromatic photons resulting in their decays would serve as a measure for the density of the lepto-hadronic matter. Also the presence of lepto-pionic condensates can be considered.

These findings force to take seriously the identification of the dark matter as lepto-hadrons. This is however not the only possibility. The TGD based model for tetra-neutrons discussed in [K80] is based on the hypothesis that mesons made of scaled down versions of quarks corresponding to Mersenne prime M_{127} (ordinary quarks correspond to $k = 107$) and having masses around one MeV could correspond to the color electric flux tubes binding the neutrons to form a tetra-neutron. The same force would be also relevant for the understanding of alpha particles.

There are also good theoretical arguments for why lepto-hadrons should be dark matter in the sense of having a non-standard value of Planck constant.

1. Since particles with different Planck constant correspond to different pages of the book like structure defining the generalization of the imbedding space, the decays of intermediate gauge bosons to colored excitations of leptons would not occur and would thus not contribute to their decay widths.

2. In the case of electro-pions the large value of the coupling parameter $Z_1Z_2\alpha_{em} > 1$ combined with the hypothesis that a phase transition increasing Planck constant occurs as perturbative QFT like description fails would predict that electro-pions represent dark matter. Indeed, the power series expansion of the $exp(iS)$ term might well fail to converge in this case since S is proportional to Z_1Z_2. For τ-pion production one has $Z_1 = -Z_2 = 1$ and in this case one can consider also the possibility that τ-pions are not dark in the sense of having large Planck constant. Contrary to the original expectations darkness does not affect the lowest order prediction for the production cross section of lepto-pion.

The proposed identification raises several questions.

1. Why the ratio of the lepto-hadronic mass density to the mass density of the ordinary hadrons would be so high, of order 7? Could an entire hierarchy of asymptotically non-free QCDs be responsible for the dark matter so that lepto-hadrons would explain only a small portion of the dark matter?

2. Under what conditions one can regard lepto-hadronic matter as a dark matter? Could short life-times of lepto-hadrons make them effectively dark matter in the sense that there would be no stable enough atom like structures consisting of say charged leptobaryons bound electromagnetically to the ordinary nuclei or electrons? But what would be the mechanism producing lepto-hadrons in this case (nuclear collisions produce lepto-pions only under very special conditions)?

3. What would be the role of the many-sheeted space-time: could lepto-hadrons and atomic nuclei reside at different space-time sheets so that leptobaryons could be long-lived? Could dark matter quite generally correspond to the matter at different space-time sheets and thus serve as a direct signature of the many-sheeted space-time topology?

Lightnings and lepto-pions

The latest discovery of Fermi space-telescope [C25] is the finding of .511 MeV gamma rays in the the spectrum of photons associated with lightnings. It was discovered already years ago that lightnings are accompanied by X-rays [C90] and even gamma rays [C99] . For instance, the strong electric fields created by a positively charged region of cloud could accelerate electron from both downwards and upwards to this region. The problem is that atmosphere is not empty and dissipation would restrict the energies to be much lower than gamma ray energies which are in MeV range. Note that the temperatures in lightning are about 3×10^4 K and correspond to electron energy of 2.6 eV which is by a factor 10^5 smaller than electron mass and gamma ray energy scale!

Situation changes if dissipation is absent so that the electrons are accelerated without any energy losses. This is the case if the electrons reside in large $\hbar$ quantum phase at magnetic flux tubes so that dissipative losses are small and electrons can reach relativistic energies. This is the explanation that I provided years ago for the [K28] .

Fermi however observed also something completely new. There is also a peaking of gamma rays around energy .511 MeV. The decay of electro-pion is an obvious explanation for this peaking. If electro-pions are there, collisions of highly energetic particles lasting for time of about $\tau \sim \hbar/\text{MeV}$ are expected. The natural candidates for the colliding charged particles are electrons. The center of mass system -the system in which total momentum of colliding electron pair vanishes- should be in a good approximation at rest with respect to Fermi space telescope. Otherwise the energy of gamma rays would be higher or lower than .511 MeV.

The only possibility that I can imagine is that the second electron comes from below and second from above the positively charged region of the thunder cloud. Both arrive as dark electrons with a large value of $\hbar$ and are accelerated to relativistic energies since dissipation is very small. They could collide as dark electrons (the more probable option as will be found below) or suffer a phase transition transforming them to ordinary electrons before the collision. Electro-pion coherent state is created in the strong $E \cdot B$ created for a a period of time of order $\tau \sim \hbar_0/\text{MeV}$. This state annihilates rapidly to pairs of gamma rays which are ordinary or transform to ordinary ones depending on whether electrons where dark or not.

What the phase transition of dark electrons to ordinary electrons means, needs some explaining. The generalized imbedding space is obtained by gluing almost copies of 8-D imbedding space $M^4 \times CP_2$ along their common back to get a book like structure. Particles at different pages of the book are dark with respect to each other in the sense that they have no local interactions. This is enough to explain what is actually known about dark matter. Particles at different pages can however interact via classical fields and photon exchange (for instance). The phase transition of electron from dark to visible form preceding the collision of dark electrons would simply mean the leakage from large $\hbar$ page to the "visible" page with ordinary value of Planck constant.

Alert reader might be ready to ask the obvious question. Why not to test the hypothesis in laboratory? It should not be too difficult to allow two electrons to collide with a relativistic energy and find whether gamma pairs with energy .511 MeV are produced in rest system. Maybe gamma ray pairs have been missed for some reason? If not (the probable option), then colored electrons and lepto-pions are always dark. This would explain why the colored leptons do not contribute to the decay widths of weak gauge bosons which pose very strong constraints for the existence of light exotic particles.

Experimental evidence for colored muons

Also μ and τ should possess colored excitations. About fifteen years after this prediction was made. Direct experimental evidence for these states finally emerges (the year I am adding this comment is 2007) [C126, C127] . The mass of the new particle, which is either scalar or pseudo-scalar, is 214.4 MeV whereas muon mass is 105.6 MeV. The mass is about 1.5 per cent higher than two times muon mass. The proposed interpretation is as a light Higgs. I do not immediately resonate with this interpretation although p-adically scaled up variants of also Higgs bosons live happily in the fractal Universe of TGD. The most natural TGD inspired interpretation is as a pion like bound state of colored excitations of muon completely analogous to lepto-pion (or rather, electro-pion).

Scaled up variants of QCD appear also in nuclear string model [K80, L2] , [L2] , where scaled variant of QCD for exotic quarks in p-adic length scale of electron is responsible for the binding of ^4He nuclei to nuclear strings. One cannot exclude the possibility that the fermion and anti-fermion at the ends of color flux tubes connecting nucleons are actually colored leptons although the working hypothesis is that they are exotic quark and anti-quark. One can of course also turn around the argument: could it be that lepto-pions are "leptonuclei", that is bound states of ordinary leptons bound by color flux tubes for a QCD in length scale considerably shorter than the p-adic length scale of lepton.

6.4.3 Evidence for τ-hadrons

The evidence for τ-leptons came in somewhat funny but very pleasant manner. During my friday morning blog walk, the day next to my birthday October 30, I found that Peter Woit had told in his blog about a possible discovery of a new long-lived particle by CDF experiment [C123] emphasizing how revolutionary finding is if it is real. There is a detailed paper [C26] with title *Study of multi-muon events produced in p-pbar collisions at* $\sqrt(s) = 1.96$ *TeV* by CDF collaboration added to the ArXiv October 29 - the eve of my birthday. I got even second gift posted to arXiv the very same day and reporting an anomalously high abundance of positrons in cosmic ray radiation [C32] . Both of these article give support for basic predictions of TGD differentiating between TGD and standard model and its generalizations.

The first gift

A brief summary of Peter Woit about the finding gives good idea about what is involved.

The article originates in studies designed to determine the b-bbar cross-section by looking for events, where a b-bbar pair is produced, each component of the pair decaying into a muon. The b-quark lifetime is of order a picosecond, so b-quarks travel a millimeter or so before decaying. The tracks from these decays can be reconstructed using the inner silicon detectors surrounding the beam-pipe, which has a radius of 1.5 cm. They can be characterized by their impact parameter, the closest distance between the extrapolated track and the primary interaction vertex, in the plane transverse to the beam.

If one looks at events where the b-quark vertices are directly reconstructed, fitting a secondary vertex, the cross-section for b-bbar production comes out about as expected. On the other hand, if one just tries to identify b-quarks by their semi-leptonic decays, one gets a value for the b-bbar cross-section that is too large by a factor of two. In the second case, presumably there is some background being misidentified as b-bbar production.

The new result is based on a study of this background using a sample of events containing two muons, varying the tightness of the requirements on observed tracks in the layers of the silicon detector. The background being searched for should appear as the requirements are loosened. It turns out that such events seem to contain an anomalous component with unexpected properties that disagree with those of the known possible sources of background. The number of these anomalous events is large (tens of thousands), so this cannot just be a statistical fluctuation.

One of the anomalous properties of these events is that they contain tracks with large impact parameters, of order a centimeter rather than the hundreds of microns characteristic of b-quark decays. Fitting this tail by an exponential, one gets what one would expect to see from the decay of a new, unknown particle with a lifetime of about 20 picoseconds. These events have further unusual properties, including an anomalously high number of additional muons in small angular cones about the primary ones.

The lifetime is estimated to be considerably longer than b quark life time and below the lifetime 89.5 ps of $K_{0,s}$ mesons. The fit to the tail of "ghost" muons gives the estimate of 20 picoseconds.

The second gift

In October 29 also another remarkable paper [C32] had appeared in arXiv. It was titled *Observation of an anomalous positron abundance in the cosmic radiation*. PAMELA collaboration finds an excess of cosmic ray positron at energies $10 \rightarrow 50$ GeV. PAMELA anomaly is discussed in Resonaances blog [C9] . ATIC collaboration in turn sees an excess of electrons and positrons going all the way up to energies of order 500-800 GeV [C79] .

Also Peter Woit refers to these cosmic ray anomalies and also to the article *LHC Signals for a SuperUnified Theory of Dark Matter* by Nima Arkadi-Hamed and Neal Weiner [C19] , where a model of dark matter inspired by these anomalies is proposed together with a prediction of lepton jets with invariant masses with mass scale of order GeV. The model assumes a new gauge interaction for dark matter particles with Higgs and gauge boson masses around GeV. The prediction is that LHC should detect "lepton jets" with smaller angular separations and GeV scale invariant masses.

Explanation of the CDF anomaly

Consider first the CDF anomaly. TGD predicts a fractal hierarchy of QCD type physics. In particular, colored excitations of leptons are predicted to exist. Neutral lepto-pions would have mass only slightly above two times the charged lepton mass. Also charged lepto-pions are predicts and their masses depend on what is the p-adic mass scale of neutrino and it is not clear whether it is much longer than that for charge colored lepton as in the case of ordinary leptons.

1. There exists a considerable evidence for colored electrons as already found. The anomalous production of electron positron pairs discovered in heavy ion collisions can be understood in terms of decays of electro-pions produced in the strong non-orthogonal electric and magnetic fields created in these collisions. The action determining the production rate would be proportional to the product of the lepto-pion field and highly unique "instanton" action for electromagnetic field determined by anomaly arguments so that the model is highly predictive.

2. Also the .511 MeV emission line [C50, C53] from the galactic center can be understood in terms of decays of neutral electro-pions to photon pairs. Electro-pions would reside at magnetic flux tubes of strong galactic magnetic fields. It is also possible that these particles are dark in TGD sense.

3. There is also evidence for colored excitations of muon and muo-pion [C126, C127] . Muo-pions could be produced by the same mechanism as electro-pions in high energy collisions of charged particles when strong non-orthogonal magnetic and electric fields are generated.

Also τ-hadrons are possible and CDF anomaly can be understood in terms of a production of higher energy τ-hadrons as the following argument demonstrates.

1. τ-QCD at high energies would produce "lepton jets" just as ordinary QCD. In particular, muon pairs with invariant energy below $2m(\tau) \sim 3.6$ GeV would be produced by the decays of neutral τ-pions. The production of monochromatic gamma ray pairs is predicted to dominate the decays. Note that the space-time sheet associated with both ordinary hadrons and τ lepton correspond to the p-adic prime $M_{107} = 2^{107} - 1$.

2. The model for the production of electro-pions in heavy ion collisions suggests that the production of τ-pions could take place in higher energy collisions of protons generating very strong non-orthogonal magnetic and electric fields. This This would reduce the model to the quantum model for electro-pion production.

3. One can imagine several options for the detailed production mechanism.

   (a) The decay of *virtual* τ-pions created in these fields to pairs of leptobaryons generates lepton jets. Since colored leptons correspond to color octets, leptobaryons could correspond to states of form LLL or $L\overline{L}L$.

   (b) The option inspired by a blog discussion with Ervin Goldfein is that a coherent state of τ-pions is created first and is then heated to QCD plasma like state producing the lepton jets like in QCD. The linear coupling to $E \cdot B$ defined by em fields of colliding nucleons would be analogous to the coupling of harmonic oscillator to constant force and generate the coherent state.

   (c) The option inspired by CDF model [C73] is that a p-adically scaled up variant of *on mass shell* neutral τ-pion having $k = 103$ and 4 times larger mass than $k = 107$ τ-pion is produced and decays to three $k = 105$ τ-pions with $k = 105$ neutral τ-pion in turn decaying to three $k = 107$ τ-pions.

4. The basic characteristics of the anomalous muon pair prediction seems to fit with what one would expect from a jet generating a cascade of τ-pions. Muons with both charges would be produced democratically from neutral τ-pions; the number of muons would be anomalously high; and the invariant masses of muon pairs would be below 3.6 GeV for neutral τ-pions and below 1.8 GeV for charged τ-pions if colored neutrinos are light.

5. The lifetime of 20 ps can be assigned with charged τ-pion decaying weakly only into muon and neutrino. This provides a killer test for the hypothesis. In absence of CKM mixing for colored neutrinos, the decay rate to lepton and its antineutrino is given by

$$\Gamma(\pi_\tau \rightarrow L + \overline{\nu}_L) = \frac{G^2 m(L)^2 f^2(\pi)(m(\pi_\tau)^2 - m(L)^2)^2}{4\pi m^3(\pi_\tau)} . \quad (6.4.1)$$

The parameter $f(\pi_\tau)$ characterizing the coupling of pion to the axial current can be written as $f(\pi_\tau) = r(\pi_\tau)m(\pi_\tau)$. For ordinary pion one has $f(\pi) = 93$ MeV and $r(\pi) = .67$. The decay rate for charged τ-pion is obtained by simple scaling giving

$$\begin{aligned} \Gamma(\pi_\tau \rightarrow L + \overline{\nu}_L) &= 8x^2u^2y^3(1-z^2)\frac{1}{cos^2(\theta_c)}\Gamma(\pi \rightarrow \mu + \overline{\nu_\mu}) \ , \\ x &= \frac{m(L)}{m(\mu)} \ , \ y = \frac{m(\tau)}{m(\pi)} \ , \ z = \frac{m(L)}{2m(\tau)} \ , \ u = \frac{r(\pi_\tau)}{r(\pi)} \ . \end{aligned} \tag{6.4.2}$$

If the p-adic mass scale of the colored neutrino is same as for ordinary neutrinos, the mass of charged lepto-pion is in good approximation equal to the mass of τ and the decay rates to τ and electron are for the lack of phase space much slower than to muons so that muons are produced preferentially.

6. For $m(\tau) = 1.8$ GeV and $m(\pi) = .14$ GeV and the same value for f_π as for ordinary pion the lifetime is obtained by scaling from the lifetime of charged pion about 2.6×10^{-8} s. The prediction is 3.31×10^{-12} s to be compared with the experimental estimate about 20×10^{-12} s. $r(\pi_\tau) = .41 r_\pi$ gives a correct prediction. Hence the explanation in terms of τ-pions seems to be rather convincing unless one is willing to believe in really nasty miracles.

7. Neutral τ-pion would decay dominantly to monochromatic pairs of gamma rays. The decay rate is dictated by the product of τ-pion field and "instanton" action, essentially the inner product of electric and magnetic fields and reducing to total divergence of instanton current locally. The rate is given by

$$\begin{aligned} \Gamma(\pi_\tau \rightarrow \gamma + \gamma) &= \frac{\alpha_{em}^2 m^3(\pi_\tau)}{64\pi^3 f(\pi_\tau)^2} = 2x^{-2}y \times \Gamma(\pi \rightarrow \gamma + \gamma) \ , \\ x &= \frac{f(\pi_\tau)}{m(\pi_\tau)} \ , \ y = \frac{m(\tau)}{m(\pi)} \ .\Gamma(\pi \rightarrow \gamma + \gamma) = 7.37 \ eV \ . \end{aligned} \tag{6.4.3}$$

The predicted lifetime is 1.17×10^{-17} seconds.

8. Second decay channel is to lepton pairs, with muon pair production dominating for kinematical reasons. The invariant mass of the pairs is 3.6 GeV of no other particles are produced. Whether the mass of colored neutrino is essentially the same as that of charged lepton or corresponds to the same p-adic scale as the mass of the ordinary neutrino remains an open question. If colored neutrino is light, the invariant mass of muon-neutrino pair is below 1.78 GeV.

PAMELA and ATIC anomalies

TGD predicts also a hierarchy of hadron physics assignable to Mersenne primes. The mass scale of M_{89} hadron physics is by a factor 512 higher than that of ordinary hadron physics. Therefore a very rough estimate for the nucleons of this physics is 512 GeV. This suggest that the decays of M_{89} hadrons are responsible for the anomalous positrons and electrons up to energies 500-800 GeV reported by ATIC collaboration. An equally naive scaling for the mass of pion predicts that M_{89} pion has mass 72 GeV. This could relate to the anomalous cosmic ray positrons in the energy interval 10-50 GeV reported by PAMELA collaboration. Be as it may, the prediction is that M_{89} hadron physics exists and could make itself visible in LHC.

The surprising finding is that positron fraction (the ratio of flux of positrons to the sum of electron and positron fluxes) increases above 10 GeV. If positrons emerge from secondary production during the propagation of cosmic ray-nuclei, this ratio should decrease if only standard physics is be involved with the collisions. This is taken as evidence for the production of electron-positron pairs, possibly in the decays of dark matter particles.

Leptohadron hypothesis predicts that in high energy collisions of charged nuclei with charged particles of matter it is possible to produce also charged electro-pions, which decay to electrons or positrons depending on their charge and produce the electronic counterparts of the jets discovered in CDF. This proposal - and more generally lepto-hadron hypothesis - could be tested by trying to find whether also electronic jets can be found in proton-proton collisions. They should be present at considerably lower energies than muon jets. I decided to check whether I have said something about this earlier and found that I have noticed years ago that there is evidence for the production of anomalous electron-positron pairs in hadronic reactions [C70, C41, C65, C20] : some of it dates back to seventies.

The first guess is that the center of mass energy at which the jet formation begins to make itself visible is in a constant ratio to the mass of charged lepton. From CDF data this ratio satisfies $\sqrt{s}/m_\tau = x < 10^3$. For electro-pions the threshold energy would be around $10^{-3}x \times .5$ GeV and for muo-pions around $10^{-3}x \times 100$ GeV.

Comparison of TGD model with the model of CDF collaboration

Few days after the experimental a theoretical paper by CDF collaboration proposing a phenomenological model for the CDF anomaly appeared in the arXiv [C73], and it is interesting to compare the model with TGD based model (or rather, one of them corresponding to the third option mentioned above).

The paper proposes that three new particles are involved. The masses for the particles - christened h_3, h_2, and h_1 - are assumed to be 3.6 GeV, 7.3 GeV, and 15 GeV. h_1 is assumed to be pair produced and decay to h_2 pair decaying to h_3 pair decaying to a τ pair.

h_3 is assumed to have mass 3.6 GeV and life-time of 20×10^{-12} seconds. The mass is same as the TGD based prediction for neutral τ-pion mass, whose lifetime however equals to 1.12×10^{-17} seconds ($\gamma+\gamma$ decay dominates). The correct prediction for the lifetime provides a strong support for the identification of long-lived state as charged τ-pion with mass near τ mass so that the decay to μ and its antineutrino dominates. Hence the model is not consistent with lepto-hadronic model.

p-Adic length scale hypothesis predicts that allowed mass scales come as powers of $\sqrt{2}$ and these masses indeed come in good approximation as powers of 2. Several p-adic scales appear in low energy hadron physics for quarks and this replaces Gell-Mann formula for low-lying hadron masses. Therefore one can ask whether the proposed masses correspond to neutral tau-pion with $p = M_k = 2^k - 1$, $k = 107$, and its p-adically scaled up variants with $p \simeq 2^k$, $k = 105$, and $k = 103$ (also prime). The prediction for masses would be 3.6 GeV, 7.2 GeV, 14.4 GeV.

This co-incidence cannot of course be taken too seriously since the powers of two in CDF model have a rather mundane origin: they follow from the assumed production mechanism producing 8 τ-leptons from h_1. One can however spend some time by looking whether it could be realized somehow allowing p-adically scaled up variants of τ-pion.

1. The proposed model for the production of muon jets is based on production of k=103 neutral τ-pion (or several of them) having 4 times larger mass than k=107 τ-pion in strong EB background of the colliding proton and antiproton and decaying via weak boson and gluon exchanges to k=105 and k=107 τ-pions. The simplest decays are parity breaking $1 \to 2$ decays and must involve exchange of virtual W or Z boson. Three-pion coupling λ with dimensions of mass determines the decay rates for neutral τ-pions appearing in the cascade. For the four-pion decay the coupling is dimensionless. Rates are proportional to phase space-volumes, which are rather small by kinetic reasons and also reduced by weak coupling.

2. For a neutral initial state the first step could be one of the following ones:

$$\begin{aligned}
\pi_\tau^0(103) &\to \pi_\tau^+(105) + \pi_\tau^-(105) \\
\pi_\tau^0(103) &\to \pi_\tau^0(105) + \pi_\tau^0(105) \\
\pi_\tau^0(103) &\to 2\gamma \\
\pi_\tau^0(103) &\to \pi_\tau^+(105) + \pi_\tau^-(107) + \pi_\tau^0(107)
\end{aligned}$$

In the last decay permutations of the final state charges are possible. Since the last reaction is parity conserving and governed by strong interactions it dominates. This step is not kinematically possible if masses are obtained by exact scaling and if $m(\pi_\tau^0) < m(pi_\tau^\pm)$ holds true as for ordinary pion. p-Adic mass formulas do not however predict exact scaling. In the case that reaction is not kinematically possible, it must be replaced with a reaction in which one final state pion is virtual.

3. At the second step charged pion would decay to two pions

$$\pi_\tau^\pm(105) \to \pi_\tau^0(107) + \pi_\tau^\pm(107) \ ,$$

Neutral pion could decay to two gammas or to two pions

$$\pi_\tau^0(105) \to 2\gamma \ or \ \pi_\tau^+(107) + \pi_\tau^-(107) \ or \ \pi_\tau^0(107) + \pi_\tau^0(107) \ .,$$

Here second charged pion also can be virtual and decay weakly, and the weak decays of the $\pi_\tau^\pm(105)$ with mass $2m(\tau)$ to lepton pairs. The rates for these are obtained from previous formulas by scaling. For neutral pion the deay to two gammas dominates now.

4. The last step would involve the decays of both charged and neutral $\pi_\tau(107)$. The signature of the mechanism would be anomalous γ pairs with invariant masses $2^k \times m(\tau)$, $k = 1, 2, 3$ coming from the decays of neutral τ-pions.

The total cross section for producing single lepto-pion can be estimated by using the quantum model for lepto-pion production. Production amplitude is essentially Coulomb scattering amplitude for a given value of the impact parameter b for colliding proton and anti-proton multiplied by the amplitude $U(b,p)$ for producing on mass shell $k = 103$ lepto-pion with given four-momentum in the fields E and B and given essentially by the Fourier transform of $E \cdot B$. The replacement of the motion with free motion should be a good approximation.

UV and IR cutoffs for the impact parameter appear in the model and are identifiable as appropriate p-adic length scales. UV cutoff could correspond to the Compton size of nucleon ($k = 107$) and IR cutoff to the size of the space-time sheets representing topologically quantized electromagnetic fields of colliding nucleons (perhaps $k = 113$ corresponding to nuclear p-adic length scale and size for color magnetic body of constituent quarks or $k = 127$ for the magnetic body of current quarks with mass scale of order MeV). If one has $\hbar/\hbar_0 = 2^7$ one could also guess that the IR cutoff corresponds to the size of dark em space-time sheet equal to $2^7 L(113) = L(127)$ (or $2^7 L(127) = L(141)$), which corresponds to electron's p-adic length scale. These are of course rough guesses.

Quantitatively the jet-likeness of muons means that the additional muons are contained in the cone $\theta < 36.8$ degrees around the initial muon direction. If the decay of $\pi_\tau^0(k)$ can occur to on mass shell $\pi_\tau^0(k+2)$, $k = 103, 105$, it is possible to understand jets as a consequence of the decay kinematics forcing the pions resulting as decay products to be almost at rest.

1. Suppose that the decays to three pions can take place as on mass shell decays so that pions are very nearly at rest. The distribution of decay products $\mu\overline{\nu}$ in the decays of $\pi^\pm(105)$ is spherically symmetric in the rest frame and the energy and momentum of the muon are given by

$$[E, p] = [m(\tau) + \frac{m^2(\mu)}{4m(\tau)}, m(\tau) - \frac{m^2(\mu)}{4m(\tau)}] \ .$$

The boost factor $\gamma = 1/\sqrt{1 - v^2}$ to the rest system of muon is $\gamma = \frac{m(\tau)}{m(\mu)} + \frac{m(\mu)}{4m(\tau})) \sim 18$.

2. The momentum distribution for μ^+ coming from π_τ^+ is spherically symmetric in the rest system of π^+ . In the rest system of μ^- the momentum distribution is non-vanishing only for when the angle θ between the direction of velocity of μ^- is below a maximum value of given by $tan(\theta_{max}) = 1$ corresponding to a situation in which the momentum μ^+ is orthogonal to the momentum of μ^- (the maximum transverse momentum equals to $m(\mu)v\gamma$ and longitudinal

momentum becomes $m(\mu)v\gamma$ in the boost). This angle corresponds to 45 degrees and is not too far from 36.8 degrees.

3. At the next step the energy of muons resulting in the decays of $\pi^{\pm}(103)$

$$[E,p] = [\frac{m(\tau)}{2} + \frac{m^2(\mu)}{2m(\tau)}, \frac{m(\tau)}{2} - \frac{m^2(\mu)}{2m(\tau)}] \ ,$$

and the boost factor is $\gamma_1 = \frac{m(\tau)}{2m(\mu)} + \frac{m(\mu)}{2m(\tau)} \sim 9$. θ_{max} satisfies the condition $tan(\theta_{max}) = \gamma_1 v_1/\gamma v \simeq 1/2$ giving $\theta_{max} \simeq 26.6$ degrees.

If on mass shell decays are not allowed the situation changes since either of the charged pions is off mass shell. In order to obtain similar result the virtual should occur dominantly via states near to on mass shell pion. Since four-pion coupling is just constant, this option does not seem to be realized.

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3. At the next step the energy of muons resulting in the decays of $\pi^{\pm}(103)$

$$[E,p] = [\frac{m(\tau)}{2} + \frac{m^2(\mu)}{2m(\tau)}, \frac{m(\tau)}{2} - \frac{m^2(\mu)}{2m(\tau)}] \ ,$$

and the boost factor is $\gamma_1 = \frac{m(\tau)}{2m(\mu)} + \frac{m(\mu)}{2m(\tau)} \sim 9$. θ_{max} satisfies the condition $tan(\theta_{max}) = \gamma_1 v_1/\gamma v \simeq 1/2$ giving $\theta_{max} \simeq 26.6$ degrees.

If on mass shell decays are not possible, the situation changes since either of the charged pions is off mass shell. In order to obtain similar result the virtual should occur dominantly via states near to on mass shell pion. Since four-pion coupling is just constant, this option does not seem to be realized.

Numerical estimate for the production cross section

The numerical estimate of the cross section involves some delicacies. The model has purely physical cutoffs which must be formulated in a precise manner.

1. Since energy conservation is not coded into the model, some assumption about the maximal τ-pion energy in cm system expressed as a fraction ϵ of proton's center of mass energy is necessary. Maximal fraction corresponds to the condition $m(\pi_\tau) \leq m(\pi_\tau)\gamma_1 \leq \epsilon m_p \gamma_{cm}$ in cm system giving $[m(\pi_\tau)/(m_p\gamma_{cm}) \leq \epsilon \leq 1$. γ_{cm} can be deduced from the center of mass energy of proton as $\gamma_{cm} = \sqrt{s}2m_p$, $\sqrt{s} = 1.96$ TeV. This gives $1.6 \times 10^{-2} < \epsilon < 1$ in a reasonable approximation. It is convenient to parameterize ϵ as

$$\epsilon = (1+\delta) \times \frac{m(\pi_\tau)}{m_p} \times \frac{1}{\gamma_{cm}} \quad .$$

The coordinate system in which the calculations are carried out is taken to be the rest system of (say) antiproton so that one must perform a Lorentz boost to obtain upper and lower limits for the velocity of τ-pion in this system. In this system the range of γ_1 is fixed by the maximal cm velocity fixed by ϵ and the upper/lower limit of γ_1 corresponds to a direction parallel/opposite to the velocity of proton.

2. By Lorentz invariance the value of the impact parameter cutoff b_{max} should be expressible in terms τ-pion Compton length and the center of mass energy of the colliding proton and the assumption is that $b_{max} = \gamma_{cm} \times \hbar/m(\pi_\tau)$, where it is assumed $m(\pi_\tau) = 8m(\tau)$. The production cross section does not depend much on the precise choice of the impact parameter cutoff b_{max} unless it is un-physically large in which case b_{max}^2 proportionality is predicted.

The numerical estimate for the production cross section involves some delicacies.

1. The power series expansion of the integral of CUT_1 using partial fraction representation does not converge since that roots $c_\pm$ are very large in the entire integration region. Instead the approximation $A_1 \simeq iBcos(\psi)/D$ simplifying considerably the calculations can be used. Also the value of b_1L is rather small and one can use stationary phase approximation for CUT_2. It turns out that the contribution of CUT_2 is negligible as compared to that of CUT_1.

2. Since the situation is singular for $\theta = 0$ and $\phi = 0$ and $\phi = \pi/2$ (by symmetry it is enough to calculate the cross section only for this kinematical region), cutoffs

$$\theta \in [\epsilon_1, (1-\epsilon_1)] \times \pi \ \ , \quad \phi \in [\epsilon_1, (1-\epsilon_1)] \times \pi/2 \ \ , \quad \epsilon_1 = 10^{-3} \quad .$$

The result of the calculation is not very sensitive to the value of the cutoff.

3. Since the available numerical environment was rather primitive (MATLAB in personal computer), the requirement of a reasonable calculation time restricted the number of intervals in the discretization for the three kinematical variables γ, θ, ϕ to be below $N_{max} = 80$. The result of calculation did not depend appreciably on the number of intervals above $N = 40$ for γ_1 integral and for θ and ϕ integrals even $N = 10$ gave a good estimate.

The calculations were carried for the $exp(iS)$ option since in good approximation the estimate for $exp(iS) - 1$ model is obtained by a simple scaling. $exp(iS)$ model produces a correct order of magnitude for the cross section whereas $exp(iS) - 1$ variant predicts a cross section, which is by several orders of magnitude smaller by downwards α_{em}^2 scaling. As I asked Tommaso Dorigo for an estimate for the production cross section in his first blog posting [C38] , he mentioned that authors refer to a production cross section is 100 nb which looks to me suspiciously large (too large by three orders of magnitude), when compared with the production rate of muon pairs from b-bbar. $\delta = 1.5$ which corresponds to τ-pion energy 36 GeV gives the estimate $\sigma = 351$ nb. The energy is suspiciously high.

In fact, in the recent blog posting of Tommaso Dorigo [C37] a value of order .1 nb for the production cross section was mentioned. Electro-pions in heavy ion collisions are produced almost at rest and one has $\Delta v/v \simeq .2$ giving $\delta = \Delta E/m(\pi) \simeq 2 \times 10^{-3}$. If one believes in fractal scaling, this should be at least the order of magnitude also in the case of τ-pion. This would give the estimate $\sigma = 1$ nb. For $\delta = \Delta E/m(\pi) \simeq 10^{-3}$ a cross section $\sigma = .16$ nb would result.

One must of course take the estimate cautiously but there are reasons to hope that large systematic errors are not present anymore. In any case, the model can explain also the order of magnitude of the production cross section under reasonable assumptions about cutoffs.

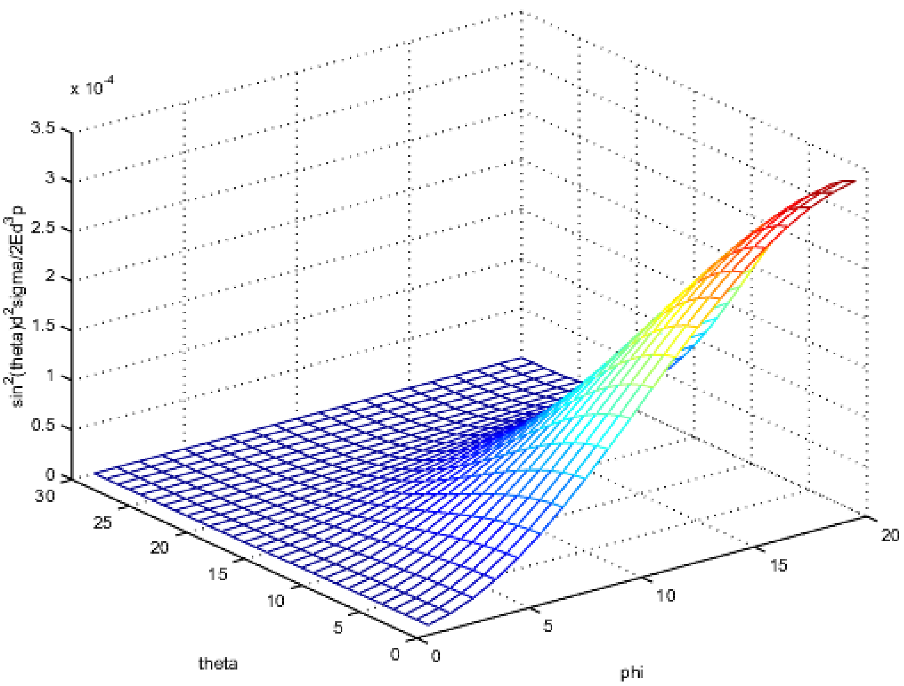

Figure 6.2: Differential cross section $sin^2(\theta) \times \frac{d^2\sigma}{2Ed^3p}$ for τ-pion production for $\gamma_1 = 1.090 \times 10^3$ in the rest system of antiproton for $\delta = 1.5$. $m(\pi_\tau)$ defines the unit of energy and nb is the unit for cross section. The ranges of θ and ϕ are $(0, \pi)$ and $(0, \pi/2)$.

Does the production of lepto-pions involve a phase transition increasing Planck constant?

The critical argument of Tommaso Dorigo in his blog inspired an attempt to formulate more precisely the hypothesis $\sqrt{s}/m_\tau > x < 10^3$. This led to the realization that a phase transition increasing Planck constant might happen in the production process as also the model for the production of electro-pions requires.

Suppose that the instanton coupling gives rise to *virtual* neutral lepto-pions which ultimately produce the jets (this is first of the three models that one can imagine). E and B could be associated with the colliding proton and antiproton or quarks.

1. The amplitude for lepto-pion production is essentially Fourier transform of $E \cdot B$, where E and B are the non-orthogonal electric and magnetic fields of the colliding charges. At the level of scales one has $\tau \sim \hbar/E$, where τ is the time during which $E \cdot B$ is large enough during collision and E is the energy scale of the virtual lepto-pion giving rise to the jet.

2. In order to have jets one must have $m(\pi_\tau) << E$. If the scaling law $E \propto \sqrt{s}$ hold true, one indeed has $\sqrt{s}/m(\pi_\tau) > x < 10^3$.

3. If proton and antiproton would move freely, τ would be of the order of the time for proton to move through a distance, which is 2 times the Lorentz contracted radius of proton: $\tau_{free} = 2 \times \sqrt{1-v^2}R_p/v = 2\hbar/E_p$. This would give for the energy scale of virtual τ-pion the estimate $E = \hbar/\tau_{free} = \sqrt{s}/4$. $x = 4$ is certainly quite too small value. Actually $\tau > \tau_{free}$ holds true but one can argue that without new physics the time for the preservation of $E \cdot B$ cannot be by a factor of order 2^8 longer than for free collision.

4. For a colliding quark pair one would have $\tau_{free} = 4\hbar/\sqrt{s_{pair}(s)}$, where $\sqrt{s_{pair}(s)}$ would be the typical invariant energy of the pair which is exponentially smaller than $\sqrt{s}$. Somewhat paradoxically from classical physics point of view, the time scale would be much longer for the collision of quarks than that for proton and antiproton.

The possible new physics relates to the possibility that lepto-pions are dark matter in the sense that they have Planck constant larger than the standard value.

1. Suppose that the produced lepto-pions have Planck constant larger than its standard value $\hbar_0$. Originally the idea was that larger value of $\hbar$ would scale up the production cross section. It turned out that this is not the case. For $exp(iS)$ option the lowest order contribution is not affected by the scaling of $\hbar$ and for $exp(iS)-1$ option the lowest order contribution scales down as $1/hbar^2$. The improved formulation of the model however led to a correct order of magnitude estimates for the production cross section.

2. Assume that a phase transition increasing Planck constant occurs during the collision. Hence τ is scaled up by a factor $y = \hbar/\hbar_0$. The inverse of the lepto-pion mass scale is a natural candidate for the scaled up dark time scale. $\tau(\hbar_0) \sim \tau_{free}$, one obtains $y \sim \sqrt{s_{min}}/4m(\pi_\tau) \leq 2^8$ giving for proton-antiproton option the first guess $\sqrt{s}/m(\pi_\tau) > x < 2^{10}$. If the value of y does not depend on the type of lepto-pion, the proposed estimates for muo- and electro-pion follow.

3. If the fields E and B are associated with colliding quarks, only colliding quark pairs with $\sqrt{s_{pair}(s)} > (>)m(\pi_\tau)$ contribute giving $y_q(s) = \sqrt{s_{pair}(s)/s} \times y$.

If the τ-pions produced in the magnetic field are on-mass shell τ-pions with $k = 113$, the value of $\hbar$ would satisfy $\hbar/\hbar_0 < 2^5$ and $\sqrt{s}/m(\pi_\tau) > x < 2^7$.

Tau-pions again but now as dark matter candidate in galactic center

The standard view about dark matter is that it has only gravitational interactions with ordinary matter so that high densities of dark matter are required to detect its signatures. On the average the density of dark matter is about 80 per cent of ordinary matter. Clearly, Milky Way's center is an excellent place for detecting the signatures of dark matter. The annihilation of pairs of dark matter particles to gamma rays is one possible signature and one could study the anomalous features of gamma ray spectrum from the galactic center (a region with radius about 100 light years).

Europe's INTEGRAL satellite launched in 2002 indeed found bright gamma ray radiations coming from the center of galaxy with energy of .511 MeV, which is slightly above electron mass (see the references below). The official interpretation is that the gammas are produced in the annihilations of particles of positrons and electrons in turn created in dark matter annihilations. TGD suggests much simpler mechanism. Gamma rays would be produced in the decay of what I call electro-pions having mass which is slightly larger than $m = 2m_e$.

The news of the day [C35] was that the data from Fermi Gamma Ray telescope give analyzed by Dan Hooper and Lisa Goodenough [C86] gives evidence for a dark matter candidate with mass between 7.3-9.2 GeV decaying predominantly into a pair of τ leptons. The estimate for the mass region is roughly 4 times τ mass. What puts bells ringing that a mass of a charged lepton appears again!

1. Explanation in TGD framework

The new finding fits nicely to a bigger story based on TGD.

1. TGD predicts that both quarks and leptons should have colored excitations devoted to the lepto-hadron model). In the case of leptons lowest excitations are color octets. In the case of electro-pion this hypothesis finds support from the anomalous production of electron positron pairs in heavy ion collisions discovered already at seventies but forgotten for long ago since the existence of light particle at this mass scale simply was in total complete with standard model and what was known about the decay widths of intermediate gauge bosons. Also ortopositronium decay width anomaly -forgotten also-has explanation in terms of lepto-pion hypothesis [C44, C103] .

2. The colored leptons would be dark in TGD sense, which means that they live in dark sector of the "world of classical worlds" (WCW) meaning that they have no direct interactions (common vertices of Feynman diagrams) with ordinary matter. They simply live at different

space-time sheets. A phase transition which is geometrically a leakage between dark sector and ordinary sector are possible and make possible interactions between ordinary and dark matter based on exchanged particles suffering this phase transition. Therefore the decay widths of intermediate gauge bosons do not kill the model. TGD based model of dark matter in terms of hierarchy of values of Planck constants coming as multiples of its smallest possible value (the simplest option) need not to be postulated separately and can be regarded as a prediction of quantum TGD reflecting directly the vacuum degeneracy and extreme non-linearity of Kähler action (Maxwell action for induced CP_2 Kähler form).

3. CDF anomaly which created a lot of discussion in blogs for two years ago can be understood in terms of taupion. Taupion and its p-adically scaled up versions with masses about $2^k m_\tau$, $k = 1, 2, 3$ and $m_\tau \simeq 1.8$ GeV explains the findings reported by CDF in TGD framework. The masses of taupions would be 3.6 GeV, 7.2 GeV, and 14.2 GeV in good approximation and come as octaves of the mass of tau-lepton pair.

2. Predictions

The mass estimate for the dark matter particle suggests by Fermi Gamma Ray telescope corresponds to $k = 2$ octave for taupion and the predict mass is about 7.2 GeV which at the lower boundary of the range 7.3-9.2 GeV. Also dark matter particles decaying to tau pairs and having masses 3.6 GeV and 14.2 GeV should be found.

Also muo-pion should exist there and should have mass slightly above $2m_\mu = 210.4$ MeV so that a gamma rays peak slightly above the energy $m_\mu = 105.2$ MeV should be discovered. Also octaves of this mass can be imagined. There is also evidence also for the existence of muo-pion [C126, C127].

LHC should provide excellent opportunities to test tau-pion and muo-pion hypothesis. Electro-pion was discovered in heavy ion collisions and also at LHC they study have heavy ion collisions but at much higher energies generating the required very strong non-orthogonal electric and magnetic fields for which the "instanton density" defined as the inner product of electric and magnetic fields is large and rapidly varying. As an optimist I hope that muo-pion and tau-pion could be discovered despite the fact that their decay signatures are very different from those for ordinary particles and despite that fact that at these energies one must know precisely what one is trying to find in order to disentangle it from the enormous background.

3. Also DAMA, CoGeNT, and PAMELA give indications for tau-pion

Note that also DAMA experiment [C30] suggests the existence of dark matter particle in this mass range but it is not clear whether it can have anything to do with tau-pion state. One could of course imagine that dark tau-pions are created in the collisions of highly energetic cosmic rays with the nuclei of atmosphere. Also Coherent Germanium Neutrino Technology (CoGeNT) experiment [C28] has released data that are best explained in terms of a dark matter particle with mass in the range 7-11 GeV.

The decay of tau-pions produce lepton pairs, mostly tau but also muons and electrons. The subsequent decays of tau-leptons to muons and electrons produce also electrons and positrons. This relates interestingly to the positron excess reported by PAMELA collaboration [C32] at the same time as CDF anomaly was reported. The anomaly started at positron energy about 3.6 GeV, which is one just one half of 7. 2 GeV for tau-pion mass! What was remarkable that no antiproton excess predicted by standard dark matter candidates was observed. Therefore the interpretation as decay products of tau-pions seems to make sense!

Could it have been otherwise?

To sum up, the probability that a correct prediction for the lifetime of the new particle using only known lepton masses and standard formulas for weak decay rates follows by accident is extremely low. Throwing billion times coin and getting the same result every time might be something comparable to this. Therefore my sincere hope is that colleagues would be finally mature to take TGD seriously. If TGD based explanation of the anomalous production of electron positron pairs in heavy ion collisions would have been taken seriously for fifteen years ago, particle physics might look quite different now.

6.4.4 Dark matter puzzle

Sean Carroll has explained in Cosmic Variance (http://blogs.discovermagazine.com/cosmicvariance/) the latest rather puzzling situation in dark matter searches. Some experiments support the existence of dark matter particles with mass of about 7 GeV, some experiments exclude them. The following arguments show that TGD based explanation might allow to understand the discrepancy.

How to detect dark matter and what's the problem?

Consider first the general idea behind the attempts to detect dark matter particles and how one ends up with the puzzling situation.

1. Galactic nucleus serves as a source of dark matter particles and these one should be able to detect. There is an intense cosmic ray flux of ordinary particles from galactic center which must be eliminated so that only dark matter particles interacting very weakly with matter remain in the flux. The elimination is achieved by going sufficiently deep underground so that ordinary cosmic rays are shielded but extremely weakly interacting dark matter particles remain in the flux. After this one can in the ideal situation record only the events in which dark matter particles scatter from nuclei provided one eliminates events such as neutrino scattering.

2. DAMA experiment does not detect dark matter events as such but annual variations in the rate of events which can include besides dark matter events and other kind of events. DAMA finds an annual variation interpreted as dark matter signal since other sources of events are not expected to have this kind of variation [C29]. Also CoGENT has reported the annual variation with 2.8 sigma confidence level [C81]. The mass of the dark matter particle should be around 7 GeV rather than hundreds of GeVs as required by many models. An unidentified noise with annual variation having nothing to do with dark matter could of course be present and this is the weakness of this approach.

3. For a few weeks ago we learned that XENON100 experiment detects no dark matter [C33] (http://blogs.discovermagazine.com/cosmicvariance/2011/04/14/no-dark-matter-seen-by-xenon/). Also CDMS has reported a negative result [C27]. According to Sean Carroll, the detection strategy used by XENON100 is different from that of DAMA: individual dark matter scatterings on nuclei are detected. This is a very significant difference which might explain the discrepancy since the theory laden prejudices about what dark matter particle scattering can look like, could eliminate the particles causing the annual variations. For instance, these prejudices are quite different for the habitants of the main stream Universe and TGD Universe.

TGD based explanation of the DAMA events and related anomalies

I have commented earlier the possible interpretation of DAMA events in terms of tau-pions (http://matpitka.blogspot.com/2010/10/tau-pions-again-but-now-in-galactic.html). The spirit is highly speculative.

1. Tau-pions would be identifiable as the particles claimed by Fermi Gamma Ray telescope with mass around 7 GeV and decaying into tau pairs so that one could cope with several independent observations instead of only single one.

2. Recall that the CDF anomaly gave for two and half years ago support for tau-pions whereas earlier anomalies dating back to seventies give support for electro-pions and mu-pions. The existence of these particles is purely TGD based phenomenon and due to the different view about the origin of color quantum numbers. In TGD colored states would be partial waves in CP_2 and spin like quantum numbers in standard theories so that leptons would not have colored excitations.

3. Tau-pions are of course highly unstable and would not come from the galactic center. Instead, they would be created in cosmic ray events at the surface of Earth and if they can penetrate the shielding eliminating ordinary cosmic rays they could produce events responsible for the annual variation caused by that for the cosmic ray flux from galactic center.

Can one regard tau-pion as dark matter in some sense? Or must one do so? The answer is affirmative to both questions on both theoretical and experimental grounds.

1. The existence of colored variants of leptons is excluded in standard physics by intermediate gauge boson decay widths. They could however appear as states with non-standard value of Planck constant and therefore not appearing in same vertices with ordinary gauge bosons so that they would not contribute to the decay widths of weak bosons. In this minimal sense they would be dark and this is what is required in order to understand what we know about dark matter.

   Of course, all particles can in principle appear in states with non-standard value of Planck constant so that tau-pion would be one special instance of dark matter. For instance, in living matter the role of dark variants of electrons and possibly also other stable particles would be decisive. To put it bluntly: in mainstream approach dark matter is identified as some exotic particle with ad hoc properties whereas in TGD framework dark matter is outcome of a generalization of quantum theory itself.

2. DAMA experiment requires that the tau-pions behave like dark matter: otherwise they would never reach the strongly shielded detector. The interaction with the nuclei of detector would be preceded by a transformation to a particle-tau-pion or something else- with ordinary value of Planck constant.

TGD based explanation for the dark matter puzzle

The criteria used in experiments to eliminate events which definitely are not dark matter events - according to the prevailing wisdom of course - dictates to high degree what interactions of tau pions with solid matter detector are used as a signature of dark matter event. It could well be that the criteria used in XENON100 do not allow the scatterings of tau-pions with nuclei. This is indeed the case. The clue comes from the comments of Jester in Resonaances. From a comment of Jester one learns that CoGENT - and also DAMA utilizing the same detections strategy - "does not cut on ionization fraction". Therefore, if dark matter mimics electron recoils (as Jester says) or if dark matter produced in the collisions of cosmic rays with the nuclei of the atmosphere decays to charged particles one can understand the discrepancy.

The TGD based model [K88] explaining the more than two years old CDF anomaly [C26, C63] indeed explains also the discrepancy between XENON100 and CDMS on one hand and DAMA and CoGENT on the other hand. The TGD based model for the CDF anomaly can be found in [K88].

1. To explain the observations of CDF [C26, C63] one had to assume that tau-pions and therefore also color excited tau-leptons inside them appear as several p-adically scaled up variants so that one would have several octaves of the ground state of tau-pion with masses in good approximation equal to 3.6 GeV (two times the tau-lepton mass), 7.2 GeV, 14.4 GeV. The 14.4 GeV tau-pion was assumed to decay in a cascade like manner via lepto-strong interactions to lighter tau-pions- both charged and neutral- which eventually decayed to ordinary charged leptons and neutrinos.

2. Also other decay modes -say the decay of neutral tau-pions to gamma pair and to a pair of ordinary leptons- are possible but the corresponding rates are much slower than the decay rates for cascade like decay via multi-tau-pion states proceeding via lepto-strong interactions.

3. Just this cascade would take place also now after the collision of the incoming cosmic ray with the nucleus of atmosphere. The mechanism producing the neutral tau-pions -perhaps a coherent state of them- would degenerate in the collision of charged cosmic ray with nucleus generating strong non-orthogonal electric and magnetic fields and the production amplitude would be essentially the Fourier transform of the "instanton density" $E \cdot B$. The decays of 14 GeV neutral tau-pions would produce 7 GeV charged tau-pions, which would scatter from the protons of nuclei and generate the events excluded by XENON100 but not by DAMA and Cogent.

4. In principle the model predicts to a high degree quantitatively the rate of the events. The scattering rates are proportional to an unknown parameter characterizing the transformation

probability of tau-pion to a particle with ordinary value of Planck constant and this allows to perform some parameter tuning. This parameter would correspond to a mass insertion in the tau-pion line changing the value of Planck constant and have dimensions of mass squared.

The overall conclusion is that the discrepany between DAMA and XENON100 might be interpreted as favoring TGD view about dark matter and it is fascinating to see how the situation develops. This confusion is not the only confusion in recent day particle physics. All believed-to-be almost-certainties are challenged.

Has Fermi observed dark matter?

Resonaances (http://resonaances.blogspot.com/2012/04/dark-matter-signal-in-fermi.html) reports about a possible dark matter signal at Fermi satellite [C119]. Also Lubos (http://motls.blogspot.com/2012/04/fermi-fifty-dark-matter-photons-at-130.html) has a posting about the finding and mentions that the statistical significance is 3.3 sigma.

The proposed dark matter interpretation for the signal would be pair of monochromatic photons with second one detected at Earth. The interpretation would be that dark matter particles with mass m nearly at rest in galactic center annihilate to a pair of photons so that one obtains a pair of photons with energy equal to the cm energy which is in a good approximation the sum $E = 2 \times m$ for the masses of the particles. The mass value would be around m=130 GeV if the final state involves only 2 photons.

In TGD framework I would consider as a first guess a pion like state decaying to two photons with standard coupling given by the coupling to the "instanton density" $E \cdot B$ of electromagnetic field. The mass of this particle would be 260 GeV, in reasonable approximation 2 times the mass m=125 GeV of the Higgs candidate.

1. Similar coupling was assumed to [K88]. The anomaly would have been produced by tau-pions, which are pionlike states formed by pairs of colored excitations of tau and its antiparticle (or possibly their super-partners). What was remarkable that the mass had three values coming as powers of two: $M = 2^k \times 2m(\tau;)$, $k = 0, 1, 2$. The interpretation in terms of p-adic length scale hypothesis would be obvious: also the octaves of the basic state are there. The constraint from intermediate gauge boson decay widths requires that these states are dark in TGD sense and therefore correspond to a non-standard value of Planck constant coming as an integer multiple of the standard value.

2. Also the explanation of the findings of Pamela discussed in this chapter require octaves of tau-pion produced in Earth's atmosphere.

3. Even ordinary pion should have 2-adic octaves. But doesn't this kill the hypothesis? We "know" that pion does not have any octaves! Maybe not, there is recent evidence for satellites of ordinary pion with energy scale of 40 MeV interpreted in terms of IR Regge trajectories assignable to the color magnetic flux tubes assignable to pion. There has been several wrong alarms about Higgs: at 115 GeV and 155 GeV at least. Could it be that there there is something real behind these wrong alarms: the scale for IR Regge trajectories would be about 20 GeV now!

So: could the dark matter candidates with mass around 260 GeV correspond to the first octave of M_{89} pion with mass around 125 GeV, the particle that colleagues want to call Higgs boson although its decay signatures suggest something different?

1. In this case it does not seem necessary to assume that the Planck constant has non-standard value although this is possible.

2. This particle should be produced in M_{89} strong interactions in the galactic center. This would require the presence of matter consisting of M_{89} nucleons emitting these pions in strong interactions. Galactic center (http://en.wikipedia.org/wiki/Galactic_center) is very exotic place and believed to contain even super-massive black hole. Could this environment accommodate also a scaled up copy of hadron physics? Presumably this would require very high temperatures with thermal energy of order .5 TeV correspond to the mass of M_{89}

proton to make possible the presence of M_{89} matter. Or could M_{89} pion be produced in ultrastrong non-orthogonal electric and magnetic fields in the galactic center by the coupling to the instanton density. The needed field strengths would be extremely high. I have indeed proposed long time ago an explanation of very high energy cosmic rays in terms of the decay products of scaled up hadron physics (see "Cosmic Rays and Mersenne primes" in this chapter).

One can of course imagine that the photon pair is produced in the annilation of M_{89} pions with opposite charges via standard electromagnetic coupling. Also the annihilation of M_{89} spions consisting of squark pair can be considered in TGD framework where squarks could have same mass scale as quarks. In this case mass would be near 125 GeV identified as mass of neutral M_{89} pion. By scaling up the mass difference 139.570-134.976 MeV of the ordinary charged and neutral pion by the ratio of the pion M_{89} and M_{107} pion masses equal to $(125/140) \times 10^3$ one obtains that the charged M_{89} pion should have mass equal to 129.6 MeV to be compared with the 130 GeV mass suggested by experimental evidence.

The story did not end here as so often when observations cannot be replicated. The Estonian researchers Elmo Tempel, Andi Hektora and Martti Raidala have found a confirmation for the 130 GeV Fermi excess in gamma radiation from galactic center discovered by Cristoph Weniger [E8]. An important conclusion of these researchers is that best fit is obtained if the dark matter candidates decay by two-body annihilation to photons and have mass 145 GeV. The reason for why the gamma peak is at 130 GeV rather than 145 GeV would be due to the emission light particle pairs by the photons. There are also indications for a peak at 111 GeV: this could be assigned to γZ finals state of two-body decay.

In TGD framework the annihilating particles with mass about 145 GeV mass could be charged pion-like states of M_{89} hadron physics. They could be dark in the sense of having large value of Planck constant but it is not clear whether this is necessarily so. The TGD based on view about galactic dark matter locates in cosmic string like objects containing galaxies as pearls in necklace and no halo is needed to explain galactic rotation spectrum [K26]. An ultrahigh temperature would be needed to excite M_{89} hadron physics and if there is giant blackhole in galactic nucleus, there are hopes about this. M_{89} hadron physics could also produce ultrahigh energy cosmic rays as described in this chapter.

It is amusing that also CDF found for a couple of years ago evidence for a bump at the same 145 GeV energy (this has been forgotten long time ago by bloggers in 125 GeV Higgs hysteria). Estonians propose that also a particle with 290 GeV (mass would twice that of 145 GeV state) is needed. This brings further support for the idea about mass octaves of ground state of pionlike states needed to explain various anomalies (see this chapter and [K88]).

If one takes seriously the evidence for 125 GeV state and its identification as Eucdlian pion together with the evidence for galactic pionlike state with mass of 145 GeV identified as M_{89}, one has a nice support for the overall TGD based view about situation described in this chapter. The small splitting between pionlike states has possible counterpart in the ordinary hadron physics: there is evidence for satellites of pion, mesons, and baryons in 20-40 MeV scale for mass splittings and in TGD framework they would correspond to IR Regge trajectories with the scale of 10-20 GeV mass splittings (see this chapter).

We are living exciting times!

6.4.5 Has Pamela observed evidence for the non-dark electro-pion of M_{89} lepto-hadron physics?

Resonaances tells that the Fermi collaboration confirms the claim of PAMELA collaboration about anomalous e^+e^- pairs in cosmic ray radiation (see that abstract *Consistency of fermi-lat and pamela cosmic ray lepton measurement* by P. Grandi et al at
http://fermi.gsfc.nasa.gov/science/ symposium/2011/Fermi_Symposium_2011_Abstracts.pdf).

The announcement of Pamela was my second birthday gift at October 30 for two and half years ago. The first gift was CDF anomaly which found a beautiful explanation in terms of tau-pions and the p-adically scaled up variants with color tau- lepton having mass scale by power of two. The tau-pion of mass about 14 GeV decaying in cascade like manner to lower octaves of basic tau-pion explained elegantly the observations reported by CDF.

For some time ago the dilemma posed by the contradictory claims of DAMA and Cogent collaborations on one hand and XENON100 collaboration on one hand finds also nice solution in terms of 14 GeV taupion decaying to charged taupions with mass about 7 GeV [K88].

The decays of electro-pions to gamma pair can explain the observed anomalous gammas from galactic nucleus with energy very nearly to electron rest mass. Could one understand also the anomalous positrons reported by PAMELA as decay products of lepto-pion like states, say taupions? Intriguingly, the first figures of the article by Alessandro Strumia [C113] discussing the constraints on the possible explanations of the PAMELA anomaly show that the anomalous positron excess starts around 10 GeV, possible it starts already at 7 GeV. It is not possible to say anything certain below 10 GeV since the measurements are affected by the solar actvity below 10 GeV. What is however clear is that the excess cannot be explained by taupion decays with 14 GeV mass since the excess would be localized around energy of about 7 GeV. Higher mass is required.

The article by Alessandro Strumia summarizes various theoretical constraints on the new particle explaining positron and electron excesses. The conclusions are following.

1. DM should result in a decay of quite a narrow particle with a mass very near to 2M, which is nearly at rest. What narrow means quantitatively is not clear to me.

2. DM should carry a charge mediating long range interaction with the mediating boson which is must lighter than the particle itself: photon is the obvious candidate. Electromagnetically charged dark matter is however in conflict with the standard prejudices about dark matter and actually in dramatic conflict with its basic property of being invisible. Hierarchy of Planck constants is the only solution to the paradox of charged invisible dark matter.

3. DM must prefer the decays to leptons since otherwise there would be also antiproton and proton excess which has not been observed.

4. The mass of DM should be above 100 GeV.

These conditions encourage the identification of DM as a decay product of lepto-pion like state but with mass considerably higher than the 14 GeV mass. Tau-pions could of course be present but would not contribute to the anomaly at energies not too much above 7 GeV. Tau-pions would also give muon pair anomaly. Heavier lepto-pion like states are required and electro-pion would be the most natural candidate.

1. If a scaled up variant of ordinary hadron physics characterized by M_{89} is there as the recent bumps having interpretation as mesons of this physics suggest, there is no deep reason preventing the presence of also the scale variant of lepto-hadron physics in this scale. Even more, one can argue that colored leptons must appear as both dark and ordinary variants. Dark variants with non-standard value of Planck constant can have masses of ordinary leptons plus possibly their octaves as in the case of tau at least. The decay widths of intermediate gauge bosons require ordinary colored leptons to have mass higher than 45 GeV.

2. The mass of scaled up electro-pion would be obtained by scaling the mass of the dark electro-pion which for M_{89} electro-pion physics is in a good approximation $2m_e$=1 MeV by a factor $2^{(127-89)/2} = 2^{19}$. This gives electro-pion mass equal to 500 GeV. Ordinary colored electron would therefore have mass of 250 GeV consistent with the lower bound. The conclusion would be rather ironic: we would have seen dark colored electron (in TGD sense) already at seventies and covered it carefully under the rug and would be seeing now the ordinary colored electron and stubbornly trying to identify it as DM without caring about the fact that if dark matter is invisible in the standard sense it cannot be electromagnetically charged!

3. By stretching one's imagination one might play with the thought that superpartners of colored leptons with mass scale of order 100 GeV could form pion like states. The superpartners decay to partner and neutrino since R-parity is not exact invariance in TGD and all depends on how fast this process occurs.

4. Skeptic could wonder why the counterparts for colored excitations of quarks are not there and induce the increase of proton and antiproton fluxes.

To summarize, entire Zoo of not only new particles but even of new physics could be waiting for us at LHC energies if we live in TGD Universe!

6.4.6 Could lepto-hadrons be replaced with bound states of exotic quarks?

Can one then exclude the possibility that electron-hadrons correspond to colored quarks condensed around $k = 127$ hadronic space-time sheet: that is M_{127} hadron physics? There are several objections against this identification.

1. The recent empirical evidence for the colored counterpart of μ and τ supports the view that colored excitations of leptons are in question.

2. The octet character of color representation makes possible the mixing of leptons with leptobaryons of form $L\nu_L\overline{\nu}_L$ by color magnetic coupling between leptogluons and ordinary and colored lepton. This is essential for understanding the production of electron-positron pairs.

3. In the case CDF anomaly also the assumption that colored variant of τ neutrino is very light is essential. In the case of colored quarks this assumption is not natural.

6.4.7 About the masses of lepto-hadrons

The progress made in understanding of dark matter hierarchy [K32] and non-perturbative aspects of hadron physics [K58, K52] allow to sharpen also the model of lepto-hadrons.

The model for the masses of ordinary hadrons [K58] applies also to the scaled up variants of the hadron physics. The two contributions to the hadron mass correspond to quark contribution and a contribution from super-symplectic bosons. For quarks labeled identical p-adic primes mass squared is additive and for quarks labeled by different primes mass is additive. Quark contribution is calculable once the p-adic primes of quarks are fixed.

super-symplectic contribution comes from super-symplectic bosons at hadronic space-time sheet labeled by Mersenne prime and is universal if one assumes that the topological mixing of the super-symplectic bosons is universal. If this mixing is same as for U type quarks, hadron masses can be reproduced in an excellent approximation if the super-symplectic boson content of hadron is assumed to correlate with the net spin of quarks.

In the case of baryons and pion and kaon one must assume the presence of a negative color conformal weight characterizing color binding. The value of this conformal weight is same for all baryons and super-symplectic contribution dominates over quark contribution for nucleons. In the case of mesons binding conformal weight can be assumed to vanish for mesons heavier than kaon and one can regard pion and kaon as Golstone bosons in the sense that quark contribution gives the mass of the meson.

This picture generalizes to the case of lepto-hadrons.

1. By the additivity of the mass squared leptonic contribution to lepto-pion mass would be $\sqrt{2}m_e(k)$, where k characterizes the p-adic length scale of colored electron. For $k = 127$ the mass of lepto-pion would would be .702 MeV and too small. For $k = 126$ the mass would be $2m_e = 1.02$ MeV and is very near to the mass of the lepto-pion. Note that for ordinary hadrons quarks can appear in several scaled up variants inside hadrons and the value of k depends on hadron. The prediction for the mass of leptoρ would be $m_{\pi_L} + \sqrt{7}m_{127} \simeq 1.62$ MeV ($m_{127} = m_e/\sqrt{5}$).

2. The state consisting of three colored electrons would correspond to leptonic variant of Δ_{++} having charge $q = -3$. The quark contribution to the mass of $\Delta_L \equiv \Delta_{L,3-}$ would be by the additivity of mass squared $\sqrt{3} \times m_e(k = 126) = 1.25$ MeV. If super-symplectic particle content is same as for Δ_L, super-symplectic contribution would be $m_{SC} = 5 \times m_{127}$, and equal to $m_{SC} = .765$ MeV so that the mass of Δ_L would be $m_{\Delta_L} = 2.34$ MeV. If colored neutrino corresponds to the same p-adic prime as colored electron, also leptoproton has mass in MeV scale.

6.4.8 Do X and Y mesons provide evidence for color excited quarks or squarks?

Now and then come the days when head is completely empty of ideas. One just walks around and gets more and more frustrated. One can of course make authoritative appearances in blog groups and express strong opinions but sooner or later one is forced to look for web if one could find some problem. At this time I had good luck. By some kind of divine guidance I found myself immediately in Quantum Diaries and found a blog posting with title *Who ordered that?! An X-traordinary particle?* [C15].

Not too many unified theorists take meson spectroscopy seriously. Although they are now accepting low energy phenomenology (*the* physics for the rest of us) as something to be taken seriously, meson physics is for them a totally uninteresting branch of botany. They could not care less. As a crackpot I am however not well-informed about what good theoretician should do and shouldn't do and got interested. Could this give me a problem that my poor crackpot brain is crying for?

The posting told me that in the spectroscopy of $c\overline{c}$ type mesons is understood except for some troublesome mesons christened imaginatively with letters X and Y plus brackets containing their mass in MeVs. X(3872) is the firstly discovered troublemaker and what is known about it can be found in the blog posting and also in Particle Data Tables [C10]. The problem is that these mesons should not be there. Their decay widths seem to be narrow taking into account their mass and their decay characteristics are strange: in particular the kinematically allow decays to $D\overline{D}$ dominating the decays of $\Psi(3770)$ with branching ratio 93 per cent has not been observed whereas the decay to $D\overline{D}\pi^0$ occurs with a branching fraction $> 3.2 \times 10^{-3}$. Why the pion is needed? $X(3872)$ should decay to photon and charmonium state in a predictable way but it does not.

Could these be the good questions?

TGD predicts a lot of exotic physics and I of course started to exclude various alternatives. First one must however try to invent a good question. Maybe the following questions might satisfy the criterion of goodness.

1. Why these exotic states appear only for mesons made of heavy quark and antiquark? Why not for light mesons? Why not for mesons containing one heavy quark and light quark? Could it be that also $b\overline{b}$ mesons could have exotic partners not yet detected? Could it be that also exotic $b\overline{c}$ type mesons could be there? Why the presence of light quark would eliminate the exotic partner from the spectrum?

2. Do the decays obey some selection rules? There is indeed this kind of rule: the numbers of c and $\overline{c}$ quarks in the final state are equal to one.

   (a) If c and $\overline{c}$ exist in the initial state and the decay involves only strong interactions, the rule holds true.

   (b) If c and $\overline{c}$ are not present in the initial state the only option that one can imagine is the exhange of two W bosons transforming d type quarks to c type quarks must be present. If this were the case the initial state should correspond to $d\overline{d}$ like state rather than $c\overline{c}$ and this looks very strange from the standard physics point of view. Also the rate for this kind of decays would be very small and it seems that this option cannot make sense.

Both leptons and quarks have color excitations in TGD Universe

TGD predicts that both leptons and quarks have color excitations [K88]. For leptons they correspond to color octets and there is a lot of experimental evidence for them. Why we do not have any evidence for color excited quarks? Or do we actually have?! Could these strange X:s and Y:s provide this evidence?

Ordinary quarks correspond to triality one color triplet partial waves in CP_2. The higher color partial waves would also correspond to triality one states but in higher color partial waves in CP_2. The representations of the color group are labelled by two integers (p,q) and the dimension of the representation is given by

$$d = \frac{(p+1)(q+1)(p+q+2)}{2} \quad .$$

A given $t = \pm 1$ representation is accompanied by its conjugate with the same dimension and opposite triality $t = \mp 1$. $t = 1$ representations satisfy $p - q = 1$ modulo 3 and come as (1,0), (0,2), (3,0), (2,1), with dimensions 3, 6, 10, 15,... The simplest candidate for the color excitations would correspond to the representation $\overline{6}$. It does not correspond directly the a solution of the Dirac equation in CP_2 since physical states involve also color Kac-Moody generators [K48].

Some remarks are in order:

1. The tensor product of gluon octet with $t = 1$ with color triplet representation contains $8 \times 3 = 24$ states and decomposes into $t = 1$ representations as $3 \oplus \overline{6} \oplus 15$. The coupling of gluons by Lie algebra action can couple given representation only with itself. The coupling between triplet and $\overline{6}$ and 15 is therefore not by Lie algebra action. The coupling constant between quarks and color excited quarks is *assumed* to be proportional to color coupling.

2. The existence of this kind of coupling would explain the selection rules elegantly. If this kind of coupling is not allowed then only the annihilation of exotic quark to gluon decaying to quark pair can transform exotic mesons to ordinary ones and I have not been able to explain selection rules using this option.

The basic constraint applying to all variants based on exotic states of quarks comes from the fact that the decay widths of intermediate gauge bosons do not allow new light particles. This objection is encountered already in the model of lepto-hadrons [K88]. The solution is that the light exotic states are possible only if they are dark in TGD sense having therefore non-standard value of Planck constant and behaving as dark matter. The value of Planck constant is only effective and has purely geometric interpretation in TGD framework. This implies that a phase transition transforming quarks and gluons to their dark counterparts is the key element of the model. After this a phase transition a gluon exchange would transform the quark pair to an exotic quark pair.

Also squarks could explain exotic charmonium states

Supersymmetry provides an alternative mechanism. Right-handed neutrino generates super-symmetries in TGD Universe and quarks are accompanied by squarks consisting in a well-defined sense of of quark and right-handed neutrino. Super-symmetry would allow completely standard couplings to gluons by adding to the spectrum squarks and gluinos.Exactly the same selection rules result if these new states are mesonlike states from from squark and anti-squark and the exchange of gluino after the $\hbar$ changing phase transition transforms exotic meson to ordinary one and vice versa.

In the sequel it will be shown that the existence of color excited quarks or of their superpartners could indeed allow to understand the origin of X and Y mesons and also the absence of analogous states accompanying mesons containing light quarks or antiquarks.

This picture would lead to a completely new view about detection of squarks and gluinos.

1. In the standard scenario the basic processes are production of squark and gluino pair. The creation of squark-antisquark pair is followed by the decay of squark (anti-squark) to quark (antiquark) and neutralino or chargino. If R-parity is conserved, the decay chain eventually gives rise to at least two hadron jets and lightest neutralinos identifiable as missing energy. Gluinos in turn decay to quark and anti-squark (squark and antiquark) and squark (anti-squark) in turn to quark (anti-quark) and neutralino or chargino. At least four hadron jets and missing energy is produced. In TGD framework neutralinos would decay eventually to zinos or photinos and right-handed neutrino transforming to ordinary neutrino (R-parity is not conserved). This process might be however slow.

2. In the recent case quite different scenario relying on color confinement and "shadronization" suggests itself. By definition smesons consist of squarks and antisquark. Sbaryons could consist of two squarks containing right-handed neutrino and its antineutrino ($\mathcal{N} = 2$ SUSY) and one quark and thus have same quantum numbers as baryon. Note that the squarks are dark in TGD sense.

   Also now dark squark or gluino pair would be produced at the first step and would require $\hbar$ changing phase transition of gluon. These would shadronize to form a dark shadron. One can indeed argue that the required emisson of winos and zinos and photinos is too slow a process as compared to shadronization. Shadrons (mostly smesons) would in turn decay to hadrons by the exchange of gluinos between squarks. No neutralinos (missing energy) would be produced. This would explain the failure to detect squarks and gluinos at LHC.

This mechanism does not however apply to sleptons so that it seems that the p-adic mass scale of sleptons must be much higher for sleptons than that for squarks as I have indeed proposed.

Could exotic charmonium states consist of color excited c and $\overline{c}$ or of their spartners?

Could one provide answers to the questions presented in the beginning assuming that exotic charmonium states consists of dark color excited c and $\overline{c}$: or more generally, a mixture of ordinary charmonium and exotic charmonium state? The mixing is expected since $\hbar$ changing phase transition followed by a gluon exchange can transform these meson states to each other. Also annihilation to gluon and back to quark pair can induce this mixing. The mixing is however small for heavy quarks for which $\alpha_s \simeq .1$ holds true. Exactly the same arguments apply to the meson like bound states of squarks and in the following only the first option will be discussed.

1. In the case of charged leptons colored excitations have have same p-adic mass scale: for τ however several p-adic mass scales appear as the model if the two year old CDF anomaly is taken seriously [K88]. Assume that p-adic mass scales - but not necessarily masses- are the same also now. This assumption might be non-sensical since also light mesons would have exotic counterparts and somehow they should disappear from the spectrum. To simplify the estimates one could even assume even that the masses are same.

2. In the presence of small mixing the decay amplitude would come solely from the small contribution of the ordinary $c\overline{c}$ state present in the state dominated by color excited pair. The two manners to see the situation should give essentially the same answer.

3. The decays would take place via strong interactions.

The challenge is to understand why the dominating decays to $D\overline{D}$ with branching fraction of 93 per cent are not allowed whereas $D\overline{D}\pi^0$ takes place. Why the pion is needed? The second challenge is to understand why X does not decay to charmonium and photon.

1. For ordinary charmonium the decay to $D\overline{D}$ could take place by the emission of gluon from either c or $\overline{c}$ which then decays to light quark pair whose members combine with c and $\overline{c}$ to form D and $\overline{D}$. Now this mechanism does not work. At least *two* gluons must be emitted to transform colored excited $c\overline{c}$ to ordinary $c\overline{c}$. If these gluons decay to light quark pairs one indeed obtains an additional pion in hadronization. The emission of two gluons instead of only one is expected to reduce the rate roughly by $\alpha_s^2 \simeq 10^{-2}$ factor.

2. Also ordinary decays are predicted to occur but with a slower rate. The first step would be an exchange of gluon transforming color excited charmed quark pair to an ordinary charmed quark pair. After the transformation to off mass shell $c\overline{c}$ pair, the only difference to the decays of charmonium states would be due to the fact that charmonium would be replaced with $c\overline{c}$ pair. The exchange of the gluon preceding this step could reduce the decay rate with respect to charmonium decay rates by a factor of order $\alpha_s^2 \simeq 10^{-2}$. Therefore also the ordinary decay modes should be there but with a considerably reduced rate.

3. Why the direct decays to photon and charmonium state do not occur in the manner predicted by the model of charmonium? For ordinary charmonium the decay proceeds by an emission of photon by either quark or antiquark. Same mechanism applies for exotic charmonium states but leads to final state which consists of *exotic* charmonium and photon. In the case of $X(3872)$ there exists no lighter exotic charmonium state so that the decay is forbidden in this order of perturbation theory. Heavier exotic charmonium states can however decay to photon plus exotic charmonium state in this order of perturbation theory if discrete symmetries favor this.

Essentially identical arguments go through if c and $\overline{c}$ are replaced with their dark spartners and exchange of gluon by the emission of gluino. The transformation of gluon to its dark variants is an essential element in the process.

Why the color excitations/spartners of light quarks would be effectively absent?

Can one understand the effective absence of mesons consisting of color excited light quarks or squarks if the excitations have same mass scale and even mass as the light quarks? The following arguments are for color excited quarks but they apply also to squarks.

1. Suppose that the mixing induced by $\hbar$ changing phase transition followed by a gluon exchange and annihilation is described by mass squared matrix containing besides diagonal components $M_1^2 = M_2^2$ also non-diagonal component $M_{12}^2 = \overline{M_{21}^2}$. The eigenstates of the mass squared matrix correspond to the physical states which are mixtures of states consisting of ordinary quark pair and pair of color excited quarks. The non-diagonal elements of the mass squared matrix corresponds to gluon exchange and since color interactions get very strong at low energy scales, one expects that these elements get very large. In the degenerate case $M_1^2 = M_2^2$ the mass squared eigen values are given by

$$M_\pm^2 = M_0^2 \pm |M_{12}|^2 \ . \tag{6.4.4}$$

2. Suppose that $M_0^2 = 0$ holds true in accordance with approximate pseudo Goldstone nature of pion and more generally all light pseudo-scalar mesons. In fact assume that this is the case before color magnetic spin-spin splitting has taken place so that in this approximation pion and ρ would have same mass $m_\pi^2 = m_\rho^2 = M_0^2$. In TGD based model for color magnetic spin-spin splitting M_0^2 energy is replaced with mass squared [K58] and M_0^2 is obtained in terms of physical masses of π and ρ from the basic formulas

$$\begin{aligned} m_\pi^2 &= M_0^2 - \frac{1}{4}\Delta \ , \ m_\rho^2 = M_0^2 + \frac{3}{4}\Delta \ , \\ M_0^2 &= \frac{m_\rho^2 + 3m_\pi^2}{2} \ , \ \Delta = m_\rho^2 - m_\pi^2 \ . \end{aligned} \tag{6.4.5}$$

The exotic π and ρ would have masses

$$\begin{aligned} m_{\pi_{ex}}^2 &= -M_0^2 - \frac{1}{4}\Delta = m_\pi^2 - 2M_0^2 \ , \\ m_{\rho_{ex}}^2 &= -M_0^2 + \frac{3}{4}\Delta = m_{rho}^2 - 2M_0^2\Delta \ . \end{aligned} \tag{6.4.6}$$

For $m_\pi = 140MeV$ and $m_\rho = 770$ MeV the calculation gives $m_{\pi_{ex}} = i \times 685$ MeV so a tachyon would be in question. For ρ one would have $m_{\pi_{ex}} = 323$ MeV so that the mass would not be tachyonic.

One can try to improve the situation by allowing $M_1^2 \neq M_2^2$ giving additional flexibility and hopes about tachyonicity of the exotic ρ.

1. In this case one obtains the equations

$$\begin{aligned} m_\pi^2 &= M_+^2 - \frac{1}{4}\Delta \ , \ m_\rho^2 = M_+^2 + \frac{3}{4}\Delta \\ m_{\pi_{ex}}^2 &= M_-^2 - \frac{1}{4}\Delta \ , \ m_{\rho_{ex}}^2 = M_-^2 + \frac{3}{4}\Delta \ ., \\ M_+^2 &= \frac{M_1^2 + M_2^2}{2} + \sqrt{(\frac{M_1^2 + M_2^2}{2})^2 + M_{12}^4} = \frac{m_\rho^2 + 3m_\pi^2}{2} \ , \\ M_-^2 &= \frac{M_1^2 + M_2^2}{2} - \sqrt{(\frac{M_1^2 + M_2^2}{2})^2 + M_{12}^4} = M_+^2 - 2\sqrt{(\frac{M_1^2 + M_2^2}{2})^2 + M_{12}^4} \end{aligned} \tag{6.4.7}$$

2. The condition that ρ_{ex} is tachyonic gives

$$m^2_{\rho_{ex}} = M_-^2 + \frac{3}{4}\Delta < 0 \ , \tag{6.4.8}$$

giving

$$\begin{aligned} m_\rho^2 &< 2\sqrt{(\frac{(M_1^2+M_2^2}{2})^2 + M_{12}^4} \ , \\ M_+^2 &= \frac{M_1^2+M_2^2}{2} + \sqrt{(\frac{(M_1^2+M_2^2}{2})^2 + M_{12}^4} = \frac{m_\rho^2 + 3m_\pi^2}{2} \ , \end{aligned} \tag{6.4.9}$$

3. In the parameterization $(m_1^2, m_2^2, M_{12}^2) = (x,y,z)m_\rho^2$ one obtains the conditions

$$\begin{aligned} D \equiv \sqrt{(x+y)^2 + z^2} &> 1/2 \ , \\ \frac{x+y}{2} + D = \frac{1}{2} + \frac{3}{2}\frac{m_\pi^2}{m_\rho^2} & \ . \end{aligned} \tag{6.4.10}$$

4. These equations imply the conditions

$$\begin{aligned} x+y &< 3\frac{m_\pi^2}{m_\rho^2} \simeq .099 \ , \\ .490 &< z < .599 \ . \end{aligned} \tag{6.4.11}$$

The first condition implies $\sqrt{m_1^2+m^2} < 242.7$ MeV. Second condition gives $339 < M_{12}/MeV < 595.9$ so that rather stringent bounds on the parameters are obtained. The simplest solution to the conditions corresponds to $x = y = 0$ and $z = .599$. This solution would mean vanishing masses in the absence of mixing and spin-spin splitting and could be defended by the Golstone boson property of pions mass degenerate with ρ mesons.

This little calculation encourages to consider the possibility that all exotic counterparts of light mesons are tachyonic and that this due the very large mixing induced by gluon exchange (gluino exchange squark option) at low energies. It would be nice if also mesons containing only single heavy quark were tachyonic and this could be the case if the p-adic length scale defining the strength of color interactions corresponds to that of the light quark so that the mass matrix has large enough non-diagonal component. Here one must be however very cautious since experimental situation is far from clear.

The model suggests that ordinary charmonium states and their exotic partners are in 1-1 correspondence. If so then many new exotic states are waiting to be discovered.

The option based on heavy color excitations/spartners of light quarks

An alternative option is that color excitations/spartners of light quarks have large mass: this mass should not be however larger than the mass of c quarks if we want to explain X:s and Y:s as pairs of color excitations of light quarks. Suppose that the p-adic mass scale is same as that for c quarks or near it (not that the scales come as powers of $\sqrt{2}$). This raises the question whether exotic $c\overline{c}$ mesons really consist of exotic c and $\overline{c}$: why not color excitations of u,d,s and their antiquarks? As a matter fact, we cannot be sure about the quark content of X and Y mesons. Could these states be $d\overline{d}$ and $u\overline{u}$ states for their color excitations? It however seems that the presence of two W exchanges makes the decay rate quite too low so that this option seems to be out of question.

One can however consider the option in which the squarks associated with light quarks are heavy. This option is indeed realized in standard SUSY were the mass scales of particles families are inverted so that stop and sbottom are the lightest squarks and super-partners of u and d the heaviest ones. This would would predict that the smesons associated with $\overline{t}$ and $b\overline{b}$ are lighter than X and Y (s)mesons. This option does not look at all natural in TGD but of course deserves experimential checking.

How to test the dark squark option?

The identification of X and Y as dark smesons looks like a viable option and explains the failure to find SUSY at LHC if shadronization is a fast process as compared to the selectro-weak decays. The option certainly deserves an experimental testing. One could learn a lot about SUSY in TGD sense (or maybe in some other sense!) by just carefully scanning the existing data at lower energies. For instance, one could try to answer the following questions by analyzing the already existing experimental data.

1. Are X and Y type mesons indeed in 1-1 correspondence with charmonium states? One could develop numerical models allowing to predict the precise masses of scharmonium states and their decay rates to various final states and test the predictions experimentally.

2. Do $b\overline{b}$ mesons have smesonic counterparts with the same mass scale? What about B_c type smesons containing two heavy squarks?

3. Do the mesons containing one heavy quark and one light quark have smesonic counterparts? My light-hearted guess that this is not the case is based on the assumption that the general mass scale of the mass squared matrix is defined by the p-adic mass scale of the heavy quark and the non-diagonal elements are proportional to the color coupling strength at p-adic length scale associated with the light quark and therefore very large: as a consequence the second mass eigenstate would be tachyonic.

4. What implications the strong mixing of light mesons and smesons would have for CP breaking? CP breaking amplitudes would be superpositions of diagrams representing CP breaking for mesons *resp.* smesons. Could the presence of smesonic contributions perhaps shed light on the poorly understood aspects of CP breaking?

Objection against covariantly constant neutrinos as SUSY generators

TGD SUSY in its simplest form assumes that covariantly constant right-handed neutrino generates SUSY. The second purely TGD based element is that squarks would correspond to the same p-adic mass scale as partners.

This looks nice but there are objections.

1. The first objection relates to the tachyonicity needed to get rid of double degeneracy of light mesons consisting of u, d, and s quarks. Mesons and smesons consisting of squark pair mix and for large α_s the mixing is large and can indeed make second eigenvalue of the mass squared matrix negative. If so, these states disappears from spectrum. At least to me this looks however somewhat unaesthetic.

   Luckily, the transformation of second pion-like state to tachyon and disappearance from spectrum is not the only possibility. After a painful search I found experimental work [C115] claiming the existence of states analogous to ordinary pion with masses 60, 80, 100, 140,.... MeV. Also nucleons have this kind of satellite states. Could it be that one of these states is spion predicted by TGD SUSY for ordinary hadrons? But what about other states? They are not spartners: what are they?

2. The second objection relates to the missing energy. SUSY signatures involving missing energy have not been observed at LHC. This excludes standard SUSY candidates and could do the same in the case of TGD. In TGD framework the missing energy would be eventually right

handed neutrinos resulting from the decays of sfermions to fermion and sneutrino in turn decaying to neutrino and right handed neutrino. The naive argument is that shadronization would be much faster process than the decay of squarks to quarks and spartners of electro-weak gauge bosons and missing energy so that these events would not be observed. Shadrons would in turn decay to hadrons by gluino exchanges. The problem with this argument is that the weak decays of squarks producing right handed neutrinos as missing energy are still there!

This objection forces to consider the possibility that covariantly constant right handed neutrino which generates SUSY is replaced with a color octet. Color excitations of leptons of lepto-hadron hypothesis would be sleptons which are color octets so that SUSY for leptons would have been seen already at seventies in the case of electron. The whole picture would be nicely unified. Sleptons and squark states would contain color octet right handed neutrino the same wormhole throats as their em charge resides. In the case of squarks the tensor product $3 \otimes 8 = 3 + \overline{6} + 15$ would give several colored exotics. Triplet squark would be like ordinary quark with respect to color.

Covariantly constant right-handed neutrino as such would represent pure gauge symmetry, a super-generator annihilating the physical states. Something very similar can occur in the reduction of ordinary SUSY algebra to sub-algebra familiar in string model context. By color confinement missing energy realized as a color octet right handed neutrino could not be produced and one could overcome the basic objections against SUSY by LHC.

What about the claimed anomalous trilepton events at LHC interpreted in terms of SUSY, which however breaks either the conservation of lepton or baryon number. I have proposed TGD based interpretation [K52] is in terms of the decays of W to $\tilde{W}$ and $\tilde{Z}$, which in turn decay and produce the three lepton signature. Suppose that $\tilde{W}$ and $\tilde{Z}$ are color octets and that sleptons replace the color octet excitations of leptons responsible for lepto-hadron physics [K88]. One possible decay chain would involve the decays $\tilde{W}^+ \rightarrow \tilde{L}^+ + \overline{\nu}_L$ and $\tilde{Z} \rightarrow L^+ + \tilde{L}^-$. Color octet sleptons pair combine to form lepto-pion which decays to lepton pair. This decay cascade would produce missing energy as neutrino and this seems to be the case for other options too.e could overcome the basic objections against SUSY by LHC.

This view about TGD SUSY clearly represents a hybrid of the two alternative views about X and Y bosons as composites of either color excitations of quarks or of squarks and is just one possibility. The situation is not completely settled and one must keep mind open.

6.5 APPENDIX

6.5.1 Evaluation of lepto-pion production amplitude

General form of the integral

The amplitude for lepto-pion production with four momentum

$$\begin{aligned} p &= (p_0, \bar{p}) = m\gamma_1(1, vsin(\theta)cos(\phi), vsin(\theta)sin(\phi), vcos(\theta)) \ , \\ \gamma_1 &= 1/(1-v^2)^{1/2} \ , \end{aligned} \tag{6.5.1}$$

is essentially the Fourier component of the instanton density

$$U(b,p) = \int e^{ip\cdot x} E\cdot B d^4x \tag{6.5.2}$$

associated with the electromagnetic field of the colliding nuclei.

In order to avoid cumbersome numerical factors, it is convenient to introduce the amplitude $A(b,p)$ as

$$
\begin{aligned}
A(b,p) &= N_0 \times \frac{4\pi}{Z_1 Z_2 \alpha_{em}} \times U(b,p) \ , \\
N_0 &= \tfrac{(2\pi)^7}{i}
\end{aligned}
\tag{6.5.3}
$$

Coordinates are chosen so that target nucleus is at rest at the origin of coordinates and colliding nucleus moves along positive z direction in $y = 0$ plane with velocity β. The orbit is approximated with straight line with impact parameter b.

Instanton density is just the scalar product of the static electric field E of the target nucleus and magnetic field B the magnetic field associated with the colliding nucleus, which is obtained by boosting the Coulomb field of static nucleus to velocity β. The flux lines of the magnetic field rotate around the direction of the velocity of the colliding nucleus so that instanton density is indeed non vanishing.

The Fourier transforms of E and B for nuclear charge 4π (chose for convenience) giving rise to Coulomb potential $1/r$ are given by the expressions

$$
\begin{aligned}
E_i(k) &= N\delta(k_0)k_i/k^2 \ , \\
B_i(k) &= N\delta(\gamma(k_0 - \beta k_z))k_j\varepsilon_{ijz}e^{ik_x b}/((\frac{k_z}{\gamma})^2 + k_T^2) \ , \\
N &= \frac{1}{(2\pi)^2} \ .
\end{aligned}
\tag{6.5.4}
$$

The normalization factor corresponds to momentum space integration measure d^4p. The Fourier transform of the instanton density can be expressed as a convolution of the Fourier transforms of E and B.

$$
\begin{aligned}
A(b,p) &\equiv = N_0 N_1 \int E(p-k) \cdot B(k) d^4k \ , \\
N_1 &= \frac{1}{(2\pi)^4} \ .
\end{aligned}
\tag{6.5.5}
$$

Where the fields correspond to charges $\pm 4\pi$. In the convolution the presence of two delta functions makes it possible to integrate over k_0 and k_z and the expression for U reduces to a two-fold integral

$$
\begin{aligned}
A(b,p) &= \beta\gamma \int dk_x dk_y exp(ikxb)(k_x p_y - k_y p_x)/AB \ , \\
A &= (p_z - \frac{p_0}{\beta})^2 + p_T^2 + k_T^2 - 2k_T \cdot p_T \\
B &= k_T^2 + (\frac{p_0}{\beta\gamma})^2 \ , \\
p_T &= (p_x, p_y) \ .
\end{aligned}
\tag{6.5.6}
$$

To carry out the remaining integrations one can apply residue calculus.

1. k_y integral is expressed as a sum of two pole contributions
2. k_x integral is expressed as a sum of two pole contributions plus two cut contributions.

k_y-integration

Integration over k_y can be performed by completing the integration contour along real axis to a half circle in upper half plane (see Fig. 6.5.1).

The poles of the integrand come from the two factors A and B in denominator and are given by the expressions

$$
\begin{aligned}
k_y^1 &= i(k_x^2 + (\frac{p_0}{\beta\gamma})^2)^{1/2} \ , \\
k_y^2 &= p_y + i((p_z - \frac{p_0}{\beta})^2 + p_x^2 + k_x^2 - 2p_xk_x)^{1/2} \ .
\end{aligned} \tag{6.5.7}
$$

One obtains for the amplitude an expression as a sum of two terms

$$
A(b,p) = 2\pi i \int e^{ik_xb}(U_1 + U_2)dk_x \ , \tag{6.5.8}
$$

corresponding to two poles in upper half plane.

The explicit expression for the first term is given by

$$
\begin{aligned}
U_1 &= RE_1 + iIM_1 \ , \\
RE_1 &= (k_x\frac{p_0}{\beta}y - p_xre_1/2)/(re_1^2 + im_1^2) \ , \\
IM_1 &= (-k_xp_yre_1/2K_1^{1/2} - p_xp_yK_1^{1/2})/(re_1^2 + im_1^2) \ , \\
re_1 &= (p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2 - 2p_xk_x \ , \\
im_1 &= -2K_1^{1/2}p_y \ , \\
K_1 &= k_x^2 + (\frac{p_0}{\beta\gamma})^2 \ .
\end{aligned} \tag{6.5.9}
$$

The expression for the second term is given by

$$
\begin{aligned}
U_2 &= RE_2 + iIM_2 \ , \\
RE_2 &= -((k_xp_y - p_xp_y)p_y + p_xre_2/2)/(re_2^2 + im_2^2) \ , \\
IM_2 &= (-(k_xp_y - p_xp_y)re_2/2K_2^{1/2} + p_xp_yK_2^{1/2})/(re_2^2 + im_2^2) \ , \\
re_2 &= -(p_z - \frac{p_0}{\beta})^2 + (\frac{p_0}{\beta\gamma})^2 + 2p_xk_x + \frac{p_0}{\beta}y - \frac{p_0}{\beta}x \ , \\
im_2 &= 2p_yK_2^{1/2} \ , \\
K_2 &= (p_z - \frac{p_0}{\beta})^2 + \frac{p_0}{\beta}x + k_x^2 - 2p_xk_x \ .
\end{aligned} \tag{6.5.10}
$$

A little inspection shows that the real parts cancel each other:$RE_1 + RE_2 = 0$. A further useful result is the identity $im_1^2 + re_1^2 = re_2^2 + im_2^2$ and the identity $re_2 = -re_1 + 2p_y^2$.

k_x-integration

One cannot perform k_x-integration completely using residue calculus. The reason is that the terms IM_1 and IM_2 have cuts in complex plane. One can however reduce the integral to a sum of pole terms plus integrals over the cuts.

The poles of U_1 and U_2 come from the denominators and are in fact common for the two integrands. The explicit expressions for the pole in upper half plane, where integrand converges exponentially are given by

$$
\begin{aligned}
re_i^2 &+ im_i^2 = 0 \ , \ i = 1,2 \ , \\
k_x &= (-b + i(-b^2 + 4ac)^{1/2})/2a \ , \\
a &= 4p_T^2 \ , \\
b &= -4((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)p_x \ per, \\
c &= ((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)^2 + 4(\frac{p_0}{\beta\gamma})^2p_y^2 \ .
\end{aligned} \tag{6.5.11}
$$

A straightforward calculation using the previous identities shows that the contributions of IM_1 and IM_2 at pole have opposite signs and the contribution from poles vanishes identically!

The cuts associated with U_1 and U_2 come from the square root terms K_1 and K_2. The condition for the appearance of the cut is that K_1 (K_2) is real and positive. In case of K_1 this condition gives

$$k_x = it,\ t \in (0, \frac{p_0}{\beta\gamma}) \ . \quad (6.5.12)$$

In case of K_2 the same condition gives

$$k_x = p_x + it,\ t \in (0, \frac{p_0}{\beta} - p_z) \ . \quad (6.5.13)$$

Both cuts are in the direction of imaginary axis.

The integral over real axis can be completed to an integral over semi-circle and this integral in turn can be expressed as a sum of two terms (see Fig. 6.5.1).

$$A(b,p) = 2\pi i(CUT_1 + CUT_2) \ . \quad (6.5.14)$$

The first term corresponds to contour, which avoids the cuts and reduces to a sum of pole contributions. Second term corresponds to the addition of the cut contributions.

In the following we shall give the expressions of various terms in the region $\phi \in [0, \pi/2]$. Using the symmetries

$$\begin{aligned} A(b, p_x, -p_y) &= -A(b, p_x, p_y) \ , \\ A(b, -p_x, -p_y) &= \bar{A}(b, p_x, p_y) \ . \end{aligned} \quad (6.5.15)$$

of the amplitude one can calculate the amplitude for other values of ϕ.

The integration variable for cuts is the imaginary part t of complexified k_x. To get a more convenient form for cut integrals one can perform a change of the integration variable

$$\begin{aligned} cos(\psi) &= \frac{t}{(\frac{p_0}{\beta\gamma})} \ , \\ cos(\psi) &= \frac{t}{(\frac{p_0}{\beta} - p_z)} \ , \\ \psi &\in [0, \pi/2] \ . \end{aligned} \quad (6.5.16)$$

1. The contribution of the first cut

By a painstaking calculation one verifies that the expression for the contribution of the first cut is given by

$$\begin{aligned} CUT_1 &= D_1 \times \int_0^{\pi/2} exp(-\frac{b}{b_0} cos(\psi)) A_1 d\psi \ , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \ , \ b_0 = \frac{\hbar}{m}\frac{\beta\gamma}{\gamma_1} \ , \\ A_1 &= \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\ A &= sin(\theta)cos(\phi) \ , \ B = K \ , \\ C &= K\frac{cos(\phi)}{sin(\theta)} \ , \ D = -sin^2(\phi) - \frac{K^2}{sin^2(\theta)} \ , \\ K &= \beta\gamma(1 - \frac{v_{cm}}{\beta} cos(\theta)) \ , \ v_{cm} = \frac{2v}{1+v^2} \ . \end{aligned} \quad (6.5.17)$$

The definitions of the various kinematical variables are given in previous formulas. The notation is tailored to express that A_1 is rational function of $cos(\psi)$.

1. The exponential $exp(-bcos(\psi)/b_0)$ is very small in the condition

$$cos(\psi) \geq cos(\psi_0) \equiv \frac{\hbar}{mb}\frac{\beta\gamma}{\gamma_1 cos(\phi)} \quad (6.5.18)$$

   holds true. Here $\hbar = 1$ convention has been given up to make clear that the increase of the Compton length of lepto-pion due to the scaling of $\hbar$ increase the magnitude of the contribution. If the condition $cos(\psi_0) << 1$ holds true, the integral over ψ receives contributions only from narrow range of values near the upper boundary $\psi = \pi/2$ plus the contribution corresponding to the pole of X_1. The practical condition is in terms of critical parameter b_{max} above which exponential approaches zero very rapidly.

2. For $cos(\psi_0) << 1$, that is for $b > b_{max}$ and in the approximation that the function multiplying the exponent is replaced with its value for $\psi = \pi/2$, one obtains for CUT_1 the expression

$$\begin{aligned} CUT_1 &\simeq D_1 A_1(\psi = \pi/2)\frac{\hbar}{mb} \\ &= \frac{1}{2} \times \frac{\beta\gamma}{\gamma_1} \times \frac{\hbar}{mb} \times \frac{sin^2(\theta)cos(\phi)sin(\phi)}{sin^2(\theta)sin^2(\phi) + K^2} \ . \end{aligned} \quad (6.5.19)$$

3. For $cos(\psi_0) >> 1$ exponential factor can be replaced by unity in good approximation and the integral reduces to an integral of rational function of $cos(\psi)$ having the form

$$D_1 \frac{A + iBcos(\psi)}{cos^2(\psi) + 2iC \times cos(\psi) + D} \ . \quad (6.5.20)$$

   which can be expressed in terms of the roots $c_\pm$ of the denominator as

$$D_1 \times \sum_\pm \frac{A \mp iBc_\pm}{cos(\psi) - c_\pm} \ , \ c_\pm) = -iC \pm \sqrt{-C^2 - D} \ . \quad (6.5.21)$$

   Integral reduces to an integral of rational function over the interval $[0, 1]$ by the standard substitution $tan(\psi/2) = t$, $d\psi = 2dt/(1+t^2)$, $cos(\psi) = (1-t^2)/(1+t^2)$, $sin(\psi) = 2t/(1+t^2)$.

$$I = 2D_1 \sum_\pm \int_0^1 dt \frac{A \mp iBc_\pm}{1 - c_\pm - (1 + c_\pm)t^2} \quad (6.5.22)$$

   This gives

$$I = 2D_1 \sum_\pm \frac{A \mp iBc_\pm}{s_\pm} \times arctan(\frac{1 + c_\pm}{1 - c_\pm}) \ . \quad (6.5.23)$$

$s_\pm$ is defined as $\sqrt{1-c_\pm^2}$ and one must be careful with the signs. This gives for CUT_1 the approximate expression

$$\begin{aligned} CUT_1 &= D_1 \sum_\pm \frac{sin(\theta)cos(\phi) \mp iKc_\pm}{s_\pm} \times arctan(\frac{1+c_\pm}{1-c_\pm}) \ , \\ c_\pm &= \frac{-iKcos(\phi) \pm sin(\phi)\sqrt{sin^2(\theta)+K^2}}{sin(\theta)} \ . \end{aligned} \tag{6.5.24}$$

Arcus tangent function must be defined in terms of logarithm functions since the argument is complex.

4. In the intermediate region, where the exponential differs from unity one can use expansion in Taylor polynomial to sum over integrals of rational functions of $cos(\psi)$ and one obtains the expression

$$\begin{aligned} CUT_1 &= D_1 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(\frac{b}{b_0})^n I_n \ , \\ I_n &= \sum_\pm (A \mp iBc_\pm I_n(c_\pm) \ , \\ I_n(c) &= \int_0^{\pi/2} \frac{cos^n(\psi)}{cos(\psi)-c} \ . \end{aligned} \tag{6.5.25}$$

$I_n(c)$ can be calculated explicitly by expanding in the integrand $cos(\psi))^n$ to polynomial with respect to $cos(\psi))-c$, $c \equiv c_\pm$

$$\frac{cos^n(\psi)}{cos(\psi)-c} = \sum_{m=0}^{n-1} \left(\begin{array}{c} n \\ m \end{array} \right) c^m (cos(\psi)-c)^{n-m-1} + \frac{c^n}{cos(\psi)-c} \ . \tag{6.5.26}$$

After the change of the integration variable the integral reads as

$$\begin{aligned} I_n(c) &= \sum_{m=0}^{n-1} \sum_{k=0}^{n-m-1} \left(\begin{array}{c} n \\ m \end{array} \right) \left(\begin{array}{c} n-m-1 \\ k \end{array} \right) (-1)^k (1-c)^{n-m-1-k}(1+c)^k c^m I(k,n-m) \\ &+ \frac{c^n}{1-c} \times log[\frac{\sqrt{1-c}+\sqrt{1+c}}{\sqrt{1-c}-\sqrt{1+c}}] \ , \\ I(k,n) &= 2\int dt \frac{t^{2k}}{(1+t^2)^n} \ . \end{aligned} \tag{6.5.27}$$

Partial integration for $I(k,n)$ gives the recursion formula

$$I(k,n) = -\frac{2^{-n+1}}{n-1} + \frac{2k-1}{2(n-1)} \times I(k-1,n-1) \ . \tag{6.5.28}$$

The lowest term in the recursion formula corresponds to $I(0, n-k)$, can be calculated by using the expression

$$\begin{aligned} (1+t^2)^{-n} &= \sum_{k=0}^{n} c(n,k)[(1+it)^{-k} + (1-it)^{-k}] \ , \\ c(n,k) &= \sum_{l=0}^{n-k-1} c(n-1,k+l)2^{-l-2} + c(n-1,n-1)2^{-n+k-1} \ . \end{aligned} \quad (6.5.29)$$

The formula is deducible by assuming the expression to be known for n and multiplying the expression with $(1+t^2)^{-1} = [(1+it)^{-1} + (1-it)^{-1}]/2$ and applying this identity to the resulting products of $(1+it)^{-1}$ and $(1-it)^{-1}$. This gives

$$I(0,n) = -2i \sum_{k=2,n} \frac{c(n,k)}{(k-1)}[1 + 2^{(k-1)/2} sin((k-1)\pi/4)] + c(n,1) log(\frac{1+i}{1-i}) \quad (6.5.30)$$

This boils down to the following expression for CUT_1

$$\begin{aligned} CUT_1 &= D_1 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\frac{b}{b_0})^n I_n \ , \\ I_n &= \sum_{\pm} [A \mp iBc_{\pm}] I_n(cos(c_{\pm}) \ , \\ I_n(c) &= \sum_{m=1}^{n-1} \sum_{k=0}^{n-m-1} \begin{pmatrix} n \\ m \end{pmatrix} \begin{pmatrix} n-m-1 \\ k \end{pmatrix} (1-c)^{n-m-1-k}(1+c)^k c^m I(k, n-m-1) \\ &+ \frac{c^n}{1-c} \times log[\frac{\sqrt{1-c}+\sqrt{1+c}}{\sqrt{1-c}-\sqrt{1+c}}] \ , \\ I(k,n) &= -\frac{2^{-n+1}}{n-1} + \frac{2k-1}{2(n-1)} \times I(k-1,n-1) \ , \\ I(0,n) &= -2i \sum_{k=2}^{n} \frac{c(n,k)}{(k-1)} [1 + 2^{(k-1)/2} sin((k-1)\pi/4)] - c(n,1) \ , \\ c(n,k) &= \sum_{l=0}^{n-k-1} c(n-1,k+l)2^{-l-2} + c(n-1,n-1)2^{-n+k-1} \ . \end{aligned} \quad (6.5.31)$$

This expansion in powers of $c_{\pm}$ fails to converge when their values are very large. This happens in the case of τ-pion production amplitude. In this case one typically has however the situation in which the conditions $A_1 \simeq iBcos(\psi)/D$ holds true in excellent approximation and one can write

$$\begin{aligned} CUT_1 &\simeq i\frac{D_1 B}{D} \times \sum_{n=0,1,...} \frac{(-1)^n}{n! 2^n} (\frac{b}{b_0})^n I_n \times \ , \\ I_n &= \int_0^{\pi/2} cos(\psi)^{n+1} d\psi = \sum_{k=0}^{n+1} \begin{pmatrix} n+1 \\ k \end{pmatrix} \frac{i^{n-2k}-1}{n+1-2k} \ . \end{aligned} \quad (6.5.32)$$

The denominator X_1 vanishes, when the conditions

$$\begin{aligned} cos(\theta) &= \frac{\beta}{v_{cm}} \ , \\ sin(\phi) &= cos(\psi) \end{aligned} \tag{6.5.33}$$

hold. In forward direction the conditions express the vanishing of the z-component of the lepto-pion velocity in velocity cm frame as one can realize by noticing that condition reduces to the condition $v = \beta/2$ in non-relativistic limit. This corresponds to the production of lepto-pion with momentum in scattering plane and with direction angle $cos(\theta) = \beta/v_{cm}$.

CUT_1 diverges logarithmically for these values of kinematical variables at the limit $\phi \to 0$ as is easy to see by studying the behavior of the integral near as K approaches zero so that X_1 approaches zero at $sin(\phi) = cos(\Phi)$ and the integral over a small interval of length $\Delta\Psi$ around $cos(\Psi) = sin(\phi)$ gives a contribution proportional to $log(A + B\Delta\Psi))/B$, $A = K[K - 2isin(\theta)sin^2(\phi)]$ and $B = 2sin(\theta)cos(\phi)[sin(\theta)sin(\phi) - iKcos(\phi)]$. Both A and B vanish at the limit $\phi \to 0$, $K \to 0$. The exponential damping reduces the magnitude of the singular contribution for large values of $sin(\phi)$ as is clear form the first formula.

2. The contribution of the second cut

The expression for CUT_2 reads as

$$\begin{aligned} CUT_2 &= D_2 exp(-\frac{b}{b_2}) \times \int_0^{\pi/2} exp(i\frac{b}{b_1}cos(\psi))A_2 d\psi \ , \\ D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \ , \\ b_1 &= \frac{\hbar}{m}\frac{\beta}{\gamma_1} \ , \ b_2 = \frac{\hbar}{mb}\frac{1}{\gamma_1 \times sin(\theta)cos(\phi)} \\ A_2 &= \frac{Acos(\psi) + B}{cos^2(\psi) + 2iCcos(\psi) + D} \ , \\ A &= sin(\theta)cos(\phi)u \ , \ B = \frac{w}{v_{cm}} + \frac{v}{\beta}sin^2(\theta)[sin^2(\phi) - cos^2(\phi)] \ , \\ C &= \frac{\beta w}{uv_{cm}}\frac{cos(\phi)}{sin(\theta)} \ , \ D = -\frac{1}{u^2}(\frac{sin^2(\phi)}{\gamma^2} + \beta^2(v^2 sin^2(\theta) - \frac{2vw}{v_{cm}})cos^2(\phi)) \\ &+ \frac{w^2}{v_{cm}^2 u^2 sin^2(\theta)} + 2i\frac{\beta v}{u}sin(\theta)cos(\phi) \ , \\ u &= 1 - \beta v cos(\theta) \ , \ w = 1 - \frac{v_{cm}}{\beta}cos(\theta) \ . \end{aligned} \tag{6.5.34}$$

(6.5.35)

The denominator X_2 has no poles and the contribution of the second cut is therefore always finite.

1. The factor $exp(-b/b_2)$ gives an exponential reduction and the contribution of CUT_2 is large only when the criterion

$$b < \frac{\hbar}{m} \times \frac{1}{v\gamma_1 sin(\theta)cos(\phi)}$$

for the impact parameter b is satisfied. Large values of $\hbar$ increase the range of allowed impact parameters since the Compton length of lepto-pion increases.

2. At the limit when the exponent becomes very large the variation of the phase factor implies destructive interference and one can perform stationary phase approximation around $\psi = \pi/2$. This gives

$$
\begin{aligned}
CUT_2 &\simeq \sqrt{\frac{2\pi b_1}{b}} \times D_2 \times exp(\frac{b}{b_2}) A_2(\psi = 0) \ , \\
D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \ , \ A_2 = \frac{A}{D} \ .
\end{aligned} \tag{6.5.36}
$$

3. As for CUT_1, the integral over ψ can be expressed as a finite sum of integrals of rational functions, when the value of $(b/b_1)cos(\psi)$ is so small that $exp(i(b/b_1)cos(\psi))$ can be approximated by a Taylor polynomial. More generally, one obtains the expansion

$$
\begin{aligned}
CUT_2 &= D_2 exp(-\frac{b}{b_2}) \times \sum_{n=0}^{\infty} \frac{1}{n!} i^n (\frac{b}{b_1})^n I_n(A, B, C, D) \ , \\
I_n(A, B, C, D) &= \int_0^{\pi/2} cos(\psi)^n \frac{A + iBcos(\psi)}{cos^2(\psi) + Ccos(\psi) + D} \ .
\end{aligned} \tag{6.5.37}
$$

The integrand of $I_n(A, B, C, D)$ is same rational function as in the case of CUT_1 but the parameters A, B, C, D given in the expression for CUT_2 are different functions of the kinematical variables. The functions appearing in the expression for integrals $I_n(c)$ correspond to the roots of the denominator of A_2 and are given by $c_\pm = -iC \pm \sqrt{-C^2 - D}$, where C and D are the function appearing in the general expression for CUT_2 in Eq. 6.5.35.

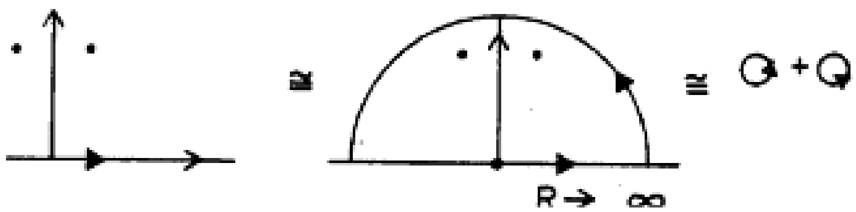

Figure 6.3: Evaluation of k_y-integral using residue calculus.

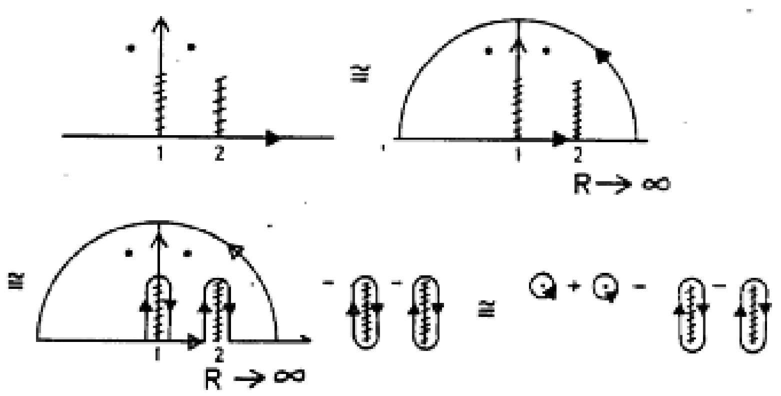

Figure 6.4: Evaluation of k_x-integral using residue calculus.

6.5.2 Production amplitude in quantum model

The previous expressions for CUT_1 and CUT_2 as such give the production amplitude for given b in the classical model and the cross section can be calculated by integrating over the values of b. The finite Taylor expansion of the amplitude in powers of b allows explicit formulas when impact parameter cutoff is assumed.

General expression of the production amplitude

In quantum model the production amplitude can be reduced to simpler form by using the defining integral representation of Bessel functions

$$\begin{aligned} f_B &= i\int F(b)J_0(\Delta kb)(CUT_1+CUT_2)bdb \ , \\ F &= 1 \quad \text{for } exp(i(S)) \text{ option} \ , \\ F(b\geq b_{cr}) &= \int dz\frac{1}{\sqrt{z^2+b^2}} = 2ln(\frac{\sqrt{a^2-b^2}+a}{b}) \ \text{ for } exp(i(S))-1 \text{ option} \ , \\ \Delta k &= 2ksin(\frac{\alpha}{2}) \ , \quad k = M_R\beta \quad . \end{aligned} \tag{6.5.38}$$

Note that F is a rather slowly varying function of b and in good approximation can be replaced by its average value $A(b,p)$, which has been already explicitly calculated as power series in b. α_{em} corresponds to the value of α_{em} for the standard value of Planck constant.

The limit $\Delta k = 0$

The integral of the contribution of CUT_1 over the impact parameter b involves integrals of the form

$$\begin{aligned} J_{1,n} &= b_0^2\int J_0(\Delta kb)F(b)x^{n+1}dx \ , \\ x &= \frac{b}{b_0} \ . \end{aligned} \tag{6.5.39}$$

Here a is the upper impact parameter cutoff. For CUT_2 one has integrals of the form

$$\begin{aligned} J_{2,n} &= b_1^2(\frac{b_2}{b_1})^{n+2}\int J_0(\Delta kb)F(b)exp(-x)x^{n+1}dx \ , \\ x &= \frac{b}{b_2} \ . \end{aligned} \tag{6.5.40}$$

Using the following approximations it is possible to estimate the integrals analytically.

1. The logarithmic term is slowly varying function and can be replaced with its average value

$$F(b) \rightarrow \langle F(b)\rangle \equiv F \ . \tag{6.5.41}$$

2. Δk is fixed once the value of the impact parameter is known. At the limit $\Delta k = 0$ making sense for very high energy collisions one can but the value of Bessel function to $J_0(0) = 1$. Hence it is advantageous to calculated the integrals of $\int CUT_ibdb$.

Consider first the integral $\int CUT_1bdb$. If exponential series converges rapidly one can use Taylor polynomial and calculate the integrals explicitly. When this is not the case one can calculate integral approximately and the total integral is sum over two contributions:

$$\int CUT_1bdb = I_a + I_b \ . \tag{6.5.42}$$

1. The region in which Taylor expansion converges rapidly gives rise integrals

$$\begin{aligned} I_{1,n} &\simeq b_0^2 \int x^{n+1}dx = b_0^2 \frac{1}{n+2}[(\frac{b_{max}}{b_0})^{n+2} - (\frac{b_{cr}}{b_0})^{n+2}] \simeq b_0^2 \frac{1}{n+2}(\frac{b_{max}}{b_0})^{n+2} \quad , \\ I_{2,n} &\simeq b_1^2(\frac{b_2}{b_1})^{n+2} \int exp(-x)x^{n+1}dx = b_1^2(\frac{b_2}{b_1})^{n+2}(n+1)! \quad . \end{aligned} \tag{6.5.43}$$

2. For the perturbative part of CUT_1 one obtains the expression

$$\begin{aligned} I_a &= \int_0^{b_{max}} CUT_1 bdb = D_1 \times b_0^2 \times \sum_{n=0}^{\infty} \frac{1}{n!(n+2)}(\frac{b_{max}}{b_0})^{n+2} I_n(A,B,C,D) \quad , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \quad , \quad b_0 = \frac{\hbar\beta\gamma}{m\gamma_1} \quad . \end{aligned} \tag{6.5.44}$$

   There b_{max} is the largest value of b for which the series converges sufficiently rapidly.

3. The convergence of the exponential series is poor for large values of b/b_0, that is for $b > b_m$. In this case one can use the approximation in which the multiplier of exponent function in the integrand is replaced with its value at $\psi = \pi/2$ so that amplitude becomes proportional to b_0/b. In this case the integral over b gives a factor proportional to ab_0, where a is the impact parameter cutoff.

$$\begin{aligned} I_b \equiv \int_{b_m}^{a} CUT_1 bdb &\simeq b_0(a-b_m)D_1 \times A_1(\psi=\pi/2) \\ &= \frac{\beta\gamma}{\gamma_1} \times \frac{\hbar}{m} \times \frac{sin^2(\theta)cos(\phi)sin(\phi)}{sin^2(\theta)sin^2(\phi)+K^2} \quad , \\ D_1 &= -\frac{1}{2}\frac{sin(\phi)}{sin(\theta)} \quad , \quad A_1(\psi=\pi/2) = \frac{A}{D} \quad . \end{aligned} \tag{6.5.45}$$

4. As already explained, the expansion based on partial fractions does not converge, when the roots $c_\pm$ have very large values. This indeed occurs in the case of τ-pion production cross section. In this case one has $A_1 \simeq iBcos(\psi)/D$ in excellent approximation and one can calculate CUT_1 in much easier manner. Using the formula of Eq. 6.5.32 for CUT_1, one obtains

$$\begin{aligned} \int CUT_1 bdb &\simeq b_0^2 \frac{D_1 B}{D} \times \sum_{n=0,1,...} \frac{(-1)^n}{n!(n+2)2^n} \times \sum_{k=0}^{n+1} \begin{pmatrix} n+1 \\ k \end{pmatrix} c_{n,k} \times (\frac{b_{max}}{b_0})^n \quad , \\ c_{n,k} &= \frac{i^{n+1-2k}-1}{n+1-2k} for \;\; n \neq 2k-1 \quad , \quad c_{n,k} = \frac{i\pi}{2} \;\; for \;\; n = 2k-1 \quad , \end{aligned} \tag{6.5.46}$$

   Note that for $n = 2k+1 = k$ the coefficient diverges formally and actua

Highly analogous treatment applies to the integral of CUT_2.

1. For the perturbative contribution to $\int CUT_2 bdb$ one obtains

$$\begin{aligned} I_a &= \int_0^{b_{1,max}} CUT_2 bdb = b_1^2 D_2 \sum_{n=0}^{\infty} (n+1) i^n I_n(A,B,C,D) \times (\frac{b_2}{b_1})^{n+2} \ , \\ D_2 &= -\frac{sin(\frac{\phi}{2})}{usin(\theta)} \ , \\ b_1 &= \frac{\hbar\beta}{m\gamma_1} \ , \ b_2 = \frac{\hbar}{m\gamma_1} \frac{1}{sin(\theta)cos(\phi)} \ . \end{aligned} \quad (6.5.47)$$

2. Taylor series converges slowly for

$$\frac{b_1}{b_2} = \frac{sin(\theta)cos(\phi)}{\beta} \rightarrow 0 \ .$$

In this case one can replace $exp(-b/b_2)$ with unity or expand it as Taylor series taking only few terms. This gives the expression for the integral which is of the same general form as in the case of CUT_1

$$I_a = \int_0^{b_{max}} CUT_2 bdb = b_1^2 D_2 \sum_{n=0}^{\infty} \frac{i^n}{n!(n+2)} I_n(A,B,C,D) (\frac{b_{max}}{b_1})^{n+1} \ . \quad (6.5.48)$$

3. Also when b/b_1 becomes very large, one must apply stationary phase approximation to calculate the contribution of CUT_2 which gives a result proportional to $\sqrt{b_1/b}$. Assume that $b_m >> b_1$ is the value of impact parameter above which stationary phase approximation is good. This gives for the non-perturbative contribution to the production amplitude the expression

$$\begin{aligned} I_b &= \int_{b_m}^{a} CUT_2 bdb = k\sqrt{\frac{2\pi b_1}{b_2}} b_2^2 \times D_2 \times A_2(\psi = \pi/2) \ , \\ k &= \int_{x_1}^{x_2} exp(-x) x^{1/2} dx = 2\int_{\sqrt{x_1}}^{\sqrt{x_2}} exp(-u^2) u^2 du \ , \\ x_1 &= \frac{b_m}{b_2} \ , \ x_2 = \frac{a}{b_2} \ . \end{aligned} \quad (6.5.49)$$

In good approximation one can take $x_2 = \infty$. $x_1 = 0$ gives the upper bound $k \leq \sqrt{\pi}$ for the integral.

Some remarks relating to the numerics are in order.

1. The contributions of both CUT_1 and CUT_2 are proportional to $1/sin(\theta)$ in the forward direction. The denominators of A_i however behave like $1/sin^2(\theta)$ at this limit so that the amplitude behaves as $sin(\theta)$ at this limit and the amplitude approaches to zero like $sin(\theta)$ Therefore the singularity is only apparent but must be taken into account in the calculation since one has $c_\pm \rightarrow i\infty$ at this limit for CUT_2 and for CUT_1 the roots approach to $c_+ = c_- = i\infty$. One must pose a cutoff θ_{min} below which the contribution of CUT_1 and CUT_2 are calculated directly using approximate he expressions for $D_i A_i$.

$$
\begin{aligned}
D_1A_1 &\rightarrow -\frac{i}{K}cos(\psi) \times sin(\theta) \rightarrow 0 \\
D_2A_2 &\rightarrow -\frac{uv_{cm}}{w} \times sin(\theta) \rightarrow 0 \ .
\end{aligned} \quad (6.5.50)
$$

In good approximation both contributions vanish since also $sin^2(\theta)$ factor from the phase space integration reduces the contribution.

2. A second numerical problem is posed by the possible vanishing of

$$
K = \beta\gamma(1 - \frac{v_cm}{\beta}cos(\theta)) \ .
$$

In this case the roots $c_{\pm} = \pm sin(\phi)$ are real and c_+ gives rise to a pole in the integrand.

The singularity to the amplitude comes from the logarithmic contributions in the Taylor series expansion of the amplitude. The sum of the singular contributions coming from c_+ and c_- are of form

$$
\frac{c_n}{2}(\sqrt{1 - sin(\phi)} + \sqrt{1 + sin(\phi)}log(\frac{1+u}{1-u}) \ , \ u = \sqrt{\frac{1 + sin(\phi)}{1 - sin(\phi)}} \ .
$$

Here c_n characterizes the $1/(cos(\psi) - c_{\pm})$ term of associated with the $cos(\psi)^n$ term in the Taylor expansion. Logarithm becomes singular for the two terms in the sum at the limit $\phi \rightarrow 0$. The sum however behaves as

$$
\frac{c_n}{2}sin(\phi)log(\frac{sin(\phi)}{2}) \ .
$$

so that the net result vanishes at the limit $\phi \rightarrow 0$. It is essential that the logarithmic singularities corresponding to the roots c_+ and c_- cancel each other and this must be taken into account in numerics. There is also apparent singularity at $\phi = \pi/2$ canceled by $cos(\phi)$ factor in D_1. The simplest manner to get rid of the problem is to exclude small intervals $[0, \epsilon]$ and $[\pi/2 - \epsilon, \pi/2]$ from the phase space volume.

Improved approximation to the production cross section

The approximation $J_0(\Delta k_T(b)b) = 1$ and $F(b) = F = constant$ allows to perform the integrations over impact parameter explicitly (for $exp(iS)$ option $F = 1$ holds true identically in the lowest order approximation). An improved approximation is obtained by diving the range of impact parameters to pieces and performing the integrals over the impact parameter ranges exactly using the average values of these functions. This requires only a straightforward generalization of the formulas derived above involving integrals of the functions x^n and $exp(-x)x^n$ over finite range. Obviously this is still numerically well-controlled procedure.

6.5.3 Evaluation of the singular parts of the amplitudes

The singular parts of the amplitudes $CUT_{1,sing}$ and $B_{1,sing}$ are rational functions of $cos(\psi)$ and the integrals over ψ can be evaluated exactly.

In the classical model the expression for $U_{1,sing}$ appearing as integrand in the expression of $CUT_{1,sing}$ reads as

$$\begin{aligned}
A_{1,sing} &= -\frac{1}{2\sqrt{K^2+sin^2(\theta)}}(sin(\theta)cos(\phi)A_a + iKA_b) \ , \\
A_a &= I_1(\beta,\pi/2) = \int_0^{\pi/2} d\psi f_1 \ , \\
A_b &= I_2(\beta,\pi/2) = \int_0^{\pi/2} d\psi f_2 \ , \\
f_1 &= \frac{1}{(cos(\psi)-c_1)(cos(\psi)-c_2)} \ , \\
f_2 &= cos(\psi) f_1 \ , \\
c_1 &= \frac{-iKcos(\phi) + sin(\phi)\sqrt{K^2+sin^2(\theta)}}{sin(\theta)} \ , \\
c_2 &= -\bar{c}_1 \ .
\end{aligned} \tag{6.5.51}$$

Here c_i are the roots of the polynomial X_1 appearing in the denominator of the integrand.

In quantum model the approximate expression for the singular contribution to the production amplitude can be written as

$$\begin{aligned}
B_{1,sing} &\simeq k_1 \frac{sin(\theta)sin(\phi)}{2\sqrt{K^2+sin^2(\theta)}} \sum_n \langle F\rangle_n (I(x(n+1)) - I(x(n))) \ , \\
I(x) &= exp(-\frac{sin(\phi)x}{sin(\phi_0)})(sin(\theta)cos(\phi)A_a(\Delta ka,x) + iKA_b(\Delta ka,x)) \ , \\
k_1 &= 2\pi^2 M_R Z_1 Z_2 \alpha_{em} \frac{\sqrt{2}}{\sqrt{\Delta k\pi}} sin(\phi_0) \ .
\end{aligned} \tag{6.5.52}$$

The expressions for the amplitudes $A_a(k,x)$ and $A_b(k,x)$ read as

$$\begin{aligned}
A_a(k,x) &= cos(kx)I_3(k,0,\pi/2) + isin(\phi_0)ksin(kx)I_5(k,0,\pi/2) \ , \\
A_b(k,x) &= cos(kx)I_4(k,0,\pi/2) + isin(\phi_0)ksin(kx)I_3(k,0,\pi/2) \ , \\
I_i(k,\alpha,\beta) &= \int_\alpha^\beta f_i(k) d\psi \ , \\
f_3(k) &= \frac{cos(\psi)}{(cos^2(\psi)+sin^2(\phi_0)k^2)} f_1(k) \ , \\
f_4(k) &= cos(\psi) f_3(k) \ , \\
f_5(k) &= \frac{1}{(cos^2(\psi)+sin^2(\phi_0)k^2)} f_1(k) \ .
\end{aligned} \tag{6.5.53}$$

The expressions for the integrals I_i as functions of the endpoints α and β can be written as

$$\begin{aligned}
I_1(k,\alpha,\beta) &= I_0(c_1,\alpha,\beta) - I_0(c_2,\alpha,\beta) \ , \\
I_2(\alpha,\beta) &= c_1 I_0(c_1,\alpha,\beta) - c_2 I_0(c_2,\alpha,\beta) \ , \\
I_3 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i-c_j)}(c_i I_0(c_i,\alpha,\beta) - c_j I_0(c_j,\alpha,\beta)) \ , \\
I_4 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i-c_j)}((c_i-c_j)(\beta-\alpha) - c_i^2 I_0(c_i,\alpha,\beta) + c_j^2 I_0(c_j,\alpha,\beta)) \ , \\
I_5 &= C_{34} \sum_{i=1,2,j=3,4} \frac{1}{(c_i-c_j)}(I_0(c_i,\alpha,\beta) - I_0(c_j,\alpha,\beta)) \ , \\
C_{34} &= \frac{1}{c_3-c_4} = \frac{1}{2ikasin(\phi_0)} \ .
\end{aligned} \tag{6.5.54}$$

The parameters c_1 and c_2 are the zeros of X_1 as function of $cos(\psi)$ and c_3 and c_4 the zeros of the function $cos^2(\psi) + k^2a^2sin^2(\phi_0)$:

$$\begin{aligned}
c_1 &= \frac{-iKcos(\phi) + sin(\phi)\sqrt{K^2+sin^2(\theta)}}{sin(\theta)} \ , \\
c_2 &= \frac{-iKcos(\phi) - sin(\phi)\sqrt{K^2+sin^2(\theta)}}{sin(\theta)} \ , \\
c_3 &= ikasin(\phi_0) \ , \\
c_4 &= -ikasin(\phi_0) \ .
\end{aligned} \tag{6.5.55}$$

The basic integral $I_0(c,\alpha,\beta)$ appearing in the formulas is given by

$$\begin{aligned}
I_0(c,\alpha,\beta) &= \int_\alpha^\beta d\psi \frac{1}{(cos(\psi)-c)} \ , \\
&= \frac{1}{\sqrt{1-c^2}}(f(\alpha) - f(\beta)) \ , \\
f(x) &= ln(\frac{(1+tan(x/2)t_0)}{(1-tan(x/2)t_0)}) \ , \\
t_0 &= \sqrt{\frac{1-c}{1+c}} \ .
\end{aligned} \tag{6.5.56}$$

From the expression of I_0 one discovers that scattering amplitude has logarithmic singularity, when the condition $tan(\alpha/2) = 1/t_0$ or $tan(\beta/2) = 1/t_0$ is satisfied and appears, when c_1 and c_2 are real. This happens at the cone $K = 0$ ($\theta = \theta_0$), when the condition

$$\begin{aligned}
\sqrt{\frac{(1-sin(\phi))}{(1+sin(\phi))}} &= tan(x/2) \ , \\
x &= \alpha \text{ or } \beta \ .
\end{aligned} \tag{6.5.57}$$

holds true. The condition is satisfied for $\phi \simeq x/2$. $x = 0$ is the only interesting case and gives singularity at $\phi = 0$. In the classical case this gives logarithmic singularity in production amplitude for all scattering angles.

Chapter 7

TGD and Nuclear Physics

7.1 Introduction

Despite the immense amount of data about nuclear properties, the first principle understanding of the nuclear strong force is still lacking. The conventional meson exchange description works at qualitative level only and does not provide a viable perturbative approach to the description of the strong force. The new concept of atomic nucleus forced by TGD suggests quite different approach to the quantitative description of the strong force in terms of the notion of field body, join along boundaries bond concept, long ranged color gauge fields associated with dark hadronic matter, and p-adic length scale hierarchy.

7.1.1 p-Adic length scale hierarchy

p-Adic length scale hypothesis

The concept of the p-adic topological condensate is the corner stone of p-adic TGD. Various levels of the topological condensate obey effective p-adic topology and are assumed to form a p-adic hierarchy ($p_1 \leq p_2$ can condense on p_2). By the length scale hypothesis, the physically interesting length scales should come as square roots of powers of 2: $L(k) \simeq 2^{\frac{k}{2}} l$, $l \simeq 1.288E + 4\sqrt{G}$ and prime powers of k are especially interesting. In biological scales the scaled up Compton length scales of electron given by $L_e(k) = \sqrt{5}L(k)$ seem to be more relevant.

For nuclear physics applications the most interesting values of k are: $k = 107$ (hadronic space-time sheet at which quarks feed their color gauge fluxes), $k = 109$ (radius of light nucleus such as alpha particle, $k = 113$ (the space-time at which quarks feed their electromagnetic gauge fluxes), $k = k_{em} = 127$ or 131 (electronic or atomic space-time sheet receiving electromagnetic gauge fluxes of nuclei).

The so called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne. Also the primes $k = 151, 157, 163, 167$ defining biologically important length scales correspond to Gaussian Mersennes. Thus the electromagnetic p-adic length scales associated with quarks, hadrons, and nuclear physics as well as with muon are in well defined sense also Mersenne length scales.

Particles are characterized by a collection of p-adic primes

It seems that is not correct to speak about particle as a space-time sheet characterized by single p-adic prime. Already p-adic mass calculations suggest that there are several sizes corresponding to space-time sheets at which particle feeds its gauge charges. p-Adic length scale hypothesis provides further insight: the length scale is more like the size of field body and possibly also de-localization volume of particle determining the p-adic mass scale in p-adic thermodynamics rather than the geometric size for the elementary particle.

What one can definitely say that each particle is characterized by a collection of p-adic primes and one of them characterizes the mass scale of the particle whereas other characterize its interactions. There are two possible interpretations and both of them allow to resolve objections against p-adic hierarchies of color and electro-weak physics.

1. These primes characterize the space-time sheets at which it feeds its gauge fluxes and particles can interact only via their common space-time sheets and are otherwise dark with respect to each other.

2. Number theoretical vision supports the notion of multi-p p-adicity and the idea that elementary particles correspond to infinite primes, integers, or perhaps even rationals [K37, K82] . To infinite primes, integers, and rationals it is possible to associate a finite rational $q = m/n$ by a homomorphism. q defines an effective q-adic topology of space-time sheet consistent with p-adic topologies defined by the primes dividing m and n ($1/p$-adic topology is homeomorphic to p-adic topology). The largest prime dividing m determines the mass scale of the space-time sheet in p-adic thermodynamics. m and n are exchanged by super-symmetry and the primes dividing m (n) correspond to space-time sheets with positive (negative) time orientation. Two space-time sheets characterized by rationals having common prime factors can be connected by a $\#_B$ contact and can interact by the exchange of particles characterized by divisors of m or n.

   The nice feature of this option is that single multi-p p-adic space-time sheet rather than a collection of them characterizes elementary particle. Concerning the description of interaction vertices as generalization of vertices of Feynman graphs (vertices as branchings of 3-surfaces) this option is decisively simpler than option 1) and is consistent with earlier number theoretic argument allowing to evaluate gravitational coupling strength [K37, K82] . It is also easier to to understand why the largest prime in the collection determines the mass scale of elementary particle.

Interestingly, these two options are not necessarily mutually exclusive: single multi-p p-adic space-time sheet could correspond to many-sheeted structure with respect to real topology.

What is the proper interpretation of p-adic length scales

One of the surprises of p-adic mass calculations was that for u and d quarks electromagnetic size corresponds to $k = 113$ which corresponds to the length scale of 2×10^{-14} m. This leads to the view that also hadrons and nuclei have this size in some sense. The charge radii of even largest nuclei without neutron halo are smaller than this.

1. If electromagnetic charges of quarks inside nucleons were separately de-localized in the scale $L(113)$, also the distributions of electromagnetic charges of nuclei would be non-trivial in surprisingly long length scale. Em charges would exhibit fractionality in this length scale and Rutherford scattering cross sections would be modified. The fact that the height of the Coulomb wall at $L(113)$ is lower than the observed heights of the Coulomb wall would lead to a paradox.

   This suggests that the p-adic length scale $L(113)$ does not characterize the geometric size of neither nucleons nor nuclei but to the size, perhaps height, of the electromagnetic field body associated with quark/hadron/nucleus.

2. If protons feed their electric em gauge fluxes to the same space-time sheet, there is an electromagnetic harmonic oscillator potential contributing to the nuclear energies. The Mersenne prime M_{127} as a characterizer of the field body of nucleus is natural and it also corresponds to the space-time sheet of electron.

3. For weak forces the size of the field body would be given by electro-weak length scale $L(89)$. The size scale would also correspond to the p-adic de-localization length scale of ordinary sized nucleons and nuclei.

4. It turns out that the identification of nuclear strong interactions in terms of dark QCD with large value of $\hbar$ and color length scale scaled up to $L_c \simeq 2^{11}L(107) \simeq .5 \times 10^{-11}$ m (!) predicts for the nuclei same electromagnetic sizes as in the conventional theory: scaled up

sizes appear only in the dark sector and characterize the size of color field body so that paradoxes are avoided. There are also reasons to believe that dark quarks are dark also with respect to electromagnetic and weak interactions so that the sizes of corresponding field bodies are scaled up by a factor $1/v_0$.

The hypothesis that the collection of primes corresponds to multi-p p-adicity rather than collection of space-time sheets implies this. For this option various field bodies could form single field body in q-adic sense with superposed p-adic fractalities much like waves of shorter wavelength scale superposed on waves of longer wavelength scale. As noticed, this might be consistent with the existence of several p-adic field bodies with respect to real topology.

Field/magnetic bodies would represent the space-time correlate for the formation of bound states. It is even possible to think that bound state entanglement corresponds to the linking of magnetic flux tubes. The contributions of say color interactions between nucleons to the binding energy would be estimated using the field magnitudes at position of exotic quarks and the hypothesis is made that these intensities correlate with the shortest distance between dark quarks although the distance along the field body is of order L_c.

This picture finds experimental support. In the following all the scales are given as electron Compton scales also for scales shorter than electron scale $L_e(k) = \sqrt{5}L(k)$. Reader can calculate $L(k)$ using this formula. This is due a longstanding mis-identification of $L(k)$ with $L_e(k)$ and for the fact that recalculation of scales $L(k)$ is not practical.

1. Neutron proton scattering at low energies gives however surprisingly clear evidence for the presence of the p-adic length scales $L(109)$ ($k = 109$ is prime) and $L(113)$ in nuclear physics. The scattering lengths for s and p waves are $a_s = -2.37 \times 10^{-14}$ m and $a_t = 5.4 \times 10{-}15$ m [C108] . a_s is anomalously large and the standard explanation is that deuteron almost allows singlet wave bound state. The p-adic length scales $L(113)$ and $L(109)$ are by more than factor of 2 smaller than these scales. Interestingly, a_t is near to $L_e(109) = 2L_e(107) \simeq 5.0 \times 10^{-15}$ m. a_s is of same order of magnitude as $L_e(113) = 2 \times 10^{-14}$ m so that the interpretation in terms of the $k = 113$ space-time sheet is suggestive.

   If $L(k)$ is replaced with $L_e(k)$, there is a qualitative accordance with the assumption that in triplet state neutron and proton are glued by color bond together to form structure with size or order $L_e(109) = 2L_e(107)$. Does this mean that p-adically scaled up variants of $L_e(127)$ define fundamental length scales besides $L(k)$? Could they correspond to and algebraic extension of p-adic numbers involving $\sqrt{5}$ and therefore also Golden Mean?

2. Neutron halos at distance of about 2.5×10^{-14} m longer than even $L_e(113) = 2 \times 10^{-14}$ m are difficult to understand in the standard nuclear physics framework and provide support for the large value of L_c. They could be understood in terms of de-localization of quarks in the length scale $L_e(113)$ and color charges in the length scale of L_c. For instance, the nucleus in the center could be color charged and neutron halo would be analogous to a colored matter around the central halo.

What these considerations suggest is that $L_e(k) = \sqrt{5}L(k)$ could define another hierarchy of p-adic length scales perhaps naturally associated with causal diamonds and that electron Compton length just happens to co-incide with one of these length scales. A possible interpretation would be in terms of an algebraic extension allowing also to understand the role of Golden Mean in biology.

7.1.2 TGD based view about dark matter

TGD suggests an explanation of dark matter as a macroscopically quantum coherent phase residing at larger space-time sheets [K28] .

1. TGD suggests that $\hbar$ is dynamical and possesses a spectrum expressible as integer multiples of the ordinary Planch constant. A good guess is that the criticality condition reads as $Q_1Q_2\alpha \simeq 1$ where Q_i are gauge charges and α gauge coupling strength. This leads to universal properties of the large $\hbar$ phase. For instance, $\hbar$ is scaled in the transition to dark phase by a harmonic or subharmonic of parameter $1/v_0 \simeq 2^{11}$ which is essentially the ratio of CP_2 length scale and Planck length [K75, K28] . The criticality condition can be applied

also to dark matter itself and entire hierarchy of dark matters is predicted corresponding to the spectrum of values of $\hbar$.

2. The particles of dark matter can also carry phase carry complex conformal weights but the net conformal weights for blocks of this kind of dark matter would be real. This implies macroscopic quantum coherence. It is not absolutely necessary that $\hbar$ is large for this phase.

3. From the point of view of nuclear physics application of this hypothesis is to QCD. The prediction is that the electromagnetic Compton sizes of dark quarks are scaled from $L(107)$ to about $2^{11}L(107) = L(129) = 2L(127) = (2/\sqrt{5})L_e(127)$, which is almost as long as Compton length of electron! The classical scattering cross sections are not changed but changes the geometric sizes of dark quarks, hadrons, and nuclei. The original hypothesis that ordinary valence quarks are dark whereas sea quarks correspond to ordinary value of $\hbar$ is taken as a starting point. In accordance with the earlier model, nucleons in atomic nuclei are assumed to be accompanied by color bonds connecting exotic quark and anti-quark characterized p-adic length scale $L(127)$ with ordinary value of $\hbar$ and having thus scaled down mass of order MeV. The strong binding would be due the color bonds having exotic quark and anti-quark at their ends.

4. Quantum classical correspondence suggests that classical long ranged electro-weak gauge fields serve as classical space-time correlates for dark electro-weak gauge bosons, which are massless. This hypothesis could explain the special properties of bio-matter, in particular the chiral selection as resulting from the coupling to dark Z^0 quanta. Long range weak forces present in TGD counterpart of Higgs=0 phase should allow to understand the differences between biochemistry and the chemistry of dead matter.

   The basic implication of the new view is that the earlier view about nuclear physics applies now to dark nuclear physics and large parity breaking effects and contribution of Z^0 force to scattering and interaction energy are not anymore a nuisance.

5. For ordinary condensed matter quarks and leptons Z^0 charge are screened in electro-weak length scale whereas in dark matter $k = 89$ electro-weak space-time sheet have suffered a phase transition to a p-adic topology with a larger value of k. Gaussian Mersennes, in particular those associated with $k = 113, 151, 157, 163, 167$ are excellent candidates in this respect.

   In dark matter phase weak gauge fluxes could be feeded to say $k = k_Z = 169$ space-time sheet corresponding to neutrino Compton length and having size of cell. For this scenario to make sense it is essential that p-adic thermodynamics predicts for dark quarks and leptons essentially the same masses as for their ordinary counterparts [K48] .

7.1.3 The identification of long range classical weak gauge fields as correlates for dark massless weak bosons

Long ranged electro-weak gauge fields are unavoidably present when the dimension D of the CP_2 projection of the space-time sheet is larger than 2. Classical color gauge fields are non-vanishing for all non-vacuum extremals. This poses deep interpretational problems. If ordinary quarks and leptons are assumed to carry weak charges feeded to larger space-time sheets within electro-weak length scale, large hadronic, nuclear, and atomic parity breaking effects, large contributions of the classical Z^0 force to Rutherford scattering, and strong isotopic effects, are expected. If weak charges are screened within electro-weak length scale, the question about the interpretation of long ranged classical weak fields remains.

During years I have discussed several solutions to these problems.

Option I: The trivial solution of the constraints is that Z^0 charges are neutralized at electro-weak length scale. The problem is that this option leaves open the interpretation of classical long ranged electro-weak gauge fields unavoidably present in all length scales when the dimension for the CP_2 projection of the space-time surface satisfies $D > 2$.

Option II: Second option involves several variants but the basic assumption is that nuclei or even quarks feed their Z^0 charges to a space-time sheet with size of order neutrino Compton length. The large parity breaking effects in hadronic, atomic, and nuclear length scales is not the only difficulty. The scattering of electrons, neutrons and protons in the classical long range Z^0 force contributes to the Rutherford cross section and it is very difficult to see how neutrino screening could make these effects small enough. Strong isotopic effects in condensed matter due to the classical Z^0 interaction energy are expected. It is far from clear whether all these constraints can be satisfied by any assumptions about the structure of topological condensate.

Option III: During 2005 third option solving the problems emerged based on the progress in the understanding of the basic mathematics behind TGD.

In ordinary phase the Z^0 charges of elementary particles are indeed neutralized in intermediate boson length scale so that the problems related to the parity breaking, the large contributions of classical Z^0 force to Rutherford scattering, and large isotopic effects in condensed matter, trivialize.

Classical electro-weak gauge fields in macroscopic length scales are identified as space-time correlates for the gauge fields created by dark matter, which corresponds to a macroscopically quantum coherent phase for which elementary particles possess complex conformal weights such that the net conformal weight of the system is real.

In this phase $U(2)_{ew}$ symmetry is not broken below the scaled up weak scale except for fermions so that gauge bosons are massless below this length scale whereas fermion masses are essentially the same as for ordinary matter. By charge screening gauge bosons look massive in length scales much longer than the relevant p-adic length scale. The large parity breaking effects in living matter (chiral selection for bio-molecules) support the view that dark matter is what makes living matter living.

Classical long ranged color gauge fields always present for non-vacuum extremals are interpreted as space-time correlates of gluon fields associated with dark copies of hadron physics. It seems that this picture is indeed what TGD predicts.

One cannot deny that the above scene is still somewhat unsatisfactory since it is difficult to understand why the classical weak fields present in say atomic nuclei would not couple to induced spinor fields and cause large parity breaking effects. The solution of this problem case after the realization that well-definedness of em charge for the modes of induced spinor field forces their modes to be localized at 2-D surfaces in the generic case. Induced W fields vanish at these surfaces and also Z^0 fields can vanish and naturally do so above weak scale proportional to h_{eff}. Therefore no large parity breaking effects come from fermionic sector for ordinary matter.

7.1.4 Dark color force as a space-time correlate for the strong nuclear force?

Color confinement suggests a basic application of the basic criteria for the transition to large $\hbar$ phase. The obvious guess is that valence quarks are dark [K30, K28] . Dark matter phase for quarks does not change the lowest order classical strong interaction cross sections but reduces dramatically higher order perturbative corrections and resolves the problems created by the large value of QCD coupling strength in the hadronic phase.

The challenge is to understand the strong binding solely in terms of dark QCD with large value of $\hbar$ reducing color coupling strength of valence quarks to $v_0 \simeq 2^{-11}$. The best manner to introduce the basic ideas is as a series of not so frequently asked questions and answers.

Rubber band model of strong nuclear force as starting point

The first question is what is the vision for nuclear strong interaction that one can start from. The sticky toffee model of Chris Illert [C88] is based on the paradox created by the fact alpha particles can tunnel from the nucleus but that the reversal of this process in nuclear collisions does not occur. Illert proposes a classical model for the tunnelling of alpha particles from nucleus based on dynamical electromagnetic charge. Illert is forced to assume that virtual pions inside nuclei have considerably larger size than predicted by QCD and the model. Strikingly, the model favors fractional alpha particle charges at the nuclear surface. The TGD based interpretation would be based on the identification of the rubber bands of Illert as long color bonds having exotic light quark and anti-quark at their ends and connecting escaping alpha particle to the mother nucleus. The challenge is to give meaning to the attribute "exotic".

How the darkness of valence quarks can be consistent with the known sizes of nuclei?

The assumption about darkness of valence quarks in the sense of of large $\hbar$ ($\hbar_s = \hbar/v_0$) is very natural if one takes the basic criterion for darkness seriously. The obvious question is how the dark color force can bind the nucleons to nuclei of ordinary size if the strength of color force is v_0 and color sizes of valence quarks are about $L_e(129)$?

It seems also obvious that $L(107)$ in some sense defines the size for nucleons, and somehow this should be consistent with scaled up size scale $L(k_{eff} = 129)$ implied by the valence quarks with large $\hbar$. The proposal of [K30, K28] inspired by RHIC findings [C97] is that valence quarks are dark in the sense of having large value of $\hbar$ and thus correspond to $k_{eff} = 129$ whereas sea quarks correspond to ordinary value for $\hbar$ and give rise to the QCD size $\sim L(107)$ of nucleon.

If one assumes that the typical distances between sea quark space-time sheets of nucleons is obtained by scaling down the size scale of valence quarks, the size scale of nuclei comes out correctly.

Valence quarks and exotic quarks cannot be identical

The hypothesis is that nucleons contain or there are associated with them pairs of exotic quarks and flux tubes of color field bodies of size $\sim L(129)$ connecting the exotic quark and anti-quark in separate nuclei. Nucleons would be structures with the size of ordinary nucleus formed as densely packed structures of size $L(129)$ identifiable as the size of color magnetic body.

The masses of exotic quarks must be however small so that they must differ from valence quarks. The simplest possibility is that exotic quarks are not dark but p-adically scaled down versions of sea quarks with ordinary value of $\hbar$ having $k = 127$ so that masses are scaled down by a factor 2^{-10}.

Energetic considerations favor the option that exotic quarks associate with nucleons via the $k_{eff} = 111$ space-time sheets containing nucleons and dark quarks. Encouragingly, the assumption that nucleons topologically condense at the weak $k_{eff} = 111$ space-time sheet of size $L(111) \simeq 10^{-14}$ m of exotic quarks predicts essentially correctly the mass number of the highest known super-massive nucleus. Neutron halos are outside this radius and can be understood in terms color Coulombic binding by dark gluons. Tetraneutron can be identified as alpha particle containing two negatively charged color bonds.

What determines the binding energy per nucleon?

The binding energies per nucleon for $A \geq 4$ to not vary too much from 7 MeV but the lighter nuclei have anomalously small binding energies. The color bond defined by a color magnetic flux tube of length $\sim L(k = 127)$ or $\sim L(k_{eff} = 129)$ connecting exotic quark and anti-quark in separate nucleons with scaled down masses $m_q(dark) \sim xm_q$, with $x = 2^{-10}$ for option for $k = 127$, is a good candidate in this respect. Color magnetic spin-spin interaction would give the dominant contribution to the interaction energy as in the case of hadrons. This interaction energy is expected to depend on exotic quark pair only. The large zero point kinetic energy of light nuclei topologically condensed at $k_{eff} = 111$ space-time sheet having possible identification as the dark variant of $k = 89$ weak space-time sheet explains why the binding energies of D and ^{3}He are anomalously small.

What can one assume about the color bonds?

Can one allow only quark anti-quark type color bonds? Can one allow the bonds to be also electromagnetically charged as the earlier model for tetra-neutron suggests (tetra-neutron would be alpha particle containing two negatively charged color bonds so that the problems with the Fermi statistics are circumvented). Can one apply Fermi statistics simultaneously to exotic quarks and anti-quarks and dark valence quarks?

Option I: Assume that exotic and dark valence quarks are identical in the sense of Fermi statistics. This assumption sounds somewhat non-convincing but is favored by p-adic mass calculations supporting the view that the p-adic mass scale of hadronic quarks can vary. If this hypothesis holds true at least effectively, very few color bonds from a given nucleon are allowed by statistics and there are good reasons to argue that nucleons are arranged to highly tangled string like structures filling nuclear volume with two nucleons being connected by color bonds having of length of order $L(129)$. The organization into closed strings is also favored by the conservation of magnetic flux.

The notion of nuclear string is strongly supported by the resulting model explaining the nuclear binding energies per nucleon. It is essential that nucleons form what might be called nuclear strings rather than more general tangles. Attractive p-p and n-n bonds must correspond to colored ρ_0 type bonds with spin one and attractive p-n type bonds to color singlet pion type bonds. The quantitative estimates for the spin-spin interaction energy of the lightest nuclei lead to more precise estimates for the lengths of color bonds. The resulting net color quantum numbers must be compensated by dark gluon condensate, the existence of which is suggested by RHIC experiments [C97]. This option is strongly favored by the estimate of nuclear binding energies.

Option II: If Fermi statistics is not assumed to apply in the proposed manner, then color magnetic flux tubes bonds between any pair of nucleons are possible. The identification of color isospin as strong isospin still effective removes color degree of freedom. As many as 8 color tubes can leave the nucleus if exotic quarks and anti-quarks are in the same orbital state and a cubic lattice like structure would become possible. This picture would be consistent with the idea that in ordinary field theory all particle pairs contribute to the interaction energy. The large scale of the magnetic flux tubes would suggest that the contributions cannot depend much on particle pair. The behavior of the binding energies favors strongly the idea of nuclear string and reduces this option to the first one.

What is the origin of strong force and strong isospin?

Here the answer is motivated by the geometry of CP_2 allowing to identify the holonomy group of electro-weak spinor connection as U(2) subgroup of color group. Strong isospin group SU(2) is identified as subgroup of isotropy group U(2) for space-time surfaces in a sub-theory defined by $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere of CP_2 and second factor of $U(1) \times U(1)$ subgroup of the holonomies for the induced Abelian gauge fields corresponds to strong isospin component I_3. The extremely tight correlations between various classical fields lead to the hypothesis that the strong isospin identifiable as color isospin I_3 of exotic quarks at the ends of color bonds attached to a given nucleon is identical with the weak isospin of the nucleon. Note that this does not require that exotic and valence quarks are identical particles in the sense of Fermi statistics.

Does the model explain the strong spin orbit coupling ($L \cdot S$ force)? This force can be identified as an effect due to the motion of fermion string containing the effectively color charged nucleons in the color magnetic field $v \times E$ induced by the motion of string in the color electric field at the dark $k = 107$ space-time sheet.

How the phenomenological shell model with harmonic oscillator potential emerges?

Nucleus can be seen as a collection of of long color magnetic flux tubes glued to nucleons with the mediation of exotic quarks and anti-quarks. If nuclei form closed string, as one expects in the case of Fermi statistics constraint, also this string defines a closed string or possibly a collection of linked and knotted closed strings. If Fermi statistics constraint is not applied, the nuclear strings form a more complex knotted and linked tangle. The stringy space-time sheets would be the color magnetic flux tubes connecting exotic quarks belonging to different nucleons.

The color bonds between the nucleons are indeed strings connecting them and the averaged interaction between neighboring nucleons in the nuclear string gives in the lowest order approximation 3-D harmonic oscillator potential although strings have $D = 2$ transversal degrees of freedom. Even in the case that nucleons for nuclear strings and thus have only two bonds to neighbors the average force around equilibrium position is expected to be a harmonic force in a good approximation. The nuclear wave functions fix the restrictions of stringy wave functionals to the positions of nucleons at the nuclear strings. Using M-theory language, nucleons would represent branes connected by color magnetic flux tubes representing strings whose ends co-move with branes.

Which nuclei are the most stable ones and what is the origin of magic numbers?

$P = N$ closed strings correspond to energy minima and their deformations obtained by adding or subtracting nucleons in general correspond to smaller binding energy per nucleon. Thus the observed strong correlation between P and N finds a natural explanation unlike in the harmonic oscillator model. For large values of A the generation of dark gluon condensate and corresponding color Coulombic binding energy favors the surplus of neutrons and the generation of neutron halos. The model explains also the spectrum of light nuclei, in particular the absence of pp, nn, ppp, and nnn nuclei.

In the standard framework spin-orbit coupling explains the magic nuclei and color Coulomb force gives rise to this kind of force in the same manner as in atomic physics context. Besides the standard magic numbers there are also non-standard ones (such as $Z, N = 6, 12$) if the maximum of binding energy is taken as a definition of magic, there are also other magic numbers than the standard ones. Hence can consider also alternative explanations for magic numbers. The geometric view about nucleus suggests that the five Platonic regular solids might defined favor nuclear configurations and it indeed turns that they explain non-standard magic numbers for light nuclei.

New magic nuclei might be obtained by linking strings representing doubly magic nuclei. An entire hierarchy of linkings becomes possible and could explain the new magic numbers 14, 16, 30, 32 discovered for neutrons [C39] . Linking of the nuclear strings could be rather stable by Pauli Exclusion Principle. For instance, ^{16}O would corresponds to linked ^4He and ^{12}C nuclei. Higher magic numbers $28, 50, ...$ allow partitions to sums of lower magic numbers which encourages to consider the geometric interpretation as linked nuclei. p-Adic length scale hypothesis in turn suggest the existence of magic numbers coming as powers of 2^3.

What about the description of nuclear reactions?

The identification of nuclei as linked and knotted strings filling the nuclear volume for constant nuclear density leads to a topological description for the nuclear reactions with simplest reactions corresponding to fusion and fission of closed nuclear strings. The microscopic description is in terms of nucleon collisions in which exotic quarks and anti-quarks are re-shared between nucleons and also new pairs are created. The distinction to ordinary string model is that the topological reactions for strings can occur only when the points at which where they are attached to nucleons collide.

The old fashioned description of the nuclear strong force is based on the meson exchange picture. The perturbation theory based on the exchange of pions doesn't however make sense in practice. In the hadronic string model this description would be replaced by hadronic string diagrams. The description of nuclear scattering in terms of nuclear strings allows phenomenological interpretation in terms of stringy diagrams but color bonds between nucleons do not correspond to meson exchanges but are something genuinely new.

7.1.5 Tritium beta decay anomaly

The proposed model explains the anomaly associated with the tritium beta decay. What has been observed [C77, C47] is that the spectrum intensity of electrons has a narrow bump near the endpoint energy. Also the maximum energy E_0 of electrons is shifted downwards.

I have considered two explanations for the anomaly. The original models are based on TGD variants of original models [C34, C91] involving belt of dark neutrinos or antineutrinos along the orbit of Earth. Only recently (towards the end of year 2008) I realized that nuclear string model provides much more elegant explanation of the anomaly and has also the potential to explain much more general anomalies [E15, C57, C89] , [E15] .

7.1.6 Cold fusion and Trojan horse mechanism

Cold fusion [C112] has not been taken seriously by the physics community but the situation has begun to change gradually. There is an increasing evidence for the occurrence of nuclear transmutations of heavier elements besides the production of ^4He and ^3H whereas the production rate of ^{3}He and neutrons is very low. These characteristics are not consistent with the standard nuclear physics predictions. Also Coulomb wall and the absence of gamma rays and the lack of a mechanism transferring nuclear energy to the electrolyte have been used as an argument against cold fusion.

An additional piece to the puzzle came when Ditmire *et al* [C74] observed that the spectrum of electron energies in laser induced explosions of ion clusters extends up to energies of order MeV (rather than 10^2 eV!): this suggests that strong interactions are involved.

The possibility of charged color bonds explaining tetra-neutron allows to construct a model explaining both the observations of Ditmire *et al* and cold fusion and nuclear transmutations. 'Trojan horse mechanism' allows to circumvent the Coulomb wall, and explains various selection rules and the absence of gamma rays, and also provides a mechanism for the heating of electrolyte.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Applications of TGD [L22]
- Geometrization of fields [L26]
- Widom-Larsen theory from TGD point of view [L44]
- Nuclear string model [L33]

7.2 Model for the nucleus based on exotic quarks

The challenge is to understand the strong binding solely in terms of the color bonds and large value of $\hbar$ for valence quarks reducing color coupling strength to v_0 and scaling there sizes to $L(107)/v_0 = L(129)$. There are many questions to be answered. How exotic quarks with scaled down masses differ from dark valence quarks? How the model can be consistent with the known nuclear radii of nuclei if valence quarks have Compton length of order $L(129)$?

7.2.1 The notion of color bond

The basic notion is that of color bond having exotic quark and anti-quark at its ends. Color bonds connecting nucleons make them effectively color charged so that nuclei can be regarded as color bound states of nucleons glued together using color bonds.

The motivation for the notion of color bond comes from the hypothesis that valence quarks are in large $\hbar$ phase, and also from the ideas inspired by the work of Chris Illert [C88] suggesting that long virtual pions act as "rubber bands" connecting nucleons to each other. There are indications that the quark distribution functions for the nucleons inside nuclei differ from those for free nucleons [C124, C42] . QCD based estimates show that color van der Waals force is not involved [C124] . The contribution of the quark pairs associated with color bonds is a possible explanation for this phenomenon.

7.2.2 Are the quarks associated with color bonds dark or p-adically scaled down quarks?

What seems clear is that color bonds with light quark and antiquark, to be referred as exotic quarks in the sequel, at their ends could explain strong nuclear force. Concerning the identification of the exotic quarks there are frustratingly many options. In lack of deeper understanding, the only manner to proceed is to try to make a detailed comparison of various alternatives in hope of identifying a unique internally consistent option.

The basic observation is that if four-momentum is conserved in the phase transition to the dark phase, the masses of quarks in large $\hbar$ phase should not differ from those in ordinary phase, which means that Compton lengths and p-adic length scale are scaled up by a factor $1/v_0$. This assumption explains elegantly cold fusion and many other anomalies [K30, K28] . The quarks at the ends of color bonds must however have scaled down masses to not affect too much the masses of nuclei. This option would also allow to identify valence and possibly also sea quarks as dark quarks in accordance with the general criterion for the transition to dark phase as proposed in the model for RHIC events [K28] .

Exotic quarks must be light. Hence there should be some difference between exotic and valence quarks. This leaves two options to consider.

Are the exotic quarks p-adically scaled down versions of ordinary quarks with ordinary value of $\hbar$?

Exotic quarks could simply correspond to longer p-adic length scale, say M_{127} and thus having masses scaled down by a factor 2^{-10} but ordinary value of $\hbar$. One can also consider the possibility that they correspond to a QCD associated with M_{127} as proposed earlier. They could also correspond to their own weak length scale and weak bosons. This would resolve the objections against new elementary particles coming from the decay widths of intermediate gauge bosons even without assumption about the loss of asymptotic freedom implying that the QCD in question effectively exists only in finite length scale range.

p-Adic mass calculations indeed support the view to that hadronic quarks appear as several scaled up variants and there is no reason to assume that also scaled down variants could not appear. This hypothesis leads to correct order of magnitude estimates for the color magnetic spin-spin interaction energy.

For this option valence (and possibly also sea) quarks could be dark and have color sizes of order $L(k_{eff} = 129)$ as suggested by the criterion $\alpha_s Q_c^2 \simeq 1$ for color confinement as a transition to a dark phase.

Do exotic quarks correspond to large $\hbar$ and reduced c?

If valence quarks are dark one can wonder why not also exotic quarks are dark and whether there exists a mechanism reducing their masses by a factor v_0.

If one questions the assumption that $\hbar$ is a fundamental constant, sooner or later also the question "What about c?" pops up. There are indeed motivations for expecting that c has a discrete spectrum in a well-defined sense. TGD predicts an infinite variety of warped vacuum extremals defining imbeddings of M^4 to $M^4 \times CP_2$ with $g_{tt} = \sqrt{1 - R^2\omega^2}, g_{ij} = -\delta_{ij}$, and if common M^4 time coordinate is used for them the maximal signal velocity is for them given by $c_\#/c = \sqrt{1 - R^2\omega^2}$.

Physically this means that the time taken for light to travel between point A and B depends on what space-time sheet the light travels even in the case that gravitational and gauge fields are absent. The fact that the analog of Bohr quantization occurs for the deformed vacuum extremals of Kähler action suggests that $c_\#$ has a discrete spectrum.

This inspires the question whether also light velocity c besides $\hbar$ is quantized in powers of v_0 so that the rest energies of dark quarks would be given by $E_0 = \hbar_s c_\#/L(k_{eff} = k + 22) = \hbar c_\#/L(k)$ and scale down because of the scaling $c \to v_0 \times c$. A distinction between rest mass and rest energy should be made since rest mass is scaled up as $M \to M/v_0$. Compton time would be by a factor $1/v_0^2$ longer than the ordinary Compton time.

If c and $\hbar$ can scale up separately but in powers of v_0 (or its harmonics and sub-harmonics) it is possible to have a situation in which $\hbar c$ remains invariant because mass scale is reduced v_0 and $\hbar$ is increased by $1/v_0$. In the case of dark quarks this would mean that light would propagate with velocity $2^{-11}c$ along various space-time sheets associated with dark quarks.

This admittedly complex looking option would mean that valence quarks have large $\hbar$ but ordinary c and exotic quarks have large $\hbar$ but small c due to the warping of their space-time sheet in time direction.

7.2.3 Electro-weak properties of exotic and dark quarks

Are exotic quarks scaled down with respect to electromagnetic interactions?

The earlier models involving large $\hbar$ rely on the assumption that the transition to large $\hbar$ phase with respect to electromagnetic interactions occurs only under special conditions (models for cold fusion and structure of water represent basic examples). Hence valence quarks can be in large $\hbar$ phase only with respect to strong and possibly weak interactions.

1. For p-adically scaled down exotic quarks also the electromagnetic space-time sheet should correspond to scaled up value of k since $k = 113$ would give too large contribution to the quark mass. It is not clear whether both em and color space-time sheets can correspond to $k = 127$ or whether one must have $k_{em} = 131$.

2. For exotic quarks with large $\hbar$ and small c the situation can be different $k = 107$ contribution to quark mass is scaled down by v_0 factor: $m_q(dark) = v_0 m_q \sim .05$ MeV. Since $k = 113$ contributes a considerable fraction to hadron mass, one can argue that also the $k = 113$ contribution to the mass must be scaled down so that dark quarks would be also electromagnetically dark. If so, the size of $k = 113$ dark electromagnetic field body would be of order atomic size and nuclei would represent in their structure also atomic length scale.

Are exotic and dark quarks scaled down with respect to weak interactions?

What about darkness of exotic and dark quarks with respect to weak interactions? The qualitative behavior of the binding energies of $A \leq 4$ nuclei can be understood if they possess zero point kinetic energy associated with space-time sheet with size characterized by $L(k = 111 = 3 \times 37) \simeq 10^{-14}$ m. Also the maximal mass number of super-heavy nuclei without neutron halo is predicted correctly. $k_{eff} = 111$ happens to correspond to the scaled weak length scale M_{89} which raises the possibility that dark quarks correspond to large value of $\hbar$ with respect to weak interactions. This could be the case for dark valence quarks and both identifications of exotic quarks.

1. For $k = 127$ quarks with dark weak interactions no large parity breaking effects are induced neither below mass scale m_W.

2. For large $\hbar$-small c option the scale invariance of gauge interactions would mean that the masses of the corresponding weak bosons are of order 50 MeV but the weak interaction rates of are scaled down by a factor v_0^2 since the ratios m_q/m_W invariant under the transition to dark phase appear in the rates: this at energy scale smaller than $v_0 m_w$. This disfavors this option.

7.2.4 How the statistics of exotic and ordinary quarks relate to each other?

Exotic and ordinary quarks should be identical or in some sense effectively identical in order that nuclear string picture would result.

Can one regard exotic quarks and ordinary quarks as identical fermions?

The first guess would be that this is not the case. One must be however cautious. The fact that p-adically scaled up variants of quarks appear in the model of hadrons suggested by p-adic mass calculations, suggests that the scaled up versions must be regarded as identical fermions. Since also the scaling of $\hbar$ induces only a scaling up of length scale, one might argue that this conclusion holds true quite generally.

Identity is also favored by a physical argument. If identity holds true, Fermi statistics forces the nucleons to form closed nuclear strings to maximize their binding energies. The notion nuclear string explains nicely the behavior nuclear binding energies per nucleon and also suggests that linking and knotting could define mechanisms for nuclear binding.

Could dark quarks and ordinary quarks be only effectively identical?

The idea of regarding quarks and dark quarks as identical fermions does not sound convincing, and one can ask the idea could make sense in some effective sense only.

1. The effective identity follows from a model for matter antimatter symmetry assuming that ordinary quarks form strongly correlated pairs with dark anti-quarks so that nucleons would be accompanied by dark antinucleons and quarks and dark quarks would be effectively identical. This option looks however rather science fictive and involves un-necessarily strong assumption.

2. A weaker hypothesis is inspired by the model of topological condensation based on # (/wormhole/ topological sum) contacts [K37] . # contact can be modelled as a CP_2 type extremal with Euclidian signature of induced metric forming topological sum with the two space-time sheets having Minkowskian signature of induced metric. # contact is thus accompanied by two light-like 3-D causal horizons at which the metric determinant vanishes. These causal horizons carry of quantum numbers and are identified as partons. If the contact is passive in the sense that it mediates only gauge fluxes, the quantum numbers of the two partons cancel each other. This can be true also for four-momentum in the case that time orientations of the space-time sheets are opposite.

   This kind of # contacts between $k_{eff} = 129$ and $k = 127$ space-time sheets would force effective identity of $k = 127$ and $k_{eff} = 129$ quarks. The implication would be that in many-sheeted sense nucleons inside nuclei would have ordinary quantum numbers whereas in single sheeted point sense they would carry quantum numbers of quark or anti-quark.

7.3 Model of strong nuclear force based on color bonds between exotic quarks

In this section the color bond model of strong nuclear force is developed in more detail.

7.3.1 A model for color bonds in terms of color flux tubes

Simple model for color bond

Consider next a simple model for color bond.

1. The first guess would be that the color bond has quantum numbers of neutral pion so that also the pair of nucleons connected by a color bond would behave like a pion. This gives attractive color magnetic interaction energy and an attractive identification is as p-n bond.

2. Also the bonds with identical spins and identical color charges at the ends of the bond yield an attractive color magnetic spin-spin interaction energy. This kind of bonds would be responsible for p-p and n-n pairing. In this case color magnetic energy is however by a factor 1/3 smaller and could explain the non-existence of pp and nn bound states. An even number of neutral ρ type bonds could be allowed without anomalous contribution to the spin. High spin nuclei could contain many ρ type bonds so that antimatter would play important role in the physics of heavy nuclei.

3. A further generalization by allowing also electromagnetically charged color bonds with em quantum numbers of pion and ρ would explain tetra-neutron [C51, C22] as alpha particle (pnpn) with two π_- type color bonds. This would predict a rich variety of exotic nuclei. Long color bonds connecting quark and anti-quark attached to different nucleons would also allow to understand the observation of Chris Illert [C88] that the classical description of quantum tunnelling suggests that nucleons at the surface of nucleus have charges which are fractional.

This picture would suggest that the color isospin of the quark at the end of the bond equals to the weak isospin of the nucleon and is also identifiable as the strong isospin of the nucleon inside nucleus. To achieve an overall color neutrality the presence a dark gluon condensate compensating for the net color charge of colored bonds must be assumed. This could also compensate the net spin of the colored bonds.

The surplus of neutrons in nuclei would tend to create a non-vanishing color isospin which could be cancelled by the dark gluon condensate. The results of RHIC experiment [C97] can be understood in TGD framework as a generation of a highly tangled string like structure containing large number of p-p and n-n type bonds and thus also dark gluon condensate neutralizing the net color charge. This would suggest that in a good approximation the nuclei could be seen as tangled string like structures formed from protons and neutrons. If the distances between nuclei are indeed what standard nuclear physics suggests, kind of nuclear strings would be in question.

Simple model for color magnetic flux tubes

Color magnetic flux tubes carrying also color electric fields would define the color magnetic body of the nucleus having size of order $L(129)$. Dark quarks would have also weak and electromagnetic field bodies with sizes $L(111)$ and $L(135)$. The color magnetic body codes information about nucleus itself but also has independent degrees of freedom, in particular those associated with linking and knotting of the flux tubes (braiding plays a key role in the models of topological quantum computation [K94]).

Color flux tubes carry a non-trivial color magnetic flux and one can wonder whether the color flux tubes can end of whether they form closed circuits. Since CP_2 geometry allows homological magnetic charges, color magnetic flux tubes could have ends with quarks and anti-quark at them acting as sources of the color magnetic field. The model for binding energies however favors closed strings. In the general case the color magnetic flux tubes would have a complex sub-manifold of CP_2 with boundary as a CP_2 projection.

The spin-spin interaction energies depend crucially on the value of the color magnetic field strength experienced by the exotic quark at the end of color flux tube, and one can at least try make educated guesses about it. The conservation of the color magnetic flux gives the condition $g_sB \propto 1/S$, where S is the area of the cross section of the tube. $S \geq L^2(107)$ is the first guess for the area if valence quarks are ordinary. $S \geq L^2(k_{eff} = 129)$ is the natural guess if valence quarks are dark.

The quantization of the color magnetic flux using the scaled up value of $\hbar$ would give $\int g_sBdS = n/v_0$ implying $g_sB \simeq n/v_0S$. When applied to $S \sim L^2(107)$ the quantization condition would give quite too large estimate for the spin-spin interaction energy. For $S \sim L^2(129)$ the scale of the interaction energy would come out correctly. For $k = 127$ option $S \sim L^2(127)$ is forced by the quantization condition.

This observation favors strongly dark valence quarks for both options. The magnetic flux of exotic quarks would be feeded to flux tubes of transverse area $\sim L^2(k)$, $k = 127$ or $k = 129$, coupling naturally with the color magnetic flux tubes of valence quarks with size $L(129)$.

A further constraint could come from the requirement that the flux tubes is such that locally the magnetic field looks like a dipole field. This would mean that the flux tube would become thicker at larger distances roughly as $S(r) \propto r^3$. An alternative restriction would come from the requirement that the energy of the color magnetic flux tube is same irrespective of its cross section at dark quark position. This would give $S \propto L$ where L is the length of the flux tube.

Quantum classical correspondence requires color bonds

Non-vacuum extremals are always accompanied by a non-vanishing classical electro-weak and color gauge fields. This is an obvious challenge for quantum classical correspondence. The presence of a suitable configuration of color bonds with dark quarks at their ends starting from nucleon gives hopes of resolving this interpretational problem. Dark quarks and anti-quarks would serve as sources of the color and weak electric gauge fluxes and quarks and nucleons would create the classical em field.

The requirement that classical Abelian gauge fluxes are equal to the quantum charges would pose very strong conditions on the physical states. For instance, quantization condition for Weinberg angle is expected to appear. The fact that classical fluxes are inversely proportional to the inverse of the corresponding gauge coupling strength $1/\alpha_i$ gives additional flexibility and with a proper choice of gauge coupling strengths the conditions might be satisfied and space-time description would also code for the values of gauge coupling strengths. Color bonds should be present in all length scales for non-vacuum extremals encouraging the hypothesis about the p-adic hierarchy of dark QCD type phases.

Identification of dark quarks and valence quarks as identical fermions forces the organization of nucleons to nuclear strings?

Quantum classical correspondence in strong form gives strong constraints on the construction. The model explaining the nuclear binding energies per nucleon strongly favors the option in which nucleons arrange to form closed nuclear strings. If dark quarks and ordinary valence quarks can be regarded as identical fermions this hypothesis follows as a prediction. Therefore this hypothesis, which admittedly looks ad hoc and might make sense only effectively (see the discussion below), deserves a detailed consideration.

Fermi statistics implies that the quark at the end of the color bond must be in a spin state which is different from the spin state of the nucleon (spin of d quark in the case of p=uud and u quark in the case of n) to allow local S-wave. For anti-quarks there are no constraints. Only d (u) quark with spin opposite to that of p (n) can be associated with p (n) end of the color bond. Hence at most five different bonds can begin from a given nucleon. In the case of proton $p_\downarrow$ they are give by $d_\uparrow\overline{d}_\downarrow$, $\overline{q}_\downarrow q_\uparrow$, $q = u, d$.

Only two bonds between given nuclei are possible as following examples demonstrate.

1. $p_\downarrow - n_\uparrow$: $d_\uparrow\overline{d}_\downarrow$, $\overline{u}_\downarrow u_\uparrow$.
2. $p_\downarrow - p_\uparrow$: $d_\uparrow\overline{d}_\uparrow$,$\overline{d}_\uparrow d_{\ uparrow}$.

The experimentation with the rules in case of neutral color bonds supports the view that although branchings are possible, they do not allow more than $A = Z + N$ bonds. One example is 6 nucleon state with p at center connected by 5 bonds to p+ 4n at periphery and an additional bond connecting peripheral p and n. This kind of configuration could be considered as one possible configuration in the case of ^{6}Li and ^{6}He. It would seem that there is always a closed string structure with A bonds maximizing the color magnetic binding energy. The allowance of also charged color bonds makes possible to understand tetra-neutron as alpha particle with two charged color bonds.

The fact that neutron number for nuclei tends to be larger than proton number implies that the number of n-n type ρ bonds for stringy configurations is higher than p-p type bonds so that net color isospin equal equal to $I_3 = -(A - 2Z)$ is generated in case of stringy nuclei and is most naturally cancelled by a dark gluon condensate. Neutralizing gluon condensate allows neutron halo with a non-vanishing value of I_3.

7.3.2 About the energetics of color bonds

To build a more quantitative picture about the anatomy of the color bond it is necessary to consider its energetics. The assumption that in lowest order in $\hbar$ the binding energy transforms as rest energy under the p-adic scaling and scaling of $\hbar$ makes it easy to make order of magnitude estimates by scaling from the hadronic case.

Color field energy of the bond

At the microscopic level the harmonic oscillator description should correspond to the color energy associated with color bonds having u or d type quark and corresponding anti-quarks at their ends. For simplicity restrict the consideration in the sequel to electromagnetically neutral color bonds.

Besides spin-spin interaction energy and color Coulombic interaction energy there are contributions of color fields coded by the string tension $T_d = v_0 T$ of the color bond, where $T \simeq 1/\mathrm{GeV}^2$ is hadronic string tension. The energy of string with given length remains invariant in the combined scaling of $\hbar$, string tension, and length L of the color bond represented by color magnetic flux tube (which contain also color electric fields).

1. The mass of the color bonds between valence quarks assumed to have $\hbar_s = \hbar/v_0$ of length $L = xL(129)$ are given by $M(107) \sim x \times \hbar/L(107) \sim x \times .5$ GeV and correspond naturally to the energy scale of hadronic strong interactions.
2. The rest energy of the color bonds between $k = 127$ quarks with ordinary value of $\hbar$ having length $L = xL(127)$ are given by $M(127) \sim x \times \hbar/L(127) = 2^{-10}M(107)$ so that the order order of magnitude is $x \times .5$ MeV.
3. The rest energy of the color bonds between $k_{eff} = 129$ dark quarks with $c_\# = v_0 c$ is given by the same expression. Note however that rest mass would be scaled up by a factor $1/v_0$.

The resulting picture seems to be in a dramatic conflict with the electromagnetic size of nucleus which favors the $L \sim L(107) < 2$ fm rather than $L \sim L(129)$ and which is smaller by a factor 2^{-11} and which favors also the notion of nuclear string. The resolution of the paradox is based on the notion of color magnetic body. Color bonds behave like color magnetic dipoles and bonds correspond to flux tubes of a topologically quantized dipole type color magnetic field having length of order $L(129) \simeq 5 \times 10^{-12}$ m connecting nucleons at distance $L < L(107)$.

Color magnetic spin-spin interaction energy, the structure of color bonds, and the size scale of the nucleus

Color magnetic spin-spin interaction allows to understand $\rho - \pi$ mass splitting in terms of color magnetic spin-spin interaction expected to give the dominating contribution to the nuclear binding energy. The quantitative formulation of this idea requiring consistency with p-adic mass calculations and with existing view about typical electromagnetic nuclear size scale fixed by the height of Coulomb wall leads to a rather unique picture about color magnetic bonds.

1. Questions

One can pose several questions helping to develop a detailed model for the structure of the color bond.

1. The contributions $k = 113$ and $k = 107$ space-time sheets to the mass squared are of same order of magnitude [K58] . The contributions to the mass squared add coherently inside a given space-time sheet. This requires that nucleonic space-time sheet are not directly connected by join along boundaries bonds and the assumption that color bond connect dark quarks is consistent with this. This means that it makes sense to estimate contributions to the mass squared at single nucleon level.

2. The contribution of color magnetic spin-spin interaction to the mass squared of nucleon can be regarded as coming from $k = 107$ space-time sheets as p-adic contribution but with a large value of $\hbar$. If $k = 107$ contribution would vanish, only a positive contribution to mass would be possible since the real counterpart Δm_R^2 of p-adic Δm^2 is always positive whereas $(m^2 + \Delta m^2)_R < m_R^2$ can hold true.

3. What has been said about color magnetic body and color bonds applies also to electromagnetic field body characterized by $k = 113$. The usual electromagnetic size of nucleus is defined by the relative distances of nucleons in M^4 can be much smaller than $L(113)$ so that the prediction for Coulomb wall is not reduced to the Coulomb potential at distance $L(113)$. Nucleon mass could be seen as due to p-adic thermodynamics for mass squared (or rather, conformal weight) with the real counterpart of the temperature being determined by p-adic length scale $L(113)$.

4. The model inspired by p-adic mass calculations [K58] forced the conclusion that valence quarks have join along boundaries bonds between $k = 107$ and $k = 113$ space-time sheets possibly feeding color fluxes so that closed flux loops between the two space-time sheets result. The counter intuitive conclusion was that roughly half of quark mass is contributed by the $k = 113$ space-time sheet which is by a scale factor 8 larger than the color size of quarks. If valence quarks are dark, scaled up $k = 107$ space-time sheet having $k_{eff} = 129$ becomes the larger space-time sheet, and the situation would not look so counter-intuitive anymore.

5. How the ends of the color bonds are attached to the $k = 113$ nucleon space-time sheets? The simplest assumption is that color bonds correspond to color magnetic flux tubes of length scale $L(129)$ starting at or being closely associated with $k = 107$ space-time sheets of nucleons. Hence the contribution to the mass squared would come from scaled up $k_{eff} = 129$ space-time sheet and add coherently to the dominating p-adic $k = 107$ contribution to the mass squared of nucleon.

6. If exotic quarks are $k = 127$ quarks with ordinary value of $\hbar$, one encounters the problem how their contributions can add coherently with $k_{eff} = 129$ color contribution to reduce the rest energy of nucleus. One possibility allowed by the appearance of harmonics of v_0 is that $\hbar$ is scaled up by $1/(2v_0) \simeq 2^{10}$ so that space-time sheets have same size or that p-adic additivity of mass squared is possible for effective p-adic topologies which do not differ too much from each other.

2. Estimate for color magnetic spin-spin interaction energy

Suppose the scaling invariance in the sense that the binding energies transform in the lowest order just like rest masses so that one can estimate the color magnetic spin-spin splittings from the corresponding splittings for hadrons without any detailed modelling. This hypothesis is very attractive predicts for both options that the scale of color magnetic spin-spin splitting is 2^{-n} times lower than for $\pi - rho$ system, where $n = 10$ for $n = 127$ option and $n = 11$ for $k_{eff} = 129$ option. For scaled down spin-spin interaction energy for π type bond is $E \sim .4$ MeV for $k = 127$ and $\sim .2$ MeV for $k_{eff} = 11$, which would mean that the bond is shorter than scaled up length $L(\pi)$ of color bond between valence quarks of pion.

The further assumption that color magnetic spin-spin interaction energy behaves as $\alpha_s/m_q^2L^3$, $L = x2^nL(\pi)$. This gives $E \simeq x^{-3}2^{-n}E(107)$. The value of x can be estimated from the requirement that the energy is of order few MeV. This gives $x \sim 10^{-1/3}$ for $k = 127$ option and $x \sim (20)^{-1/3}$ for $k_{eff} = 129$ option.

7.4 Model of strong nuclear force based on color bonds between exotic quarks

In this section the color bond model of strong nuclear force is developed in more detail.

7.4.1 A model for color bonds in terms of color flux tubes

Simple model for color bond

Consider next a simple model for color bond.

1. The first guess would be that the color bond has quantum numbers of neutral pion so that also the pair of nucleons connected by a color bond would behave like a pion. This gives attractive color magnetic interaction energy and an attractive identification is as p-n bond.

2. Also the bonds with identical spins and identical color charges at the ends of the bond yield an attractive color magnetic spin-spin interaction energy. This kind of bonds would be responsible for p-p and n-n pairing. In this case color magnetic energy is however by a factor 1/3 smaller and could explain the non-existence of pp and nn bound states. An even number of neutral ρ type bonds could be allowed without anomalous contribution to the spin. High spin nuclei could contain many ρ type bonds so that antimatter would play important role in the physics of heavy nuclei.

3. A further generalization by allowing also electromagnetically charged color bonds with em quantum numbers of pion and ρ would explain tetra-neutron [C51, C22] as alpha particle (pnpn) with two π_- type color bonds. This would predict a rich variety of exotic nuclei. Long color bonds connecting quark and anti-quark attached to different nucleons would also allow to understand the observation of Chris Illert [C88] that the classical description of quantum tunnelling suggests that nucleons at the surface of nucleus have charges which are fractional.

This picture would suggest that the color isospin of the quark at the end of the bond equals to the weak isospin of the nucleon and is also identifiable as the strong isospin of the nucleon inside nucleus. To achieve an overall color neutrality the presence a dark gluon condensate compensating for the net color charge of colored bonds must be assumed. This could also compensate the net spin of the colored bonds.

The surplus of neutrons in nuclei would tend to create a non-vanishing color isospin which could be cancelled by the dark gluon condensate. The results of RHIC experiment [C97] can be understood in TGD framework as a generation of a highly tangled string like structure containing large number of p-p and n-n type bonds and thus also dark gluon condensate neutralizing the net color charge. This would suggest that in a good approximation the nuclei could be seen as tangled string like structures formed from protons and neutrons. If the distances between nuclei are indeed what standard nuclear physics suggests, kind of nuclear strings would be in question.

Simple model for color magnetic flux tubes

Color magnetic flux tubes carrying also color electric fields would define the color magnetic body of the nucleus having size of order $L(129)$. Dark quarks would have also weak and electromagnetic field bodies with sizes $L(111)$ and $L(135)$. The color magnetic body codes information about nucleus itself but also has independent degrees of freedom, in particular those associated with linking and knotting of the flux tubes (braiding plays a key role in the models of topological quantum computation [K94]).

Color flux tubes carry a non-trivial color magnetic flux and one can wonder whether the color flux tubes can end of whether they form closed circuits. Since CP_2 geometry allows homological magnetic charges, color magnetic flux tubes could have ends with quarks and anti-quark at them acting as sources of the color magnetic field. The model for binding energies however favors closed strings. In the general case the color magnetic flux tubes would have a complex sub-manifold of CP_2 with boundary as a CP_2 projection.

The spin-spin interaction energies depend crucially on the value of the color magnetic field strength experienced by the exotic quark at the end of color flux tube, and one can at least try make educated guesses about it. The conservation of the color magnetic flux gives the condition $g_sB \propto 1/S$, where S is the area of the cross section of the tube. $S \geq L^2(107)$ is the first guess for the area if valence quarks are ordinary. $S \geq L^2(k_{eff} = 129)$ is the natural guess if valence quarks are dark.

The quantization of the color magnetic flux using the scaled up value of $\hbar$ would give $\int g_sBdS = n/v_0$ implying $g_sB \simeq n/v_0S$. When applied to $S \sim L^2(107)$ the quantization condition would give quite too large estimate for the spin-spin interaction energy. For $S \sim L^2(129)$ the scale of the interaction energy would come out correctly. For $k = 127$ option $S \sim L^2(127)$ is forced by the quantization condition.

This observation favors strongly dark valence quarks for both options. The magnetic flux of exotic quarks would be feeded to flux tubes of transverse area $\sim L^2(k)$, $k = 127$ or $k = 129$, coupling naturally with the color magnetic flux tubes of valence quarks with size $L(129)$.

A further constraint could come from the requirement that the flux tubes is such that locally the magnetic field looks like a dipole field. This would mean that the flux tube would become thicker at larger distances roughly as $S(r) \propto r^3$. An alternative restriction would come from the requirement that the energy of the color magnetic flux tube is same irrespective of its cross section at dark quark position. This would give $S \propto L$ where L is the length of the flux tube.

Quantum classical correspondence requires color bonds

Non-vacuum extremals are always accompanied by a non-vanishing classical electro-weak and color gauge fields. This is an obvious challenge for quantum classical correspondence. The presence of a suitable configuration of color bonds with dark quarks at their ends starting from nucleon gives hopes of resolving this interpretational problem. Dark quarks and anti-quarks would serve as sources of the color and weak electric gauge fluxes and quarks and nucleons would create the classical em field.

The requirement that classical Abelian gauge fluxes are equal to the quantum charges would pose very strong conditions on the physical states. For instance, quantization condition for Weinberg angle is expected to appear. The fact that classical fluxes are inversely proportional to the inverse of the corresponding gauge coupling strength $1/\alpha_i$ gives additional flexibility and with a proper choice of gauge coupling strengths the conditions might be satisfied and space-time description would also code for the values of gauge coupling strengths. Color bonds should be present in all length scales for non-vacuum extremals encouraging the hypothesis about the p-adic hierarchy of dark QCD type phases.

Identification of dark quarks and valence quarks as identical fermions forces the organization of nucleons to nuclear strings?

Quantum classical correspondence in strong form gives strong constraints on the construction. The model explaining the nuclear binding energies per nucleon strongly favors the option in which nucleons arrange to form closed nuclear strings. If dark quarks and ordinary valence quarks can be regarded as identical fermions this hypothesis follows as a prediction. Therefore this hypothesis, which admittedly looks ad hoc and might make sense only effectively (see the discussion below), deserves a detailed consideration.

Fermi statistics implies that the quark at the end of the color bond must be in a spin state which is different from the spin state of the nucleon (spin of d quark in the case of p=uud and u quark in the case of n) to allow local S-wave. For anti-quarks there are no constraints. Only d (u) quark with spin opposite to that of p (n) can be associated with p (n) end of the color bond. Hence at most five different bonds can begin from a given nucleon. In the case of proton $p_{\downarrow}$ they are give by $d_{\uparrow}\overline{d}_{\downarrow}$, $\overline{q}_{\downarrow}q_{\uparrow}$, $q = u, d$.

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The experimentation with the rules in case of neutral color bonds supports the view that although branchings are possible, they do not allow more than $A = Z + N$ bonds. One example is 6 nucleon state with p at center connected by 5 bonds to p+ 4n at periphery and an additional bond connecting peripheral p and n. This kind of configuration could be considered as one possible configuration in the case of ^{6}Li and ^{6}He. It would seem that there is always a closed string structure with A bonds maximizing the color magnetic binding energy. The allowance of also charged color bonds makes possible to understand tetra-neutron as alpha particle with two charged color bonds.

The fact that neutron number for nuclei tends to be larger than proton number implies that the number of n-n type ρ bonds for stringy configurations is higher than p-p type bonds so that net color isospin equal equal to $I_3 = -(A - 2Z)$ is generated in case of stringy nuclei and is most naturally cancelled by a dark gluon condensate. Neutralizing gluon condensate allows neutron halo with a non-vanishing value of I_3.

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Besides spin-spin interaction energy and color Coulombic interaction energy there are contributions of color fields coded by the string tension $T_d = v_0 T$ of the color bond, where $T \simeq 1/\mathrm{GeV}^2$ is hadronic string tension. The energy of string with given length remains invariant in the combined scaling of $\hbar$, string tension, and length L of the color bond represented by color magnetic flux tube (which contain also color electric fields).

1. The mass of the color bonds between valence quarks assumed to have $\hbar_s = \hbar/v_0$ of length $L = xL(129)$ are given by $M(107) \sim x \times \hbar/L(107) \sim x \times .5$ GeV and correspond naturally to the energy scale of hadronic strong interactions.

2. The rest energy of the color bonds between $k = 127$ quarks with ordinary value of $\hbar$ having length $L = xL(127)$ are given by $M(127) \sim x \times \hbar/L(127) = 2^{-10}M(107)$ so that the order order of magnitude is $x \times .5$ MeV.

3. The rest energy of the color bonds between $k_{eff} = 129$ dark quarks with $c_{\#} = v_0 c$ is given by the same expression. Note however that rest mass would be scaled up by a factor $1/v_0$.

The resulting picture seems to be in a dramatic conflict with the electromagnetic size of nucleus which favors the $L \sim L(107) < 2$ fm rather than $L \sim L(129)$ and which is smaller by a factor 2^{-11} and which favors also the notion of nuclear string. The resolution of the paradox is based on the notion of color magnetic body. Color bonds behave like color magnetic dipoles and bonds correspond to flux tubes of a topologically quantized dipole type color magnetic field having length of order $L(129) \simeq 5 \times 10^{-12}$ m connecting nucleons at distance $L < L(107)$.

Color magnetic spin-spin interaction energy, the structure of color bonds, and the size scale of the nucleus

Color magnetic spin-spin interaction allows to understand $\rho - \pi$ mass splitting in terms of color magnetic spin-spin interaction expected to give the dominating contribution to the nuclear binding energy. The quantitative formulation of this idea requiring consistency with p-adic mass calculations and with existing view about typical electromagnetic nuclear size scale fixed by the height of Coulomb wall leads to a rather unique picture about color magnetic bonds.

1. Questions

One can pose several questions helping to develop a detailed model for the structure of the color bond.

1. The contributions $k = 113$ and $k = 107$ space-time sheets to the mass squared are of same order of magnitude [K58] . The contributions to the mass squared add coherently inside a given space-time sheet. This requires that nucleonic space-time sheet are not directly connected by join along boundaries bonds and the assumption that color bond connect dark quarks is consistent with this. This means that it makes sense to estimate contributions to the mass squared at single nucleon level.

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4. The model inspired by p-adic mass calculations [K58] forced the conclusion that valence quarks have join along boundaries bonds between $k = 107$ and $k = 113$ space-time sheets possibly feeding color fluxes so that closed flux loops between the two space-time sheets result. The counter intuitive conclusion was that roughly half of quark mass is contributed by the $k = 113$ space-time sheet which is by a scale factor 8 larger than the color size of quarks. If valence quarks are dark, scaled up $k = 107$ space-time sheet having $k_{eff} = 129$ becomes the larger space-time sheet, and the situation would not look so counter-intuitive anymore.

5. How the ends of the color bonds are attached to the $k = 113$ nucleon space-time sheets? The simplest assumption is that color bonds correspond to color magnetic flux tubes of length scale $L(129)$ starting at or being closely associated with $k = 107$ space-time sheets of nucleons. Hence the contribution to the mass squared would come from scaled up $k_{eff} = 129$ space-time sheet and add coherently to the dominating p-adic $k = 107$ contribution to the mass squared of nucleon.

6. If exotic quarks are $k = 127$ quarks with ordinary value of $\hbar$, one encounters the problem how their contributions can add coherently with $k_{eff} = 129$ color contribution to reduce the rest energy of nucleus. One possibility allowed by the appearance of harmonics of v_0 is that $\hbar$ is scaled up by $1/(2v_0) \simeq 2^{10}$ so that space-time sheets have same size or that p-adic additivity of mass squared is possible for effective p-adic topologies which do not differ too much from each other.

2. Estimate for color magnetic spin-spin interaction energy

Suppose the scaling invariance in the sense that the binding energies transform in the lowest order just like rest masses so that one can estimate the color magnetic spin-spin splittings from the corresponding splittings for hadrons without any detailed modelling. This hypothesis is very attractive predicts for both options that the scale of color magnetic spin-spin splitting is 2^{-n} times lower than for $\pi - rho$ system, where $n = 10$ for $n = 127$ option and $n = 11$ for $k_{eff} = 129$ option. For scaled down spin-spin interaction energy for π type bond is $E \sim .4$ MeV for $k = 127$ and $\sim .2$ MeV for $k_{eff} = 11$, which would mean that the bond is shorter than scaled up length $L(\pi)$ of color bond between valence quarks of pion.

The further assumption that color magnetic spin-spin interaction energy behaves as $\alpha_s/m_q^2L^3$, $L = x2^nL(\pi)$. This gives $E \simeq x^{-3}2^{-n}E(107)$. The value of x can be estimated from the requirement that the energy is of order few MeV. This gives $x \sim 10^{-1/3}$ for $k = 127$ option and $x \sim (20)^{-1/3}$ for $k_{eff} = 129$ option.

7.5 How the color bond model relates to the ordinary description of nuclear strong interactions?

How the notion of strong isospin emerges from the color bond model? What about shell model description based on harmonic oscillator potential? Does the model predict spin-orbit interaction? Is it possible to understand the general behavior of the nuclear binding energies, in particular the anomalously small binding energies of light nuclei? What about magic numbers? The following discussion tries to answer these questions.

7.5.1 How strong isospin emerges?

The notion of strong isospin is a crucial piece of standard nuclear physics. Could it emerge naturally in the transition to the phase involving dark quarks? Could the transition to color confined phase mean a reduction of color group as isotropy group of CP_2 type extremals representing elementary particles to $U(2)$ identifiable as strong isospin group. Could $U(1)_Y \times U(1)_{I_3}$ or $U(1)_{I_3}$ be identifiable as the Abelian holonomy group of the classical color field responsible for the selection of a preferred direction of strong isospin?

This picture would not mean breaking of the color symmetry at the WCW level where it would rotate space-time surfaces in CP_2 like rigid bodies. Rather, the breaking would be analogous to the breaking of rotational symmetry of individual particles by particle interactions. Strong isospin would correspond to the isotropy group of the space-time surface and the preferred quantization direction to the holonomy group of the induced color gauge field. The topological condensation of quarks and gluons at hadronic and nuclear space-time surfaces would freeze the color rotational degrees of freedom apart from isotropies providing thus the appropriate description for the reduced color symmetries.

Mathematical support for the picture from classical TGD

There is mathematical support for the proposed view and closely relating to the long-standing interpretational problems of TGD.

1. CP_2 holonomy group is identifiable as U(2) subgroup of color group and well as electro-weak gauge group. Hitherto the possible physical meaning of this connection has remained poorly understood. $U(2)$ subgroup as as isotropies of space-time surfaces with $D = 2$-dimensional CP_2 projection, which belongs to a homologically non-trivial geodesic sphere S^2, and defines a sub-theory for which all induced gauge fields are Abelian and a natural selection of a preferred strong isospin direction occurs. Thus one might identify strong isospin symmetry as the $SU(2)$ subgroup of color group acting as the isotropy group of the space-time surface and strong isospin would not correspond to the group of isometries but to space-time isotropies.

2. Color isospin component of gluon field, em field and Z^0 field which corresponds to weak isospin, are proportional to each other for solutions having 2-dimensional CP_2 projection. In

fact, both Z^0 and I_3 component of gluon field are proportional to the induced Kähler form with a positive coefficient. If the proposed quantum classical correspondence for color bonds holds true, this means that the signs of these charges are indeed correlated also for nucleon and quark/ anti-quark. The ratios of these charges are fixed for the extremals for which CP_2 projection is homologically non-trivial geodesic sphere S^2.

3. It is far from clear whether the classical Z^0 field can vanish for any non-vacuum extremals. If this is not the case, dark weak bosons would be unavoidable and strong isospin could be identifiable as color isospin and dark weak isospin. The predicted parity breaking effects need not be easily detectable since dark quarks would be indeed dark matter. An open question is whether some kind of duality holds true in the sense that either color field or vectorial part of Z^0 field could be used to describe the nuclear interaction. This duality brings in mind the $SO(4) \leftrightarrow SU(3)$ duality motivated by the number theoretical vision [K9, K8, K79, K84] .

4. The minimal form of the quantum classical correspondence is that at least the signs of the I_3 and Y components of the color electric flux correlate with the dark quark at the end of color bond and the signs of the Z^0 field and Kähler field correlate with the sign of weak isospin and weak hyper-charge of nucleon. A stronger condition is that these classical gauge fluxes are identical with a proper choice of the values of gauge coupling strengths and that in the case of color fluxes the quark at the end of the bond determines the color gauge fluxes in the bond whereas electromagnetic would distribute freely between the bonds.

Correlation between weak isospin and color isospin

The weakest assumption motivated by this picture would be that the sign of color isospin correlates with the sign of weak isospin so that the quarks at the ends of color bonds starting from nucleon would have color isospin equal to the weak isospin of the nucleon:

$$I_{3,s} = I_{3,w} = I_{3,c} \ .$$

This assumption would allow to interpret the attractive strong interaction between nucleons in terms of color magnetic interaction. p-n bond would be neutral π_0 type color singlet bonds. n-n and p-p bonds would have spin ± 1 and color isospin equal to strong iso-spin of $\rho_\pm$. Note that QCD type color singlet states invariant under $I_3 \to -I_3$ would not be possible. Color magnetic interaction mediated by the pion type color bond would be attractive for p and n since color isospins would be opposite sign but repulsive for pp and nn since color isospins would have same sign. The ρ type color bond with identical spins and color isospins I_3 would generate attractive interaction between identical nucleons. The color magnetic spin-spin interaction energy would be 3 times larger for π type bond so that the formation of deuterium as bound state of p and n and absence of pp and nn bound states might be understood.

It is not possible to exclude charged color bonds, and as will be found, their presence provides an elegant explanation for tetra-neutron [C51, C22] .

7.5.2 How to understand the emergence of harmonic oscillator potential and spin-orbit interaction?

Shell model based on harmonic oscillator potential and spin-orbit interaction provide rather satisfactory model of nuclei explaining among other things magic numbers.

Harmonic oscillator potential as a phenomenological description

It would be a mistake to interpret nuclear harmonic oscillator potential in terms Coulomb potential for the I_3 component of the classical gluon field having color isospin as its source. Interaction energy would have correct sign only for proton+quark/ anti-quark or neutron+quark/ anti-quark at the end of the color bond so that only neutrons or protons would experience an attractive force.

Rather, the harmonic oscillator potential codes for the presence of color Coulombic and color magnetic interaction energies and is thus only a phenomenological notion. Harmonic oscillator potential emerges indeed naturally since the nucleus can be regarded as a collection of nucleons connected by color flux tubes acting rather literally as strings. The expansion the interaction energy around equilibrium position naturally gives a collection of harmonic oscillators. The average force experience by a nucleon is expected to be radial and this justifies the introduction of external harmonic oscillator potential depending on A via the oscillator frequency.

At the deeper level the system could be seen as a tangle formed by bosonic strings represented by magnetic flux tubes connecting $k = 111$ space-time sheets containing dark quarks closely associated with nucleons. The oscillations of nucleons in harmonic oscillator potential induce the motion of dark quark space-time sheets play the role of branes in turn inducing motion of the ends of flux tubes fix the boundary values for the vibrations of the flux tubes. The average force experienced by nucleons around equilibrium configuration is expected to define radial harmonic force. This holds true even in the case of nuclear string.

In this picture $k = 111$ space-time sheets could contain the nucleons of even heaviest nuclei if the nucleon size is taken to be $2L_e(107)/3 \simeq 1.5$ fm. The prediction for the highest possible mass number without neutron halo, which is at radius $2.5 \times L_e(111)$, would be $A = 296$ assuming that nuclear radius is $R = 1.4$ fm. $A = 298$ is the mass number of the heaviest known superheavy nucleus [C85] so that the prediction can be regarded as a victory of the model.

Could conformal invariance play a key role in nuclear physics?

The behavior the binding energies of $A \leq 4$ nucleons strongly suggest that nucleons are arranged to closed string like structure and have thus only two color bonds to the neighboring nucleons in the nuclear string. The thickness of the string at the positions of fermions defines the length scale cutoff defining the minimal volume taken by a single localized fermion characterized by given p-adic prime.

The conformal invariance for the sections of the string defined by color bonds is should allow a deeper formulation of the model in terms of conformal field theory. The harmonic oscillator spectrum for single particle states could be interpreted in terms of stringy mass squared formula $M^2 = M_0^2 + m_1^2 n$ which gives in good approximation

$$M = m_0 + \frac{m_1^2}{2M_0} n \ . \tag{7.5.1}$$

The force constant would be determined by M_0 which would be equal to nucleon mass.

Presumably this would bring to the mind of M-theorist nucleus as a system of A branes connected by strings. The restriction of the wave functional of the string consisting of portions connecting nucleons to each other at the junction points would be induced by the wave functions of nucleons. The bosonic excitations of the color magnetic strings would contribute to color magnetic energy of the string characterized by its string tension. This energy scale might be considerably smaller than the fermionic energy scale determined by the color magnetic spin-spin interaction.

Dark color force as the origin of spin-orbit interaction?

The deviation of the magic numbers associated with protons ($Z = 2, 8, 20, 28, 40, 82$) and neutrons ($A - Z = 2, 8, 20, 28, 50, 82, 126$) from the predictions of harmonic oscillator model provided the motivation for the introduction of the spin orbit interaction V_{L-S} [C124] with the following general form

$$V_{L-S}(r) = \bar{L} \cdot \bar{S} \frac{1}{r} (\frac{dV_s}{dr} + \frac{dV_I}{dr} \bar{\tau}_1 \cdot \bar{\tau}_2) \ . \tag{7.5.2}$$

The interaction implies the splitting of (j, l, s) eigen states so that states $j, l = j \pm \frac{1}{2}$ have different energies. If the energy splitting is large enough, some states belonging to a higher shell come down and combine with the states of the lower shell to form a new shell with a larger magic number. This is what should happen for both proton and neutron single particle states.

The origin of spin-orbit interaction would be the classical color field created by the color isospin in p-p and n-n color bonds and dark gluons compensating the color charge. Spin-orbit interaction results in the atomic physics context from the motion of electrons in the electric field of the nucleus. The moving particle experiences in its rest frame a magnetic field $\overline{B} = \overline{v} \times \overline{E}$, which in the spherically symmetric case can by little manipulations can be cast into the form

$$\overline{B} = \frac{\overline{p}}{m} \times \frac{\overline{r}}{r}\frac{dV}{dr} = \overline{L}\frac{1}{m}\frac{1}{r}\frac{dV}{dr} \ .$$

The interaction energy is given by

$$E = -\overline{\mu} \cdot \overline{B} = -\frac{ge}{2m^2}\overline{S} \cdot \overline{L} \times \frac{1}{r}\frac{dV}{dr} \ . \quad (7.5.3)$$

Here magnetic magnetic moment is expressed in terms of spin using the standard definitions. g denotes Lande factor and equals in good approximation to $g = 2$ for point like fermion.

Classical color field can be assumed to contain only I_3 component and be derivable from spherically symmetric potential. In the recent case the color bonds moving made from two quarks and moving with nuclear string experience the force.

The color magnetic moments of the quark and anti-quark are of same sign in for both ρ and π type bond so that the isospin component of the net color magnetic moment can be written as

$$\overline{\mu}_c = g\frac{g_s}{m_q(dark)}I_3\overline{S} \ . \quad (7.5.4)$$

Here g_s denotes color coupling constant, g is the Lander factor equal to $g = 2$ in ideal case, and m is mass parameter. Since the color bond is color magnetic flux tube attached from its ends to dark quarks it seems that the mass parameter $m_q(dark)$ in the magnetic moment is that of dark quark and should be $m_q(dark) = v_0 m_q$.

An additional factor of 2 is present because both quark and anti-quark of the bond give same contribution to the color moment of the bond. I_3 equals to the strong isospin of the nucleon to which the quark is attached and spin is opposite to the spin of this quark so that a complete correlation with the quantum numbers of the second nucleon results and one can effectively assign the spin orbit interaction with nucleons. The net interaction energy is small for spin paired states. The sign of the interaction is same for both neutrons and protons.

Using the general form of the spin orbit interaction potential in the non-relativistic limit, one can cast the $L - S$ interaction term in a the form

$$V_{L-S}(r) = \frac{16\pi\mu_c}{m_q(dark)}\bar{L} \cdot \bar{S}\frac{1}{r}\frac{dV_{I_3}}{dr} \ . \quad (7.5.5)$$

In the first order perturbation theory the energy change for (j, l, s) eigen state with $l = j + \epsilon\frac{1}{2}$, $\epsilon = \pm 1$ and spherically symmetric electromagnetic gauge potential $V(r)$ [B21] given by

$$\Delta E(j, l = j + \epsilon\frac{1}{2}) = \frac{4g_s^2}{m_q^2(dark)L^3}(N - P)c(l)\left[\epsilon(j + \frac{1}{2}) + 1\right] \ ,$$

$$c(l) = -\frac{4\pi L^3}{g_s(N - P)}\langle l|\frac{1}{r}\frac{dV_{I_3}}{dr}|l\rangle \ . \quad (7.5.6)$$

The coefficient $g_s/4\pi$ and factor $N - P$ have been extracted from the color gauge potential in the expression of $a(l)$ to get a more kinematical expression. $N - P$-proportionality is expected since the system has net nuclear color isospin proportional to $N - P$ neutralized by dark gluons which can be thought of as creating the potential in which the nuclear string moves. The constant $c(l)$ contains information about the detailed distribution of the color isospin. $c(l)$ depends also on the details of the model (the behavior of single particle radial wave function $R_{n,l}(r)$ in case of wave mechanical model and now on its analog defined by the wave function of nucleon induced by nuclear string). R denotes the nuclear charge radius.

The general order of magnitude of L is $L \sim L_e(129)$. What comes in mind first is the scaling $L \sim v_0^{-1}R_0$, $R_0 \sim (3/5) \times L_e(107) \simeq 1.5 \times 10^{-15}$ m. This is not consistent with the fact that for light nuclei with $A \leq 4$ L decreases with A but conforms with the fact that spin-spin interaction energies which are very sensitive to L can depend only slightly on A so that L must be more or less independent on A. Assume $g_s^2/4\pi = .1$ and $m_q = m_u \sim .1$ GeV, $g = 1$ in the formula for the color magnetic moment. By using $2\pi/L_e(107) \simeq .5$ GeV these assumptions lead to the estimate

$$\begin{aligned} \Delta E(j, l = j + \epsilon\frac{1}{2}) &= \frac{4g_s^2}{m_q^2 L^3(107)}(\frac{5}{3})^3 \times v_0 \times (N-P) \times c(l) \times \left[\epsilon(j+\frac{1}{2})+1\right] \\ &\simeq (\epsilon(j+\frac{1}{2})+1) \times (N-P) \times c(l) \times 2.9 \ \ MeV \ \ . \end{aligned} \tag{7.5.7}$$

The splitting is predicted to be same for protons and neutrons and also the magnitude looks reasonable. If the dark gluons are at the center and create a potential which is gradually screened by the dark quark pairs, the sign of the spin-orbit interaction term is correct meaning that the contribution to binding energy is positive for $j + 1/2$ state. In the case of neutron halo the unscreened remainder of the dark gluon color charge would define $1/r$ potential at the halo possibly responsible for the stability.

This estimate should be compared to the general estimate for the energy scale in the harmonic oscillator model given by $\omega_0 \simeq 41 \cdot A^{-1/3}$ MeV [C124] so that the general orders of magnitude make sense.

7.5.3 Binding energies and stability of light nuclei

Some examples are in order to see whether the proposed picture might have something to do with reality.

Binding energies of light nuclei

The estimate for the binding energies of light nuclei is based on the following assumptions.

1. Neglect the contribution of the string tension and dark gluon condensate to the binding energy.

2. Suppose that the number of bonds equals to A for $A \leq 4$ nuclei and that the the bonds are arranged to maximize color magnetic spin-spin interaction energy. A possible interpretation is in terms of a closed color magnetic flux tube connecting nucleons. The presence of close color magnetic flux tubes is necessary unless one allows homological color magnetic monopoles. This option favors the maximization of the number of n-p type bonds since their spin-spin interaction energy is 3 times higher than that for p-p and n-n type bonds. This is just a working hypothesis and would mean that nuclei could be seen as nuclear strings.

   The alternative interpretation is that the number bonds per nucleon is constant so that the binding energy would not depend on nucleon. The number of bonds could be quite large. Scaling the c quark mass of about 4 GeV gives gives dark mass of about 2 MeV so that two dark generations might be possible. For two dark quark generations 8+8 different quarks can appear at the ends of color flux tubes and 64 different color bonds are in principle possible (which brings in mind the idea of nuclear genetic code and TGD proposal for quantum computation utilizing braided flux tubes!). Also in this case the bond energy can depend on whether p or n is question for $P \neq N$ nuclei since p-p and n-n bonds have smaller bind energy than p-n type bonds.

3. Assume that the nucleons are topologically condensed at $k = 111$ space-time sheet with zero point kinetic energy

$$E_0(A) \sim \frac{3n}{2} \frac{\pi^2}{A m_p L^2(111)} \equiv \frac{n}{A} \times E_0(A=1) \ \ ,$$

   where n is a numerical factor and $E_0(A = 1) \simeq 23$ MeV. Let ΔE denote the color magnetic spin-spin interaction energy per nucleon for π type bond. The zero point kinetic energy is largest for $A \leq 3$ and explains why the binding energy is so small. For $n = 1$ the zero point kinetic energy would be 5.8 MeV for $A = 4$, 7.7 MeV for $A = 3$, and 11.5 MeV for $A = 2$.

With these assumptions the binding energy per bond can be written for $A \leq 4$ as

$$E = r \times \Delta E - \frac{nE_0(p)}{A^2} \quad ,$$

where Δ denotes the color magnetic spin-spin interaction energy per bond. The parameter r codes for the fact that color magnetic spin-spin interaction energy depends on whether p-p or n-n type bond is in question. The values of r are $r(^4He) = 1$,$r(^3He) = 7/9$, $r(^2H) = 1$.

Estimates for n and Δ can be deduced from the binding energies of ^2H and ^{3}He . The result is $n = 1.0296$ and $\Delta E = 7.03$ MeV. The prediction for ^{4}He biding energy is 6.71 MeV which is slightly smaller than the actual energy 7.07 MeV. The value of the binding energy per nucleon is in the range 7.4-8.8 MeV for heavier nuclei which compares favorably with the prediction 7.66 MeV at the limit $A \rightarrow \infty$. The generation of dark gluon condensate and color Coulomb energy per nucleon increasing with the number of nucleons could explain the discrepancy.

(A,Z)	(2,1)	(3,1)	(3,2)	(4,2)
E_B/MeV	1.111	2.826	2.572	7.0720

Table 1. The binding energies per nucleon for the lightest nuclei.

Why certain light nuclei do not exist?

The model should also explain why some light nuclei do not exist. In the case of proton rich nuclei electromagnetic Coulomb interaction acts as un-stabilizer. For heavy nuclei with non-vanishing value of $P-N$ the positive contribution of dark gluons to the energy tends to in-stabilize the nuclei. The color Coulombic interaction energy is expected to behave as $(N-P)^2$ whereas the energy of dark gluons behaves as $|N-P|$. Hence one expects that for some critical value of $|N-P|$ color Coulombic interaction is able to compensate the contribution of dark gluon energy. One the other hand, the larger number of nn type bonds tends reduced the color magnetic spin-spin interaction energy.

1. Coulomb repulsion for pp is estimated to be .76 MeV from ^{3}He -^{3}H mass difference whereas the color magnetic binding energy would be $E_D/3 = .74$ MeV from the fact that the energy of ρ type bond is 1/3 from that for π type bond. Hence pp bound state would not be possible. The fact that nn bound state does not exist, suggests that the energy of the color neutralizing dark gluon overcomes the color Coulombic interaction energy of dark gluon and dark quarks and spin-spin interaction energy of ρ_0 type bond.

2. For ppp and nnn protons cannot be in S wave. The color magnetic bond energy per nucleon would be predicted to be $E_D = 2.233$ MeV whereas a rough order of magnitude estimate for Coulombic repulsion as

$$E_{em} = Z(Z-1) \times \left[E_B(^3H) - E_B(^3He)\right] = Z(Z-1) \times .76 \ \ MeV$$

gives $E_{em} \simeq 4.56$ MeV so that ppp bound state is not possible. nnn bound state would not be possible because three dark gluons would not be able to create high enough color Coulomb interaction energy E_c which together with color magnetic spin-spin interaction energy E_D would compensate their own negative contribution $3E_g$:

$$3E_g > E_D + E_c \quad .$$

3. For pppp and nnnn Fermi statistics forces two nucleons to higher partial waves so that the states are not stable. Tetraneutron need not correspond to nnnn state in TGD framework but has more natural interpretation as an alpha particle containing two negatively charged dark quark pairs.

7.5.4 Strong correlation between proton and neutron numbers and magic numbers

The estimates for the binding energies suggest that nucleons arrange into closed nuclear strings in which nucleons are connected by long color magnetics with one dark quark anti-quark pair per nucleon. Nuclear string approach allows to understand the strong correlation between proton and neutron numbers as well as magic numbers.

Strong correlation between Z and N

$N = Z$ nuclei with maximal color magnetic spin-spin interaction energy arranged into closed nuclear strings contain only colored π type bonds between p and n and should be especially stable. The question is how to create minimum energy configurations with $N \neq Z$.

1. If only stringy configurations are allowed, the removal of the proton would create ρ type n-n bond and lead to a reduction of binding energy per nucleon. This would predict that (Z,Z) type isotopes correspond to maxima of binding energy per nucleon. The increase of the Coulombic energy disfavors the removal of neutrons and addition of protons.

2. For a given closed string structure one can always link any given proton by $\overline{u}u$ bond to neutron and by $\overline{d}d$ bonds to two protons (same for neutron). The addition of only neutron to a branch from proton gives nuclei (Z,N=Z+k), $k = 1, ..., Z$, having only π type bonds. In a similar manner nuclei with $(Z+k, Z)$, $k = 1, .., Z$, containing only π type bonds are obtained. This mechanism would predict isotopes in the ranges (Z,Z)-(Z,2Z) and (Z,Z)-(2Z,Z) with the same strong binding energy per nucleon apart from increase of the binding energy caused by the generation of dark gluon condensate which in the case of protons seems to be overcome by Coulomb repulsion. Very many of these isotopes are not observed so that this mechanism is not favored.

Consider how this picture compares with experimental facts.

1. Most $Z = N$ with $Z \leq 29$ nuclei exist and are stable against strong decays but can decay weakly. The interpretation for the absence of $Z > 29$ $Z = N$ nuclei would be in terms of Coulomb repulsion. Binding energy per nucleon is usually maximum for $N = Z$ or $N = Z+1$ for nuclei lighter than Si. The tendency $N > Z$ for heavier nuclei could be perhaps understood in terms of the color Coulombic interaction energy of dark gluon condensate with color charges in n-n type color bonds. This would allow also to understand why for $Z = 20$ all isotopes with $(Z = 20, N > 20)$ have higher binding energy per nucleon than $(Z = 20, N = 20)$ isotope in conflict with the idea that doubly magic nucleus should have a maximal binding energy.

   The addition of neutrons to ^{40}Ca nucleus, besides increasing the binding energy per nucleon, also decreases the charge radius of the nucleus contrary to the expectation that the radius of the nucleus should be proportional to $A^{1/3}$ [C124] . A possible interpretation is in terms of the color Coulombic interaction energy due to the generation of dark gluon condensate, the presence of which reduces the equilibrium charge radius of the nucleus.

2. ^{8}Be having $(Z, N) = (4, 4)$ decaying by alpha emission (to two alpha particles) is an exception to the rule. The binding energy per nucleon 7.0603 MeV of Be is slightly lower than the binding energy 7.0720 MeV of alpha particle and the pinching of the Be string to form two alpha strings could be a possible topological decay mechanism.

Magic nuclei in shell model and TGD context

Spin-spin pairing for identical nucleons in the harmonic oscillator potential is an essential element of the harmonic oscillator model explaining among other things shell structure and lowest magic numbers 2, 8, 20 but failing for higher magic numbers 28, 50, 82, 126 (the prediction is 2,8,20 and 40, 68, 82, 122). Spin-orbit coupling [C100] reproduces effectively the desired shell structure by drawing some states of the higher shell to the lower shell, and it is indeed possible to reproduce the magic numbers in this manner for 3-D harmonic oscillator model.

This picture works nicely if magic nuclei are identified as nuclei which have exceptionally high abundances. ^{28}Fe, the most abundant element, is however an exception to the rule since neither Z nor N are magic in this case. The standard explanation for the stable nuclei of this kind is as endpoints of radioactive series. This explanation does not however remove the problem of understanding their large binding energy, which is after all what matters.

The surprise of recent years has been that even for neutron rich unstable nuclei 28 appears as a magic number for unstable neutrons in very neutron rich nuclei such as Si(14,28) [C39] so that the notion of magic number does not seem to be so dependent on spin-orbit interactions with the nuclear environment as believed. Also new magic numbers such as N=14,16,30,32 have been discovered in the neutron sector [C39] . Already the stable isotope Mg(12,14) has larger binding energy per nucleon than doubly magic Mg(12,12) and could be perhaps understood in terms of dark gluons. ^{56}Fe and ^{58}Fe correspond to N=30 and 32. The linking of two N=8 magic nuclei would give N=16 and various linkings of N=14 and N=16 nuclei would reduce the stability N = 28, 30, 32 magic nuclei to the stability of their building blocks. Perhaps these findings could provide motivations for considering whether the stringy picture might provide an alternative approach to understanding of the magic numbers.

1. *The identification of magic nuclei as minima of binding energy predicts new magic numbers*

The identification of the magic nuclei as minima of the binding energy as function of Z and N provides an alternative definition for magic numbers but this would predict among other things that also $Z = N = 4, 6, 12$ also correspond to doubly magic nuclei in the sense that $E_b(^8Be) = 7.0603)$, $E_b(^{12}C) = 7.677$ MeV and $E_B(^{24}Mg) = 8.2526$ are maxima for the binding energy per nucleon as a function of Z and N. For higher nuclei addition of neutrons to a doubly magic nucleus typically increases the binding energy up to some critical number of added neutrons (the generation of the dark gluon condensate would explain this in TGD framework). The maximum for the excitation energy of the first excitation seems to be the definition of magic in the shell model.

2. *Platonic solids and magic numbers*

The TGD picture suggest that light magic nuclei could have a different, purely geometric, interpretation in terms of five regular Platonic solids. $Z = N = 4, 6, 8, 12, 20$ could correspond to tetrahedron, octahedron (6 vertices), hexahedron (8 vertices), dodecahedron (12 vertices), and icosahedron (20) vertices. Each vertex would contain a bonded neutron and proton in the case of doubly magic nucleus. This model would predict correctly all the maxima of the binding energy per nucleon for $Z, N \leq 20$.

3. *p-Adic length scale hypothesis and magic numbers*

$Z = N = 8$ could be also interpreted as a maximal number of nucleons which $k = 109$ space-time sheet associated with dark quarks can contain. p-Adic length scale hypothesis would suggest that strings with length coming as p-adic length scale $L_e(k)$ are especially stable. Strings with thickness $L_e(109)$ would correspond to Z=N=2 for length $L = L_e(109)$, $L_e(k = 109 + 2n)$ would correspond to $Z = 2^{n+1}$ explaining $N = 2, 8, 16, 32$.

4. *Could the linking of magic nuclei produce new magic nuclei?*

Nuclear strings can become knotted and linked with fermion statistics guaranteeing that the links cannot be destroyed by a 3-dimensional topological transition.

An interesting question is whether the magic numbers $N = 14, 16, 30, 32$ could be interpreted in terms of lower level magic numbers: 14=8+6,16=8+8, 30=16+40,32=16+16. This would make sense if $k = 111$ space-time sheets containing $Z, N \leq 4, 6, 8$ neutrons and protons define basic nucleon clusters forming closed nuclear strings. The linking these structures could give rise to higher magic nuclei whose stability would reduce that of the building blocks, and it would be possible to interpret magic number $Z, N = 28 = 20 + 8$ as linked lower level magic nuclei.

The partitions 28=20+8, 50=20+2+28 = 20+2+8+20, 82=50+2+28, 126=50+50+20+6=82+28+8+8 inspire the question whether higher doubly magic nuclei and their deformations could correspond to linked lower level magic nuclei so that a linking hierarchy would result.

Could the transition to the electromagnetically dark matter cause the absence of higher shells?

Spin orbit coupling explains the failure of the shell model as an explanation of the magic numbers. Transition to electromagnetic dark matter at critical charge number $Z = 12$ suggests an alternative explanation for the failure in the case of protons. The phase transition of Pd nuclei (Z=46) to electromagnetically dark nuclear phase inducing in turn the transition of D nuclei to dark matter phase has been proposed as an explanation for cold fusion [K28] .

On basis of $Z^2\alpha_{em} \simeq 1$ criterion $Z = 12$ would correspond to the critical value for the nuclear charge causing this transition. One can argue that due to the Fermi statistics nuclear shells behave as weakly interacting units and the transition occurs for the first time for $Z = 20$ nucleus, which corresponds to Ca, one of the most important ions biologically and neurophysiologically. These necessarily completely filled structures would become structural units of nuclei at electromagnetically dark level.

An alternative interpretation is that the criterion to dark matter phase applies only to a pair of two systems and reads thus $Z_1Z_2\alpha_{em} \simeq 1$ implying that only the nuclei $Z, N \geq 40$ can perform the transition to the dark phase (what this really means is an interesting question). This would explain why Pd with $Z = 46$ has so special role in cold fusion.

Interestingly, the number of protons at $n = 2$ shell of harmonic oscillator is $Z = 12$ and thus corresponds to a critical value for em charge above which a transition to an electromagnetic dark matter phase increasing the size of the electromagnetic $k = 113$ space-time sheet of nucleus by a factor $\simeq 2^{11}$ could occur. This could explain why $n = 2$ represents the highest allowed harmonic oscillator shell with higher level structures consisting of clusters of $n < 3$ shells. Neutrons halos could however allow higher shells.

Could only the hadronic space-time sheet be scaled up for light nuclei?

The model discussed in this chapter is based on guess work and leaves a lot of room for different scenarios. One of them emerged only after a couple of months finishing the work with this chapter.

1. Is only the $\hbar$ associated with hadronic space-time sheet large?

The surprising and poorly understood conclusion from the p-adic mass calculations was that the p-adic primes characterizing light quarks u,d,s satisfy $k_q < 107$, where $k = 107$ characterizes hadronic space-time sheet [K58] .

1. The interpretation of $k = 107$ space-time sheet as a hadronic space-time sheet implies that quarks topologically condense at this space-time sheet so that $k = 107$ cannot belong to the collection of primes characterizing quark.

2. Since hadron is expected to be larger than quark, quark space-time sheets should satisfy $k_q < 107$ unless $\hbar$ is large for the hadronic space-time sheet so that one has $k_{eff} = 107+22 = 129$. This would predict two kinds of hadrons. Low energy hadrons consists of u, d, and s quarks with $k_q < 107$ so that hadronic space-time sheet must correspond to $k_{eff} = 129$ and large value of $\hbar$. One can speak of confined phase. This allows also $k = 127$ light variants of quarks appearing in the model of atomic nucleus. The hadrons consisting of c,t,b and the p-adically scaled up variants of u,d,s having $k_q > 107$, $\hbar$ has its ordinary value in accordance with the idea about asymptotic freedom and the view that the states in question correspond to short-lived resonances.

This picture is very elegant but would mean that it would be light *hadron* rather than quark which should have large $\hbar$ and scaled up Compton length. This does not affect appreciably the model of atomic nucleus since the crucial length scales $L_e(127)$ and $L_e(129)$ are still present.

2. Under what conditions quarks correspond to large $\hbar$ phase?

What creates worries is that the scaling up of $k = 113$ quark space-time sheets of quarks forms an essential ingredient of condensed matter applications [K30] assuming also that these scaled up space-time sheets couple to scaled up $k = 113$ variants of weak bosons. Thus one must ask under what conditions $k = 113$ quarks, and more generally, all quarks can make a transition to a dark phase accompanying a simultaneous transition of hadron to a doubly dark phase.

The criterion for the transition to a large $\hbar$ phase at the level of valence quarks would require that the criticality criterion is satisfied at $k = 111$ space-time sheet and would be expressible as $Z^2\alpha_{em} = 1$ or some variant of this condition discussed above.

The scaled up $k = 127$ quark would correspond to $k = 149$, the thickness of the lipid layer of cell membrane. The scaled up hadron would correspond to $k = 151$, the thickness of cell membrane. This would mean that already the magnetic bodies of hadrons would have size of cell membrane thickness so that the formation of macroscopic quantum phases would be a necessity since the average distance between hadrons is much smaller than their Compton length.

7.5.5 A remark about stringy description of strong reactions

If nucleons are arranged into possibly linked and knotted closed nuclear strings, nuclear reactions could be described in terms of basic string diagrams for closed nuclear strings.

The simplest fusion/fission reactions $A_1 + A_2 \leftrightarrow (A_1 + A_2)$, $A_i > 2$, could correspond to reactions in which the $k = 111$ dark space-time sheets fuse or decay and re-distribution of dark quarks and anti-quarks between nucleons occurs so that system can form a new nucleus or decay to a new nuclei. This also means re-organizes the linking and knotting of the color flux tubes.

The reactions $p/n+A \rightarrow ..$ would involve the topological condensation of the nucleon to $k = 111$ space-time sheet after which it can receive quark anti-quark pair, which can be also created by dark gluon emission followed by annihilation to a dark quark pair.

7.5.6 Nuclear strings and DNA strands

Nuclear strings consisting of protons and neutrons bring in mind bit arrays. Their dark mirror counterparts in turn brings in mind the structure of DNA double strand. This idea does not look so weird once one fully accepts the hierarchies associated with TGD. The hierarchy of space-time sheets quantified by p-adic fractality, the hierarchy of infinite primes representable as a repeated second quantization of a super-symmetric arithmetic quantum field theory, the self hierarchy predicted by TGD inspired theory of consciousness, the Jones inclusion hierarchy for von Neumann factors of type II_1 appearing in quantum TGD and allowing to formulate what might be called Feynman rules for cognition, and the hierarchy of dark matters would all reflect the same reflective hierarchy.

The experience with DNA suggests that nuclear strings could form coiled tight double helices for which only transversal degrees of freedom would appear as collective degrees of freedom. DNA allows a hierarchy of coilings and DNA molecules can also link and this could happen also now. Nuclei as collections of linked nuclear strings could perhaps be said to code the electromagnetic and color field bodies and it is difficult to avoid the idea that DNA would code in the same manner field bodies at which matter condenses to form much larger structures. The hierarchy of dark matters would give rise to a hierarchy of this kind of codings.

The linking and knotting of string like structures is the key element in the model of topological quantum computation and the large value of $\hbar$ for dark matter makes it ideal for this purpose. I have already earlier proposed a model of DNA based topological quantum computation inspired by some strange numerical co-incidences [K94] . If dark matter is the essence of intelligent and intentional life at the level of molecular physics, it is difficult to see how it could not serve a similar role even at the level of elementary particle physics and provide kind of zoomed up "cognitive" representation for the ordinary matter.

The precise dark-visible correspondence might fail at the level of nuclei and nucleons because the lifetimes of the scaled down dark matter nucleons and nuclei are different from those of ordinary nucleons if dark matter is dark also with respect to weak interactions. The weak interaction rates in the lowest order are scaled up by the presence of $1/m_W^4$ factors by a factor 2^{-44} so that weak interactions are not so weak anymore. If dark electron and neutrino have their ordinary masses, dark proton and neutron would be stable. If also they appear as scaled down versions situation changes, but only a small change of the mass ratio of dark proton and neutron can make the weak decay of free dark neutron impossible kinematically and the one-to-one correspondence would make sense for stable nuclei. The beta decays of dark nuclei could however as a third order process with a considerable rate and change dramatically the weak decay rates of dark nuclei.

7.6 Neutron halos, tetra-neutron, and "sticky toffee" model of nucleus

Neutron halos and tetra neutron represent two poorly understood features of nuclear physics which all have been seen as suggesting the existence of an unknown long range force or forces.

7.6.1 Tetraneutron

There is evidence for the existence of tetra-neutrons [C51] . Standard theory does not support their existence [C22] so that the evidence for them came as a complete experimental surprise. Tetra-neutrons are believed to consist of 4 neutrons. In particular their lifetime, which is about 100 nanoseconds, is almost an eternity in the natural time scale of nuclear physics. The reason why the existing theory of nuclear force does not allow tetra-neutrons relates to Fermi statistics: the second pair of neutrons is necessarily in a highly energetic state so that a bound state is not possible.

Exotic quarks and charged color bonds provide perhaps the most natural explanation for tetra-neutron in TGD framework. In the model discussed hitherto only electromagnetically neutral color bonds have been considered but one can consider also charged color bonds in analogy allowing instead of neutral π and ρ also their charged companions. This would make possible to construct from two protons and neutrons the analog of alpha particle by replacing two neutral color bonds with negatively charged bonds so that one would have two $\overline{u}d$ p-n bonds and two $\overline{u}u$ p-n bonds. Statistics difficulty would be circumvented and the state would decay to four neutrons via W boson exchange between quark of charged p-n bonds and protons. The model suggests the existence of also neutral variant of deuteron.

One can consider two options according to whether the exotic quarks have large $\hbar$ but small c (Option II) or whether they are just p-adically scaled up quarks with $k = 127$ (Option I). I have considered earlier a model analogous to option II but based on the hypothesis about existence of scaled down variant of QCD associated with Mersenne prime M_{127}. The so called lepto-hadron physics would also be associated with M_{127} and involve colored excitations of leptons [K88] which might also represent dark matter: in this case dark valence leptons with color would correspond to $keff = 149$, which happens to correspond to the thickness of the lipid layer of cell membrane.

The notion of many-sheeted space-time predicts the possibility of fractal scaled up/down versions of QCD which, by the loss of asymptotic freedom, exist only in certain length scale range and energy range. Thus the prediction does not lead to contradictions elementary particle physics limits for the number of colored elementary particles. The scaled up dark variants of QCD like theory allow to circumvent these problems even when asymptotic freedom is assumed.

In particular, pions and other mesons could exist for $k = 127$ option as scaled down versions having much smaller masses. This lead to the earlier model of tetra-neutron as an ordinary alpha particle bound with two exotic pions with negative charges and having very small masses. This state looks like tetra-neutron and decays to neutrons weakly. The statistics problem is thus circumvented and the model makes precise quantitative predictions.

7.6.2 The formation of neutron halo and TGD

One counter argument against TGD inspired nuclear model is the short range of the nuclear forces: the introduction of the p-adic length scale $L(113) \simeq 1.6E^{-14}$ m is in conflict with this classical wisdom. There exists however direct evidence for the proposed length scale besides the evidence from the p-n low energy scattering. Some light nuclei such as ^8He, ^{11}Li and ^{11}Be possess neutron halo with radius of size $\sim 2.5E^{-14}$ m [C84] . The width of the halo is rather large if the usual nuclear length scale is used as unit and the neutrons in the halo seem to behave as free particles. The short range of the nuclear forces makes it rather difficult to understand the formation of the neutron halo although the existing models can circumvent this difficulty. The proposed picture of the nucleus suggests a rather simple model for the halo.

For ordinary nuclei the densities of nucleons tend to be concentrated near the center of the nucleus. One can however consider the possibility of adding nucleons in vicinity of the boundary of the $k = 111$ space-time sheet associated with the nucleus itself. The binding force would be color interaction between the color charges of color bonds and neutralizing color charge of colored gluons in the center (or in halo itself). Neutron halo would define a separate nucleus in the sense that states could be constructed by starting from the ground state. Halo would correspond to a quantum de-localized cluster of size of alpha particle.

The case ^{11}Be provides support for the theory. Standard shell model suggests that six neutrons of ^{11}Be fill completely $1s_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ states while $1p_{\frac{1}{2}}$ state holds one neutron so that ^{11}Be ground state has $J^\pi = \frac{1}{2}^-$ whereas experimentally ground state is known to have $J^\pi = \frac{1}{2}^+$. The system can be regarded as ^{10}Be + halo neutron. The first guess is that the state could be simply of the following form

$$|0^+\rangle \times |2s_{1/2}\rangle \ . \tag{7.6.1}$$

Color force would stabilize this state. A more general state is a superposition of higher $ns_{1/2}$ states in order to achieve more sharp localization near boundary. This increases the kinetic energy of the neutron and the small binding energy of the halo neutron about 2.5 MeV implies that the kinetic energy should be of order $5-6$ MeV. For instance, in the model described in [C114] the halo neutron property and correct spin-parity for ^{11}Be can be realized if the state is superposition of form

$$\begin{aligned} |^{11}Be\rangle &= a|0^+\rangle \times |2s_{1/2}\rangle + ba|2^+\rangle \times |21d_{5/2}\rangle \ , \\ a &\simeq .74 \ , \\ b &\simeq .63 \ . \end{aligned} \tag{7.6.2}$$

The correlation between the core and and halo neutron is necessary in the model of Otsuka [C114] to produce bound $1/2^+$ state. The halo neutron must also rotate.

The second example is provided by two-neutron halo nuclei, such as ^{11}Li and ^{12}Be, which do not bind single neutron but bind two neutrons. This looks mysterious since free neutrons do not allow bound states. A possible explanation is that the increase of the color Coulomb energy of neutron color bonds with at least N-P dark gluons makes possible binding of neutron halo to the center nucleus. The situation would be analogous to the formation of planetary system. Order of magnitude estimate for color Coulomb energy of halo neutron is $E \sim (N-P)\alpha_s/L(113) \simeq (N-P) \times .8$ MeV. For $N-P=3$ the binding energy would be about 2.3 MeV and smaller than the experimental estimate 2.5 MeV. For $N-P=4$ this gives 3.2 MeV and larger than 2.5 MeV so that there is some room for the reduction of binding energy by the contribution from kinetic energy.

7.6.3 The "sticky toffee" model of Chris Illert for alpha decays

Chris Illert [C88] has proposed what he calls "sticky toffee" model of alpha decay. The starting point of the work is a criticism of the wave-mechanical model for alpha decay of nuclei as occurring through tunnelling. The proposal is that tunnelling might allow a classical particle description after all. Quantum classical correspondence suggests the same in TGD framework.

The proposed description is based on the idea that the tunnelling alpha particle has abnormally small charge inside the tunnelling region. This reduces the electrostatic interaction of alpha particle with nucleus so that it can penetrate to otherwise classically non-allowed region separating it from the external world and can leak out of the parent nucleus. More quantitatively, the momentum given by $p = \sqrt{2m(E-V)}$ of alpha particle remains real during tunnelling. As the alpha particle escapes, it gradually increases its charge to its full value of 2 units possessed by the ordinary alpha particle.

What is interesting is that the model predicts the charge of the proto-alpha particle at the surface of the decaying nucleus from the knowledge of alpha particle energy, nuclear radius, and charge by using just energy conservation in Coulomb field. What is assumed that the charge of the particle is such that Coulomb energy remains equal to the alpha particle energy all the way from the nuclear surface through the Coulomb wall to the distance where alpha particle can have full charge. This is a slight idealization since it would mean that the alpha particle kinetic energy vanishes.

To my opinion, the dynamical charge of alpha particle is a manner to articulate what happens in the tunnelling. Thus the model cannot replace quantum description but only become a part of it. In particular, the successful prediction of the decay rates exponentially sensitive to the alpha particle energy cannot be deduced from a purely classical theory.

The charges at the surface of the nucleus tend to be near 1/3 and 2/3. What is amazing is that these charges correspond to the charges of the quark and anti-quark composing pion. That quarks should reveal themselves in the classical model for alpha decay is a complete surprise.

From this finding Illert concludes that during the decay the alpha particle is connected to the parent nucleus by rubber band like strings having quark and anti-quark at their ends, that is color flux tubes. These strings are interpreted as virtual pions. These strings get stretched and eventually must split since the color force between the quark and anti-quark at the ends of the string grows very strong.

This model is very attractive but has a deep problem: color forces mediate very short ranged and rapidly occurring interactions and should not be important for alpha decay which is a very slow process involving electromagnetic interactions in an essential manner. This does not diminish the pioneering value of Illert's work, just the opposite: pioneers must often have the courage to go against rationality as defined by the existing dogmas.

My earlier suggestion was that these pions serving as "rubber strings" are not ordinary pions but fractal copies of ordinary pions being much lighter and having much larger size. TGD indeed predicts the possibility of fractal copies of quantum chromo-dynamics (QCD). Thus there would exist a fractal copy of ordinary hadron physics operating in much longer length scales and having its own, much lighter, particle spectrum. The proposal was that this QCD corresponds to Mersenne prime M_{127}.

The dark QCD based on scaled up copies of ordinary quarks leads to a more elegant model in which virtual are replaced by π and ρ type color bonds, the latter being colored. Also an explanation of tetra-neutron emerges as a by-product since two pionic bonds can have negative charges. The identification of the nucleus as a nuclear string predicts the decay mechanism in which alpha particle pinches off and indeed has quarks and/or anti-quark attached to the ends of two nucleons.

Summarizing, although the model discussed in [C88] does not predict tetra-neutron, it represents findings and ideas, which might be of crucial importance in the topological and geometric modelling of nuclear decays. The finding that alpha decay could be described in terms of pions, although wrong as such, opens the way to a realization that ordinary pions and thus also ordinary hadron and nuclear physics might have lighter fractal copies.

7.7 Tritium beta decay anomaly

The determination of neutrino mass from the beta decay of tritium leads to a tachyonic mass squared [C77, C47]. I have considered several alternative explanations for this long standing anomaly.

1. ^{3}He nucleus resulting in the decay could be fake (tritium nucleus with one positively charged color bond making it to look like ^{3}He). The idea that slightly smaller mass of the fake ^{3}He might explain the anomaly: it however turned out that the model cannot explain the variation of the anomaly from experiment to experiment.

2. Much later I realized that also the initial ^3H nucleus could be fake (^{3}He nucleus with one negatively charged color bond). It turned out that fake tritium option can explain all aspects of the anomaly and also other anomalies related to radioactive and alpha decays of nuclei.

3. The alternative based on the assumption of dark neutrino or antineutrino belt surrounding Earth's orbit and explain satisfactorily several aspects of the anomaly but fails in its simplest form to explain the dependence of the anomaly on experiment. Since the fake tritium scenario is based only on the basic assumptions of the nuclear string model [L2] , [L2] and brings in only new values of kinematical parameters it is definitely favored.

7.7.1 Tritium beta decay anomaly

A brief summary of experimental data before going to the detailed models is in order.

Is neutrino tachyonic?

Nuclear beta decay allows in principle to determine the value of the neutrino mass since the energy distribution function for electrons is sensitive to neutrino mass at the boundary of the kinematically allowed region corresponding to the situation in which final neutrino energy goes to zero [C95].

The most useful quantity for measuring the neutrino mass is the so called Kurie plot for the function

$$\begin{aligned} K(E) &\equiv \left[\frac{d\Gamma/dE}{pEF(Z,E)}\right]^{1/2} \sim (E_\nu k_\nu)^{1/2} = \left[E_\nu\sqrt{E_\nu^2 - m(\nu)^2}\right]^{1/2} \ , \\ E_\nu &= E_0 - E \ , \ E_0 = M_i - M_f - m(\nu) \ . \end{aligned} \tag{7.7.1}$$

Here E denotes electron energy and E_0 is its upper bound from energy and momentum conservation (for a configuration in which final state nucleon is at rest). Mass shell condition lowers the upper bound to $E \leq E_0 - m(\nu)$. For $m(\nu) = 0$ Kurie plot is straight line near its endpoint. For $m(\nu) > 0$ the end point is shifted to $E_0 - m(\nu)$ and $K(E)$ behaves as $m(\nu)^{1/2} k_\nu^{1/2}$ near the end point.

The problem is that the determination of $m(\nu)$ from this parameterization in tritium beta decay experiments gives a negative mass squared varies and is $m(\nu)^2 = -147 \pm 68 \pm 41$ eV^2 according to [C95]! This behavior means that the derivative of $K(E)$ is infinite at the end point E_0 and $K(E)$ increases much faster near end point than it should. One can quite safely argue that tachyonicity gives only an ad hoc parameterization for the change of the shape of the function K deriving from some unidentified physical effect: in particular, the value of the tachyonic mass must correspond to a parameter related to new physics and need not have anything to do with neutrino mass.

More detailed experimental data

The results of Troitsk and Mainz experiments can be taken as constraints of the model. In Troitsk experiments [C77] gas phase tritium is used whereas in Mainz experiments [C47] liquid tritium film is used.

Troitsk experiments are described in [C77]. In 1944 Troitsk experiment, the enhancement of the spectrum intensity was found to begin roughly at $V_b \simeq 7.6$ eV below E_0. The conclusion was that the rise of the spectrum intensity below 18,300 eV with respect to the standard model prediction takes place (this is illustrated in fig. 4 of [C77]). No bump was claimed in this paper. In the analysis of 1996 experiment Troitsk group however concluded that the trapping of electrons gives rise to the enrichment of the low energy spectrum intensity of electrons and that when takes this effect into account, a narrow bump results.

Figure 4 of [C77] demonstrates that spectrum intensity is below the theoretical value near the endpoint (right from the bump). In [C77] the reduction of the spectrum intensity was assumed to be due to non-vanishing neutrino mass in [C77]. The determination of $m(\nu)$ from the data near the end point assuming that beta decay is in question [C77] gives $m(\nu) \sim 5$ eV.

The data can be parameterized by a parameter V_b which in the model context can be interpreted as repulsive interaction energy of antineutrinos with condensed matter suggested to explain the bump. Accordingly, the parameterization of $K(E)$ near the end point is

$$K(E) \sim (E - E_0)\theta(E - E_0) \rightarrow (E - E_0)\theta(E - E_0 + V_b) \ .$$

The end point is shifted to energy $E_\nu = V_b$ and $K(E)$ drops from the value V_b to zero at at this energy.

The values of V_b deduced from Troitsk and Mainz experiments are in the range $5 - 100$ eV. The value of V_b observed in Troitsk experiments using gas phase tritium [C77] was of order 10 eV. In Mainz experiment [C47] tritium film was used and the excess of counts around energy $V_b \simeq 100$ eV below E_0 was observed.

There is also a time variation involved with the value of V_b. In 1944 experiment [C77] the bump was roughly $V_b \simeq 7.6$ eV below E_0. In 1996 experiment [C77] the value of V_b was found to be

$V_b \simeq 12.3$ eV [C47]. Time variation was observed also in the Mainz experiment. In 'Neutrino 98' conference an oscillatory time variation for the position of the peak with a period of 1/2 years in the amplitude was reported by Troitsk group.

7.7.2 Could TGD based exotic nuclear physics explain the anomaly?

Nuclear string model explains tetra-neutron as alpha particle with two negatively charged color bonds. This inspires the question whether some fraction of decays could correspond to the decays of tritium to fake ^{3}He (tritium with one positively charged color bond) or fake tritium (^{3}He with one negatively charged color bond) to ^{3}He.

Could the decays of tritium decay to fake ^{3}He explain the anomaly?

Consider first the fake ^{3}He option. Tritium (pnn) would decay with some rate to a fake ^{3}He, call it $^3\mathrm{He}_f$, which is actually tritium nucleus containing one positively charged color bond and possessing mass slightly different than that of ^{3}He (ppn).

1. In this kind of situation the expression for the function $K(E,k)$ differs from $K(stand)$ since the upper bound E_0 for the maximal electron energy is modified:

$$\begin{aligned} E_0 &\rightarrow E_1 = M(^3\mathrm{H}) - M(^3\mathrm{He}_f) - m_\mu = M(^3\mathrm{H}) - M(^3\mathrm{He}) + \Delta M - m_\mu \ , \\ \Delta M &= M(^3\mathrm{He}) - M(^3\mathrm{He}_f) \ . \end{aligned} \quad (7.7.2)$$

   Depending on whether $^3\mathrm{He}_f$ is heavier/lighter than ^{3}He E_0 decreases/decreases. From $V_b \in [5-100]$ eV and from the TGD based prediction order $m(\overline{\nu}) \sim .27$ eV one can conclude that ΔM should be in the range 10-200 eV.

2. In the lowest approximation $K(E)$ can be written as

$$K(E) = K_0(E, E_1, k))\theta(E_1 - E) \simeq (E_1 - E)\theta(E_1 - E) \ . \quad (7.7.3)$$

   Here $\theta(x)$ denotes step function and $K_0(E, E_0, k)$ corresponds to the massless antineutrino.

3. If a fraction p of the final state nuclei correspond to a fake ^{3}He the function $K(E)$ deduced from data is a linear combination of functions K(E,^{3}He) and K(E,$^3\mathrm{He}_f$) and given by

$$\begin{aligned} K(E) &= (1-p)K(E,^3\mathrm{He}) + pK(E,^3\mathrm{He}_f) \\ &\simeq (1-p)(E_0 - E)\theta(E_0 - E) + p(E_1 - E)\theta(E_1 - E) \end{aligned} \quad (7.7.4)$$

   in the approximation $m_\nu = 0$.

   For $m(^3\mathrm{He}_f)<m(^3\mathrm{He})$ one has $E_1 > E_0$ giving

$$K(E) = (E_0 - E)\theta(E_0 - E) + p(E_1 - E_0)\theta(E_1 - E)\theta(E - E_0) \ . \quad (7.7.5)$$

   $K(E, E_0)$ is shifted upwards by a constant term $p\Delta M$ in the region $E_0 > E$. At $E = E_0$ the derivative of $K(E)$ is infinite which corresponds to the divergence of the derivative of square root function in the simpler parameterization using tachyonic mass. The prediction of the model is the presence of a tail corresponding to the region $E_0 < E < E_1$.

4. The model does not as such explain the bump near the end point of the spectrum. The decay ^{3}H→^{3}He$_f$ can be interpreted in terms of an exotic weak decay $d \rightarrow u + W^-$ of the exotic d quark at the end of color bond connecting nucleons inside ^3H. The rate for these interactions cannot differ too much from that for ordinary weak interactions and W boson must transform to its ordinary variant before the decay $W \rightarrow e + \overline{\nu}$. Either the weak decay at quark level or the phase transition could take place with a considerable rate only for low enough virtual W boson energies, say for energies for which the Compton length of massless W boson correspond to the size scale of color flux tubes predicted to be much longer than nuclear size. Is so the anomaly would be absent for higher energies and a bump would result.

5. The value of $K(E)$ at $E = E_0$ is $V_b \equiv p(E_1 - E_0)$. The variation of the fraction p could explain the observed dependence of V_b on experiment as well as its time variation. It is however difficult to understand how p could vary.

Could the decays of fake tritium to ^{3}He explain the anomaly?

Second option is that fraction p of the tritium nuclei are fake and correspond to ^{3}He nuclei with one negatively charged color bond.

1. By repeating the previous calculation exactly the same expression for $K(E)$ in the approximation $m_\nu = 0$ but with the replacement

$$\Delta M = M(^3\mathrm{He}) - M(^3\mathrm{He}_f) \quad \rightarrow \quad M(^3\mathrm{H}_f) - M(^3\mathrm{H}) \ . \tag{7.7.6}$$

2. In this case it is possible to understand the variations in the shape of $K(E)$ if the fraction of ^{3}H$_f$ varies in time and from experiment to experiment. A possible mechanism inducing this variation is a transition inducing the transformation ^{3}H$_f$ →^{3}H by an exotic weak decay $d + p \rightarrow u + n$, where u and d correspond to the quarks at the ends of color flux tubes. This kind of transition could be induced by the absorption of X-rays, say artificial X-rays or X-rays from Sun. The inverse of this process in Sun could generate X rays which induce this process in resonant manner at the surface of Earth.

3. The well-known poorly understood X-ray bursts from Sun during solar flares in the wavelength range 1-8 A [C6] corresponds to energies in the range 1.6-12.4 keV, 3 octaves in good approximation. This radiation could be partly due to transitions between ordinary and exotic states of nuclei rather than brehmstrahlung resulting in the acceleration of charged particles to relativistic energies. The energy range suggests the presence of three p-adic length scales: nuclear string model indeed predicts several p-adic length scales for color bonds corresponding to different mass scales for quarks at the ends of the bonds [L2] , [L2] . This energy range is considerably above the energy range $5-100$ eV and suggests the range $[4\times10^{-4}, 6\times10^{-2}]$ for the values of p. The existence of these excitations would mean a new branch of low energy nuclear physics, which might be dubbed X-ray nuclear physics. The energy scale of for the excitation energies of exotic nuclei could corresponds to Coulomb interaction energy $\alpha_{em}m$, where m is mass scale of the exotic quark. This means energy scale of 10 keV for MeV mass scale.

4. The approximately 1/2 year period of the temporal variation would naturally correspond to the $1/R^2$ dependence of the intensity of X-ray radiation from Sun. There is evidence that the period is few hours longer than 1/2 years which supports the view that the origin of periodicity is not purely geometric but relates to the dynamics of X-ray radiation from Sun. Note that for 2 hours one would have $\Delta T/T \simeq 2^{-11}$, which defines a fundamental constant in TGD Universe and is also near to the electron proton mass ratio.

5. All nuclei could appear as similar anomalous variants. Since both weak and strong decay rates are sensitive to the binding energy, it is possible to test this prediction by finding whether nuclear decay rates show anomalous time variation.

6. The model could explain also other anomalies of radioactive reaction rates including the findings of Shnoll [E15] , [E15] and the unexplained fluctuations in the decay rates of ^{32}Si and ^{226}Ra reported quite recently [C57] and correlating with $1/R^2$, R distance between Earth and Sun. ^{226}Ra decays by alpha emission but the sensitive dependence of alpha decay rate on binding energy means that the temporal variation of the fraction of fake ^{226}Ra isotopes could explain the variation of the decay rates. The intensity of the X-ray radiation from Sun is proportional to $1/R^2$ so that the correlation of the fluctuation with distance would emerge naturally.

7. Also a dip in the decay rates of ^{54}Mn coincident with a peak in proton and X-ray fluxes during solar flare [C89] has been observed: the proposal is that neutrino flux from Sun is also enhanced during the solar flare and induces the effect. A peak in X-ray flux is a more natural explanation in TGD framework.

8. The model predicts interaction between atomic physics and nuclear physics, which might be of relevance in biology. For instance, the transitions between exotic and ordinary variants of nuclei could yield X-rays inducing atomic transitions or ionization. The wave length range 1-8 Angstroms for anomalous X-rays corresponds to the range $Z \in [11, 30]$ for ionization energies. The biologically important ions Na^+, Mg^{++}, P^-, Cl^-, K^+, Ca^{++} have $Z = (11, 15, 17, 19, 20)$. I have proposed that Na^+, Cl^-, K^+ (fermions) are actually bosonic exotic ions forming Bose-Einstein condensates at magnetic flux tubes [K69] . The exchange of W bosons between neutral Ne and A(rgon) atoms (bosons) could yield exotic bosonic variants of Na^+ (perhaps even Mg^{++}, which is boson also as ordinary ion) and Cl^- ions. Similar exchange between A atoms could yield exotic bosonic variants of Cl^- and K^+ (and even Ca^{++}, which is also boson as ordinary variant). This hypothesis is testable by measuring the nuclear weights of these ions. X-rays from Sun are not present during night time and this could relate to the night-day cycle of living organisms. Note that magnetic bodies are of size scale of Earth and even larger so that the exotic ions inside them could be subject to intense X-ray radiation. X-rays could also be dark X-rays with large Planck constant and thus with much lower frequency than ordinary X-rays so that control could be possible.

7.7.3 The model based on dark neutrinos

A common origin of the tritium beta decay anomaly was independently suggested by several groups (see [C34]): a broad spike or bump like excess of counts centered $5-100$ eV below the end point energy E_0. In [C34] it was suggested that a repulsive interaction of antineutrinos with condensed matter with interaction energy of order $V_b \simeq 5-100$ eV could explain the bump.

It has been pointed out by Stevenson [C91] that the process in which neutrinos are absorbed from a background of electron neutrinos

$$\nu_e +^3 \mathrm{H} \rightarrow^3 \mathrm{He} + e^-$$

leads to electrons in the anomalous endpoint region. This gives an essentially constant addition to the region $E_0 - E_F < E < E_0$. The density of cosmic neutrino background is however far too small to give the required large background density of order $1/m(\nu)^3$.

The earlier -wrong- hypothesis that nuclei are Z^0 charged are consistent with both options described above as explanations of the anomaly. One can modify these models to apply also in the new framework. The problem of these models is that one is forced to make ad hoc assumptions about dynamics in long length scales. They might make sense in TGD Universe but would require experimental justification. These models in their simplest form fail also to explain the dependence of V_b on experiment and fail to provide provide insights about more general time variations of nuclear decay rates.

Neutrino belt or antineutrino belt?

The model corresponding to mechanism of [C34] is that the belt consists of dark antineutrinos and the repulsive interaction energy of antineutrino with the these neutrinos explains the anomaly. The model based on dark neutrinos assumes that Earth's orbit is surrounded by a belt of dark neutrinos and that the mechanism proposed in [C91] could be at work. The periodic variation of the dark neutrino density along the orbit of Earth around Sun could also explain the periodic variations of the bump.

1. The first mechanism corresponds to that suggested in [C34]. The antineutrino emitted in the beta decay can transform to a dark neutrino by mixing and experiences a repulsive Z^0 force which effectively shifts the electron energy spectrum downwards. In this case the repulsive interaction energy V_b of dark anti-neutrinos with the dark antineutrinos of the solar belt would replace ΔM in the previous formula:

$$E_0 = M(^3\text{H}) - M(^3\text{He}) - m(\nu) \rightarrow M(^3\text{H}) - M(^3\text{He}) - V_b - m(\nu_d) \ . \tag{7.7.7}$$

2. Second option corresponds to the mechanism proposed in [C91]. Dark neutrino transforms to ordinary one and induces by ordinary W exchange ordinary tritium beta decay. In this case the Fermi energy E_F of dark neutrino determines the width of the bump and one has $V_b = E_F$:

$$E_0 = M(^3\text{H}) - M(^3\text{He}) - m(\nu) \rightarrow M(^3\text{H}) - M(^3\text{He}) - E_F + m(\nu_d) \ . \tag{7.7.8}$$

   The rate of the process would be given by the standard model and only the density of dark neutrinos and the ratio $M^2(\nu, \nu(dark))/M^2(\nu)$ appear as free parameters.

Notice that these models are simpler than the original models which assumed that the interaction of neutrinos with condensed matter carrying Z^0 charge is involved. The explanation for the dependence of V_b on experiment poses a difficulty for both models. For the antineutrino belt the repulsive interaction energy is proportional to the density of antineutrinos. For neutrino belt V_b corresponds to the Fermi energy proportional to the density of neutrinos. In both cases large variation of V_b requires a large variation of the density of antineutrinos (neutrinos) of the belt in the scale smaller than Earth size. This does not look too plausible.

Can one understand time variation of V_b?

The periodic variation of the density of neutrinos or antineutrinos in the belt should induce the variation of V_b. The ordering of the two models trying to explain this variation reflects the evolution of the general ideas about quantum TGD.

1. First model

The value of the period and the fact that maximum shift occurs when Earth is near to its position nearest to Sun suggests that the physics of solar system must be involved somehow. The simplest explanation is that gravitational acceleration tends to drive dark neutrinos (antineutrinos) as near as possible to Sun inside the belt. In thermal equilibrium with temperature T the Boltzmann factor

$$exp(-\frac{V_{gr}}{T}) = exp(-\frac{GMm(\nu_d)}{rT}) \tag{7.7.9}$$

for the dark neutrino would determine the density profile of dark neutrinos along the belt as function of the distance r to the Sun.

The existence of the dark neutrino belt conforms with the model of for the formation of solar system from dark matter with a gigantic value of Planck constant discussed in [K75, K28] . The model indeed assumes that the dark matter is located at space-time sheet surrounding the orbit of Earth. The requirement that dark neutrino density is few neutrinos per atomic volume in the belt leads to a lower bound for the mass of the belt:

$$M(belt) \simeq m(\nu_d)\frac{Vol(belt)}{a^3} > 10^{-11}M(Sun) \quad (a \simeq 10^{-10} \text{ meters}) \ . \tag{7.7.10}$$

Here it is assumed that the dark neutrino mass is same as neutrino mass, which of course is an un-necessarily strong assumption. If the belt is at rest, the time period for the variation of the tritium beta decay anomaly is exactly half year. The period seems to be few hours longer than one half year (as reported in Neutrino98 conference in Tokyo by Lobashev *et al*) [C77], which suggests that belt rotates slowly relative to Earth in the same direction as Earth.

2. Second model

The model for radioactive decay rate anomalies requires that neutrinos and Earth move respect to each other and that the density of neutrinos in the laboratory volume varies along the orbit.

1. Assume first that ordinary quantum mechanics applies and neutrinos are ordinary. The simplest expectation from Equivalence Principle assuming that neutrinos and Earth move independently along geodesic lines is that the velocity is same for Earth and neutrinos. No effect results even if the density of neutrinos along the orbit varies.

2. Suppose that the neutrinos are dark in the sense of having gigantic gravitational Planck constant and are in a macroscopically quantum coherent phase de-localized along the entire orbit and described by a wave function (also neutrino Cooper pairs can be considered). If the neutrino ring is exactly circular as Bohr orbit picture suggests and contains Earth's orbit, the thickness of the ring must be at least $d = a - b$, where a and b are major and minor axis. Exact rotational symmetry implies that dark neutrinos are characterized by a phase factor characterizing the angular momentum eigen state in question (the unit of the quantized angular momentum is now very large). Thus neutrino density depends only on the transversal coordinates of the tube and vanishes at the boundary of the tube. Since the Earth's orbit is ellipse, the transversal variation of the neutrino density inside the tube induces periodic variations of the neutrino density in the detector and could explain the effects on radioactive decay rates.

Although the model might explain the time variation of V_b it does not provide any obvious explanation for beta decay rates in general and fails to explain the variation of the alpha decay rate of ^{226}Ra nor the correlation of decay rates with solar flares. Hence it is clear that the model involving only the notion of nuclear string is favored.

7.7.4 Some other apparent anomalies made possible by dark neutrinos

The appearance of dark neutrinos in the final states of beta decays allow to imagine also some other apparent anomalies.

Apparent anomaly in the inverse beta decay

For the antineutrino belt option one can consider also the possibility of an apparent anomaly in the inverse beta decay in which positron and neutrino are emitted but only electron observed. The apparent anomaly would result from the absorption of a dark antineutrino with repulsive Z^0 interaction energy with condensed matter.

In this case the value of E_0 increases

$$E_0 = M_i - M_f - m(\nu) \rightarrow \hat{E}_0 = M_i - M_f + m(\nu_d) + V_b \ , \tag{7.7.11}$$

which means that positron spectrum extends above the kinematic limit if V_b has the value predicted by the explanation of tritium beta decay anomaly.

A second anomalous situation results if the emitted neutrino transforms to a dark neutrino with negative binding energy. In this case the value of E_0 would change as

$$E_0 \rightarrow \hat{E}_0 = M_i - M_f - m(\nu_d) + V_b \ . \tag{7.7.12}$$

Apparently neutrinoless beta decay and double beta decay

Neutrinoless double beta decay (NDB) is certainly one of the most significant nuclear physics processes from the point of view of unified theories (the popular article of New Scientist [E10] provides a good view of NDB and the recent rather exciting experimental situation). In the standard physics framework NDB can occur only if neutrinos are Majorana neutrinos so that neutrino number is conserved only modulo 2 meaning that neutrino and antineutrino are one and the same particle. Since no antineutrinos are emitted in the NDB, the total energy of the two electrons is larger than in the normal double beta decay, and serves as an experimental signature of the process.

There are several collaborations studying NDB. The team formed by Hans Klapdor-Kleingrothaus and colleagues from the the Max Planck Institute for Nuclear Physics in Heidelberg have been studying this process since 1990 in Gran Sasso laboratory. The decays studied are decays of Germanium-76 isotope known to be one of the few isotopes undergoing ordinary double beta decay transforming it into Selenium. The energy of the emitted two electrons is absorbed by the surrounding Ge atoms. The total energy which is larger for NDB decay serves as a signature of the process.

Three years ago came the first paper of the Heidelberg group reporting the observation of 15 NDB decays [C54] . The analysis of the experiments however received a very critical response from colleagues. The Kurchatov Institute quitted the collaboration at 2001 and represented its own analysis with the conclusion that the data do not support NDB. Three years later Heidelberg group represented 14 new candidates for NDB and a new analysis [C55] . It is now admitted that the team is not obviously wrong but that there are still doubts whether the background radioactivity has been handled correctly.

In TGD Universe neutrinos are Dirac neutrinos and NDB is not possible. The possibility of dark neutrinos however allow to consider the possibility of apparently neutrinoless beta decay and double beta decay.

What would happen that the ordinary neutrino emitted in the beta decay of proton transforms into a dark neutrino by mixing. The dark neutrino would not be observed so that apparently neutrinoless beta decay would be in question. Dark neutrino has a negative interaction energy with condensed matter assuming that the explanation of tritium beta decay anomaly is correct so that electron would have an anomalously high energy. The process cannot occur if the negative energy states of the Fermi sea are filled as indeed suggested by energetic considerations.

The generalization of this process would be double beta decay involving strong interaction between decaying neutrons mediated by color bond between them and the transformation of second neutrino to dark neutrino with negative energy so that the electrons would have anomalously high energy. The same objection applies to this process as to the apparently neutrinoless beta decay.

7.8 Cold fusion and Trojan horse mechanism

The model for cold fusion has developed gradually as the understanding of quantum TGD and many-sheeted space-time has developed. Trojan horse mechanism has served as the connecting thread between various models. The last step of progress relates to the new vision about nuclear physics but it is still impossible to fix the model completely unless one poses the condition of minimality and the requirement that single mechanism is behind various anomalies.

7.8.1 Exotic quarks and charged color bonds as a common denominator of anomalous phenomena

There should exist a common denominator for anomalous behavior of water, cold fusion, the findings of Ditmire suggesting cold fusion, sono-fusion, exotic chemistries, strange properties of living matter including chiral selection, and also phenomena like low compressibility of condensed matter which standard physicist would not be worried about.

It seems that compression inducing the generation of charged color bonds between nucleons and leading to a formation of super-nuclei with atomic distances between building blocks might be the sought for common denominator. For super nuclei the repulsive weak interactions between exotic quark and anti-quark belonging to the two bonded nuclei would compensate the attractive color force so that a stable configuration of atomic size would result. Note that the weak coupling strength would be actually strong by the general criterion for transition to the large $\hbar$ phase.

The charging of color bonds would occur via W boson exchange between exotic and valence quarks with exotic W boson transforming to ordinary W via mixing.

The alternative option is a phase transition of nuclei transforming $k = 113$ em space-time sheets of valence quarks to em dark space-time sheets with a large value of $\hbar$ suggested for heavier nuclei by the general criteria. This phase transition could be avoided if the criticality forces surplus protons to transfer the electromagnetic charge of valence quarks to color bonds so that the situation reduces to the first option. In this picture standard nuclear physics would remain almost untouched and nothing new expect exotic quarks and charged color bonds is introduced.

The following examples suggest that this general picture indeed might unify a large class of phenomena.

1. The super-nuclei formed by the dark protons of water would be a basic example about this phenomenon. The occurrence of the process is plausible if also nucleons possess or can generate closed loops with exotic quark and anti-quark at the ends of the loop belonging to the same nucleon. The fact that these protons are dark with respect to electromagnetic interactions suggests that the charge of protons is transferred to the color bonds so that the outcome is a nuclear string formed from neutrons connected by positively charged color bonds. Darkness with respect to weak interactions suggests that valence quarks are doubly dark. This would mean that the p-adic length scale of color bonds would correspond to $k_{eff} = 107 + 2 \times 22 = 151$ for $\hbar_s = n^2\hbar/v_0^2$, $n = 1$. This corresponds to the thickness of cell membrane so that the structure of water would contain information about the basic biological length scale.

2. In condensed matter the super-nuclei would form at some critical pressure when weakly charged color bonds between neighboring nuclei become possible and compensate the attractive color force. This would explain the low compressibility of condensed matter.

3. Bio-polymers in vivo might correspond to super-nuclei connected by charged color bonds whose weak charges would explain the large parity breaking involve with chiral selection. Hydrogen bond might be a basic example of a charged color bond. It could be that the value of integer n in $\hbar_s = n\hbar/v_0$ is $n = 3$ in living matter and $n = 1$ in ordinary condensed matter.

   Trojan horse mechanism might work also at the level of chemistry making possible to circumvent electronic Coulomb wall and might be an essential characteristic of the catalytic action. Note that Pd is also a powerful catalyst. $n = 1$ might however distinguish it from bio-catalysts. In separate context I have dubbed this mechanism as 'Houdini effect'.

   The reported occurrence of nuclear transmutations [C94, C116] such as $^{23}Na +^{16} O \rightarrow^{39} K$ in living matter allowing growing cells to regenerate elements K, Mg, Ca, or Fe, could be understood as fusion of neighboring nuclei connected by charged color bond which becomes neutral by W emission so that collapse to single nucleus results in absence of the repulsive weak force. Perhaps it is someday possible to produce metabolic energy by bio-fusion or perhaps Nature has already discovered the trick!

4. In cold fusion the nuclei of target D and Pd would combine to form super-nuclei connected by charged color bonds. This would explain why the heavy loading of Pd nuclei with D (for a review of loading process see [C111]) does not generate enormous pressures. Cold fusion would occur in some critical interval of loadings allowing ordinary and exotic nuclei to transform to each other. The transfer of the em charge of D to the color bond connecting D and Pd would make D effectively nn state. Together with the fact that the color bond would have length of order atomic radius would mean that the Coulomb wall of Pd and D is not felt by beam nuclei and Trojan horse mechanism would become possible. The prediction is that Coulomb wall disappears only only when deuterium or tritium target is used. If nuclei

can transform to dark em phase cold fusion could occur for arbitrary target nuclei. That it is observed only for D and possibly H does not support this option.

If valence quarks are doubly dark, their magnetic bodies have size of order $L(151) = 10$ nm, which is also the size scale of the nano-scaled Pd particles, color force would become long ranged. In sono-luminescence and son-fusion and also in nuclear transmutations similar formation of super-nuclei would occur and the collapse of super-nucleus to single nucleus could occur by the proposed mechanism.

5. In the experiments of Ditmire *et al* laser pulse induces very dense phase of Xenon atoms having $Z = 54$ which is heated to energies in which electron energies extend to MeV region and expands rapidly. $Z = 54$ means that Xe satisfies the most stringent condition of criticality for the transition to electromagnetic large $\hbar$ phase. This transition does not occur if protons feed the surplus em charge to the color bonds so that Xe nuclei also weakly charged. Assume that some fraction of Xe is in this kind of phase. The compression of Xe gas by laser pulse compresses Xe super-nuclei. If the connecting charged color bonds emit their em and weak charge by emission of W boson the super-nuclei collapse to single nucleus and nuclear fusion reactions become possible. The repulsive weak force becoming manifest in the compression generates brehmstrahlung heating the system and induces a violent explosion much like in sono-fusion.

In the sequel the experiments Ditmire *et al* and cold fusion are discussed in detail using this model.

7.8.2 The experiments of Ditmire *et al*

An important stimulation in the development of the model for cold fusion came from the observations of Ditmire *et al* [C74] published in Nature. The discovery was that the energy spectrum of electrons in ionic explosions induced by the laser heating of ionic clusters extends up to energies of order MeV (rather 10^2 eV(!)): this suggests strongly a mechanism making strong interactions possible.

In the experimental arrangement of Ditmire *et al* clusters of Xenon atoms are hit by ultrashort (150 fsec), high-intensity (2×10^{16} W/cm^2) laser pulses [C74] . This leads to superheating and production of high energy ions in the explosions of the superheated clusters. The highest ion energies are by 4 orders of magnitude larger than expected and of order MeV, the typical energy scale of nuclear strong interaction. The average ion energy is 45 ± 5 keV for cluster size of 6.5 nm and decreases slowly with the size of the cluster. No hot ions are produced for small clusters containing less than ~ 100 Xe atoms. It is not yet understood why the clusters explode so much more violently than molecules (producing 1 MeV ions as opposed to 100 eV ions) and small clusters. Another striking feature of the laser-cluster interaction is ionization to very high charge states, much higher than in the ionization, which can be produced by simple field ionization.

Consider first a more detailed model of the superheating as it is described in [C74] .

In an intensely irradiated cluster, optically and collisionally ionized electrons undergo rapid collisional heating for the short time (< ps) before the cluster disassembles in the laser field. Our recent studies of the electron energy spectra produced by the high intensity irradiation of large Xe clusters with 150 fs *laser pulse indicate that collisional heating within the cluster can produce electrons with energy up to* 3 keV*, an energy much higher than that that typical of solid target plasmas.*

A sharp peak in the measured electron energy spectrum suggested that the cluster micro-plasma exhibited a resonance in the heating by laser pulse similar to the giant resonance seen in metallic clusters. A small amount of cluster expansion during the laser pulse lowers the electron density to bring the near-infrared laser light into resonance with the free-electron plasma frequency in the cluster. This resonance greatly increases the laser electric field density within the cluster, and the laser absorption rate is enhanced, rapidly heating the electrons on a very fast (< 10 fs) *time scale to a highly non-equilibrium superheated state with mean energies of many* keV*. Charge separation of hot electrons inevitably leads to a very fast expansion of the cluster ions. This process is fundamentally different from low-intensity photo-fragmentation of cluster and far more energetic than the Coulomb explosion of small molecules.*

Authors believe (rather naturally!) that the production of hot ions is made possible by the high ion-temperatures produced by the not yet properly understood heating mechanism and suggest that this mechanism might make even table-top fusion possible.

In TGD framework the proposed general vision suggests following picture.

1. Laser pulse induces a compression of clusters of Xe atoms already containing super-nuclei with charged color bonds so that repulsive weak interaction compensated by color force in equilibrium situation becomes manifest and induces the expansion of the system much like the expansion of the bubble in sono-luminescence. The resulting brehmstrahlung heats the system.

2. The critical cluster size 6.5 nm could correspond to the p-adic length length scale $L(k_{eff} = 151) = 10$ nm for doubly dark valence quarks with $n = 1$.

3. The nuclear fusions resulting when color bonds between nuclei become neutral and induce collapse to single nucleus. Anomalously high charge states could be a byproduct of violent and very rapid fusions of neighboring color bonded Xe nuclei tearing Xe nuclei and outer electrons apart. These fusions would generate quantum coherent dark gamma ray beams transforming to ordinary gamma rays by de-coherence transition reducing the wavelength of gamma rays by a factor of 2^{-11} for $\hbar_s = \hbar/v_0$. It is also possible that dark gamma rays are absorbed by Pd super-nuclei. The wavelengths of dark gamma rays with energy of MeV would of order 2 nanometers in this case so that a coherent heating would happen in rather large volume.

7.8.3 Brief summary of cold fusion

In the following history and signatures of cold fusion are briefly summarized.

History of cold fusion

The first claim for cold fusion [C98] dates back to March 23, 1989, when Pons and Fleischmann announced that nuclear fusion, producing usable amounts of heat, could be induced to take place on a table-top by electrolyzing heavy water and using electrodes made of Pd and platinum. Various laboratories all-over the world tried to reproduce the experiments. The poor reproducibility and the absence of the typical side products of nuclear fusion (gamma rays and neutrons) led soon to the conclusion (represented in the dramatic session of American Physical Society May 1, 1989) that nuclear fusion cannot explain the heat production. Main stream scientists made final conclusions about the subject of 'cold fusion' and cold fusion people became a pariah class of the scientific community.

The work with cold fusion however continued and gradually situation has changed. It became clear that nuclear reaction products, mainly ^4He, are present. Gradually also the reasons for the poor reproducibility of the experiments became better understood. A representative example about the change of the attitudes is the article of Schwinger [C107] in which cold fusion is taken seriously. The article also demonstrates that the counter arguments of hot fusion people are based on the implicit assumption that hot fusion theory describes cold fusion despite the fact that the physical situations are radically different.

The development on the experimental side has been based on techniques involving the use of catalysis, nanotechnology, electrolysis, glow discharge and ultrasonic cavitation. There are now public demonstrations of cold fusion reactors, whose output energy far exceeds input energy and commercial applications are under intensive development.

Rather remarkably, also the production of heavier elements has been detected [C105] and this makes the explanation of the effect even more difficult in standard physics context and definitely excludes the explanations claiming that some chemical process is the source of the excess heat. The possibility of nuclear transmutation also suggests the possibility to transform ordinary nuclear wastes into non-radioactive nuclei and the first method achieving this has already been reported [C21] . There are claims [C94, C116] that cold fusion indeed occurs in bio-systems.

There is also some evidence for high temperature super conductivity associated with deuterium loaded palladium [C105] . Good representations about the subject of cold fusion and references to the experimental work can be found at various cold fusion web-sites [C7, C2] . Also the articles of J. Rothwell [C105] and the excellent review article of E. Storms [C110] are recommended.

It has become clear that cold fusion differs from hot fusion in several respects: gamma rays are not produced and the flux of neutrons is much lower than predicted by standard nuclear physics (these features are very well-come from the point of view of the technological applications). Together with the fact that Coulomb wall does not allow the occurrence of cold fusion at all in the standard physics context, this forces the conclusion that new physics must be involved.

It seems that TGD indeed could provide this new physics. The key elements of the model to be discussed are Trojan horse mechanism and coherent photon exchange action of D nuclei with Cooper pairs of the exotic super conductor formed by the D-loaded cathode material (say Palladium).

In the sequel the consideration is restricted to the case of Pd cathode: the model generalizes trivially to the case of a more general cathode material.

Signatures of cold fusion

In the following the consideration is restricted to cold fusion in which two deuterium nuclei react strongly since this is the basic reaction type studied.

In hot fusion there are three reaction types:
1) $D + D \rightarrow^4 He + \gamma$ $(23.8MeV)$
2) $D + D \rightarrow^3 He + n$
3) $D + D \rightarrow^3 H + p$.
The rate for the process 1) predicted by standard nuclear physics is more than 10^{-3} times lower than for the processes 2) and 3) [C112] . The reason is that the emission of the gamma ray involves the relatively weak electromagnetic interaction whereas the latter two processes are strong.

The most obvious objection against cold fusion is that the Coulomb wall between the nuclei makes the mentioned processes extremely improbable at room temperature. Of course, this alone implies that one should not apply the rules of hot fusion to cold fusion. Cold fusion indeed differs from hot fusion in several other aspects.

1. No gamma rays are seen.

2. The flux of energetic neutrons is much lower than expected on basis of the heat production rate an by interpolating hot fusion physics to the recent case.

   These signatures can also be (and have been!) used to claim that no real fusion process occurs. It has however become clear that the isotopes of Helium and also some tritium accumulate to the Pd target during the reaction and already now prototype reactors for which the output energy exceeds input energy have been built and commercial applications are under development. Therefore the situation has turned around. The rules of standard physics do not apply so that some new nuclear physics must be involved and it has become an exciting intellectual challenge to understand what is happening. A representative example of this attitude and an enjoyable analysis of the counter arguments against fold fusion is provided by the article 'Energy transfer in cold fusion and sono-luminescence' of Julian Schwinger [C107] . This article should be contrasted with the ultra-skeptical article 'ESP and Cold Fusion: parallels in pseudoscience' of V. J. [C109] [C109] .

Cold fusion has also other features, which serve as valuable constraints for the model building.

1. Cold fusion is not a bulk phenomenon. It seems that fusion occurs most effectively in nano-particles of Pd and the development of the required nano-technology has made possible to produce fusion energy in controlled manner. Concerning applications this is a good news since there is no fear that the process could run out of control.

2. The ratio x of D atoms to Pd atoms in Pd particle must lie the critical range [.85, .90] for the production of ^4He to occur [D41] . This explains the poor repeatability of the earlier experiments and also the fact that fusion occurred sporadically.

3. Also the transmutations of Pd nuclei are observed [C105] .

Below a list of questions that any theory of cold fusion should be able to answer.

1. Why cold fusion is not a bulk phenomenon?
2. Why cold fusion of the light nuclei seems to occur only above the critical value $x \simeq .85$ of D concentration?
3. How fusing nuclei are able to effectively circumvent the Coulomb wall?
4. How the energy is transferred from nuclear degrees of freedom to much longer condensed matter degrees of freedom?
5. Why gamma rays are not produced, why the flux of high energy neutrons is so low and why the production of ^4He dominates (also some tritium is produced)?
6. How nuclear transmutations are possible?

7.8.4 TGD inspired model of cold fusion

The model to be discussed is based on Trojan horse mechanism and explains elegantly all those aspects of cold fusion which are in conflict with standard nuclear physics. The reaction mechanism explains also the sensitivity of the occurrence of cold fusion to small external perturbations.

Model for D-loaded Pd nano-particle

It seems that cold fusion is a critical phenomenon. The average D/Pd ratio must be in the interval $(.85, .90)$. The current must be over-critical and must flow a time longer than a critical time. The effect occurs in a small fraction of samples. Deuterium at the surface of the cathode is found to be important and activity tends to concentrate in patches. The generation of fractures leads to the loss of the anomalous energy production. Even the shaking of the sample can have the same effect. The addition of even a small amount of H_2O to the electrolyte (protons to the cathode) stops the anomalous energy production.

All these findings support the catastrophe theoretic picture according to which the decomposition into patches corresponds to criticality allowing the presence of ordinary and exotic phase whose transformation to the ordinary phases makes possible cold fusion reactions. The added ordinary protons and fractures could serve as a seed for a phase transition leading to a region where only single phase is possible.

In TGD framework Pd nano-particles correspond to space-time sheets of size of order $10^{-9}-10^{-8}$ m and fusion is restricted inside these structures. Cold fusion can be regarded as a fusion of incoming ordinary D with target D attached to the surface of Pd rather than between two free D:s as suggested by the standard nuclear physics wisdom. Thus cold fusion could be regarded as 'burning' of D associated with a finite space-time sheets so that cold fusion is not a bulk phenomenon and is very sensitive to the in-homogenities of the Pd particle. Note that this in principle makes the control of cold fusion easier.

The critical loading fraction varies in the range $.85-.90$. This value is so large that enormous pressures would be generated unless the deuterium nuclei lose their translational degrees of freedom by forming some kind of bound states with Pd nuclei. The guess is that the bound states correspond to the formation of super-nuclei with em and weakly charged color bonds connecting Pd and D nuclei. $k = 113$ dark weak force, which is actually strong by the criticality condition, compensates the color force between the exotic quarks. This makes D nuclei effectively nn nuclei so that Coulomb wall does not produce difficulties.

The challenge is to understand the origin of the criticality condition for super-nucleus. The question is why the number of D nuclei per Pd nuclei varies in so narrow range for the phase transition leading to the formation of super-nuclei to occur. Catastrophe theoretic thinking suggests that a cusp catastrophe typical for phase transitions is in question. In the critical range there are two phases, exotic and ordinary phase, which can easily transform to each other. Criticality is essential for the cold fusion reactions to occur since initial state involves exotic D+Pd complex and final state involves ordinary nuclei.

The ratio x would correspond to the variable which varies in a direction transversal to cusp and whose variation therefore leads to a catastrophic jump inducing the phase transition or its reverse. x would be a pressure type variable which plays similar role also in phase transitions like liquid-gas phase transition. The critical range for x would correspond to the critical range of pressure in which liquid and gas are in equilibrium.

Second variable varies along the cusp so that the transition is possible above certain critical value. This variable presumably relates to the energetics of the transition so that transition would liberate energy above critical value of the parameter. Temperature is a natural candidate for this variable. Catastrophe theoretic model implies that for a given value x in catastrophe region both ordinary phase and exotic phase are possible. In these regions cold fusion can occur. In regions where the system is outside the catastrophe region so that system is stably in either phase, cold fusion cannot occur. This explains why Pd contains only patches were cold fusion occurs. The control variable, be it local temperature or something else, could be perhaps identified by studying the local conditions guaranteeing the occurrence of cold fusion. It is indeed known that the increase of temperature favors the occurrence of cold fusion.

The behavior variable could distinguish between the two phases and could correspond to the surface density n of D nuclei bound to Pd nuclei and transformed to fake D. The potential function could be free energy minimized when the system is in constant temperature and the two phases would correspond to local minima of free energy.

Anomalous reaction kinetics of cold fusion

One can deduce a more detailed model for cold fusion from observations, which are discussed systematically in [C112] and in the references discussed therein.

1. When D_2O is used as an electrolyte, the process occurs when PdD acts as a cathode but does not seem to occur when it is used as anode. This suggests that the basic reaction is between the ordinary deuterium $D = pn$ of electrolyte with the the exotic D=nn + charged color bond attached to Pd in the cathode.

2. For ordinary nuclei fusions to tritium and ^{3}He occur with approximately identical rates. The first reaction produces neutron and ^{3}He via $D + D \rightarrow n +^3 He$, whereas second reaction produces proton and tritium via $D + D \rightarrow p +^3 H$. The standard nuclear physics prediction is that one neutron per each tritium nucleus should be produced. Tritium can be observed by its beta decay to ^{3}He and the ratio of neutron flux is several orders of magnitude smaller than tritium flux as found for instance by Tadahiko Mizuno and his collaborators (Mizuno describes the experimental process leading to this discovery in his book [C75]). Hence the reaction producing ^{3}He cannot occur significantly in cold fusion which means a conflict with the basic predictions of the standard nuclear physics.

   The explanation is following. If D is fake D (nn+charged color bond connecting it with Pd), one expects that the production of ^{3}He is hindered since there is no proton directly available. Also in the case that the reaction n+color bond $\rightarrow p$ occurs, one expects that Coulomb wall makes the process slow.

3. The production of ^4He, which should not occur practically at all, is reported to dominate and the fraction of tritium is below .1 per cent. The explanation could be that also multiple attachments to target can occur such that D attaches to (D+Pd) by forming a charged color bond. Thus would have nnnnn state with two charged color bonds attached to Pd. This state could split from Pd and transform via exchange of two light weak bosons between exotic and valence quarks to ^4He (assuming that dark $W(113)$ can mix with $W(89)$). It is also possible that the super-nuclear string formed by Pd and D splits and emits ^4He as in ordinary alpha decay. Gamma rays need not be generated since the recoil momentum could be transferred to the Pd target like in Mössbauer effect.

4. Also more complex reactions between D and Pd and between Pd nuclei can occur. These can lead to the reactions transforming the nuclear charge of Pd and thus to nuclear transmutations.

5. The mechanism also explains why the cold fusion producing ^{3}He and neutrons does not occur using water instead of heavy water. There are reports about cold fusion also in this case [C110] . If one fourth of protons in water are arranged to nuclear strings consisting of neutrons connected by positively charged color bonds as the TGD based model explaining the anomalies of water suggests [K30] , these strings could attach to fake D and induce cold fusion reactions.

6. The proposed reaction mechanism explains why neutrons are not produced in amounts consistent with the anomalous energy production. The addition of water to the electrolyte however induces neutron bursts. Suppose that one fourth of protons in water forms similar dark phase being transformed to neutrons connected by positively charged color bonds, as assumed in the model of water explaining various anomalies of water [K30] . What comes in mind is that neutrons are generated when a neutron string from H_2O containing only charged color bonds attaches to D+Pd (nn + charged color bond +Pd). Neutrons of nn are connected by a neutral color bond. If charged color bonds between neutrons are energetically more favorable than neutral color bonds, nn could emit a free neutron in the process so that the outcome would be a neutron string containing only charged color bonds attached to Pd.

How objections against cold fusion are circumvented?

It has already become clear that the model allows to circumvent the basic reaction kinetic arguments against cold fusion [C112] .

1. Coulomb wall makes nuclear fusions impossible.

2. ^{3}He and ^3H should be produced in equal amounts. The fraction of ^4He should be smaller than 10^{-3}.

3. The claimed nuclear transmutation reactions (reported to occur also in living matter [C94]) should not occur.

Consider next the objections related to energetics.

1. Gamma rays, which should be produced in most nuclear reactions such as ^4He production to guarantee momentum conservation are not observed. The explanation is that the recoil momentum goes to the macroscopic quantum phase defined by the Pd lattice as in Mössbauer effect, and eventually heats the electrolyte system. This provides the mechanism by which the liberated nuclear energy is transferred to the electrolyte difficult to imagine in standard nuclear physics framework.

2. If a nuclear reaction should occur, the immediate release of energy can not be communicated to the lattice in the time available. In the recent case the time scale is however multiplied by the factor $r = \hbar_s/\hbar$ giving scaling factor 2^{11} so that the situation changes dramatically.

7.8.5 Do nuclear reaction rates depend on environment?

Claus Rolfs and his group have found experimental evidence for the dependence of the rates of nuclear reactions on the condensed matter environment [C46] . For instance, the rates for the reactions ^{50}V(p,n)^{50}Cr and ^{176}Lu(p,n) are fastest in conductors. The model explaining the findings has been tested for elements covering a large portion of the periodic table.

Debye screening of nuclear charge by electrons as an explanation for the findings

The proposed theoretical explanation [C46] is that conduction electrons screen the nuclear charge or equivalently that incoming proton gets additional acceleration in the attractive Coulomb field of electrons so that the effective collision energy increases so that reaction rates below Coulomb wall increase since the thickness of the Coulomb barrier is reduced.

The resulting Debye radius

$$R_D = 69\sqrt{\frac{T}{n_{eff}\rho_a}} , \tag{7.8.1}$$

where ρ_a is the density of atoms per cubic meter and T is measured in Kelvins. R_D is of order .01 Angstroms for $T = 373$ K for $n_{eff} = 1$, $a = 10^{-10}$ m. The theoretical model [C82, C104] predicts that the cross section below Coulomb barrier for $X(p,n)$ collisions is enhanced by the factor

$$f(E) = \frac{E}{E+U_e}exp(\frac{\pi\eta U_e}{E}) . \tag{7.8.2}$$

E is center of mass energy and η so called Sommerfeld parameter and

$$U_e \equiv U_D = 2.09\times 10^{-11}(Z(Z+1))^{1/2}\times(\frac{n_{eff}\rho_a}{T})^{1/2}\ eV \tag{7.8.3}$$

is the screening energy defined as the Coulomb interaction energy of electron cloud responsible for Debye screening and projectile nucleus. The idea is that at R_D nuclear charge is nearly completely screened so that the energy of projectile is $E + U_e$ at this radius which means effectively higher collision energy.

The experimental findings from the study of 52 metals support the expression for the screening factor across the periodic table.

1. The linear dependence of U_e on Z and $T^{-1/2}$ dependence on temperature conforms with the prediction. Also the predicted dependence on energy has been tested [C46] .

2. The value of the effective number n_{eff} of screening electrons deduced from the experimental data is consistent with $n_{eff}(Hall)$ deduced from quantum Hall effect.

The model suggests that also the decay rates of nuclei, say beta and alpha decay rates, could be affected by electron screening. There is already preliminary evidence for the reduction of beta decay rate of ^{22}Na β decay rate in Pd [C45] , metal which is utilized also in cold fusion experiments. This might have quite far reaching technological implications. For instance, the artificial reduction of half-lives of the radioactive nuclei could allow an effective treatment of radioactive wastes. An interesting question is whether screening effect could explain cold fusion [C112] and sono-fusion [C66] .

Electron screening and Trojan horse mechanism

These experimental findings allow to quantify the Trojan horse mechanism. The idea is that projectile nucleus enters the region of the target nucleus along a larger space-time sheet and in this manner avoids the Coulomb wall. The nuclear reaction itself occurs conventionally. In conductors the space-time sheet of conduction electrons is a natural candidate for the larger space-time sheet.

At conduction electron space-time sheet there is a constant charged density consisting of n_{eff} electrons in the atomic volume $V = 1/n_a$. This creates harmonic oscillator potential in which incoming proton accelerates towards origin. The interaction energy at radius r is given by

$$V(r) = \alpha n_{eff}\frac{r^2}{2a^3} , \tag{7.8.4}$$

where a is atomic radius.

The proton ends up to this space-time sheet by a thermal kick compensating the harmonic oscillator energy. This occurs below with a high probability below radius R for which the thermal energy $E = T/2$ of electron corresponds to the energy in the harmonic oscillator potential. This gives the condition

$$R = \sqrt{\frac{Ta}{n_{eff}\alpha}}a \ . \tag{7.8.5}$$

This condition is exactly of the same form as the condition given by Debye model for electron screening but has a completely different physical interpretation.

Since the proton need not travel through the nuclear Coulomb potential, it effectively gains the energy

$$E_e = Z\frac{\alpha}{R} = \frac{Z\alpha^{3/2}}{a}\sqrt{\frac{n_{eff}}{Ta}} \ . \tag{7.8.6}$$

which would be otherwise lost in the repulsive nuclear Coulomb potential. Note that the contribution of the thermal energy to E_e is neglected. The dependence on the parameters involved is exactly the same as in the case of Debye model. For $T = 373$ K in the ^{176}Lu experiment and $n_{eff}(Lu) = 2.2 \pm 1.2$, and $a = a_0 = .52 \times 10^{-10}$ m (Bohr radius of hydrogen as estimate for atomic radius), one has $E_e = 28.0$ keV to be compared with $U_e = 21 \pm 6$ keV of [C46] ($a = 10^{-10}$m corresponds to 1.24×10^4 eV and 1 K to 10^{-4} eV). A slightly larger atomic radius allows to achieve consistency. The value of $\hbar$ does not play any role in this model since the considerations are purely classical.

An interesting question is what the model says about the decay rates of nuclei in conductors. For instance, if the proton from the decaying nucleus can enter directly to the space-time sheet of the conduction electrons, the Coulomb wall corresponds to the Coulomb interaction energy of proton with conduction electrons at atomic radius and is equal to $\alpha n_{eff}/a$ so that the decay rate should be enhanced.

Trojan horse mechanism realized in this manner does not seem to explain the basic findings about cold fusion. Trojan horse mechanism applied to deuterium projectile and D-Pd target would predict standard nuclear physics. The reported strong suppression of ^{3}He production with respect to ^3H production however requires non-standard nuclear physics and the model discussed in the previous subsection provides this physics. Both mechanisms could of course be involved.

Chapter 8

Nuclear String Hypothesis

8.1 Introduction

Nuclear string hypothesis [K80] is one of the most dramatic almost-predictions of TGD [K72]. The hypothesis in its original form assumes that nucleons inside nucleus organize to closed nuclear strings with neighboring nuclei of the string connected by exotic meson bonds consisting of color magnetic flux tube with quark and anti-quark at its ends. The lengths of flux tubes correspond to the p-adic length scale of electron and therefore the mass scale of the exotic mesons is around 1 MeV in accordance with the general scale of nuclear binding energies. The long lengths of em flux tubes increase the distance between nucleons and reduce Coulomb repulsion. A fractally scaled up variant of ordinary QCD with respect to p-adic length scale would be in question and the usual wisdom about ordinary pions and other mesons as the origin of nuclear force would be simply wrong in TGD framework as the large mass scale of ordinary pion indeed suggests. The presence of exotic light mesons in nuclei has been proposed also by Illert [C88] based on evidence for charge fractionization effects in nuclear decays.

8.1.1 $A > 4$ nuclei as nuclear strings consisting of $A \leq 4$ nuclei

In the sequel a more refined version of nuclear string hypothesis is developed.

1. The first refinement of the hypothesis is that ^4He nuclei and $A < 4$ nuclei and possibly also nucleons appear as basic building blocks of nuclear strings instead of nucleons which in turn can be regarded as strings of nucleons. Large number of stable lightest isotopes of form $A = 4n$ supports the hypothesis that the number of ^4He nuclei is maximal. One can hope that even also weak decay characteristics could be reduced to those for $A < 4$ nuclei using this hypothesis.

2. One can understand the behavior of nuclear binding energies surprisingly well from the assumptions that total *strong* binding energy associated with $A \leq 4$ building blocks is *additive* for nuclear strings and that the addition of neutrons tends to reduce Coulomb energy per string length by increasing the length of the nuclear string implying increase binding energy and stabilization of the nucleus. This picture does not explain the variation of binding energy per nucleon and its maximum appearing for ^{56}Fe.

3. In TGD framework tetra-neutron [C51, C22] is interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants [K80] . For heavier nuclei tetra-neutron is needed as an additional building brick and the local maxima of binding energy E_B per nucleon as function of neutron number are consistent with the presence of tetra-neutrons. The additivity of magic numbers $2, 8, 20, 28, 50, 82, 126$ predicted by nuclear string hypothesis is also consistent with experimental facts and new magic numbers are predicted [C100, C39] .

8.1.2 Bose-Einstein condensation of color bonds as a mechanism of nuclear binding

The attempt to understand the variation of the nuclear binding energy and its maximum for Fe leads to a quantitative model of nuclei lighter than Fe as color bound Bose-Einstein condensates of ^4He nuclei or rather, of pion like colored states associated with color flux tubes connecting ^4He nuclei. The crucial element of the model is that color contribution to the binding energy is proportional to n^2 where n is the number of color bonds. Fermi statistics explains the reduction of E_B for the nuclei heavier than Fe. Detailed estimate favors harmonic oscillator model over free nucleon model with oscillator strength having interpretation in terms of string tension.

Fractal scaling argument allows to understand ^4He and lighter nuclei as strings formed from nucleons with nucleons bound together by color bonds. Three fractally scaled variants of QCD corresponding $A > 4$ nuclei, $A = 4$ nuclei and $A < 4$ nuclei are thus involved. The binding energies of also lighter nuclei are predicted surprisingly accurately by applying simple p-adic scaling to the parameters of model for the electromagnetic and color binding energies in heavier nuclei.

8.1.3 Giant dipole resonance as de-coherence of Bose-Einstein condensate of color bonds

Giant (dipole) resonances [C5, C71, C49] , and so called pygmy resonances [C12, C52] interpreted in terms of de-coherence of the Bose-Einstein condensates associated with $A \leq 4$ nuclei and with the nuclear string formed from $A \leq 4$ nuclei provide a unique test for the model. The key observation is that the splitting of the Bose-Einstein condensate to pieces costs a precisely defined energy due to the n^2 dependence of the total binding energy. For ^4He de-coherence the model predicts singlet line at 12.74 MeV and triplet (25.48, 27.30,29.12) MeV at ~ 27 MeV spanning 4 MeV wide range which is of the same order as the width of the giant dipole resonance for nuclei with full shells.

The de-coherence at the level of nuclear string predicts 1 MeV wide bands 1.4 MeV above the basic lines. Bands decompose to lines with precisely predicted energies. Also these contribute to the width. The predictions are in a surprisingly good agreement with experimental values. The so called pygmy resonance appearing in neutron rich nuclei can be understood as a de-coherence for $A = 3$ nuclei. A doublet (7.520,8.4600) MeV at ~ 8 MeV is predicted. At least the prediction for the position is correct.

8.1.4 Dark nuclear strings as analogs of as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code

One biological speculation [K87] inspired by the dark matter hierarchy is that genetic code as well as DNA-, RNA- and amino-acid sequences should have representation in terms of dark nuclear strings. The model for dark baryons indeed leads to an identification of these analogs and the basic numbers of genetic code including also the numbers of amino-acids coded by a given number of codons are predicted correctly. Hence it seems that genetic code is universal rather than being an accidental outcome of the biological evolution.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Applications of TGD [L22]
- Geometrization of fields [L26]
- Widom-Larsen theory from TGD point of view [L44]
- Nuclear string model [L33]

8.2 Some variants of the nuclear string hypothesis

The basic assumptions of the nuclear string model could be made stronger in several testable ways. One can make several alternative hypothesis.

8.2.1 Could linking of nuclear strings give rise to heavier stable nuclei?

Nuclear strings (Z_1, N_1) and (Z_2, N_2) could link to form larger nuclei $(Z_1 + Z_2, N_1 + N_2)$. If one can neglect the interactions between linked nuclei, the properties of the resulting nuclei should be determined by those of composites. Linking should however be the confining interaction forbidding the decay of the stable composite. The objection against this option is that it is difficult to characterize the constraint that strings are not allowed to touch and there is no good reason forbidding the touching.

The basic prediction would be that if the nuclei (Z_1, N_1) and (Z_2, N_2) which are stable, very long-lived, or possess exceptionally large binding energy then also the nucleus $(Z_1 + Z_2, N_1 + N_2)$ has this property. If the linked nuclear strings are essentially free then the expectation is that the half-life of a composite of unstable nuclei is that of the shorter lived nucleus. This kind of regularity would have been probably observed long time ago.

8.2.2 Nuclear strings as connected sums of shorter nuclear strings?

Nuclear strings can form connected sum of the shorter nuclear strings. Connected sum means that one deletes very short portions of nuclear string A and B and connects the resulting ends of string A and B together. In other words: A is inserted inside B or vice versa or A and B are cut to open strings and connected and closed again. This outcome would result when A and B touch each other at some point. If touching occurs at several points more complex fusion of nuclei to a larger nucleus to a composite occurs with piece of A followed by a piece of B followed... For this option there is a non-trivial interaction between strings and the properties of nuclei need not be simply additive but one might still hope that stable nuclei fuse to form stable nuclei. In particular, the prediction for the half-life based on binding by linking does not hold true anymore.

Classical picture would suggest that the two strings cannot rotate with respect to each other unless they correspond to rather simple symmetric configurations: this applies also to linked strings. If so then the relative angular momentum L of nuclear strings vanishes and total angular momentum J of the resulting nucleus satisfies $|J_1 - J_2| \leq J \leq J_1 + J_2$.

8.2.3 Is knotting of nuclear strings possible?

One can consider also the knotting of nuclear strings as a mechanism giving rise to exotic excitations of nuclear. Knots decompose to prime knots so that kind of prime nuclei identified in terms of prime knots might appear. Fractal thinking suggests an analogy with the poorly understood phenomenon of protein folding. It is known that proteins always end up to a unique highly folded configuration and one might think that also nuclear ground states correspond to unique configurations to which quantum system (also proteins would be such if dark matter is present) ends up via quantum tunnelling unlike classical system which would stick into some valley representing a state of higher energy. The spin glass degeneracy suggests an fractal landscape of ground state configurations characterized by knotting and possibly also linking.

8.3 Could nuclear strings be connected sums of alpha strings and lighter nuclear strings?

The attempt to kill the composite string model leads to a stronger formulation in which nuclear string consists of alpha particles plus a minimum number of lighter nuclei. To test the basic predictions of the model I have used the rather old tables of [C87] for binding energies of stable and long-lived isotopes and more modern tables [C14] for basic data about isotopes known recently.

8.3.1 Does the notion of elementary nucleus make sense?

The simplest formulation of the model assumes some minimal set of *stable* "elementary nuclei" from which more complex *stable* nuclei can be constructed.

1. If heavier nuclei are formed by *linking* then alpha particle $^4He = (Z, N) = (2, 2)$ suggests itself as the lightest stable composite allowing interpretation as a closed string. For connected sum option even single nucleon n or p can appear as a composite. This option turns out to be the more plausible one.

2. In the model based on linking $^6Li = (3, 3)$ and $^7Li = (3, 4)$ would also act as "elementary nuclei" as well as $^9Be = (4, 5)$ and $^{10}Be = (4, 6)$. For the model based on connected sum these nuclei might be regarded as composites $^6Li = (3, 3) = (2, 2) + (1, 1)$, $^7Li = (3, 4) = (2, 2) + (1, 2)$, $^9Be = (4, 5) = 2 \times (2, 2) + (0, 1)$ and $^{10}Be = (4, 6) = (2, 2) + 2 \times (1, 2)$. The study of binding energies supports the connected sum option.

3. ^{10}B has total nuclear spin $J = 3$ and $^{10}B = (5, 5) = (3, 3) + (2, 2) =^6 Li +^4 He$ makes sense if the composites can be in relative $L = 2$ state (^6Li has $J = 1$ and ^4He has $J = 0$). ^{11}B has $J = 3/2$ so that $^{11}B = (5, 6) = (3, 4) + (2, 2) =^7 Li +^4 He$ makes sense because ^7Li has $J = 3/2$. For the model based on disjoint linking also 10^B would be also regarded as "elementary nucleus". This asymmetry disfavors the model based on linking.

8.3.2 Stable nuclei need not fuse to form stable nuclei

The question is whether the simplest model predicts stable nuclei which do not exist. In particular, are the linked ^4He composites stable? The simplest case corresponds to $^8B = (4, 4) =^4 He +^4 He$ which is not stable against alpha decay. Thus stable nuclei need not fuse to form stable nuclei. On the other hand, the very instability against alpha decay suggests that ^4B can be indeed regarded as composite of two alpha particles. A good explanation for the instability against alpha decay is the exceptionally large binding energy $E = 7.07$ MeV per nucleon of alpha particle. The fact that the binding energy per nucleon for ^8Be is also exceptionally large and equal to 7.06 MeV $< E_B(^4He)$ supports the interpretation as a composite of alpha particles.

For heavier nuclei binding energy per nucleon increases and has maximum 8.78 MeV for Fe. This encourages to consider the possibility that alpha particle acts as a fundamental composite of nuclear strings with minimum number of lighter isotopes guaranteeing correct neutron number. Indeed, the decomposition to a maximum number of alpha particles allows a qualitative understanding of binding energies assuming that additional contribution not larger than 1.8 MeV per nucleon is present.

The nuclei ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{36}A, and ^{40}Ca are lightest stable isotopes of form $(Z, Z) = n \times^4 He$, $n = 3, ..., 10$, for which E_B is larger than for ^4He. For the first four nuclei E_B has a local maximum as function of N. For the remaining the maximum of E_B is obtained for $(Z, Z + 1)$. $^{44}Ti = (22, 22)$ does not exist as a long-lived isotope whereas ^{45}Ti does. The addition of neutron could increase E_B by increasing the length of nuclear string and thus reducing the Coulomb interaction energy per nucleon. This mechanism would provide an explanation also for neutron halos [C84] .

Also the fact that stable nuclei in general have $N \geq Z$ supports the view that $N = Z$ state corresponds to string consisting of alpha particles and that $N > Z$ states are obtained by adding something between. $N < Z$ states would necessarily contain at least one stable nucleus lighter than ^4He with smaller binding energy. ^3He is the only possible candidate as the only stable nucleus with $N < Z$. ($E_B(^2H) = 1.11$ MeV and $E_B(^3He) = 2.57$ MeV). Individual nucleons are also possible in principle but not favored. This together with increase of Coulomb interaction energy per nucleon due to the greater density of em charge per string length would explain their smaller binding energy and instability.

8.3.3 Formula for binding energy per nucleon as a test for the model

The study of ^8B inspires the hypothesis that the total binding energy for the nucleus (Z_1+Z_2, N_1+N_2) is in the first approximation the sum of total binding energies of composites so that one would have for the binding energy per nucleon the prediction

$$E_B = \frac{A_1}{A_1 + A_2} \times E_{B_1} + \frac{A_2}{A_1 + A_2} \times E_{B_2}$$

in the case of 2-nucleus composite. The generalization to N-nucleus composite would be

$$E_B = \sum_k \frac{A_k}{\sum_r A_r} \times E_{B_k} \ .$$

This prediction would apply also to the unstable composites. The increase of binding energy with the increase of nuclear weight indeed suggests a decomposition of nuclear string to a sequence alpha strings plus some minimum number of shorter strings.

The first objection is that for both Li, B, and Be which all having two stable isotopes, the lighter stable isotope has a slightly smaller binding energy contrary to the expectation based on additivity of the total binding energy. This can be however understood in terms of the reduction of Coulomb energy per string length resulting in the addition of neutron (protons have larger average distance along nuclear string along mediating the electric flux) . The reduction of Coulomb energy per unit length of nuclear string could also partially explain why one has $E_B > E_B(^4He)$ for heavier nuclei.

The composition $^6Li = (3,3) = (2,2) + (1,1)$ predicts $E_B \simeq 5.0$ MeV not too far from 5.3 MeV. The decomposition $^7Li = (3,4) = (2,2) + (1,2)$ predicts $E_B = 5.2$ MeV to be compared with 5.6 MeV so that the agreement is satisfactory. The decomposition $^8Be = (4,4) = 2\times\ ^4He$ predicts $E_B = 7.07$ MeV to be compared with the experimental value 7.06 MeV. ^9Be and ^{10}Be have $E_B = 6.46$ MeV and $E_B = 6.50$ MeV. The fact that binding energy slightly increases in addition of neutron can be understood since the addition of neutrons to ^8Be reduces the Coulomb interaction energy per unit length. Also neutron spin pairing reduces E_B. The additive formula for E_B is satisfied with an accuracy better than 1 MeV also for ^{10}B and ^{11}B.

8.3.4 Decay characteristics and binding energies as signatures of the decomposition of nuclear string

One might hope of reducing the weak decay characteristics to those of shortest unstable nuclear strings appearing in the decomposition. Alternatively, one could deduce the decomposition from the weak decay characteristics and binding energy using the previous formulas. The picture of nucleus as a string of alpha particles plus minimum number of lighter nuclei ^3He having $E_B = 2.57$ MeV, ^3H unstable against beta decay with half-life of 12.26 years and having $E_B = 2.83$ MeV, and ^2H having $E_B = 1.1$ MeV gives hopes of modelling weak decays in terms of decays for these light composites.

1. β^- decay could be seen as a signature for the presence of ^3H string and alpha decay as a signature for the presence of ^4He string.

2. β^+ decay might be interpreted as a signature for the presence of ^3He string which decays to ^3H (the mass of ^3H is only .018 MeV higher than that of ^3He). For instance, $^8B = (5,3) = (3,2) + (2,1) = {}^5Li + {}^3He$ suffers β^+ decay to $^8Be = (4,4)$ which in turn decays by alpha emission which suggests the re-arrangement to $(3,2) + (1,2) \rightarrow (2,2) + (2,2)$ maximizing binding energy.

3. Also individual nucleons can appear in the decomposition and give rise to β^- and possible also β^+ decays.

8.3.5 Are magic numbers additive?

The magic numbers $2, 8, 20, 28, 50, 82, 126$ [C100] for protons and neutrons are usually regarded as a support for the harmonic oscillator model. There are also other possible explanations for magic nuclei and there are deviations from the naive predictions. One can also consider several different criteria for what it is to be magic. Binding energy is the most natural criterion but need not always mean stability. For instance $^8B = (4,4) = {}^4He + {}^4He$ has high binding energy but is unstable against alpha decay.

Nuclear string model suggests that the fusion of magic nuclear strings by connected sum yields new kind of highly stable nuclei so that also $(Z_1 + Z_2, N_1 + N_2)$ is a magic nucleus if (Z_i, N_i) is such. One has $N = 28 = 20 + 8$, $50 = 28 + 20 + 2$, and $N = 82 = 50 + 28 + 2 \times 2$. Also other magic numbers are predicted. There is evidence for them [C39] .

1. $^{16}O = (8, 8)$ and $^{40}Ca = (20, 20)$ corresponds to doubly magic nuclei and $^{60}Ni = (28, 32) = (20, 20) + (8, 8) +^4 n$ has a local maximum of binding energy as function of neutron number. This is not true for ^{56}Ni so that the idea of magic nucleus in neutron sector is not supported by this case. The explanation would be in terms of the reduction of E_B due to the reduction of Coulomb energy per string length as neutrons are added.

2. Also $^{80}Kr = (36, 44) = (36, 36) +^4 n = (20, 20) + (8, 8) + (8, 8) +^4 n$ corresponds to a local maximum of binding energy per nucleon as also does $^{84}Kr =^{80} Kr +^4 n$ containing two tetra-neutrons. Note however that $^{88}Zr = (40, 48)$ is not a stable isotope although it can be regarded as a composite of doubly magic nucleus and of two tetra-neutrons.

8.3.6 Stable nuclei as composites of lighter nuclei and necessity of tetra-neutron?

The obvious test is to look whether stable nuclei can be constructed as composites of lighter ones. In particular, one can check whether tetra-neutron ^4n interpreted as a variant of alpha particle obtained by replacing two meson-like stringy bonds connecting neighboring nucleons of the nuclear string with their negatively charged variants is necessary for the understanding of heavier nuclei.

1. $^{48}Ca = (20, 28)$ with half-life $> 2 \times 10^{16}$ years has neutron excess of 8 units and the only reasonable interpretation seems to be as a composite of the lightest stable Ca isotope $Ca(20, 20)$, which is doubly magic nucleus and two tetra-neutrons: $^{48}Ca = (20, 28) =^{40} Ca + 2 \times^4 n$.

2. The next problematic nucleus is ^{49}Ti.

   i) $^{49}Ti = (22, 27)$ having neutron excess of 5 one cannot be expressed as a composite of lighter nuclei unless one assumes non-vanishing and large relative angular momentum for the composites. For $^{50}Ti = (22, 28)$ no decomposition can be found. The presence of tetra-neutron would reduce the situation to $^{49}Ti = (22, 27) =^{45} Ti +^4 n$. Note that ^{45}Ti is the lightest Ti isotope with relatively long half-life of 3.10 hours so that the addition of tetra-neutron would stabilize the system since Coulomb energy per length of string would be reduced.

   ii) ^{48}Ti could not involve tetra-neutron by this criterion. It indeed allows decomposition to standard nuclei is also possible as $^{48}Ti = (22, 26) =^{41} K +^7 Li$.

   iii) The heaviest stable Ti isotope would have the decomposition $^{50}Ti =^{46} Ti +^4 n$, where ^{46}Ti is the lightest stable Ti isotope.

3. The heavier stable nuclei $^{50+k}V = (23, 27 + k)$, $k = 0, 1$, $^{52+k}Cr = (24, 28 + k)$, $k = 0, 1, 2$, $^{55}Mn = (25, 30)$ and $^{56+k}Fe = (26, 30 + k)$, $k = 0, 1, 2$ would have similar interpretation. The stable isotopes $^{50}Cr = (24, 26)$ and $^{54}Fe = (26, 28)$ would not contain tetra-neutron. Also for heavier nuclei both kinds of stable states appear and tetra-neutron would explain this.

4. $^{112}Sn = (50, 62) = (50, 50) + 3 \times^4 n$, ^{116}Sn, ^{120}Sn, and ^{124}Sn are local maxima of E_B as a function of neutron number and the interpretation in terms of tetra-neutrons looks rather natural. Note that $Z = 50$ is a magic number.

Nuclear string model looks surprisingly promising and it would be interesting to compare systematically the predictions for E_B with its actual values and look whether the beta decays could be understood in terms of those of composites lighter than ^4He.

8.3.7 What are the building blocks of nuclear strings?

One can also consider several options for the more detailed structure of nuclear strings. The original model assumed that proton and neutron are basic building blocks but this model is too simple.

Option Ia)

A more detailed work in attempt to understand binding energies led to the idea that there is fractal structure involved. At the highest level the building blocks of nuclear strings are $A \leq 4$ nuclei. These nuclei in turn would be constructed as short nuclear strings of ordinary nucleons.

The basic objection against the model is the experimental absence of stable $n-n$ bound state analogous to deuteron favored by lacking Coulomb repulsion and attractive electromagnetic spin-spin interaction in spin 1 state. Same applies to tri-neutron states and possibly also tetra-neutron state. There has been however speculation about the existence of di-neutron and poly-neutron states [C3, C11] .

The standard explanation is that strong force couples to strong isospin and that the repulsive strong force in nn and pp states makes bound states of this kind impossible. This force, if really present, should correspond to shorter length scale than the isospin independent forces in the model under consideration. In space-time description these forces would correspond to forces mediated between nucleons along the space-time sheet of the nucleus whereas exotic color forces would be mediated along the color magnetic flux tubes having much longer length scale. Even for this option one cannot exclude exotic di-neutron obtained from deuteron by allowing color bond to carry negative em charge. Since em charges $0, 1, -1$ are possible for color bonds, a nucleus with mass number $A > 2$ extends to a multiplet containing $3A$ exotic charge states.

Option Ib)

One might ask whether it is possible to get rid of isospin dependent strong forces and exotic charge states in the proposed framework. One can indeed consider also other explanations for the absence of genuine poly-neutrons.

1. The formation of negatively charged bonds with neutrons replaced by protons would minimize both nuclear mass and Coulomb energy although binding energy per nucleon would be reduced and the increase of neutron number in heavy nuclei would be only apparent.
2. The strongest hypothesis is that mass minimization forces protons and negatively charged color bonds to serve as the basic building bricks of all nuclei. If this were the case, deuteron would be a di-proton having negatively charged color bond. The total binding energy would be only $2.222 - 1.293 = .9290$ MeV. Di-neutron would be impossible for this option since only one color bond can be present in this state.

The small mass difference $m(^3He) - m(^3H) = .018$ MeV would have a natural interpretation as Coulomb interaction energy. Tri-neutron would be allowed. Alpha particle would consist of four protons and two negatively charged color bonds and the actual binding energy per nucleon would be by $(m_n - m_p)/2$ smaller than believed. Tetra-neutron would also consist of four protons and the binding energy per nucleon would be smaller by $m_n - m_p$ than what obtains in the standard model of nucleus. Beta decays would be basically beta decays of exotic quarks associated with color bonds.

Note that the mere assumption that the di-neutrons appearing inside nuclei have protons as building bricks means a rather large apparent binding energy this might explain why di-neutrons have not been detected. An interesting question is whether also higher n-deuteron states than ^4He consisting of strings of deuteron nuclei and other $A \leq 3$ nuclei could exist and play some role in the nuclear physics of $Z \neq N$ nuclei.

If protons are the basic building bricks, the binding energy per nucleon is replaced in the calculations with its actual value

$$E_B \quad \rightarrow \quad E_B - \frac{N}{A}\Delta m \ , \ \ \Delta m = m_n - m_p = 1.2930 \ \ MeV \ . \tag{8.3.1}$$

This replacement does not affect at all the parameters of the of $Z = 2n$ nuclei identified as ^4He strings.

One can of course consider also the option that nuclei containing ordinary neutrons are possible but that are unstable against beta decay to nuclei containing only protons and negatively charged bonds. This would suggest that di-neutron exists but is not appreciably produced in nuclear reactions and has not been therefore detected.

Options IIa) and IIb)

It is not clear whether the fermions at the ends of color bonds are exotic quarks or leptons. Lepto-pion (or electro-pion) hypothesis [K88] was inspired by the anomalous e^+e^- production in heavy ion collisions near Coulomb wall and states that electro-pions which are bound states of colored excitations of electrons with ground state mass 1.062 MeV are responsible for the effect. The model predicts that also other charged leptons have color excitations and give rise to exotic counterpart of QCD.

Also μ and τ should possess colored excitations. About fifteen years after this prediction was made, direct experimental evidence for these states finally emerges [C126, C127] . The mass of the new particle, which is either scalar or pseudo-scalar, is 214.4 MeV whereas muon mass is 105.6 MeV. The mass is about 1.5 per cent higher than two times muon mass. The most natural TGD inspired interpretation is as a pion like bound state of colored excitations of muon completely analogous to lepto-pion (or rather, electro-pion) [K88] .

One cannot exclude the possibility that the fermion and anti-fermion at the ends of color flux tubes connecting nucleons are actually colored leptons although the working hypothesis is that they are exotic quark and anti-quark. One can of course also turn around the argument: could it be that lepto-pions are "leptonuclei", that is bound states of ordinary leptons bound by color flux tubes for a QCD in length scale considerably shorter than the p-adic length scale of lepton.

Scaling argument applied to ordinary pion mass suggests that the masses of exotic quarks at the ends of color bonds are considerably below MeV scale. One can however consider the possibility that colored electrons with mass of ordinary electron are in question in which case color bonds identifiable as colored variants of electro-pions could be assumed to contribute in the first guess the mass $m(\pi) = 1.062$ MeV per each nucleon for $A > 2$ nuclei. This implies the general replacement

$$
\begin{aligned}
E_B &\rightarrow E_B + m(\pi_L) - \frac{N}{A}\Delta m \;\text{ for }\; A > 2 \;, \\
E_B &\rightarrow E_B + \frac{m(\pi_L)}{2} - \frac{N}{A}\Delta m \;\text{ for }\; A = 2 \;.
\end{aligned} \tag{8.3.2}
$$

This option will be referred to as option IIb). One can also consider the option IIa) in which nucleons are ordinary but lepto-pion mass $m(\pi_L) = 1.062$ MeV gives the mass associated with color bond.

These options are equivalent for $N = Z = 2n$ nuclei with $A > 4$ but for $A \leq 4$ nuclei assumed to form nucleon string they options differ.

8.4 Light nuclei as color bound Bose-Einstein condensates of ^4He nuclei

The attempt to understand the variation of nuclear binding energy and its maximum for Fe leads to a model of nuclei lighter than Fe as color bound Bose-Einstein condensates of ^4He nuclei or meson-like structures associated with them. Fractal scaling argument allows to understand ^4He itself as analogous state formed from nucleons.

8.4.1 How to explain the maximum of E_B for iron?

The simplest model predicts that the binding energy per nucleon equals to $E_B(^4He)$ for all $Z = N = 2n$ nuclei. The actual binding energy grows slowly, has a maximum at ^{52}Fe, and then begins to decrease but remains above $E_B(^4He)$. The following values give representative examples for $Z = N$ nuclei.

nucleus	^4He	^8Be	^{40}Ca	^52Fe
E_B/MeV	7.0720	7.0603	8.5504	8.6104

For nuclei heavier than Fe there are no long-lived $Z = N = 2n$ isotopes and the natural reason would be alpha decay to ^{52}Fe. If tetra-neutron is what TGD suggests it to be one can guess that tetra-neutron mass is very nearly equal to the mass of the alpha particle. This would allow to regard states $N = Z + 4n$ as states as analogous to unstable states $N_1 = Z_1 = Z + 2n$ consisting of alpha particles. This gives estimate for E_B for unstable $N = Z$ states. For $^{256}Fm = (100, 156)$ one has $E_B = 7.433$ MeV which is still above $E_B(^4He) = 7.0720$ MeV. The challenge is to understand the variation of the binding energy per nucleon and its maximum for Fe.

8.4.2 Scaled up QCD with Bose-Einstein condensate of ^4He nuclei explains the growth of E_B

The first thing to come in mind is that repulsive Coulomb contribution would cause the variation of the binding energy. Since alpha particles are building blocks for $Z = N$ nuclei, ^8Be provides a test for this idea. If the difference between binding energies per nucleon for ^8Be and ^4He were due to Coulomb repulsion alone, one would have $E_c = E_B(^4He) - E_B(^8Be) = .0117$ MeV, which is of order $\alpha_{em}/L(127)$. This would conform with the idea that flux tubes mediating em interaction have length of order electron Compton length. Long flux tubes would provide the mechanism minimizing Coulomb energy. A more realistic interpretation consistent with this mechanism would be that Coulomb and color interaction energies compensate each other: this can of course occur to some degree but it seems safe to assume that Coulomb contribution is small.

The basic question is how one could understand the behavior of E_B if its variation corresponds to that for color binding energy per nucleon. The natural scale of energy is MeV and this conforms with the fact that the range of variation for color binding energy associated with $L(127)$ QCD is about 1.5 MeV. By a naive scaling the value of M_{127} pion mass is by a factor $2^{(127-107)/2} = 10^{-3}$ times smaller than that of ordinary pion and thus .14 MeV. The scaling of QCD Λ is a more reliable estimate for the binding energy scale and gives a slightly larger value but of the same order of magnitude. The total variation of E_B is large in the natural energy scale of M_{127} QCD and suggests strong non-linear effects.

In the absence of other contributions em and color contributions to E_B cancel for ^8Be. If color and Coulomb contributions on total binding energy depend roughly linearly on the number of ^4He nuclei, the cancellation to E_B should occur in a good approximation also for them. This does not happen which means that color contribution to E_B is in lowest approximation linear in n meaning n^2-dependence of the total color binding energy. This non-linear behavior suggests strongly the presence of Bose-Einstein condensate of ^4He nuclei or structures associated with them. The most natural candidates are the meson like colored strings connecting ^4He nuclei together.

The additivity of n color magnetic (and/or electric) fluxes would imply that classical field energy is n^2-fold. This does not yet imply same for binding energy unless the value of α_s is negative which it can be below confinement length scale. An alternative interpretation could be in terms of color magnetic interaction energy. The number of quarks and anti-quarks would be proportional to n as would be also the color magnetic flux so that n^2- proportionality would result also in this manner.

If the addition of single alpha particle corresponds to an addition of a constant color contribution E_s to E_B (the color binding energy per nucleon, not the total binding energy!) one has $E_B(^{52}Fe) = E_B(^4He) + 13E_s$ giving $E_s = .1834$ MeV, which conforms with the order of magnitude estimate given by M_{127} QCD.

The task is to find whether this picture could explain the behavior of E_B. The simplest formula for $E_B(Z = N = 2n)$ would be given by

$$E_B(n) = -\frac{n(n-1)}{L(A)n}k_s + nE_s \ . \tag{8.4.1}$$

Here the first term corresponds to the Coulomb interaction energy of n ^4He nuclei proportional to $n(n-1)$ and inversely proportional to the length $L(A)$ of nuclear string. Second term is color binding energy per nucleon proportional to n.

The simplest assumption is that each ^4He corresponds always to same length of nuclear string so that one has $L \propto A$ and one can write

$$E_B(n) = E_B(^4He) - \frac{n(n-1)}{n^2}E_c + nE_s \ . \tag{8.4.2}$$

The value of $E_B(^8Be) \simeq E_B(^4He)$ $(n=2)$ gives for the unit of Coulomb energy

$$E_c = 4E_s + 2[E_B(^4He) - E_B(^8Be)] \simeq 4E_s \ . \tag{8.4.3}$$

The general formula for the binding energy reads as

$$\begin{aligned} E_B(n) &= E_B(^4He) - 2\frac{n(n-1)}{n^2}[E_B(^4He) - E_B(^8Be)] \\ &+ [-4\frac{n(n-1)}{n^2} + n]E_s \ . \end{aligned} \tag{8.4.4}$$

The condition that $E_B(^{52}Fe)$ $(n=13)$ comes out correctly gives

$$E_s = \frac{13}{121}(E_B(^{52}Fe) - E_B(^4He)) + \frac{13\times 24}{121}[E_B(^4He) - E_B(^8Be)] \ . \tag{8.4.5}$$

This gives $E_s \simeq .1955$ MeV which conforms with M_{127} QCD estimate. For the E_c one obtains $E_c = 1.6104$ MeV and for Coulomb energy of ^4He nuclei in ^8Be one obtains $E = E_c/2 = .8052$ MeV. The order of magnitude is consistent with the mass difference of proton and neutron. The scale suggests that electromagnetic flux tubes are shorter than color flux tubes and correspond to the secondary p-adic length scale $L(2,61) = L(127)/2^{5/2}$ associated with Mersenne prime M_{61}. The scaling factor for the energy scale would be $2^{5/2} \simeq 5.657$.

The calculations have been carried out without assuming which are actual composites of ^4He nuclei (neutrons and protons plus neutral color bonds or protons and neutral and negatively charged color bonds) and assuming the masses of color bonds are negligible. As a matter fact, the mass of color bond does not affect the estimates if one uses only nuclei heavier than ^4He to estimate the parameters. The estimates above however involve ^4He so that small change on the parameters is induced.

8.4.3 Why E_B decreases for heavier nuclei?

The prediction that E_B increases as $(A/4)^2$ for $Z=N$ nuclei is unrealistic since E_B decreases slowly for $A \geq 52$ nuclei. Fermi statistics provides a convincing explanation assuming that fermions move in an effective harmonic oscillator potential due to the string tension whereas free nucleon model predicts too large size for the nucleus. The splitting of the Bose-Einstein condensate to pieces is second explanation that one can imagine but fails at the level of details.

Fermi statistics as a reason for the reduction of the binding energy

The failure of the model is at least partially due to the neglect of the Fermi statistics. For the lighter nuclei description as many boson state with few fermions is expected to work. As the length of nuclear string grows in fixed nuclear volume, the probability of self intersection increases and Fermi statistics forces the wave function for stringy configurations to wiggle which reduces binding energy.

1. For the estimation purposes consider $A = 256$ nucleus ^{256}Mv having $Z = 101$ and $E_B = 7.4241$ MeV. Assume that this unstable nucleus is nearly equivalent with a nucleus consisting of $n = 64$ ^4He nuclei $(Z=N)$. Assuming single color condensate this would give the color contribution

$$E_s^{tot} = (Z/2)^2 \times E_s = 64^2 \times E_s$$

with color contribution to E_B equal to $(Z/2)E_s \simeq 12.51$ MeV.

2. Suppose that color binding energy is canceled by the energy of nucleon identified as kinetic energy in the case of free nucleon model and as harmonic oscillator energy in the case of harmonic oscillator model.

3. The number of states with a given principal quantum number n for both free nucleons in a spherical box and harmonic oscillator model is by spherical symmetry $2n^2$ and the number of protons/neutrons for a full shell nuclei behaves as $N_1 \simeq 2n_{max}^3/3$. The estimate for the average energy per nucleon is given in the two cases as

$$\begin{aligned} \langle E\rangle_H &= 2^{-4/3} \times N^{1/3} E_0 \ , \quad E_0 = \omega_0 \ , \\ \langle E\rangle_F &= \frac{2}{5}(\frac{3}{2})^{5/3} N^{2/3} E_0 \ , \quad E_0 = \frac{\pi^2}{2m_p L^2} \ . \end{aligned} \tag{8.4.6}$$

   Harmonic oscillator energy $\langle E\rangle_H$ increases as $N^{1/3}$ and $\langle E\rangle_F$ as $N^{2/3}$. Neither of these cannot win the contribution of the color binding energy increasing as N.

4. Equating this energy with the total color binding energy gives an estimate for E_0 as

$$\begin{aligned} E_0 &= (2/3)^{1/3} \times Z^{-4/3} \times (Z/2)^2 \times E_s \ , \\ E_0 &= \frac{5}{4}(\frac{2}{3})^{5/3} \times Z^{-5/3} \times (Z/2)^2 \times E_s \ , \\ E_s &= .1955 \ MeV \ . \end{aligned} \tag{8.4.7}$$

   The first case corresponds to harmonic oscillator model and second to free nucleon model.

5. For the harmonic oscillator model one obtains the estimate $E_0 = \hbar\omega_0 \simeq 2.73 \ MeV$. The general estimate for the energy scale in the harmonic oscillator model given by $\omega_0 \simeq 41 \cdot A^{-1/3}$ MeV [C124] giving $\omega_0 = 6.5$ MeV for $A = 256$ (this estimate implies that harmonic oscillator energy per nucleon is approximately constant and would suggest that string tension tends to reduce as the length of string increases). Harmonic oscillator potential would have roughly twice too strong strength but the order of magnitude is correct. Color contribution to the binding energy might relate the reduction of the oscillator strength in TGD framework.

6. Free nucleon model gives the estimate $E_0 = .0626$ MeV. For the size of a $A = 256$ nucleus one obtains $L \simeq 3.8L(113) \simeq 76$ fm. This is by one order of magnitude larger that the size predicted by the standard formula $r = r_0 A^{1/3}$, $r_0 = 1.25$ fm and 8 fm for $A = 256$.

   Harmonic oscillator picture is clearly favored and string tension explains the origin of the harmonic oscillator potential. Harmonic oscillator picture is expected to emerge at the limit of heavy nuclei for which nuclear string more or less fills the nuclear volume whereas for light nuclei the description in terms of bosonic ^4He nuclei should make sense. For heavy nuclei Fermi statistics at nuclear level would begin to be visible and excite vibrational modes of the nuclear string mapped to the excited states of harmonic oscillator in the shell model description.

Could upper limit for the size of ^4He Bose-Einstein condensate explain the maximum of binding energy per nucleon?

One can imagine also an alternative explanation for why E_B to decrease after $A = 52$. One might that $A = 52$ represents the largest ^4He Bose-Einstein condensate and that for heavier nuclei Bose-Einstein condensate de-coheres into two parts. Bose-Einstein condensate of $n = 13$ ^4He nuclei would the best that one can achieve.

This could explain the reduction of the binding energy and also the emergence of tetra-neutrons as well as the instability of $Z = N$ nuclei heavier than ^{52}Fe. A number theoretical interpretation related to the p-adic length scale hypothesis suggests also itself: as the size of the tangled nuclear string becomes larger than the next p-adic length scale, Bose-Einstein condensate might lose its coherence and split into two.

If one assumes that ^4He Bose-Einstein condensate has an upper size corresponding to $n = 13$, the prediction is that after $A = 52$ second Bose-Einstein condensate begins to form. E_B is obtained as the average

$$E_B(Z,N) = \frac{52}{A}E_B(^{52}Fe) + \frac{A-52}{A}E_B(^{A-52}X(Z,N)) \ .$$

The derivative

$$dE_B/dA = (52/A)[-E_B(^{52}Fe) + E_B(^{A-52}X)] + \frac{A-52}{A}dE_B(^{A-52}X(Z,N))/dA$$

is first negative but its sign must change since the nuclei consisting of two copies of ^{52}Fe) condensates have same E_B as ^{52}Fe). This is an un-physical result. This does not exclude the splitting of Bose-Einstein condensate but the dominant contribution to the reduction of E_B must be due to Fermi statistics.

8.5 What QCD binds nucleons to $A \leq 4$ nuclei?

The obvious question is whether scaled variant(s) of color force could bind nucleons to form $A \leq 4$ nuclei which in turn bind to form heavier nuclei. Since the binding energy scale for ^3He is much smaller than for ^4He one might consider the possibility that the p-adic length scale for QCD associated with ^4He is different from that for $A < 4$ nuclei.

8.5.1 The QCD associated with nuclei lighter than ^4He

It would be nice if one could understand the binding energies of also $A \leq 4$ nuclei in terms of a scaled variant of QCD applied at the level of nucleons. Here one has several options to test.

Various options to consider

Assume that neutral color bonds have negligible fermion masses at their ends: this is expected if the exotic quarks appear at the ends of color bonds and by the naive scaling of pion mass. One can also consider the possibility that the p-adic temperature for the quarks satisfies $T = 1/n \leq 1/2$ so that quarks would be massless in excellent approximation. $T = 1/n < 1$ holds true for gauge bosons and one might argue that color bonds as bosonic particles indeed have $T < 1$.

Option Ia): Building bricks are ordinary nucleons.

Option IIa): Building blocks are protons and neutral and negatively charged color bonds. This means the replacement $E_B \rightarrow E_B - \Delta m$ for $A > 2$ nuclei and $E_B \rightarrow E_B - \Delta m/2$ for $A = 2$ with $\Delta m = n_n - m_p = 1.2930$ MeV.

Options Ib and IIb are obtained by assuming that the masses of fermions at the ends of color bonds are non-negligible. Electro-pion mass $m(\pi_L) = 1.062$ MeV is a good candidate for the mass of the color bond. Option Ia allow 3 per cent accuracy for the predicted binding energies. Option IIb works satisfactorily but the errors are below 22 per cent only.

Ordinary nucleons and massless color bonds

It turns out that for the option Ia), ordinary nucleons and massless color bonds, is the most plausible candidate for $A < 4$ QCD is the secondary p-adic length scale $L_e(2,59)$ associated with prime $p \simeq 2^k$, $k = 59$ with $k_{eff} = 2 \times 59 = 118$. The proper scaling of the electromagnetic p-adic length scale corresponds to a scaling factor 2^3 meaning that one has $k_{eff} = 122 \rightarrow k_{eff} - 6 = 116 = 4 \times 29$ corresponding to $L_e(4,29)$.

1. Direct p-adic scaling of the parameters

E_s would be scaled up p-adically by a factor $2^{(127-118)/2} = 2^{9/2}$. E_c would be scaled up by a factor $2^{(122-116)/2} = 2^3$. There is also a scaling of E_c by a factor 1/4 due to the reduction of charge unit and scaling of both E_c and E_s by a factor 1/4 since the basic units are now nucleons. This gives

$$\hat{E}_s = 2^{5/2}E_s = 1.1056 \ MeV \ , \ \hat{E}_c = 2^{-1}E_c = .8056 \ MeV \ . \quad (8.5.1)$$

The value of electromagnetic energy unit is quite reasonable.

The basic formula for the binding energy reads now

$$E_B = -\frac{(n(p)(n(p)-1))}{A^2}\hat{E}_c + n\hat{E}_s \ , \quad (8.5.2)$$

where $n(p)$ is the number of protons $n = A$ holds true for $A > 2$. For deuteron one has $n = 1$ since deuteron has only single color bond. This delicacy is a crucial prediction and the model fails to work without it.

This gives

$$E_B(^2H) = \hat{E}_s \ , \ E_B(^3H) = 3\hat{E}_s \ , \ E_B(^3He) = -\frac{2}{9}\hat{E}_c + 3\hat{E}_s \ . \quad (8.5.3)$$

The predictions are given by the third row of the table below. The predicted values given are too large by about 15 per cent in the worst case.

The reduction of the value of α_s in the p-adic scaling would improve the situation. The requirement that $E_B(^3H)$ comes out correctly predicts a reduction factor .8520 for α_s. The predictions are given in the fourth row of the table below. Errors are below 15 per cent.

nucleus	^2H	^3H	^3He
$E_B(exp)/MeV$	1.111	2.826	2.572
$E_B(pred_1)/MeV$	1.106	3.317	3.138
$E_B(pred_2)/MeV$	.942	2.826	2.647

The discrepancy is 15 per cent for ^2H. By a small scaling of E_c the fit for ^3He can be made perfect. Agreement is rather good but requires that conventional strong force transmitted along nuclear space-time sheet is present and makes *nn* and *pp* states unstable. Isospin dependent strong interaction energy would be only .17 MeV in isospin singlet state which suggests that a large cancelation between scalar and vector contributions occurs. *pnn* and *ppn* could be regarded as Dn and Dp states with no strong force between D and nucleon. The contribution of isospin dependent strong force to E_B is scaled down by a factor 2/3 in $A = 3$ states from that for deuteron and is almost negligible. This option seems to allow an almost perfect fit of the binding energies. Note that one cannot exclude exotic nn-state obtained from deuteron by giving color bond negative em charge.

Other options

Consider next other options.

1. *Option IIb*

For option IIb) the basic building bricks are protons and $m(\pi) = 1.062$ is assumed. The basic objection against this option is that for protons as constituents *real* binding energies satisfy $E_B(^3He) < E_B(^3H)$ whereas Coulombic repulsion would suggest $E_B(^3He) > E_B(^3H)$ unless magnetic spin-spin interaction effects affect the situation. One can however look how good a fit one can obtain in this manner.

As found, the predictions of direct scaling are too large for $E_B(^3H)$ and $E_B(^3He)$ (slight reduction of α_s cures the situation). Since the actual binding energy increases by $m(\pi_L)-(2/3)(m_n-m_p)$ for ^3H and by $m(\pi_L)-(1/3)(m_n-m_p)$ for ^3He, it is clear that the assumption that lepto-pion mass is of order 1 MeV improves the fit. The results are given by the table below.

nucleus	^2H	^3H	^3He
$E_B(exp)/MeV$	1.111	2.826	2.572
$E_B(pred)/MeV$	.875	3.117	2.507

Here $E_B(pred)$ corresponds to the effective value of binding energy assuming that nuclei effectively consist of ordinary protons and neutrons. The discrepancies are below 22 percent.

What is troublesome that neither the scaling of α_s nor modification of E_c improves the situation for ^2H and ^3H. Moreover, magnetic spin-spin interaction energy for deuteron is expected to reduce $E_B(pred)$ further in triplet state. Thus option IIb) does not look promising.

2. Option Ib)

For option Ib) with $m(\pi) = 1.062$ MeV and ordinary nucleons the actual binding $E_B(act)$ energy increases by $m(\pi)$ for $A = 3$ nuclei and by $m(\pi)/2$ for deuteron. Direct scaling gives a reasonably good fit for the p-adic length scale $L_e(9,13)$ with $k_{eff} = 117$ meaning $\sqrt{2}$ scaling of E_s. For deuteron the predicted E_B is too low by 30 per cent. One might argue that isospin dependent strong force between nucleons becomes important in this p-adic length scale and reduces deuteron binding energy by 30 per cent. This option is not un-necessary complex as compared to the option Ia).

nucleus	^2H	^3H	^3He
$E_B(act)/MeV$	1.642	3.880	3.634
$E_B(pred)/MeV$	1.3322	3.997	3.743

For option IIa) with $m(\pi) = 0$ and protons as building blocks the fit gets worse for $A = 3$ nuclei.

8.5.2 The QCD associated with ^4He

^4He must somehow differ from $A \leq 3$ nucleons. If one takes the argument based on isospin dependence strong force seriously, the reasonable looking conclusion would be that ^4He is at the space-time sheet of nucleons a bound state of two deuterons which induce no isospin dependent strong nuclear force. One could regard the system also as a closed string of four nucleons such that neighboring p and n form strong iso-spin singlets. The previous treatment applies as such.

For ^4He option Ia) with a direct scaling would predict $E_B(^4He) < 4 \times \hat{E}_s = 3.720$ MeV which is by a factor of order 2 too small. The natural explanation would be that for ^4He both color and em field body correspond to the p-adic length scale $L_e(4,29)$ ($k_{eff} = 116$) so that E_s would increase by a factor of 2 to 1.860 MeV. Somewhat surprisingly, $A \leq 3$ nuclei would have "color field bodies" by a factor 2 larger than ^4He.

1. For option Ia) this would predict $E_B(^4He) = 7.32867$ MeV to be compared with the real value 7.0720 MeV. A reduction of α_s by 3.5 per cent would explain the discrepancy. That α_s decreases in the transition sequence $k_{eff} = 127 \rightarrow 118 \rightarrow 116$ which is consistent with the general vision about evolution of color coupling strength.

2. If one assumes option Ib) with $m(\pi) = 1.062$ MeV the actual binding energy increases to 8.13 MeV. The strong binding energy of deuteron units would give an additional .15 MeV binding energy per nucleon so that one would have $E_B(^4He) = 7.47$ MeV so that 10 per cent accuracy is achieved. Obviously this option does not work so well as Ia).

3. If one assumes option IIb), the actual binding energy would increase by .415 MeV to 7.4827 MeV which would make fit somewhat poorer. A small reduction of E_c could allow to achieve a perfect fit.

8.5.3 What about tetra-neutron?

One can estimate the value of $E_B(^4n)$ from binding energies of nuclei (Z,N) and $(Z,N+4)$ $(A = Z + N)$ as

$$E_B(^4n) = \frac{A+4}{4}[E_B(A+4) - \frac{A}{A+4}E_B(A)] \ .$$

In the table below there are some estimate for $E_B(^4n)$.

(Z,N)	(26,26)(^{52}Fe)	(50,70)(^{120}Sn)	(82,124) (^{206}Pb)
$E_B(^4n)/MeV$	6.280	7.3916	5.8031

The prediction of the above model would be $E(^4n) = 4\hat{E}_s = 3.760$ MeV for $\hat{E}_s = .940$ MeV associated with $A < 4$ nuclei and $k_{eff} = 118 = 2\times 59$ associated with $A < 4$ nuclei. For $k_{eff} = 116$ associated with ^4He $E_s(^4n) = E_s(^4He) = 1.82$ MeV the prediction would be 7.28 MeV. 14 percent reduction of α_s would give the estimated value for of E_s for ^{52}Fe.

If tetra-neutron is ppnn bound state with two negatively charged color bonds, this estimate is not quite correct since the actual binding energy per nucleon is $E_B(^4He) - (m_n - m_p)/2$. This implies a small correction $E_B(A+4) \rightarrow E_B(A+4) - 2(m_n - m_p)/(A+4)$. The correction is negligible.

One can make also a direct estimate of ^4n binding energy assuming tetra-neutron to be ppnn bound state. If the masses of charged color bonds do not differ appreciably from those of neutral bonds (as the p-adic scaling of $\pi + -\pi^0$ mass difference of about 4.9 MeV strongly suggests) then model Ia) with $E_s = E_B(^3H)/3$ implies that the actual binding energy $E_B(^4n) = 4E_s = E_B(^3H)/3$ (see the table below). The apparent binding energy is $E_{B,app} = E_B(^4n) + (m_n - m_p)/2$. Binding energy differs dramatically from what one can imagine in more conventional models of strong interactions in which even the existence of tetra-neutron is highly questionable.

k_{eff}	2×59	4×29
$E_B(act)(^4n)/MeV$	3.7680	
$E_{B,app}(4n)/MeV$	4.4135	8.1825

The higher binding energy per nucleon for tetra-neutron might directly relate to the neutron richness of heavy nuclei in accordance with the vision that Coulomb energy is what disfavors proton rich nuclei.

According to [C117] , tetra-neutron might have been observed in the decay $^8He \rightarrow^4 He+^4n$ and the accepted value for the mass of ^8He isotope gives the upper bound of $E(^4n) < 3.1$ MeV, which is one half of the the estimate. One can of course consider the possibility that free tetra-neutron corresponds to $L_e(2,59)$ and nuclear tetra-neutron corresponds to the length scale $L_e(4,29)$ of ^4He. Also light quarks appear as several p-adically scaled up variants in the TGD based model for low-lying hadrons and there is also evidence that neutrinos appear in several scales.

8.5.4 What could be the general mass formula?

In the proposed model nucleus consists of $A \leq 4$ nuclei. Concerning the details of the model there are several questions to be answered. Do $A \leq 3$ nuclei and $A = 4$ nuclei ^4He and tetra-neutron form separate nuclear strings carrying their own color magnetic fields as the different p-adic length scale for the corresponding "color magnetic bodies" would suggest? Or do they combine by a connected sum operation to single closed string? Is there single Bose-Einstein condensate or several ones.

Certainly the Bose-Einstein condensates associated with nucleons forming $A < 4$ nuclei are separate from those for $A = 4$ nuclei. The behavior of E_B in turn can be understood if ^4He nuclei and tetra-neutrons form separate Bose-Einstein condensates. For $Z > N$ nuclei poly-protons constructed as exotic charge states of stable $A \leq 4$ nuclei could give rise to the proton excess.

Before continuing it is appropriate to list the apparent binding energies for poly-neutrons and poly-protons.

poly-neutron	n	^2n	^3n	^4n
$E_{B,app}/MeV$	0	$E_B(^2H)+\frac{\Delta}{2}$	$E_B(^3H)+\frac{2\Delta}{3}$	$E_B(^4He)+\frac{\Delta}{2}$
poly-proton	p	^2p	^3p	^4p
$E_{B,app}/MeV$	0	$E_B(^2H)-\frac{\Delta}{2}$	$E_B(^3He)-\frac{\Delta}{3}$	$E_B(^4He)-\frac{\Delta}{2}$

For heavier nuclei $E_{B,app}(^4n)$ is smaller than $E_B(^4He)+(m_p-m_n)/2$.

The first guess for the general formula for the binding energy for nucleus (Z,N) is obtained by assuming that for maximum number of ^4He nuclei and tetra-neutrons/tetra-protons identified as ^4H nuclei with 2 negatively/positively charged color bonds are present.

1. $N \geq Z$ nuclei

Even-Z nuclei with $N \geq Z$ can be expressed as $(Z=2n, N=2(n+k)+m)$, $m=0,1,2$ or 3. For $Z \leq 26$ (only single Bose-Einstein condensate) this gives for the apparent binding energy per nucleon (assuming that all neutrons are indeed neutrons) the formula

$$\begin{aligned} E_B(2n,2(n+k)+m) &= \frac{n}{A}E_B(^4He)+\frac{k}{A}E_{B,app}(^4n)+\frac{1}{A}E_{B,app}(^mn) \\ &+ \frac{n^2+k^2}{n+k}E_s-\frac{Z(Z-1)}{A^2}E_c \ . \end{aligned} \quad (8.5.4)$$

The situation for the odd-Z nuclei $(Z,N)=(2n+1,2(n+k)+m)$ can be reduced to that for even-Z nuclei if one can assume that the $(2n+1)^{th}$ proton combines with 2 neutrons to form ^3He nucleus so that one has still $2(k-1)+m$ neutrons combining to $A \leq 4$ poly-neutrons in above described manner.

2. $Z \geq N$ nuclei

For the nuclei having $Z > N$ the formation of a maximal number of ^4He nuclei leaves k excess protons. For long-lived nuclei $k \leq 2$ is satisfied. One could think of decomposing the excess protons to exotic variants of $A \leq 4$ nuclei by assuming that some charged bonds carry positive charge with an obvious generalization of the above formula.

The only differences with respect to a nucleus with neutron excess would be that the apparent binding energy is smaller than the actual one and positive charge would give rise to Coulomb interaction energy reducing the binding energy (but only very slightly). The change of the binding energy in the subtraction of single neutron from $Z=N=2n$ nucleus is predicted to be approximately $\Delta E_B = -E_B(^4He)/A$. In the case of ^{32}S this predicts $\Delta E_B = .2209$ MeV. The real value is .2110 MeV. The fact that the general order of magnitude for the change of the binding energy as Z or N changes by one unit supports the proposed picture.

8.5.5 Nuclear strings and cold fusion

To summarize, option Ia) assuming that strong isospin dependent force acts on the nuclear space-time sheet and binds pn pairs to singlets such that the strong binding energy is very nearly zero in singlet state by the cancelation of scalar and vector contributions, is the most promising one. It predicts the existence of exotic di-,tri-, and tetra-neutron like particles and even negatively charged exotics obtained from $^2H,^3H,^3He$, and ^4He by adding negatively charged color bond. For instance, ^3H extends to a multiplet with em charges $1,0,-1,-2$. Of course, heavy nuclei with proton neutron excess could actually be such nuclei.

The exotic states are stable under beta decay for $m(\pi) < m_e$. The simplest neutral exotic nucleus corresponds to exotic deuteron with single negatively charged color bond. Using this as target it would be possible to achieve cold fusion since Coulomb wall would be absent. The empirical evidence for cold fusion thus supports the prediction of exotic charged states.

Signatures of cold fusion

In the following the consideration is restricted to cold fusion in which two deuterium nuclei react strongly since this is the basic reaction type studied.

In hot fusion there are three reaction types:
1) $D + D \rightarrow^4 He + \gamma$ $(23.8MeV)$
2) $D + D \rightarrow^3 He + n$
3) $D + D \rightarrow^3 H + p$.
The rate for the process 1) predicted by standard nuclear physics is more than 10^{-3} times lower than for the processes 2) and 3) [C112] . The reason is that the emission of the gamma ray involves the relatively weak electromagnetic interaction whereas the latter two processes are strong.

The most obvious objection against cold fusion is that the Coulomb wall between the nuclei makes the mentioned processes extremely improbable at room temperature. Of course, this alone implies that one should not apply the rules of hot fusion to cold fusion. Cold fusion indeed differs from hot fusion in several other aspects.

1. No gamma rays are seen.
2. The flux of energetic neutrons is much lower than expected on basis of the heat production rate an by interpolating hot fusion physics to the recent case.

These signatures can also be (and have been!) used to claim that no real fusion process occurs. It has however become clear that the isotopes of Helium and also some tritium accumulate to the Pd target during the reaction and already now prototype reactors for which the output energy exceeds input energy have been built and commercial applications are under development. Therefore the situation has turned around. The rules of standard physics do not apply so that some new nuclear physics must be involved and it has become an exciting intellectual challenge to understand what is happening. A representative example of this attitude and an enjoyable analysis of the counter arguments against fold fusion is provided by the article 'Energy transfer in cold fusion and sono-luminescence' of Julian Schwinger [C107] . This article should be contrasted with the ultra-skeptical article 'ESP and Cold Fusion: parallels in pseudoscience' of V. J. [C109] [C109] .

Cold fusion has also other features, which serve as valuable constraints for the model building.

1. Cold fusion is not a bulk phenomenon. It seems that fusion occurs most effectively in nano-particles of Pd and the development of the required nano-technology has made possible to produce fusion energy in controlled manner. Concerning applications this is a good news since there is no fear that the process could run out of control.
2. The ratio x of D atoms to Pd atoms in Pd particle must lie the critical range $[.85, .90]$ for the production of ^4He to occur [D41] . This explains the poor repeatability of the earlier experiments and also the fact that fusion occurred sporadically.
3. Also the transmutations of Pd nuclei are observed [C105] .

Below a list of questions that any theory of cold fusion should be able to answer.

1. Why cold fusion is not a bulk phenomenon?
2. Why cold fusion of the light nuclei seems to occur only above the critical value $x \simeq .85$ of D concentration?
3. How fusing nuclei are able to effectively circumvent the Coulomb wall?
4. How the energy is transferred from nuclear degrees of freedom to much longer condensed matter degrees of freedom?
5. Why gamma rays are not produced, why the flux of high energy neutrons is so low and why the production of ^4He dominates (also some tritium is produced)?
6. How nuclear transmutations are possible?

Could exotic deuterium make cold fusion possible?

One model of cold fusion has been already discussed in [K80] and the recent model is very similar to that. The basic idea is that only the neutrons of incoming and target nuclei can interact strongly, that is their space-time sheets can fuse. One might hope that neutral deuterium having single negatively charged color bond could allow to realize this mechanism.

1. Suppose that part of the deuterium in Pd catalyst corresponds to exotic deuterium with neutral nuclei so that cold fusion would occur between neutral exotic D nuclei in the target and charged incoming D nuclei and Coulomb wall in the nuclear scale would be absent.

2. The exotic variant of the ordinary $D + D$ reaction yields final states in which ^4He, ^3He and ^3H are replaced with their exotic counterparts with charge lowered by one unit. In particular, exotic ^3H is neutral and there is no Coulomb wall hindering its fusion with Pd nuclei so that nuclear transmutations can occur.

Why the neutron and gamma fluxes are low might be understood if for some reason only exotic ^3H is produced, that is the production of charged final state nuclei is suppressed. The explanation relies on Coulomb wall at the nucleon level.

1. Initial state contains one charged and one neutral color bond and final state $A = 3$ or $A = 4$ color bonds. Additional neutral color bonds must be created in the reaction (one for the production $A = 3$ final states and two for $A = 4$ final state). The process involves the creation of neural fermion pairs. The emission of one exotic gluon per bond decaying to a neutral pair is necessary to achieve this. This requires that nucleon space-time sheets fuse together. Exotic D certainly belongs to the final state nucleus since charged color bond is not expected to be split in the process.

2. The process necessarily involves a temporary fusion of nucleon space-time sheets. One can understand the selection rules if only neutron space-time sheets can fuse appreciably so that only ^3H would be produced. Here Coulomb wall at nucleon level should enter into the game.

3. Protonic space-time sheets have the same positive sign of charge always so that there is a Coulomb wall between them. This explains why the reactions producing exotic ^4He do not occur appreciably. If the quark/antiquark at the neutron end of the color bond of ordinary D has positive charge, there is Coulomb attraction between proton and corresponding negatively charged quark. Thus energy minimization implies that the neutron space-time sheet of ordinary D has positive net charge and Coulomb repulsion prevents it from fusing with the proton space-time sheet of target D. The desired selection rules would thus be due to Coulomb wall at the nucleon level.

About the phase transition transforming ordinary deuterium to exotic deuterium

The exotic deuterium at the surface of Pd target seems to form patches (for a detailed summary see [K80]). This suggests that a condensed matter phase transition involving also nuclei is involved. A possible mechanism giving rise to this kind of phase would be a local phase transition in the Pd target involving both D and Pd. In [K80] it was suggested that deuterium nuclei transform in this phase transition to "ordinary" di-neutrons connected by a charged color bond to Pd nuclei. In the recent case di-neutron could be replaced by neutral D.

The phase transition transforming neutral color bond to a negatively charged one would certainly involve the emission of W^+ boson, which must be exotic in the sense that its Compton length is of order atomic size so that it could be treated as a massless particle and the rate for the process would be of the same order of magnitude as for electro-magnetic processes. One can imagine two options.

1. Exotic W^+ boson emission generates a positively charged color bond between Pd nucleus and exotic deuteron as in the previous model.

2. The exchange of exotic W^+ bosons between ordinary D nuclei and Pd induces the transformation $Z \rightarrow Z+1$ inducing an alchemic phase transition $Pd \rightarrow Ag$. The most abundant Pd isotopes with $A = 105$ and 106 would transform to a state of same mass but chemically equivalent with the two lightest long-lived Ag isotopes. ^{106}Ag is unstable against β^+ decay to Pd and ^{105}Ag transforms to Pd via electron capture. For ^{106}Ag (^{105}Ag) the rest energy is 4 MeV (2.2 MeV) higher than for ^{106}Pd (^{105}Pd), which suggests that the resulting silver cannot be genuine.

This phase transition need not be favored energetically since the energy loaded into electrolyte could induce it. The energies should (and could in the recent scenario) correspond to energies typical for condensed matter physics. The densities of Ag and Pd are 10.49 gcm^{-3} and 12.023 gcm^{-3} so that the phase transition would expand the volume by a factor 1.0465. The porous character of Pd would allow this. The needed critical packing fraction for Pd would guarantee one D nucleus per one Pd nucleus with a sufficient accuracy.

Exotic weak bosons seem to be necessary

The proposed phase transition cannot proceed via the exchange of the ordinary W bosons. Rather, W bosons having Compton length of order atomic size are needed. These W bosons could correspond to a scaled up variant of ordinary W bosons having smaller mass, perhaps even of the order of electron mass. They could be also dark in the sense that Planck constant for them would have the value $\hbar = n\hbar_0$ implying scaling up of their Compton size by n. For $n \sim 2^{48}$ the Compton length of ordinary W boson would be of the order of atomic size so that for interactions below this length scale weak bosons would be effectively massless. p-Adically scaled up copy of weak physics with a large value of Planck constant could be in question. For instance, W bosons could correspond to the nuclear p-adic length scale $L_e(k = 113)$ and $n = 2^{11}$.

Few weeks after having written this chapter I learned that cold fusion is in news again: both Nature and New Scientists commented the latest results [C1] . It seems that the emission of highly energetic charged particles which cannot be due to chemical reactions and could emerge from cold fusion has been demonstrated beyond doubt by Frank Cordon's team [C4] using detectors known as CR-39 plastics of size scale of coin used already earlier in hot fusion research. The method is both cheap and simple. The idea is that travelling charged particles shatter the bonds of the plastic's polymers leaving pits or tracks in the plastic. Under the conditions claimed to make cold fusion possible (1 deuterium per 1 Pd nucleus making in TGD based model possible the phase transition of D to its neutral variant by the emission of exotic dark W boson with interaction range of order atomic radius) tracks and pits appear during short period of time to the detector.

8.5.6 Strong force as a scaled and dark electro-weak force?

The fiddling with the nuclear string model has led to following conclusions.

1. Strong isospin dependent nuclear force, which does not reduce to color force, is necessary in order to eliminate polyneutron and polyproton states. This force contributes practically nothing to the energies of bound states. This can be understood as being due to the cancelation of isospin scalar and vector parts of this force for them. Only strong isospin singlets and their composites with isospin doublet (n,p) are allowed for $A \leq 4$ nuclei serving as building bricks of the nuclear strings. Only *effective* polyneutron states are allowed and they are strong isospin singlets or doublets containing charged color bonds.

2. The force could act in the length scalar of nuclear space-time sheets: $k = 113$ nuclear p-adic length scale is a good candidate for this length scale. One must be however cautious: the contribution to the energy of nuclei is so small that length scale could be much longer and perhaps same as in case of exotic color bonds. Color bonds connecting nuclei correspond to much longer p-adic length scale and appear in three p-adically scaled up variants corresponding to $A < 4$ nuclei, $A = 4$ nuclei and $A > 4$ nuclei.

3. The prediction of exotic deuterons with vanishing nuclear em charge leads to a simplification of the earlier model of cold fusion explaining its basic selection rules elegantly but requires a scaled variant of electro-weak force in the length scale of atom.

What is then this mysterious strong force? And how abundant these copies of color and electro-weak force actually are? Is there some unifying principle telling which of them are realized?

From foregoing plus TGD inspired model for quantum biology involving also dark and scaled variants of electro-weak and color forces it is becoming more and more obvious that the scaled up variants of both QCD and electro-weak physics appear in various space-time sheets of TGD Universe. This raises the following questions.

1. Could the isospin dependent strong force between nucleons be nothing but a p-adically scaled up (with respect to length scale) version of the electro-weak interactions in the p-adic length scale defined by Mersenne prime M_{89} with new length scale assigned with gluons and characterized by Mersenne prime M_{107}? Strong force would be electro-weak force but in the length scale of hadron! Or possibly in length scale of nucleus ($k_{eff} = 107 + 6 = 113$) if a dark variant of strong force with $h = nh_0 = 2^3 h_0$ is in question.

2. Why shouldn't there be a scaled up variant of electro-weak force also in the p-adic length scale of the nuclear color flux tubes?

3. Could it be that all Mersenne primes and also other preferred p-adic primes correspond to entire standard model physics including also gravitation? Could be be kind of natural selection which selects the p-adic survivors as proposed long time ago?

Positive answers to the last questions would clean the air and have quite a strong unifying power in the rather speculative and very-many-sheeted TGD Universe.

1. The prediction for new QCD type physics at M_{89} would get additional support. Perhaps also LHC provides it within the next half decade.

2. Electro-weak physics for Mersenne prime M_{127} assigned to electron and exotic quarks and color excited leptons would be predicted. This would predict the exotic quarks appearing in nuclear string model and conform with the 15 year old lepto-hadron hypothesis [K88] . M_{127} dark weak physics would also make possible the phase transition transforming ordinary deuterium in Pd target to exotic deuterium with vanishing nuclear charge.

The most obvious objection against this unifying vision is that hadrons decay only according to the electro-weak physics corresponding to M_{89}. If they would decay according to M_{107} weak physics, the decay rates would be much much faster since the mass scale of electro-weak bosons would be reduced by a factor 2^{-9} (this would give increase of decay rates by a factor 2^{36} from the propagator of weak boson). This is however not a problem if strong force is a dark with say $n = 8$ giving corresponding to nuclear length scale. This crazy conjecture might work if one accepts the dark Bohr rules!

8.6 Giant dipole resonance as a dynamical signature for the existence of Bose-Einstein condensates?

The basic characteristic of the Bose-Einstein condensate model is the non-linearity of the color contribution to the binding energy. The implication is that the the de-coherence of the Bose-Einstein condensate of the nuclear string consisting of ^4He nuclei costs energy. This de-coherence need not involve a splitting of nuclear strings although also this is possible. Similar de-coherence can occur for ^4He $A < 4$ nuclei. It turns out that these three de-coherence mechanisms explain quite nicely the basic aspects of giant dipole resonance (GDR) and its variants both qualitatively and quantitatively and that precise predictions for the fine structure of GDR emerge.

8.6.1 De-coherence at the level of ^4He nuclear string

The de-coherence of a nucleus having n ^4He nuclei to a nucleus containing two Bose-Einstein condensates having $n - k$ and $k > 2$ ^4He nuclei requires energy given by

$$
\begin{aligned}
\Delta E &= (n^2-(n-k)^2-k^2)E_s = 2k(n-k)E_s \ , \quad k>2 \ , \\
\Delta E &= (n^2-(n-2)^2-1)E_s = (4n-5)E_s \ , \quad k=2 \ , \\
E_s &\simeq .1955 \ MeV \ .
\end{aligned} \qquad (8.6.1)
$$

Bose-Einstein condensate could also split into several pieces with some of them consisting of single ^4He nucleus in which case there is no contribution to the color binding energy. A more general formula for the resonance energy reads as

$$
\begin{aligned}
\Delta E &= (n^2-\sum_i k^2(n_i))E_s \ , \quad \sum_i n_i = n \ , \\
k(n_i) &= \left\{ \begin{array}{l} n_i \text{ for } n_i>2 \ , \\ 1 \text{ for } n_i=2 \ , \\ 0 \text{ for } n_i=1 \ . \end{array} \right.
\end{aligned} \qquad (8.6.2)
$$

The table below lists the resonance energies for four manners of ^{16}O nucleus ($n=4$) to lose its coherence.

final state	3+1	2+2	2+1+1	1+1+1+1
$\Delta E/MeV$	1.3685	2.7370	2.9325	3.1280

Rather small energies are involved. More generally, the minimum and maximum resonance energy would vary as $\Delta E_{min} = (2n-1)E_s$ and $\Delta E_{max} = n^2E_s$ (total de-coherence). For $n = n_{max} = 13$ one would have $\Delta E_{min} = 2.3640$ MeV and $\Delta E_{max} = 33.099$ MeV.

Clearly, the loss of coherence at this level is a low energy collective phenomenon but certainly testable. For nuclei with $A > 60$ one can imagine also double resonance when both coherent Bose-Einstein condensates possibly present split into pieces. For $A \geq 120$ also triple resonance is possible.

8.6.2 De-coherence inside ^4He nuclei

One can consider also the loss of coherence occurring at the level ^4He nuclei. Predictions for resonances energies and for the dependence of GR cross sections on mass number follow.

Resonance energies

For ^4He nuclei one has $E_s = 1.820$ MeV. In this case de-coherence would mean the decomposition of Bose-Einstein condensate to $n = 4 \rightarrow \sum n_i = n$ with $\Delta E = n^2 - \sum_{n_i} k^1(n_i) = 16 - \sum_{n_i} k^2(n_i)$. The table below gives the resonance energies for the four options $n \rightarrow \sum_i n_i$ for the loss of coherence.

final state	3+1	2+2	2+1+1	1+1+1+1
$\Delta E/MeV$	12.74	25.48	27.30	29.12

These energies span the range at which the cross section for $^{16}O(\gamma, xn)$ reaction has giant dipole resonances [C5] . Quite generally, GDR is a broad bump with substructure beginning around 10 MeV and ranging to 30 MeV. The average position of the bump as a function of atomic number can be parameterized by the following formula

$$
E(A)/MeV = 31.2A^{-1/3} + 20.6A^{-1/6} \qquad (8.6.3)
$$

given in [C71] . The energy varies from 36.6 MeV for $A = 4$ (the fit is probably not good for very low values of A) to 13.75 MeV for $A = 206$. The width of GDR ranges from 4-5 MeV for closed shell nuclei up to 8 MeV for nuclei between closed shells.

The observation raises the question whether the de-coherence of Bose-Einstein condensates associated with ^4He and nuclear string could relate to GDR and its variants. If so, GR proper would be a collective phenomenon both at the level of single ^4He nucleus (main contribution to the resonance energy) and entire nucleus (width of the resonance). The killer prediction is that even ^4He should exhibit giant dipole resonance and its variants: GDR in ^4He has been reported [C76] .

Some tests

This hypothesis seems to survive the basic qualitative and quantitative tests.

1. The basic prediction of the model peak at 12.74 MeV and at triplet of closely located peaks at $(25.48, 27.30, 29.12)$ MeV spanning a range of about 4 MeV, which is slightly smaller than the width of GDR. According to [C49] there are two peaks identified as iso-scalar GMR at $13.7 \pm .3$ MeV and iso-vector GMR at 26 ± 3 MeV. The 6 MeV uncertainty related to the position of iso-vector peak suggests that it corresponds to the triplet $(25.48, 27.30, 29.12)$ MeV whereas singlet would correspond to the iso-scalar peak. According to the interpretation represented in [C49] iso-scalar *resp.* iso-vector peak would correspond to oscillations of proton and neutron densities in same *resp.* opposite phase. This interpretation can make sense in TGD framework only inside single ^4He nucleus and would apply to the transverse oscillations of ^4He string rather than radial oscillations of entire nucleus.

2. The presence of triplet structure seems to explain most of the width of iso-vector GR. The combination of GDR internal to ^4He with GDR for the entire nucleus (for which resonance energies vary from $\Delta E_{min} = (2n-1)E_s$ to $\Delta E_{max} = n^2E_s$ $(n = A/4)$) predicts that also latter contributes to the width of GDR and give it additional fine structure. The order of magnitude for ΔE_{min} is in the range [1.3685,2.3640] MeV which is consistent with the with of GDR and predicts a band of width 1 MeV located 1.4 MeV above the basic peak.

3. The de-coherence of $A < 4$ nuclei could increase the width of the peaks for nuclei with partially filled shells: maximum and minimum values of resonance energy are $9E_s(^4He)/2 = 8.19$ MeV and $4E_s(^4He) = 7.28$ MeV for ^3He and ^3H which conforms with the upper bound 8 MeV for the width.

4. It is also possible that n ^4He nuclei simultaneously lose their coherence. If multiplet de-coherence occurs coherently it gives rise to harmonics of GDR. For de-coherent de-coherence so that the emitted photons should correspond to those associated with single ^4He GDR combined with nuclear GDR. If absorption occurs for $n \leq 13$ nuclei simultaneously, one obtains a convoluted spectrum for resonant absorption energy

$$\Delta E = [16n - \sum_{j=1}^{n} \sum_{i_j} k^2(n_{i_j})]E_s \quad . \tag{8.6.4}$$

   The maximum value of ΔE given by $\Delta E_{max} = n \times 29.12$ MeV. For $n = 13$ this would give $\Delta E_{max} = 378.56$ MeV for the upper bound for the range of excitation energies for GDR. For heavy nuclei [C71] GDR occurs in the range 30-130 MeV of excitation energies so that the order of magnitude is correct. Lower bound in turn corresponds to a total loss of coherence for single ^4He nucleus.

5. That the width of GDR increases with the excitation energy [C71] is consistent with the excitation of higher GDR resonances associated with the entire nuclear string. $n \leq n_{max}$ for GDR at the level of the entire nucleus means saturation of the GDR peak with excitation energy which has been indeed observed [C5] .

One can look whether the model might work even at the level of details. Figure 3 of [C5] compares total photoneutron reaction cross sections for $^{16}O(\gamma, xn)$ in the range 16-26 MeV from some experiments so that the possible structure at 12.74 MeV is not visible in it. It is obvious that the resonance structure is more complex than predicted by the simplest model. It seems however possible to explain this.

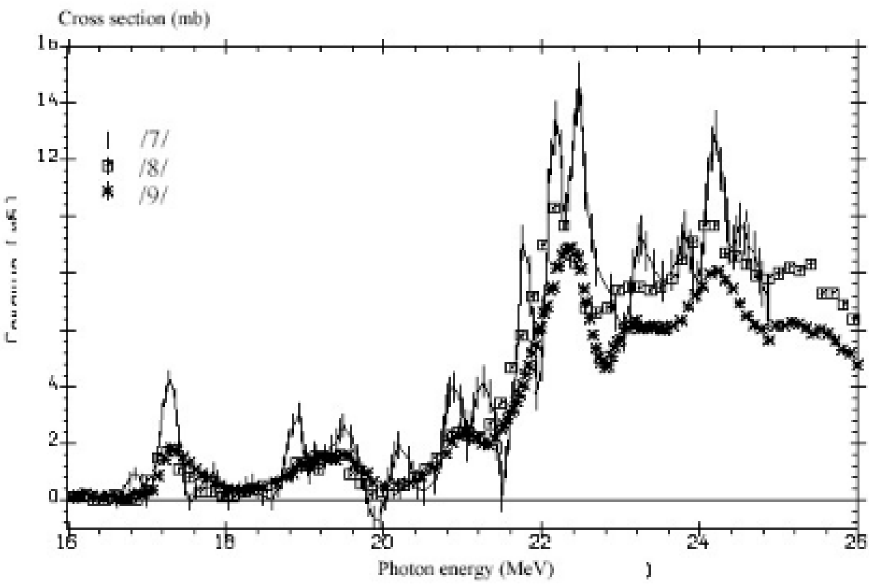

Figure 8.1: The comparison of photoneutron cross sections $^{16}O(\gamma, xn)$ obtained in one BR-experiment (Moscow State University) and two QMA experiments carried out at Saclay (France) Livermoore (USA). Figure is taken from [C5] where also references to experiments can be found.

1. The main part of the resonance is a high bump above 22 MeV spanning an interval of about 4 MeV just as the triplet at (25.48,27.30,29.12) MeV does. This suggest a shift of the predicted 3-peak structure in the range 25-30 MeV range downwards by about 3 MeV. This happens if the photo excitation inducing the de-coherence involves a dropping from a state with excitation energy of 3 MeV to the ground state. The peak structure has peaks roughly at the shifted energies but there is also an additional structure which might be understood in terms of the bands of width 1 MeV located 1.4 MeV above the basic line.

2. There are three smaller bumps below the main bump which also span a range of 4 MeV which suggests that also they correspond to a shifted variant of the basic three-peak structure. This can be understood if the photo excitation inducing de-coherence leads from an excited state with excitation energy 8.3 MeV to ground state shifting the resonance triplet (25.48, 27.30, 29.12) MeV to resonance triplet at (17.2, 19.00, 20.82) MeV.

On basis of these arguments it seems that the proposed mechanism might explain GR and its variants. The basic prediction would be the presence of singlet and triplet resonance peaks corresponding to the four manners to lose the coherence. Second signature is the precise prediction for the fine structure of resonance peaks.

Predictions for cross sections

The estimation of collision cross sections in nuclear string model would require detailed numerical models. One approach to modelling would be to treat the colliding nuclear strings as random coils with finite thickness defined by the size of $A \leq 4$ strings. The intersections of colliding strings would induce fusion reactions and self intersections fissions. Simple statistical models for the intersections based on geometric probability are possible and allow to estimate branching ratios to various channels.

In the case of GR the reduction to ^4He level means strong testable predictions for the dependence of GR cross sections on the mass number. GR involves formation of eye-glass type configuration at level of single ^4He and in the collision of nuclei with mass numbers A_1 and A_2 GR means formation of these configurations for some $A = 4$ unit associated with either nucleus.

Hence the GR cross section should be in a reasonable approximation proportional to $n_1 + n_2$ where n_i are the numbers of $A = 4$ sub-units, which can be either ^4He, tetra-neutron, or possible other variants of ^4He having charged color bonds. For $Z_i = 2m_i$, $N = 2n_i$, $A_i = 4(m_i + n_i)$ nuclei one has $n_1 + n_2 = (A_1 + A_2)/4$. Also a characteristic oscillatory behavior as a function of A is expected if the number of $A = 4$ units is maximal. If GR reactions are induced by the touching of ^4He units of nuclear string implying transfer of kinetic energy between units then the GR cross sections should depend only on the energy per ^4He nucleus in cm system, which is also a strong prediction.

8.6.3 De-coherence inside $A = 3$ nuclei and pygmy resonances

For neutron rich nuclei the loss of coherence is expected to occur inside ^4He, tetra-neutron, ^3He and possibly also ^3n which might be stable in the nuclear environment. The de-coherence of tetra-neutron gives in the first approximation the same resonance energy spectrum as that for ^4He since $E_B(^4n) \sim E_B(^4He)$ roughly consistent with the previous estimates for $E_B(^4n)$ implies $E_s(^4n) \sim E_s(^4He)$.

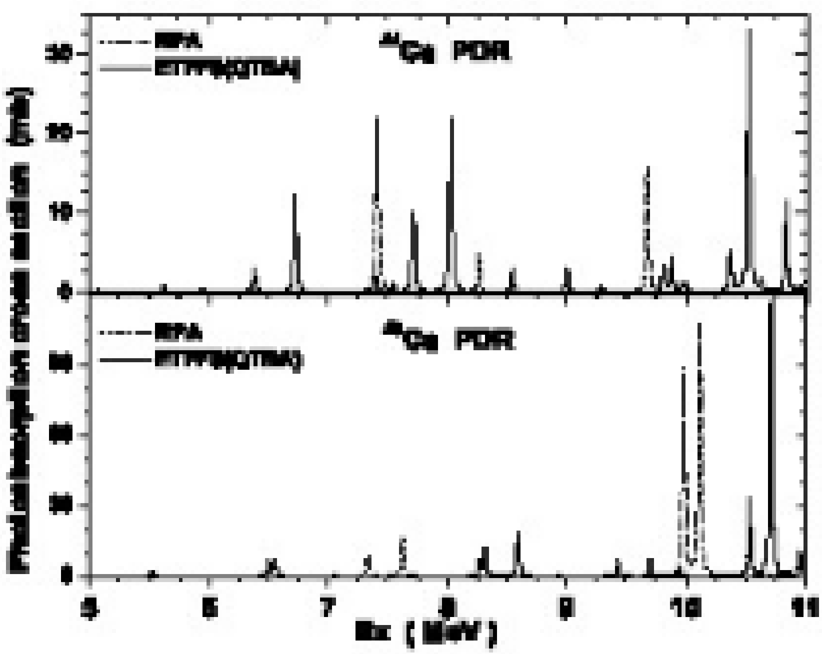

Figure 8.2: Pygmy resonances in ^{44}Ca and ^{48}Ca up to 11 MeV. Figure is taken from [C52] .

The de-coherence inside $A = 3$ nuclei might explain the so called pygmy resonance appearing in neutron rich nuclei, which according to [C12] is wide bump around $E \sim 8$ MeV. For $A = 3$ nuclei only two de-coherence transitions are possible: $3 \rightarrow 2{+}1$ and $3 \rightarrow 1{+}1{+}1$ and $E_s = E_B(^3H) = .940$ MeV the corresponding energies are $8E_s = 7.520$ MeV and $9 * Es = 8.4600$ MeV. Mean energy is indeed ~ 8 MeV and the separation of peaks about 1 MeV. The de-coherence at level of ^4He string might add to this 1 MeV wide bands about 1.4 MeV above the basic lines.

The figure of [C52] illustrating photo-absorption cross section in ^{44}Ca and ^{48}Ca shows three peaks at 6.8, 7.3, 7.8 and 8 MeV in ^{44}Ca. The additional two peaks might be assigned with the excitation of initial or final states. This suggests also the presence of also $A = 3$ nuclear strings in ^{44}Ca besides $^H4$ and ^4n strings. Perhaps neutron halo wave function contains $^3n + n$ component besides ^4n. For ^{48}Ca these peaks are much weaker suggesting the dominance of $2{\times}^4n$ component.

8.6.4 De-coherence and the differential topology of nuclear reactions

Nuclear string model allows a topological description of nuclear decays in terms of closed string diagrams and it is interesting to look what characteristic predictions follow without going to detailed quantitative modelling of stringy collisions possibly using some variant of string models.

In the de-coherence eye-glass type singularities of the closed nuclear string appear and make possible nuclear decays.

1. At the level of ^4He sub-strings the simplest singularities correspond to $4 \rightarrow 3+1$ and $4 \rightarrow 2+2$ eye-glass singularities. The first one corresponds to low energy GR and second to one of higher energy GRs. They can naturally lead to decays in which nucleon or deuteron is emitted in decay process. The singularities $4 \rightarrow 2+1+1$ *resp.* $4 \rightarrow 1+1+1+1$ correspond to eye-glasses with 3 *resp.* four lenses and mean the decay of ^4He to deuteron and two nucleons *resp.* 4 nucleons. The prediction is that the emission of deuteron requires a considerably larger excitation energy than the emission of single nucleon. For GR at level of $A = 3$ nuclei analogous considerations apply. Taking into account the possible tunnelling of the nuclear strings from the nuclear space-time sheet modifies this simple picture.

2. For GR in the scale of entire nuclei the corresponding singular configurations typically make possible the emission of alpha particle. Considerably smaller collision energies should be able to induce the emission of alpha particles than the emission of nucleons if only stringy excitations matter. The excitation energy needed for the emission of α particle is predicted to increase with A since the number n of ^4He nuclei increases with A. For instance, for $Z = N = 2n$ nuclei $n \rightarrow n-1+1$ would require the excitation energy $(2n-1)E_c = (A/2-1)E_c$, $E_c \simeq .2$ MeV. The tunnelling of the alpha particle from the nuclear space-time sheet can modify the situation.

The decay process allows a differential topological description. Quite generally, in the decoherence process $n \rightarrow (n-k)+k$ the color magnetic flux through the closed string must be reduced from n to $n-k$ units through the first closed string and to k units through the second one. The reduction of the color color magnetic fluxes means the reduction of the total color binding energy from n^2E_c $((n-k)^2+k^2)E_c$ and the kinetic energy of the colliding nucleons should provide this energy.

Faraday's law, which is essentially a differential topological statement, requires the presence of a time dependent color electric field making possible the reduction of the color magnetic fluxes. The holonomy group of the classical color gauge field $G^A_{\alpha\beta}$ is always Abelian in TGD framework being proportional to $H^AJ_{\alpha\beta}$, where H^A are color Hamiltonians and $J_{\alpha\beta}$ is the induced Kähler form. Hence it should be possible to treat the situation in terms of the induced Kähler field alone. Obviously, the change of the Kähler (color) electric flux in the reaction corresponds to the change of (color) Kähler (color) magnetic flux. The change of color electric flux occurs naturally in a collision situation involving changing induced gauge fields.

8.7 Cold fusion, plasma electrolysis, biological transmutations, and burning salt water

The article of Kanarev and Mizuno [D67] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

One can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the cathode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E11]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.
2. The so called $H_{1.5}O$ anomaly of water [D30, D27, D32, D52] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.
3. The mysterious behavior burning salt water [D1] can be also understood in the same framework.
4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

8.7.1 The data

Findings of Kanarev

Kanarev has found that the volume of produced H_2 and O_2 gases is much larger than the volume resulting in the electrolysis of the water used in the process. If one knows the values of p and T one can estimate the volumes of H_2 and O_2 using the equation of state $V = nT/p$ of ideal gas. This gives

$$V(H_2;p,T) = \frac{A(H_2)}{A(H_2O)} \times \frac{M(H_20)}{m_p} = \frac{1}{9}\frac{M(H_20)}{m_p} \times \frac{T}{p} \ .$$

Here $M(H_20)$ is the total mass of the water (.272 kg for KOH and .445 kg for NaOH).

In the situation considered one should be able to produce from one liter of water 1220 liters of hydrogen and 622 liters of oxygen giving

$$V(H_2)/V(H_2O) = 1.220 \times 10^3 \ , \qquad V(O_2)/V(H_2O) = .622 \times 10^3 \ ,$$

$$r(gas) = V(H_2 + O_2)/V(H_2O) = 1.844 \times 10^3 \ , \quad V(H_2)/V(O_2)) \simeq 1.96 \ .$$

$V(H_2)/V(O_2) \simeq 1.96$ is 4 per cent smaller than the prediction $V(H_2)/V(O_2) = 2$ of the ideal gas approximation.

The volumes of O_2 and H_2 are not reported separately. The table gives the total volumes of gas produced and ratios to the volume of water used.

	$M(H_2O)/kg$	$V(gas)/m^3$	$\frac{V(gas)}{V(H_2O)}$	$\frac{[V(gas)/V(H_2O)]}{r(gas)}$
KOH	.272	8.75	3.2×10^4	17.4
NaOH	.445	12.66	2.8×10^4	15.2

Table 1. The weight of water used in the electrolysis and the total volume of gas produced for KOH and NaOH electrolysis. $r(gas)$ denotes the naive prediction for the total volume of gas per water volume appearing in previous table. For KOH *resp.* NaOH the volume ratio $[V(gas)/V(H_2O)]$ is by a factor $r = 17.4$ *resp.* $r = 15.2$ higher than the naive estimate.

Findings of Mizuno

Mizuno in turn found that the Fe cathode contains Si, K, Cr, Fe, Cu for both KOH and NaOH electrolysis and in case of NaOH also Al, Sl, Ca. The fraction of these nuclei is of order one per cent. The table below gives the fractions for both KOH and NaOH.

KOH				
Element(Z,N)	Al(13,27)	Si(14,28)	Cl(17,18)	K(19,20)
		0.94		4.50
Element(Z,N)	Ca(20,20)	Cr(24,28))	Fe(26,29)	Cu(29,34)
		1.90	93.0	0.45
NaOH				
Element(Z,N)	Al(13,27)	Si(14,28)	Cl(17,18)	K(19,20)
	1.10	0.55	0.20	0.60
Element(Z,N)	Ca(20,20)	Cr(24,28))	Fe(26,29)	Cu(29,34)
	0.40	1.60	94.0	0.65

Table 2. The per cent of various nuclei in cathode for KOH and NaOH electrolysis.

The results supports the view that nuclear reactions involving new nuclear physics are involved and that part of H_2 and O_2 could be produced by nuclear reactions at the cathode.

1. For Si, K, Cr, Fe, and Cu the mechanism could be common for both $NaOH$ and KOH electrolysis and presumably involve fission of Fe nuclei. The percent of K in KOH is considerably larger than in $NaOH$ case and this is presumably due to the absorption of K^+ ions by the cathode.

2. For Al, Si, and Ca the reaction occurring only for Na should involve Na ions absorbed by the cathode and suffering cold fusion with some particles -call them just X - to be identified.

3. Cu is the only element heavier than Fe and is expected to be produced by fusion with X. Quite generally, the fractions are of order one per cent.

4. The authors suggests that the extra volume of H_2 and O_2 molecules is due to nuclear reactions in the cathode. A test for this hypothesis would be the ratio of H_2 and O_2 volumes. Large deviation from value 2 would support the hypothesis. The value near 2 would in turn support the hypothesis that the water produced by electrolysis is considerably denser than ordinary water.

8.7.2 $H_{1.5}O$ anomaly and nuclear string model

It would seem that some exotic nuclei, perhaps consisting of protons, should be involved with the cold fusion. Concerning the identification of these exotic particles there are several guidelines. $H_{1.5}O$ anomaly, anomalous production of e^+e^- pairs in heavy ion collisions, and nuclear string model.

$H_{1.5}O$ anomaly and anomalous production of electron-positron pairs in heavy ion collisions

There exists an anomaly which could be explained in terms of long open nuclear strings. The explanation of $H_{1,5}O$ anomaly [D30, D27, D32, D52] discussed in [K30] as a manifestation of dark protons was one of the first applications of TGD based ideas about dark matter. The proposed explanation is that the fraction of 1/4 of protons is in atto-second time scale dark and invisible in electron scattering and neutron diffraction. Note that atto-second time scale corresponds to the time during which light travels a length of order atomic size.

A natural identification of the dark protons would be in terms of protonic strings behaving like nuclei having anomalously large size, which would be due to the anomalously large value of Planck constant. A partial neutralization by negatively charge color bonds would make these states stable.

The TGD based explanation of anomalous production of electron-positron pairs in the collisions of heavy nuclei just above the Coulomb wall [K88] is in terms of lepto-pions consisting of pairs of color octet electron and positron allowed by TGD and having mass slightly below $2m_e \simeq 1$ MeV. The strong electromagnetic fields created in collision create coherent state of lepto-pions decaying into electron positron pairs.

Nuclear string model

The nuclear string model describes nuclei as string like structures with nucleons connected by color magnetic flux tubes whose length is of order electron Compton length about 10^{-12} meters and even longer and thus much longer than the size scale of nuclei themselves which is below 10^{-14} meters. Color magnetic flux tubes define the color magnetic body of nucleus and each flux tube has colored fermion and anti-fermion at its ends. The net color of pair is non-vanishing so that color confinement binds the nucleons to the nuclear string. Nuclei can be visualized as structures analogous to plants with nucleus taking the role of seed and color magnetic body of much larger size taking the role of plant with color flux tubes however returning back to another nucleon inside nucleus.

One can imagine two basic identifications of the fermions.

1. For the first option fermions are identified as quarks. The color flux tube can have three charge states $q = +1, 0, -1$ according to whether it corresponds to $u\overline{d}, u\overline{u} + d\overline{d}$, or $\overline{u}d$ type state for quarks. This predicts a rich spectrum of exotic nuclei in which neutrons consist actually of proton plus negatively charged flux tube. The small mass difference between neutron and proton and small mass of the quarks (of order MeV) could quite well mean that these exotic nuclei are identified as ordinary nuclei. The findings of [C88] [C88] support the identification as quarks.

2. Lepto-hadron hypothesis [K88] encourages to consider also the possibility that color bonds have color octet electrons at their ends. This would make it easier to understand why lepto-pions are produced in the collisions of heavy nuclei.

3. One can also consider the possibility that the color bonds are superpositions of quark-antiquark pairs and colored electron-positron pairs.

Two options

One can consider two options for protonic strings. Either their correspond to open strings connected by color magnetic flux tubes or protons are dark so that giant nuclei are in question.

1. Protonic strings as open strings?

Color flux tubes connecting nucleons are long and one can ask whether it might be possible also open nuclear strings with long color flux tubes connecting widely separate nucleons even at atomic distance. These kind of structures would be favored if the ends of nuclear string are charged.

Even without assumption of large values of Planck constant for the color magnetic body and quarks the net length of flux tubes could be of the order of atomic size. Large value $\hbar$ would imply an additional scaling.

The simplest giant nuclei constructible in this manner would consist of protons connected by color magnetic flux tubes to from an open string. Stability suggest that the charge per length is not too high so that some minimum fraction of the color bonds would be negatively charged. One could speak of exotic counterparts of ordinary nuclei differing from them only in the sense that size scale is much larger. A natural assumption is that the distance between charged protonic space-time sheets along string is constant.

In the sequel the notation $X(z, n)$ will be is used for the protonic string containing net charge z and n negatively charged bonds. $a = z + n$ will denote the number of protons. z, n and a are analogous to nuclear charge Z, neutron number N, and mass number A. For open strings the charge is $z \geq 1$ and for closed strings $z \geq 0$ holds true.

This option has however problem. It is difficult imagine how the nuclear reactions could take place. One can imagine ordinary stringy diagrams in which touching of strings means that proton of protonic string and ordinary nucleus interact strongly in ordinary sense of the word. It is however difficult to imagine how entire protonic string could be absorbed into the ordinary nucleus.

2. Are protons of the protonic string dark?

Second option is that protonic strings consist of dark protons so that nuclear space-time-sheet has scale up size, perhaps of order atomic size. This means that fermionic charge is distributed in much larger volume and possibly also the fermions associated with color magnetic flux tubes have scaled up sized. The value $\hbar = 2^{11}\hbar_0$ would predict Compton length of order 10^{-12} m for nucleon and upper size of order 10^{-11} for nuclei.

Cold nuclear reactions require a transformation of dark protons to ordinary ones and this requires leakage to the sector of the imbedding space in which the ordinary nuclei reside (here the book metaphor for imbedding space is very useful). This process can take place for a neutral part of protonic string and involves a reduction of proton and fermion sizes to normal ones. The phase transition could occur first only for a neutral piece of the protonic string having charges at its ends and initiate the nuclear reaction. Part of protonic string could remain dark and remaining part could be "eaten" by the ordinary nucleus or dark protonic string could "eat" part of the ordinary nuclear string. If the leakage occurs for the entire dark proton string, the nuclear reaction itself is just ordinary nuclear reaction and is expected to give out ordinary nuclei. What is important that apart from the crucial phase transition steps in the beginning and perhaps also in the end of the reaction, the model reduces to ordinary nuclear physics and is in principle testable.

The basic question is how plasma phase resulting in electrolysis leads to the formation of dark protons. The proposal [K32] that the transition takes place with perturbative description of the plasma phase fails, might be more or less correct. Later a more detailed nuclear physics picture about the situation emerges.

3. What happens to electrons in the formation of protonic strings?

One should answer two questions.

1. What happens to the electrons of hydrogen atoms in the formation of dark protonic strings?
2. In plasma electrolysis the increase of the input voltage implies a mysterious reduction of the electron current with the simultaneous increase of the size of the plasma region near the cathode [C75] . This means reduction of conductance with voltage and thus non-linear behavior. Where does electronic charge go?

Obviously the negatively charged color bond created by adding one proton to a protonic string could take the charge of electron and transform electrons as charge carriers to color bonds of dark Li isotopes which charge $Z = 3$ by gluing to existing protons sequence proton and negatively charged color bond. If the proton comes from H_2O OH^- replaces electron as a charge carrier. This would reduced the conductivity since OH^- is much heavier than electron. This kind of process and its reversal would take place in the transformation of hydrogen atoms to dark proton strings and back in atto-second time scale.

The color bond could be either $\overline{u}d$ pair or $e_8\overline{\nu}_8$ pair or quantum superposition of these. The basic vertex would involve the exchange of color octet super-symplectic bosons and their neutrino counterparts. Lepton number conservation requires creation of color singlet states formed of color octet neutrinos which ar bosons and carrying lepton number -2. One color confined neutrino pair would be created for each electron pair consumed in the process and might escape the system: if this happens, the process is not reversible above the time scale defined by colored neutrino mass scale of order .1 eV which happens to be of order .1 atto-seconds for ordinary neutrinos. Also ordinary nuclei could consist of nucleons connected by identical neutral color bonds (mostly).

The exchange of light counterparts of charged ρ mesons having mass of order MeV could lead to the transformation of neutral color bonds to charged ones. In deuterium cold fusion the exchange of charged ρ mesons between D and Pd nuclei could transform D nuclei to states behaving like di-neutrons so that cold fusion for D could take place. In the earlier proposal exchange of W^+ boson of scaled variant of weak interactions was proposed as a mechanism.

The formation of charged color bonds binding new dark protons to existing protonic nuclear strings or giving rise to the formation of completely new protonic strings would also increase of the rates of cold nuclear reactions.

Note that this picture leaves open the question whether the fermions associated with color bonds are quarks or electrons.

Nuclei and their dark variants must have same binding energy scale at nuclear quantum criticality

The basic question is what happens to the scale of binding energy of nuclei in the zooming up of nuclear space-time sheet. Quantum criticality requires that the binding energies scales must be same.

1. Consider first the binding energy of the nuclear strings. The highly non-trivial prediction of the nuclear string model is that the contributions of strong contact interactions at nuclear space-time sheet (having size $L < 10^{-14}$ m) to the binding energy vanish in good approximation for ground states with vanishing strong isospin. This means that the binding energy comes from the binding energy assignable to color bonds connecting nucleons together.

2. Suppose that this holds true in a good approximation also for dark nuclei for which the distances of nucleons at zoomed up nuclear space-time sheet (having originally size below 10^{-14} meters) are scaled up. As a matter fact, since the scale of binding energy for contact interactions is expected to reduce, the situation is expected to improve. Suppose that color bonds with length of order 10^{-12} m preserve their lengths. Under these assumptions the nuclear binding energy scale is not affected appreciably and one can have nuclear quantum criticality. Note that the length for the color bonds poses upper limit of order 100 for the scaling of Planck constant.

It is essential that the length of color bonds is not changed and only the size of the nuclear space-time sheet changes. If also the length and thickness of color bonds is scaled up then a naive scaling argument assuming that color binding energy related to the interaction of transforms as color Coulombic binding energy would predict that the energy scales like $1/\hbar$. The binding energies of dark nuclei would be much smaller and transformation of ordinary nuclei to dark nuclei would not take place spontaneously. Quantum criticality would not hold true and the argument explaining the transformation of ordinary Li to its dark counterpart and the model for the deuterium cold fusion would be lost.

8.7.3 A model for the observations of Mizuno

The basic objection against cold nuclear reactions is that Coulomb wall makes it impossible for the incoming nuclei to reach the range of strong interactions. In order that the particle gets to the cathode from electrolyte it should be positively charged. Positive charge however implies Coulomb wall which cannot be overcome with the low energies involved.

These two contradictory conditions can be satisfied if the electrolysis produces exotic phase of water satisfying the chemical formula $H_{1.5}O$ with 1/4 of protons in the form of almost neutral protonic strings can possess only few neutral color bonds. The neutral portions of the protonic string, which have suffered phase transition to a phase with ordinary Planck constant could get very near to the target nucleus since the charges of proton can be neutralized in the size scale of proton by the charges $\overline{u}$ and d quarks or e and $\overline{\nu}$ associated with the two bonds connecting proton to the two neighboring protons. This could make possible cold nuclear reactions.

It turns out that the model fixes protonic strings to isotopes of dark Lithium (with neutrons replaced with proton plus negatively charged color bond). What is intriguing is that the biologically most important ions (besides Na^+) Cl^-, K^+, and Ca^{++} appear at the cathode in Kanarev's plasma electrolysis actually result as outcomes of cold nuclear reactions between dark Li and Na^+.

General assumptions of the model

The general assumptions of the model are following.

1. Ordinary nuclei are nuclear strings, which can contain besides neutrons also "pseudo-neutrons" consisting of pairs of protons and negatively charged color bonds. The model for D cold fusion requires that the Pd nuclei contain also "pseudo-neutrons".

2. Reaction products resulting in the fusion of exotic protonic string transforming partially to ordinary nuclear matter (if originally in dark phase) consist of the nuclei detected in the cathode plus possibly also nuclei which form gases or noble gases and leak out from the cathode.

3. Si, K, Cr, and Cu are produced by the same mechanism in both KOH and NaOH electrolysis.

4. Al, Cl, and Ca is produced by a mechanism which must involve cold nuclear reaction between protonic string and Na ions condensed on the cathode.

5. $Cu(Z,N) = Cu(29,34)$ is the only product nucleus heavier than $Fe(26,29)$. If no other nuclei are involved and Cu is produced by cold fusion

$$X(z,n) + Fe(26,29) \rightarrow Cu(29,34) \ ,$$

the anatomy of protonic string must be

$$X(z,n) = X(3,5)$$

so that dark variant $Li(3,5)$ having charge 3 and mass number 8 would be in question. $X(3,5)$ would have 2 neutral color bonds and 5 negatively charged color bonds. To minimize Coulomb interaction the neutral color bonds must reside at the ends of the string. For quark option one would have charge $1+2/3$ at the first end and $1+1/3$ at the second end and charges of all protons between them would be neutralized. For color octet lepton color bond one would have charge 2 at the other end and zero at the other end.

For quark option the net protonic charge at the ends of the string causing repulsive interaction between the ends could make protonic string unstable against transition to dark phase in which the distance between ends is much longer even if the ends are closed within scaled up variant of the nuclear volume.

Arbitrarily long strings $X(3,n)$ having neutral bonds only at their ends are possible and their fusions lead to neutron rich isotopes of Cu nucleus decaying to the stable isotope. Hence the prediction that only Cu is produced is very general.

The simplest dark protonic strings $X(3,n)$ have quantum numbers of $Li(3,n)$. One of the hard problems of Big Bang cosmology is that the measured abundance of lithium is by a factor of about 2.5 lower than the predicted abundance [E11] . The spontaneous transformation of $Li(3,n)$ isotopes to their dark variants could explain the discrepancy. Just by passign notice that Li has mood stabilizing effect [C8] : the spontaneous transformation of Li^+ to its dark variant might relate to this effect.

Production mechanisms for the light nuclei common to Na and K

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These nuclei must be produced by a fission of Fe nuclei.

1. For $Si(14,14)$ production the mechanism would be cold fission of Fe nucleus to two parts in the collision with the protonic string:

$$X(3,5) + Fe(26,29) \rightarrow Si(14,14) + Al(13,14) + X(2,6) \ .$$

$X(2,6)$ represent dark or ordinary $He(2,6)$. As a noble gas He isotope would leave the cathode.

Note that arbitrarily long proton strings with two neutral bonds at their ends give neutron rich isotope of Si and exotic or ordinary isotope of He so that again the prediction is very general.

2. $K(19,20)$ is produced much more in KOH which most probably means that part of K^+ is absorbed from the electrolyte. In this case the reaction could proceed as follows:

$$X(3,5) + Fe(26,29) \rightarrow K(19,20) + Ne(7,7) + X(3,7) \ .$$

Note that the neutron number could be distributed in many manners between final states. For arbitrarily long proton string with two neutral bonds at ends higher neutron rich isotopes of K and Ne are produced. As noble gas Ne would leak out from the cathode.

Ordinary $Li(3,7)$ would decay by neutron emission to stable isotopes of Li. The temperature of the system determines whether Li boils out (1615 K under normal pressure). Li is not reported to appear in the cathode. In plasma electrolysis the temperature is in the interval $.5 \times 10^4$- 10^4 C and around 10^3 C in the ordinary electrolysis so that the high temperature might explain the absence of Li. Also the in-stability of Li isotopes against transition to dark Li in electrolyte would imply the absence of Li.

3. For $Cr(24,28)$ production the simplest reaction would be

$$X(3,5) + Fe(26,29) \rightarrow Cr(24,28) + He(2,2) + X(3,4) \ .$$

Helium would leak out as noble gas. Proton string would shorten by one unit and keep its charge. $X(3,4)$ would represent the stable isotope $Li(3,4)$ or its dark counterpart and what has been said in 2) applies also now.

How to understand the difference between KOH and NOH?

One should understand why Al, Cl, and Ca are not detected in the case of KOH electrolysis.

Al, Cl, and Ca would be created in the fusion of protonic strings with $Na(11,12)$ nuclei absorbed by the cathode. With this assumption the rates are expected to be of same order of magnitude for all these processes as suggested by the one per cent order of magnitude for all fractions.

One can imagine two reaction mechanisms.

I: One could understand the production assuming only $X(3,5)$ protonic strings if the number of $X(3,5)$ strings absorbed by single Na nucleus can be $k = 1,2,3$ and that nuclear fission can take place after each step with a rate which is slow as compared to the rate of absorptions involving also the phase transition to dark matter. This is however highly implausible since ordinary nuclear interactions are in question.

II: Second possibility is that the protonic strings appearing with the highest probability are obtained by fusing copies of the basic string $X(3,5)$ by using neutral color bond between the strings. The minimization of electrostatic energy requires that that neutral color bonds are equally spaced so that there are three completely neutralized protons between non-neutralized protons.

One would have thus at least the strings $X(3,5)$, $X(6,10)$, and $X(9,15)$, which correspond to dark $Li(3,5)$ and dark variants of the unstable isotopes $C(6,10)$ and $F(9,15)$. In nuclear string model also ordinary nuclei are constructed from $He(2,2)$ strings and lighter strings in completely analogous manner, and one could perhaps see the dark nuclei constructed from $Li(3,5)$ as the next level of hierarchy realized only at the level of dark matter.

The charge per nucleon would be 3/8 and the length of the string would be a multiple of 8. Interestingly, the numbers 3, 5, and 8 are subsequent Fibonacci numbers appearing very frequently also in biology (micro-tubules, sunflower patterns). The model predicts also the occurrence of cold fusions $X(z = 3k, n = 5k) + Fe(26,29) \rightarrow (Z,N) = (26+3k, 29+5k)$. For $k = 2$ this would give $Ge(32,39)$ which is stable isotope of Ge. For $k = 3$ one would have $(Z,N) = (35,44)$ which is stable isotope of Br [C87, C14] .

Consider now detailed description of the reactions explaining the nuclei detected in the cathode.

1. $Al(13,14)$ would be produced in the reaction

$$X(3,5) + Na(11,12) \rightarrow Al(13,14) + X(1,3) \ .$$

$H(1,3)$ or its dark variant could be in question. Also the reaction $X(3,5) + Na(11,12) \rightarrow Al(13,17) + p$, where $Al(12,17)$ is an unstable isotope of Al is possible.

The full absorption of protonic string would yield $Si(14,17)$ beta-decaying to $P(15,16)$, which is stable. Either P leaks out from the cathode or full absorption does not take place appreciably.

2. $Cl(17,18)$ would be produced by the sequence

$$\begin{array}{lrcl} \textbf{I1}: & 2X(3,5)+Na(11,12) & \rightarrow & Cl(17,18)+X(0,4)\ , \\ \textbf{I2}: & X(6,10)+Na(11,12) & \rightarrow & Cl(17,18)+X(0,4)\ . \end{array}$$

$X(0,4)$ represents ordinary or dark tetra-neutron [C117, C51, C22] . The instability of the transformation of tetra-neutron to dark matter could explain why its existence has remained controversial.

If the protonic string were absorbed completely, the resulting $Cl(17,22)$ - if equivalent to ordinary nucleus - would transform via beta-decays to $A(18,23)$ and then to $K(19,22)$, which is stable and detected in the target.

3. $Ca(20,20)$ would be produced in the reaction

$$\begin{array}{lrcl} \textbf{I1}: & 3X(3,5)+Na(11,12) & \rightarrow & Ca(20,20)+X(0,7)\ , \\ \textbf{I2}: & X(9,15)+Na(11,12) & \rightarrow & Ca(20,20)+X(0,7)\ . \end{array}$$

$X(0,7)$ would be dark counterpart of "septa-neutron". The complete absorption of nuclear string would produce $Ca(20,27)$, which (if ordinary nucleus) transforms via beta decays to $Sc(21,26)$ and then to $Ti(22,25)$, which is stable.

8.7.4 Comparison with the model of deuterium cold fusion

It is interesting to compare the model with the model for cold fusion [C105, C1] reported using deuterium target and D_2O instead of water.

Earlier model

1. The model is based on the assumption that D nuclei in the target suffer a phase transition to a state in which D nuclei become neutral so that the color bond between neutron and proton becomes negatively charged: one has effectively di-neutrons.

2. The mechanism of charging of color bond must either involve weak interactions or exchange of lepto-ρ mesons already discussed briefly. The proposal is that the exchange of W bosons of scaled up version of weak physics is involved with the range of interactions given by atomic length scale. The exchange of W^+ bosons was assumed to take place between Pd and D nuclei. This mechanism could lead to the formation of negatively charged color bonds in also ordinary nuclei.

3. The neutrality of exotic D nuclei allows to overcome Coulomb wall. One can understand the reported selection rules: in particular the absence of Helium isotopes (only isotopes of H are detected). The absence of gamma rays can be understood if the resulting gamma rays are dark and leak out before a transformation to ordinary gamma rays.

Are D nuclei in Pd target dark or not?

The question whether the exotic D nuclei are dark was left pending. The recent model suggests that the answer is affirmative.

1. The basic difference between the two experiments would be that in Kanarev's experiments incoming nuclei are dark whereas in D fusion cathode contains the dark nuclei and cold nuclear reactions occur at the "dark side" and is preceded by ordinary-to-dark phase transition for incoming D.

2. D cold fusion occurs for a very restricted range of parameters characterizing target: the first parameter is doping ratio: essentially one D nucleus per one Pd nucleus is needed which would fit with the assumption that scaled up size is of the order of atom size. Temperature is second parameter. This and the fact that the situation is highly sensitive to perturbations conforms with the interpretation as a phase transition to dark matter occurring at quantum criticality.

3. The model for Kanarev's findings forces to consider the possibility that dark D nuclei combine to form longer strings and can also give rise to dark $Li(3,5)$ explaining the observed nuclear transmutations in the target.

4. In cold nuclear reactions incoming nuclei would transform to dark nuclei (the picture as a leakage between different pages of a book like structure defined by the generalized imbedding space is helpful). The reaction would take place for dark nuclei in zoomed up nuclear physics and the reaction products would be unstable against phase transition to ordinary nuclei.

5. Is it then necessary to assume that target D nuclei are transformed to neutral ones (di-neutrons effectively) in order to have cold nuclear reactions? Nuclear space-time sheets are scaled up. If nucleon space-time sheets are not scaled up, p and n are connected by color magnetic flux tubes of same length as in the case of ordinary nuclei but located at much larger nuclear space-time sheet. The classical analog for the quantal distribution of nucleon charges is even charge distribution in a sphere or radius R defined by the charge of the scaled up nucleus. The height of the Coulomb wall is $E_c = e^2/R$. If $R = a$, a the atomic radius, one has $E_c \sim .1$ keV. The wall is by a factor 10^{-4} lower than in ordinary nuclear collision so that the incoming D nucleus might overcome the Coulomb wall.

   If Coulomb wall can be overcome, all dark variants of $D + D$ reaction are possible. Helium nuclei have not been however detected, which supports the view that D in target is transformed to its neutral variant. Gamma rays would be dark and could leak out without detection which would explain the absence of gamma rays.

Nuclear quantum criticality is essential

A note about the energetics of cold nuclear reactions is in order. The nuclear quantum criticality deriving from the cancelation of the contact interaction energies between nucleons for isospin singlets and scaling up of *only* nuclear space-time sheet is an absolutely essential assumption. Otherwise dark D would have much smaller binding energy scale than the visible one, and ordinary D in the Pd target could not transform to dark "di-neutron" state. Also the transformation of incoming D to its dark variant D at cathode could not take place.

8.7.5 What happens to OH bonds in plasma electrolysis?

For an innocent novice one strange aspect of hydrolysis is how the OH bonds having energies of order 8 eV can be split in temperatures corresponding to photon energies of order .5 eV. Kanarev has suggested his own theory for how this could happen [D66] . TGD suggests that OH bonds are transformed to their dark variants with scaled down bond energy and that there might be no essential difference between OH bond and hydrogen bond.

The reduction of energy of OH bonds in plasma electrolysis

Kanarev has found that in plasma electrolysis the energy of OH bonds is reduced from roughly 8 eV to about .5 eV, which corresponds to the fundamental metabolic energy quantum identifiable as the zero point kinetic energy liberate as proton drops from $k = 137$ space-time sheet to much larger space-time sheet. In pyrolysis [D16] similar reduction could occur since the pyrolysis occurs above temperature about 4000 C conforming with the energy scale of hydrogen bond.

The explanation discussed in [K86] is that there is some mechanism exciting the bonds to a state with much lower bond energy. Dark matter hierarchy [K32] suggests that the excitation corresponds to the transformation of OH bond to dark bond so that the energy scale of the state is reduced.

Also in the ordinary electrolysis of water [D4] the energy of OH bonds is reduced to about 3.3 eV meaning a reduction factor of order 2. The simplest interpretation would be as a transformation of OH bonds to dark OH bond with $\hbar \rightarrow 2\hbar$ (the scaling could be also by some other integer or even rational). The energy needed to transform the bond to dark bond could come from remote metabolism via the dropping of dark protons from a dark variant of some sub-atomic space-time sheet with size not smaller than the size of the atomic space-time sheet to a larger space-time sheet.

In many-sheeted space-time (see fig. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg` or fig. 9 in the appendix of this book) particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

$H_{1.5}O$ anomaly suggests that 1/4 of protons of water are dark in atto-second time scale [K30] and one can imagine that both protons of water molecule can become dark under conditions defined by plasma electrolysis. Also the atomic space-time sheets and electron associated with OH bonds could become dark.

Atomic binding energies transform as $1/\hbar^2$. If the energy of hydrogen bond transforms like Coulombic interaction energy as given by the perturbative calculation, it is scaled down as $1/\hbar$ since the length of the bond scales up like $\hbar$. Effectively $\alpha_{em} \propto 1/\hbar$ is replaced by its scaled down value. For $\hbar \rightarrow 2^4\hbar_0$ the energy would scale from 8 eV to .5 eV and the standard metabolic energy quantum could induce the splitting of the dark OH bond. If 2^4 is the scale factor of $\hbar$ for dark nuclear space-time sheets, their size would be of order 10^{-3} meters. The model for cold fusion is consistent with this since what matters is different value of Planck constant for the dark nuclear space-time sheets.

There is an objection against the reduction of OH bond energy. The bonds could be split by a process in which dark nuclear reactions kick protons to $k = 133$ dark space-time sheet. In this case the maximal zero point kinetic energy liberated in the dropping back would be 8 eV and could induce breaking of OH bond. For $\hbar/\hbar_0 \geq 4$ the size of $k = 133$ dark space-time sheet would be larger than the size of $k = 137$ atomic space-time sheet.

Are hydrogen bonds dark OH bonds?

The fact that the energy of hydrogen bonds [D9] is typically around .5 eV forces to ask what distinguishes hydrogen bond from dark OH bond. Could it be that the two bonds are one and the same thing so that dark OH bonds would form standard part of the standard chemistry and molecular biology? In hydrogen bond same hydrogen would be shared by the oxygen atoms of the neighboring atoms. For the first O the bond would be ordinary OH bond and for the second O its dark variant with scaled down Coulomb energy. Under conditions making possible pyrolysis and plasma electrolysis both bonds would become dark. The variation of the hydrogen bond energy could reflect the variation of the scaling factor of $\hbar$.

The concentration of the spectrum of bond energies on integer multiples of fundamental energy scale - or even better, on powers of 2 - would provide support for the identification. There is evidence for two kinds of hydrogen bonds with bond energies in ratio 1:2 [D70] : the TGD based model is discussed in [K30] .

Mechanism transforming OH bonds to their dark counterparts

The transformation of OH bonds to dark bonds would occur both in ordinary and plasma electrolysis and only the change of Planck constant would distinguish between the two situations.

1. Whatever the mechanism transforming OH bonds to their dark counterparts is, metabolic energy is needed to achieve this. Kanarev also claims over-unity energy production [D66] . Cold fusion researchers make the same claim about ordinary electrolysis. Cold nuclear reactions between Na^+ (K^+) and dark protons and dark Li could obviously serve as the primary energy source. This would provide the fundamental reason for why $NaOH$ or KOH must be present. Cold nuclear reactions would thus occur also in the ordinary electrolysis of water and provide the energy inducing the transition of OH bonds to dark ones by (say) $\hbar \rightarrow 2\hbar$ transition.

2. One can imagine several metabolic mechanisms for the visible-to-dark transformation of HO bonds. The energy spectrum of cold nuclear reactions forms a continuum whereas the energies needed to transform OH bonds to their dark variants presumably are in narrow

bands. Therefore the energy liberated in cold nuclear reactions is not probably used as such. It is more plausible that standard metabolic energy quanta liberated in the dropping of protons (most naturally) to larger space-time sheets are utilized. The most important metabolic energy quanta for the dropping of proton come as $E_k = 2^{k-137} k E_0$: $E_0 = .5$ eV is liberated in the dropping of proton from atomic space-time sheet ($k = 137$) to much larger space-time sheet (the discrete spectrum of increments of the vacuum energy in the dropping approaches this energy [K62]). The energy liberated in the dark nuclear reactions would "load metabolic batteries" by kicking the dark protons to the dark variants of $k < 137$ space-time sheet (the size of dark atomic space-time sheet scales like $\hbar$). Their dropping to larger space-time sheets would liberate photons with energies near to those transforming OH bonds to hydrogen bonds.

3. A signature for the standard metabolic energy quanta would be visible light at $2eV$ and also discrete lines below it accumulating to $2eV$. Kanarev's indeed reports the presence of red light [D66] as a signature for the occurrence of process.

8.7.6 A model for plasma electrolysis

Kanarev's experiments involve also other strange aspects which lead to the view that cold nuclear reactions and dark matter physics are essential aspects of not only plasma electrolysis of Kanarev but also of ordinary electrolysis and responsible for the claimed over unity energy production. Biologically important ions are produced in reactions of dark Li and Na^+ and there is very strong electric voltage over the cell membrane. This inspires the question whether cold nuclear reactions serve as a metabolic energy source in living cell and are also responsible for production of ions heavier than Na^+.

Brief description of plasma electrolysis

Electrolysis [D4] , pyrolysis [D16] , and plasma electrolysis [C75] , [D66] of water are methods of producing free hydrogen. In pyrolysis the temperature above 4000 C leads to hydrogen and oxygen production. Oxygen production occurs also at cathode and hydrogen yield is higher than given by Faraday law for ordinary electrolysis [D4] .

The article of Mizuno and collaborators [C75] about hydrogen production by plasma electrolysis contains a brief description of plasma electrolysis. A glow discharge occurs as the input voltage used in electrolysis is above a critical value and plasma is formed near cathode. In the arrangement of [C75] plasma state is easily achieved above 140 V. If the values of temperature and current density are right, hydrogen generation in excess of Faraday's law as well as a production of oxygen at cathode (not possible in ideal electrolysis) are observed. Above 350 V the control of the process becomes difficult.

What really happens in electrolysis and plasma electrolysis?

1. Ordinary electrolysis

To understand what might happen in the plasma electrolysis consider first the ordinary electrolysis of water.

1. The arrangement involves typically the electrolyte consisting of water plus $NaOH$ or KOH without which hydrolysis is impossible for thermodynamical reasons.

2. Electronic current flows from the anode to cathode along a wire. In electrolyte there is a current of positively charged ions form anode to cathode. At the cathode the reaction $2H_2O + 2e^- \rightarrow 2H_2 + 2OH^-$ yields hydrogen molecules seen as bubbles in water. At the anode the reaction $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$ is followed by the reaction $2H^+ + 2e^- \rightarrow H_2$ and the flow of $2e^-$ to the cathode along wire. The net outcome is hydrolysis: $H_2O \rightarrow 2H_2 + 2O_2$. Note that O_2 is produced only at anode and H_2 at both anode and cathode.

2. What happens in plasma electrolysis?

In plasma electrolysis something different might happen.

1. Cold nuclear reactions should take place at cathode in presence of Na^+ ions plus dark Li and should be in equilibrium under ordinary conditions and contribute mainly to the formation of dark OH bonds. The rate of cold nuclear reactions increases with input voltage V since the currents of Na^+ and dark Li to the cathode increase. Obviously the increased rate of energy yield from dark nuclear reactions could be the real reason for the formation of plasma phase above critical voltage.

2. By previous considerations the reduction of electron current above critical voltage has interpretation as a transition in which electronic charge is transferred to negative charge of color bonds of dark proton strings. Existing protonic strings could grow longer and also new strings could be created from the ionized hydrogen resulting in the electrolysis of water. The increase of the size of the dark nuclei would mean increase of the cross sections for cold nuclear reactions. The liberated energy would ionized hydrogen atoms and give rise to a positive feedback loop somewhat like in ordinary nuclear reactions.

3. The increased energy yield in cold nuclear reactions suggests that OH bonds are transformed very effectively to dark OH bonds in the plasma region. This means that the thermal radiation can split the hydrogen bonds and induce the splitting of two water molecules to $4H$ and $2O$ and therefore production of $2H_2 + O_2$ everywhere in this kind of region. The temperature used by Kanarev corresponds to energy between .5-1 eV [D66] which conforms with the fact that OH bond energy is reduced to about .5 eV. Note that the presence of anode and cathode is not absolutely necessary if cold nuclear reactions can take place in the entire electrolyte volume and generate plasma phase by positive feedback loop.

4. The prediction is that Faraday's law for hydrogen production does not hold true. O/H ratio has the value $r = O/H = 0$ for the ordinary electrolysis at cathode. $r = 1/2$ holds true if local dissociation of water molecules dominates. According to [C75] r increases from electrolysis value $r = .066$ above $V = 140$ V achieving the value $r = .45$ for $V = 350$ V where the system becomes unstable. Also cold nuclear reactions could contribute to hydrogen and oxygen production and affect the value of r as suggested by the large volume of gas produced in Kanarev's experiments [D67] .

Over-unity energy production?

Over-unity energy production with output power 2- or even 3-fold as compared with input power has been reported from plasma electrolysis. The effectiveness is deduced from the heating of of the system. Note that Mizuno reports in [C75] that 10 per cent effectiveness but this is for the storage of energy to hydrogen and does not take into account the energy going to the heating of water.

The formation of higher isotopes of Li by fusing dark protons to existing dark proton strings is a good candidate for the dominant energy production mechanism. An estimate for the energy liberate in single process $Li(3,n) + m_p + e \to Li(3,n+1) + 2\nu_8$ is obtained by using energy conservation. Here $2\nu_8$ denotes color singlet bound state of two color octet excitations of neutrino.

Since e_8 and ν_8 are analogous to u and d quarks one expects that their masses are very nearly the same. This gives as the first guess $m_{\nu_8} = m_e$ and since lepto-pion (color bound state of color octet electrons, [K88]) has mass $m = 2m_e$ a good guess is $m(2\nu_8) = 2m_{\nu_8} = 2m_e$. The energy conservation would give

$$m(Li(3,n)) + m_p \quad = \quad m(Li(3,n+1)) + m_e + T(2\nu_8) + E(\gamma) \ . \tag{8.7.1}$$

Here $T(2\nu_8)$ is the kinetic energy of $2\nu_8$ state and E_γ is the energy of photon possibly also emitted in the process.

The process is kinematically possible if the condition

$$\Delta m \quad = \quad m(Li(3,n)) + m_p - m(Li(3,n+1) \geq m_e \ . \tag{8.7.2}$$

is satisfied. All incoming particles are approximated to be at rest, which is a good approximation taking into account that chemical energy scales are much lower than nuclear ones. For the left hand side one obtains from the mass difference of $Li(3, n=4)$ and $Li(3,5)$ isotopes the estimate $\Delta m = 1.2312$ MeV for the liberated binding energy which is considerably larger than $m_e = .51$ MeV. Hence the process is kinematically possible and $2\nu_8$ would move with a relativistic velocity $v = .81c$ and presumably leave the system without interacting with it.

The process can involve also the emission of photons and the maximal amount of energy that photon can carry out corresponds to $E = \Delta m = 1.2312$ MeV. Let us denote by $\langle E\rangle < \Delta m$ the average photonic energy emitted in the process and express it as

$$\langle E\rangle = z\Delta m \ , \quad z < 1. \tag{8.7.3}$$

One obtains an estimate for the production rate of photon energy (only this heats the system) from the incoming electron current I. If a fraction $x(V)$ of the current is transformed to negatively charged color bonds the rate for energy production becomes by a little manipulation

$$\frac{P/kW}{I/A} = x(V)z \times 3.5945 \ . \tag{8.7.4}$$

This formula allows to estimate the value of the parameter $x(V)z$ from experimental data. Since simplest Feynman graph producing also photons is obtained by adding photon line to the basic graph, one expects that z is of order fine structure constant:

$$z \sim \alpha_{em} = 1/137 \ . \tag{8.7.5}$$

The ratios of the excess power for a pair of (V, I) values should satisfy the condition

$$\frac{P(V_1)I(V_2)}{P(V_2)I(V_1)} = \frac{x(V_1)}{x(V_2)} \ . \tag{8.7.6}$$

$x(V)$ should be deducible as a function of voltage using these formulas if the model is correct.

These formulae allow to compare the predictions of the model with the experimental results of Naudin for Mizuno-Omori Cold Fusion reactor [C17] . The following table gives the values of $\epsilon = x(V)z$ and ratios $x(V(n))/x(V(n_1)$ deduced from the data tabulated by Naudin [C102] for the various series of experiments using the formulae above.

1. Most values of $x(V)z$ are in the range $.03 - .12$. $z = 1/137$ would give $x(V)z \leq 1/137$ so that order of magnitude is predicted correctly. One cannot over-emphasize this result.

2. Apart from some exceptions the values look rather reasonable and do not vary too much. If one neglects the exceptional values, ones has $x_{max}(V)/x_{min}(V) < 4$. $n = 1, 5, 8, 9, 29$ correspond to exceptionally small values of $x(V)$. Perhaps cold fusion is not present for some reason. The output power is smaller than input power for $n = 9$ and $n = 29$.

n	$Voltage/V$	$Current/A$	$x(V)z$	$x(V(n))/x(V(2))$
1	185	8.56	0.005	.145
2	147	2.45	0.036	1.00
3	215	2.10	0.046	1.30
4	220	9.32	0.044	1.22
5	145	1.06	0.001	.03
6	213	1.40	0.05	1.34
7	236	1.73	0.08	2.18
8	148	.83	0.01	.21
9	148	1.01	-0.00	-0.008
10	221	1.31	0.03	.87
11	279	3.03	0.05	1.46
12	200	8.58	0.03	0.89
13	199	7.03	0.07	1.91
14	215	9.78	0.04	1.07
15	207	8.34	0.03	0.74
16	247	2.19	0.06	1.69
17	260	2.20	0.02	0.55
18	257	2.08	0.03	0.71
19	195	2.95	0.06	1.59
20	198	2.62	0.07	1.98
21	182	2.40	0.05	1.26
22	212	2.27	0.06	1.74
23	259	2.13	0.12	3.22
24	260	4.83	0.04	1.05
25	209	3.53	0.04	1.16
26	230	4.99	0.10	2.79
27	231	5.46	0.09	2.53
28	233	5.16	0.10	2.85
29	155	4.60	-0.00	-0.04
30	220	4.44	0.11	2.95
31	256	5.25	0.05	1.36
32	211	3.68	0.03	.97
33	201	3.82	0.04	1.06

Table 3. The values of $x(V)z$ and $x(V(n))/x(V(1))$ deduced from the data of *Cold Fusion reaction-Experimental test results on June 25, 2003 by JL Naudin* at `http://jlnlabs.online.fr/cfr/html/cfrdatas.htm`.

Has living matter invented cold nuclear physics?

Intriguingly, the ions Na^+, Cl^-, K^+, Ca^{++} detected by Mizuno in the cathode in Kanarev's experiments [D67] correspond to the most important biological ions. There is also a considerable evidence for the occurrence of nuclear transmutations in living matter [C94, C116] . For instance, Kervran claims that it is not possible to understand where the Ca needed to form the shells of eggs comes from. A possible explanation is that dark nuclear reactions between Na^+ and dark Litium produced the needed Ca.

There is extremely strong electric field through cell membrane (resting voltage is about .06 V). The acceleration of electrons in this field could generate plasma phase and creation of dark Li nuclei via a positive feedback loop. This could mean that cold nuclear reactions serve also in living cell as a basic metabolic energy source (possibly in the dark sector) and that also biologically important ions result as products of cold nuclear reactions.

8.7.7 Tests and improvements

Test for the hypothesis about new physics of water

The model involves hypothesis about new physics and chemistry related to water.

1. The identification of hydrogen bond as dark OH bond could be tested. One could check whether the qualitative properties of bonds are consistent with each. One could try to find evidence for quantization of bond energies as integer multiples of same energy (possible power of two multiples).

2. $H_{1.5}O$ formula in atto-second scale should be tested further and one could look whether similar formula holds true for heavy water so that sequences of dark protons might be replaced with sequences of dark deuterons.

3. One could find whether plasma electrolysis takes place in heavy water.

Testing of the nuclear physics predictions

The model in its simplest form assumes that only dark Li, C, F, etc. are present in water. This predicts quite specific nuclear reactions in electrolyte and target and reaction product. For both target and electrolyte isotopes of nuclei with atomic number $Z+k3$ are predicted to result in cold fusion reactions if energetically possible. For a target heavier than Fe also fission reactions might take place.

The estimates for the liberated energies are obtained assuming that dark nuclei have same binding energies as ordinary ones. In some cases the liberated energy is estimated using the binding energy per nucleon for a lighter isotope. Ordinary nuclei with maximal binding energy correspond to nuclear strings having ^4He or its variants containing negatively charged color bonds as a basic structural unit. One could argue that gluing $nLi(3,5)$ or its isotope does not give rise to a ground state so that the actual energy liberated in the process is reduced so that process might be even impossible energetically. This could explain the absence of Ge from Fe cathode and the absence of Ti, Mn, and Ni in KOH plasma electrolysis [D67] .

Cathode: For cathode Fe and W have been used. For Fe the fusions $Fe+Li \rightarrow Cu + 28.84\ MeV$ and $Fe+C \rightarrow Ge+21.64\ MeV$ are possible energetically. Mizuno does not report the presence of Ge in Fe target. The reduction of the binding energy of dark $C(6,10)$ by 21.64 MeV (1.35 MeV per nucleon) would make second reaction impossible but would still allow $Li+C$ and $Na+C$ fusion. Second possibility is that Ge containing negatively charged color bonds has smaller binding energy per nucleon than ordinary Ge. $W+Li \rightarrow Ir$ would liberate 8.7 MeV if binding energy of dark Li is same as of ordinary Li.

Electrolyte: Consider electrolytes containing ions X^+ with atomic number Z. If X is lighter than Fe, the isotopes of nuclei with atomic number $Z+3k$ might be produced in fusion reactions $nLi+X$. $X=Li, K, Na$ has one electron at s-shell whereas $B, Al, Cr, ...$ has one electron at p-shell.

Reaction	$\mathbf{Li}+Li \rightarrow C$	$\mathbf{C}+Li \rightarrow F$	$\mathbf{F}+Li \rightarrow Mg$
E/MeV	27.1	24.0	31.5
	$\mathbf{Li}+Na \rightarrow Si$	$\mathbf{C}+Na \rightarrow Cl$	$\mathbf{F}+Na \rightarrow Ca$
E/MeV	34.4	30.5	33.7
	$\mathbf{Li}+K \rightarrow Ti$	$\mathbf{C}+K \rightarrow Mn$	$\mathbf{F}+K \rightarrow Ni$
E/MeV	32.2	33.6	32.7

Table 5. The estimates for the energies liberated in fusions of dark nuclei of water and the ion of electrolyte. Boldface refers to dark nuclei $Li(3,5), C(6,10)$, and $F(9,15)$.

Relationship to the model of Widom and Larsen and further tests

W. Guglinski kindly informed me about the theory of cold fusion by Widom and Larsen [C122] . This theory relies on standard nuclear physics. The theory is reported to explain cold fusion reaction products nicely in terms of the transformation of electrons and protons to very low energy neutrons which can overcome the Coulomb barrier. The problem of the theory is that very high energy electrons are required since one has $Q = .78$ MeV for $e + p \rightarrow n$ and $Q = -3.0$ MeV for $e + D \rightarrow n + n$. It is difficult to understand how so energetic electrons could result in ordinary condensed matter.

Since proton plus color bond is from the point of view of nuclear physics neutron and the fusion reactions would obey ordinary nuclear physics rules, the predictions of TGD are not expected to deviate too much from those of the model of Widom and Larsen.

An important class of predictions relate to ordinary nuclear physics. Tetra-neutron could be alpha particle with two negatively charged color bonds and neutron halos could consist of protons connected to nucleus by negatively charged color bonds. This could reduce the binding energy considerably.

Cold nuclear fusion might also provide an in situ mechanism for the formation of ores. Nuclear ores in places where they should not exist but involving remnants of organic matter would be the prediction. Cold fusion has a potential for a technology allowing to generate some metals artificially.

How to optimize the energy production?

The proposed model for the plasma electrolysis suggests following improvements to the experimental arrangement.

The production of energy in process is due to three reactions: 1) Li+p in plasma. 2) Li+Fe/W... in target, and 3) $Li + Na/K$... in plasma. The model suggests that 1) dominates so that basic process would occur in plasma rather than cathode.

1. Since W does not evaporate so easily, it is better material for cathode if the production of dark Li dominates energy production.

2. Cathode could be replaced with a planar electrode with fractal peaky structure generating the required strong electric fields. This could increase the effectiveness of the energy production by increasing the effective area used.

3. Since $H_2O \rightarrow OH^- + p$ is required by the generation of dark Li sequences. The energy feed must be able to follow the rapidly growing energy needs of this reaction which seems to occur as bursts.

4. The prediction is that the output power is proportional to electron current rather than input power. This suggests minimization of input power by minimizing voltage. This requires maximization of electron conductivity. Unfortunately, the transformation of electrons to OH^- ions as charge carriers reduces conductivity.

8.8 Anomalies possible related to electrolysis of water and cold fusion

8.8.1 Comparison with the reports about biological transmutations

Kervran's book "Biological Transmutations" [C94] contains a surprisingly detailed summary about his work with biological transmutations and it is interesting to find whether the proposed model could explain the findings of Kervran. TGD suggests two general mechanisms.

1. The nuclear reactions involving dark Li, C, and F predicted to be present in living matter.

2. Nuclear fusions made possible by a temporary transformation of ordinary nuclear space-time sheets to dark ones with much larger size so that Coulomb wall is reduced considerably. The nuclear reaction might proceed if it is energetically possible. Almost any reaction $A+B \rightarrow C$ is possible via this mechanism unless the nuclei are not too heavy.

Fortuitous observations

In his childhood Kervran started to wonder why hens living in a limestone poor region containing thus very little calcium in ground and receiving no calcium in their nutrition could develop the calcium required by eggs and by their own bones. He noticed that hens had the habit of eating mica, which contains silicon. Later this led to the idea that Si could somehow transmute to Ca in living matter. In the proposed model this could correspond to fusion of $Si(14,14)+\mathbf{C(6,6)} \rightarrow Ca(20,20)$ which occurs spontaneously.

Second fortuitous observation were the mysterious CO poisonings by welders working in factory. After careful studies Kervran concluded that CO must be produced endogenously and proposed that the inhaled air which had been in contact with incandescent iron induces the transformation $N_2 \rightarrow CO$ conserving both neuron and neutron number. This transformation might be understood in TGD context if the nuclear space-time sheets are part of time in dark with much larger size so that a direct contact becomes possible for nuclear space-time sheets and Coulomb wall is reduced so that the reaction can proceed with some probability if energetically possible. The thermal energy received from hot iron might help to overcome the Coulomb barrier. The mass difference $m(2N) - m(O) - m(C) = 10.45$ MeV allows this reaction to occur spontaneously.

Examples of various anomalies

Kervran discusses several plant anomalies. The ashes of plants growing in Si rich soil contain more Ca than they should: this transmutation has been already discussed. The ashes of a plant growing on Cu fibres contain no copper but 17 per cent of iron oxides in addition to other elements which could not have come from the rain water. The reaction $Cu(58) + \mathbf{Li(3,4)} \rightarrow Fe(26,32) + \mathbf{C(6,6)}$ would liberate energy of 11.5 MeV.

There are several mineral anomalies.

1. Dolomite rock is formed inside limestone rocks which would suggest the transmutation of $Ca(20,20)$ into $Mg(12,12)$. The nuclear reaction $Ca(20,20) + \mathbf{Li(3,4)} \rightarrow Mg(12,12) + Na(11,12)$ would liberate energy of 3.46 MeV. Ca emerges from Si in soil and in what Kervran refers to a "sickness of stone". The candidate reaction has been already discussed.

2. Graphite is found in siliceous rocks. Kervran proposes the reaction $Si \rightarrow C + O$. $m(Si) - m(C) - M(O) = -16.798$ MeV does not allow this reaction to proceed spontaneously but the reaction $Si + \mathbf{Li} \rightarrow C + Na$ liberates the energy 2.8880 MeV.

3. Kervran mentions the reaction $O + O \rightarrow S$ as a manner to produce sulphur from oxygen. This reaction is obviously energetically favored.

Kervran discusses the transmutations $Na \rightarrow K$ and $Na \rightarrow Ca$ occurring also in plasma electrolysis and explained by TGD based model. Further transmutations are $Na \rightarrow Mg$ and $Mg \rightarrow Ca$. $Na \rightarrow Mg$ could correspond to the reaction $Na(11,12)+\mathbf{Li(3,2)} \rightarrow Mg(12,12)+He(2,2)$ favored by the high binding energy per nucleon for ^4He (7.072 MeV). $Mg \rightarrow Ca$ would correspond to the reaction $Mg + O \rightarrow Ca$, which obviously liberates energy.

8.8.2 Are the abundances of heavier elements determined by cold fusion in interstellar medium?

According to the standard model, elements not heavier than Li were created in Big Bang. Heavier elements were produced in stars by nuclear fusion and ended up to the interstellar space in supernova explosions and were gradually enriched in this process. Lithium problem forces to take this theoretical framework with a grain of salt.

The work of Kervran [C94] suggests that cold nuclear reactions are occurring with considerable rates, not only in living matter but also in non-organic matter. Kervran indeed proposes that also the abundances of elements at Earth and planets are to high degree determined by nuclear transmutations and discusses some examples. For instance, new mechanisms for generation of O and Si would change dramatically the existing views about evolution of planets and prebiotic evolution of Earth.

Where did the Lithium go?

Ulla - one of the commentators in my blog - sent an interesting link concerning Lithium problem to an article by Elisabetta Caffau et al titled "An extremely primitive halo star" [E13].

What has been found is a star which is extremely poor on metallic elements: ("metallic" refers to elements heavier than Li). The mystery is that not only elements heavier than Li but also Li itself, whose average abundance is believed to be determined by cosmological rather than stellar nucleosynthesis, is very scarcely present in these stars.

This finding can be coupled with too other observations about anomalies in Li abundance.

1. The average abundance of Li in Cosmos is lower than predicted by standard cosmology by a factor between 2 and 3 [E7].

2. Also Sun has too low Li abundance [E5].

Authors think that some process could have created very high temperature destroying the Li in this kind of stars: maybe dark matter annihilation might have caused this. This looks rather artificial to me and would not explain too low Li abundance for other stars and for interstellar medium.

The transformation of Li to dark matter (ordinary Lithium in a phase with larger value of Planck constant) would mean its effective disappearance. This process would have occurred both in interstellar medium and in stars so that all three Li problems would be solved at once. Many question marks remain. What about the rate for the phase transition to dark matter? Also lighter elements should be able to transform to dark form. Why the cosmological abundances for them are however essentially those predicted by the standard model of primordial nucleosynthesis? Is the reason that Li their fusion to Li was much faster than transformation to dark matter during primordial nucleosynthesis whereas Li fused very slowly and had time to transform to dark Li?

Li problem would rather sharply distinguish between two very different views about dark matter: dark matter as some exotic elementary particles on one hand and dark matter as phases of ordinary matter implied by generalization of quantum theory on the other hand.

Are heavier nuclei produced in the interstellar space?

TGD based model is consistent with the findings of Kervran and encourages to a consider a simple model for the generation of heavier elements in interstellar medium. The assumptions are following.

1. Dark nuclei $X(3k,n)$, that is nuclear strings of form $Li(3,n)$, $C(6,n)$, $F(9,n)$, $Mg(12,n)$, $P(15,n)$, $A(18,n)$, etc..., form as a fusion of Li strings. $n = Z$ is the most plausible value of n. There is also ^4He present but as a noble gas it need not play an important role in condensed matter phase (say interstellar dust). The presence of water necessitates that of $Li(3,n)$ if one accepts the proposed model as such.

2. The resulting nuclei are in general stable against spontaneous fission by energy conservation. The binding energy of $He(2,2)$ is however exceptionally high so that alpha decay can occur in dark nuclear reactions between $X(3k,n)$ allowed by the considerable reduction of the Coulomb wall. The induced fissions $X(3k,n) \rightarrow X(3k-2,n-2) + He(2,2)$ produces nuclei with atomic number $Z \ mod \ 3 = 1$ such as $Be(4,5)$, $N(7,7)$, $Ne(10,10)$, $Al(13,14)$, $S(16,16)$, $K(19,20)$,... Similar nuclear reactions make possible a further alpha decay of $Z \ mod \ 3 = 1$ nuclei to give nuclei with $Z \ mod \ 2$ such as $B(5,6)$, $O(8,8)$, $Na(11,12)$, $Si(14,14)$, $Cl(17,18)$, $Ca(20,20)$,... so that most stable isotopes of light nuclei could result in these fissions.

3. The dark nuclear fusions of already existing nuclei can create also heavier Fe. Only the gradual decrease of the binding energy per nucleon for nuclei heavier than Fe poses restrictions on this process.

The table below allows the reader to build a more concrete view about how the heavier nuclei might be generated via the proposed mechanisms.

H(1,0)							He(2,2)
Li(3,4)	Be(4,5)	B(5,6)	C(6,6)	N(7,7)	O(8,8)	F(9,10)	Ne(10,10)
Na(11,12)	Mg(12,12)	Al(13,14)	Si(14,14)	P(15,16)	S(16,16)	Cl(17,18)	A(18,22)
K(19,20)	Ca(20,20)						

Table 4. The table gives the most abundant isotopes of stable nuclei.

The abundances of nuclei in interstellar space should not depend on time

The basic prediction of TGD inspired model is that the abundances of the nuclei in the interstellar space should not depend on time if the rates are so high that equilibrium situation is reached rapidly. The $\hbar$ increasing phase transformation of the nuclear space-time sheet determines the time scale in which equilibrium sets on. Standard model makes different prediction: the abundances of the heavier nuclei should gradually increase as the nuclei are repeatedly re-processed in stars and blown out to the interstellar space in super-nova explosion.

Amazingly, there is empirical support for this highly non-trivial prediction [E20] . Quite surprisingly, the 25 measured elemental abundances (elements up to $Sn(50,70)$ (tin) and $Pb(82,124)$ (lead)) of a 12 billion years old galaxy turned out to be very nearly the same as those for Sun. For instance, oxygen abundance was 1/3 from that from that estimated for Sun. Standard model would predict that the abundances should be .01-.1 from that for Sun as measured for stars in our galaxy. The conjecture was that there must be some unknown law guaranteeing that the distribution of stars of various masses is time independent. The alternative conclusion would be that heavier elements are created mostly in interstellar gas and dust.

Could also "ordinary" nuclei consist of protons and negatively charged color bonds?

The model would strongly suggest that also ordinary stable nuclei consist of protons with proton and negatively charged color bond behaving effectively like neutron. Note however that I have also consider the possibility that neutron halo consists of protons connected by negatively charged color bonds to main nucleus. The smaller mass of proton would favor it as a fundamental building block of nucleus and negatively charged color bonds would be a natural manner to minimizes Coulomb energy. The fact that neutron does not suffer a beta decay to proton in nuclear environment provided by stable nuclei would also find an explanation.

1. Ordinary shell model of nucleus would make sense in length scales in which proton plus negatively charged color bond looks like neutron.

2. The strictly nucleonic strong nuclear isospin is not vanishing for the ground state nuclei if all nucleons are protons. This assumption of the nuclear string model is crucial for quantum criticality since it implies that binding energies are not changed in the scaling of $\hbar$ if the length of the color bonds is not changed. The quarks of charged color bond however give rise to a compensating strong isospin and color bond plus proton behaves in a good approximation like neutron.

3. Beta decays might pose a problem for this model. The electrons resulting in beta decays of this kind nuclei consisting of protons should come from the beta decay of the d-quark neutralizing negatively charged color bond. The nuclei generated in high energy nuclear reactions would presumably contain genuine neutrons and suffer beta decay in which d quark is nucleonic quark. The question is whether how much the rates for these two kinds of beta decays differ and whether existing facts about beta decays could kill the model.

8.8.3 Burning salt water by radio-waves and cold fusion by plasma electrolysis

John Kanzius has made a strange discovery [D1] : salt water in the test tube radiated by radio waves at harmonics of a frequency f=13.56 MHz burns. Temperatures about 1500 C, which correspond to .17 eV energy have been reported. One can radiate also hand but nothing happens. The original discovery of Kanzius was the finding that radio waves could be used to cure cancer by destroying the cancer cells. The proposal is that this effect might provide new energy source by liberating chemical energy in an exceptionally effective manner. The power is about 200 W so that the power used could explain the effect if it is absorbed in resonance like manner by salt water. In the following it is proposed that the cold nuclear reactions are the source of the energy.

Do radio waves of large Planck constant transform to microwave photons or visible photons in the process?

The energies of photons involved are very small, multiples of 5.6×10^{-8} eV and their effect should be very small since it is difficult to imagine what resonant molecular transition could cause the effect. This leads to the question whether the radio wave beam could contain a considerable fraction of dark photons for which Planck constant is larger so that the energy of photons is much larger. The underlying mechanism would be phase transition of dark photons with large Planck constant to ordinary photons with shorter wavelength coupling resonantly to some molecular degrees of freedom and inducing the heating. Microwave oven of course comes in mind immediately. The fact that photosynthesis means burning of water and the fact that visible light is emitted in turn suggests that the radio wave photons are transformed to visible or nearly visible photons corresponding to the energy scale of photons involved with photosynthesis.

The original argument inspired by the analogy with microwave oven is discussed below. The generalization to the case of visible photons is rather straightforward and is discussed in [K30] .

1. The fact that the effects occur at harmonics of the fundamental frequency suggests that rotational states of molecules are in question as in microwave heating. The formula for the rotational energies [D17] is

$$E(l) = E_0 \times (l(l+1) \ , \ E_0 = \hbar_0^2/2\mu R^2 \ , \ \mu = m_1m_2/(m_1+m_2) \ .$$

   Here R is molecular radius which by definition is deduced from the rotational energy spectrum. The energy inducing the transition $l \to l+1$ is $\Delta E(l) = 2E_0 \times (l+1)$.

2. $NaCl$ molecules crystallize to solid so that the rotational heating of $NaCl$ molecules cannot be in question.

3. The microwave frequency used in microwave ovens is 2.45 GHz giving for the Planck constant the estimate 180.67 equal to 180 with error of .4 per cent. The values of Planck constants for $(\hat{M}^4/G_a)\times\hat{CP}_2\hat{\times}G_b$ option (factor space of M^4 and covering space of CP_2 maximizing Planck constant for given G_a and G_b) are given by $\hbar/\hbar_0 = n_an_b$. $n_an_b = 4\times9\times5 = 180$ can result from the number theoretically simple values of quantum phases $exp(i2\pi/n_i)$ corresponding to polygons constructible using only ruler and compass. For instance, one could have $n_a = 2\times3$ and $n_b = 2\times3\times5$.

Connection with plasma electrolysis?

The burning of salt water involves also the production of O_2 and H_2 gases. Usually this happens in the electrolysis of water [D4] . The arrangement involves typically electrolyte consisting of water plus $NaOH$ or KOH present also now but anode, cathode and electronic current absent. The proposed mechanism of electrolysis involving cold nuclear reactions however allows the splitting of water molecules to H_2 and O_2 even without these prerequisites.

The thermal radiation from the plasma created in the process has temperature about 1500 C which correspond to energy about .17 eV: this is not enough for splitting of bonds with energy .5 eV. The temperature in salt water could be however considerably higher.

The presence of visible light suggests that plasma phase is created as in plasma electrolysis. Dark nuclear reactions would provide the energy leading to ionization of hydrogen atoms and subsequent transformation of the electronic charge to that of charged color bonds in protonic strings. This in turn would increase the rate of cold nuclear reactions and the liberated energy would ionize more hydrogen atoms so that a positive feedback loop would result.

Cold nuclear reactions should provide the energy transforming hydrogen bonds to dark bonds with energy scaled down by a factor of about 2^{-6} from say 8 eV to .125 eV if $T = 1500C$ is accepted as temperature of water. If Planck constant is scaled up by the factor $r = 180$ suggested by the interpretation in terms of microwave heating, the scaling of the Planck constant would reduce the energy of OH bonds to about .04 eV, which happens to be slightly below the energy assignable to the cell membrane resting potential. The scaling of the size of nuclear space-time sheets of D by factor $r = 180$ is consistent with the length of color bonds of order 10^{-12} m. The role of microwave heating would be to preserve this temperature so that the electrolysis of water can continue. Note that the energy from cold nuclear reactions could partially escape as dark photons.

There are some questions to be answered.

1. Are the radio wave photons dark or does water - which is a very special kind of liquid - induce the transformation of ordinary radio wave photons to dark photons by fusing 180 radio wave massless extremals (MEs) to single ME. Does this transformation occur for all frequencies? This kind of transformation might play a key role in transforming ordinary EEG photons to dark photons and partially explain the special role of water in living systems.

2. Why the radiation does not induce a spontaneous combustion of living matter which also contains Na^+ and other ions. A possible reason is that $\hbar$ corresponds to Planck constant of dark Li which is much higher in living water. Hence the energies of dark photons do not induce microwave heating.

3. The visible light generated in the process has yellow color. The mundane explanation is that the introduction Na or its compounds into flame yields bright yellow color due to so called sodium D-lines [D54] at 588.9950 and 589.5924 nm emitted in transition from 3p to 3s level. Visible light could result as dark photons from the dropping of dark protons from dark space-time sheets of size at least atomic size to larger dark space-time sheets or to ordinary space-time sheets of same size and de-cohere to ordinary light. In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system.

   Yellow light corresponds roughly to the rather narrow energy range .96-2.1 eV ($.59-.63\ \mu m$). The metabolic quanta correspond to jumps to space-time sheets of increasing size give rise to the fractal series $E/eV = 2 \times (1 - 2^{-n})$ for transitions $k = 135 \to 135 + n$, $n = 1, 2, ...$ [K62] . For $n = 3, 4, 5$ the lines have energies $1.74, 1.87, 1.93$ eV and are in the visible red ($\lambda/\mu m = .71, .66, .64$). For $n > 5$ the color is yellow. In Kanarev's experiments the color is red which would mean the dominance of $n < 6$ lines: this color is regarded as a signature of the plasma electrolysis. In the burning of salt water the light is yellow [D1] , which allows to consider the possibility that yellow light is partially due to $n > 5$ lines. Yellow color could also result from the dropping $k = 134 \to 135$ ($n = 1$).

8.8.4 GSI anomaly

"Jester" wrote a nice blog posting titled *Hitchhikers-guide-to-ghosts-and-spooks in particle physics* summarizing quite a bundle of anomalies of particle physics and also one of nuclear physics- known as GSI anomaly. The abstract of the article *Observation of Non-Exponential Orbital Electron Capture Decays of Hydrogen-Like* ^{140}Pr *and* ^{142}Pm *Ions* [C31] describing the anomaly is here.

We report on time-modulated two-body weak decays observed in the orbital electron capture of hydrogen-like ^{140}Pr*¡sup¿59+¡/sup¿ and* ^{142}Pm*¡sup¿60+¡/sup¿ ions coasting in an ion storage ring. Using non-destructive single ion, time-resolved Schottky mass spectrometry we found that the expected exponential decay is modulated in time with a modulation period of about 7 seconds for both systems. Tentatively this observation is attributed to the coherent superposition of finite mass eigenstates of the electron neutrinos from the weak decay into a two-body final state.*

This brings in mind the nuclear decay rate anomalies which I discussed earlier in the blog posting *Tritium beta decay anomaly and variations in the rates of radioactive processes* and in [K80] . These variations in decay rates are in the scale of year and decay rate variation correlates with the distance from Sun. Also solar flares seem to induce decay rate variations.

The TGD based explanation [K80] relies on nuclear string model in which nuclei are connected by color flux tubes having exotic variant quark and antiquark at their ends (TGD predicts fractal hierarchy of QCD like physics). These flux tubes can be also charged: the possible charges $\pm 1, 0$. This means a rich spectrum of exotic states and a lot of new low energy nuclear physics. The energy scale corresponds to Coulomb interaction energy $\alpha_{em} m$, where m is mass scale of the exotic quark. This means energy scale of 10 keV for MeV mass scale. The well-known poorly understood X-ray bursts from Sun during solar flares in the wavelength range 1-8 A correspond to energies in the range 1.6-12.4 keV -3 octaves in good approximation- might relate to this new nuclear physics and in turn might excite nuclei from the ground state to these excited states and the small mixture of exotic nuclei with slightly different nuclear decay rates could cause the effective variation of the decay rate. The mass scale $m \sim 1$ MeV for exotic quarks would predict Coulomb energy of order $\alpha_{em} m$ which is of order 10 keV.

The question is whether there could be a flux of X rays in time scale of 7 seconds causing the rate fluctuation by the same mechanism also in GSI experiment. For instance, could this flux relate to synchrotron radiation. I could no identify any candidate for this periodicity from the article. In any case, the prediction is what might be called X ray nuclear physics and artificial X ray irradiation of nuclei would be an easy manner to kill or prove the general hypothesis.

One can imagine also another possibility.

1. The first guess is that the transitions between ordinary and exotic states of the ion are induced by the emission of exotic W boson between nucleon and exotic quark so that the charge of the color bond is changed. In standard model the objection would be that classical W fields do not make sense in the length scale in question. The basic prediction deriving from induced field concept (classical ew gauge fields correspond to the projection of CP_2 spinor curvature to the space-time surface) is however the existence of classical long range gauge fields- both ew and color. Classical W field can induce charge entanglement in all length scales and one of the control mechanisms of TGD inspired quantum biology relies on remote control of charge densities in this manner. Also the model of cold fusion could involve similar oscillating time like entanglement allowing the bombarding nucleus to penetrate to the nucleus when proton has transformed to neuron in good approximation and charge is de-localized to the color bond having much larger size.

2. In the approximation that one has two-state system, this interaction can be modelled by using as interaction Hamiltonian hermitian non-diagonal matrix V, which can be written as $V\sigma_x$, where σ_x is Pauli sigma matrix. If this process occurs coherently in time scales longer than $\hbar/V$, an oscillation with frequency $\omega = V/\hbar$ results. Since weak interactions are in question 7 second modulation period might make sense.

The hypothesis can be tested quantitatively.

1. The weak interaction Coulomb potential energy is of form

$$\frac{V(r)}{\hbar} = \alpha_W \frac{exp(-m_W r)}{r} \ , \tag{8.8.1}$$

   where r is the distance between nucleon center of mass and the end of color flux tube and therefore of order proton Compton length r_p so that one can write

$$r = x \times r_p \ .$$

   where x should be of order unity but below it.

2. The frequency $\omega = 2\pi/\tau = V/\hbar$ must correspond to 14 seconds, twice the oscillation period of the varying reaction rate. By taking W boson Compton time t_W as time unit this condition can be written as

$$\frac{\alpha_W exp(-y)}{y} = \frac{t_W}{\tau} \ ,$$
$$y = x\frac{r_p}{r_W} = x\frac{m_W}{m_p} \simeq 80 \times x \ ,$$
$$\alpha_W = \alpha_{em}/sin^2\theta_W \ .$$

3. This gives the condition

$$\frac{exp(-y)}{y} = \frac{t_p}{\tau} \times \frac{sin^2\theta_W}{80 \times \alpha_{em}} . \quad (8.8.2)$$

   This allows to solve y since the left hand side is known. Feeding in proton Compton length $r_p = 1.321 \times 10^{-15}$ m and $sin^2\theta_W = .23$ one obtains that the distance between flux tube end and proton cm is $x = .6446$ times proton Compton length, which compares favorably with the guess $x \simeq 1$ but smaller than 1. One must however notice that the oscillation period is exponentially sensitive to the value of x. For instance, if the charge entanglement were between nucleons, $x > 1$ would hold true and the time scale would be enormous. Hence the simple model requires new physics and predicts correctly the period of the oscillation under very reasonable assumptions.

4. One could criticize this by saying that the masses of two states differ by amount which is of order 10 keV or so. This does not however affect the argument since the mass corresponds to the diagonal non-interaction part of the Hamiltonian contributing only rapidly oscillating phases whereas interaction potential induces oscillating mixing as is easy to see in interaction picture.

5. If one believes in the hierarchy of Planck constants and p-adically scaled variants of weak interaction physics, charge entanglement would be possible in much longer length scales and the time scale of it raises the question whether qubits could be realized using proton and neutron in quantum computation purposes. I have also proposed that charge entanglement could serve as a mechanism of bio-control allowing to induce charge density gradients from distance in turn acting as switches inducing biological functions.

So: it happened again! Again I have given a good reason for my learned critics to argue that TGD explains everything so that I am a crackpot and so on and so on. Well... after a first feeling of deep shame I dare to defend myself. In the case of standard model explanatory power has not been regarded as an argument against the theory but my case is of course different since I do not have any academic position since my fate is to live in the arctic scientific environment of Finland. And if my name were Feynman, this little argument would be an instant classic. But most theoreticians are just little opportunists building their career and this does not leave much room for intellectual honesty.

8.8.5 New evidence for anomalies of radio-active decay rates

Lubos Motl told about new evidence for periodic variations of nuclear decay rates reported by Sturrock et al in their article *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence* [C62]. The abstract of the article summarizes the results.

This article presents an analysis of about 29,000 measurements of gamma radiation associated with the decay of radon in a sealed container at the Geological Survey of Israel (GSI) Laboratory in Jerusalem between 28 January 2007 and 10 May 2010. These measurements exhibit strong variations in time of year and time of day, which may be due in part to environmental influences. However, time-series analysis reveals a number of periodicities, including two at approximately 11.2 year^{-1} *and 12.5 year*$^{-1}$*. We have previously found these oscillations in nuclear-decay data acquired at the Brookhaven National Laboratory (BNL) and at the Physikalisch-Technische Bundesanstalt (PTB), and we have suggested that these oscillations are attributable to some form of solar radiation that has its origin in the deep solar interior. A curious property of the GSI data is that the annual oscillation is much stronger in daytime data than in nighttime data, but the opposite is true for all other oscillations. This may be a systematic effect but, if it is not, this property should help narrow the theoretical options for the mechanism responsible for decay-rate variability.*

Quantitative summary of findings

The following gives a brief quantitative summary of the findings. Radioactive decays of nuclei have been analyzed in three earlier studies and also in the recent study.

1. BNL data are about ^{36}Cl and ^{32}Si nuclei. Strong day-time variation in month time scale was observed. Two frequency bands ranging from 11.0 to 11.2 year^{-1} and from 12.6 to 12.9 year^{-1} were observed.

2. PTB data are about ^{226}Ra nuclei. Also now strong day-time variation was observed with frequency bands ranging from 11.0 to 11.3 year^{-1} and from 12.3 to 12.5 year^{-1} .

3. GIS data are about ^{222}Ra nuclei. Instead of strong day-time variation a strong night-time variation was observed. Annual oscillation was centered on mid-day. 2 year^{-1} is the next strongest feature. Also a night time feature with a peak at 17 hours was observed. There are also features at 12.5 year^{-1} and 11.2 year^{-1} and 11.9 year^{-1}. All these three data sets lead to oscillations in frequency bands ranging from 11.0 to 11.4 year^{-1} and from 12.1 to 12.9 year^{-1}.

4. Bellotti et al studied ^{137}Cl nuclei deep underground in Gran Sasso. No variations were detected.

Could exotic nuclear states explain the findings?

The TGD based new physics involved with the effect could relate to the excitations of exotic nuclear states induced by em radiation arriving from Sun. This would change the portions of various excited nuclei with nearly the same ground state energy and affect the average radio-active decay rates.

1. The exotic nuclei emerge in the model of nucleus as a nuclear string with nucleons connected by color flux tubes having quark and antiquark at ends [L2]. The excitations could be also involved with cold fusion. For the normal nuclei color flux tubes would be neutral but one can consider also excitations for which quark pair carries a net change $\pm e$. This would give rise to a large number of nuclei with same em charge and mass number but having actually abnormal proton and neutron numbers. If the energy differences for these excitations are in keV range they might represent a fine structure of nuclear levels not detected earlier.

   Could these exchanges take place also between different nuclei? For instance, could it be that in the collision of deuterium nuclei the second nucleus can be neutralized by the exchange of scaled down W boson leading to neutralization of second deuterium nucleus so that Coulomb wall could disappear and make possible cold nuclear reaction. It seems that the range of this scaled variant of weak interaction is quite too short. M_{127} variant of weak interactions with W boson mass very near to electron mass could make possible this mechanism.

2. The exchange of weak bosons could be responsible for generating these excitations: in this case two neutral color bonds would become charged with opposite charges. If one takes seriously the indications for 38 MeV new particle [C60], one can even consider a scaled variant of weak interaction physics with weak interaction length scale given by a length scale near hadronic length scale [K99]. E(38) could be scaled down Z boson with mass of about 38 MeV.

Em radiation from Sun inducing transitions of ordinary nuclei to their exotic counterparts could be responsible for the variation of the radio-active decay rates. If course, exotic nuclei in the above sense are only one option and the following argument below applies quite generally.

Kinetic model for the evolution for the number of excited nuclei

A simple model for the evolution of the number of excited nuclei is as follows:

$$\begin{aligned} \frac{dN}{dt} &= kJ - k_1N \text{ for } t \in [t_0, t_1] \ , \\ \frac{dN}{dt} &= -k_1N \text{ for } t \in [t_1, t_0 + T] \ . \end{aligned} \quad (8.8.3)$$

J denotes the flux of incoming radiation and N the number of excited nuclei. t_0 corresponds to the time of sunrise and t_1 to the time of sunset and T is 24 hours in the approximation that sun rises at the same time every morning. The time evolution of $N(t)$ is given by

$$\begin{aligned} N(t) &= \frac{k}{k_1}J + (N(t_0) - \frac{k}{k_1}J)exp\left[-k_1(t-t_0\right] \text{ for } t \in [t_0, t_1] \ , \\ N(t) &= N(t_1)exp\left[-k_1(t-t_1)\right] \quad \text{for } t \in [t_1, t_0 + T] \ . \end{aligned} \quad (8.8.4)$$

Explanation for the basic features of the data

The model can explain the qualitative features of the data rather naturally.

1. The period of 1 year obviously correlates with the distance from Sun. .5 year period correlates with the fact that the distance from Sun is minimal twice during a year. Day-time night-time difference can be explained with the fact that em radiation at night-time does not penetrate Earth. This explains also why Gran Sasso in deep underground observes nothing.

2. The large long time scale variation for the day-time data for BNL and PTB seems to be in apparent contrast with that for the night-dime data at GIS. It is however possible to understand the difference.

   (a) If the rate parameter k_1 is large, one can understand why variations are strong at day-time in BNL and BTB. For large value of k_1 $N(t)$ increases rapidly to its asymptotic value $N_{max} = kJ/k_1$ and stays in it during day so that day-time variations due to solar distance are large. At night-time $N(t)$ rapidly decreases to zero so that night-time variation due to the variation of the solar distance is small.

   (b) For GIS the strong variation is associated with the night-dime data. This can be understood in terms of small value of k_1 which can be indeed smaller for ^{226}Ra than for the nuclei used in the other studies. During daytime $N(t)$ slowly increases to its maximum at $N(t_1)$ and decreases slowly during night-time. Since $N(t_1)$ depends on the time of the year, the night-time variation is large.

   (c) The variations in time scales of roughly the time scale of month should be due to the variations in the intensity of the incoming radiation. The explanation suggested in [C62] is that the dynamics of solar core has these periodicities manifested also as the periodicities of the emission of radiation at the frequencies involved. These photons would naturally correspond to the photons emitted in the transitions between excited states of nuclei in the solar core or possibly in solar corona having temperature of about 300 eV. One could in fact think that the mysterious heating of solar corona [E4] to a temperature of 3 million K could be due to the exotic excitations of the nuclei by radiation coming from Sun. At this temperature the maximum of black body distribution with respect to frequency corresponds to energy of .85 keV consistent with the proposal that the energy scale for excitations is keV.

   (d) The difference of frequencies 12.49 year^{-1} and 11.39 year^{-1} is in good approximation 1 year^{-1}, which suggests modulation of the average frequencies with a period of year being due to the rotation of Earth around Sun. The average frequency is 11.89 year^{-1} that is 1/month. The explanation proposed in the article is in terms of rotation velocity of the inner core which would be smaller but same order of magnitude as that of outer core (frequency range from 13.7 to 14.7 year^{-1}). It is however not plausible that the keV photons could propagate from the inner core of Sun unless they are dark in TGD sense. In TGD framework it would be natural to assign the frequency band to solar Corona.

Can one assign the observed frequency band to the rotation of solar corona?

The rotation frequency band assignable to photosphere is too high by about $\Delta f = 3$ year^{-1} as compared to that appearing in decay rate variation. Could one understand this discrepancy?

1. One must distinguish between the synodic rotation frequency f_S measured in the rest system of Sun and the rotation frequency observed in Earth rotating with frequency $f = 1$ year^{-1} around Sun: these frequencies relate by $f_E = f_S - f$ giving frequency range 12.7 to 13.7 year^{-1}. This is still too high by about $\Delta f = 2$ year^{-1}.

2. Could corona rotate slower than photosphere? The measurements by Mehta [E21] give the value range 22 - 26.5 days meaning that the the coronal synodic frequency f_C would be in the range 14.0-16.6 year^{-1}. The range of frequencies observed at Earth would be 13-15.6 year^{-1} and too high by about $\Delta = 2$ year^{-1}.

   If I have understood correctly, the coronal rotational velocity is determined by using solar spots as markers and therefore refers to the magnetic field rather than the gas in the corona. Could the rotation frequency of the gas in corona be about $\Delta f = 2$ year^{-1} lower than that for the magnetic spots?

One can develop a theoretical argument in order to understand the rotational periods of photosphere and corona and why they could differ by about $\Delta f = 2$ year^{-1}.

1. Suppose that one can distinguish between the rotation frequencies of magnetic fields (magnetic body in many-sheeted space-time) and gas. Suppose that photosphere (briefly 'P') and corona (briefly 'C') can be treated in the first approximation as rigid spherical shells having thus moment of inertia $I = (2/3)mR^2$ around the rotational axis. The angular momentum per unit mass is $dL/dm = (2/3)R^2\omega$. Suppose that the value of dL/dm is same for the photosphere and Corona. If the rotation velocity magnetic fields determined from magnetic spots is same as the rotation velocity of gas in corona, this implies $f_C/f_P = (R_S/R_C)^2$, where R_S is solar radius identifiable as the radius of photosphere. The scaling of 13 year^{-1} down to 11 year^{-1} would require $R_C/R_S \simeq 1.09$. This radius should correspond to the hottest part of the corona at temperature about 1-2 million K.

   The inner solar corona extends up to $(4/3)R_S$. This would give average radius of the inner coronal shell about $1.15R_S$. The constancy of $dL/dm(R)$ would give a differential rotation with frequency varying as $1/R^2$. If the frequency band reflects the presence of differential rotation, one has $R_{max}/R_{min} \simeq (f_{max}/f_{min})^{1/2} \simeq (15/13)^{1/2} \simeq 1.07$.

2. One can understand why angular momentum density per mass is constant if one accepts a generalization of the Bohr quantization of planetary orbits originally proposed by Nottale and based on the notion of gravitational Planck constant $\hbar_{gr}$ [K75, K62]. One has $\hbar_{gr} = GMm/v_0$ and is assigned with the flux sheets mediating gravitational interaction between Sun and the planet or some other astrophysical object near Sun. The dependence on solar mass and planetary mass is is fixed by Equivalence Principle. v_0 has dimensions of velocity and therefore naturally satisfies $v_0 < c$. For the three inner planets one has $v_0/c \simeq 2^{-11}$. Angular momentum quantization gives $mR^2\omega = n\hbar_{gr}$ giving $R^2\omega = nGM/v_0$ so that the angular momentum per mass is integer valued. For the inner planets n has values 3,4,5.

3. One could argue that for the photosphere and corona regarded as rigid bodies a similar quantization holds true but with the same vale of n since the radii are so near to each other. Also v_0 should be larger. Consider first photosphere. One can apply the angular momentum quantization condition to photosphere approximate as a spherical shell and rigid body. $I\omega_P = nGmM/v_{0P}$ for $n = 1$ gives $(2/3)R^2\omega = GM/v_{0P}$. For $v_{0P} = c$ one would obtain $\omega_P/\omega_E = (3/2)(R_E/R)^2(v_0/v_{0P}$. For $R_P = .0046491R_E$ (solar radius) this gives $\omega_P/\omega_E \simeq 12.466$ for the $v_0/c = 4.6\times10^{-4}$ used by Nottale [K75]: I have often used the approximate nominal value $v_0/c = 2^{-11}$ but now it this approximation is too rough. Taking into account the frequency shift due to Earth's orbital motion one obtains $\omega_P/\omega_E \simeq 11.466$ which is consistent with the lower bound of the observed frequency band and would correspond to R_{max}. The value $v_{0P} = v_{0C} = c$ looks unrealistic if interpreted as a physical velocity of some kind the increase

of R_C allows however to reduce the value of v_{0C} so that it seems possible to understand the situation quantitatively.

If one wants to generalize this argument to differential rotation, one must decompose the system spherical shells or more general elements rotating at different velocities and having different value of $\hbar_{gr}$ assignable to the flux tubes connecting them to Sun and mediating gravitational interaction. This decomposition must be physical.

8.9 Dark nuclear strings as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code?

Water memory is one of the ugly words in the vocabulary of the main stream scientist. The work of pioneers is however now carrying fruit. The group led by Jean-Luc Montagnier, who received Nobel prize for discovering HIV virus, has found strong evidence for water memory and detailed information about the mechanism involved [K41, K87] , [I5] . The work leading to the discovery was motivated by the following mysterious finding. When the water solution containing human cells infected by bacteria was filtered in purpose of sterilizing it, it indeed satisfied the criteria for the absence of infected cells immediately after the procedure. When one however adds human cells to the filtrate, infected cells appear within few weeks. If this is really the case and if the filter does what it is believed to do, this raises the question whether there might be a representation of genetic code based on nano-structures able to leak through the filter with pores size below 200 nm.

The question is whether dark nuclear strings might provide a representation of the genetic code. In fact, I posed this question year before the results of the experiment came with motivation coming from the attempts to understand water memory. The outcome was a totally unexpected finding: the states of dark nucleons formed from three quarks can be naturally grouped to multiplets in one-one correspondence with 64 DNAs, 64 RNAS, and 20 amino-acids and there is natural mapping of DNA and RNA type states to amino-acid type states such that the numbers of DNAs/RNAs mapped to given amino-acid are same as for the vertebrate genetic code.

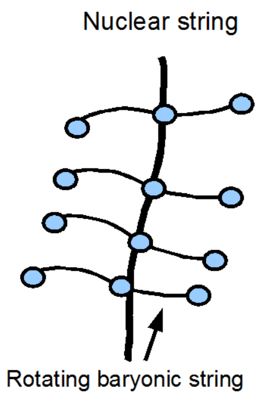

Figure 8.3: Illustration of a possible vision about dark nucleus as a nuclear string consisting of rotating baryonic strings.

The basic idea is simple. Since baryons consist of 3 quarks just as DNA codons consist of three nucleotides, one might ask whether codons could correspond to baryons obtained as open strings with quarks connected by two color flux tubes. This representation would be based on entanglement rather than letter sequences. The question is therefore whether the dark baryons constructed as string of 3 quarks using color flux tubes could realize 64 codons and whether 20 amino-acids could be identified as equivalence classes of some equivalence relation between 64 fundamental codons in a natural manner.

The following model indeed reproduces the genetic code directly from a model of dark neutral baryons as strings of 3 quarks connected by color flux tubes.

1. Dark nuclear baryons are considered as a fundamental realization of DNA codons and constructed as open strings of 3 dark quarks connected by two colored flux tubes, which can be also charged. The baryonic strings cannot combine to form a strictly linear structure since strict rotational invariance would not allow the quark strings to have angular momentum with respect to the quantization axis defined by the nuclear string. The independent rotation of quark strings and breaking of rotational symmetry from SO(3) to SO(2) induced by the direction of the nuclear string is essential for the model.

   (a) Baryonic strings could form a helical nuclear string (stability might require this) locally parallel to DNA, RNA, or amino-acid) helix with rotations acting either along the axis of the DNA or along the local axis of DNA along helix. The rotation of a flux tube portion around an axis parallel to the local axis along DNA helix requires that magnetic flux tube has a kink in this portion. An interesting question is whether this kink has correlate at the level of DNA too. Notice that color bonds appear in two scales corresponding to these two strings. The model of DNA as topological quantum computer [K31] allows a modification in which dark nuclear string of this kind is parallel to DNA and each codon has a flux tube connection to the lipid of cell membrane or possibly to some other bio-molecule.

   (b) The analogs of DNA -, RNA -, and of amino-acid sequences could also correspond to sequences of dark baryons in which baryons would be 3-quark strings in the plane transversal to the dark nuclear string and expected to rotate by stringy boundary conditions. Thus one would have nuclear string consisting of short baryonic strings not connected along their ends (see Fig. 8.9). In this case all baryons would be free to rotate.

2. The new element as compared to the standard quark model is that between both dark quarks and dark baryons can be charged carrying charge $0, \pm 1$. This is assumed also in nuclear string model and there is empirical support for the existence of exotic nuclei containing charged color bonds between nuclei.

3. The net charge of the dark baryons in question is assumed to vanish to minimize Coulomb repulsion:

$$\sum_q Q_{em}(q) = - \sum_{flux\ tubes} Q_{em}(flux\ tube) \quad . \tag{8.9.1}$$

   This kind of selection is natural taking into account the breaking of isospin symmetry. In the recent case the breaking cannot however be as large as for ordinary baryons (implying large mass difference between Δ and nucleon states).

4. One can classify the states of the open 3-quark string by the total charges and spins associated with 3 quarks and to the two color bonds. Total em charges of quarks vary in the range $Z_B \in \{2, 1, 0, -1\}$ and total color bond charges in the range $Z_b \in \{2, 1, 0, -1, -2\}$. Only neutral states are allowed. Total quark spin projection varies in the range $J_B = 3/2, 1/2, -1/2, -3/2$ and the total flux tube spin projection in the range $J_b = 2, 1, -1, -2$. If one takes for a given total charge assumed to be vanishing one representative from each class (J_B, J_b), one obtains $4 \times 5 = 20$ states which is the number of amino-acids. Thus genetic code might be realized at the level of baryons by mapping the neutral states with a given spin projection to single representative state with the same spin projection. The problem is to find whether one can identify the analogs of DNA, RNA and amino-acids as baryon like states.

8.9.1 States in the quark degrees of freedom

One must construct many-particle states both in quark and flux tube degrees of freedom. These states can be constructed as representations of rotation group SU(2) and strong isospin group SU(2) by using the standard tensor product rule $j_1 \times j_2 = j_1 + j_2 \oplus j_1 + j_2 - 1 \oplus ... \oplus |j_1 - j_2|$ for the representation of SU(2) and Fermi statistics and Bose-Einstein statistics are used to deduce correlations between total spin and total isospin (for instance, $J = I$ rule holds true in quark degrees of freedom). Charge neutrality is assumed and the breaking of rotational symmetry in the direction of nuclear string is assumed.

Consider first the states of dark baryons in quark degrees of freedom.

1. The tensor product $2 \otimes 2 \otimes 2$ is involved in both cases. Without any additional constraints this tensor product decomposes as $(3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$: 8 states altogether. This is what one should have for DNA and RNA candidates. If one has only identical quarks uuu or ddd, Pauli exclusion rule allows only the 4-D spin 3/2 representation corresponding to completely symmetric representation -just as in standard quark model. These 4 states correspond to a candidate for amino-acids. Thus RNA and DNA should correspond to states of type uud and ddu and amino-acids to states of type uuu or ddd. What this means physically will be considered later.

2. Due to spin-statistics constraint only the representations with $(J, I) = (3/2, 3/2)$ (Δ resonance) and the second $(J, I) = (1/2, 1/2)$ (proton and neutron) are realized as free baryons. Now of course a dark -possibly p-adically scaled up - variant of QCD is considered so that more general baryonic states are possible. By the way, the spin statistics problem which forced to introduce quark color strongly suggests that the construction of the codons as sequences of 3 nucleons - which one might also consider - is not a good idea.

3. Second nucleon like spin doublet - call it 2_{odd} - has wrong parity in the sense that it would require $L = 1$ ground state for two identical quarks (uu or dd pair). Dropping 2_{odd} and using only $4 \oplus 2$ for the rotation group would give degeneracies $(1, 2, 2, 1)$ and 6 states only. All the representations in $4 \oplus 2 \oplus 2_{odd}$ are needed to get 8 states with a given quark charge and one should transform the wrong parity doublet to positive parity doublet somehow. Since open string geometry breaks rotational symmetry to a subgroup SO(2) of rotations acting along the direction of the string and since the boundary conditions on baryonic strings force their ends to rotate with light velocity, the attractive possibility is to add a baryonic stringy excitation with angular momentum projection $L_z = -1$ to the wrong parity doublet so that the parity comes out correctly. $L_z = -1$ orbital angular momentum for the relative motion of uu or dd quark pair in the open 3-quark string would be in question. The degeneracies for spin projection value $J_z = 3/2, ..., -3/2$ are $(1, 2, 3, 2)$. Genetic code means spin projection mapping the states in $4 \oplus 2 \oplus 2_{odd}$ to 4.

8.9.2 States in the flux tube degrees of freedom

Consider next the states in flux tube degrees of freedom.

1. The situation is analogous to a construction of mesons from quarks and antiquarks and one obtains the analogs of π meson (pion) with spin 0 and ρ meson with spin 1 since spin statistics forces $J = I$ condition also now. States of a given charge for a flux tube correspond to the tensor product $2 \otimes 2 = 3 \oplus 1$ for the rotation group.

2. Without any further constraints the tensor product $3 \otimes 3 = 5 \oplus 3 \oplus 1$ for the flux tubes states gives 8+1 states. By dropping the scalar state this gives 8 states required by DNA and RNA analogs. The degeneracies of the states for DNA/RNA type realization with a given spin projection for $5 \oplus 3$ are $(1, 2, 2, 2, 1)$. $8\times$ 8 states result altogether for both uud and udd for which color bonds have different charges. Also for ddd state with quark charge -1 one obtains $5 \oplus 3$ states giving 40 states altogether.

3. If the charges of the color bonds are identical as the are for uuu type states serving as candidates for the counterparts of amino-acids bosonic statistics allows only 5 states ($J = 2$

state). Hence 20 counterparts of amino-acids are obtained for uuu. Genetic code means the projection of the states of $5 \oplus 3$ to those of 5 with the same spin projection and same total charge.

8.9.3 Analogs of DNA,RNA, amino-acids, and of translation and transcription mechanisms

Consider next the identification of analogs of DNA, RNA and amino-acids and the baryonic realization of the genetic code, translation and transcription.

1. The analogs of DNA and RNA can be identified dark baryons with quark content uud, ddu with color bonds having different charges. There are 3 color bond pairs corresponding to charge pairs $(q_1, q_2) = (-1, 0), (-1, 1), (0, 1)$ (the order of charges does not matter). The condition that the total charge of dark baryon vanishes allows for uud only the bond pair $(-1, 0)$ and for udd only the pair $(-1, 1)$. These thus only single neutral dark baryon of type uud *resp.* udd: these would be the analogous of DNA and RNA codons. Amino-acids would correspond to uuu states with identical color bonds with charges $(-1, -1)$, $(0, 0)$, or $(1, 1)$. uuu with color bond charges (-1,-1) is the only neutral state. Hence only the analogs of DNA, RNA, and amino-acids are obtained, which is rather remarkable result.

2. The basic transcription and translation machinery could be realized as processes in which the analog of DNA can replicate, and can be transcribed to the analog of mRNA in turn translated to the analogs of amino-acids. In terms of flux tube connections the realization of genetic code, transcription, and translation, would mean that only dark baryons with same total quark spin and same total color bond spin can be connected by flux tubes. Charges are of course identical since they vanish.

3. Genetic code maps of $(4 \oplus 2 \oplus 2) \otimes (5 \oplus 3)$ to the states of 4×5. The most natural map takes the states with a given spin to a state with the same spin so that the code is unique. This would give the degeneracies $D(k)$ as products of numbers $D_B \in \{1, 2, 3, 2\}$ and $D_b \in \{1, 2, 2, 2, 1\}$: $D = D_B \times D_b$. Only the observed degeneracies $D = 1, 2, 3, 4, 6$ are predicted. The numbers $N(k)$ of amino-acids coded by D codons would be

$$[N(1), N(2), N(3), N(4), N(6)] = [2, 7, 2, 6, 3] \ .$$

   The correct numbers for vertebrate nuclear code are $(N(1), N(2), N(3), N(4), N(6)) = (2, 9, 1, 5, 3)$. Some kind of symmetry breaking must take place and should relate to the emergence of stopping codons. If one codon in second 3-plet becomes stopping codon, the 3-plet becomes doublet. If 2 codons in 4-plet become stopping codons it also becomes doublet and one obtains the correct result $(2, 9, 1, 5, 3)$!

4. Stopping codons would most naturally correspond to the codons, which involve the $L_z = -1$ relative rotational excitation of uu or dd type quark pair. For the 3-plet the two candidates for the stopping codon state are $|1/2, -1/2\rangle \otimes \{|2, k\rangle\}$, $k = 2, -2$. The total spins are $J_z = 3/2$ and $J_z = -7/2$. The three candidates for the 4-plet from which two states are thrown out are $|1/2, -3/2\rangle \otimes \{|2, k\rangle, |1, k\rangle\}$, $k = 1, 0, -1$. The total spins are now $J_z = -1/2, -3/2, -5/2$. One guess is that the states with smallest value of J_z are dropped which would mean that $J_z = -7/2$ states in 3-plet and $J_z = -5/2$ states 4-plet become stopping codons.

5. One can ask why just vertebrate code? Why not vertebrate mitochondrial code, which has unbroken $A - G$ and $T - C$ symmetries with respect to the third nucleotide. And is it possible to understand the rarely occurring variants of the genetic code in this framework? One explanation is that the baryonic realization is the fundamental one and biochemical realization has gradually evolved from non-faithful realization to a faithful one as kind of emulation of dark nuclear physics. Also the role of tRNA in the realization of the code is crucial and could explain the fact that the code can be context sensitive for some codons.

8.9.4 Understanding the symmetries of the code

Quantum entanglement between quarks and color flux tubes would be essential for the baryonic realization of the genetic code whereas chemical realization could be said to be classical. Quantal aspect means that one cannot decompose to codon to letters anymore. This raises questions concerning the symmetries of the code.

1. What is the counterpart for the conjugation $ZYZ \rightarrow X_cY_cZ_c$ for the codons?
2. The conjugation of the second nucleotide Y having chemical interpretation in terms of hydrophoby-hydrophily dichotomy in biology. In DNA as TQC model it corresponds to matter-antimatter conjugation for quarks associated with flux tubes connecting DNA nucleotides to the lipids of the cell membrane. What is the interpretation in now?
3. The A-G, T-C symmetries with respect to the third nucleotide Z allow an interpretation as weak isospin symmetry in DNA as TQC model. Can one identify counterpart of this symmetry when the decomposition into individual nucleotides does not make sense?

Natural candidates for the building blocks of the analogs of these symmetries are the change of the sign of the spin direction for quarks and for flux tubes.

1. For quarks the spin projections are always non-vanishing so that the map has no fixed points. For flux tube spin the states of spin $S_z = 0$ are fixed points. The change of the sign of quark spin projection must therefore be present for both $XYZ \rightarrow X_cY_cZ_c$ and $Y \rightarrow Y_c$ but also something else might be needed. Note that without the symmetry breaking $(1,3,3,1) \rightarrow (1,2,3,2)$ the code table would be symmetric in the permutation of 2 first and 2 last columns of the code table induced by both full conjugation and conjugation of Y.
2. The analogs of the approximate $A-G$ and $T-C$ symmetries cannot involve the change of spin direction in neither quark nor flux tube sector. These symmetries act inside the A-G and T-C sub-2-columns of the 4-columns defining the rows of the code table. Hence this symmetry must permute the states of same spin inside 5 and 3 for flux tubes and 4 and 2 for quarks but leave 2_{odd} invariant. This guarantees that for the two non-degenerate codons coding for only single amino-acid and one of the codons inside triplet the action is trivial. Hence the baryonic analog of the approximate $A-G$ and $T-C$ symmetry would be exact symmetry and be due to the basic definition of the genetic code as a mapping states of same flux tube spin and quark spin to single representative state. The existence of full 4-columns coding for the same amino-acid would be due to the fact that states with same quark spin inside $(2,3,2)$ code for the same amino-acid.
3. A detailed comparison of the code table with the code table in spin representation should allow to fix their correspondence uniquely apart from permutations of n-plets and thus also the representation of the conjugations. What is clear that Y conjugation must involve the change of quark spin direction whereas Z conjugation which maps typically 2-plets to each other must involve the permutation of states with same J_z for the flux tubes. It is not quite clear what X conjugation correspond to.

8.9.5 Some comments about the physics behind the code

Consider next some particle physicist's objections against this picture.

1. The realization of the code requires the dark scaled variants of spin 3/2 baryons known as Δ resonance and the analogs (and only the analogs) of spin 1 mesons known as ρ mesons. The lifetime of these states is very short in ordinary hadron physics. Now one has a scaled up variant of hadron physics: possibly in both dark and p-adic senses with latter allowing arbitrarily small overall mass scales. Hence the lifetimes of states can be scaled up.
2. Both the absolute and relative mass differences between Δ and N *resp.* ρ and π are large in ordinary hadron physics and this makes the decays of Δ and ρ possible kinematically. This is due to color magnetic spin-spin splitting proportional to the color coupling strength $\alpha_s \sim .1$,

which is large. In the recent case α_s could be considerably smaller - say of the same order of magnitude as fine structure constant 1/137 - so that the mass splittings could be so small as to make decays impossible.

The color magnetic spin interaction energy give rise to hyperfine splitting of quark in perturbative QCD is of form $E_c \propto \hbar g B/m$, where m is mass parameter which is of the order of baryon mass. Magnetic flux scales as $\hbar$ by flux quantization and if flux tube thickness scales as $\hbar^2$, one has $B \propto 1/\hbar$. Mass splittings would not depend on $\hbar$, which does not make sense. Mass splitting becomes small for large $\hbar$ if the area of flux quantum scales as $\hbar^{2+n}$, $n > 0$ so that color magnetic hyper-fine splitting scales as $1/\hbar^n$ from flux conservation. The magnetic energy for a flux tube of length L scaling as $\hbar$ and thickness $S \propto \hbar^{2+n}$ has order of magnitude g^2B^2LS and does not depend on $\hbar$ for $n = 1$. Maybe this could provide first principle explanation for the desired scaling.

The size scale of DNA would suggest that single DNA triplet corresponds to 3 Angstrom length scale. Suppose this corresponds to the size of dark nucleon. If this size scales as $\sqrt{\hbar}$ as p-adic mass calculations suggest, one obtains a rough estimate $\hbar/hbar_0 = 2^{38}$. The proton-Δ mass difference due to hyper-fine splitting would be scaled down to about $2^{-38} \times 300$ MeV $\sim 10^{-9}$ eV, which is completely negligible in the metabolic energy scale .5 eV. If the size of dark nucleon scales as $\hbar$ the mass difference is about 12 eV which corresponds to the energy scale for the ionization energy of hydrogen. Even this might be acceptable.

3. Dark hadrons could have lower mass scale than the ordinary ones if scaled up variants of quarks in p-adic sense are in question. Note that the model for cold fusion that inspired the idea about genetic code requires that dark nuclear strings have the same mass scale as ordinary baryons. In any case, the most general option inspired by the vision about hierarchy of conscious entities extended to a hierarchy of life forms is that several dark and p-adic scaled up variants of baryons realizing genetic code are possible.

4. The heaviest objection relates to the addition of $L_z = -1$ excitation to $S_z = |1/2, \pm 1/2\rangle_{odd}$ states which transforms the degeneracies of the quark spin states from $(1, 3, 3, 1)$ to $(1, 2, 3, 2)$. The only reasonable answer is that the breaking of the full rotation symmetry reduces $SO(3)$ to $SO(2)$. Also the fact that the states of massless particles are labeled by the representation of $SO(2)$ might be of some relevance. The deeper level explanation in TGD framework might be as follows. The generalized imbedding space is constructed by gluing almost copies of the 8-D imbedding space with different Planck constants together along a 4-D subspace like pages of book along a common back. The construction involves symmetry breaking in both rotational and color degrees of freedom to Cartan sub-group and the interpretation is as a geometric representation for the selection of the quantization axis. Quantum TGD is indeed meant to be a geometrization of the entire quantum physics as a physics of the classical spinor fields in the "world of classical worlds" so that also the choice of measurement axis must have a geometric description.

The conclusion is that genetic code can be understand as a map of stringy baryonic states induced by the projection of all states with same spin projection to a representative state with the same spin projection. Genetic code would be realized at the level of dark nuclear physics and biochemical representation would be only one particular higher level representation of the code. A hierarchy of dark baryon realizations corresponding to p-adic and dark matter hierarchies can be considered. Translation and transcription machinery would be realized by flux tubes connecting only states with same quark spin and flux tube spin. Charge neutrality is essential for having only the analogs of DNA, RNA and amino-acids and would guarantee the em stability of the states.

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Chapter 9

Dark Nuclear Physics and Condensed Matter

9.1 Introduction

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether electro-weak charges of elementary particles are screened in electro-weak length scale or not. The TGD based view about dark matter assumes that weak charges are indeed screened for ordinary matter in electro-weak length scale but that dark electro-weak bosons correspond to much longer symmetry breaking length scale.

The large value of $\hbar$ in dark matter phase implies that Compton lengths and -times are scaled up. In particular, the sizes of nucleons and nuclei become of order atom size so that dark nuclear physics would have direct relevance for condensed matter physics. It becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore.

In its original form this chapter was an attempt to concretize and develop ideas related to dark matter by using some experimental inputs with emphasis on the predicted interaction between the new nuclear physics and condensed matter. As the vision about dark matter became more coherent and the nuclear string model developed in its recent form, it became necessary to update the chapter and throw away the obsolete material. I have also divided the material to two chapters such that second chapter focuses to dark weak and color forces and their implications. I dare hope that the recent representation is more focused than the earlier one.

9.1.1 Dark rules

I have done a considerable amount of trials and errors in order to identify the basic rules allowing to understand what it means to be dark matter is and what happens in the phase transition to dark matter. It is good to try to summarize the basic rules of p-adic and dark physics allowing to avoid obvious contradictions.

Could basic quantum TGD imply the hierarchy of Planck constants?

The implications of the hierarchy of Planck constants depend on whether one assumes it as an independent additional postulate or as a consequence of basic quantum TGD. The first option originally motivated by physical anomalies would allow both singular coverings and factor spaces. The latter option, which emerged five years after the basic idea, would allow only singular coverings. They would provide only a convenient tool to describe the fact that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates at the ends of space-time sheets is not one-to-one. As a matter fact, this observation forced the idea about quantum physics as classical physics in the "world of classical worlds" for two decades ago. The quantization of Planck constant as integer multiples of its standard value would be an effective phenomenon for this option holding true at the sheets of the covering. These options lead to different predictions and one can in principle test whether either of them is correct.

The notion of field body

The notion of "field body" implied by topological field quantization is essential piece of classical TGD. It seems possible to assign to physical systems field identities- that is separate magnetic and electric field bodies identifiable as flux quanta. This is not possible in Maxwell's electrodynamics. The first naive guess was that one can speak of separate em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its own p-adic prime. The tight constraints coming from the fact that the induced gauge fields are expressible in terms of CP_2 coordinates and their derivatives implies however strong correlations between classical gauge fields. For instance, the vanishing of classical Kähler field for vacuum extremals implies that em and Z^0 fields are proportional to each other. The non-vanishing of induced Kähler field in turn implies non-vanishing classical color fields. This gives rise at least to two basic types of field bodies predicting a lot of new physics even in macroscopic length scales. For instance, electric and magnetic flux tubes must have at their ends quarks and antiquarks serving as sources of classical color fields unless one believes that vacuum charge densities serve as sources of these fields. In the similar manner neutrinos and antineutrinos are needed to create classical Z^0 fields associated with almost vacuum extremal flux tubes. These fields could be interpreted also as vacuum polarization effects and one could distinguish them from fields created by genuine sources. For instance, the unavoidable classical color fields associated with the flux tubes of electromagnetic field body which is not vacuum extremal would represent vacuum polarization in macroscopic scale.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

The p-adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

What dark variant of elementary particle means

It is not at all clear what the notion of dark variant of elementary particle or of larger structures could mean.

1. *Are only field bodies dark?*

One variety of dark particle is obtained by making some of the field bodies dark by increasing the value of Planck constant. This hypothesis could be replaced with the stronger assumption that elementary particles are maximally quantum critical systems so that they are same irrespective of the value of the Planck constant. Elementary particles would be represented by partonic 2-surfaces, which belong to the universal orbifold singularities remaining invariant by all groups

$G_a \times G_b$ for a given choice of quantization axes. If $G_a \times G_b$ is assumed to leave invariant the choice of the quantization axes, it must be of the form $Z_{n_a} \times Z_{n_b} \subset SO(3) \times SU(3)$. Partonic 2-surface would belong to $M^2 \times CP_2/U(1) \times U(1)$, where M^2 is spanned by the quantization axis of angular momentum and the time axis defining the rest system.

A different manner to say this is that the CP_2 type extremal representing particle would suffer multiple topological condensation on its field bodies so that there would be no separate "particle space-time sheet".

Darkness would be restricted to particle interactions. The value of the Planck constant would be assigned to a particular interaction between systems rather than system itself. This conforms with the original finding that gravitational Planck constant satisfies $\hbar = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$. Since each interaction can give rise to a hierarchy dark phases, a rich variety of partially dark phases is predicted. The standard assumption that dark matter is visible only via gravitational interactions would mean that gravitational field body would not be dark for this particular dark matter.

Complex combinations of dark field bodies become possible and the dream is that one could understand various phases of matter in terms of these combinations. All phase transitions, including the familiar liquid-gas and solid-liquid phase transitions, could have a unified description in terms of dark phase transition for an appropriate field body. At mathematical level Jones inclusions would provide this description.

The book metaphor for the interactions at space-time level is very useful in this framework. Elementary particles correspond to ordinary value of Planck constant analogous to the ordinary sheets of a book and the field bodies mediating their interactions are the same space-time sheet or at dark sheets of the book.

2. Can also elementary particles be dark?

Also dark elementary particles themselves rather than only the flux quanta could correspond to dark space-time sheet defining multiple coverings of $H/G_a \times G_b$. This would mean giving up the maximal quantum criticality hypothesis in the case of elementary particles. These sheets would be exact copies of each other. If single sheet of the covering contains topologically condensed space-time sheet, also other sheets contain its exact copy.

The question is whether these copies of space-time sheet defining classical identical systems can carry different fermionic quantum numbers or only identical fermionic quantum numbers so that the dark particle would be exotic many-fermion system allowing an apparent violation of statistics (N fermions in the same state).

Even if one allows varying number of fermions in the same state with respect to a basic copy of sheet, one ends up with the notion of N-atom in which nuclei would be ordinary but electrons would reside at the sheets of the covering. The question is whether symbolic representations essential for understanding of living matter could emerge already at molecular level via the formation of N-atoms.

What happens in charge fractionization?

The hierarchy of Planck constants suggests strongly charge fractionization. What happens for binding energies is however not obvious. The first guess is that one just replaces $\hbar$ with its scaled value in the standard formulas. One can however ask whether the resulting expression applies to single sheet of covering or to the sum of binding energies associated with the sheets of covering. In the case of factor space analogous problem is not encountered.

If the coverings follow from basic quantum TGD one can deduce unique rules for what happens. These rules can be assumed also in the more general case. Since the sheets of the singular covering co-incide at the partonic 2-surfaces associated with ends of CD the time evolution and also "evolution" in space-like direction means instability of in the sense that partonic 2-surface decomposes to $r = \hbar/\hbar_0 = n_a n_b$ sheets. This implies fractionization of all total quantum numbers such as energy and momentum. From this one can also deduce what happens to various binding energies. For instance, the total (!) cyclotron energy is indeed multiplied by factor and the total(!) binding energy of dark hydrogen atom is what the naive scaling of $\hbar$ would give. The reason is that the mass of particle is fractionized: $m \rightarrow m/n_a n_b$. Therefore the original guesses would be correct. In particular, the expression of the total gravitational binding energy essential for the original Bohr model of planetary orbits is consistent with the new more precise rules.

Criterion for the transition to dark phase

The naive criterion $\alpha Q_1 Q_2 > 1$ (or its generalization) for the transition to dark matter phase relates always to the interaction between two systems and the interpretation is that when the field strength characterizing the interaction becomes too strong, the interaction is mediated by dark space-time sheets which define $n = n(G_a) \times n(G_b)$-fold covering of $M^4 \times CP_2/G_a \times G_b$. The sharing of flux between different space-time sheets reduces the field strength associated with single sheet below the critical value.

For the option in which singular coverings follow from basic quantum TGD this criterion or its appropriate generalization has very concrete interpretation. At the ends of CD the partonic 2-surface is unstable against decay to n_a sheets when some of the quantum numbers of the partonic 2-surface are too large. A similar decay to n_b sheets would happen also when one moves in space-like direction.

One can ask whether this instability could have something to do with N-vertices of generalized Feynman diagrams in which decay of a partonic 2-surfaces to N-1 surfaces takes place. For instance, could it be that 3-vertex- possibly the only fundamental vertex, correspond to this process and could higher vertices have an interpretation in terms of the hierarchy of Planck constants? This would mean analogy with Jones inclusions for which $n \geq 3$ holds true. The assumption that exact fractionization of quantum numbers takes place is not consistent with the identification in terms of Feynman diagrams. Also the huge values of $n_a n_b$ disfavor this identification unless one restricts it to $n_a n_b = 2$.

The considerations of [K33] suggest that in the vertices of generalized Feynman diagrams a re-distribution of the sheets of the coverings can take place in such a manner that the total number of sheets is conserved. The leakage of between different sectors of WCW would in turn mean analogs of self-energy vertices in which n_a and n_b are replaced with their factors or with integers containing them as factors.

Mersenne hypothesis

The generalization of the imbedding space means a book like structure for which the pages are products of singular coverings or factor spaces of CD (causal diamond defined as intersection of future and past directed light-cones) and of CP_2 [K32] . This predicts that Planck constants are rationals and that given value of Planck constant corresponds to an infinite number of different pages of the Big Book, which might be seen as a drawback. If only singular covering spaces are allowed the values of Planck constant are products of integers and given value of Planck constant corresponds to a finite number of pages given by the number of decompositions of the integer to two different integers.

TGD inspired quantum biology and number theoretical considerations suggest preferred values for $r = \hbar/\hbar_0$. For the most general option the values of $\hbar$ are products and ratios of two integers n_a and n_b. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n_i)$, $i \in \{a, b\}$, in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of r.

One can however ask whether a more precise characterization of preferred Mersennes could exist and whether there could exists a stronger correlation between hierarchies of p-adic length scales and Planck constants. Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ are expected to be physically highly interesting and up to $k = 127$ indeed correspond to elementary particles. The number theoretical miracle is that all the four scaled up electron Compton lengths with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm). The question has been whether these define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$. The proposal that this is the case and that these physics are in a well-defined sense induced by the dark scaled up variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$.

What induction means is that dark variant of exotic nuclear physics induces exotic physics with ordinary value of Planck constant in the new scale in a resonant manner: dark gauge bosons transform to their ordinary variants with the same Compton length. This transformation is natural since in length scales below the Compton length the gauge bosons behave as massless and free particles. As a consequence, lighter variants of weak bosons emerge and QCD confinement scale becomes longer.

This proposal will be referred to as Mersenne hypothesis. It leads to strong predictions about EEG [K29] since it predicts a spectrum of preferred Josephson frequencies for a given value of membrane potential and also assigns to a given value of $\hbar$ a fixed size scale having interpretation as the size scale of the body part or magnetic body. Also a vision about evolution of life emerges. Mersenne hypothesis is especially interesting as far as new physics in condensed matter length scales is considered: this includes exotic scaled up variants of the ordinary nuclear physics and their dark variants. Even dark nucleons are possible and this gives justification for the model of dark nucleons predicting the counterparts of DNA,RNA, tRNA, and amino-acids as well as realization of vertebrate genetic code [K90] .

These exotic nuclear physics with ordinary value of Planck constant could correspond to ground states that are almost vacuum extremals corresponding to homologically trivial geodesic sphere of CP_2 near criticality to a phase transition changing Planck constant. Ordinary nuclear physics would correspond to homologically non-trivial geodesic sphere and far from vacuum extremal property. For vacuum extremals of this kind classical Z^0 field proportional to electromagnetic field is present and this modifies dramatically the view about cell membrane as Josephson junction. The model for cell membrane as almost vacuum extremal indeed led to a quantitative breakthrough in TGD inspired model of EEG and is therefore something to be taken seriously. The safest option concerning empirical facts is that the copies of electro-weak and color physics with ordinary value of Planck constant are possible only for almost vacuum extremals - that is at criticality against phase transition changing Planck constant.

9.1.2 Some implications

As already noticed, the detailed implications of the hierarchy of Planck constants depend on whether one brings in the hierarchy of singular coverings and factor spaces of the imbedding space as an independent postulate or whether one assumes that singular coverings emerge as an effective description from basic quantum TGD

Dark variants of nuclear physics

One can imagine endless variety of dark variants of ordinary nuclei and every piece of data is well-come in attempts to avoid a complete inflation of speculative ideas. The book metaphor for the extended imbedding space is useful in the attempts to imagine various exotic phases of matter. For the minimal option atomic nuclei would be ordinary whereas field bodies could be dark and analogous to n-sheeted Riemann surfaces. One can imagine that the nuclei are at the "standard" page of the book and color bonds at different page with different p-adic length scale or having different Planck constant $\hbar$. This would give two hierarchies of nuclei with increasing size.

Color magnetic body of the structure would become a key element in understanding the nuclear binding energies, giant dipole resonances, and nuclear decays. Also other field bodies are in a key role and there seems to be a field body for every basic interaction (classical gauge fields are induced from spinor connection and only four independent field variables are involved so that this is indeed required).

Nothing prevents from generalizing the nuclear string picture so that color bonds could bind also atoms to molecules and molecules to larger structures analogous to nuclei. Even hydrogen bond might be interpreted in this manner. Molecular physics could be seen as a scaled up variant of nuclear physics in a well-defined sense. The exotic features would relate to the hierarchy of various field bodies, including color bonds, electric and weak bonds. These field bodies would play key role also in biology and replaced molecular randomness with coherence in much longer length scale.

In the attempt to make this vision quantitative the starting point is nuclear string model [L2] and the model of cold fusion based on it forcing also to conclude the scaled variants of electro-weak bosons are involved. The model of cold fusion requires the presence of a variant electro-weak interactions for which weak bosons are effectively massless below the atomic length scale.

$k = 113$ p-adically scaled up variant of ordinary weak physics which is dark and corresponds to $\hbar = r\hbar_0$, $r = 2^{k_d}$, $k_d = 14 = 127 - 113$ is an option consistent with Mersenne hypothesis and gives weak bosons in electron length scale. Another possibility is defined by $k = 113$ and $k_d = 24 = 113 - 89 = 151 - 127$ and corresponds to the p-adic length scale $k = 137$ defining atomic length scale. This would give rise to weak bosons with masses in keV scale and these would be certainly relevant for the physics of condensed matter.

Anomalies of water could be understood if one assumes that color bonds can become dark with suitable values of $r = 2^{k_d}$ and if super-nuclei formed by connecting different nuclei by the color bonds are possible. Tetrahedral and icosahedral water clusters could be seen as magic super-nuclei in this framework. Color bonds could connect either proton nuclei or water molecules.

The model for partially dark condensed matter deriving from exotic nuclear physics and exotic weak interactions could allow to understand the low compressibility of the condensed matter as being due to the repulsive weak force between exotic quarks, explains large parity breaking effects in living matter (chiral selection), and suggests a profound modification of the notion of chemical bond having most important implications for bio-chemistry and understanding of bio-chemical evolution.

Could the notion of dark atom make sense?

One can also imagine several variants of dark atom. Book metaphor suggest one variant of dark atom.

1. Nuclei and electrons could be ordinary but classical electromagnetic interactions are mediated via dark space-time sheet "along different page of the book". The value of Planck constant would be scaled so that one would obtain a hierarchy of scaled variants of hydrogen atom. The findings of [D51] could find an explanation in terms of a reduced Planck constant if singular factor spaces are assumed to be possible. An alternative explanation is based on the notion of quantum-hydrogen atom obtained as q-deformation of the ordinary hydrogen atom.

2. A more exotic variant if atom is obtained by assuming ordinary nuclei but dark, not totally quantum critical, electrons. Dark space-time surface is analogous to n-sheeted Riemann surface and if one assumes that each sheet could carry electron, one ends up with the notion of N-atom. This variant of dark atom is more or less equivalent with that following from the option for which the singular coverings of imbedding space are effective manner to describe the many-valuedness of the time derivatives of the imbedding space coordinates as functions of canonical momentum densities.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L27]

- Nuclear string model [L33]

9.2 A generalization of the notion of imbedding space as a realization of the hierarchy of Planck constants

9.2.1 Hierarchy of Planck constants and the generalization of the notion of imbedding space

In the following the recent view about structure of imbedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E25] that the orbits of inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K75] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K76] . TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the imbedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K87] .

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K64] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E25] can be understood. Dark matter would resemble

to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L2, K87] , [L2] .

The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the modified gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD} \hat{\times} G_a) \times (\hat{CP}_2 \hat{\times} G_b)$, $(\hat{CD} \hat{\times} G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2 \hat{\times} G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2

or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K29].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K29] . This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various super-conformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

9.3 General ideas about dark matter

In the sequel general ideas about the role of dark matter in condensed matter physics are described.

9.3.1 How the scaling of $\hbar$ affects physics and how to detect dark matter?

It is relatively easy to deduce the basic implications of the scaling of $\hbar$.

1. If the rate for the process is non-vanishing classically, it is not affected in the lowest order. For instance, scattering cross sections for say electron-electron scattering and e^+e^- annihilation are not affected in the lowest order since the increase of Compton length compensates for the reduction of α_{em}. Photon-photon scattering cross section, which vanishes classically and is proportional to $\alpha_{em}^4\hbar^2/E^2$, scales down as $1/\hbar^2$.

2. Higher order corrections coming as powers of the gauge coupling strength α are reduced since $\alpha = g^2/4\pi\hbar$ is reduced. Since one has $\hbar_s/\hbar = \alpha Q_1Q_2/v_0$, αQ_1Q_2 is effectively replaced with a universal coupling strength v_0. In the case of QCD the paradoxical sounding implication is that α_s would become very small.

9.3.2 General view about dark matter hierarchy and interactions between relatively dark matters

The identification of the precise criterion characterizing dark matter phase is far from obvious. TGD actually suggests an infinite number of phases which are dark relative to each other in some sense and can transform to each other only via a phase transition which might be called de-coherence or its reversal and which should be also characterized precisely.

A possible solution of the problem comes from the general construction recipe for S-matrix. Fundamental vertices correspond to partonic 2-surfaces representing intersections of incoming and outgoing light-like partonic 3-surfaces.

1. If the characterization of the interaction vertices involves all points of partonic 2-surfaces, they must correspond to definite value of Planck constants and more precisely, definite groups G_a and G_b characterizing dark matter hierarchy. Particles of different G_b phases could not appear in the same vertex since the partons in question would correspond to vacuum extremals. Hence the phase transition changing the particles to each other analogous could not be described by a vertex and would be analogous to a de-coherence.

   The phase transition could occur at the incoming or outgoing particle lines. At space-time level the phase transition would mean essentially a leakage between different sectors of imbedding space and means that partonic 2-surface at leakage point has CP_2 projection reducing to the orbifold point invariant under G or alternatively, its $M^4_\pm$ projection corresponds to the tip of $M^4_\pm$. Relative darkness would certainly mean different groups G_a and G_b. Note that $\hbar(M^4)$ *resp.* $\hbar(CP_2)$ can be same for different groups G_a *resp.* G_b and that only the ratio of $\hbar(M^4)/\hbar(M^4)$ appears in the Kähler action.

2. One can represent a criticism against the idea that relatively dark matters cannot appear at the same interaction vertex. The point is that the construction of S-matrix for transitions transforming partonic 2-surfaces in different number fields involves only the rational (algebraic) points in the intersection of the 2-surfaces in question. This idea applies also to the case in which particles correspond to different values of Planck constant. What is only needed that all the common points correspond to the orbifold point in M^4 or CP_2 degrees of freedom and are thus intermediate between two sectors of imbedding space. In this picture phase transitions would occur through vertices and S-matrix would characterize their probabilities. It seems that this option is the correct one.

If the matrix elements for real-real transitions involve all or at least a circle of the partonic 2-surface as stringy considerations suggest [K24] , then one would have clear distinction between quantum phase transitions and ordinary quantum transitions. Note however that one could understand the weakness of the quantal interactions between relatively dark matters solely from the fact that the CP_2 type extremals providing space-time correlates for particle propagators must in this case go through an intermediate state with at most point-like CP_2 projection.

What does one mean with dark variants of elementary particle?

It is not at all clear what one means with the dark variant of elementary particle. In this respect p-adic mass calculations provide a valuable hint. According to the p-adic mass calculations [K58] , $k = 113$ characterizes electromagnetic size of u and d quarks, of nucleons, and nuclei. $k = 107$

characterizes the QCD size of hadrons. This is somewhat paradoxical situation since one would expect that quark space-time sheets would be smaller than hadronic space-time sheets.

The simplest resolution of the problem suggested by the basic characteristics of electro-weak symmetry breaking is that $k = 113$ characterizes the size of the electro-magnetic field body of the quark and that the prime characterizing p-adic mass scale labels the em field body of the particle. One can assign mass also the Z^0 body but this would be much smaller as the small scale of neutrino masses suggests. This size scale correspond to a length scale of order 10 μm, which conforms with the expectation that classical Z^0 force is important in biological length scales. The size of Z^0 body of neutrino could relate directly to the chirality selection in living matter. An interesting question is whether the Z^0 field bodies of also other elementary fermions are of this size.

If this picture is correct then dark variant of elementary particle would differ from ordinary only in the sense that its field body would be dark. This conforms with the general working hypothesis is that only field bodies can be dark.

Are particles characterized by different p-adic primes relatively dark?

Each particle is characterized by a collection of p-adic primes corresponding to the partonic 2-surfaces associate with the particle like 3-surface. Number theoretical vision supports the notion of multi-p p-adicity and the idea that elementary particles correspond to infinite primes, integers, or perhaps even rationals [K37, K82] . To infinite primes, integers, and rationals it is possible to associate a finite rational $q = m/n$ by a homomorphism. This would suggest generalization of p-adicity with q-adicity (q-adic topology does not correspond to number field) but this does not seem to be a promising idea.

The crucial observation is that one can decompose the infinite prime, call it P, to finite and infinite parts and distinguish between bosonic and fermionic finite primes of which infinite prime can be said to consist of [K95, K82, K59]. The interpretation is that bosonic and fermionic finite primes in the *infinite* part of P code for p-adic topologies of light-like partonic 3-surfaces associated with a given *real* space-time sheet whereas the primes in the *finite* part of P code for p-adic light-like partonic 3-surfaces.

This raises two options.

1. Two space-time sheets characterized by rationals having common prime factors can be connected by a $\#_B$ contact and can interact by the exchange of particles characterized by divisors of m or n since in this case partonic 2-surface with same p-adic or effective p-adic topology can be found. This is the only possible interaction between them.

2. The number theoretic vision about the construction of S-matrix however allows to construct S-matrix also in the case that partons belong to different number fields and one ends up with a very elegant description involving only finite number of points of partonic 2-surfaces belonging to their intersection consisting of rational (algebraic points of imbedding space), which by algebraic universality could apply also to diagonal transitions. Also now the interactions mediated between propagators connecting partons with different effective p-adic topologies might be very slow so that this would give rise to relative darkness.

Hierarchy of infinite primes and dark matter hierarchy

In previous consideration only the simplest infinite primes at the lowest level of hierarchy were considered. Simple infinite primes allow a symmetry changing the sign of the finite part of infinite prime. A possible interpretation in terms of phase conjugation. One can consider also more complex infinite primes at this level and a possible interpretation in terms of bound states of several particles. One can also consider infinite integers and rationals: the interpretation would be as many particle states. Rationals might correspond to states containing particles and antiparticles. At the higher levels of the hierarchy infinite primes of previous take the role of finite primes at the previous level and physically these states correspond to higher level bound states of the particles of the previous level.

Thus TGD predicts an entire hierarchy of dark matters such that the many particle states at previous level become particles at the next level. This hierarchy would provide a concrete physical identification for the hierarchy of infinite primes identifiable in terms of a repeated second quantization of an arithmetic super-symmetric QFT [K82] including both free many-particle states and their bound states. The finite primes about which infinite prime is in a well defined sense a composite of would correspond to the particles in the state forming a unit of dark matter. Particles belonging to different levels of this hierarchy would obviously correspond to different levels of dark matter hierarchy but their interactions must reduce to the fundamental partonic vertices.

9.3.3 How dark matter and visible matter interact?

The hypothesis that the value of $\hbar$ is dynamical, quantized and becomes large at the verge of a transition to a non-perturbative phase in the ordinary sense of the word has fascinating implications. In particular, dark matter, would correspond to a large value of $\hbar$ and could be responsible for the properties of the living matter. In order to test the idea experimentally, a more concrete model for the interaction of ordinary matter and dark matter must be developed and here of course experimental input and the consistency with the earlier quantum model of living matter is of considerable help.

How dark photons transform to ordinary photons?

The transitions of dark atoms naturally correspond to coherent transitions of the entire dark electron BE condensate and thus generate N_{cr} dark photons and behave thus like laser beams. Dark photons do not interact directly with the visible matter. An open question is whether even ordinary laser beams could be identified as beams of dark photons: the multiple covering property at the level of imbedding space and the fact that MEs are possible in all sectors suggests that this is not the case. Note that the transition from dark to ordinary photons implies the scaling of wave length and thus also of coherence length by a factor n_b/n_a.

Dark $\leftrightarrow$ visible transition should have also a space-time correlate. The so called topological light rays or MEs ("massless extremals") represent a crucial deviation of TGD from Maxwell's ED and have all the properties characterizing macroscopic classical coherence. Therefore MEs are excellent candidates for the space-time correlate of BE condensate of dark photons.

MEs carry in general a superposition of harmonics of some basic frequency determined by the length of ME. A natural expectation is that the frequency of classical field corresponds to the generalized de Broglie frequency of dark photon and is thus $\hbar/\hbar_s$ times lower than for ordinary photons. In completely analogous manner de Broglie wave length is scaled up by $k = \hbar_s/\hbar$. Classically the decay of dark photons to visible photons would mean that an oscillation with frequency f inside topological light ray transforms to an oscillation of frequency f/k such that the intensity of the oscillation is scaled up by a factor k. Furthermore, the ME in question could naturally decompose into $1 < N_{cr} \leq 137$ ordinary photons in the case that dark atoms are in question. Of course also MEs could decay to lower level MEs and this has an interpretation in terms of hierarchy of dark matters to be discussed next.

About the criterion for the transition increasing the value of Planck constant

An attractive assumption is that the transition to dark matter phase occurs when the interaction strength satisfies the criticality condition $Q_1Q_2\alpha \simeq 1$. A special case corresponds to self interaction with $Q_1 = Q_2$. This condition applies only to gauge interactions so that particles can be characterized by gauge charges. A more general characterization would be that transition occurs when perturbation theory ceases to converge. The criterion cannot be applied to phenomenological QFT description of strong force in terms of, say, pion exchange.

Some examples are in order to test this view.

1. Transition from perturbative phase in QCD to hadronic phase is the most obvious application. The identification of valence quarks and gluons as dark matter would predict for them QCD size ($k = 107$ space-time sheet) of about electron Compton length. This does not change the QCD cross sections in the lowest order perturbation theory but makes them excellent predictions. It also provides completely new view about how color force determines the nuclear strong force indeed manifesting itself as long ranged harmonic oscillator potential, the long range of which becomes manifest in the case of neutron halos of size of 2.5×10^{-14} m [C84] . One can also understand tetraneutron in this framework. This criterion applies also

in QCD plasma and explains the formation of liquid like color glass condensate detected in RHIC [C97]. A possible interpretation for QCD size would be as a length of the cylindrical magnetic walls defining the magnetic body associated with u and d type valence quarks, nucleons, and nuclei.

2. QCD size of quark must be distinguished from the electromagnetic size of quark associated with $k = 113$ space-time sheets of u and d quarks and assignable to the height of the magnetic body and defining the length scale of join along boundaries contacts feeding quark charges to $k = 113$ space-time sheets.

3. In the case of atomic nuclei the criterion would naturally apply to the electromagnetic interaction energy of two nucleon clusters inside nucleus or to to self energy ($Q^2\alpha_{em} = 1$). Quite generally, the size of the electromagnetic $k = 113$ space-time sheet would increase by a $n_F = 2^k \prod_s F_s$, where F_s are different Fermat primes (the known ones being $3, 5, 17, 257, 2^{16} + 1$), in the transition to large $\hbar$ phase. Especially interesting values of n_F seem to be of form $n_F = 2^{k11}$ and possibly also $n_F = 2^{k11} \prod_s F_s$. Similar criterion would apply in the plasma phase. Note that many free energy anomalies involve the formation of cold plasma [K86] .

   The criterion would give in the case of single nucleus and plasma $Z \geq 12$ if the charges are within single space-time sheet. This is consistent with cold fusion involving Palladium nuclei [C112] . Since u and d quarks have $k = 113$, they both and thus both neutrons and protons could make a transition to large $\hbar$ phase. This is consistent with the selection rules of cold fusion since the production of ^3He involves a phase transition pnp$_d$ $\rightarrow$ pnp and the contraction of p_d to p is made un-probable by the Coulomb wall whereas the transition nnp$_d$ $\rightarrow$nnp producing tritium does not suffer from this restriction.

Strong and weak physics of nuclei would not be affected in the phase transition. Electromagnetic perturbative physics of nuclei would not be affected in the process in the lowest order in $\hbar$ (classical approximation) but the height of the Coulomb wall would be reduced by a factor $1/n_F$ by the increase in the electromagnetic size of the nucleus. Also Pd nuclei could make the transition and Pd nuclei could catalyze the transition in the case the deuterium nuclei.

9.3.4 Could one demonstrate the existence of large Planck constant photons using ordinary camera or even bare eyes?

If ordinary light sources generate also dark photons with same energy but with scaled up wavelength, this might have effects detectable with camera and even with bare eyes. In the following I consider in a rather light-hearted and speculative spirit two possible effects of this kind appearing in both visual perception and in photos. For crackpotters I want to make clear that I love to play with ideas to see whether they work or not, and that I am ready to accept some convincing mundane explanation of these effects and I would be happy to hear about this kind of explanations. I was not able to find any such explanation from Wikipedia using words like camera, digital camera, lense, aberrations [D3] .

Why light from an intense light source seems to decompose into rays?

If one also assumes that ordinary radiation fields decompose in TGD Universe into topological light rays ("massless extremals", MEs) even stronger predictions follow. If Planck constant equals to $\hbar = q \times \hbar_0$, $q = n_a/n_b$, MEs should possess Z_{n_a} as an exact discrete symmetry group acting as rotations along the direction of propagation for the induced gauge fields inside ME.

The structure of MEs should somewhat realize this symmetry and one possibility is that MEs has a wheel like structure decomposing into radial spokes with angular distance $\Delta\phi = 2\pi/n_a$ related by the symmetries in question. This brings strongly in mind phenomenon which everyone can observe anytime: the light from a bright source decomposes into radial rays as if one were seeing the profile of the light rays emitted in a plane orthogonal to the line connecting eye and the light source. The effect is especially strong if eyes are stirred. It would seem that focusing makes the effect stronger.

Could this apparent decomposition to light rays reflect directly the structure of dark MEs and could one deduce the value of n_a by just counting the number of rays in camera picture, where the phenomenon turned to be also visible? Note that the size of these wheel like MEs would be macroscopic and diffractive effects do not seem to be involved. The simplest assumption is that most of photons giving rise to the wheel like appearance are transformed to ordinary photons before their detection.

The discussions about this led to a little experimentation with camera at the summer cottage of my friend Samppa Pentikäinen, quite a magician in technical affairs. When I mentioned the decomposition of light from an intense light source to rays at the level of visual percept and wondered whether the same occurs also in camera, Samppa decided to take photos with a digital camera directed to Sun. The effect occurred also in this case and might correspond to decomposition to MEs with various values of n_a but with same quantization axis so that the effect is not smoothed out.

What was interesting was the presence of some stronger almost vertical "rays" located symmetrically near the vertical axis of the camera. In old-fashioned cameras the shutter mechanism determining the exposure time is based on the opening of the first shutter followed by closing a second shutter after the exposure time so that every point of sensor receives input for equally long time. The area of the region determining input is bounded by a vertical line. If macroscopic MEs are involved, the contribution of vertical rays is either nothing or all unlike that of other rays and this might somehow explain why their contribution is enhanced. The shutter mechanism is unnecessary in digital cameras since the time for the reset of sensors is what matters. Something in the geometry of the camera or in the reset mechanism must select vertical direction in a preferred position. For instance, the outer "aperture" of the camera had the geometry of a flattened square.

Anomalous diffraction of dark photons

Second prediction is the possibility of diffractive effects in length scales where they should not occur. A good example is the diffraction of light coming from a small aperture of radius d. The diffraction pattern is determined by the Bessel function

$$J_1(x) \ , \ x = kdsin(\theta) \ , \ k = 2\pi/\lambda .$$

There is a strong light spot in the center and light rings around whose radii increase in size as the distance of the screen from the aperture increases. Dark rings correspond to the zeros of $J_1(x)$ at $x = x_n$ and the following scaling law for the nodes holds true

$$sin(\theta_n) = x_n \frac{\lambda}{2\pi d} per.$$

For very small wavelengths the central spot is almost point-like and contains most light intensity.

If photons of visible light correspond to large Planck constant $\hbar = q \times \hbar_0$ transformed to ordinary photons in the detector (say camera film or eye), their wavelength is scaled by q, and one has

$$sin(\theta_n) \rightarrow q \times sin(\theta_n)$$

The size of the diffraction pattern for visible light is scaled up by q.

This effect might make it possible to detect dark photons with energies of visible photons and possibly present in the ordinary light.

1. What is needed is an intense light source and Sun is an excellent candidate in this respect. Dark photon beam is also needed and n dark photons with a given visible wavelength λ could result when dark photon with $\hbar = n \times q \times \hbar_0$ decays to n dark photons with same wavelength but smaller Planck constant $\hbar = q \times \hbar_0$. If this beam enters the camera or eye one has a beam of n dark photons which forms a diffraction pattern producing camera picture in the de-coherence to ordinary photons.

2. In the case of an aperture with a geometry of a circular hole, the first dark ring for ordinary visible photons would be at $sin(\theta) \simeq (\pi/36)\lambda/d$. For a distance of $r = 2$ cm between the sensor plane ("film") and effective circular hole this would mean radius of $R \simeq rsin(\theta) \simeq 1.7$ micrometers for micron wave length. The actual size of spots is of order $R \simeq 1$ mm so that the value of q would be around 1000: $q = 2^{10}$ and $q = 2^{11}$ belong to the favored values for q.

3. One can imagine also an alternative situation. If photons responsible for the spot arrive along single ME, the transversal thickness R of ME is smaller than the radius of hole, say of of order of wavelength, ME itself effectively defines the hole with radius R and the value of $sin(\theta_n)$ does not depend on the value of d for $d > R$. Even ordinary photons arriving along MEs of this kind could give rise to an anomalous diffraction pattern. Note that the transversal thickness of ME need not be fixed however. It however seems that MEs are now macroscopic.

4. A similar effect results as one looks at an intense light source: bright spots appear in the visual field as one closes the eyes. If there is some more mundane explanation (I do not doubt this!), it must apply in both cases and explain also why the spots have precisely defined color rather than being white.

5. The only mention about effects of diffractive aberration effects are colored rings around say disk like objects analogous to colors around shadow of say disk like object. The radii of these diffraction rings in this case scale like wavelengths and distance from the object.

6. Wikipedia contains an article from which one learns that the effect in question is known as lens flares [D11] . The article states that flares typically manifest as several starbursts, circles, and rings across the picture and result in internal reflection and scattering from material inhomogenuities in lens (such as multiple surfaces). The shape of the flares also depends on the shape of aperture. These features conform at least qualitatively with what one would expect from a diffraction if Planck constant is large enough for photons with energy of visible photon.

   The article [D20] defines flares in more restrictive manner: lense flares result when *non-image* forming light enters the lens and subsequently hits the camera's film or digital sensor and produces typically polygonal shape with sides which depend on the shape of lense diaphgram. The identification as a flare applies also to the apparent decomposition to rays and this dependence indeed fits with the observations.

The experimentation of Samppa using digital camera demonstrated the appearance of colored spots in the pictures. If I have understood correctly, the sensors defining the pixels of the picture are in the focal plane and the diffraction for large Planck constant might explain the phenomenon. Since I did not have the idea about diffractive mechanism in mind, I did not check whether fainter colored rings might surround the bright spot.

1. In any case, the readily testable prediction is that zooming to bright light source by reducing the size of the aperture should increase the size and number of the colored spots. As a matter fact, experimentation demonstrated that focusing brought in large number of these spots but we did not check whether the size was increased.

2. Standard explanation predicts that the bright spots are present also with weaker illumination but with so weak intensity that they are not detected by eye. The positions of spots should also depend only on the illumination and camera. The explanation in terms of beams of large Planck constant photons predicts this if the flux of dark photons from any light source is constant.

9.3.5 Dark matter and exotic color and electro-weak interactions

The presence of classical electro-weak and color gauge fields in all length scales is an unavoidable prediction of TGD and the interpretation in terms of p-adic and dark matter hierarchies is also more or less unavoidable. The new element in the interpretation is based on the observation that the quark and antiquarks at the ends of flux tubes serving as sources of classical color gauge fields could be seen as a vacuum polarization effect. In the same manner neutrino pairs at the ends of flux tubes serving as sources of classical Z^0 fields could be seem as a vacuum polarization effect.

One of the many open questions is whether also p-adic hierarchy defines a hierarchy of confinement scales for color interactions and screening scales for weak interactions or whether only the hierarchy of Planck constants gives rise to this kind of hierarchy. It would look strange if all flux tubes of macroscopic size scale would always correspond to a large value of $\hbar$ and therefore singular covering and fractionized quantum numbers. Also the proposed dark rules involving hierarchy of Mersenne rules would support the view that both hierarchies are present and there is an interaction between them in the sense that phase transitions between dark and thus scaled up counterpart of p-adic length scale and non-dark scaled up p-adic length scale can take place. The proposed stability criteria certainly allow this.

Do p-adic and dark matter hierarchies provide a correct interpretation of long ranged classical electro-weak gauge fields?

For two decades one of the basic interpretational challenges of TGD has been to understand how the un-avoidable presence of long range classical electro-weak gauge fields can be consistent with the small parity breaking effects in atomic and nuclear length scales. Also classical color gauge fields are predicted, and I have proposed that color qualia correspond to increments of color quantum numbers [K38] . The proposed model for screening cannot banish the unpleasant feeling that the screening cannot be complete enough to eliminate large parity breaking effects in atomic length scales so that one one must keep mind open for alternatives.

p-Adic length scale hypothesis suggests the possibility that both electro-weak gauge bosons and gluons can appear as effectively massless particles in several length scales and there indeed exists evidence that neutrinos appear in several scaled variants [C36] (for TGD based model see [K48]).

This inspires the working hypothesis that long range classical electro-weak gauge and gluon fields are correlates for light or massless p-adically scaled up and dark electro-weak gauge bosons and gluons. Thus both p-adic and dark hierarchies would be involved. For the p-adic hierarchy the masses would be scaled up whereas for the dark hierarchy masses would be same. The essentially new element in the interpretation would be that these fields assignable to flux quanta could be seen as vacuum polarization effects in even macroscopic length scales. This vision would definitely mean new physics effects but the interpretation would be consistent with quantum field theoretic intuition.

1. In this kind of scenario ordinary quarks and leptons could be essentially identical with their standard counterparts with electro-weak charges screened in electro-weak length scale so that the problems related to the smallness of atomic parity breaking would be trivially resolved. The weak form of electric-magnetic duality allows to identify the screening mechanism as analog of confinement mechanism for weak isospin

2. In condensed matter blobs of size larger than neutrino Compton length (about 5 μm if $k = 169$ determines the p-adic length scale of condensed matter neutrinos) the situation could be different. Also the presence of dark matter phases with sizes and neutrino Compton lengths corresponding to the length scales defined as p-adically scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$, $k = 151, 157, 163, 167$ in the range 10 nm-2.5 μm are suggested by the number theoretic considerations (these values of k correspond to so called Gaussian Mersennes [K43]). Only a fraction of the condensed matter consisting of regions of size $L_e(k)$ need to be in the dark phase.

3. Dark quarks and leptons would have masses essentially identical to their standard model counterparts. Only the electro-weak boson masses which are determined by a different mechanism than the dominating contribution to fermion masses [K48, K48] would be small or vanishing. Below the dark or p-adic length scale in question gauge bosons would behave like massless quanta.

4. The large parity breaking effects in living matter would be due to the presence of dark nuclei and leptons. Later the idea that super-fluidity corresponds to Z^0 super-conductivity will be discussed it might be that also super-fluid phase corresponds to dark neutron phase.

The basic prediction of TGD based model of dark matter as a phase with a large value of Planck constant is the scaling up of various quantal length and time scales. Mersenne hypothesis allows a wide range of scales so that very rich structures are possible.

Dark photon many particle states behave like laser beams decaying to ordinary photons by de-coherence meaning a transformation of dark photons to ordinary ones. Also dark electro-weak bosons and gluons would be massless or have small masses determined by the p-adic length scale in question. The decay products of dark electro-weak gauge bosons would be ordinary electro-weak bosons decaying rapidly via virtual electro-weak gauge boson states to ordinary leptons. Topological light rays ("massless extremals") for which all classical gauge fields are massless are natural space-time correlates for the dark boson laser beams. Obviously this means that the basic difference between the chemistries of living and non-living matter would be the absence of electro-weak symmetry breaking in living matter (which does not mean that elementary fermions would be massless).

Criterion for the presence of exotic electro-weak bosons and gluons

Classical gauge fields directly are space-time correlates of quantum states. The gauge fields associated with massless extremals ("topological light rays") decompose to free part and a part having non-vanishing divergence giving rise to a light-like Abelian gauge current. Free part would correspond to Bose-Einstein condensates and current would define a coherent state of dark photons.

The dimension D of the CP_2 projection of the space-time sheet serves as a criterion for the presence of long ranged classical electro-weak and gluon fields. D also classifies the (possibly asymptotic) solutions of field equations [K12] .

1. For $D=2$ induced gauge fields are Abelian and induced Kähler form vanishes for vacuum extremals: in this case classical em and Z^0 fields are proportional to each other. The non-vanishing Kähler field implies that induced gluon fields are non-vanishing in general. This raises the question whether long ranged color fields and by quantum classical correspondence also long ranged QCD accompany non-vacuum extremals in all length scales. This makes one wonder whether color confinement is possible at all and whether scaled down variants of QCD appear in all length scales.

   The possibility to add constants to color Hamiltonians appearing in the expression of the classical color gauge fields allows to have vanishing color charges in the case of an arbitrary space-time sheet. The requirement that color quantum numbers of the generator vanish allows to add the constant only to the Hamiltonians of color hyper charge and isospin so that for $D=2$ extremals color charges can be made vanishing. This might allow to understand how color confinement is consistent with long ranged induced Kähler field.

2. For $D \geq 3$ all classical long ranged electro-weak fields and non-Abelian color fields are present. This condition is satisfied when electric and magnetic fields are not orthogonal and the instanton density $A \wedge J$ for induced Kähler form is non-vanishing. The rather strong conclusion is that in length scales in which exotic electro-weak bosons are not present, one has $D=2$ and gauge fields are Abelian and correspond trivially to fixed points of renormalization group realized as a hydrodynamic flow at space-time sheets [K5] .

Quantum classical correspondence suggests the existence of electro-weak gauge bosons with mass scale determined by the size of the space-time sheets carrying classical long range electro-weak fields. This would mean the existence of new kind of gauge bosons.

The obvious objection is that the existence of these gauge bosons would be reflected in the decay widths of intermediate gauge bosons. The remedy of the problem is based on the notion of space-time democracy suggested strongly by the fact that the interactions between space-time sheets possessing different p-adic topologies proceed with very slow rates simply because the number of common rational (algebraic) points of partonic 2-surfaces appearing in the vertex is small.

For light exotic electro-weak bosons also the corresponding leptons and quarks would possess a large weak space-time sheet but lack the ordinary weak partonic 2-surface so that there would be no direct coupling to electro-weak gauge bosons. These space-time sheets are dark in weak sense but need not have a large value of $\hbar$. This picture implies the notion of partial darkness since any space-time sheets with different ordinary of Gaussian primes are dark with respect to each other.

Do Gaussian Mersennes define a hierarchy of dark electro-weak physics?

Gaussian Mersennes are defined as Gaussian primes of form $g_n = (1+i)^n - 1$, where n must be prime. They have norm squared $g\overline{g} = 2^n - 1$. The list of the first Gaussian Mersennes corresponds to the following values of n.

2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113, 151, 157, 163, 167, 239, 241, 283, 353, 367, 379, 457, 997, 1367, 3041, 10141, 14699, 27529, 49207, 77291, 85237, 106693, 160423 and 203789.

The Gaussian primes $k = 113, 151, 157, 163, 167$ correspond to length scales which are of most obvious interest but in TGD framework one cannot exclude the twin prime $239, 241$ corresponds to length scales $L_e(k) \simeq 160$ km and 320 km. Also larger primes could be of relevant for bio-systems and consciousness. Also the secondary and higher length scales associated with $k < 113$ could be of importance and their are several length scales of this kind in the range of biologically interesting length scales. Physics and biology inspired considerations suggests that particular Gaussian primes correspond to a particular kind of exotic matter, possibly also to large $\hbar$ phase.

$k = 113$ corresponds to the electromagnetic length scale of u and d quarks and nuclear p-adic length scale. For dark matter these length scales are scaled up by a factor $r \sim 2^{k_d}$, with k_d fixed by Mersenne hypothesis.

On basis of biological considerations (large parity breaking in living matter) there is a temptation to assign to these length scales a scaled down copy of electro-weak physics and perhaps also of color physics. The mechanism giving rise to these states would be a phase transition transforming the ordinary $k = 89$ Mersenne of weak space-time sheets to a Gaussian Mersenne and thus increasing its size dramatically.

If given space-time sheet couples considerably only to space-time sheets characterized by same prime or Gaussian prime, the bosons of these physics do not couple directly to ordinary particles, and one avoids consistency problems due to the presence of new light particles (consider only the decay widths of intermediate gauge bosons [K52]) even in the case that the loss of asymptotic freedom is not assumed.

A question arises about the interpretation of structures of the predicted size. The strong interaction size of u and d quarks, hadrons, and nuclei is smaller than $L(k = 113) \simeq 2 \times 10^{-4}$ m for even heaviest nuclei if one accepts the formula $R \sim A^{1/3} \times 1.5 \times 10^{-15}$ m. A natural interpretation for this length scale would be as the size of the field body/magnetic body of system defined by its topologically quantized gauge fields/magnetic parts of gauge fields. The (possibly dark) p-adic length scale characterizes also the lengths of join along boundaries bonds feeding gauge fluxes from elementary particle to the space-time sheet in question. The de-localization due these join along boundaries bonds in p-adic length scale in question would determine the scale of the contribution to the mass squared of the system as predicted by p-adic thermodynamics.

9.3.6 Anti-matter and dark matter

The usual view about matter anti-matter asymmetry is that during early cosmology matter-antimatter asymmetry characterized by the relative density difference of order $r = 10^{-9}$ was somehow generated and that the observed matter corresponds to what remained in the annihilation of quarks and leptons to bosons. A possible mechanism inducing the CP asymmetry is based on the CP breaking phase of CKM matrix.

The TGD based view about energy [K89, K76] forces the conclusion that all conserved quantum numbers including the conserved inertial energy have vanishing densities in cosmological length scales. Therefore fermion numbers associated with matter and antimatter must compensate each other. Therefore the standard option seems to be excluded in TGD framework.

The way out could be based on the many-sheeted space-time and the possibility of cosmic strings. One particular TGD inspired model involves a small matter-antimatter asymmetry induced by the Kähler electric fields of cosmic strings [K26] . The topological condensation of fermions and anti-fermions at space-time sheets carrying Kähler electric field of say cosmic string gives rise to a binding energy which is of different sign for fermions and anti-fermions and therefore should induce the asymmetry. The outcome of the annihilation period would be matter outside cosmic strings and antimatter inside them.

One can also imagine that in a given Kähler electric field matter develops large binding energy and antimatter large positive interaction energy which induces instability leading to the splitting of partonic 2-surfaces to dark space-time sheets implying fractionization and reduction of the energy at given sheet of the covering. Dark antimatter would interact very weakly with ordinary matter so that the non-observability of antimatter would find an elegant explanation. One can imagine also the generation of local asymmetries inside Kähler electric flux tubes leading to flux tube states with matter and antimatter condensed at the opposite ends of the flux tubes.

9.4 Dark variants of nuclear physics

The book metaphor for the extended imbedding space can be utilized as a guideline as one tries to imagine various exotic phases of matter. For the minimal option atomic nuclei can be assumed to be ordinary (in the sense of nuclear string model [K26] !) and only field bodies can be dark. If only singular coverings of M^4 and CP_2 are allowed the value of Planck constant is product of two integers. Ruler and compass hypothesis restricts these integers considerably and Mersenne hypothesis provides further constraints on the model. Nuclei can be visualized as residing at the "standard" pages of the book and dark color-/weak-/em- bonds are at different pages with different p-adic length scale or having different Planck constant. This would give two hierarchies of nuclei with increasing size.

9.4.1 Constraints from the nuclear string model

In the case of exotic nuclei nuclear string model [L2] , [L2] is a safe starting point. In this model nucleons are connected by color flux tubes having exotic light fermion and anti-fermion at their ends. Whether fermion is quark or colored excitation of lepton remains open question at this stage. The mass of the exotic fermion is much smaller than 1 MeV (p-adic temperature $T = 1/n < 1$). This model predicts large number of exotic states since color bonds, which can be regarded as colored pions, can have em charges (1,-1,0). In particular, neutral variant of deuterium is predicted and this leads to a model of cold fusion explaining its basic selection rules. The earlier model for cold fusion discussed in [K80] , which served as a constraint in the earlier speculations, is not so simple than the model of [L2] , [L2] .

What is important that the model requires that weak bosons for which Compton length is of order atomic size are involved. Weak bosons would behave as massless particles below the Compton and the rates for the exchanges of weak bosons would be high in the length scales considered. Weak bosons would correspond to scaled up variants of the ordinary weak bosons: scaling could be p-adic in which mass scale is reduced and weak interaction rates even above Compton length would be scaled up as $1/M_W^4$. The scaling could result also from the scaling of Planck constant in which case masses of weak bosons nor weak interaction rates in the lowest order would not be affected. If only dark scaling is involved, weak interactions would be still extremely weak above dark Compton length of weak bosons. Of course, both scalings can be imagined.

The scale of the color binding energy is $E_s = .2$ MeV for ordinary ^4He strings [K26] . $k = 151, 157, 163, 167$ define Gaussian Mersennes $G_{M,k} = (1+i)^k - 1$ and excellent candidates for biologically important p-adic length scales. There are also higher Gaussian Mersennes such as those corresponding to $k = 239, 241$ and also these seem to be interesting biologically (see [K29] where a vision about evolution and generalized EEG based on Gaussian Mersennes is described). Let us assume that these scales and also those corresponding to $k = 89, 107, 113, 127$ allow scaled variants of electroweak and color interactions with ordinary value of Planck constant. If M_{127} is scaled up to Gaussian Mersenne $M_{G,167}$, one obtains cell-nucleus sized (2.58 μm) exotic nuclei and the unit of color binding energy is still .2 eV. For p-adic length scale of order 100 μm (size of large neuron) the energy scale is still around thermal energy at room temperature.

In the case of dark color bonds it is not quite clear how the unit E_s of the color binding energy scales. If color Coulomb energy is in question, one expects $1/\hbar^2$ scaling. Rather remarkably, this scaling predicts that the unit for the energy of $A < 4$ color bond scales down to .5 eV which is the energy of hydrogen bond so that hydrogen bonds, and also other molecular bonds, might involve color bonds between proton and oxygen.

9.4.2 Constraints from the anomalous behavior of water

$H_{1.5}O$ behavior of water with respect to neutron and electron scattering is observed in atto-second time scale which corresponds to 3 Angstrom length scale, defining an excellent candidate for the size scale of exotic nuclei and Compton length of exotic weak interactions.

What happens to the invisible protons?

A possible explanation for the findings is that one fourth of protons forms neutral multi-proton states connected by possibly negatively charged color bonds of length differing sufficiently from the length of ordinary O-H bond. Although the protons are ordinary, neutron diffraction reflecting the crystal like order of water in atomic length scales would not see these poly-proton super-nuclei if they form separate closed strings.

1. For the ordinary nuclei the p-adic length scale associated with the color bonds between ^4He corresponds to M_{127}, and one can imagine exotic nuclear strings obtained by connecting two ordinary nuclei with color bonds. If second exotic nucleus is neutral (the model of cold fusion assumes that D nucleus is neutral) this could work since the Coulomb wall is absent. If the exotic nuclei have opposite em charges, the situation improves further. New super-dense phases of condensed matter would be predicted.

   If one fourth of hydrogen nuclei of water combine to form possibly neutral nuclear strings with average distance of nuclei of order $L(127)$, they are not visible in diffraction at atomic length scale because the natural length scale is shortened by a factor of order 32 but could be revealed in neutron diffraction at higher momentum exchanges. The transition between this kind of phase and ordinary nuclei would be rather dramatic event and the exchanges of exotic weak bosons with Compton lengths of order atomic size induce the formation of this kind of nuclei (this exchange is assumed in the model of cold fusion).

2. If dark color magnetic bonds are allowed, a natural distance between the building blocks of super-nuclei is given by the size scale of the color magnetic body. In nuclear string model the size scales of color magnetic bodies associated with nuclear strings consisting of ^4He and $A < 4$ nuclei color magnetic bodies correspond to $k = 127$ and $k = 118$ whereas em magnetic body corresponds to $k = 116$ [L2] , [L2] . For dark variants of magnetic bodies the sizes of these magnetic bodies are scaled. There are several options to consider: consider only $k_d = 113 - 89 = 24$, $k_d = 127 - 107 = 20$ and $k_d = 107 - 89 = 18$. Note that one has $h_{eff} = nh$, where n is product of distinct Fermat primes and power 2^{k_d}. Table 1 below summarizes the effective dark p-adic length scales involved.

3. Consider $k_d = 24$ as an example. From Table 1 the scaled up p-adic length scales of the magnetic bodies would be $L(127 + 24 = 151) = 10$ nm, $L(118 + 24 = 142) = 4.4$ Angstrom, and $L(116 + 24 = 140) = 2.2$ Angstrom. The first scale equals to the thickness of cell membrane which suggests a direct connection with biology. The latter two scales correspond to molecular length scales and it is not clear why the protons of dark nuclear strings of this kind would not be observed in electron and neutron scattering. This would leave only nuclear strings formed from ^4He nuclei into consideration.

   The crucial parameter is the the unit E_s of the color binding energy. Since this parameter should correspond to color Coulombic potential it could transform like the binding energy of hydrogen atom and therefore scale as $1/\hbar^2$. This would mean that $E_s = 2.2$ MeV deduced from the deuteron binding energy would scale down to .12 eV for $r = 2^{24}$.

k_d	24	20	18
$k_{eff} = 116 + k_d$	140	136	134

Table 1. The integers k_{eff} characterize the effective p-adic length scales for some dark variants of color magnetic bodies for ^4He and $A < 4$ color magnetic bodies corresponding to $k \in \{127, 118\}$ and for the dark variants of $k = 116$ electromagnetic body for nuclear strings . Dark variants correspond to $k_d \in \{24 = 113 - 89 = 151 - 127, 20 = 127 - 107, 18 = 107 - 89\}$ allowed by Mersenne hypothesis.

The transition between the dark and ordinary nuclei would be favored by the minimization of Coulomb energy and energy differences would be small because of darkness. The transitions in which ordinary proton becomes dark and fuses to super-nuclear string or vice versa could be the basic control mechanism of bio-catalysis. Metabolic energy quantum .5 eV should relate to this transition.

Magic nuclei could have fractally scaled up variants in molecular length scale and tetrahedral and icosahedral water clusters could correspond to $A = 8$ and $A = 20$ magic nuclei with color bonds connecting nucleons belonging to different dark nuclei.

About the identification of the exotic weak physics?

The model of cold fusion requires exotic weak physics with the range of weak interaction of order atomic radius.

One can consider the possibility of $k = 113$ dark weak physics with $r = 2^{24}$ ($89 \rightarrow 113$ in Mersenne hypothesis) implying that the dark weak scale corresponds to p-adic length scale $k = 137$. Weak Compton length for $k = 113$ dark weak bosons would be about 3 Angstrom. Below $L(137)$ weak bosons would behave as massless particles. Above $L(137)$ weak bosons would have the mass scale $2^{-12}m_W \sim 25$ MeV and weak rates would be scaled up by 2^{48}. Bohr radius would represent a critical transition length scale and exotic weak force could have dramatic implications for the behavior of the condensed matter in high pressures when exotic weak force would become visible. In particular, chiral selection in living matter could be understood in terms of large parity breaking implied. These physics would manifest themselves only at criticality for the phase transitions changing Planck constant and would correspond to almost vacuum extremals defining a phase different from that assignable to standard model physics.

To sum up, it would seem that the variant of ordinary nuclear physics obtained by making color bonds and weak bonds dark is the most promising approach to the $H_{1.5}O$ anomaly and cold fusion. Exotic weak bosons with Compton wave length of atomic size and the most natural assumption is that they are dark $k = 113$ weak bosons with $k_d = 24 = 113 - 89$. One variant of exotic atoms is as atoms for which electromagnetic interaction between ordinary nuclei and ordinary electrons is mediated along dark topological field quanta.

9.4.3 Exotic chemistries and electromagnetic nuclear darkness

The extremely hostile and highly un-intellectual attitude of skeptics stimulates fear in anyone possessing amygdala, and I am not an exception. Therefore it was a very pleasant surprise to receive an email telling about an article published in April 16, 2005 issue of New Scientist [D25] . The article gives a popular summary about the work of the research group of Walter Knight with Na atom clusters [D54] and of the research group of Welford Castleman with Al atom clusters [D34] .

The article tells that during last two decades a growing evidence for a new kind of chemistry have been emerging. Groups of atoms seem to be able to mimic the chemical behavior of single atom. For instance, clusters of 8, 20, 40, 58 or 92 sodium atoms mimic the behavior of noble gas atoms [D54] . By using oxygen to strip away electrons one by one from clusters of Al atoms it is possible to make the cluster to mimic entire series of atoms [D34] . For aluminium cluster-ions made of 13, 23 and 37 atoms plus an extra electron are chemically inert.

One can imagine two explanations for the findings.

1. The nuclei are dark in the sense that the sizes of nuclear space-time sheets are scaled up implying the smoothing out of the nuclear charge.

2. Only electrons are dark in the sense of having scaled up Compton lengths so that the size of multi-electron bound states is not smaller than electron Compton length and electrons "see" multi-nuclear charge distribution.

If darkness and Compton length is assigned with the em field body, it becomes a property of interaction, and it seems impossible to distinguish between options 1) and 2).

What one means with dark nuclei and electrons?

Can the idea about dark nuclei and electrons be consistent with the minimalist picture in which only field bodies are dark? Doesn't the darkness of nucleus or electron mean that also multi-electron states with n electrons are possible?

The proper re-interpretation of the notion Compton length would allow a consistency with the minimalist scenario. If the p-adic prime labelling the particle actually labels its electromagnetic body as p-adic mass calculations for quark masses encourage to believe, Compton length corresponds to the size scale of the electromagnetic field body and the models discussed below would be consistent with the minimal scenario. Electrons indeed "see" the external charge distribution by their electromagnetic field body and field body also carries this distribution since CP_2 extremals do not carry it. One could also defend this interpretation by saying that electrons is operationally only what can be observed about it through various interactions and therefore Compton length (various Compton length like parameters) must be assigned with its field body (bodies).

Also maximal quantum criticality implies that darkness is restricted to field bodies but does not exclude the possibility that elementary particle like structures can possess non-minimal quantum criticality and thus possess multi-sheeted character.

Option I: nuclei are electromagnetically dark

The general vision about nuclear dark matter suggests that the system consists of super-nuclei analogous to ordinary nuclei such that electrons are ordinary and do not screen the Coulomb potentials of atomic nuclei.

The simplest possibility is that the electromagnetic field bodies of nuclei or quarks become dark implying de-localization of nuclear charge. The valence electrons would form a kind of mini-conductor with electrons de-localized in the volume of the cluster. The electronic analog of the nuclear shell model predicts that full electron shells define stable configurations analogous to magic nuclei. The model explains the numbers of atoms in chemically inert Al and Ca clusters and generalizes the notion of valence to the level of cluster so that the cluster would behave like single super-atom.

The electromagnetic $k = 113$ space-time sheets (em field bodies) of quarks could have scaled up size $\sqrt{r}L(113) = L(113 + k_d) = 2^{k_d/2} \times 2 \times 10^{-14}$ m. One would have atomic size scale .8 Angstroms for $r = 2^{k_d}$, $k_d = 24$- an option already introduced. A suggestive interpretation is that the electric charge of nuclei or valence quarks assignable to their field bodies is de-localized quantum mechanically to atomic length scale. Electrons would in a good approximation experience quantum mechanically the nuclear charges as a constant background, jellium, whose effect is indeed modellable using harmonic oscillator potential.

One can test the proposed criterion for the phase transition to darkness. The unscreened electromagnetic interaction energy between a block of partially ionized nuclei with a net em charge Z with Z electrons would define the relevant parameter as $r \equiv Z^2\alpha$. For the total charge $Z \geq 12$ the condition $r \geq 1$ is satisfied. For a full shell with 8 electrons this condition is not satisfied.

Option II: Electrons are electro-magnetically dark

Since the energy spectrum of harmonic oscillator potential is invariant under the scaling of $\hbar$ accompanied by the opposite scaling of the oscillator frequency ω, one must consider also the em bodies of electrons are in large $\hbar$ phase (one can of course ask whether they could be observed in this phase!). The rule would be that the size of the bound states is larger than the scaled up electron Compton length.

The Compton wavelength of electrons would be scaled up by a factor r where r is product of different Fermat primes and power of 2 for ruler and compass hypothesis. For Mersenne hypothesis one would have $r = 2^{k_d}$. For $k_d = 24$ the effective p-adic scale of electron would be to about $L(151) = 10$ nm. The atomic cluster of this size would contain roughly $10^6 \times (a_0/a)^3$ atoms where a is atomic volume and $a_0 = 1$ Angstrom is the natural unit.

The shell model of nucleus is in TGD framework a phenomenological description justified by nuclear string model with string tension responsible for the oscillator potential. This leads to ask whether the electrons of jellium actually form analogs of nuclear strings with electrons connected by color bonds.

9.5 Has dark matter been observed?

In this section two examples about anomalies perhaps having interpretation in terms of quantized Planck constant are discussed. The first anomaly belongs to the realm of particle physics and hence does not quite fit the title of the chapter. Second anomaly relates to nuclear physics.

9.5.1 Optical rotation of a laser beam in a magnetic field

The group of G. Cantatore has reported an optical rotation of a laser beam in a magnetic field [D42] . The experimental arrangement involves a magnetic field of strength $B = 5$ Tesla. Laser beam travels 22000 times forth and back in a direction orthogonal to the magnetic field travelling 1 m during each pass through the magnet. The wavelength of the laser light is 1064 nm (the energy is 1.1654 eV). A rotation of $(3.9 \pm .5) \times 10^{-12}$ rad/pass is observed.

Faraday effect [D5] is optical rotation which occurs when photon beam propagates in a direction parallel to the magnetic field and requires parity breaking guaranteeing that the velocities of propagation for two circular polariations are different. Now however the laser beam is orthogonal to the magnetic field so that Faraday effect cannot be in question.

The proposed interpretation for the rotation would be that the component of photon having polarization parallel to the magnetic field mixes with QCD axion, one of the many candidates for dark matter. The mass of the axion would be about 1 meV. Mixing would imply a reduction of the corresponding polarization component and thus in the generic case induce a rotation of the polarization direction. Note that the laser beam could partially transform to axions, travel through a non-transparent wall, and appear again as ordinary photons.

The disturbing finding is that the rate for the rotation is by a factor 2.8×10^4 higher than predicted. This would have catastrophic astrophysical implications since stars would rapidly lose their energy via axion radiation.

What explanations one could imagine for the observations in TGD framework if one accepts the hierarchy of Planck constants?

1. The simplest model that I have been able to imagine does not assume axion like states. The optical rotation would be due to the leakage of the laser photons to dark pages of the Big Book at the ends of the magnet where the space-time sheet carrying the magnetic field becomes locally a vacuum extremal. This explanation would not mean direct seeing of dark matter but the observation of a transformation of ordinary matter to dark matter. Quite generally, this experimental approach might be much better strategy to the experimental proof of the existence of the dark matter than the usual approaches and is especially attractive in living matter.

2. TGD could also provide a justification for the axion based explanation of the optical rotation involving parity breaking. TGD predicts the existence of a hierarchy of QCD type physics based on the predicted hierarchy of scaled up variants of quarks and also those of color excited leptons. The fact that these states are not seen in the decay widths of intermediate gauge bosons can be understood if the particles in question are dark matter with non-standard value of Planck constant and hence residing at different page of the book like structure formed by the imbedding space. I have discussed in detail the general model in the case of lepto-hadrons consisting of colored excitation of ordinary lepton and explaining quite an impressive bundle of anomalies [K88] . Since lepto-pion has quantum numbers of axion and similar couplings, it is natural to propose that the claimed axion like particle -if it indeed exists- is a pion like state consisting either exotic light quarks or leptons.

   The dark variants of hadron physics are suggestive in living matter. By p-adic length scale hypothesis one expects that the mass of axion-like state identifiable as a scaled variant of pion would relate by a power of $\sqrt{2}$ to pion mass. For 1 meV axion like particle, call it A, the mass ratio is $m(\pi)/m(A) = 2^{37} \times 1.004$ and indeed very near to a power of 2.

3. Rather interestingly, years later emerged evidence for an axion like particle interpreted as dark matter and having mass $m(A) = .11$ meV. The decays of this particle in the electric field of Josephson junction generate photon absorbed by Cooper pair are claimed to induce resonantly an anomalous Josephson current [D26] (`http://www.newscientist.com/article/`

dn24689#.Uql3eKWvLB0). If there exists several dark copies of hadron physics, it would not be surprising if the pions of these copies would behave like axions. Interpretation as scaled variant of electro-pion however yields a mass ratio nearer to a power of two: it consists of electron and positron and has mass $m(\pi_L) \simeq 2m_e$ given $m(\pi_L)(m(A) \simeq 1.08$. For ordinary pion the ratio is $m(\pi)(m(A) \simeq 1.14$.

4. The TGD inspired model would differ from the above model only in that the leakage to the dark sector would take place by a transformation of the laser photon to a pionlike state so that no parity breaking would take place. But the basic point is that vacuum extremals through which the leakage can occur, break the parity strongly by the presence of classical Z^0 fields. The idea about leakage together with the non-constancy of pion-type field appearing in the coupling to the instanton density imply that that the space-time sheet representing the magnetic field is vacuum extremal -at least in some regions- and this assumption looks unnecessarily strong. Also detailed assumptions about the dependence of the basic parameters appearing in PCAC hypothesis must be made.

   What raised the hopes was the intriguing observation that the ratio of laser photon frequency to the cyclotron frequency of electron in the magnetic field considered equals to $r = 2^{11}$: this put bells ringing in the p-adically tuned mind and inspired the question whether one could have $\hbar/\hbar_0 = 2^{11}$. It must be however emphasized that this assumption about the values of $\hbar$ might be too restrictive. The assumption of cyclotron condensate of electron pairs at dark space-time sheet must be however justified and one must answer at least the question why it is needed. A possible answer would be that the leakage occurs via Bose-Einstein condensation to a coherent state of cyclotron photons. But this would mean return to the original model where laser photons leak! Obviously the model becomes too complicated for Occam and therefore I have dropped out the model.

The simplest model should start just from the finding that the linear polarization parallel to the magnetic field seems to leak with a certain rate as it traverses the magnet. The leakage of laser photons to a dark matter space-time sheet is what comes mind first in TGD context. A killer test for this explanation is to use polarization parallel to the magnetic field: in this case no optical rotation should take place.

1. The leakage should take place along the intersection of the pages of the Big Book which correspond to geodesically trivial geodesic sphere of CP_2 so that induced Kähler field vanishes and vacuum extremals or nearly vacuum extremals are in question. Leakage could occur within magnet or the ends of the magnet could involve this kind of critical membrane like region and as the photon passes through them the leakage could occur.

2. Since parity breaking takes place, the instanton density for the electromagnetic field provides a natural description of the situation. The interaction term is obtained by replacing either E in $E \cdot B$ with its quantized counterpart describing laser photons. This gives a linear coupling to photon oscillator operators completely analogous to a coupling to an external current and one can calculate the leakage rate using the standard rules.

3. The interaction term is total divergence and reduces to a 3-D Chern-Simons type term associated with the boundaries of the membrane like region or magnet in the general case and the leakage can be said to occur at the ends of the magnet for non-vacuum exremals.

One can ask whether one should use the instanton density of Kähler field rather than that of em field in the model. In this case Kähler gauge potential would couple the quantized em field via U(1) part of em charge. One would not have gauge invariance since for the induced Kähler field gauge degeneracy is replaced with spin glass degeneracy and gauge transformations of the vacuum extremals induced by symplectic transformations of CP_2 deform the space-time surface. In this case E in $E \cdot A$ would be replaced with the radiation field at the ends of the magnet. In order to have a non-vanishing leakage the instanton density within magnet must be non-vanishing meaning that CP_2 projection of the magnet's space-time sheet must be 4-D at least somewhere. For the first option it can be 2-D.

The coefficient K of the instanton term defining the action should depend on the value of Planck constant. $1/e^2$ proportionality of the ordinary Maxwell action means that the coefficient of the instanton term could be propertional to $\hbar$. The most general dependence $K = k(e^2\hbar/4\pi)/e^2 \equiv f(\alpha_{em}r)/e^2$, $r = \hbar/\hbar_0$. Since non-perturbative effect is in question $k((\alpha_{em}r) \propto 1/(\alpha_{em}r)$ is suggestive and guarantees that the leakage probability becomes small for large values of Planck constant.

This option will not be discussed further but it might have also relevance to the parity breaking in biology. In fact, I have proposed that the realization of genetic code based on nuclelotide dependent optical rotation of polarization of photons proposed by Gariaev [I6] could be based on Faraday effect or its analogy [K87] .

One can consider also a generalization of this model by assuming that photon transforms to dark pion-like state in the leakage. In this case the action does not however reduce to a total divergence and the condition that the entire magnet corresponds to vacuum extremal seems to be unrealistic.

9.5.2 Do nuclear reaction rates depend on environment?

Claus Rolfs and his group have found experimental evidence for the dependence of the rates of nuclear reactions on the condensed matter environment [C46] . For instance, the rates for the reactions $^{50}V(p,n)^{50}Cr$ and $^{176}Lu(p,n)$ are fastest in conductors. The model explaining the findings has been tested for elements covering a large portion of the periodic table.

Debye screening of nuclear charge by electrons as an explanation for the findings

The proposed theoretical explanation [C46] is that conduction electrons screen the nuclear charge or equivalently that incoming proton gets additional acceleration in the attractive Coulomb field of electrons so that the effective collision energy increases so that reaction rates below Coulomb wall increase since the thickness of the Coulomb barrier is reduced.

The resulting Debye radius

$$R_D = 69\sqrt{\frac{T}{n_{eff}\rho_a}} \ , \qquad (9.5.1)$$

where ρ_a is the density of atoms per cubic meter and T is measured in Kelvins. R_D is of order .01 Angstroms for $T = 373$ K for $n_{eff} = 1$, $a = 10^{-10}$ m. The theoretical model [C82, C104] predicts that the cross section below Coulomb barrier for $X(p,n)$ collisions is enhanced by the factor

$$f(E) = \frac{E}{E+U_e}exp(\frac{\pi\eta U_e}{E}) \ . \qquad (9.5.2)$$

E is center of mass energy and η so called Sommerfeld parameter and

$$U_e \equiv U_D = 2.09\times 10^{-11}(Z(Z+1))^{1/2}\times(\frac{n_{eff}\rho_a}{T})^{1/2} \ eV \qquad (9.5.3)$$

is the screening energy defined as the Coulomb interaction energy of electron cloud responsible for Debye screening and projectile nucleus. The idea is that at R_D nuclear charge is nearly completely screened so that the energy of projectile is $E + U_e$ at this radius which means effectively higher collision energy.

The experimental findings from the study of 52 metals support the expression for the screening factor across the periodic table.

1. The linear dependence of U_e on Z and $T^{-1/2}$ dependence on temperature conforms with the prediction. Also the predicted dependence on energy has been tested [C46] .

2. The value of the effective number n_{eff} of screening electrons deduced from the experimental data is consistent with $n_{eff}(Hall)$ deduced from quantum Hall effect.

The model suggests that also the decay rates of nuclei, say beta and alpha decay rates, could be affected by electron screening. There is already preliminary evidence for the reduction of beta decay rate of ^{22}Na β decay rate in Pd [C45] , metal which is utilized also in cold fusion experiments. This might have quite far reaching technological implications. For instance, the artificial reduction of half-lives of the radioactive nuclei could allow an effective treatment of radioactive wastes. An interesting question is whether screening effect could explain cold fusion [C112] and sono-fusion [C66] : I have proposed a different model for cold fusion based on large $\hbar$ in [K80] .

Could quantization of Planck constant explain why Debye model works?

The basic objection against the Debye model is that the thermodynamical treatment of electrons as classical particles below the atomic radius is in conflict with the basic assumptions of atomic physics. On the other hand, it is not trivial to invent models reproducing the predictions of the Debye model so that it makes sense to ask whether the quantization of Planck constant predicted by TGD could explain why Debye model works.

TGD predicts that Planck constant is quantized in integer multiples: $\hbar = n\hbar_0$, where $\hbar_0$ is the minimal value of Planck constant identified tentatively as the ordinary Planck constant. The preferred values for the scaling factors n of $\hbar$ correspond to n-polygons constructible using ruler and compass. The values of n in question are given by $n_F = 2^k \prod_i F_{s_i}$, where the Fermat primes $F_s = 2^{2^s}+1$ appearing in the product are distinct. The lowest Fermat primes are $3, 5, 17, 257, 2^{16}+1$. In the model of living matter the especially favored values of $\hbar$ come as powers 2^{k11} [K28, K29] .

It is not quite obvious that ordinary nuclear physics and atomic physics should correspond to the minimum value $\hbar_0$ of Planck constant. The predictions for the favored values of n are not affected if one has $\hbar(stand) = 2^k\hbar_0$, $k \geq 0$. The non-perturbative character of strong force suggests that the Planck constant for nuclear physics is not actually the minimal one [K80] . As a matter fact, TGD based model for nucleus implies that its "color magnetic body" has size of order electron Compton length. Also valence quarks inside hadrons have been proposed to correspond to non-minimal value of Planck constant since color confinement is definitely a non-perturbative effect. Since the lowest order classical predictions for the scattering cross sections in perturbative phase do not depend on the value of the Planck constant one can consider the testing of this issue is not trivial in the case of nuclear physics where perturbative approach does not really work.

Suppose that one has $n = n_0 = 2^{k_0} > 1$ for nuclei so that their quantum sizes are of order electron Compton length or perhaps even larger. One could even consider the possibility that both nuclei and atomic electrons correspond to $n = n_0$, and that conduction electrons can make a transition to a state with $n_1 < n_0$. This transition could actually explain how the electron conductivity is reduced to a finite value. In this state electrons would have Compton length scaled down by a factor n_0/n_1.

For instance, if one has $n_0 = 2^{11k_0}$ as suggested by the model for quantum biology [K29] and by the TGD based explanation of the claimed detection of dark matter [D42] , the Compton length $L_e = 2.4 \times 10^{-12}$ m for electron would reduce in the transition $k_0 \to k_0 - 1$ to $L_e = 2^{-11}L_e \simeq 1.17$ fm, which is rather near to the proton Compton length since one has $m_p/m_e \simeq .94 \times 2^{11}$. It is not too difficult to believe that electrons in this state could behave like classical particles with respect to their interaction with nuclei and atoms so that Debye model would work.

The basic objection against this model is that anyonic atoms should allow more states that ordinary atoms since very space-time sheet can carry up to n electrons with identical quantum numbers in conventional sense. This should have been seen.

Electron screening and Trojan horse mechanism

An alternative mechanism is based on Trojan horse mechanism suggested as a basic mechanism of cold fusion [K80] . The idea is that projectile nucleus enters the region of the target nucleus along a larger space-time sheet and in this manner avoids the Coulomb wall. The nuclear reaction itself occurs conventionally. In conductors the space-time sheet of conduction electrons is a natural candidate for the larger space-time sheet.

At conduction electron space-time sheet there is a constant charged density consisting of n_{eff} electrons in the atomic volume $V = 1/n_a$. This creates harmonic oscillator potential in which incoming proton accelerates towards origin. The interaction energy at radius r is given by

$$V(r) = \alpha n_{eff}\frac{r^2}{2a^3} , \quad (9.5.4)$$

where a is atomic radius.

The proton ends up to this space-time sheet by a thermal kick compensating the harmonic oscillator energy. This occurs below with a high probability below radius R for which the thermal energy $E = T/2$ of electron corresponds to the energy in the harmonic oscillator potential. This gives the condition

$$R = \sqrt{\frac{Ta}{n_{eff}\alpha}}a . \quad (9.5.5)$$

This condition is exactly of the same form as the condition given by Debye model for electron screening but has a completely different physical interpretation.

Since the proton need not travel through the nuclear Coulomb potential, it effectively gains the energy

$$E_e = Z\frac{\alpha}{R} = \frac{Z\alpha^{3/2}}{a}\sqrt{\frac{n_{eff}}{Ta}} . \quad (9.5.6)$$

which would be otherwise lost in the repulsive nuclear Coulomb potential. Note that the contribution of the thermal energy to E_e is neglected. The dependence on the parameters involved is exactly the same as in the case of Debye model. For $T = 373$ K in the ^{176}Lu experiment and $n_{eff}(Lu) = 2.2 \pm 1.2$, and $a = a_0 = .52 \times 10^{-10}$ m (Bohr radius of hydrogen as estimate for atomic radius), one has $E_e = 28.0$ keV to be compared with $U_e = 21 \pm 6$ keV of [C46] ($a = 10^{-10}$m corresponds to 1.24×10^4 eV and 1 K to 10^{-4} eV). A slightly larger atomic radius allows to achieve consistency. The value of $\hbar$ does not play any role in this model since the considerations are purely classical.

An interesting question is what the model says about the decay rates of nuclei in conductors. For instance, if the proton from the decaying nucleus can enter directly to the space-time sheet of the conduction electrons, the Coulomb wall corresponds to the Coulomb interaction energy of proton with conduction electrons at atomic radius and is equal to $\alpha n_{eff}/a$ so that the decay rate should be enhanced.

9.5.3 Refraction of gamma rays from silicon prism?

The following considerations were inspired by to a popular article [D23] (`http://www.mpg.de/5799885/gold_lenses_gamma_optics?`) telling about refraction of gamma rays from silicon prisms. This should not be possible and since I love anomalies I got interested. Below I discuss the discovery from the point of standard physics and TGD point of view.

What happens in refraction?

Absorption, reflection, and refraction are basic phenomena of geometric optics [D7] describing the propagation of light in terms of light rays and neglecting interference and diffraction making it possible for light to "go around the corner". The properties of medium are described in terms of refraction index n which in general is a complex quantity. The real part of n gives the phase velocity of light in medium using vacuum velocity c as unit, which - contrary to a rather common misconception - can be also larger than c as a phase velocity which cannot be assigned to energy transfer. The imaginary part characterizes absorption. n depends in general on frequency of the incoming light and the resonant interactions of light with the atoms of medium make themselves manifest in the frequency dependence of n - in particular in absorption described by the imaginary part of n.

What happens in the boundary of two media - reflection or refraction - is characterized the the refraction index boundary conditions for radiation fields at the boundary, which are essentially Maxwell's equations at the discontinuity. Snell's law tells what happens to the direction of the beam and states essentially that only the momentum component of incoming photon normal to the boundary changes in these processes since only the translational symmetry in normal direction is changed.

How refractive index is determined?

What determines the index of refraction [D10]? To build a microscopic theory for n one must model what happens for the incoming beam of light in medium. One must model the scattering of light from the atoms of the medium.

In the case of condensed matter X ray diffraction is excellent example about this kind of theory. In this case the lattice structure of the condensed matter system makes the situation simple. For infinitely large medium and for an infinitely wide incoming beam the scattering amplitude is just the Fourier transform of the density of atoms for the change of the wave vector (or equivalently momentum) of photon, which must be a vector in the resiprocal lattice of the crystal lattice. Therefore the beam is split into beams in precisely defined directions. The diffracted beam has a sharp maximum in forward direction and the amplitude in this direction is essentially the number of atoms.

In less regular situation such as for water or bio-matter for which regular lattice structure typically exists only locally the peaking to forward direction, is even more pronounced, and in the first approximation the beam travels in the direction that it has after entering to the system and only the phase velocity is changed and attenuation takes place. Diffraction patterns are however present also now and allow to deduce information about the structure of medium in short length scales. For instance, Delbrueck diffraction from biological matter allowed to deduce structural information about DNA and deduce its structure.

This description contains an important implicit assumption. The width and length of the incoming photon beam must be so large that the number of atoms inside it is large enough. If this condition is not satisfied, the large scale interference effects crucial for diffraction do not take place. For very narrow beams the situation approaches to a scattering from single atom and one expects that the beam is gradually widened but that it does not make sense to speak about refraction index and that the application of Snell's law does not make sense. Incoming photons see individual atoms rather than the lattice of atoms. For this reason the prevailing wisdom has been that it does not make sense to speak about bending of gamma rays from solid state. A gamma ray photon with energy of one MeV corresponds to a wavelength λ of about 10^{-12} meters which is of same order as electron Compton length. One expects that the width and length of gamma ray beam is measured using λ as a natural unit. Even width of 100 wavelengths corresponds to 1 Angstrom which corresponds to the size scale of single atom.

Surprise

The real surprise was that gamma rays bend in prisms made from silicon! The discovery was made by a group of scientists working in Ludwig-Maximilians-Universität in Munich [D23, D24, D38]. The group was led by Dietrich Habs. The gamma ray energies where in the range .18-2 MeV. The bending known as refraction was very small using every day standards. The value of the refractive index which gives the ratio c/v for light velocity c to the light velocity v in silicon is $1 + 10^{-9}$ as one learns from another popular article [D24]. When compared to the predictions of the existing theory, the bending was however anomalously large. By the previous argument it should not be even possible to talk about bending.

Dietrich Habs suggests that so called Delbrueck scattering of gamma rays from virtual electron positron pairs created in the electric fields of atoms could explain the result. This scattering would be diffraction (scattering almost totally in forward direction as for light coming through a hole). This cannot however give rise to an effective scattering from a many-atom system unless the gamma ray beam is effectively or in real sense scaled up. The scattering would be still from single atom or even part of single atom. One could of course imagine that atoms themselves have hidden structure analogous to lattice structure but why virtual electron pairs could give rise to it?

In the following I discuss two TGD inspired proposals for how the diffraction that should not occur could occur after all?

Could gamma rays scatter from quarks?

There is another strange anomaly that I discussed for a couple of years ago christened as the incredibly shrinking proton [K53]. It was found that protons charge distribution deviates slightly from the expected one. The TGD inspired explanation was based on the observation that quarks in proton are rather light having masses of 5 and 20 MeV. These correspond to gamma ray energies. Therefore the Compton wave lengths of quarks are also rather long, much longer than the Compton length of proton itself! Parts would be larger than the whole! The explanation for this quantum mystical fact would be that the Compton length corresponds to length scale assignable to color magnetic body of quark. Could it be that the scattering gamma rays see the magnetic bodies of $3 \times 14 = 42$ valence quarks of 14 nucleons of Si nucleus. The regular structure of atomic nucleus as composite of quark magnetic would induce the diffractive pattern. If so, we could do some day nuclear physics and perhaps even study the structure of proton by studying diffraction patterns of gamma rays on nuclei!

Could part of gamma beam transform to large $\hbar$ gamma rays?

Also the hierarchy of Planck constants [K32] comes in mind. Scaling of $\hbar$ for a fixed photon energy scales up the wavelength of gamma ray. Could some fraction of incoming gamma rays suffer a phase transition increasing their Planck constant? The scaling of Planck constant make gamma rays to behave like photons with scaled up wavelength. Also the width of the beam would be zoomed up. As a result the incoming gamma ray beam would see a group of atoms instead of single atom and for a large enough value of Planck constant one could speak of diffraction giving rise to refraction.

For years ago I considered half jokingly the possibility that hierarchy of Planck constants could imply quantum effects in much longer scales than usually [K32]. Diffraction would be a typical quantum effect involving interference. Perhaps even the spots seen sometimes in ordinary camera lense could be analogous to diffractive spots generated by diffraction of large $\hbar$ visible photons through a hole (they should usually appear in the scale of visible wavelength about few microns [K30]. Take this as a joke!

I also proposed that strong classical em fields provide the environment inducing increase of Planck constant at some space-time sheets. The proposal was that Mother Nature is theoretician friendly [K32]. As perturbation expansion in powers of $1/\hbar$ fails, Mama Nature scales up $\hbar$ to make the life of her theorizing children easier, one might say. Strong electric and magnetic fields of atomic nuclei believed by Habs to be behind the diffraction might provide the manner to generate large Planck constant phases and dark matter.

9.6 Water and new physics

In this section the previous ideas are applied in an attempt to understand the very special properties of water.

9.6.1 The 41 anomalies of water

The following list of 41 anomalies of water taken from [D30] should convince the reader about the very special nature of water. The detailed descriptions of the anomalies can be found in [D30] . As a matter fact, the number of anomalies had grown to 63 when I made my last visit to the homepage of Chaplin.

The many anomalies of water need not be all due to the presence of the dark matter. As suggested already fifteen years ago, p-adic length scale hierarchy forces to replace ordinary thermodynamics with a p-adic fractal hierarchy of thermodynamics and this means that one must speak about thermodynamics in a given length scale rather than mere thermodynamics of continuous matter.

Instead of listing just the anomalies I suggest also a possible interpretation based on the assumption that some fraction of protons (and perhaps also OH^- ions) is dark. This hypothesis is motivated by the scattering data suggesting that $H_{1.5}O$ is the proper chemical formula for water in atto-second time scale and explained by assuming that about 1/4 of protons are dark in the experimental situation. It is natural to assume that the increase of temperature or pressure reduces the dark portion. Unless the establishment of equilibrium ratio for dark and ordinary phase is very fast process, water can be regarded as a two-phase system mathematically. A continuous spectrum of metastable forms of water and ice distinguished by the ratio of the densities of ordinary and dark phase is expected. Complex phase diagrams is also a natural outcome.

Dark portion is expected to induce long range correlations affecting melting/boiling/critical points, viscosity, and heats of vaporization and fusion. Anomalous behaviors under the changes of temperature and pressure and anomalies in compressibility and thermal expansivity are expected. Specific heats and transport properties are affected by the presence of dark degrees of freedom, and the coupling of electromagnetic radiation to dark degrees of freedom influences the di-electric properties of water.

In order to systematisize the discussion I have classified the anomalous to different groups.

1. Anomalies suggesting the presence of dark phase inducing long range correlations.

   (a) Water has unusually high melting point.

   (b) Water has unusually high boiling point.

   (c) Water has unusually high critical point.

   (d) Water has unusually high surface tension and can bounce.

   (e) Water has unusually high viscosity.

   (f) Water has unusually high heat of vaporization.

   Comment: The presence of dark portion implies long range correlations and they could help to restore solid/liquid phase, raise the the critical point, increase surface tension, increase viscosity and require more energy to achieve vaporization. The ability to bounce would suggest that dark portion of water -at least near the surface- is in solid phase. Dark water is in rubber-like phase also in the interior below a length scale defined by the length of dark flux tubes.

2. Anomalies related to the effect of temperature increase.

   (a) Water shrinks on melting.

   (b) Water has a high density that increases on heating (up to 3.984°C).

   (c) The number of nearest neighbors increases on melting.

   (d) The number of nearest neighbors increases with temperature.

   (e) Water shows an unusually large viscosity increase but diffusion decrease as the temperature is lowered.

   (f) At low temperatures, the self-diffusion of water increases as the density and pressure increase.

   (g) Water has a low coefficient of expansion (thermal expansivity).

   (h) Water's thermal expansivity reduces increasingly (becoming negative) at low temperatures.

   Comment: The increase of temperature induces shrinking of the flux tubes connecting water molecules in the phase transition reducing Planck constant and brings the molecules closer to each other. This could explain shrinking on melting, the increase of the density in some temperature range above which the normal thermal expansion would win the shrinking tendency, the increase of nearest neighbors on melting and with the increase of temperature. Concerning the shrinking on melting one can however argue that the regular lattice like structure of ice is not that with minimum volume per molecule so that no new physics would be needed unless it is needed to explain why the volume per molecule is not minimum.

The unusually large viscosity increase with reduce temperature would be due to the increase of the large $\hbar$ portion inducing long range correlations. If the diffusion takes place only in the normal phase the anomalous reduction of diffusion could be due to the reduction of the density of the normal phase. Similar explanation applies to the behavior of self-diffusion.

The low value of coefficient of thermal expansion could be understood in terms of the phase transitions reducing the flux tube lengths and bringing the molecules near to each other and thus reducing the normal thermal expansion. At low enough temperatures the expansivity would become negative since this effect would overcome the normal thermal expansion.

3. Anomalies related to the effects of pressure.

   (a) Pressure reduces its melting point (13.35 MPa about 133.5 times the standard atmospheric pressure) gives a melting point of -1°C)
   (b) Pressure reduces the temperature of maximum density.
   (c) D_2O and T_2O differ from H_2O in their physical properties much more than might be expected from their increased mass; e.g. they have increasing temperatures of maximum density (11.185°C and 13.4°C respectively).
   (d) Water's viscosity decreases with pressure (at temperatures below 33°C).

   Comment: The reduction of melting point, temperature of maximum density, and viscosity with pressure could be due to the reduction of the dark portion as pressure increases. Pressure would induce the phase transition reducing the value of Planck constant for the flux tubes connecting water molecules. That the situation is different for D_2O and T_2O could be understood if dark D and T are absent. The question is what happens in the transition to solid phase. The reduction of the density would conform with the idea that the portion of dark phase increases. The reduction of viscosity with pressure would would follow from the reduction of dark phase causing long range correlations.

4. Anomalies related to compressibility.

   (a) Water has unusually low compressibility.
   (b) The compressibility drops as temperature increases down to a minimum at about 46.5°C. Below this temperature, water is easier to compress as the temperature is lowered.

      Comment: The anomalously high compressibility below 46.5°C could be understood if only the standard phase responds to pressure appreciably. In this case the effective density is smaller than the net density and make it easier to compress the water as the temperature is lowered. The increase of temperature would increase the effective density as dark matter is transformed to ordinary one and reduce the compressibility. Above 46.5°C the effect of dark matter would be overcome by the increase of compressibility due to the increase of temperature.
   (c) The speed of sound increases with temperature (up to a maximum at 73°C).

      Comment: The speed of sound is given by the expression

      $$c^2 = \frac{\partial p}{\partial \rho} \ .$$

      Pressure p is essentially the density of thermal energy associate with the ordinary matter. When the fraction of ordinary matter increases the pressure effectively increases and this leads to the increase of c.
   (d) Under high pressure water molecules move further away from each other with increasing pressure.

      Comment: The behavior under increasing high pressure is in conflict with the hypothesis that pressure tends to reduce the portion of dark phase. The question is why the increase of pressure at high enough pressures would induce phase transition increasing the value of Planck constant for the flux tubes connecting the molecules? If the dark matter does not respond to pressure appreciably, the increase of the portion of dark matter might allow the minimization of energy. Does this mean that the work done by the high enough pressure to reduce the volume is larger than the energy needed to induce the tunnelling to the dark phase?

5. Anomalies related to the heat capacity.

   (a) Water has over twice the specific heat capacity of ice or steam.

   (b) The specific heat capacity (C_P and C_V) is unusually high.

   (c) Specific heat capacity C_P has a minimum.

   Comment: The anomalously high heat capacity of water could be understood in terms of dark non-translational degrees of freedom even if the dark phase is rubber-like below the length scale of the dark flux tubes. The energy pumped to the system would go to these degrees of freedom. The small heat capacity of solid phase would suggest that the freezing means also freezing of these degrees of freedom meaning the reduction of the contribution to heat capacity.

6. Anomalies related to phase transitions

   (a) Supercooled water has two phases and a second critical point at about -91°C.

   (b) Liquid water may be supercooled, in tiny droplets, down to about -70°C. It may also be produced from glassy amorphous ice between -123°C and - 149°C and may coexist with cubic ice up to -63°C.

   (c) Solid water exists in a wider variety of stable (and metastable) crystal and amorphous structures than other materials.

   (d) The heat of fusion of water with temperature exhibits a maximum at -17°C.

   Comment: The presence of both dark and ordinary phase with varying ratio of densities could help to understand the richness of the structures below freezing point. For instance, one can imagine that either the ordinary or dark phase is super-cooled and the other freezes.

7. Anomalies of solutions of water.

   (a) Solutes have varying effects on properties such as density and viscosity.

   (b) None of its solutions even approach thermodynamic ideality; even D_2O in H_2O is not ideal.

   (c) The solubilities of non-polar gases in water decrease with temperature to a minimum and then rise.

   Comment: The different interactions of solutes with the dark phase could explain these findings. For instance, the probability that the presence of solute induces a phase transition reducing the portion of the dark phase could depend on solute. The decrease of the solubilities of non-polar gases in water with temperature could be due to the fact that the solubility is at low temperatures basically due to the presence of the dark phase. At higher temperatures higher thermal energies of the solute molecules would increase the solubility.

8. Anomalies in transport properties.

   (a) NMR spin-lattice relaxation time is very short at low temperatures.

   Comment: The transfer of magnetic energy to the dark degrees of freedom could dominate the relaxation process. If synchrotron Bose-Einstein condensates are present in dark degrees of freedom this might make sense.

   (b) Hot water may freeze faster than cold water; the Mpemba effect [D13] . For instance, water sample in 100 C freezes faster than that in 35 C.

   Comment: This effect seems to be in conflict with thermodynamics and remains poorly understood. The possibility of having continuum of metastable two-phase systems suggests a possible solution to the mystery. The freezing of the dark portion of water should occur slower than the freezing of the ordinary portion since the heat transfer rate is expected to be lower a larger value of Planck constant. The very naive just-for-definiteness estimate is that the transfer rate for energy to the cold system is inversely proportional to $1/\hbar$. If the formation of dark phase is a slow process as compared to the transfer of energy to the cold phase, the freezing of hot water would lead to a metastable ice consisting mostly of ordinary water molecules and takes place faster than the freezing of cold water already containing the slowly freezing dark portion.

(c) Proton and hydroxide ion mobilities are anomalously fast in an electric field.

Comment: Mobility is of form $a\tau$, where a the acceleration a in the electric field times the characteristic time τ for motion without collisions. If part of protons move along dark flux tubes this time is longer. The high mobility of OH_- ions would suggest that also these can be in dark phase.

(d) The electrical conductivity of water rises to a maximum at about 230°C and then falls.

Comment: Electrical conductivity is closely related to mobility so that the same argument applies.

(e) The thermal conductivity of water is high and rises to a maximum at about 130°C.

Comment: The anomalously high thermal conductivity could be due to the motion of heat carriers along dark flux tubes with low dissipation.

(f) Warm water vibrates longer than cold water.

Comment: This could be due to the faster transfer of vibrational energy to the dark vibrational of magnetic degrees of freedom. If the number of these degrees of freedom is higher than the number of ordinary degrees of freedom, one can understand also the anomalously high heat capacity. Vibration could continue in dark degrees of freedom in which case the effect would be apparent. If its only the ordinary water which vibrates in the original situation then equipartition of energy with dark degrees of freedom implies apparent dissipation.

9. Anomalous electromagnetic properties of water.

(a) X-ray diffraction shows an unusually detailed structure.

Comment: This would not be surprising if two phases with possibly varying ratio are present. For instance, the different X-ray diffraction patterns for water obtained by a rapid freezing from high and low temperatures could serve as a test for the proposed explanation of Mpemba effect.

(b) The dielectric constant is high and behaves anomalously with temperature.

Comment: This could relate to the interaction of photons with dark portion of water. Dielectric constant characterizes the coupling of radiation to oscillatory degrees of freedom and is sum of terms proportional to $1/(\omega^2 - \omega_i^2)$, where ω_i is resonance frequency. If the resonance frequencies ω_i scale as $1/\hbar$, dark portion gives a larger contribution at frequencies $\omega < \omega_i$. In particular the static dielectric constant increases.

(c) The refractive index of water has a maximum value at just below 0°C.

Comment: It is not quite clear whether this maximum corresponds to room pressure or appears quite generally. Let us assume the first option. In any case the dependence of the freezing temperature on pressure is very weak. The maximal interaction with the dark portion of water at freezing point combined with the above argument would predict that refractive index increases down to the freezing point. The reduction of the density at freezing point would reduce the refractive index since dynamic susceptibility is proportional to the density of atom so that a maximum would be the outcome.

These examples might serve as a motivation for an attempt to build a more detailed model for the dark portion of water. The model to be discussed was one of the first attempts to understand the implications of the idea about hierarchy of Planck constants. Since five years have passed is badly in need of updating.

9.6.2 The model

Networks of directed hydrogen bonds $H - O - H \cdots OH_2$ with positively charged H acting as a binding unit between negatively charged O (donor) and OH_2 (acceptor) bonds explaining clustering of water molecules can be used to explain qualitatively many of the anomalies at least qualitatively [D30] .

The anomaly giving evidence for anomalous nuclear physics is that the physical properties D_2O and T_2O differ much more from H_2O than one might expect on basis of increased masses of water molecules. This suggests that dark protons could be responsible for the anomalies. That heavy water in large concentrations acts as a poison is consistent with the view that the macroscopic quantum phase of dark protons is responsible for the special biological role of water.

What proton darkness could mean?

In the experimental situation one fourth of protons of water are not seen in neither electron nor neutron scattering in atto-second time scale which translates 3 Angstrom wavelength scale suggesting that in both cases diffraction scattering is in question. This of course does not mean that the fraction of dark protons is always 1/4 and it is indeed natural to assume that it is reduced at higher temperatures. Both nuclear strong interactions and magnetic scattering contribute to the diffraction which is sensitive to the intra-atomic distances. The minimal conclusion is that the protons form a separate phase with inter-proton distance sufficiently different from that between water molecules and are not seen in neutron and electron diffraction in the atto-second time scale at which protons of water molecule are visible. The stronger conclusion is that they are dark with respect to nuclear strong interactions.

The previous considerations inspired by the model of nuclei as nuclear strings suggests possible explanations.

1. Hydrogen atoms form analogs of nuclear strings connected by color bonds.

2. Nuclear protons form super-nuclei connected by dark color bonds or belong to such super-nuclei (possibly consisting of ^4He nuclei). If color bonds are negatively charged, closed nuclear strings of this kind are neutral and not visible in electron scattering: this assumption is however un-necessarily strong for invisibility in diffractive scattering in atto-second time scale. Only the field bodies of proton carrying weak and color fields could be dark and electromagnetic field body has ordinary value of Planck constant so that dark protons could give rise to ordinary hydrogen atoms.

Could also the color flux tubes connecting quarks inside dark protons be dark?

The first option is that only the color flux tubes connecting protons are dark and of length of atomic size scale. The second possibility is that also the color flux tubes connecting quarks are dark and have length of order atomic size scale. Dark nucleons could be visualized as strings formed from three quarks of order atom size scale connected by color flux tubes. The generalization of the nuclear string model leads to a model of dark nucleon discussed in detail [L2, K41, K90] , [L2] . Dark nucleons would in turn form dark nuclei as string like objects.

The amazing finding is that the states of nucleon assumed to be neutral (for definiteness) are in one-one-correspondence with DNA, RNA, mRNA, tRNA and amino-acids and that a physically natural pairing of DNA codons and amino-acids exists and consistent with vertebrate genetic code. Same applies also to nucleons having the charge of proton. The nuclear strings formed from either dark neutrons or dark protons could in principle realize genetic code. This realization would be more fundamental than the usual chemical realization and would force to modify profoundly the ideas about prebiotic evolution. The prebiotic evolution could be evolution of water and the recent evolution could involve genetic engineering based on virtual world experimentation with the dark variant variant of the genetic apparatus. The minimum requirement would be the transcription of at dark DNA defined by nuclear strings to ordinary DNA. Dark nuclear strings could be able to diffuse without difficulties through cell membranes and the transcription of the dark genes to ordinary ones followed by gluing and pasting to genome could make possible the genetic engineering at the level of germ cells.

Another natural hypothesis is that the magnetic bodies assignable to the nuclear strings are responsible for water memory [K41] and that the mechanism of water memory relies on the mimicry of biologically active molecules by dark proton strings. The frequencies involved with water memory are low and nothing to do with molecular energy levels. This is consistent with the identification as cyclotron frequencies so that it would be enough to mimic only the cyclotron spectrum. The mechanism would be similar to that of entrainment of brain to external frequencies and based on the variation of the thickness of magnetic flux tubes or sheets inducing the change of magnetic field and cyclotron frequency. One could perhaps say that magnetic bodies of dark genes as living creatures with some amount of intelligence and ability to planned actions. The evolution of cells up to the neurons of cortex could be accompanied by the evolution of the magnetic bodies of dark nuclear strings realized as the emergence of higher values of Planck constant.

Concerning the mechanism of the debated homeopathic effect itself the situation remains unclear. Homeopathic remedy is obtained by a repeated dilution and succussion of the solution containing the molecules causing the symptoms of the disease [K41] . If the cyclotron frequencies of the magnetic body alone are responsible for the biological effect, one can wonder why the homeopathic remedy does not have the same undesired effects as the original molecule. A more reasonable hypothesis is that the cyclotron frequency spectrums serves only as a signature of the molecule and the homeopathic remedy only activates the immune system of the organism by cheating it to believe that the undesired molecules are present. The immune system is known to be subject to very fast genetic evolution, and dark nuclear strings forming representations of biologically active molecules and dark genome could be actively involved with this evolution.

What inspires to take these speculations more than as a poor quality entertainment is that the recent findings of the group led by HIV Nobelist Montagnier related to water memory provide support for the hypothesis that a nonstandard realization of genetic code indeed exists [I5] . These findings will be discussed later in this section.

Model for super-nuclei formed from dark protons

Dark protons could form super nuclei with nucleons connected by dark color bonds with $\hbar = r\hbar_0$ with $r = 2^{k_d}$, $k_d = 151 - 127 = 24$. The large distance between protons would eliminate isospin dependent strong force so that multi-proton states are indeed possible. The interpretation would be that nuclear p-adic length scale is zoomed up to $L(113+24 = 137) \sim .78$ Angstroms. Dark color bonds could also connect different nuclei. The earlier hypothesis $r = 2^{11k}$ encourages to consider also $k_d = 22$, which is also one of the favored dark scalings allowed by Mersenne hypothesis $(22 = 18 + 4 = 107 - 89 + 167 - 163)$ giving p-adic scale .39 Angstroms.

The predictions of the model for bond energy depend on the transformation properties of E_s under the scaling of $\hbar$.

1. For small perturbations harmonic oscillator approximation $V \propto kR^2/2 \propto \alpha R^2/2$ makes sense and is invariant under the scalings $\alpha_s \rightarrow \alpha_s/r$ and $R \rightarrow \sqrt{r}R$ -at least if the scalings are not too large. Bonds with different values of Planck constant have nearly identical energies, which would be indeed consistent with the idea about criticality against the change of Planck constant.

   One can arrive the same conclusion follows also in different manner. The parameter ω corresponds to a quantity of form $\omega = v/L$, where L is a characteristic length scale and v a characteristic velocity. The scaling law of homeopathy [K41] would suggest the dependence $v = c/\sqrt{r}$ and $L \propto \sqrt{r}L$ giving predicting that energy is invariant.

   The result also conforms with the idea that classical perturbative theory does not involve Planck constant. This behavior does not however allow to identify hydrogen as color bonds since the resulting bond energies would be in MeV range.

2. The interpretation of E_s as color Coulombic potential energy α_s/R would suggest that E_s behaves under scaling like the binding energy of hydrogen atom ($1/r^2$ scaling). This interpretation implies non-perturbative effects since in semiclassical approximation energy should not depend on r. Color force is non-perturbative so that one can defend this assumption.

   (a) For $k_d = 24$ E_s would be about .12 eV and considerably lower than the nominal energy of the hydrogen bond.

(b) For $k_d = 22$ one would obtain energy .48 eV. This energy is same as the universal metabolic energy quantum so that the basic metabolic processes might involve transitions dark-ordinary transition for protons. This would however suggest that the length of color bond is same as that of hydrogen bond so that the protons in question would not be invisible in diffraction in atto-second time scale. The interpretation of color bonds between atoms as hydrogen bonds is much more attractive. Of course, for large values of Planck the invariance of oscillator spectrum implies very large force constant so that the color bond would become very rigid.

These two interpretations are not contradictory if one interprets the non-perturbative contribution to the color binding energy as an additional constant contribution to the harmonic oscillator Hamiltonian which does not contribute the spectrum of excitations energies but only to the ground state energy.

The notion of flux tube state

An approach based more heavily on first principles that the above order of magnitude estimates is inspired by two steps of progress several years after these speculations.

1. Weak form of electric-magnetic duality

The weak form of electric magnetic duality led to an identification of a concrete mechanism of electroweak screening based on the pairing of homological Kähler magnetic monopoles formed by fermion wormhole throats with oppositely magnetically charged wormhole throats carrying quantum numbers of neutrino pair and screening the weak isospin and leaving only electromagnetic charge.

1. The size scale of the Kähler magnetic flux tubes connecting the magnetic monopoles would be of order intermediate gauge boson Compton length. For dark variants of elementary fermions it would be scaled up by $\sqrt{\hbar/\hbar_0}$. The new weak physics involving long range weak fields would be associated with magnetic flux tube like structures. Same conclusion applies also to new QCD type physics since also color confinement would be accompanied Kähler magnetic confinement. This allows to pose very strong restrictions on the models. For instance, it is quite possible that the notion of neutrino atom does not make sense expect if one can assume that the dark quarks feed their weak Z^0 gauge fluxes through a spherically symmetric flux collection of radial flux tubes allowing Coulombic Z^0 gauge potential as an approximate representation inside the radius defined by the length of the flux tubes.

2. It is important to notice that the screening leaves the vectorial coupling to classical Z^0 field proportional to $sin^2(\theta_W)Q_{em}$. This could have non-trivial physical implications perhaps allowing to kill the model.

   (a) For space-time surfaces near vacuum extremals the classical Z^0 fields are strong due to the condition that the induced Kähler field is very weak. More explicitly, from the equations for classical induced gauge fields in terms of Kähler form and classical Z^0 field [L1] , [L1]

$$\gamma = 3J - \frac{p}{2}Z^0 \quad , \quad Q_Z = I_L^3 - pQ_{em} \quad , \quad p = sin^2(\theta_W) \tag{9.6.1}$$

   it follows that for the vacuum extremals the part of the classical electro-weak force proportional to the electromagnetic charge vanishes for $p = 0$ so that only the left-handed couplings to the weak gauge bosons remain. The vanishing of induced Kähler form gives

$$Z^0 = -\frac{2}{p}\gamma \ . \tag{9.6.2}$$

   The condition implies very large effective coupling to the classical electromagnetic field since electromagnetic charge is effectively replaced with

$$Q_{em,eff} = Q_{em} - \frac{2}{p}(I_L^3 - pQ_{em}) \ . \qquad (9.6.3)$$

(b) The proposed model for cell membrane as a Josephson junction relies on almost vacuum extremals and dark nuclei in the sense that the weak space-time sheet associated with the nuclei of biological important ions (at least) are dark [K69] . It is is assumed that quarks are dark in the length scale considered so that also their weak isospin remains unscreened. In the case of nuclei this means that there is contribution from the vectorial part of weak isospin given by $(Z-N)/4$ proportional to the difference of proton number and neutron number. The dominating contribution comes from Q_{em} term for heavier nuclei. It is essential that weak space-time sheets of electrons are assumed to be ordinary.

(c) One can ask whether the the nuclei could be ordinary nuclei. If so, one must still assume that the electrons of the nuclei do not couple to the classical fields assignable to the cell membrane space-time sheet since without this assumption the coupling to Z^0 field would be proportional to the total em charge of the ion rather than nuclear em charge. It is difficult to justify this assumption. In any case, for this option I_L^3 contribution would be totally absent. This affects the effective couplings of biologically important ions to the membrane potential somewhat and modifies the nice quantitative predictions of the model of photoreceptors predicting correctly the frequencies of visible light with maximal response.

2. The notion of flux tube state

The TGD inspired explanation for the finding that the measurement of Lamb shift for muonic hydrogen atom gives proton radius which is 4 per cent smaller than that deducible from ordinary hydrogen atom led to the notion of flux tube state in which muon or electric is confined inside flux tube [K52] . In non-relativistic approximation based on Schrödinger equation, the model leads to wave functions expressible in terms of Airy and "Bairy" functions and WKB approximation allows to deduce an estimate for the energy eigenvalue spectrum. This model works as such also as a model for flux tubes states in which also classical electroweak and and color fields are involved. Color holonomy is quite generally Abelian for classical color fields and for 2-D CP_2 projection electroweak fields are also Abelian so that the model is expected to be mathematically reasonably simple even when induced spinors are assumed.

The concept of flux tube state is very general and allow to model at least some chemical bonds. In particular, valence bonds might allow description as flux tube states of valence electrons. Hydrogen bonds are responsible for the clustering of water molecules and an obvious question is whether these bonds could be modeled as dark flux tube states of valence electrons. The model is testable since one can predict the energy spectrum of excited states for given thickness of the flux tube and the value of electric flux through it. Also the flux tube states of say electrons assignable to the magnetic flux tubes assumed to connect DNA nucleotides and lipids of cell membrane in the model of DNA as topological quantum computer [K31] could be relevant.

Two kinds of bonds are predicted

Duppose that dark bonds are associated with the electro-magnetic field body. If classical Z^0 field vanishes, em field is proportional to Kähler field as are also the components of the classical color field. The bonds involving classical color gauge fields could have quark and antiquark at the opposite ends of the flux tube as the source of the color gauge field. This is indeed assumed in the model of DNA as topological quantum computer [K31] .

If one wants vanishing or very weak color gauge fields, one must allow almost vacuum extremals. This implies that classical Z^0 force is strong and the situation assumed to prevail for the cell membrane would hold also for hydrogen bonds. For almost vacuum extremals the ratio of electric and Z^0 fluxes is so small- of order 1/50 for the small value of Weinberg angle $p = .0295$ (rather than $p \simeq .23$) if appearing as the parameter of the model. The molecule can serve as the source of classical Z^0 and electromagnetic fields in two manners.

1. The almost vacuum flux tubes could have many neutrino state and its conjugate at the opposite ends of the flux tube acting as the source of the classical Z^0 field. This kind of flux tubes would traverse through the cell membrane.

2. The molecule would be accompanied by two kinds of flux tubes. Some of them would be almost vacuum extremals carrying an electric flux much smaller than elementary charge e. Some of them would be accompanied by very weak Z^0 field and electromagnetic field plus color gauge fields generated by the above mechanism. These flux tubes would connect cell membrane and genome.

Two kinds of hydrogen bonds

There is experimental evidence for two different hydrogen bonds. Li and Ross represent experimental evidence for two kinds of hydrogen bonds in ice in an article published in Nature 1993 [D70]. The ratio of the force constants K associated with the bonds is 1:2.

The proposed scaling law $\omega \rightarrow \omega/r$ predicts $\omega \propto 1/r$ so that $k_d \rightarrow k_d + 1$ would explain the reduction of the force constant by factor 1/2. The presence of two kinds of bonds could be also seen as a reflection of quantum criticality against change of Planck constant.

Can one understand the finding in terms of dark color bonds?

1. The model is consistent with the identification of the two bonds in terms different values of Planck constant. The proposed scaling law for ω predicts $\omega \propto 1/r$ so that $k_d \rightarrow k_d + 1$ would explain the reduction of the force constant by factor 1/2. above described general model for which bond energy contains perturbative harmonic oscillator contribution and non-perturbative Coulombic contribution.

2. The identification of hydrogen bond as dark color bond is however questionable. If bond energy contains a color binding energy scaling as $1/r^2$ contributing only constant shift to the harmonic oscillator Hamiltonian, the behavior of the force constant is consistent with the model. If one assumes that the harmonic oscillator spectrum remains invariant under large scalings of $\hbar$, the force constant becomes extremely strong and the color bond would be by a factor r^2 more rigid than hydrogen bond if one takes seriously the proposed estimates for the value of r. The alternative interpretation would be in terms of almost vacuum extremal property reducing the force constant to a very small value already from the beginning.

The possibility to divide the bonds to two kinds of bonds in an arbitrary manner brings in a large ground state degeneracy given by $D = 16!/(8!)^2$ unless additional symmetries are assumed and give for the system spin glass like character and explain large number of different amorphous phases for ice [D30] . This degeneracy would also make possible information storage and provide water with memory.

Hydrogen bonds as color bonds between nuclei?

The original hypothesis was that there are two kinds of hydrogen bonds: dark and "ordinary". The finding that the estimate for the energy of dark nuclear color bond with $k_d = 22$ equals to the energy of typical hydrogen bond raises the question whether all hydrogen bonds are associated with color bonds between nuclei. Color bond would bind the proton to electronegative nucleus and this would lead to to the formation of hydrogen bond at the level of valence electrons as hydrogen donates its electron to the electronegative atom. The electronic contribution would explain the variation of the bond energy.

If hydrogen bonds connect H-atom to O-atom to acceptor nucleus, if E_s for p-O bond is same as for p-n color bond, and if color bonds are dark with $k_d = 22$, the bond energy $E_s = .5$ eV. Besides this one must assume that the oscillator energy is very small and comparable to the energy of hydrogen bond - this could be due to almost vacuum extremal property.

Dark -possibly (almost unavoidably) colored or weakly charged- bonds could serve as a prerequisite for the formation of electronic parts of hydrogen bonds and could be associated also with other molecular bonds so that dark nuclear physics might be essential part of molecular physics. Dark color bonds could be also charged which brings in additional exotic effects. The long range order of hydrogen bonded liquids could due to the ordinary hydrogen bonds. An interesting question is whether nuclear color bonds could be responsible for the long range order of all liquids. If so dark nuclear physics would be also crucial for the understanding of the condensed matter.

In the case of water the presence of dark color bonds between dark protons would bring in additional long range order in length scale of order 10 Angstrom characteristic for DNA transversal scale: also hydrogen bonds play a crucial role in DNA double strand. Two kinds of bond networks could allow to understand why water is so different from other molecular liquids containing also hydrogen atoms and the long range order of water molecule clusters would reflect basically the long range order of two kinds of dark nuclei.

Recall that the model for dark nucleons predicts that nucleon states can be grouped to states in one-one correspondence with DNA, RNA, tRNA, and amino-acids and that the degeneracies of the vertebrate genetic code are predicted correctly. This led to suggestion that genetic code is realized already at the level of dark nuclei consisting of sequences of neutrons [L2, K90] , [L2] . Neutrons were assumed in order to achieve stability and could be replaced with protons.

Tedrahedral and icosahedral clusters of water molecules and dark color bonds

Water molecules form both tetrahedral and icosahedral clusters. ^4He corresponds to tetrahedral symmetry so that tetrahedral cluster could be the condensed matter counterpart of ^4He. It the nuclear string model nuclear strings consist of maximum number of ^4He nuclei themselves closed strings in shorter length scale.

The p-adic length scales associated with ^4He nuclei and nuclear string are $k = 116$ and $k = 127$ The color bond between ^4He units has $E_s = .2$ MeV and $r = 2^{22}$ would give by scaling $E_s = .05$ eV which is the already familiar energy associated with cell membrane potential at the threshold for the nerve pulse generation. The binding energy associated with a string formed by n tetrahedral clusters would be n^2E_s. This observation raises the question whether the neural firing is accompanied by the re-organization of strings formed by the tetrahedral clusters and possibly responsible for a representation of information and water memory.

The icosahedral model [D30] for water clusters assumes that 20 tetrahedral clusters, each of them containing 14 molecules, combine to form icosahedral clusters containing 280 water molecules. Concerning the explanation of anomalies, the key observation is that icosahedral clusters have a smaller volume per water molecule than tetrahedral clusters but cannot form a lattice structure.

The number 20 for the dark magic dark nuclei forming the icosahedron is also a magic number and a possible interpretation for tetrahedral and icosahedral water clusters would be as magic super-nuclei and the prediction would be that binding energy behaves as n^2E_s rather than being just the sum of the binding energies of hydrogen bonds (nE_s).

It is interesting to compare this model with the model for hexagonal ice which assumes four hydrogen bonds per water molecule: for two of them the molecule acts as a donor and for two of them as an acceptor. Each water molecule in the vertices of a tetrahedron containing 14 hydrogen atoms has a hydrogen bond to a water molecule in the interior, each of which have 3 hydrogen bonds to molecules at the middle points of the edges of the tetrahedron. This makes 16 hydrogen bonds altogether. If all of them are of first type with bonding energy $E_s = .5$ eV and if the bond network is connected one would obtain total bond energy equal to $n^2E_s = 258 \times .5$ eV rather than only $nE_s = 16 \times .5$ eV. Bonds of second type would have no role in the model.

Tedrahedral and icosahedral clusters and dark electrons

An interesting question is whether one could interpret tetrahedral and icosahedral symmetries in terms of symmetries of the singular coverings or factor spaces of CD. This does not seem to be the case.

1. One cannot understand discrete molecular symmetries for factor space-space option since the symmetry related points of CD would correspond to one and same space-time point.

2. For the option allowing only singular coverings of $CD \times CP_2$ interpreted in terms of many-valuedness of the time derivatives of the imbedding space coordinates as functions of canonical momentum densities this interpretation is not possible.

3. One can also consider the possibility that the singular coverings are over $(CD/G_a)\times(CP_2/G_b)$ rather than $CD \times CP_2$. This would predict Planck constant to be of form $r = n_a n_b$, with $n_a = 3$ for tetrahedral clusters and $n_a = 5$ for icosahedral clusters. n_a and n_b would correspond to the orders of maximal cyclic subgroups of the corresponding symmetry groups. There would be a deviation from the simplest proposal for preferred Planck constants. This option would require space-time surfaces to have exact discrete symmetries and this does not look plausible.

Note that synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron.

It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

9.6.3 Further comments on 41 anomalies

Some clarifying general comments -now in more standard conceptual framework- about the anomalies are in order. Quite generally, it seems that it is the presence of new degrees of freedom, the presence of icosahedral clusters, and possibly also macroscopic quantum coherence of dark matter, which are responsible for the peculiar properties of water.

The hydrogen bonds assigned to tetrahedral and icosahedral clusters should be same so that if the hydrogen bonds are assignable to dark protons this is the case for all clusters. Perhaps the number of dark protons and -perhaps equivalently- hydrogen bonds per volume is what distinguishes between these clusters and that the disappearance of dark protons leads to the disappearance of hydrogen bonds. Since it is quite possible that no new physics of proposed kind is involved, the following the explanation of anomalies uses only the notions of icosahedral and tetrahedral clusters and dark protons are mentioned only in passing.

1. Anomalies relating to the presence of icosahedral clusters

Icosahedral water clusters have a better packing ratio than tetrahedral lattice and thus correspond to a larger density. They also minimize energy but cannot cannot form a lattice [D30] .

1. This explains the unusually high melting point, boiling point, critical point, surface tension, viscosity, heat of vaporization, shrinking on melting, high density increasing on heating, increase of the number of nearest neighbors in melting and with temperature. It is also possible to understand why X-ray diffraction shows an unusually detailed structure.

   The presence of icosahedral clusters allows to understand why liquid water can be super-cooled, and why the distances of water molecules increase under high pressure. The spin glass degeneracy implied by dark and ordinary hydrogen bonds could explain why ice has many glassy amorphous phases. The two phases of super-cooled water could correspond to the binary degree of freedom brought in by two different hydrogen bonds. For the first phase both hydrogen atoms of a given water molecule would be either dark or ordinary. For the second phase the first hydrogen atom would be dark and second one ordinary.

   Since icosahedral clusters have lower energy than a piece of ice of same size, they tend to super-cool and this slows down the transition to the solid phase. The reason why hot water cools faster would be that the number of icosahedral clusters is smaller: if cooling is carried with a sufficient efficiency icosahedral clusters do not form.

2. Pressure can be visualized as a particle bombardment of water clusters tending to reduce their volume. The collisions with particles can induce local transitions of icosahedral structures to tetrahedral structures with a larger specific volume and energy. This would explain the low compressibility of water and why pressure reduces melting point and the temperature of maximum density and viscosity.

3. The increase of temperature is expected to reduce the number of icosahedral clusters so that the effect of pressure on these clusters is not so large. This explains the increase of compressibility with temperature below 46.5°C. The fact that the collapse of icosahedral clusters opposes the usual thermal expansion is consistent with the low thermal expansivity as well as the change of sign of expansivity near melting point. Since the square of sound velocity is inversely proportional to compressibility and density, also the increase of speed of sound with temperature can be understood.

2. The presence of dark degrees of freedom and spin glass degeneracy

The presence of dark degrees of freedom and the degeneracy of dark nucleus ground states could explain the high specific heat capacity of water. The reduction of dark matter degrees of freedom for ice and steam would explain why water has over twice the specific heat capacity of ice or steam. The possibility to relax by dissipating energy to the dark matter degrees of freedom would explain the short spin-lattice relaxation time. The fact that cold water has more degrees of freedom explains why warm water vibrates longer than cold water.

Also the high thermal and electric conductivity of water could be understood. The so called Grotthuss [I11] [D30] explaining OH_- and H_+ mobilities (related closely to conductivities) is based on hopping of electron of OH_- and H_+ in the network formed by hydrogen bonds and generalizes to the recent case. The reduction of conductivity with temperature would be due to the storage of the transferred energy/capture of charge carriers to the water molecule clusters.

3. Macroscopic quantum coherence

The high value of dielectric constant could derive from the fact that dark nuclei and super-nuclei are quantum coherent in a rather long length scale. For curl free electric fields potential difference must be same along space-time sheets of matter and dark matter. The synchronous quantum coherent collective motion of dark protons (and possible dark electrons) in an oscillating external electric field generates dark photon laser beams (it is not clear yet whether these dark laser beams are actually ordinary laser beams) de-cohering to ordinary photons and yield a large dynamical polarization. As the temperature is lowered the effect becomes stronger.

9.6.4 Genes and water memory

After long time I had opportunity to read a beautiful experimental article about experimental biology. Yolene Thomas, who worked with Benveniste, kindly sent the article to me. The freely loadable article is *Electromagnetic Signals Are Produced by Aqueous Nanostructures Derived from Bacterial DNA Sequences* by Luc Montagnier, Jamal Aissa, Stephane Ferris, Jean-Luc Montagnier, and Claude Lavall'e published in the journal Interdiscip. Sci. Comput. Life Sci. (2009) [I5] .

Basic findings at cell level

I try to list the essential points of the article. Apologies for biologists: I am not a specialist.

1. Certain pathogenic micro-organisms are objects of the study. The bacteria Mycoplasma Pirum and E. Choli belong to the targets of the study. The motivating observation was that some procedures aimed at sterilizing biological fluids can yield under some conditions the infectious micro-organism which was present before the filtration and absent immediately after it. For instance, one filtrates a culture of human lymphocytes infected by M. Pirum, which has infected human lymphocytes to make it sterile. The filters used have 100 nm and 20 nm porosities. M. Pirum has size of 300 nm so that apparently sterile fluids results. However if this fluid is incubated with a mycoplasma negative culture of human lymphocytes, mycoplasma re-appears within 2 or 3 weeks! This sounds mysterious. Same happens as 20 nm filtration is applied to a a minor infective fraction of HIV, whose viral particles have size in the range 100-120 nm.

2. These findings motivated a study of the filtrates and it was discovered that they have a capacity to produce low frequency electromagnetic waves with frequencies in good approximation coming as the first three harmonics of kHz frequency, which by the way plays also a

central role in neural synchrony. What sounds mysterious is that the effect appeared after appropriate dilutions with water: positive dilution fraction varied between 10^{-7} and 10^{-12}. The uninfected eukaryotic cells used as controls did not show the emission. These signals appeared for both M. Pirum and E. Choli but for M. Pirum a filtration using 20 nm filter canceled the effect. Hence it seems that the nano-structures in question have size between 20 and 100 nm in this case.

A resonance phenomenon depending on excitation by the electromagnetic waves is suggested as an underlying mechanism. Stochastic resonance familiar to physicists suggests itself and also I have discussed it while developing ideas about quantum brain [K70] . The proposed explanation for the necessity of the dilution could be kind of self-inhibition. Maybe a gel like phase which does not emit radiation is present in sufficiently low dilution but is destroyed in high dilutions after which emission begins. Note that the gel phase would not be present in healthy tissue. Also a destructive interference of radiation emitted by several sources can be imagined.

3. Also a cross talk between dilutions was discovered. The experiment involved two tubes. Donor tube was at a low dilution of E. Choli and "silent" (and carrying gel like phase if the above conjecture is right). Receiver tube was in high dilution (dilution fraction 10^{-9}) and "loud". Both tubes were placed in mu-metal box for 24 hours at room temperature. Both tubes were silent after his. After a further dilution made for the receiver tube it became loud again. This could be understood in terms of the formation of gel like phase in which the radiation does not take place. The effect disappeared when one interposed a sheath of mu-metal between the tubes. Emission of similar signals was observed for many other bacterial specials, all pathogenic. The transfer occurred only between identical bacterial species which suggests that the signals and possibly also frequencies are characteristic for the species and possibly code for DNA sequences characterizing the species.

4. A further surprising finding was that the signal appeared in dilution which was always the same irrespective of what was the original dilution.

Experimentation at gene level

The next step in experimentation was performed at gene level.

1. The killing of bacteria did not cancel the emission in appropriate dilutions unless the genetic material was destroyed. It turned out that the genetic material extracted from the bacteria filtered and diluted with water produced also an emission for sufficiently high dilutions.

2. The filtration step was essential for the emission also now. The filtration for 100 nm did not retain DNA which was indeed present in the filtrate. That effect occurred suggests that filtration destroyed a gel like structure inhibiting the effect. When 20 nm filtration was used the effect disappeared which suggests that the size of the structure was in the range 20-100 nm.

3. After the treatment by DNAse enzyme inducing splitting of DNA to pieces the emission was absent. The treatment of DNA solution by restriction enzyme acting on many sites of DNA did not suppress the emission suggesting that the emission is linked with rather short sequences or with rare sequences.

4. The fact that pathogenic bacteria produce the emission but not "good" bacteria suggests that effect is caused by some specific gene. It was found that single gene - adhesin responsible for the adhesion of mycoplasma to human cells- was responsible for the effect. When the cloned gene was attached to two plasmids and the E. Choli DNA was transformed with the either plasmid, the emission was produced.

Some consequences

The findings could have rather interesting consequences.

1. The refinement of the analysis could make possible diagnostics of various diseases and suggests bacterial origin of diseases like Alzheimer disease, Parkinson disease, Multiple Sclerosis and Rheumatoid Arthritis since the emission signal could serve as a signature of the gene causing the disease. The signal can be detected also from RNA viruses such as HIV, influenza virus A, and Hepatitis C virus.

2. Emission could also play key role in the mechanism of adhesion to human cells making possible the infection perhaps acting as a kind of password.

The results are rather impressive. Some strongly conditioned skeptic might have already stopped reading after encountering the word "dilution" and associating it with a word which no skeptic scientist in his right mind should not say aloud: "homeopathy"! By reading carefully what I wrote above, it is easy to discover that the experimenters unashamedly manufactured a homeopathic remedy out of the filtrate! And the motivating finding was that although filtrate should not have contained the bacteria, they (according to authors), or at least the effects caused by them, appeared within weeks to it! This is of course impossible in the word of skeptic.

The next reaction of the skeptic is of course that this is fraud or the experimenters are miserable crackpots. Amusingly, one of the miserable crackpots is Nobelist Luc Montagnier, whose research group discovered AIDS virus.

How TGD could explain the findings?

Let us leave the raging skeptics for a moment and sketch possible explanations in TGD framework.

1. Skeptic would argue that the filtration allowed a small portion of infected cells to leak through the filter. Many-sheeted space-time suggests a science fictive variant of this explanation. During filtration part of the infected cells is "dropped" to large space-time sheets and diffused back to the original space-time sheets during the next week. This would explain why the micro-organisms were regenerated within few weeks. Same mechanism could work for ordinary molecules and explain homeopathy. This can be tested: look whether the molecules return back to the the diluted solution in the case of a homeopathic remedy.

2. If no cells remain in the filtrate, something really miraculous looking events are required to make possible the regeneration of the effects serving as the presence of cells. This even in the case that DNA fragments remain in the filtrate.

   (a) The minimum option is that the presence of these structures contained only the relevant information about the infecting bacteria and this information coded in terms of frequencies was enough to induce the signatures of the infection as a kind of molecular conditioning. Experimentalists can probably immediately answer whether this can be the case.

   (b) The most radical option is that the infecting bacteria were actually regenerated as experimenters claim! The information about their DNA was in some form present and was transcribed to DNA and/or RNA, which in turn transformed to proteins. Maybe the small fragment of DNA (adhesin) and this information should have been enough to regenerate the DNA of the bacterium and bacterium itself. A test for this hypothesis is whether the mere nanoparticles left from the DNA preparation to the filtrate can induce the regeneration of infecting molecules.

The notion of magnetic body carrying dark matter quantum controlling living matter forms the basic element of TGD inspired model of quantum biology and suggests a more concrete model. The discovery of nanotubes connecting cells with distance up to 300 μ [I10] provides experimental support for the notion .

1. If the matter at given layer of the onion-like structure formed by magnetic bodies has large $\hbar$, one can argue that the layer corresponds to a higher evolutionary level than ordinary matter with longer time scale of memory and planned action. Hence it would not be surprising if the magnetic bodies were able to replicate and use ordinary molecules as kind of sensory receptors

and motor organs. Perhaps the replication of magnetic bodies preceded the replication at DNA level and genetic code is realized already at this more fundamental level somehow. Perhaps the replication of magnetic bodies induces the replication of DNA as I have suggested.

2. The magnetic body of DNA could make DNA a topological quantum computer [K31] . DNA itself would represent the hardware and magnetic bodies would carry the evolving quantum computer programs realized in terms of braidings of magnetic flux tubes. The natural communication and control tool would be cyclotron radiation besides Josephson radiation associated with cell membranes acting as Josephson junctions. Cyclotron frequencies are indeed the only natural frequencies that one can assign to molecules in kHz range. There would be an entire fractal hierarchy of analogs of EEG making possible the communication with and control by magnetic bodies.

3. The values of Planck constant would define a hierarchy of magnetic bodies which corresponds to evolutionary hierarchy and the emergence of a new level would mean jump in evolution. Gel like phases could serve as a correlate for the presence of the magnetic body. The phase transitions changing the value of Planck constant and scale up or down the size of the magnetic flux tubes. They are proposed to serve as a basic control mechanism making possible to understand the properties and the dynamics of the gel phases and how biomolecules can find each other in the thick molecular soup via a phase transition reducing the length of flux tubes connecting the biomolecules in question and thus forcing them to the vicinity of each other.

Consider now how this model could explain the findings.

1. Minimal option is that the the flux tubes correspond to "larger space-time sheets" and the infected cells managed to flow into the filtrate along magnetic flux tubes from the filter. This kind of transfer of DNA might be made possible by the recently discovered nanotubes already mentioned.

2. Maybe the radiation resulted as dark photons invisible for ordinary instruments transformed to ordinary photons as the gel phase assignable with the dark matter at magnetic flux tube network associated with the infected cells and corresponding DNA was destroyed in the filtration.

   This is not the only possible guess. A phase conjugate cyclotron radiation with a large value of Planck constant could also allow for the nanostructures in dilute solute to gain metabolic energy by sending negative energy quanta to a system able to receive them. Indeed the presence of ambient radiation was necessary for the emission. Maybe that for sufficiently dilute solute this mechanism allows to the nanostructures to get metabolic energy from the ambient radiation whereas for the gel phase the metabolic needs are not so demanding. In the similar manner bacteria form colonies when metabolically deprived. This sucking of energy might be also part of the mechanism of disease.

3. What could be the magnetic field inducing the kHz radiation as a synchrotron radiation?

   (a) For instance, kHz frequency and its harmonics could correspond to the cyclotron frequencies of proton in magnetic field which field strength slightly above that for Earth's magnetic field (750 Hz frequency corresponds to field strength of B_E, where $B_E = .5$ Gauss, the nominal strength of Earth's magnetic field). A possible problem is that the thickness of the flux tubes would be about cell size for Earth's magnetic field from flux quantization and even larger for dark matter with a large value of Planck constant. Of course, the flux tubes could make themselves thinner temporarily and leak through the pores.

   (b) If the flux tube is assumed to have thickness of order 20-100 nm, the magnetic field for ordinary value of $\hbar$ would be of order .1 Tesla from flux quantization and in the case of DNA the cyclotron frequencies would not depend much on the length of DNA fragment since the it carries a constant charge density. Magnetic field of order .2 Tesla would give cyclotron frequency of order kHZ from the fact that the field strength of .2 Gauss gives frequency of about .1 Hz. This correspond to a magnetic field with flux tube thickness ~ 125 nm, which happens to be the upper limit for the porosity. Dark magnetic flux tubes with large $\hbar$ are however thicker and the leakage might involve a temporary phase transition to a phase with ordinary value of $\hbar$ reducing the thickness of the flux tube. Perhaps some genes (adhesin) plus corresponding magnetic bodies representing DNA in terms of cyclotron frequencies depending slightly on precise weight of the DNA sequence and thus coding it correspond to the frequency of cyclotron radiation are the sought for nano-structures.

4. While developing a model for homeopathy based on dark matter I ended up with the idea that dark matter consisting of nuclear strings of neutrons and protons with a large value of $\hbar$ and having thus a zoomed up size of nucleon could be involved. The really amazing finding was that nucleons as three quark systems allow to realize vertebrate code in terms of states formed from entangled quarks [L2] , [L2] described also in this chapter! One cannot decompose codons to letters as in the case of the ordinary genetic code but codons are analogous to symbols representing entire words in Chinese. The counterparts of DNA, RNA, and amino-acids emerge and genetic code has a concrete meaning as a map between quantum states.

   Without any exaggeration this connection between dark hadronic physics and biology has been one of the greatest surprises of my professional life. It suggests that dark matter in macroscopic quantum phase realizes genetic code at the level of nuclear physics and biology only provides one particular (or probably very many as I have proposed) representations of it. If one takes this seriously one can imagine that genetic information is represented by these dark nuclear strings of nanoscopic size and that there exists a mechanism translating the dark nuclei to ordinary DNA and RNA sequences and thus to biological matter. This would explain the claimed regeneration of the infected cells.

5. Genetic code at dark matter level would have far reaching implications. For instance, living matter - or rather, the magnetic bodies controlling it - could purposefully perform genetic engineering. This forces me to spit out another really dirty word, "Lamarckism"! We have of course learned that mutations are random. The basic objection against Lamarckism is that there is no known mechanism which would transfer the mutations to germ cells. In the homeopathic Universe of TGD the mutations could be however performed first for the dark nucleon sequences. After this these sequences would diffuse to germ cells just like homeopathic remedies do, and after this are translated to DNA or RNA and attach to DNA.

9.6.5 Burning water and photosynthesis

For a physicist liberated from the blind belief in reductionism, biology transforms to a single gigantic anomaly about which recent day physics cannot say much. During years I have constructed several models for these anomalies helping to develop a more detailed view about how the new physics predicted by quantum TGD could allow to understand biology and consciousness.

The basic problem is of course the absence of systematic experimentation so that it is possible to imagine many new physics scenarios. For this reason the article series of Mae-Wan Ho [D63, D61, D59, D62] in ISIS was a very pleasant surprise, and already now has helped considerably in the attempts to develop the ideas further.

The first article "Water electric" [D63] told about the formation of exclusion zones around hydrophilic surfaces, typically gels in the experiments considered [D85] . The zones were in potential of about 100 meV with respect to surroundings (same order of magnitude as membrane potential) and had thickness ranging to hundreds of micrometers (the size of a large cell): the standard physics would suggests only few molecular layers instead of millions. Sunlight induced the effect. This finding allow to develop TGD based vision about how proto cells emerged and also the model for chiral selection in living matter by combining the finding with the anomalies of water about which I had learned earlier.

The article "Can water burn?" [D59] tells about the discovery of John Kanzius - a retired broadcast engineer and inventor. Kanzius found that water literally burns if subjected to a radio frequency radiation at frequency of 13.56 MHz [D1]. The mystery is of course how so low frequency can induce burning. The article "The body does burn water" [D62] notices that plant cells burn water routinely in photosynthesis and that also animal cells burn water but the purpose is now to generate hydrogen peroxide which kills bacteria (some readers might recall from childhood how hydrogen peroxide was used to sterilize wounds!). Hence the understanding of how water burns is very relevant for the understanding of photosynthesis and even workings of the immune system.

Living matter burns water routinely

Photosynthesis burns water by decomposing water to hydrogen and oxygen and liberating oxygen. Oxygen from CO_2 in atmosphere combines with the oxygen of H_2O to form O_2 molecules whereas H from H_2O combines with carbon to form hydrocarbons serving as energy sources for animals which in turn produce CO_2. This process is fundamental for aerobic life. There is also a simpler variant of photosynthesis in which oxygen is not produced and applied by an-aerobic life forms. The article "Living with Oxygen" by Mae-Wan Ho gives a nice overall view about the role of oxygen [D60] . As a matter fact, also animals burn water but they do this to produce hydrogen peroxide H_2O_2 which kills very effectively bacteria.

Burning of water has been studied as a potential solution for how to utilize the solar energy to produce hydrogen serving as a natural fuel [D61]. The reaction $O_2 + H_2 \rightarrow 2H_2O$ occurs spontaneously and liberates energy of about 1.23 eV. The reverse process $2H_2 \rightarrow H_2O_2 + H_2$ in the presence of sunlight means burning of water, and could provide the manner to store solar energy. The basic reaction $2H_2O + 4h\nu \leftrightarrow H_2O_2 + H_2$ stores the energy of four photons. What really happens in this process is far from being completely understood. Quite generally, the mechanisms making possible extreme efficiency of bio-catalysis remain poorly understood. Here new physics might be involved. I have discussed models for photosynthesis and $ADP \leftrightarrow ATP$ process involved with the utilization of the biochemical energy already earlier [K44] .

How water could burn in TGD Universe?

The new results could help to develop a more detailed model about what happens in photosynthesis. The simplest TGD inspired sketch for what might happen in the burning of water goes as follows.

1. Assume that 1/4 of water molecules are partially dark (in sense of nonstandard value of Planck constant) or at least at larger space-time sheets in atto-second scale [D30, D27, D32, D52] . This would explain the $H_{1.5}O$ formula explaining the results of neutron diffraction and electron scattering.

2. The question is what this exotic fraction of water precisely is. The models for water electret, exclusion zones and chiral selection lead to concrete ideas about this. Electrons assignable to the H atoms of (partially) dark H_2O reside at space-time sheet $k_e = 151$ (this p-adic length scale corresponds to 10 nm, the thickness of cell membrane). At least the hydrogen atom for this fraction of water molecules is exotic and findings from neutron and electron scattering suggest that both proton and electron are at non-standard space-time sheets but not necessarily at the same space-time sheet. The model for the burning requires that electron and proton are at different space-time sheets in the initial situation.

3. Suppose all four electrons are kicked to the space-time sheet of protons of the exotic hydrogen atoms labeled by k_p. This requires the energy $E_\gamma = (1 - 2^{-n})E_0(k_p)$ (the formula involves idealizations). At this space-time sheet protons and electrons are assumed to combine spontaneously to form two H_2 atoms. Oxygen atoms in turn are assumed to combine spontaneously to form O_2.

4. For $k_f = 148$ and $n = 3$ minimum energy needed would be $4E_\gamma = 4 \times .4 = 1.6$ eV. For $k_p = 149$ (thickness of lipid layer) and $n = 2$ one would have $4E_\gamma = 4 \times .3462 = 1.385$ eV whereas $H_2O_2 + H_2 \rightarrow 2H_2O$ liberates energy 1.23 eV. Therefore the model in which electrons are at cell membrane space-time sheet and protons at the space-time sheet assignable to single lipid layer of cell membrane suggests itself. This would also mean that the basic length scales of cell are already present in the structure of water. Notice that there is no need to assume that Planck constant differs from its standard value.

There is no need to add, that the model is an unashamed oversimplification of the reality. It might however catch the core mechanism of photosynthesis.

Burning of salt water induced by RF radiation

Engineer John Kanzius has made a strange discovery [D1]: salt water in the test tube radiated by radio waves at harmonics of a frequency f=13.56 MHz burns. Temperatures about 1500 K, which correspond to .15 eV energy have been reported. One can irradiate also hand but nothing happens. The original discovery of Kanzius was the finding that radio waves could be used to cure cancer by destroying the cancer cells. The proposal is that this effect might provide new energy source by liberating chemical energy in an exceptionally effective manner. The power is about 200 W so that the power used could explain the effect if it is absorbed in resonance like manner by salt water.

Mae-Wan Ho's article "Can water Burn?" [D59] provides new information about burning salt water [D1], in particular reports that the experiments have been replicated. The water is irradiated using polarized radio frequency light at frequency 13.56 MHz. The energy of radio frequency quantum is $E_{rf} = .561 \times 10^{-7}$ eV and provides only a minor fraction $E_{rf}/E = .436 \times 10^{-7}$ of the needed energy which is $E = 1.23$ eV for single $2H_2O \rightarrow H_2O_2 + H_2$ event. The structure of water has been found to change, in particular something happens to O-H bonds. The Raman spectrum of the water has changed in the energy range $[0.37, 0.43]$ eV. Recall that the range of metabolic energy quanta $E(k,n) = (1-2^-n)E_0(k)$ varies for electron in the range $[.35, .46]$ eV in the model for the formation of exclusion zone induced by light. Therefore the photons assigned to changes in Raman spectrum might be associated with the transfer of electrons between space-time sheets.

The energies of photons involved are very small, multiples of 5.6×10^{-8} eV and their effect should be very small since it is difficult to imagine what resonant molecular transition could cause the effect. This leads to the question whether the radio wave beam could contain a considerable fraction of dark photons for which Planck constant is larger so that the energy of photons is much larger. The underlying mechanism would be phase transition of dark photons with large Planck constant to ordinary photons with shorter wavelength coupling resonantly to some molecular degrees of freedom and inducing the heating. Microwave oven of course comes in mind immediately.

As I made this proposal, I did not realize the connection with photosynthesis and actual burning of water. The recent experimental findings suggest that dark radio frequency photons transform to photons inducing splitting of water as in photosynthesis so that that one should have $r = \hbar/\hbar_0 = E_{rf}/4E$. One could say that large number of radio wave photons combine to form a single bundle of photons forming a structure analogous to what mathematician calls covering space. In the burning event the dark photon would transform to ordinary photon with the same energy. This process would thus transform low energy photons to high energy protons with the ratio $r = \hbar/\hbar_0$.

Therefore the mechanism for the burning of water in the experiment of Kanzius could be a simple modification of the mechanism behind burning of water in photosynthesis.

1. Some fraction of dark radio frequency photons are dark or are transformed to dark photons in water and have energies around the energy needed to kick electrons to smaller space-time sheets .4 eV. After this they are transformed to ordinary photons and induce the above process. Their in-elastic scattering from molecules (that is Raman scattering) explains the observation of Raman scattered photons. For a fixed value of $\hbar$ the process would occur in resonant manner since only few metabolic quanta are allowed.

2. How dark radio frequency photons could be present or could be produced in water? Cyclotron radiation assignable to say electrons in magnetic field comes in mind. If the cyclotron radiation is associated with electrons it requires a magnetic field of 4.8 Gauss the cyclotron frequency is 13.56 MHz. This is roughly ten times the nominal value $B_E = .5$ Gauss of the Earth's magnetic field and 24 times the value of dark magnetic field $B_d = .4B_E = .2$ Gauss needed to explain the effects of ELF em fields on vertebrate brain. Maybe dark matter at flux tubes of Earth's magnetic field with Planck constant equal to $\hbar/\hbar_0 = \frac{1}{4}\frac{E}{E_{rf}}$ transforms radio frequency photons to dark photons or induces resonantly the generation of cyclotron photons, which in turn leak out from magnetic flux tubes and form ordinary photons inducing the burning of water. $E_\gamma = .4$ eV would give $\hbar/\hbar_0 == 1.063 \times 2^{21}$ and $E_\gamma = .36$ eV would give $\hbar/\hbar_0 = .920 \times 2^{21}$.

3. Magnetic fields of magnitude .2 Gauss are in central role in TGD based model of living matter and there are excellent reasons to expect that this mechanism could be involved also with processes involved with living matter. There is indeed evidence for this. The experiments of Gariaev demonstrated that the irradiation of DNA with 2 eV laser photons (which correspond to one particular metabolic energy quantum) induced generation of radio wave photons having unexpected effects on living matter (enhanced metabolic activity) [I7] , and that even a realization of genetic code in terms of the time variation of polarization direction could be involved. TGD based model [K14, K87] identifies radio-wave photons as dark photons with same energy as possessed by incoming visible photons so that a transformation of ordinary photons to dark photons would have been in question. The model assumed hierarchy of values of magnetic fields in accordance with the idea about onion like structure of the magnetic body.

There are several questions to be answered.

1. Is there some trivial explanation for why salt must be present or is new physics involved also here. What comes in mind are Cooper pairs dark Na^+ ions (or their exotic counterparts which are bosons) carrying Josephson currents through the cell membrane in the model of the cell membrane as a Josephson junction which is almost vacuum extremal of Kähler action. In the experimental arrangement leading to the generation of exclusion zones the pH of water was important control factor, and it might be that the presence of salt has an analogous role to that of protons.

2. Does this effect occur also for solutions of other molecules and other solutes than water? This can be tested since the rotational spectra are readily calculable from data which can be found at net.

3. Are the radio wave photons dark or does water - which is very special kind of liquid - induce the transformation of ordinary radio wave photons to dark photons by fusing $r = \hbar/\hbar_0$ radio wave massless extremals (MEs) to single ME. Does this transformation occur for all frequencies? This kind of transformation might play a key role in transforming ordinary EEG photons to dark photons and partially explain the special role of water in living systems.

4. Why the radiation does not induce spontaneous combustion of living matter which contains salt. And why cancer cells seem to burn: is salt concentration higher inside them? As a matter fact, there are reports about [D19] . One might hope that there is a mechanism inhibiting this since otherwise military would be soon developing new horror weapons unless it is doing this already now. Is it that most of salt is ionized to Na^+ and Cl^- ions so that spontaneous combustion can be avoided? And how this relates to the sensation of spontaneous burning [D18] - a very painful sensation that some part of body is burning?

5. Is the energy heating solely due to rotational excitations? It might be that also a "dropping" of ions to larger space-time sheets is induced by the process and liberates zero point kinetic energy. The dropping of proton from k=137 (k=139) atomic space-time sheet liberates about .5 eV (0.125 eV). The measured temperature corresponds to the energy .15 eV. This dropping is an essential element in the ealier of remote metabolism and provides universal metabolic energy quanta. It is also involved with TGD based models of "free energy" phenomena. No perpetuum mobile is predicted since there must be a mechanism driving the dropped ions back to the original space-time sheets.

   In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

6. The electrolysis of water and also cavitation produces what is known as Brown's gas which should consist of water vapour and there might be a connection to the burning of salt water. The properties of Brown's gas [H9] however do not support this interpretation: for instance,

Brown's gas has temperature of about 130 C but is able to melt metals so that some un-known mechanism liberating energy must be involved explaining also the claims about over-unity energy production in water splitting using electrolysis. TGD inspired model for Brown's gas [K45] suggests that activated water and Brown's gas correspond to same phase involving polymer sequences formed from exotic water molecules for which one hydrogen nucleus is dark and defining the analogs of basic biopolymers. The bond binding protons to a polymer like sequence would serve as the counterpart of covalent bond.

One also ends up with a more detailed TGD inspired view about basic mechanism of metabolism in living matter predicting a tight correlation between p-adic length scale hypothesis and hierarchy of Planck constants. The model differs in some aspects from the rough models considered hitherto assuming that metabolic energy is liberated as zero point kinetic energy when particle drops to a larger space-time sheet or as cyclotron energy when cyclotron quantum number decreases. Now a phase transition increasing the p-adic length scale of the space-time surface would liberate either kinetic energy of cyclotron energy. Quantum numbers would not change: rather, t the scale appearing as a parameter in the expression of kinetic or cyclotron energy would change adiabatically and in this manner guarantee coherence. Also a phase transition in which the changes of scale due to a reduction of Planck constant and increase of the p-adic length scale compensate each other liberate metabolic energy.

Recall that one of the empirical motivations for the hierarchy of Planck constants came from the observed quantum like effects of ELF em fields at EEG frequencies on vertebrate brain and also from the correlation of EEG with brain function and contents of consciousness difficult to understand since the energies of EEG photons are ridiculously small and should be masked by thermal noise.

9.7 Connection with mono-atomic elements, cold fusion, and sono-luminescence?

Anomalies are treasures for a theoretician and during years I have been using quite a bundle of reported anomalies challenging the standard physics as a test bed for the TGD vision about physics. The so called mono-atomic elements, cold fusion, and sonofusion represent examples of this kind of anomalies not taken seriously by most standard physicists. In the following the possibility that dark matter as large $\hbar$ phase could allow to understand these anomalies.

Of course, I hear the angry voice of the skeptic reader blaming me for a complete lack of source criticism and the skeptic reader is right. I however want to tell him that I am not a soldier in troops of either skeptics or new-agers. My attitude is "let us for a moment assume that these findings are real..." and look for the consequences in this particular theoretical framework.

9.7.1 Mono-atomic elements as dark matter and high T_c super-conductors?

The ideas related to many-sheeted space-time began to develop for a decade ago. The stimulation came from a contact by Barry Carter who told me about so called mono-atomic elements, typically transition metals (precious metals), including Gold. According to the reports these elements, which are also called ORMEs ("orbitally rearranged monoatomic elements") or ORMUS, have following properties.

1. ORMEs were discovered and patented by David [H6] [H6] are peculiar elements belonging to platinum group (platinum, palladium, rhodium, iridium, ruthenium and osmium) and to transition elements (gold, silver, copper, cobalt and nickel).

2. Instead of behaving as metals with valence bonds, ORMEs have ceramic like behavior. Their density is claimed to be much lower than the density of the metallic form.

3. They are chemically inert and poor conductors of heat and electricity. The chemical inertness of these elements have made their chemical identification very difficult.

4. One signature is the infra red line with energy of order .05 eV. There is no text book explanation for this behavior. Hudson also reports that these elements became visible in emission spectroscopy in which elements are posed in strong electric field after time which was 6 times longer than usually.

The pioneering observations of David Hudson [H6] - if taken seriously - suggest an interpretation as an exotic super-conductor at room temperature having extremely low critical magnetic fields of order of magnetic field of Earth, which of course is in conflict with the standard wisdom about super-conductivity. After a decade and with an impulse coming from a different contact related to ORMEs, I decided to take a fresh look on Hudson's description for how he discovered ORMEs [H6] with dark matter in my mind. From experience I can tell that the model to be proposed is probably not the final one but it is certainly the simplest one.

There are of course endless variety of models one can imagine and one must somehow constrain the choices. The key constraints used are following.

1. Only valence electrons determining the chemical properties appear in dark state and the model must be consistent with the general model of the enhanced conductivity of DNA assumed to be caused by large $\hbar$ valence electrons with $r = \hbar/\hbar_0 = n$, $n = 5, 6$ assignable with aromatic rings. $r = 6$ for valence electrons would explain the report of Hudson about anomalous emission spectroscopy.

2. This model cannot explain all data. If ORMEs are assumed to represent very simple form of living matter also the presence electrons having $\hbar/\hbar_0 = 2^{k11}$, $k = 1$, can be considered and would be associated with high T_c super-conductors whose model predicts structures with thickness of cell membrane. This would explain the claims about very low critical magnetic fields destroying the claimed superconductivity.

Below I reproduce Hudson's own description here in a somewhat shortened form and emphasize that must not forget professional skepticism concerning the claimed findings.

Basic findings of Hudson

Hudson was recovering gold and silver from old mining sources. Hudson had learned that something strange was going on with his samples. In molten lead the gold and silver recovered but when "I held the lead down, I had nothing". Hudson tells that mining community refers to this as "ghost-gold", a non-assayable, non-identifiable form of gold.

Then Hudson decided to study the strange samples using emission spectroscopy. The sample is put between carbon electrodes and arc between them ionizes elements in the sample so that they radiate at specific frequencies serving as their signatures. The analysis lasts 10-15 seconds since for longer times lower electrode is burned away. The sample was identified as Iron, Silicon, and Aluminium. Hudson spent years to eliminate Fe, Si, and Al. Also other methods such as Cummings Microscopy, Diffraction Microscopy, and Fluorescent Microscopy were applied and the final conclusion was that there was nothing left in the sample in spectroscopic sense.

After this Hudson returned to emission spectroscopy but lengthened the time of exposure to electric field by surrounding the lower Carbon electrode with Argon gas so that it could not burn. This allowed to reach exposure times up to 300 s. The sample was silent up to 90 s after which emission lines of Palladium (Pd) appeared; after 110 seconds Platinum (Pt); at 130 seconds Ruthenium (Ru); at about 140-150 seconds Rhodium; at 190 seconds Iridium; and at 220 seconds Osmium appeared. This is known as fractional vaporization.

Hudson reports the boiling temperatures for the metals in the sample having in mind the idea that the emission begins when the temperature of the sample reaches boiling temperature inspired by the observation that elements become visible in the order which is same as that for boiling temperatures.

The boiling temperatures for the elements appearing in the sample are given by the following table.

Element	Ca	Fe	Si	Al	Pd	Rh
$T_B/^oC$	1420	1535	2355	2327	>2200	2500
Element	Ru	Pt	Ir	Os	Ag	Au
$T_B/^oC$	4150	4300	> 4800	> 5300	1950	2600

Table 2. Boiling temperatures of elements appearing in the samples of Hudson.

Hudson experimented also with commercially available samples of precious metals and found that the lines appear within 15 seconds, then follows a silence until lines re-appear after 90 seconds. Note that the ratio of these time scales is 6. The presence of some exotic form of these metals suggests itself: Hudson talks about mono-atomic elements.

Hudson studied specifically what he calls mono-atomic gold and claims that it does not possess metallic properties. Hudson reports that the weight of mono-atomic gold, which appears as a white powder, is 4/9 of the weight of metallic gold. Mono-atomic gold is claimed to behave like super-conductor.

Hudson does not give a convincing justification for why his elements should be mono-atomic so that in following this attribute will be used just because it represents established convention. Hudson also claims that the nuclei of mono-atomic elements are in a high spin state. I do not understand the motivations for this statement.

Claims of Hudson about ORMEs as super conductors

The claims of Hudson that ORMES are super conductors [H6] are in conflict with the conventional wisdom about super conductors.

1. The first claim is that ORMEs are super conductors with gap energy about $E_g = .05$ eV and identifies photons with this energy resulting from the formation of Cooper pairs. This energy happens to correspond one of the absorption lines in high T_c superconductors.

2. ORMEs are claimed to be super conductors of type II with critical fields H_{c1} and H_{c2} of order of Earth's magnetic field having the nominal value $.5 \times 10^{-4}$ Tesla [H6] . The estimates for the critical parameters for the ordinary super conductors suggests for electronic super conductors critical fields, which are about .1 Tesla and thus by a factor $\sim 2^{12}$ larger than the critical fields claimed by Hudson.

3. It is claimed that ORME particles can levitate even in Earth's magnetic field. The latter claim looks at first completely nonsensical. The point is that the force giving rise to the levitation is roughly the gradient of the would-be magnetic energy in the volume of levitating super conductor. The gradient of average magnetic field of Earth is of order B/R, R the radius of Earth and thus extremely small so that genuine levitation cannot be in question.

Minimal model

Consider now a possible TGD inspired model for these findings assuming for definiteness that the basic Hudson's claims are literally true.

1. In what sense mono-atomic elements could be dark matter?

The simplest option suggested by the applicability of emission spectroscopy and chemical inertness is that mono-atomic elements correspond to ordinary atoms for which valence electrons are dark electrons with large value of $r = \hbar/\hbar_0$. Suppose that the emission spectroscopy measures the energies of dark photons from the transitions of dark electrons transforming to ordinary photons before the detection by de-coherence increasing the frequency by r. The size of dark electrons and temporal duration of basic processes would be zoomed up by r.

Since the time scale after which emission begins is scaled up by a factor 6, there is a temptation to conclude that $r = 6$ holds true. Note that $n = 6$ corresponds to Fermat polygon and is thus preferred number theoretically in TGD based model for preferred values of $\hbar$ [K32] . The simplest possibility is that the group G_b is trivial group and $G_a = A_6$ or D_6 so that ring like structures containing six dark atoms are suggestive.

This brings in mind the model explaining the anomalous conductivity of DNA by large $\hbar$ valence electrons of aromatic rings of DNA. The zooming up of spatial sizes might make possible exotic effects and perhaps even a formation of atomic Bose-Einstein condensates of Cooper pairs. Note however that in case of DNA $r = 6$ not gives only rise to conductivity but not super-conductivity and that $r = 6$ cannot explain the claimed very low critical magnetic field destroying the super-conductivity.

2. Loss of weight

The claimed loss of weight by a factor $p \simeq 4/9$ is a very significant hint if taken seriously. The proposed model implies that the density of the partially dark phase is different from that of the ordinary phase but is not quantitative enough to predict the value of p. The most plausible reason for the loss of weight would be the reduction of density induced by the replacement of ordinary chemistry with $r = 6$ chemistry for which the Compton length of valence electrons would increase by this factor.

3. Is super-conductivity possible?

The overlap criterion is favorable for super-conductivity since electron Compton lengths would be scaled up by factor $n_a = 6, n_b = 1$. For $r = \hbar/\hbar_0 = n_a = 6$ Fermi energy would be scaled up by $n_a^2 = 36$ and if the same occurs for the gap energy, T_c would increase by a factor 36 from that predicted by the standard BCS theory. Scaled up conventional super-conductor having $T_c \sim 10$ K would be in question (conventional super-conductors have critical temperatures below 20 K). 20 K upper bound for the critical temperature of these superconductors would allow 660 K critical temperature for their dark variants!

For large enough values of r the formation of Cooper pairs could be favored by the thermal instability of valence electrons. The binding energies would behave as $E = r^2 Z_{eff}^2 E_0/n^2$, where Z_{eff} is the screened nuclear charge seen by valence electrons, n the principal quantum number for the valence electron, and E_0 the ground state energy of hydrogen atom. This gives binding energy smaller than thermal energy at room temperature for $r > (Z_{eff}/n)\sqrt{2E_0/3T_{room}} \simeq 17.4 \times (Z_{eff}/n)$. For $n = 5$ and $Z_{eff} < 1.7$ this would give thermal instability for $r = 6$.

Interestingly, the reported .05 eV infrared line corresponds to the energy assignable to cell membrane voltage at criticality against nerve pulse generation, which suggests a possible connection with high T_c superconductors for which also this line appears and is identified in terms of Josephson energy. .05 eV line appears also in high T_c superconductors. This interpretation does not exclude the interpretation as gap energy. The gap energy of the corresponding BCS super-conductor would be scaled down by $1/r^2$ and would correspond to 14 K temperature for $r = 6$.

Also high T_c super-conductivity could involve the transformation of nuclei at the stripes containing the holes to dark matter and the formation of Cooper pairs could be due to the thermal instability of valence electrons of Cu atoms (having $n = 4$). The rough extrapolation for the critical temperature for cuprate superconductor would be $T_c(Cu) = (n_{Cu}/n_{Rh})^2 T_c(Rh) = (25/36)T_c(Rh)$. For $T_c(Rh) = 300$ K this would give $T_c(Cu) = 192$ K: according to Wikipedia cuprate perovskite has the highest known critical temperature which is 138 K. Note that quantum criticality suggests the possibility of several values of (n_a, n_b) so that several kinds of super-conductivities might be present.

ORMEs as partially dark matter, high T_c super conductors, and high T_c super-fluids

The appearance of .05 eV photon line suggest that same phenomena could be associated with ORMEs and high T_c super-conductors. The strongest conclusion would be that ORMEs are T_c super-conductors and that the only difference is that Cu having single valence electron is replaced by a heavier atom with single valence electron. In the following I shall discuss this option rather independently from the minimal model.

1. ORME super-conductivity as quantum critical high T_c superconductivity

ORMEs are claimed to be high T_c superconductors and the identification as quantum critical superconductors seems to make sense.

1. According to the model of high T_c superconductors as quantum critical systems, the properties of Cooper pairs should be more or less universal so that the observed absorption lines discussed in the section about high T_c superconductors should characterize also ORMEs. Indeed, the reported 50 meV photon line corresponds to a poorly understood absorption line in the case of high T_c cuprate super conductors having in TGD framework an interpretation as a transition in which exotic Cooper pair is excited to a higher energy state. Also Copper is a transition metal and is one of the most important trace elements in living systems [D2] . Thus the Cooper pairs could be identical in both cases. ORMEs are claimed to be superconductors of type II and quantum critical superconductors are predicted to be of type II under rather general conditions.

2. The claimed extremely low value of H_c is also consistent with the high T_c superconductivity. The supra currents in the interior of flux tubes of radius of order $L_w = .4\ \mu$m are BCS type supra currents with large $\hbar$ so that T_c is by a factor 2^{14} ($127 - 113 = 14$ is inspired by the Mersenne hypothesis for the preferred p-adic length scales) higher than expected and H_c is reduced by a factor 2^{-10}. This indeed predicts the claimed order of magnitude for the critical magnetic field.

3. The problem is that $r = 2^{14}$ is considerably higher that $r = 6$ suggested by the minimum model explaining the emission spectroscopic results of Hudson. Of course, several values of $\hbar$ are possible so that internal consistency would be achieved if ORMEs are regarded as a very simple form of living matter with relatively small value of r and giving up the claim about the low value of critical magnetic field.

4. The electronic configurations of Cu and Gold are chemically similar. Gold has electronic configuration $[Xe, 4f^{14}5d^{10}]6s$ with one valence electron in s state whereas Copper corresponds to $3d^{10}4s$ ground state configuration with one valence electron. This encourages to think that the doping by holes needed to achieve superconductivity induces the dropping of these electrons to $k = 151$ space-time sheets and gives rise to exotic Cooper pairs.

   In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

   Also this model assumes the phase transition of some fraction of Cu nuclei to large $\hbar$ phase and that exotic Cooper pairs appear at the boundary of ordinary and large $\hbar$ phase.

   More generally, elements having one electron in s state plus full electronic shells are good candidates for doped high T_c superconductors. Both Cu and Au atoms are bosons. More generally, if the atom in question is boson, the formation of atomic Bose-Einstein condensates at Cooper pair space-time sheets is favored. Thus elements with odd value of A and Z possessing full shells plus single s wave valence electron are of special interest. The six stable elements satisfying these conditions are ^{5}Li, ^{39}K, ^{63}Cu, ^{85}Rb, ^{133}Cs, and ^{197}Au.

2. "Levitation" and loss of weight

The model of high T_c superconductivity predicts that some fraction of Cu atoms drops to the flux tube with radius $L_w = .4\ \mu$m and behaves as a dark matter. This is expected to occur also in the case of other transition metals such as Gold. The atomic nuclei at this space-time sheet have high charges and make phase transition to large $\hbar$ phase and form Bose-Einstein condensate and superfluid behavior results. Electrons in turn form large $\hbar$ variant of BCS type superconductor. These flux tubes are predicted to be negatively charged because of the Bose-Einstein condensate of exotic Cooper pairs at the boundaries of the flux tubes having thickness $L(151)$. The average charge density equals to the doping fraction times the density of Copper atoms.

The first explanation would be in terms of super-fluid behavior completely analogous to the ability of ordinary superfluids to defy gravity. Second explanation is based on the electric field of Earth which causes an upwards directed force on negatively charged BE condensate of exotic Cooper pairs and this force could explain both the apparent levitation and partial loss of weight. The criterion for levitation is $F_e = 2eE/x \geq F_{gr} = Am_pg$, where $g \simeq 10$ m^2/s is gravitational acceleration at the surface of Earth, A is the atomic weight and m_p proton mass, E the strength of electric field, and x is the number of atoms at the space-time sheet of a given Cooper pair. The condition gives $E \geq 5 \times 10{-}10Ax$ V/m to be compared with the strength $E = 10^2 - 10^4$ V/m of the Earths' electric field.

An objection against the explanation for the effective loss of weight is that it depends on the strength of electric field which varies in a wide range whereas Hudson claims that the reduction factor is constant and equal to 4/9. A more mundane explanation would be in terms of a lower density of dark Gold. This explanation is quite plausible since there is no atomic lattice structure since nuclei and electrons form their own large $\hbar$ phases.

4. The effects on biological systems

Some monoatomic elements such as White Gold are claimed to have beneficial effects on living systems [H6] . 5 per cent of brain tissue of pig by dry matter weight is claimed to be Rhodium and Iridium. Cancer cells are claimed to be transformed to healthy ones in presence of ORMEs. The model for high T_c super conductivity predicts that the flux tubes along which interior and boundary supra currents flow has same structure as neuronal axons. Even the basic length scales are very precisely the same. On basis of above considerations ORMEs are reasonable candidates for high T_c superconductors and perhaps even super fluids.

The common mechanism for high T_c, ORME- and bio- super-conductivities could explain the biological effects of ORMEs.

1. In unhealthy state superconductivity might fail at the level of cell membrane, at the level of DNA or in some longer length scales and would mean that cancer cells are not anymore able to communicate. A possible reason for a lost super conductivity or anomalously weak super conductivity is that the fraction of ORME atoms is for some reason too small in unhealthy tissue.

2. The presence of ORMEs could enhance the electronic bio- superconductivity which for some reason is not fully intact. For instance, if the lipid layers of cell membrane are, not only wormhole-, but also electronic super conductors and cancer involves the loss of electronic super-conductivity then the effect of ORMEs would be to increase the number density of Cooper pairs and make the cell membrane super conductor again. Similar mechanism might work at DNA level if DNA:s are super conductors in "active" state.

5. Is ORME super-conductivity associated with the magnetic flux tubes of dark magnetic field $B_d = 0.2$ Gauss?

The general model for the ionic super-conductivity in living matter, which has developed gradually during the last few years and will be discussed in detail later, was originally based on the assumption that super-conducting particles reside at the super-conducting magnetic flux tubes of Earth's magnetic field with the nominal value $B_E = .5$ Gauss. It became later clear that the explanation of ELF em fields on vertebrate brain requires $B_d = .2$ Gauss rather than $B_E = .5$ Gauss [K29] . The interpretation was as dark magnetic field $B_d = .2$ Gauss. The model of EEG led also to the hypothesis that Mersenne primes and their Gaussian counterparts define preferred p-adic length scales and their dark counterparts. This hypothesis replaced the earlier $r = 2^{11k}$ hypothesis.

For $r = 2^{127-113=14}$ the predicted radius $L_w = .4$ μm is consistent with the radius of neuronal axons. If one assumes that the radii of flux tubes are given by this length scale irrespective of the value of r, one must replace the quantization condition for the magnetic flux with a more general condition in which the magnetic flux is compensated by the contribution of the supra current flowing around the flux tube: $\oint (p - eA) \cdot dl = n\hbar$ and assume $n = 0$. The supra currents would be present inside living organism but in the faraway region where flux quanta from organism fuse together, the quantization conditions $e \int B \cdot dS = n\hbar$ would be satisfied.

The most natural interpretation would be that these flux tubes topologically condense at the flux tubes of B_E. Both bosonic ions and the Cooper pairs of electrons or of fermionic ions can act as charge carriers so that actually an entire zoo of super-conductors is predicted. There is even some support for the view that even molecules and macromolecules can drop to the magnetic flux tubes [K44] .

Nuclear physics anomalies and ORMEs

At the homepage of Joe Champion [H11] information about claimed nuclear physics anomalies can be found.

1) The first anomaly is the claimed low temperature cold fusion. For instance, Champion claims that Mercury (Z=80), decays by emission of proton and neutrons to Gold with Z=79 in the electrochemical arrangement described in [H11] .

2) Champion mentions also the anomalous production of Cadmium isotopes electrochemically in presence of Palladium reported by Tadahiko Mizuno.

The simplest explanation of the anomalies would be based on genuine nuclear reactions. The interaction of dark nuclei with ordinary nuclei at the boundary between the two phases would make possible genuine nuclear transmutations since the Coulomb wall hindering usually cold fusion and nuclear transmutations would be absent (Trojan horse mechanism). Both cold fusion and reported nuclear transmutations in living matter could rely on this mechanism as suggested in [K80, L2, K28] , [L2] .

Possible implications

The existence of exotic atoms could have far reaching consequences for the understanding of bio-systems. If Hudson's claims about super-conductor like behavior are correct, the formation of exotic atoms in bio-systems could provide the needed mechanism of electronic super-conductivity. One could even argue that the formation of exotic atoms is the magic step transforming chemical evolution to biological evolution.

Equally exciting are the technological prospects. If the concept works it could be possible to manufacture exotic atoms and build room temperature super conductors and perhaps even artificial life some day. It is very probable that the process of dropping electron to the larger space-time sheet requires energy and external energy feed is necessary for the creation of artificial life. Otherwise the Earth and other planets probably have developed silicon based life for long time ago. Ca, K and Na ions have central position in the electrochemistry of cell membranes. They could actually correspond to exotic ions obtained by dropping some valence electrons from $k = 137$ atomic space-time sheet to larger space-time sheets. For instance, the $k = 149$ space-time sheet of lipid layers could be in question.

The status of ORMEs is far from certain and their explanation in terms of exotic atomic concept need not be correct. The fact is however that TGD predicts exotic atoms: if they are not observed TGD approach faces the challenge of finding a good explanation for their non-observability.

Interestingly, Palladium is one of the "mono-atomic" elements used also in cold fusion experiments as a target material [C110, C105]. This inspires the question whether mono-atomic phase is one of the prerequisites for cold fusion.

9.7.2 Basic ideas about cold fusion

The basic prediction of TGD is a hierarchy of fractally scaled variants of QCD like theories and that color dynamics is fundamental even for our sensory qualia (visual colors identified as increments of color quantum numbers in quantum jump). The model for ORMEs suggest that exotic protons obey QCD like theory in the size scale of atom. If this identification is correct, QCD like dynamics might be studied some day experimentally in atomic or even macroscopic length scales of order cell size and there would be no need for ultra expensive accelerators!

What makes possible cold fusion?

I have proposed that cold fusion might be based on Trojan horse mechanism in which incoming and target nuclei feed their em gauge fluxes to different space-time sheets so that electromagnetic Coulomb wall disappears [K80] . If part of Palladium nuclei are "partially dark", this is achieved. Another mechanism could be the de-localization of protons to a larger volume than nuclear volume induced by the increase of h_{eff} meaning that reaction environment would differ dramatically from that appearing in the usual nuclear reactions and the standard objections against cold fusion would not apply anymore [K80] : this de-localization could correspond to the darkness of electromagnetic and perhaps also electroweak field bodies of protons.

A third proposal is perhaps the most elegant and relies on the nuclear string model [L2] predicting a large number of exotic nuclei obtained by allowing the color bonds connecting nucleons to have all possible em charges $1, 0, 1$. Many ordinary heavy nuclei would be exotic in the sense that some protons would correspond to protons plus negatively charged color bonds. The exchange of an exotic weak boson between D and Pd nuclei transforming D nuclei to exotic neutral D nuclei would occur. The range of the exotic weak interaction correspond to atomic length scale meaning that it behaves as massless particle below this length scale. For instance, W boson could be $r = 2^{24}$ dark variant of $k = 113$ weak boson for which the dark variant of p-adic scale would correspond to the atomic scale $k = 137$ but also other options are possible.

How standard objections against cold fusion can be circumvented?

The following arguments against cold fusion are from an excellent review article by Storms [C112] .

1. Coulomb wall requires an application of higher energy. Now electromagnetic Coulomb wall disappears in both models.

2. If a nuclear reaction should occur, the immediate release of energy can not be communicated to the lattice in the time available. In the recent case the time scale is however multiplied by the factor $r = n_a$ and the situation obviously changes. For $n_a = 2^{24}$ the time scale corresponding to MeV energy becomes that corresponding to keV energy which is atomic time scale.

3. When such an energy is released under normal conditions, energetic particles are emitted along with various kinds of radiation, only a few of which are seen by various CANR (Chemically Assisted Nuclear Reactions) studies. In addition, gamma emission must accompany helium, and production of neutrons and tritium, in equal amounts, must result from any fusion reaction. None of these conditions is observed during the claimed CANR effect, no matter how carefully or how often they have been sought. The large value of $\hbar(M^4)$ implying large Compton lengths for protons making possible geometric coupling of gamma rays to condensed matter would imply that gamma rays do not leave the system. If only protons form the quantum coherent state then fusion reactions do not involve the protons of the cathode at all and production of ^3He and thus of neutrons in the fusion of D and exotic D.

4. The claimed nuclear transmutation reactions (reported to occur also in living matter [C93]) are very difficult to understand in standard nuclear physics framework.

   (a) The model of [K80] allows them since protons of different nuclei can re-arrange in many different manners when the dark matter state decays back to normal.

   (b) Nuclear string model [L2] allows transmutations too. For instance, neutral exotic tritium produced in the reactions can fuse with Pd and other nuclei.

5. Many attempts to calculate fusion rates based on conventional models fail to support the claimed rates within PdD (Palladium-Deuterium). The atoms are simply too far apart. This objections also fails for obvious reasons.

Mechanisms of cold fusion

In TGD framework exotic nuclei are needed to explain the selection rules which do not conform with standard nuclear physics. There are several options for what exotic nuclei could be.

1. Nuclei might be partially dark with some nucleons in dark state with Compton length of order atomic length scale.

2. Nuclei can also be exotic in the sense that some neutral color bonds have transformed to charged ones by exchange of dark W bosons effectively massless below atomic length scale. This could transform D nuclei to neutral ones and eliminate Coulomb wall. The presence of two oppositely charged bonds by (possibly dark) W exchange could give rise to a nucleus with same em charge as the original but different mass: presumably mass difference would be of order keV.

3. Also the emitted em radiation - say gamma rays - and particles - say protons or neutrons - could be dark and could remain undetected using standard means.

From this it is clear that it easy to invent models consistent with observations: careful consideration of data might however allow to fix the model to a high degree. One can try to deduce a more detailed model for cold fusion from observations, which are discussed systematically in [C112] and in the references discussed therein.

1. A critical phenomenon is in question. The average D/Pd ratio must be in the interval $(.85, .90)$. The current must be over-critical and must flow a time longer than a critical time. The effect occurs in a small fraction of samples. D at the surface of the cathode is found to be important and activity tends to concentrate in patches. The generation of fractures leads to the loss of the anomalous energy production. Even the shaking of the sample can have the same effect. The addition of even a small amount of H_2O to the electrolyte (protons to the cathode) stops the anomalous energy production.

   (a) These findings are consistent the view that patches correspond to a macroscopic quantum phase involving de-localized nuclear protons. The added ordinary protons and fractures could serve as a seed for a phase transition leading to the ordinary phase [K80].

   (b) An alternative interpretation is in terms of the formation of neutral exotic D and exotic Pd via exchange of exotic, possibly dark, W bosons massless below atomic length scale [L2].

2. When D_2O is used as an electrolyte, the process occurs when PdD acts as a cathode but does not seem to occur when it is used as anode. This suggests that the basic reaction is between the ordinary deuterium $D = pn$ of electrolyte with the exotic nucleus of the cathode. Denote by $\hat{p}$ the exotic proton and by $\hat{D} = n\hat{p}$ exotic deuterium at the cathode.

   For ordinary nuclei fusions to tritium and ^3He occur with approximately identical rates. The first reaction produces neutron and ^3He via $D + D \rightarrow n +^3 He$, whereas second reaction produces proton and tritium by $3H$ via $D + D \rightarrow p +^3 H$. The prediction is that one neutron per each tritium nucleus should be produced. Tritium can be observed by its beta decay to ^3He and neutron flux is several orders of magnitude smaller than tritium flux as found for instance by Tadahiko Mizuno and his collaborators (Mizuno describes the experimental process leading to this discovery in his book [C75]). Hence the reaction producing ^3He cannot occur significantly in cold fusion which means a conflict with the basic predictions of the standard nuclear physics.

   (a) The explanation discussed in [K80] does not involve exotic nuclei with charged color bonds. The assumption is that the proton in the target deuterium $\hat{D}$ is in the exotic state with large Compton length and the production of ^3He occurs very slowly since $\hat{p}$ and p correspond to different space-time sheets. Since neutrons and the proton of the D from the electrolyte are in the ordinary state, Coulomb barrier is absent and tritium production can occur. The mechanism also explains why the cold fusion producing ^3He and neutrons does not occur using water instead of heavy water.

   (b) Nuclear string model [L2] model with charged color bonds predicts that only neutral exotic tritium is produced considerably when incoming deuterium interacts with neutral exotic deuterium in the target. This requires that in target D nuclei exchange large $\hbar$ W boson with electron or Pd or other D nucleus. In the latter case the outcome is two exotic nuclei looking chemically like di-neutron and ^3He.

3. The production of ^4He has been reported although the characteristic gamma rays have not been detected.

   (a) ^4He can be produced in reactions such as $D + \hat{D} \rightarrow^4 He$ or its exotic counterpart in the model of [K80] .

   (b) Nuclear string model [K80] does not allow direct production of ^4He in D-D collisions.

4. Also more complex reactions between D and Pd for which protons are in exotic state, can occur. These can lead to the reactions transforming the nuclear charge of Pd and thus to nuclear transmutations.

   Both models allow nuclear transmutations. In nuclear string model [K80] the resulting exotic tritium can fuse with Pd and other nuclei and produce nuclear transmutations.

   The reported occurrence of nuclear transmutation such as $^{23}Na+^{16}O \rightarrow^{39} K$ in living matter [C93] allowing growing cells to regenerate elements K, Mg, Ca, or Fe, could be understood in nuclear string model if also neutral exotic charge states are possible for nuclei in living matter. The experimental signature for the exotic ions would be cyclotron energy spectrum containing besides the standard lines also lines with ions with anomalous mass number. This could be seen as a splitting of lines. For instance, exotic variants of ions such Na^+, K^+, Cl^-, Ca^{++} with anomalous mass numbers should exist. It would be easy to mis-interpret the situation unless the actual strength of the magnetic field is not checked.

5. Gamma rays, which should be produced in most nuclear reactions such as ^4He production to guarantee momentum conservation are not observed.

   (a) The explanation of the model of [K80] is that the recoil momentum goes to the macroscopic quantum phase and eventually heats the electrolyte system. This provides obviously the mechanism by which the liberated nuclear energy is transferred to the electrolyte difficult to imagine in standard nuclear physics framework. The emitted gamma rays could be also dark and observed only if they transform to ordinary ones.

   (b) In nuclear string model [L2] ^4He is not produced at all.

6. Both models explain why neutrons are not produced in amounts consistent with the anomalous energy production. The addition of water to the electrolyte is however reported to induce neutron bursts.

   (a) In the model of [K80] (no charged color bonds) a possible mechanism is the production of neutrons in the phase transition $\hat{p} \rightarrow p$. $\hat{D} \rightarrow p + n$ could occur as the proton contracts back to the ordinary size in such a manner that it misses the neutron. This however requires energy of 2.23 MeV if the rest masses of $\hat{D}$ and D are same. Also $\hat{D} + \hat{D} \rightarrow n +^3 He$ could be induced by the phase transition to ordinary matter when $\hat{p}$ transformed to p does not combine with its previous neutron partner to form D but recombines with $\hat{D}$ to form $^3\hat{He} \rightarrow^3 He$ so that a free neutron is left.

   (b) Nuclear string model [L2] would suggest that the collisions of protons of water with exotic neutral D with negatively charged color bond produce neutron and ordinary D. This requires the transformation of negatively charged color bond between p and n of target D to a neutral color bond between incoming p and neutron of target.

A cautious conclusion is that nuclear string model with exotic color bonds and dark weak bosons is the more natural option. Also dark protons suggested strongly by the model for the dark portion of water can be considered but partial darkness of nuclei is perhaps an artificial idea. Note that all nuclei might appear as dark variants with size scale of molecules and analogous to folded proteins. This intriguing similarity creates the question whether the physics of linear biomolecules mimics nuclear physics and whether dark nuclei are involved with this mimicry natural in the fractal Universe of TGD.

9.7.3 Does Rossi's reactor give rise to cold fusion?

Lubos Motl has been raging several times about the cold fusion gadget of Andrea Rossi and I decided to write the following response as he returned to the topic again. The claim of Rossi and physicist Fogardi [C80] is that the cold fusion reaction of H and Ni producing Cu takes place in the presence of some "additives" (Palladium catalyst as in may cold fusion experiments gathering at its surface Ni?).

Objections claiming that the evaporation of water does not actually take place

Lubos Motl of course "knows" before hand that the gadget cannot work: Coulomb barrier. Since Lubos is true believer in naive text book wisdom, he simply refuses to consider the possibility that the physics that we learned during student days might not be quite right. Personally I do not believe or disbelieve cold fusion: I just take it seriously as any person calling himself scientist should do. I have been developing for more than 15 years ideas about possible explanation of cold fusion in TGD framework. The most convincing idea is that large value of Planck constant associated with nuclei could be involved scaling up the range of weak interactions from 10^{-17} meters to atomic size scale and also scaling up the size of nucleus to atomic size scale so that nucleus and even quarks would like constant charge densities instead of point like charge. Therefore Coulomb potential would be smoothed and the wall would become much lower (see this and this) [K80, L2].

One must say in honor of Lubos that at this time he had detailed arguments about what goes wrong with the reactor of Rossi: this is in complete contrast with the usual arguments of skeptics which as a rule purposefully avoid saying anything about the actual content and concentrate on ridiculing the target. The reason is of course that standard skeptic is just a soldier who has got the list of targets to be destroyed and as a good soldier does his best to achieve the goal. Thinking is not what a good soldier is expected to do since the professors in the consultive board take care of this and give orders to those doing the dirty job.

As a theoretician I have learned the standard arguments used to debunk TGD: logic is circular, text is mere world salad, everything is just cheap numerology, too many self references, colleagues have not recognized my work, the work has not been published in respected journals, and so on. The additional killer arguments state that I have used certain words which are taboos and already for this reason am a complete crackpot. Examples of bad words are "water memory", "homeopathy", "cold fusion", "crop circles", "quantum biology", "quantum consciousness". There is of course no mention about the fact that I have always emphasized that I am skeptic, not a believer or disbeliever, and only make the question "What if...." and try to answer it in TGD framework. Intellectual honesty does not belong to the virtues of skeptics who are for modern science what jesuits were for the catholic church. Indeed, as Loyola said: the purpose sanctifies the deeds.

Lubos has real arguments but they suffer from the strong negative emotional background coloring so that one cannot be trust the rationality of the reasoning. The core of the arguments of Lubos is following.

1. The water inside reactor is heated to a temperature of 100.1 C. This is slightly above 100 C defining the nominal value of the boiling point temperature at normal pressure. The problem is that if the pressure is somewhat higher, the boiling point increases and the it could happen that the no evaporation of the water takes place. If this is the case, the whole energy fed into the reactor could go to the heating of the water. The input power is indeed somewhat higher than the power needed to heat the water to this temperature without boiling so that this possibility must be taken seriously and the question is whether the water is indeed evaporated.

   Comments:

   (a) This looks really dangerous. Rossi uses water only as a passive agent gathering the energy assumed to be produced in the fusion of hydrogen and nickel to copper. This would allow to assume that the water fed in is at lower temperature and also the water at outlet is below boiling boiling. Just by measuring the temperature at the outlet one can check whether the outgoing water has temperature higher than it would be if all input energy goes to its heating.

(b) This is only one particular demonstration and it might be that there are other demonstrations in which the situation is this. As a matter fact, from an excellent video interview of Nobelist Brian Josephson one learns that there are also demonstrations in which water is only heated so that the argument of Lubos does not bite here. The gadget of Rossi is already used to heat university building. The reason why the evaporation is probably that this provides an effective manner to collect the produced energy. Also by reading the Nyteknik report [C80] one learns that the energy production is directly measured rather than being based on the assumption that evaporation occurs.

2. Is the water evaporated or not? This is the question posed by Lubos. The demonstration shows explicitly that there is a flow of vapor from the outlet. As Rossi explains there is some condensation. Lubos claims that the the flow of about 2 liters of vapor per second resulting from the evaporation 2 ml of water per second should produce much more dramatic visual effect. More vapor and with a faster flow velocity. Lubos claims that water just drops from the tube and part of it spontaneously evaporates. This is what Lubos wants to see and I have no doubt that he is seeing it. Strong belief can move mountains! Or at least can make possible the impression that they are moving!

   Comments:

   (a) I do not see what Lubos sees but I am not able to tell how many liters of vapor per second comes out. Therefore the visual demonstration as such is not enough.

   (b) I wonder why Rossi has not added flow meter measuring the amount of vapor going through the tube. Second possibility is to allow the vapor condensate back to water in the tube by using heat exchanger. This would allow to calculate the energy gained without making the assumption that all that comes out is vapor. It might be that in some experiments this is done.

To sum up, Lubos in his eagerness to debunk forgets that he is concentrating on single demonstration and forgetting other demonstrations altogether and also the published report [C80] to which his argument do not apply. I remain however skeptic (I mean real skeptic, the skepticism of Lubos and -sad to say- of quite too many skeptics- has nothing to do with a real skeptic attitude). Rossi should give information about the details of his invention and quantitative tests really measuring the heat produced should be carried out and published. Presumably the financial aspects related to the invention explain the secrecy in a situation in which patenting is difficult.

Objections from nuclear physics

The reading of Rossi's paper and Wikipedia article led me to consider in more detail also various nuclear physics based objections against Rossi's reactor [C18]. Coulomb barrier, the lack of gamma rays, the lack of explanation for the origin of the extra energy, the lack of the expected radioactivity after fusing a proton with ^{58}Ni (production of neutrino and positron in beta decay of ^{59}Cu), the unexplained occurrence of 11 per cent iron in the spent fuel, the 10 per cent copper in the spent fuel strangely having the same isotopic ratios as natural copper, and the lack of any unstable copper isotopes in the spent fuel as if the reactor only produced stable isotopes.

1. *Could natural isotope ratios be determined by cold fusion?*

The presence of Cu in natural isotope ratios and the absence of unstable copper isotopes of course raise the question whether the copper is just added there. Also the presence of iron is strange. Could one have an alternative explanation for these strange co-incidences?

1. Whether unstable isotopes of Cu are present or not, depends on how fast ^{A}Cu, $A < 63$ decays by neutron emission: this decay is expected to be fast since it proceeds by strong interactions. I do not know enough about the detailed decay rates to be able to say anything about this.

2. Why the isotope ratios would be the same as for naturally occurring copper isotopes? The simplest explanation would be that the fusion cascades of two stable Ni isotopes determine the ratio of naturally occurring Cu isotopes so that cold fusion would be responsible for their production. As a matter fact, TGD based model combined with what is claimed about

bio-fusion led to the proposal that stable isotopes are produced in interstellar space by cold fusion and that this process might even dominate over the production in stellar interiors. This wold solve among other things also the well-known Lithium problem. The implications of the ability to produce technologically important elements artificially at low temperatures are obvious.

2. Could standard nuclear physics view about cold fusion allow to overcome the objections?

Consider now whether one could answer the objections in standard nuclear physics framework as a model for cold fusion processes.

1. By inspecting stable nuclides one learns that there are two fusion cascades. In the first cascade the isotopes of copper would be produced in a cascade starting from with $^{58}Ni + n \rightarrow ^{59} Cu$ and stopping at ^{63}Cu. All isotopes ^{A}Cu, $A \in \{55, 62\}$ are unstable with lifetime shorter than one day. The second fusion cascade begins from ^{63}Ni and stops at ^{65}Cu.

2. The first cascade involves five cold fusions and 4 weak decays of Cu. Second cascade involves two cold fusions and one weak decay of Cu. The time taken by the cascade would be same if there is single slow step involved having same duration. The only candidates for the slow step would be the fusion of the stable Ni isotope with the neutron or the fusion producing the stable Cu isotope. If the fusion time is long and same irrespective of the neutron number of the stable isotope, one could understand the result. Of course, this kind of co-incidence does not look plausible.

3. ^{A-5}Fe could be produced via alpha decay $^{A}Cu \rightarrow ^{A-4} Co + \alpha$ followed by $^{A-4}Co \rightarrow ^{A-5} Fe + p$.

3. Could TGD view about cold fusion allow to overcome the objections?

The claimed absence of positrons from beta decays and the absence of gamma rays are strong objections against the assumption that standard nuclear physics is enough. In TGD framework it is possible to ask whether the postulated fusion cascades really occur and whether instead of it weak interactions in dark phase of nuclear matter with range of order atomic length scale are responsible for the process because weak bosons would be effectively massless below atomic length scale. For TGD inspired model of cold fusion see `tgdtheory.fi/public_html/pdfpool/padnucl.pdf` and `tgdtheory.fi/public_html/paddark/paddark.html#nuclstring` [K80, L2].

1. The nuclear string model assumes that nucleons for nuclear strings with nucleons connected with color bonds having quark and antiquark at their ends. Color bonds could be also charged and this predicts new kind of internal structure for nuclei. Suppose that the space-time sheets mediating weak interactions between the color bonds and nucleons correspond to so large value of Planck constant that weak interaction length scale is scaled up to atomic length scale. The generalization of this hypothesis combined with the p-adic length scale hypothesis is actually standard piece of TGD inspired quantum biology (`tgdtheory.fi/public_html/paddark/paddark.html#darkforces`) [K28].

2. The energy scale of the excitations of color bond excitations of the exotic nuclei would be measured in keVs. One could even consider the possibility that the energy liberated in cold fusion would correspond to this energy scale. In particular, the photons emitted would be in keV range corresponding to wavelength of order atomic length sale rather than in MeV range. This would resolve gamma ray objection.

3. Could the fusion process $^{58}Ni + n$ actually lead to a generation of Ni nucleus ^{59}Ni with one additional positively charged color bond? Could the fusion cascade only generate exotic Ni nuclei with charged color bonds, which would transform to stable Cu by internal dark W boson exchange transferring the positive charge of color bond to neutron and thus transforming it to neutron? This would not produce any positrons. This cascade might dominate over the one suggested by standard nuclear physics since the rates for beta decays could be much slower than the rate for direct generation of Ni isotopes with positively charged color bonds.

4. In this case also the direct alpha decay of Ni with charged color bond to Fe with charged color bond decaying to ordinary Fe by positron emission can be imagined besides the proposed mechanism producing Fe.

5. If one assumes that this process is responsible for producing the natural isotope ratios, one could overcome the basic objections against Rossi's reactor.

The presence of em radiation in keV range would be a testable basic signature of the new nuclear physics as also effects of X-ray irradiation on measured nuclear decay and reaction rates due to the fact that color bonds are excited. As a matter fact, it is known that X-ray bursts from Sun in keV range has effects on the measured nuclear decay rates and I have proposed that the proposed exotic nuclear physics in keV range is responsible for the effect. Quite generally, the excitations of color bonds would couple nuclear physics with atomic physics and I have proposed that the anomalies of water could involve classical Z^0 force in atomic length scales. Also the low compressibility of condensed matter phase could involve classical Z^0 force. The possible connections with sono-luminescence and claimed sonofusion are also obvious (`http://tgdtheory.fi/public_html/paddark/paddark.html#exonuclear`) [K30].

More recent results concerning heat production in Rossi's reactor

According to the article "Indication of anomalous heat energy production in a reactor device containing hydrogen loaded nickel powder" [H5] (`http://arxiv.org/pdf/1305.3913.pdf`) cold fusion has been demonstrated quite convincingly so that "indication" in the title can be take as a humorous understatement.

The studied system is the E-Cat HT of Rossi containing Ni power plus unknown catalyst under hydrogen pressure. The durations of test runs were about 100 hours. Heat cameras were used to measure the temperature at the upper surface of the cylinder. The lower bound for the heat power estimated theoretically from the temperature distribution using estimates for radiation power, very small conduction power through the contacts with environment, and from estimate convection power through the surrounding air. In one of the runs the input power was 360 W and output power 2034 W giving $COP \simeq 5.6$. The run took 96 hours and the weight of Ni cylinder was .236 kg. On basis of this the heat energy per weight is higher than .68 MJ/kg which is higher than for any conventional energy source. This is a lower bound since only the heat energy produced during the test run is included.

To my opinion, it seems safe to conclude that low energy nuclear reactions can be regarded as an established fact and the commercialization is indeed in full swing. It is a pity that at the same time academic theoretical physics after the results from LHC has reached dead end basically due to the sticking to the reductionistic dogma, which does not allow any new physics above elementary particle length scale - and if we believe string theorists- above Planck length length scale.

9.7.4 Sono-luminescence, classical Z^0 force, and hydrodynamic hierarchy of p-adic length scales

Sono-luminescence [D35] , [D35] is a peculiar phenomenon, which might provide an application for the hydrodynamical hierarchy. The radiation pressure of a resonant sound field in a liquid can trap a small gas bubble at a velocity node. At a sufficiently high sound intensity the pulsations of the bubble are large enough to prevent its contents from dissolving in the surrounding liquid. For an air bubble in water, a still further increase in intensity causes the phenomenon of sono-luminescence above certain threshold for the sound intensity. What happens is that the minimum and maximum radii of the bubble decrease at the threshold and picosecond flash of broad band light extending well into ultraviolet is emitted. Rather remarkably, the emitted frequencies are emitted simultaneously during very short time shorter than 50 picoseconds, which suggests that the mechanism involves formation of coherent states of photons. The transition is very sensitive to external parameters such as temperature and sound field amplitude.

A plausible explanation for the sono-luminescence is in terms of the heating caused by shock waves launched from the boundary of the adiabatically contracting bubble [D35] , [D35] . The temperature jump across a strong shock is proportional to the square of Mach number and increases with decreasing bubble radius. After the reflection from the minimum radius $R_s(min)$ the outgoing shock moves into the gas previously heated by the incoming shock and the increase of the temperature after focusing is approximately given by $T/T_0 = M^4$, where M is Mach number at focusing and $T_0 \sim 300\ K$ is the temperature of the ambient liquid. The observed spectrum of sono-luminescence is explained as a brehmstrahlung radiation emitted by plasma at minimum temperature $T \sim 10^5\ K$. There is a fascinating possibility that sono-luminescence relates directly to the classical Z^0 force.

Even standard model reproduces nicely the time development of the bubble and sono-luminescence spectrum and explains sensitivity to the external parameters [D35] , [D35] . The problem is to understand how the length scales are generated and explain the jump-wise transition to sono-luminescence and the decrease of the bubble radius at sono-luminescence: ordinary hydrodynamics predicts continuous increase of the bubble radius. The length scales are the ambient radius R_0 (radius of the bubble, when gas is in pressure of 1 atm) and the minimum radius $R_s(min)$ of the shock wave determining the temperature reached in shock wave heating. Zero radius is certainly not reached since shock front is susceptible to instabilities.

p-Adic length scale hypothesis and the length scales of sono-luminescence

Since p-adic length scale hypothesis introduces a hierarchy of hydrodynamics with each hydrodynamics characterized by a p-adic cutoff length scale there are good hopes of achieving a better understanding of these length scales in TGD. The change in bubble size in turn could be understood as a change in the 'primary' condensation level of the bubble.

1. The bubble of air is characterized by its primary condensation level k. The minimum size of the bubble at level k must be larger than the electron Compton scale $L_e(k) = \sqrt{5}L(k)$. This suggests that the transition to photo-luminescence corresponds to the change in the primary condensation level of the air bubble. In the absence of photo-luminescence the level can be assumed to be $k = 163$ with $L_e(163) \sim .76\ \mu m$ in accordance with the fact that the minimum bubble radius is above $L_e(163)$. After the transition the primary condensation level of the air bubble one would have $k = 157$ with $L_e(157) \sim .07\ \mu m$. In the transition the minimum radius of the bubble decreases below $L_e(163)$ but should not decrease below $L_e(157)$: this hypothesis is consistent with the experimental data [D35] , [D35] .
2. The particles of hydrodynamics at level k have minimum size $L(k_{prev})$. For $k = 163$ one has $k_{prev} = 157$ and for $k = 157$ $k_{prev} = 151$ with $L_e(151) \sim 11.8\ nm$. It is natural to assume that the minimum size of the particle at level k gives also the minimum radius for the spherical shock wave since hydrodynamic approximation fails below this length scale. This means that the minimum radius of the shock wave decreases from $R_s(min, 163) = L_e(157)$ to $R_s(min, 157) = L_e(151)$ in the transition to sono-luminescence. The resulting minimum radius is 11 nm and much smaller than the radius .1 $\mu\ m$ needed to explain the observed radiation if it is emitted by plasma.

A quantitative estimate goes along lines described in [D35] , [D35] .

1. The radius of the spherical shock is given by

$$R_s = At^{\alpha} \ , \tag{9.7.1}$$

   where t is the time to the moment of focusing and α depends on the equation of state (for water one has $\alpha \sim .7$).

2. The collapse rate of the adiabatically compressing bubble obeys

$$\frac{dR}{dt} = c_0(\frac{2}{3\gamma}\frac{\rho_0}{\rho}(\frac{R_m}{R_0})^3)^{1/2} \ , \tag{9.7.2}$$

   where c_0 is the sound velocity in gas, γ is the heat capacity ratio and ρ_0/ρ is the ratio of densities of the ambient gas and the liquid.

3. Assuming that the shock is moving with velocity c_0 of sound in gas, when the radius of the bubble is equal to the ambient radius R_0 one obtains from previous equations for the Mach number M and for the radius of the shock wave

$$\begin{aligned} M &= \frac{\frac{dR_s}{dt}}{c_0} = (t_0/t)^{\alpha-1} \ , \\ R_s &= R_0(t/t_0)^{\alpha} \ , \\ t_0 &= \frac{\alpha R_0}{c_0} \ . \end{aligned} \quad (9.7.3)$$

where t_0 is the time that elapses between the moment, when the bubble radius is R_0 and the instant, when the shock would focus to zero radius in the ideal case. For $R_0 = L_e(167)$ (order of magnitude is this) and for $R_s(min) = L_e(151)$ one obtains $R_0/R_s(min) = 256$ and $M \simeq 10.8$ at the minimum shock radius.

4. The increase of the temperature immediately after the focusing is approximately given by

$$\frac{T}{T_0} \simeq M^4 = (\frac{R_0}{R_s})^{\frac{4(1-\alpha)}{\alpha}} \simeq 1.3 \cdot 10^4 \ . \quad (9.7.4)$$

For $T_0 = 300\ K$ this gives $T \simeq 4 \cdot 10^6\ K$: the temperature is far below the temperature needed for fusion.

In principle the further increase of the temperature can lead to further transitions. The next transition would correspond to the transition $k = 157 \rightarrow k = 151$ with the minimum size of particle changing as $L_e(k_{prev}) \rightarrow L_e(149)$. The next transition corresponds to the transition to $k = 149$ and $L_e(k_{prev}) \rightarrow L_e(141)$. The values of the temperatures reached depend on the ratio of the ambient size R_0 of the bubble and the minimum radius of the shock wave. The fact that R_0 is expected to be of the order of $L_e(k_{next})$ suggests that the temperatures achieved are not sufficiently high for nuclear fusion to take place.

Could sonoluminescence involve the formation of a phase near vacuum extremals?

In TGD inspired model of cell membrane [K69] a key role is played by almost vacuum extremals for which the induced Kähler field is very small. Vacuum extremals are accompanied by a strong classical Z^0 field proportional to classical electromagnetic field and given by $Z^0 = -2\gamma/p$, $p = sin^2(\theta_W)$. One could also imagine that em field is vanishing in which case Z^0 field is proportional to Kähler field and also strong because of $Z^0 = 6J/p$, $p = sin^2(\theta_W)$ proportionality. In this case also classical color fields are present. It is however not clear whether these fields can be realized as preferred extremals of Kähler action.

The classical Z^0 field should have a source and the vacuum polarization in the sense that flux tubes are generated with many fermion state and its conjugate at its opposite ends would generate it. The Compton scale of weak bosons must correspond to $L_e(157)$ so that either dark variants of ordinary weak bosons or their light variants would be in question. Both would be effectively massless below $L_e(157)$. The simplest situation corresponds to many-neutrino state for vacuum extremals but also many quark states are possible when em field for the flux tube vanishes.

The length scales involved correspond to Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ and together with $k = 151$ and $k = 167$ define biologically important length scales [K69] . The p-adically scaled up variants and dark variants of of QCD and weak physics have been conjectured to play key role in biology between length scales 10 nm (cell membrane thickness) and 2.5 μm (the size scale of nucleus). This motivates the question whether a nearly vacuum extremal phase (as far as induced gauge fields are considered) accompanies the transition changing the p-adic length scale associated with the bubble from $k = 163$ to $k = 157$. The acceleration in the strong Z^0 field associated with the flux tubes could generate the visible light as brehmstrahlung radiation, perhaps also Z^0 and W bremstrahlung could be generated and would decay to photons and charged particles and generate a plasma in this manner. If the weak scale is given by $k_W = 157$, the mass scale of weak bosons is $2^{-31} \simeq 10^{-9}/2$ times smaller than that of ordinary weak bosons (about 50 eV which corresponds to a temperature of 5×10^5 K). A further transition to $k = 151$ would correspond to gauge boson mass scale 400 eV and temperature or order 4×10^6 K.

Could phase transitions increasing Planck constant and p-adic prime accompany sonoluminescence

In sonoluminescence external sound source induces oscillation of the radius of a bubble of water containing noble gas atoms. The unexpected observation is generation of radiation even at gamma ray energies and it is proposed that nuclear fusion might take place.

A possible new element in the model is h_{eff} increasing phase transition of the space-time sheet containing the water vapour and other atoms to dark phase during the expansion phase and reduction back to the ordinary value during implosion period now forced by the sound wave. If implosion actually takes place spontaneously then the energy of sound wave could be liberated as luminescence. If also dark hydrogen atoms are generated, dark protons could be able to circumvent the Coulomb wall so that low energy nuclear reactions could occur. On the other hand, if the phase transition reducing the Planck constant and increasing p-adic length scale takes place for the water space-time sheet in such a manner that the two scale changes compensate each other (this requires $h_{eff} = 2^k h$ and $p \rightarrow 2^{2k}p$ (this in excellent approximation), zero point kinetic energy (ZPKE) is liberated and could heat the bubble and induce high energy radiation and perhaps even the proposed ordinary fusion. Cold fusion however seems more elegant alternative. The fact that neutron yield has not been observed in sonoluminescence suggests that ordinary hot fusion is not involved.

I have earlier considered the possibility that classical long ranged Z^0 fields predicted by TGD might be involved and give rise to a new interaction possibly related to sonoluminescence. I have proposed that classical Z^0 fields could play a role in the physics of cell membrane. The speculative proposal is that cell membrane could be in two possible states: the first ("ordinary") state would correspond to far from vacuum extremal for which electric field dominates. Second state would be near to vacuum extremal: in this case classical Z^0 field would dominate and give rise to rather radical modification of the model for cell membrane since Z^0 membrane potential would replace the ordinary one. Neurons serving as sensory receptors might correspond to this phase.

This model remains very speculative as also the possible role of classical Z^0 fields in sono-fusion. Note however that the phase transition increasing h_{eff} implies a dilution to vapour like phase ("electrically expanded water") and means that the state is near vacuum. By quantum classical correspondence classical Z^0 fields might become important. In the case of cell membrane Z^0 Coulomb energy defined by Z^0 potential is much stronger than its electronic counterpart and corresponds to voltage of order few eV and therefore to visible photon energies roughly 50 times higher than the energies assignable to the ordinary membrane potential of about .06 eV. One can wonder whether similar effect could appear also in electrolysis where also strong local electric fields appear.

9.8 The TGD variant of the model of Widom and Larsen for cold fusion

Widom and Larsen (for articles see the Widom Larsen LENR Theory Portal [C16] (`http://newenergytimes.com/v2/sr/WL/WLTheory.shtml`)) have proposed a theory of cold fusion (LENR) [C13], which claims to predict correctly the various isotope ratios observed in cold fusion and accompanying nuclear transmutations. The ability to predict correctly the isotope ratios suggests that the model is on the right track. A further finding is that the predicted isotope ratios correspond to those appearing in Nature which suggests that LENR is perhaps more important than hot fusion in solar interior as far as nuclear abundances are considered. TGD leads to the same proposal and Lithium anomaly could be understood as one implication of LENR [L2]. The basic step of the reaction would rely on weak interactions: the proton of hydrogen atom would transform to neutron by capturing the electron and therefore would overcome the Coulomb barrier.

9.8.1 Challenges of the model

The model has to meet several challenges.

1. The electron capture reaction $p + e \rightarrow n + \nu$ is not possible for ordinary atom since the mass difference of neutron is 1.3 MeV and larger than electron mass .5 MeV (electron has too small kinetic energy). The proposal is that strong electric fields at the catalyst surface imply renormalization effects for the plasmon phase at the surface of the catalyst increasing electron mass so that it has width of few MeVs [C121]. Physically this would mean that strong em radiation helps to overcome the kinematical threshold for the reaction. This assumption [C83]: the claim is that the mass renormalization is much smaller than claimed by Widom and Larsen.

2. Second problem is that weak interactions are indeed very weak. The rate is proportional to $1/m_W^4$, $m_W \sim 100$ GeV whereas for the exchange of photon with energy E it would be proportional to $1/E^4$. For $E \sim 1$ keV the ratio of the rates would be of the order of 10^{-48}!

   This problem could be circumvented if the transition from proton to neutron occurs coherently for large enough surface patch. This would give rate proportional to N^2, where N is the number electrons involved. Another mechanism hoped to help to get high enough reaction rate is based on the assumption that the neutron created by the capture process has ultra-low momentum. This is the case if the mass renormalization of electron is such that the energies of the neutrons produced in the reaction are just above the kinematical threshold. Note however that this reduces the electron capture cross section. The argument is that the absorption rate for neutron by target nucleus is by very general arguments proportional to $1/v_n$, v_n the velocity of neutron. Together these two mechanisms are hoped to give high enough rate for cold fusion.

3. The model must also explain why gamma radiation is not observed and why neutrons are produced much less than expected. Concerning gamma rays one must assume that the heavy electrons of the plasmon phase assigned to the surface of the catalyst absorb the gamma rays and re-emit them as infrared light emitted to environment as heat. Ordinary electrons cannot absorb gamma rays but heavy electrons can [C120], and the claim is that they do transform gamma rays to infrared photons. If the neutrons created in LENR have ultra-low energies their capture cross sections are enormous and the claim is that they do not get out of the system.

The assumption that electron mass is renormalized so that the capture reaction can occur but occurs only very near threshold so that the resulting neutrons are ultraslow has been criticized [C83].

9.8.2 TGD variant of the model

TGD allows to consider two basic approaches to the LENR.

1. **Option I** involves only dark nucleons and dark quarks. In this case, one can imagine that the large Compton length of dark proton - at least of order atomic scale - implies that it overlaps target nucleus, which can see the negatively charged d quark of the proton so that instead of Coulomb wall one has Coulomb well.

2. **Option II** involves involves both dark weak bosons and possibly also dark nucleons and dark electrons. The TGD inspired model for living matter - in particular, the model for cell membrane involving also Z^0 membrane potential in the case of sensory receptor neurons [K28] - favors the model involving both dark weak bosons, nucleons, and even electrons. Chiral selection for biomolecules is extremely difficult to understand in standard model but could be understood in terms of weak length scale of order atomic length scale at least: below

this scale dark weak bosons would be effectively massless and weak interactions would be as strong as em interactions. The model for electrolysis based on plasmoids identified as primitive life forms supports also this option. The presence of dark electrons is suggested by Tesla's cold currents and by the model of cell membrane.

This option is fixed quantitatively by the condition that the Compton length of dark weak bosons is of the order of atomic size scale at least. The ratio of the corresponding p-adic size scales is of order 10^7 and therefore one has $h_{eff} \sim 10^{14}$. The condition that $h_{eff}/h = 2^k$ guarantees that the phase transition reducing h_{eff} to h and increasing p-adic prime p by about 2^k and p-adic length scale by $2^{k/2}$ does not change the size scale of the space-time sheet and liberates cyclotron magnetic energy $E_n(1-2^{-k}) \simeq E_n$.

Consider next **Option II** by requiring that the Coulomb wall is overcome via the transformation of proton to neutron. This would guarantee correct isotope ratios for nuclear transmutations. There are two options to consider depending on whether a) the W boson is exchanged between proton nucleus (this option is not possible in standard model) or b) between electron and proton (the model of Widom and Larsen relying on the critical massivation of electron).

1. **Option II.1**. Proton transforms to neutron by exchanging W boson with the target nucleus.

   (a) In this case kinematics poses no obvious constraints on the process. There are two options depending on whether the neutron of the target nucleus or quark in the neutral color bond receives the W boson.

   (b) If electron and proton are dark with $h_{eff}/h = n = 2^k$ in the range $[10^{12}, 10^{14}]$ the situation can change since W boson has its usual mass from the point of view of electron and proton. $\hbar^4/m_W^4$ factor in differential cross section for 2-to-2 scattering by W exchange is scaled up by n^4 (see the appendix of [A51] so that effectively m_W would be of order 10 keV for ordinary $\hbar$.

   (c) One can argue that in the volume defined by proton Compton length $\lambda_p \simeq 2^{-11}\lambda_e \in [1.2, 12]$ nm one has a superposition of amplitudes for the absorption of dark proton by nucleus. If there are N nuclei in this volume, the rate is proportional to N^2. One can expect at most $N \in [10^3, 10^6]$ target nuclei in this volume. This would give a factor in the range $10^9 - 10^{12}$.

2. **Option II.2**: Electron capture by proton is the Widom-Larsen candidate for the reaction in question. As noticed, this process cannot occur unless one assumes that the mass of electron is renormalized to have a value in a range of few MeV. If dark electrons are heavier than ordinary, the process could be mediated by W boson exchange and if the electron and proton have their normal sizes the process occurs with same rate as em processes.

   If electron and proton are dark with $h_{eff}/h = n \in [10^{12}, 10^{14}]$ the situation can change since W boson has its usual mass from the point of view of electron and proton. 2-to-2 cross section is proportional to $\hbar^4$ and is scaled up by n^4. One the other hand, the naive expectation is that $|\Psi(0)|^2 \propto m_e^3/h_{eff}^3 \propto 1/n^{-3}$ for electron is scaled by n^{-3} so that the rate is increased by a factor of order $n \in [10^{12}, 10^{14}]$ (electron Compton length is of order cell size scale! instead of Angstrom) from its ordinary value. This is not enough.

   On the other hand, one can argue in the volume defined by proton Compton size one has a superposition of amplitudes for the absorption of electron. If there are N dark electrons in this volume, the rate is proportional to N^2. One can expect at most 10^6 dark electrons in the volume of scale 10 nm so that this could give a factor 10^{12}. This would give amplification factor 10^{26} to the weak rate so that it would be only by two orders of magnitude smaller than the rate for massless weak bosons.

There are also other strange features to be understood.

1. The absence of gamma radiation could be due to the fact that the produced gamma rays are dark. For $h_{eff}/h \in [10^{12}, 10^{14}]$ the energy frequency of 1 MeV dark gamma ray would correspond to that of photon with energy of [1,.1] μeV and thus to radio wave photon with

wavelength of order 1 m and frequency of order 3×10^8 Hz. In Widom-Larsen model the photons would be infrared photons. The decay of the dark gamma ray to a bunch of ordinary radio wave photons should be observed as radio noise. Note that Gariaev has observed transformation of laser light scattered from DNA to radio wave photons with frequencies down to 1 kHz at least.

2. The absence of the neutrons could be understood if they are dark and simply do not interact with visible matter before phase transition to ordinary neutrons. One can imagine an alternative interpretation allowing the interaction and assuming that nuclei are dark in the reaction volume. The large Compton wavelength implies that dark neutrons are absorbed by dark nuclei coherently in a volume of order 1.2-12 nm so that an additional amplification factor $N^2 \in [10^9, 10^{12}]$ would be obtained. The absorption cross section for neutrons should be proportional to $\hbar^2$ giving a huge amplification factor in the range $[10^{24}, 10^{48}]$. Effectively this corresponds to the assumption of Widom and Larsen stating that neutrons have ultra-low momentum.

The natural question is why h_{eff} us such that the resulting scale as photon wavelength corresponds to energy in scale 10-100 keV. The explanation could relate to the predicted exotic nuclei obtained by replacing some neutral color bonds connecting nucleons with charged ones and exchange of weak boson would affect this replacement. Could the weak physics associated with $h_{eff} \in [10^{12}, 10^{14}]$ be associated with dark color bonds? The reported annual variations of the nuclear reaction rates correlating with the distance of Earth from Sun suggest that these variations are induced by solar X rays [C57].

9.9 Dark atomic physics

Dark matter might be relevant also for atomic physics and in the sequel some speculations along these lines are represented. Previous considerations assumed that only field bodies can be dark and this is assumed also now. The notion of dark atom depends strongly on the precise meaning of the generalized imbedding space and I have considered several options.

1. The first option was based on the singular coverings $CD \times CP_2 \rightarrow CD/G_a \times CP_2/G_b$. This approach has a concrete connection to the quantization and the selection of quantization axes correlates closely with the identification of groups G_a and G_b. The questionable assumption is that elementary particle like partonic 2-surfaces remain invariant under the cyclic groups $G_a \times G_b$.

2. The next proposal was that both factor spaces and coverings of H are possible. For this option the notion of covering is somewhat unsatisfactory because it lacks concreteness. Singular factor of CD and CP_2 spaces make possible all rational values of Planck constant and one loses the vision about evolution as drift to the sectors of imbedding space characterized by increasing value of Planck constant.

3. The last proposal is based on the realization that basic quantum TGD could well explain the hierarchy of Planck constants in terms of singular covering spaces emerging naturally when the time derivatives of the imbedding space coordinates are many-valued functions of the canonical momentum densities. In this framework singular factors spaces are not possible and the formula $r \equiv \hbar/\hbar_0 = n_a n_b$ emerges naturally as well as charge fractionization. One also ends up to a unique recipe for how to obtain binding energies in this kind of situation and the results are consistent with the earlier formulas deduced on purely formal arguments. Groups G_a and G_b do not directly correspond to subgroups of isometry groups but the fractionization of quantum numbers implied by the scaling of Planck constant implies that wave functions for the selected quantization axes behave as if the maximal cyclic subgroups of G_a and G_b had a geometric meaning.

For covering space option fermion number is fractionized. The group algebra of $G_a \times G_b$ defines $n_a n_b$ single particle wave functions in the covering. The simplest option is that total fermion number is integer valued so that the many-sheeted structure is analogous to a full Fermi sphere containing $n_a n_b$ fermions with fractional fermion number $1/n_a n_b$. A more general option allows states with fractional total fermion number varying from from $1/n_a n_b$ to 1. One could generalize the condition about integer fermion number so that it holds for the entire quantum state involving several covering regions and the condition would correspond to the $G_a \times G_b$ singletness of the physical states.

9.9.1 Dark atoms and dark cyclotron states

The development of the notion of dark atom involves many side tracks which make me blush. The first naive guess was that dark atom would be obtained by simply replacing Planck constant with its scaled counterpart in the basic formulas and interpreting the results geometrically. After some obligatory twists and turns it became clear that this assumption is indeed the most plausible one. The main source of confusion has been the lack of precise view about what the hierarchy of Planck constants means at the level of imbedding space at space-time.

The rules are very simple when one takes the singular coverings assigned to the many-valuedness of the time-derivatives of imbedding space coordinates as functions of canonical momentum densities as a starting point.

1. The mass and charge of electron are fractionized as is also the reduced mass in Schrödinger equation. This implies the replacements $e \rightarrow e/r$, $m \rightarrow m/r$, and $\hbar \rightarrow r\hbar_0$, $r = n_a n_b$, in the general formula for the binding energy assigned with single sheet of the covering. If maximal number $n_a n_b$ are present corresponding to a full "Fermi sphere", the total binding energy is r times the binding energy associated with single sheet.

2. In the case of hydrogen atom the proportionality $E \propto m/\hbar^2$ implies that the binding energy for single sheet of the covering scales as $E \rightarrow E/(n_a n_b)^3$ and maximal binding energy scales as $E \rightarrow E/(n_a n_b)^2$. This conforms with the naive guess. For high values of the nuclear charge Z it can happen that the binding energy is larger than the rest mass and fractionization might take place when binding energy is above critical fraction of the rest mass.

3. In the case of cyclotron energies one must must decide what happens to the magnetic flux. Magnetic flux quantization states that the flux is proportional to $\hbar$ for each sheet separately. Hence one has $\Phi \rightarrow r\Phi$ for each sheet and the total flux scales as r^2. Since the dimensions of the flux quantum are scaled up by r the natural scaling of the size of flux quantum is by r^2. Therefore the quantization of the magnetic flux requires the scaling $B \rightarrow B/r$. The cyclotron energy for single sheet satisfies $E \propto \hbar qB/m$ and since both mass m and charge q become fractional , the energy E for single sheet remains invariant whereas total cyclotron energy is scaled up by r in accordance with the original guess and the assumption used in applications.

4. Dark cyclotron states are expected to be stable up to temperatures which are r times higher than for ordinary cyclotron states. The states of dark hydrogen atoms and its generalizations are expected to be stable at temperatures scaled down by $1/r^2$ in the first approximation.

5. Similar arguments allow to deduce the values of binding energies in the general case once the formula of the binding energy given by standard quantum theory is known.

The most general option option allows fractional atoms with proton and electron numbers varying from $1/r$ to 1. One can imagine also the possibility of fractional molecules. The analogs of chemical bonds between fractional hydrogen atoms with $N-k$ and k fractional electrons and protons can be considered and would give rise to a full shell of fractional electrons possessing an exceptional stability. These states would have proton and electron numbers equal to one.

Catalytic sites are one possible candidate for fractal electrons and catalyst activity might be perhaps understood as a strong tendency of fractal electron and its conjugate to fuse to form an ordinary electron.

9.9.2 Could q-Laguerre equation relate to the claimed fractionation of the principal quantum number for hydrogen atom?

The so called hydrino atom concept of Randell Mills [D51] represents one of the notions related to free energy research not taken seriously by the community of university physicists. What is claimed that hydrogen atom can exists as scaled down variants for which binding energies are much higher than usually due to the large Coulomb energy. The claim is that the quantum number n having integer values $n = 0, 1, 2, 3..$ and characterizing partially the energy levels of the hydrogen atom can have also inverse integer values $n = 1/2, 1/3, ...$ The claim of Mills is that the laboratory BlackLight Inc. led by him can produce a plasma state in which transitions to these exotic bound states can occur and liberate as a by-product usable energy.

The National Aeronautic and Space Administration has dispatched mechanical engineering professor Anthony Marchese from Rowan University to BlackLight's labs in Cranbury, NJ, to investigate whether energy plasmas-hot, charged gases- produced by Mills might be harnessed for a new generation of rockets. Marchese reported back to his sponsor, the NASA Institute for Advanced Concepts, that indeed the plasma was so far unexplainably energetic. An article about the findings of Mills and collaborators have been accepted for publication in Journal of Applied Physics so that there are reasons to take seriously the experimental findings of Mills and collaborators even if one does not take seriously the theoretical explanations.

The fractionized principal quantum number n claimed by Mills [D51] is reported to have at least the values $n = 1/k$, $k = 2, 3, 4, 5, 6, 7, 10$. First explanation would be in terms of Plack constant having also values smaller than $\hbar_0$ possible if singular factor spaces of causal diamond CD and CP_2 are allowed. q-Deformations of ordinary quantum mechanics are suggested strongly by the hierarchy of Jones inclusion associated with the hyper-finite factor of type II_1 about which WCW spinors are a basic example. This motivates the attempt to understand the claimed fractionization in terms of q-analog of hydrogen atom. The safest interpretation for them would be as states which can exist in ordinary imbedding space (and also in other branches)

The Laguerre polynomials appearing in the solution of Schrödinger equation for hydrogen atom possess quantum variant, so called q-Laguerre polynomials [A62] , and one might hope that they would allow to realize this semiclassical picture at the level of solutions of appropriately modified Schrödinger equation and perhaps also resolve the difficulty associated with $n = 1/2$. Unfortunately, the polynomials discussed in [A62] correspond to $0 < q \leq 1$ rather than complex values of $q = exp(i\pi/m)$ on circle and the extrapolation of the formulas for energy eigenvalues gives complex energies.

q-Laquerre equation for $q = exp(i\pi/m)$

The most obvious modification of the Laguerre equation for S-wave sates (which are the most interesting by semiclassical argument) in the complex case is based on the replacement

$$\begin{aligned} \partial_x &\rightarrow \frac{1}{2}(\partial_x^{q)} + \partial_x^{\overline{q})}) \\ \partial_x^{q)} f &= \frac{f(qx) - f(x)}{(q-1)x} \ , \\ q &= exp(i\pi/m) \end{aligned} \tag{9.9.1}$$

to guarantee hermiticity. When applied to the Laguerre equation

$$x\frac{d^2L_n}{dx^2} + (1-x)\frac{dL_n}{dx} = nL_n \ , \tag{9.9.2}$$

and expanding L_n into Taylor series

$$L_n(x) = \sum_{n\geq 0} l_n x^n \ , \tag{9.9.3}$$

one obtains difference equation

$$\begin{aligned}
a_{n+1}l_{n+1} &+ b_nl_n = 0 \ , \\
a_{n+1} &= \frac{1}{4R_1^2}\left[R_{2n+1} - R_{2n} + 2R_{n+1}R_1 + 3R_1)\right] + \frac{1}{2R_1}\left[R_{n+1} + R_1\right] \\
b_n &= \frac{R_n}{2R_1} - n^{q)} + \frac{1}{2} \ , \\
R_n &= 2cos\left[(n-1)\pi/m\right] - 2cos\left[n\pi/m\right] \ .
\end{aligned} \tag{9.9.4}$$

Here $n^{q)}$ is the fractionized principal quantum number determining the energy of the q-hydrogen atom. One cannot pose the difference equation on l_0 since this together with the absence of negative powers of x would imply the vanishing of the entire solution. This is natural since for first order difference equations lowest term in the series should be chosen freely.

Polynomial solutions of q-Laquerre equation

The condition that the solution reduces to a polynomial reads as

$$b_n = 0 \tag{9.9.5}$$

and gives

$$n^{q)} = \frac{1}{2} + \frac{R_n}{2R_1} \ , \tag{9.9.6}$$

For $n = 1$ one has $n^{q)} = 1$ so that the ground state energy is not affected. At the limit $N \to \infty$ one obtains $n^{q)} \to n$ so that spectrum reduces to that for hydrogen atom. The periodicity $R_{n+2Nk} = R_n$ reflects the corresponding periodicity of the difference equation which suggests that only the values $n \leq 2m - 1$ belong to the spectrum. Spectrum is actually symmetric with respect to the middle point $[N/2]$ which suggests that only $n < [m/2]$ corresponds to the physical spectrum. An analogous phenomenon occurs for representations of quantum groups [K11] . When m increases the spectrum approaches integer valued spectrum and one has $n > 1$ so that no fractionization in the desired sense occurs for polynomial solutions.

Non-polynomial solutions of q-Laquerre equation

One might hope that non-polynomial solutions associated with some fractional values of $n^{q)}$ near to those claimed by Mills might be possible. Since the coefficients a_n and b_n are periodic, one can express the solution ansatz as

$$\begin{aligned}
L_n(x) &= P_a^{2m)}(x)\sum_k a^k x^{2mk} = P_a^{2m)}(x)\frac{1}{1 - ax^{2m}} \ , \\
P_a^{2m)}(x) &= \sum_{k=0}^{2m-1} l_k x^k \ , \\
a &= \frac{l_{2m}}{l_0} \ ,
\end{aligned} \tag{9.9.7}$$

This solution behaves as $1/x$ asymptotically but has pole at $x_\infty = (1/a)^{1/2m}$ for $a > 0$.

The expression for $l_{2m}/l_0 = a$ is

$$a = \prod_{k=1}^{2m} \frac{b_{2m-k}}{a_{2m-k+1}} \ . \tag{9.9.8}$$

This can be written more explicitly as

$$\begin{aligned} a &= (2R_1)^{2m}\prod_{k=1}^{2m} X_k \ , \\ X_k &= \frac{R_{2m-k} + (-2n^{q)} + 1)R_1}{R_{4m-2k+1} - R_{4m-2k} + 4R_{2m-k+1}R_1 + 2R_1^2 + 3R_1} \ , \\ R_n &= 2cos\left[(n-1)\pi/m\right] - 2cos\left[n\pi/m\right] \ . \end{aligned} \quad (9.9.9)$$

This formula is a specialization of a more general formula for $n = 2m$ and resulting ratios l_n/l_0 can be used to construct $P_a^{2m)}$ with normalization $P_a^{2m)}(0) = 1$.

Results of numerical calculations

Numerical calculations demonstrate following.

1. For odd values of m one has $a < 0$ so that a a continuous spectrum of energies seems to result without any further conditions.
2. For even values of m a has a positive sign so that a pole results.

For even value of m it could happen that the polynomial $P_a^{2m)}(x)$ has a compensating zero at x_∞ so that the solution would become square integrable. The condition for reads explicitly

$$P_a^{2m)}\left((\frac{1}{a})^{\frac{1}{2m}}\right) = 0 \ . \quad (9.9.10)$$

If $P_a^{2m)}(x)$ has zeros there are hopes of finding energy eigen values satisfying the required conditions. Laguerre polynomials and also q-Laguerre polynomials must posses maximal number of real zeros by their orthogonality implied by the hermiticity of the difference equation defining them. This suggests that also $P_a^{2m)}(x)$ possesses them if a does not deviate too much from zero. Numerical calculations demonstrate that this is the case for $n^{q)} < 1$.

For ordinary Laguerre polynomials the naive estimate for the position of the most distant zero in the units used is larger than n but not too much so. The naive expectation is that L_{2m} has largest zero somewhat above $x = 2m$ and that same holds true a small deformation of L_{2m} considered now since the value of the parameter a is indeed very small for $n^{q)} < 1$. The ratio $x_\infty/2m$ is below .2 for $m \leq 10$ so that this argument gives good hopes about zeros of desired kind.

One can check directly whether x_∞ is near to zero for the experimentally suggested candidates for $n^{q)}$. The table below summarizes the results of numerical calculations.

1. The table gives the exact eigenvalues $1/n_{q)}$ with a 4-decimal accuracy and corresponding approximations $1/n^{q)}_{\simeq} = k$ for $k = 3, ..., 10$. For a given value of m only single eigenvalue $n^{q)} < 1$ exists. If the observed anomalous spectral lines correspond to single electron transitions, the values of m for them must be different. The value of m for which $n^{q)} \simeq 1/k$ approximation is optimal is given with boldface. The value of k increases as m increases. The lowest value of m allowing the desired kind of zero of $P^{2m)}$ is $m = 18$ and for $k \in \{3, 10\}$ the allowed values are in range $18, .., 38$.
2. $n^{q)} = 1/2$ does not appear as an approximate eigenvalue so that for even values of m quantum calculation produces same disappointing result as the classical argument. Below it will be however found that $n^{q)} = 1/2$ is a universal eigenvalue for odd values of m.

m	$1/n^{q)}_{\simeq}$	$1/n^{q)}$	m	$1/n^{q)}_{\simeq}$	$1/n^{q)}$
18	**3**	2.7568	30	8	7.5762
20	**4**	3.6748	**32**	**8**	8.3086
22	5	4.5103	**34**	**9**	9.0342
24	**5**	5.3062	**36**	**10**	9.7529
26	**6**	6.0781	38	10	10.4668
28	**7**	6.8330			

Table3. The table gives the approximations $1/n^{q)}_{\simeq} = 1/k$ and corresponding exact values $1/n^{q)}$ in the range $k = 3, ..., 10$ for which $P_a^{2m)}(x_\infty)$ is nearest to zero. The corresponding values of $m = 2k$ vary in the range, $k = 18, ..., 38$. For odd values of m the value of the parameter a is negative so that there is no pole. Boldface marks for the best approximation by $1/n^{q)}_{\simeq} = k$.

How to obtain $n^{q)} = 1/2$ state?

For odd values of m the quantization recipe fails and physical intuition tells that there must be some manner to carry out quantization also now. The following observations give a hunch about the desired condition.

1. For the representations of quantum groups only the first m spins are realized [K11] . This suggests that there should exist a symmetry relating the coefficients l_n and l_{n+m} and implying $n^{q)} = 1/2$ for odd values of m. This symmetry would remove also the double degeneracy associated with the almost integer eigenvalues of $n^{q)}$. Also other fractional states are expected on basis of physical intuition.

2. For $n^{q)} = 1/2$ the recursion formula for the coefficients l_n involves only the coefficients R_m.

3. The coefficients R_k have symmetries $R_k = R_{k+2m}$ and $R_{k+m} = -R_m$.

There is indeed this kind of symmetry. From the formula

$$\begin{aligned} \frac{l_n}{l_0} &= (2R_1)^n \prod_{k=1}^{n} X_k \ , \\ X_k &= \frac{R_{n-k} + (-2n^{q)} + 1)R_1}{[R_{2n-2k+1} - R_{n-2k} + 4R_{n-k+1}R_1 + 2R_1^2 + 3R_1} \end{aligned} \tag{9.9.11}$$

one finds that for $n^{q)} = 1/2$ the formula giving l_{n+m} in terms of l_n changes sign when n increases by one unit

$$\begin{aligned} A_{n+1} &= (-1)^m A_n \ , \\ A_n &= \prod_{k=1}^{m} \frac{b_{n+m-k}}{a_{n+m-k+1}} = \prod_{k=1}^{m} (2R_1)^m \prod_{k=1}^{m} X_{k+n} \ . \end{aligned} \tag{9.9.12}$$

The change of sign is essentially due to the symmetries $a_{n+m} = -a_n$ and $b_{n+m} = b_n$. This means that the action of translations on A_n in the space of indices n are represented by group Z_2.

This symmetry implies $a = l_{2m}/l_0 = -(l_m)(l_0)^2$ so that for $n^{q)} = 1/2$ the polynomial in question has a special form

$$\begin{aligned} P_a^{2m)} &= P_a^{m)}(1 - Ax^m) \ , \\ A &= A_0 \ . \end{aligned} \tag{9.9.13}$$

The relationship $a = -A^2$ implies that the solution reduces to a form containing the product of m^{th} (rather than $(2m)^{th}$) order polynomial with a geometric series in x^m (rather than x^{2m}):

$$L_{1/2}(x) = \frac{P_a^{m)}(x)}{1 + Ax^m} \ . \tag{9.9.14}$$

Hence the n first terms indeed determine the solution completely. For even values of m one obtains similar result for $n^{q)} = 1/2$ but now A is negative so that the solution is excluded. This result also motivates the hypothesis that for the counterparts of ordinary solutions of Laguerre equation sum (even m) or difference (odd m) of solutions corresponding to n and $2m-n$ must be formed to remove the non-physical degeneracy.

This argument does not exclude the possibility that there are also other fractional values of n allowing this kind of symmetry. The condition for symmetry would read as

$$\begin{aligned} \prod_{k=1}^{m}(R_k+\epsilon R_1) &= \prod_{k=1}^{m}(R_k-\epsilon R_1) \ , \\ \epsilon &= (2n^{q)}-1 \ . \end{aligned} \tag{9.9.15}$$

The condition states that the odd part of the polynomial in question vanishes. Both ϵ and $-\epsilon$ solutions so that $n^{q)}$ and $1-n^{q)}$ are solutions. If one requires that the condition holds true for all values of m then the comparison of constant terms in these polynomials allows to conclude that $\epsilon = 0$ is the only universal solution. Since ϵ is free parameter, it is clear that the m:th order polynomial in question has at most m solutions which could correspond to other fractionized eigenvalues expected to be present on basis of physical intuition.

This picture generalizes also to the case of even n so that also now solutions of the form of Eq. 9.9.14 are possible. In this case the condition is

$$\prod_{k=1}^{m}(R_k+\epsilon R_1) = -\prod_{k=1}^{m}(R_k-\epsilon R_1) \ . \tag{9.9.16}$$

Obviously $\epsilon = 0$ and thus $n = 1/2$ fails to be a solution to the eigenvalue equation in this case. Also now one has the spectral symmetry $n_{\pm} = 1/2 \pm \epsilon$.

The symmetry $R_n = (-1)^m R_{n+m-1} = (-1)^m R_{n-m-1} = (-1)^m R_{m-n+1}$ can be applied to show that the polynomials associated with ϵ and $-\epsilon$ contain both the terms $R_n - \epsilon$ and $R_n + \epsilon$ as factors except for odd m for $n = (m+1)/2$. Hence the values of n can be written for even values of m as

$$n^{q)}(n) = \frac{1}{2} \pm \frac{R_n}{2R_1} \ , \ n = 1, ..., \frac{m}{2} \ , \tag{9.9.17}$$

and for odd values of m as

$$\begin{aligned} n^{q)}_{\pm}(n) &= \frac{1}{2} \pm \frac{R_n}{2R_1} \ , \ n = 1, ..., \frac{m+1}{2} - 1 \ , \\ n^{q)} &= 1/2 \ . \end{aligned} \tag{9.9.18}$$

Plus sign obviously corresponds to the solutions which reduce to polynomials and to $n^{q)} \simeq n$ for large m. The explicit expression for $n^{q)}$ reads as

$$n^{q)}_{\pm}(n) = \frac{1}{2} \pm \frac{(sin^2(\pi(n-1)/2m) - sin^2(\pi n/2m))}{2sin^2(\pi/2m)} \ . \tag{9.9.19}$$

At the limit of large m one has

$$n^{q)}_{+}(n) \simeq n \ , \ n^{q)}_{-}(n) \simeq 1-n \ . \tag{9.9.20}$$

so that the fractionization $n \simeq 1/k$ claimed by Mills is not obtained at this limit. The minimum for $|n^{q)}|$ satisfies $|n^{q)}| < 1$ and its smallest value $|n^{q)}| = .7071$ corresponds to $m = 4$. Thus these zeros cannot correspond to $n^{q)} \simeq 1/k$ yielded by the numerical computation for even values of m based on the requirement that the zero of $P^{2m)}$ cancels the pole of the geometric series.

Some comments

Some closing comments are in order.

1. An open question is whether there are also zeros $|n^{q)}| > 1$ satisfying $P_a^{2m)}((1/a)^{1/2m}) = 0$ for even values of m.

2. The treatment above is not completely general since only s-waves are discussed. The generalization is however a rather trivial replacement $(1-x)d/dx \rightarrow (l+1-x)d/dx$ in the Laguerre equation to get associated Laguerre equation. This modifies only the formula for a_{n+1} in the recursion for l_n so that expression for $n^{q)}$, which depends on b_n:s only, is not affected. Also the product of numerators in the formula for the parameter $a = l_{2m}/l_0$ remains invariant so that the general spectrum has the spectral symmetry $n^{q)} \rightarrow 1 - n^{q)}$. The only change to the spectrum occurs for even values of m and is due to the dependence of $x_\infty = (1/a)^{1/2m}$ on l and can be understood in the semiclassical picture. It might happen that the value of l is modified to its q counterpart corresponding to q-Legendre functions.

3. The model could partially explain the findings of Mills and $n^{q)} \simeq 1/k$ for $k > 2$ also fixes the value of corresponding m to a very high degree so that one would have direct experimental contact with generalized imbedding space, spectrum of Planck constants, and dark matter. The fact that the fractionization is only approximately correct suggests that the states in question could be possible for all sectors of imbedding space appear as intermediate states into sectors in which the spectrum of hydrogen atom is scaled by $n_b/n_a = k = 2, 3, ...$

4. The obvious question is whether q-counterparts of angular momentum eigenstates ($idf_m/d\phi = mf_m$) are needed and whether they make sense. The basic idea of construction is that the phase transition changing $\hbar$ does not involve any other modifications except fractionization of angular momentum eigenvalues and momentum eigenvalues having purely geometric origin. One can however ask whether it is possible to identify q-plane waves as ordinary plane waves. Using the definition $L_z = 1/2(\partial_u^q + \partial_u^{\overline{q}})$, $u = exp(i\phi)$, one obtains $f_n = exp(in\phi)$ and eigenvalues as $n^{q)} = R_n/R_1 \rightarrow n$ for $m \rightarrow \infty$. Similar construction applies in the case of momentum components.

9.9.3 Shy positrons

The latest weird looking effect in atomic physics is the observation that positrium atoms consisting of positron and electron scatter particles almost as if they were lonely electrons [C101, C68] . The effect has been christened cloaking effect for positron.

The following arguments represent the first attempts to understand the cloaking of positron in terms of these notions.

1. Let us start with the erratic argument since it comes first in mind. If positron and electron correspond to different space-time sheets and if the scattered particles are at the space-time sheet of electron then they do not see positron's Coulombic field at all. The objection is obvious. If positron interacts with the electron with its full electromagnetic charge to form a bound state, the corresponding electric flux at electron's space-time sheet is expected to combine with the electric flux of electron so that positronium would look like neutral particle after all. Does the electric flux of positron return back to the space-time sheet of positronium at some distance larger than the radius of atom? Why should it do this? No obvious answer.

2. Assume that positron dark but still interacts classically with electron via Coulomb potential. In TGD Universe darkness means that positron has large $\hbar$ and Compton size much larger than positronic wormhole throat (actually wormhole contact but this is a minor complication) would have more or less constant wave function in the volume of this larger space-time sheet characterized by zoomed up Compton length of electron. The scattering particle would see point-like electron plus background charge diffused in a much larger volume. If the value of $\hbar$ is large enough, the effect of this constant charge density to the scattering is small and only electron would be seen.

3. As a matter fact, I have proposed this kind of mechanism to explain how the Coulomb wall, which is the basic argument against cold fusion could be overcome by the incoming deuteron nucleus [L2] , [L2] . Some fraction of deuteron nuclei in the palladium target would be dark and have large size just as positron in the above example. It is also possible that only the protons of these nuclei are dark. I have also proposed that dark protons explain the effective chemical formula $H_{1.5}O$ of water in scattering by neutrons and electrons in atto-second time scale [L2] , [L2] . The connection with cloaked positrons is highly suggestive.

4. Also one of TGD inspired proposals for the absence of antimatter is that antiparticles reside at different space-time sheets as dark matter and are apparently absent [K76]. Cloaking positrons (shy as also their discoverer Dirac!) might provide an experimental supports for these ideas.

The recent view about the detailed structure of elementary particles forces to consider the above proposal in more detail.

1. According to this view all particles are weak string like objects having wormhole contacts at its ends and magnetically charged wormhole throats (four altogether) at the ends of the string like objects with length given by the weak length cale connected by a magnetic flux tube at both space-time sheets. Topological condensation means that these structures in turn are glued to larger space-time sheets and this generates one or more wormhole contacts for which also particle interpretation is highly suggestive and could serve as space-time correlate for interactions described in terms of particle exchanges. As far electrodynamics is considered, the second ends of weak strings containing neutrino pairs are effectively non-existing. In the case of fermions also only the second wormhole throat carrying the fermion number is effectively present so that for practical purposes weak string is only responsible for the massivation of the fermions. In the case of photons both wormhole throats carry fermion number.

2. An interesting question is whether the formation of bound states of two charged particles at the same space-time sheet could involve magnetic flux tubes connecting magnetically charged wormhole throats associated with the two particles. If so, Kähler magnetic monopoles would be part of even atomic and molecular physics. I have proposed already earlier that gravitational interaction in astrophysical scales involves magnetic flux tubes. These flux tubes would have o interpretation as analogs of say photons responsible for bound state energy. In principle it is indeed possible that the energies of the two wormhole throats are of opposite sign for topological sum contact so that the net energy of the wormhole contact pair responsible for the interaction could be negative.

3. Also the interaction of positron and electron would be based on topological condensation at the same space-time sheet and the formation of wormhole contacts mediating the interaction. Also now bound states could be glued together by magnetically charged wormhole contacts. In the case of dark positron, the details of the interaction are rather intricate since dark positron would correspond to a multi-sheeted structure analogous to Riemann surface with different sheets identified in terms of the roots of the equation relating generalized velocities defined by the time derivatives of the imbedding space coordinates to corresponding canonical momentum densities.

Chapter 10

Dark Forces and Living Matter

10.1 Introduction

The unavoidable presence of classical long ranged weak (and also color) gauge fields in TGD Universe has been a continual source of worries for more than two decades. The basic question has been whether electro-weak charges of elementary particles are screened in electro-weak length scale or not. The TGD based view about dark matter assumes that weak charges are indeed screened for ordinary matter in electro-weak length scale but that dark electro-weak bosons correspond to much longer symmetry breaking length scale.

The large value of $\hbar$ in dark matter phase implies that Compton lengths and -times are scaled up. In particular, the sizes of nucleons and nuclei become of order atom size so that dark nuclear physics would have direct relevance for condensed matter physics. It becomes impossible to make a reductionistic separation between nuclear physics and condensed matter physics and chemistry anymore. This view forces a profound re-consideration of the earlier ideas in nuclear and condensed physics context. It however seems that most of the earlier ideas related to the classical Z^0 force and inspired by anomaly considerations survive in a modified form.

The weak form of electric-magnetic duality led to the identification of the long sought for mechanism causing the weak screening in electroweak scales. The basic implication of the duality is that Kähler electric charges of wormhole throats representing particles are proportional to Kähler magnetic charges so that the CP_2 projections of the wormhole throats are homologically non-trivial. The Kähler magnetic charges do not create long range monopole fields if they are neutralized by wormhole throats carrying opposite monopole charges and weak isospin neutralizing the axial isospin of the particle's wormhole throat. One could speak of confinement of weak isospin. The weak field bodies of elementary fermions would be replaced with string like objects with a length of order W boson Compton length. Electro-magnetic flux would be feeded to electromagnetic field body where it would be feeded to larger space-time sheets. Similar mechanism could apply in the case of color quantum numbers. Weak charges would be therefore screened for ordinary matter in electro-weak length scale but dark electro-weak bosons correspond to much longer symmetry breaking length scale for weak field body. Large values of Planck constant would make it possible to zoop up elementary particles and study their internal structure without any need for gigantic accelerators.

One can still worry about large parity breaking effects - say in nuclear physics- since the couplings of spinors to classical weak fields are there. Around 2012 it became clear that the condition that induced spinor fields have well defined em charge localizes their modes in the generic case to 2-surfaces carrying vanishing induced W gauge fields. It is quite possible that this localization is consistent with Kähler-Dirac equation only in ther Minkowskian regions were the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional.

On can pose the additional condition that also classical Z^0 field vanishes - at least above weak scale. Fundamental fermions would experience only em field so that the worries related to large parity breaking effects would disappear. The proportionality of weak scale to $h_{eff} = n \times h$ however predicts that weak fields are effectively massless belong scaled up weak scale. Therefore worries about large parity breaking effects in ordinary nuclear physics can be forgotten.

In its original form this chapter was an attempt to concretize and develop ideas related to dark matter by using some experimental inputs with emphasis on the predicted interaction between the new nuclear physics and condensed matter. As the vision about dark matter became more coherent and the nuclear string model developed in its recent form, it became necessary to update the chapter and throw away the obsolete material. I dare hope that the recent representation is more focused than the earlier one.

10.1.1 Evidence for long range weak forces and new nuclear physics

There is a lot of experimental evidence for long range electro-weak forces, dark matter, and exotic nuclear physics giving valuable guidelines in the attempts to build a coherent theoretical scenario.

Cold fusion

Cold fusion [C112] is a phenomenon involving new nuclear physics and the known selection rules give strong constraints when one tries to understand the character of dark nuclear matter. The simplest model for cold fusion found hitherto is based on the nuclear string model [L2] , [L2] and will be taken as the basis of the considerations of this chapter. Also comparisons with the earlier variant of model of cold fusion [K80] will be made in the section about cold fusion.

Large parity breaking effects

Large parity breaking effects in living matter indicate the presence of long ranged weak forces, and the reported nuclear transmutations in living matter [C94, C116] suggest that new nuclear physics plays a role also now. For instance, the Gaussian Mersennes $(1+i)^k-1$ for $k = 113, 151, 157163, 167$ could correspond to weak length scales and four biologically important length scales in the range 10 nm-25 μm, which seem to relate directly to the coiling hierarchy of DNA double strands.

Anomalies of the physics of water

The physics of water involves a large number of anomalies and life depends in an essential manner on them. As many as 41 anomalies are discussed in the excellent web page "Water Structure and Behavior" of M. Chaplin [D30] . The fact that the physics of heavy water differs much more from that of ordinary water as one might expect on basis of different masses of water molecules suggests that dark nuclear physics is involved.

1. The finding that one hydrogen atom per two water molecules remain effectively invisible in neutron and electron interactions in atto-second time scale [D30, D27] suggests that water is partially dark. These findings have been questioned in [D32] and thought to be erroneous in [D52] . If the findings are real, dark matter phase made of super-nuclei consisting of protons connected by dark color bonds could explain them as perhaps also the clustering of water molecules predicting magic numbers of water molecules in clusters. If so, dark nuclear physics could be an essential part of condensed matter physics and biochemistry. For instance, the condensate of dark protons might be essential for understanding the properties of bio-molecules and even the physical origin of van der Waals radius of atom in van der Waals equation of state.

2. The observation that the binding energy of dark color bond for $n = 2^{11} = 1/v_0$ of the scaling of $\hbar$ corresponds to the bond energy .5 eV of hydrogen bond raises the fascinating possibility that hydrogen bonds is accompanied by a color bond between proton and oxygen nucleus. Also more general chemical bonds might be accompanied by color bonds so that dark color physics might be an essential part of molecular physics. Color bonds might be also responsible for the formation of liquid phase and thus solid state. Dark weak bonds between nuclei could be involved and might be responsible for the repulsive core of van der Waals force and be part of molecular physics too. There is evidence for two kinds of hydrogen bonds [D70] : a possible identification is in terms of p-adic scaling of hydrogen bonds by a factor 2. This kind of doubling is predicted by nuclear string model [L2], [L2] .

3. Years after writing this piece of text emerged the idea that covalent bonds of biopolymers might be accompanied by color bonds carrying the metabolic energy liberated in the decay of these polymers [K45]. Polymer like sequences of "half-dark" water molecules with one dark proton with dark protons connected by color bonds to form dark nucleus could have emerged as prebiotic counterparts of biomolecules and carry metabolic energy in color bonds and realize genetic code [K41, L2]. They could accompany ordinary bio-bolymers in water environment and color bonds could carry the metabolic energy. There are of course many other options, and one must have open mind since the belief that biochemistry is understood reduces to high extent to the belief in the reductionistic dogma.

4. Tetrahedral water clusters consisting of 14 water molecules would contain 8 dark protons which corresponds to a magic number for a dark nucleus consisting of protons. Icosahedral water clusters in turn consist of 20 tetrahedral clusters. This raises the question whether fractally scaled up super-nuclei could be in question. If one accepts the vision about dark matter hierarchy based in Jones inclusions to be discussed briefly later, tetrahedral and icosahedral structures of water could correspond directly to the unique genuinely 3-dimensional $G_a = E_6$ and E_8 coverings of CP_2 with $n_a = 3$ and $n_a = 5$ assignable to dark electrons. Icosahedral structures are also very abundant in living matter, mention only viruses.

Other anomalies

There are also other anomalies which might relate to the hierarchy of Planck constants and also to dark weak forces.

1. Exotic chemistries

Exotic chemistries [D25] in which clusters of atoms of given given type mimic the chemistry of another element. These systems behave as if nuclei would form a jellium (constant charge density) defining a harmonic oscillator potential for electrons. Magic numbers correspond to full electron shells analogous to noble gas elements. It is difficult to understand why the constant charge density approximation works so well. If nuclear protons are in large $\hbar(M^4)$ phase with Fermat integer $n_F = 3 \times 2^{11}$, the electromagnetic sizes of nuclei would be about 2.4 Angstroms and the approximation would be natural.

As a matter, fact nuclear string model predicts that the nuclei can have as many as $3A$ exotic charge states obtained by giving neutral color bond charge ± 1: this would give rise to quite different kind of alchemy [L2] , [L2] revealing itself in cold fusion.

2. Free energy anomalies

The anomalies reported by free energy researchers such as over unity energy production in devices involving repeated formation and dissociation of H_2 molecules based on the original discovery of Nobelist Irwing Langmuir [D68] (see for instance [H10]) suggest that part of H atoms might end up to dark matter phase liberating additional energy. The "mono-atomic" elements of Hudson suggest also dark nuclear physics [H6] . There is even evidence for macroscopic transitions to dark phase [H2, H8, H7] .

3. Tritium beta decay anomaly and findings of Shnoll

Tritium beta decay anomaly [C77, C47, C34, C91] suggests exotic nuclear physics related to weak interactions. The evidence for the variation of the rates of nuclear and chemical processes correlating with astrophysical periods [E15] , [E15] could be understood in terms of weak fields created by dark matter and affect by astrophysical phenomena.

10.1.2 Dark rules

I have done a considerable amount of trials and errors in order to identify the basic rules allowing to understand what it means to be dark matter is and what happens in the phase transition to dark matter. It is good to try to summarize the basic rules of p-adic and dark physics allowing to avoid obvious contradictions.

The notion of field body

The notion of "field body" implied by topological field quantization is essential. There would be em, $Z0$, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

The p-adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 field body. Z^0 field body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

What dark variant of elementary particle means

It is not at all clear what the notion of dark variant of elementary particle or of larger structures could mean.

1. *Are only field bodies dark?*

One variety of dark particle is obtained by making some of the field bodies dark by increasing the value of Planck constant. This hypothesis could be replaced with the stronger assumption that elementary particles are maximally quantum critical systems so that they are same irrespective of the value of the Planck constant. Elementary particles would be represented by partonic 2-surfaces, which belong to the universal orbifold singularities remaining invariant by all groups $G_a \times G_b$ for a given choice of quantization axes. If $G_a \times G_b$ is assumed to leave invariant the choice of the quantization axes, it must be of the form $Z_{n_a} \times Z_{n_b} \subset SO(3) \times SU(3)$. Partonic 2-surface would belong to $M^2 \times CP_2/U(1) \times U(1)$, where M^2 is spanned by the quantization axis of angular momentum and the time axis defining the rest system.

A different manner to say this is that the CP_2 type extremal representing particle would suffer multiple topological condensation on its field bodies so that there would be no separate "particle space-time sheet".

Darkness would be restricted to particle interactions if it is assigned with topological field quanta mediating interactions. The value of the Planck constant would be assigned to a particular interaction between systems rather than system itself. This conforms with the original finding that gravitational Planck constant satisfies $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$. Since each interaction can give rise to a hierarchy dark phases, a rich variety of partially dark phases is predicted. The standard assumption that dark matter is visible only via gravitational interactions would mean that gravitational field body would not be dark for this particular dark matter. Note however that gravitational Planck constant h_{fr} having gigantic values could have different origin as Planck constant h_{eff} emerging from considerations related to biology: this is discussed in [K75].

Complex combinations of dark field bodies become possible and the dream is that one could understand various phases of matter in terms of these combinations. All phase transitions, including the familiar liquid-gas and solid-liquid phase transitions, could have a unified description in terms of dark phase transition for an appropriate field body. At mathematical level Jones inclusions would provide this description.

The book metaphor for the interactions at space-time level is very useful in this framework. Elementary particles correspond to ordinary value of Planck constant analogous to the ordinary sheets of a book and the field bodies mediating their interactions are the same space-time sheet or at dark sheets of the book.

2. Can also elementary particles be dark?

Also dark elementary particles themselves rather than only the flux quanta could correspond to dark space-time sheet defining multiple coverings of $H/G_a \times G_b$. This would mean giving up the maximal quantum criticality hypothesis in the case of elementary particles. These sheets would be exact copies of each other. If single sheet of the covering contains topologically condensed space-time sheet, also other sheets contain its exact copy.

The question is whether these copies of space-time sheet defining classical identical systems can carry different fermionic quantum numbers or only identical fermionic quantum numbers so that the dark particle would be exotic many-fermion system allowing an apparent violation of statistics (N fermions in the same state).

Even if one allows varying number of fermions in the same state with respect to a basic copy of sheet, one ends up with the notion of N-atom in which nuclei would be ordinary but electrons would reside at the sheets of the covering. The question is whether symbolic representations essential for understanding of living matter could emerge already at molecular level via the formation of N-atoms.

Criterion for the transition to dark phase

The criterion $\alpha Q_1 Q_2 > 1$ for the transition to dark matter phase relates always to the interaction between two systems and the interpretation is that when the field strength characterizing the interaction becomes too strong, the interaction is mediated by dark space-time sheets which define $n = n(G_a) \times n(G_b)$-fold covering of $M^4 \times CP_2/G_a \times G_b$. The sharing of flux between different space-time sheets reduces the field strength associated with single sheet below the critical value.

Mersenne hypothesis

The generalization of the imbedding space means a book like structure for which the pages are products of singular coverings or factor spaces of CD (causal diamond defined as intersection of future and past directed light-cones) and of CP_2 [K32] . This predicts that Planck constants are rationals and that given value of Planck constant corresponds to an infinite number of different pages of the Big Book, which might be seen as a drawback. If only singular covering spaces are allowed the values of Planck constant are products of integers and given value of Planck constant corresponds to a finite number of pages given by the number of decompositions of the integer to two different integers.

TGD inspired quantum biology and number theoretical considerations suggest preferred values for $r = \hbar/\hbar_0$. For the most general option the values of $\hbar$ are products and ratios of two integers n_a and n_b. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n_i)$, $i \in \{a, b\}$, in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of r.

One can however ask whether a more precise characterization of preferred Mersennes could exist and whether there could exists a stronger correlation between hierarchies of p-adic length scales and Planck constants. Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ are expected to be physically highly interesting and up to $k = 127$ indeed correspond to elementary particles. The number theoretical miracle is that all the four scaled up electron Compton lengths with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm). The question has been whether these define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$. The proposal that this is the case and that these physics are in a well-defined sense induced by the dark scaled up variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$.

What induction means is that dark variant of exotic nuclear physics induces exotic physics with ordinary value of Planck constant in the new scale in a resonant manner: dark gauge bosons transform to their ordinary variants with the same Compton length. This transformation is natural since in length scales below the Compton length the gauge bosons behave as massless and free particles. As a consequence, lighter variants of weak bosons emerge and QCD confinement scale becomes longer.

This proposal will be referred to as Mersenne hypothesis. It leads to strong predictions about EEG [K29] since it predicts a spectrum of preferred Josephson frequencies for a given value of membrane potential and also assigns to a given value of $\hbar$ a fixed size scale having interpretation as the size scale of the body part or magnetic body. Also a vision about evolution of life emerges. Mersenne hypothesis is especially interesting as far as new physics in condensed matter length scales is considered: this includes exotic scaled up variants of the ordinary nuclear physics and their dark variants. Even dark nucleons are possible and this gives justification for the model of dark nucleons predicting the counterparts of DNA,RNA, tRNA, and amino-acids as well as realization of vertebrate genetic code [K90] .

These exotic nuclear physics with ordinary value of Planck constant could correspond to ground states that are almost vacuum extremals corresponding to homologically trivial geodesic sphere of CP_2 near criticality to a phase transition changing Planck constant. Ordinary nuclear physics would correspond to homologically non-trivial geodesic sphere and far from vacuum extremal property. For vacuum extremals of this kind classical Z^0 field proportional to electromagnetic field is present and this modifies dramatically the view about cell membrane as Josephson junction. The model for cell membrane as almost vacuum extremal indeed led to a quantitative breakthrough in TGD inspired model of EEG and is therefore something to be taken seriously. The safest option concerning empirical facts is that the copies of electro-weak and color physics with ordinary value of Planck constant are possible only for almost vacuum extremals - that is at criticality against phase transition changing Planck constant.

10.1.3 Weak form of electric magnetic duality, screening of weak charges, and color confinement?

TGD predicts the presence of long range classical weak fields and color fields and one should understand classically why quarks and leptons do not couple to these fields above weak boson length scale. Why the quarks inside ordinary nuclei do not generate long range weak fields and do not couple to them? Obviously the weak charges of quarks must be screened so that only electromagnetic charge remains. The extreme non-linearity of field equations in principle allows non-vanishing vacuum charge densities making possible this kind of screening. I have not been able to develop any detailed model for this.

A rather attractive looking explanation came with the discovery of electric-magnetic duality leading to a considerable progress in the understanding of basic quantum TGD. The basic implication of the duality is that Kähler electric charges of wormhole throats representing particles are proportional to Kähler magnetic charges so that the CP_2 projections of the wormhole throats are homologically non-trivial. The Kähler magnetic charges do not create long range monopole fields if they are neutralized by wormhole throats carrying opposite monopole charges and weak isospin neutralizing the axial isospin of the particle's wormhole throat. One could speak of confinement of weak isospin. The weak field bodies of elementary fermions would be replaced with string like objects with a length of order W boson Compton length. Electro-magnetic flux would be feeded to electromagnetic field body where it would be feeded to larger space-time sheets. Similar mechanism could apply in the case of color quantum numbers.

One of the basic questions closely related to the weak screening have been whether it is s possible to have a weak analog of the ordinary atom - say neutrino atom. Formally one can of course construct this kind of model and I have indeed doe this. The recent wiew about the screening of weak forces does not however allow neutrino atoms since the weak gauge fluxes flow along flux tubes and are screened by opposite charges at their end rather than being spherically symmetric Coulomb fields. Elementary particles themselves can be regarded as string like objects neutralized above weak boson Compton length. The size of the magnetic flux tubes however scales as $\sqrt{\hbar}$ so that large values of $\hbar$ it is in principle possible to zoom up the elementary particles and see what their interior looks like. This applies to both weak and color forces and might some day make possible study of elementary particles without gigantic accelerators.

10.1.4 Dark weak forces and almost vacuum extremals

TGD suggests strongly the presence of long range weak force and the large parity breaking in living matter realized as chiral selection provides support for it. One would however like some more concrete quantitative evidence for the conjecture that the classical weak forces are indeed there. This kind of evidence comes from the model of cell membrane based on the hypothesis that cell membrane correspond to almost vacuum extremal.

1. Induced Kähler form vanishes for vacuum extremals. The condition for vanishing implies that classical Z^0 and electromagnetic fields are proportional to each other so that induced spinor field couples to both these fields. The assumption is that the quarks of nuclei and possibly also neutrinos correspond to a large value of Planck constant and therefore couple to the classical Z^0 field. Atomic electrons would not have these couplings. This modifies dramatically the model for the cell membrane as a Josephson junction and raises the scale of Josephson energies from IR range just above thermal threshold to visible and ultraviolet. The amazing finding is that the Josephson energies for biologically important ions correspond to the energies assigned to the peak frequencies in the biological activity spectrum of photoreceptors in retina suggesting. This suggests that almost vacuum extremals and thus also classical Z^0 fields could be in a central role in the understanding of the functioning of the cell membrane and of sensory qualia. This would also explain the large parity breaking effects in living matter.

   One can construct also a generalization of Josephson junction as transmembrane protein such that Josephson energy is generalized to include also the difference of cyclotron energies over the membrane. This allows to understand the role of protons in metabolism and large value about .5 eV of metabolic energy quantum roughtly 10 times larger that Josephson energy for cell membrane in terms of "square root of thermodynamics" replacing the ordinary thermodynamical model of cell membrane. In this case classical Z^0 force is not necessary. It is of course possible that cell membrane proteins can be in two phases: without or with classical Z^0 fields at string world sheets of dark fermions.

2. A further conjecture is that EEG and its predicted fractally scaled variants which same energies in visible and UV range but different scales of Josephson frequencies correspond to Josephson photons with various values of Planck constant. The decay of dark ELF photons with energies of visible photons would give rise to bunches of ordinary ELF photons. Biophotons in turn could correspond to ordinary visible photons resulting in the phase transition of these photons to photons with ordinary value of Planck constant. This leads to a very detailed view about the role of dark electromagnetic radiation in biomatter and also to a model for how sensory qualia are realized [K38, K69, K29] .

What darkness means in the case of nuclei is that the "weak" field bodies of quarks are dark so that the size scale assignable to them is of order cell size. This does not affect their electromagnetic field bodies so that it is possible to speak about ions in the ordinary sense of the word. If the size scale of a given part of field body corresponds to the Compton length proportional proportional to the p-adic length scale scaled up by $\sqrt{\hbar}$ then cell membrane thickness as a Compton scale for the field body of weak bosons means rather large value of $\hbar \sim 2^{151-89} = 2^{62}\hbar_0$. This would scale down 10^{14} Hz frequency of visible photons to about 10^{-4} Hz.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Emergent ideas and notions [L25]
- Hierarchy of Planck constants [L27]
- Quantum gravity and biology [L40]

10.2 Weak form electric-magnetic duality and its implications

The notion of electric-magnetic duality [B3] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K22] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that

all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

10.2.1 Could a weak form of electric-magnetic duality hold true?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces

and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} \quad = \quad KJ_{12} \quad . \tag{10.2.1}$$

A more general form of this duality is suggested by the considerations of [K42] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} \quad = \quad K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \quad . \tag{10.2.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m \quad = \quad (1+K)J_{12} \quad , \tag{10.2.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \quad .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{10.2.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{10.2.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{10.2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{10.2.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the modified Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K64] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z . \tag{10.2.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} . \tag{10.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{10.2.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K69]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K89]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

10.2.2 Magnetic confinement, the short range of weak forces, and color confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D40] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K34] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K51] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K52] .

10.3 Dark matter hierarchy, genetic machinery, and the un-reasonable selectivity of bio-catalysis

One of the most fascinating outcomes of ideas related to the dark matter hierarchy is the notion of inherently dark fractional atom (molecule) generalizing the notion of Bose-Einstein condensate to the fermionic case. These notions might provide an elegant manner to understand the mysteries of DNA replication, transcription, and translation, and more generally, the incredible selectivity of bio-catalysis.

As often, the original idea was not quite correct. I spoke about N-atoms rather than fractional atoms. In particular, the mass of N-molecule was N times larger than that of the ordinary molecule apart from corrections from binding energy. The more precise view about dark matter hierarchy led to the realization that fractionization of all quantum numbers occurs. In the most general case one can have fractional particles with particle number $n = k/r, k = 1,..,r$, $r = \frac{\hbar}{\hbar_0}$. This leaves the model essentially as such at formal level. The model is however much more realistic than the original one since fractional atoms have mass which is never larger than that of ordinary atom and also conforms with the recent view about the origin of the hierarchy of Planck constants.

10.3.1 Dark atoms and dark cyclotron states

The development of the notion of dark atom involves many side tracks which make me blush. The first naive guess was that dark atom would be obtained by simply replacing Planck constant with its scaled counterpart in the basic formulas and interpreting the results geometrically. After some obligatory twists and turns it became clear that this assumption is indeed the most plausible one. The main source of confusion has been the lack of precise view about what the hierarchy of Planck constants means at the level of imbedding space at space-time.

The rules are very simple when one takes the singular coverings assigned to the many-valuedness of the time-derivatives of imbedding space coordinates as functions of canonical momentum densities as a starting point.

1. The mass and charge of electron are fractionized as is also the reduced mass in Schrödinger equation. This implies the replacements $e \to e/r$, $m \to m/r$, and $\hbar \to r\hbar_0$, $r = n_a n_b$, in the general formula for the binding energy assigned with single sheet of the covering. If maximal number $n_a n_b$ are present corresponding to a full "Fermi sphere", the total binding energy is r times the binding energy associated with single sheet.

2. In the case of hydrogen atom the proportionality $E \propto m/\hbar^2$ implies that the binding energy for single sheet of the covering scales as $E \to E/(n_a n_b)^3$ and maximal binding energy scales as $E \to E/(n_a n_b)^2$. This conforms with the naive guess. For high values of the nuclear charge Z it can happen that the binding energy is larger than the rest mass and fractionization might take place when binding energy is above critical fraction of the rest mass.

3. In the case of cyclotron energies one must must decide what happens to the magnetic flux. Magnetic flux quantization states that the flux is proportional to $\hbar$ for each sheet separately. Hence one has $\Phi \to r\Phi$ for each sheet and the total flux scales as r^2. Since the dimensions of the flux quantum are scaled up by r the natural scaling of the size of flux quantum is by r^2. Therefore the quantization of the magnetic flux requires the scaling $B \to B/r$. The cyclotron energy for single sheet satisfies $E \propto \hbar qB/m$ and since both mass m and charge q become fractional , the energy E for single sheet remains invariant whereas total cyclotron energy is scaled up by r in accordance with the original guess and the assumption used in applications.

4. Dark cyclotron states are expected to be stable up to temperatures which are r times higher than for ordinary cyclotron states. The states of dark hydrogen atoms and its generalizations are expected to be stable at temperatures scaled down by $1/r^2$ in the first approximation.

5. Similar arguments allow to deduce the values of binding energies in the general case once the formula of the binding energy given by standard quantum theory is known.

The most general option option allows fractional atoms with proton and electron numbers varying from $1/r$ to 1. One can imagine also the possibility of fractional molecules. The analogs of chemical bonds between fractional hydrogen atoms with $N-k$ and k fractional electrons and protons can be considered and would give rise to a full shell of fractional electrons possessing an exceptional stability. These states would have proton and electron numbers equal to one.

Catalytic sites are one possible candidate for fractal electrons and catalyst activity might be perhaps understood as a strong tendency of fractal electron and its conjugate to fuse to form an ordinary electron.

Connection with quantum groups?

The phase $q = exp(i2\pi/r)$ brings unavoidably in mind the phases defining quantum groups and playing also a key role in the model of topological quantum computation [K94] . Quantum groups indeed emerge from the spinor structure in the "world of classical worlds" realized as the space of 3-surfaces in $M^4 \times CP_2$ and being closely related to von Neumann algebras known as hyper-finite factors of type II_1 [K95] .

Only singular coverings are allowed if the hierarchy of Planck constants and corresponding hierarchy of singular coverings follows from the basic TGD. If the integer n characterizing the quantum phase allows identification with with $r = \hbar/\hbar_0$, living matter could be perhaps understood in terms of quantum deformations of the ordinary matter, which would be characterized by the quantum phases $q = exp(i2\pi/r)$. Hence quantum groups, which have for long time suspected to have significance in elementary particle physics, might relate to the mystery of living matter and predict an entire hierarchy of new forms of matter.

How to distinguish between fractional particles and ordinary particles?

The unavoidable question is whether bio-molecules in vivo could involve actually fractional atoms molecules as their building blocks. This raises a series of related questions.

1. Could it be that we can observe only the fusion of of dark fractional fold molecules to ordinary molecules or its reversal? Is the behavior of matter matter in vivo dictated by the dark matter commentn and of matter in vitro by ordinary matter? Could just the act of observing the matter in vivo in the sense of existing science make it ordinary dead matter?

2. If fractional atoms and molecules correspond to the maximum number of fractional quanta their masses are same as for ordinary atoms and molecules and only the different binding energy photon spectrum distinguishes between them. Situation changes all fractional states are possible and one obtains scaled down spectrum as a unique signature.

3. The fusion of fractional molecules to ordinary molecules in principle allows to conclude that fractional molecule was present. Could this process mean just the replacement of DNA in vivo with DNA in vitro?

10.3.2 Spontaneous decay and completion of dark fractional atoms as a basic mechanisms of bio-chemistry?

The replication of DNA has remained for me a deep mystery and I dare to doubt that the reductionistic belief that this miraculous process is well-understood involves self deceptive elements. Of course the problem is much more general: DNA replication is only a single very representative example of the miracles of un-reasonable selectivity of the bio-catalysis. I take this fact as a justification for some free imagination inspired by the notion of dark fractional molecule.

Dark fermionic molecules can replicate via decay and spontaneous completion

Unit particle number for fractional atom or molecule means that the analog of closed electronic shell are in question so that the state is especially stable. Note that the analogy with full Fermi electronic sphere makes also sense. These atoms or molecules could decay to fractional atoms or molecules. with fractional particle numbers k/r and $(r-k)/r$.

Suppose that a fractional molecule with unit particle number decays into k/r-molecule and $(r-k)/r$-molecule. If r is even it is possible to have $k = r - k = r/2$ and the situation is especially symmetric. If fermionic $k/r < 1$ fractional atoms or molecules are present, one can imagine that they tend to be completed to full molecules spontaneously. Thus spontaneous decay and completion would favor the spontaneous replication (or rather fractionization) and dark molecules could be ideal replicators (fractionizators) The idea that the mechanisms of spontaneous decay and completion of dark fractional particles somehow lurk behind DNA replication and various high precision bio-catalytic processes is rather attractive.

Reduction of lock and key mechanism to spontaneous completion

DNA replication and molecular recognition by the lock and key mechanism are the two mysterious processes of molecular biology. As a matter fact, DNA replication reduces to spontaneous opening of DNA double strand and to the lock and key mechanism so that it could be enough to understand the opening of double strand in terms of spontaneous decay and lock and key mechanism in terms of spontaneous completion of fractional particle (-atom or -molecule).

Consider bio-molecules which fit like a lock and key. Suppose that they are accompanied by dark fractional atoms or molecules, to be called dark fractional particles in sequel, such that one has $k_1 + k_2 = r$ so that in the formation of bound state dark molecules combine to form r-molecule analogous to a full fermionic shell or full Fermi sea. This is expected to enhance the stability of this particular molecular complex and prefer it amongst generic combinations.

For instance, this mechanism would make it possible for nucleotide and its conjugate, DNA and mRNA molecule, and tRNA molecule and corresponding amino-acid to recognize each other. Spontaneous completion would allow to realize also the associations characterizing the genetic code as a map from RNAs to subset of RNAs and associations of this subset of RNAs with amino-acids (assuming that genetic code has evolved from RNA $\rightarrow$ RNA code as suggested in this chapter).

As such this mechanism allows a rather limited number of different lock and key combinations unless r is very large. There is however a simple generalization allowing to increase the representative power so that lock and key mechanism becomes analogous to a password used in computers. The molecule playing the role of lock *resp.* molecule would be characterized by a set of n fractional particles with $k_1 \in \{k_{1,1}, ...k_{1,n}\}$ *resp.* $k_2 \in \{k_{2,1} = r - k_{1,1}, ..., k_{2,n} = r - k_{1,n}\}$. The molecules with conjugate names would fit optimally together. Fractional molecules or fractional electrons or atoms appearing as their building blocks would be like letters of a text characterizing the name of the molecule.

The mechanism generalizes also to the case of $n > 2$ reacting molecules. The molecular complex would be defined by a partition of n copies of integer r to a sum of m integers $k_{k,i}$: $\sum_i k_{k,i} = r$.

This mechanism could provide a universal explanation for the miraculous selectivity of catalysts and this selectivity would have practically nothing to do with ordinary chemistry but would correspond to a new level of physics at which symbolic processes and representations based on dark fractional particles emerge.

Connection with the number theoretic model of genetic code?

The emergence of partitions of integers in the labelling of molecules by fractional particles suggests a connection with the number theoretical model of genetic code [K27] , where DNA triplets are characterized by integers $n \in \{0, ..., 63\}$ and amino-acids by integers $0, 1$ and 18 primes $p < 64$. For instance, one can imagine that the integer n means that DNA triplet is labelled by n/r-particle. $r = 64$ would be the obvious candidate for r and conjugate DNA triplet would naturally have $n_c = 64 - n$.

The model relies on number-theoretic thermodynamics for the partitions of n to a sum of integers and genetic code is fixed by the minimization of number theoretic entropy which can be also negative and has thus interpretation as information. Perhaps these partitions could correspond to states resulting in some kind of decays of n-fermion to n_k/r-fermions with $\sum_{k=1}^{r} n_k = n$. The n_k/r-fermions should however not correspond to separate particles but something different. A possible interpretation is that partition corresponds to a state in which n_1/r particle is topologically condensed at $n_2/r \geq n_1/r$ particle topologically condensed....at $n_k/r \geq n_{k-1}/r$-particle. This would also automatically define a preferred ordering of the integers n_i in the partition.

An entire ensemble of labels would be present and depending on the situation codon could be labelled not only by n/r-particle but by any partition $n = \sum_{i=1}^{k} n_i$ corresponding to the state resulting in the decay of n/r-particle to k fractional particles.

Reduction of DNA replication to a spontaneous decay of r-particle

DNA replication could be induced by a spontaneous decay of r-particle inducing the instability of the double strand leading to a spontaneous completion of the component strands.

Strand and conjugate strand would be characterized by k_1/r-particle and $(r - k_1)/r$-particle, which combine to form r-particle as the double strand is formed. The opening of the double strand is induced by the decay of r-particle to k_1/r- and $(r - k_1)/r$-particles accompanying strand and its conjugate and after this both strands would complete themselves to double strands by the completion to r-particle.

It would be basically the stability of fractional particle which would make DNA double strand stable. Usually the formation of hydrogen bonds between strands and more generally, between the atoms of stable bio-molecule, is believed to explain the stability. Since the notion of hydrogen bond is somewhat phenomenological, one cannot exclude the possibility that these two mechanisms might be closely related to each other. I have already earlier considered the possibility that hydrogen bond might involve dark protons [K30] : this hypothesis was inspired by the finding that there seems to exist two kinds of hydrogen bonds [D70] .

The reader has probably already noticed that the participating fractional molecules in the model of lock and key mechanism are like sexual partners, and if already molecules are conscious entities as TGD inspired theory of consciousness strongly suggests, one might perhaps see the formation of entangled bound states with positive number theoretic entanglement entropy accompanied by molecular experience of one-ness as molecular sex. Even more, the replication of DNA brings in also divorce and process of finding of new companions!

10.3.3 The new view about hydrogen bond and water

Concretization of the above scenario leads to a new view about hydrogen bond and the role of water in bio-catalysis.

What the fractional particles labelling bio-molecules could be?

What the dark fractional particles defining the letters for the names of various bio-molecules could be? Dark fractional hydrogen atoms are the lightest candidates for the names of bio-molecules. The fusion could give rise to the hydrogen atom appearing in hydrogen bond. One could say the fractional hydrogen atoms belong to the molecules between which the hydrogen bond is formed. In absence of bond the fractional atoms would define active catalyst sites. This mechanism would also conform with the belief that hydrogen bonds guarantee the stability of bio-molecules.

This idea is not a mere speculation. The first experimental support for the notion of dark matter [K30] came from the experimental finding that water looks in atto-second time scale from the point of view of neutron diffraction and electron scattering chemically like $H_{1.5}O$: as if one fourth of protons are dark [D30, D27, D32, D52] . Dark protons would be identifiable as fractional protons. Of course, also dark hydrogen atoms can be considered.

One can imagine also a second option. The model for [I2] [K41] leads to a rather concrete view about how magnetic body controls biological body and receives sensory input from it. The model relies on the idea that dark water molecule clusters and perhaps also dark exotically ionized super-nuclei formed as linear closed strings of dark protons [K30] perform this mimicry. Dark proton super-nuclei are ideal for mimicking the cyclotron frequencies of ordinary atoms condensed to dark magnetic flux quanta. Of course, also partially ionized hydrogen fractional ions could perform the cyclotron mimicry of molecules with the same accuracy.

One can consider the possibility fractional molecules/atoms correspond to exotic atoms formed by electrons bound to exotically ionized dark super-nuclei: the sizes of these nuclei are however above atomic size scale so that dark electrons would move in a harmonic oscillator potential rather than Coulombic potential and form states analogous to atomic nuclei. The prediction would be the existence of magic electron numbers [K30] . Amazingly, there is strong experimental evidence for the existence of this kind of many-electron states. Even more, these states are able to mimic the chemistry of ordinary atoms [D25, D54, D34] . The formation of hydrogen bonds between catalyst and substrate could be the correlate for the fusion of fractional hydrogen atoms.

If the fusion process gives rise 1/1-hydrogen, its spontaneous decay to ordinary hydrogen would liberate the difference of binding energies as metabolic energy helping to overcome the energy barrier for the reaction. The liberated energy would be rather large and correspond 3.4 eV UV photon even for $r = 2$ which suggests that it does not relate with standard metabolism. For larger values of r the liberated energy rapidly approaches to the ground state energy of hydrogen. Note that the binding energy of ordinary hydrogen atom in state $n = r$ has in the lowest order approximation same energy as the ground state of dark hydrogen atom for $\hbar/\hbar_0 = r$ so that one can consider the possibility of a resonant coupling of these states.

Fractional protons and electrons have effective charge $\pm ke/r$ so that the binding regions of catalysts and reacting molecules could carry effective fractional surface charge.

This might relate in an interesting manner to the problem of how poly-electrolytes can be stable (I am grateful for Dale Trenary for pointing me the problem and for interesting discussions). For instance, DNA carries a charge of -2 units per nucleotide due to the phosphate backbone. The models trying to explain the stability involve effective binding of counter ions to the polyelectrolyte so that the resulting system has a lower charge density. The simulations of DNA condensation by Stevens [I17] however predict that counter ion charge should satisfy $z > 2$ in the case of DNA. The problem is of course that protons with $z = 1$ are the natural counter ions. The positive surface charge defined by the fractional protons attached to the nucleotides of DNA strand could explain the stability.

The hydrogen atoms in hydrogen bonds as fractional hydrogen atoms and $H_{1.5}O$ formula for water

The simplest assumption is that the hydrogens associated with hydrogen bonds are actually associated with 1/1 type dark hydrogen atoms. This hypothesis has interesting implications and could explain the formula $H_{1.5}O$ for water in atto-second time scales suggested by neutron diffraction and electron scattering [D30, D27, D32, D52] .

The formation of hydrogen bond would correspond to a fusion of name and conjugate name between $H_{k/r}$-O-H atom and its conjugate $H_{(r-k)/r}$-O-H atom. The resulting pairs would obey the chemical formula H_3-O_2. Hence the formation of hydrogen bonds would predict the $H_{1.5}O$ formula suggested by neutron diffraction and electron scattering in atto-second time scale. This holds true only if one has complete pairing by hydrogen bonds. A more plausible explanation is that just the presence of fractioanl hydrogens implies the effect. Furthermore, the fraction of dark protons can depend on temperature.

The roles of water and ordered water in catalysis

The new view about hydrogen bond allows to understand the role of water in biology at qualitative level. For instance, one can

1. tentatively identify "ordered water" as a phase in which all $H_{k/r}$ atoms and their conjugates have combined to $H_{1/1}$ atoms,
2. understand why (or perhaps it is better to say "predict that") water containing $H_{k/r}$ atoms acts as a catalytic poison so that the binding sites of catalysts and reactants must be isolated from water unless the water is ordered,
3. justify the belief that gel phase involving ordered water is necessary for biological information processing,
4. understand why hydration causes hydrolysis,
5. understand the instability of DNA against decay to RNA outside nucleus.

A more more detailed sketch looks like following.

1. Suppose that at least part of water molecules appear in form $H_{k/r}$-OH and $H_{(r-k)/r}$-O-H. These molecules and the the molecule $H_{1/1}$-OH_2 formed in their fusion has much smaller binding energy than ordinary water molecule and is expected to be unstable against transition to H_3O. This would suggest that the feed of metabolic energy is needed to generate the dark hydrogen atoms.

   Fractional dark water molecules can join pairwise to form H-O-($H_{1/1}$)-O-H≡ H_3O_2 with $H_{1/1}$-atoms replacing hydrogen in hydrogen bond. Also $H_{k_1/r}$-O-$H_{k_2/r}$ molecules are possible and could form closed strings obeying the chemical formula $O_n(H_{1/1})_n$. Also open strings with H-O:s at ends are possible. This phase of water might allow identificaton as "ordered water" believed to be associated with gel phase and be crucial for quantal information processing inside cell. Liquid crystal phase of water could correspond to a bundle of open vertical segments H-$O_n(H_{1/1})_{n-2}$-H forming a 2-dimensional liquid (vertical freezing).

2. Exotic water molecules could spoil the action of both catalyst and reactant molecules by attaching to the "letters" in the name of catalyst or reactant so that the letters are not visible and catalyst and reactant cannot recognize each other anymore. Hence binding sites of catalyst and reactant must be isolated from water containing fractional water molecules. This is what Sidorova and Rau [I16] suggest on basis of comparison of specific and non-specific catalysts: non-specific catalysts contain water in an isolated binding volume whereas for specific catalysts this volume is empty. An alternative mechanism hindering water molecules to attach to "letters" is that water is "ordered water" with no fractional water molecules present.

3. DNA is known to be stable against decay to RNA via hydration inside the cell but not outside. Hydration could correspond to the joining of fractional water to sites of DNA transforming it to RNA. Inside nucleus this cannot occur if water is in ordered water phase permanently.

How the first self-replicators emerged?

The identification of the first self replicator can be seen as perhaps the most fascinating and challenging problem faced by the pre-biotic model builders. Self replicator is by definition an entity which catalyzes its own replication. The analogy with the self-referential statement appearing in Gödel's theorem obvious.

In TGD framework self replication would reduce to a spontaneous decay of $H_{1/1}$-atom to $H_{k/r}$- and $H_{(r-k)/r}$-atoms and their subsequent completion to $H_{1/1}$-atoms

The picture about emergence of self-replicators would be roughly following.

1. The first self-replicating entities would have been plasmoids [I12] generating $H_{1/1}$ atoms whose presence would have made possible the emergence of the first molecular self replicators. The generation of $H_{1/1}$ atoms requires metabolic energy feed. In the first approximation the decay of $H_{1,1}$ to fractional hydrogen atoms does not liberate nor require energy.

2. $H_{k/r}$ atoms would have replaced some ordinary H-atoms in some negatively charged molecules M_i (perhaps MXTP, $X = A, U, C, G$) leading to a spontaneous emergence of linear negatively charged polymers consisting of M_i. One can imagine a coding in which each X corresponds to fixed value of k or collection of the (2 hydrogen bonds or 3 hydrogen bonds depending on X). This would make the attachment of X and its conjugate to form a hydrogen bond a highly favored process.

3. $H_{k/r}$ atoms would have taken also the role of active binding sites. In ordered water conjugate molecules $M_{c,i}$ having $H_{(r-k)/r}$ atoms as labels would have had high probability to attach to the polymers made of M_i.

4. RNA molecules are good candidates for self-replicators in the presence of ordered water. The phase transition from ordinary to ordered water (which would have developed later to sol-gel phase transition) would have been an essential element of replication.

The role of water in chiral selection

In the latest New Scientist (when I am writing this) there was a news telling that chiral selection occurs in water but not in heavy water [C23] . The L form of amino-acid glutamate is more stable than R in ordinary but not so in heavy water so that water environment must be responsible for the chirality selection of bio-molecules. The proposed explanation for the finding, whose importance cannot be over-estimated, was following.

1. Water molecules have two forms: orto- and para, depending on whether the nuclear spins of protons are parallel or opposite. Deuterium nuclei are spinless so that heavy water has only single form. In thermal equilibrium the fraction of orto water is 3/4 and para water 1/4.

2. Ortho-water is magnetic and if L form of amino-acid is slightly more magnetic than R, chirality selection can be understood as result of the magnetic interaction with water.

One can of course wonder how extremely short ranged weak interactions could produce strong enough effect on the magnetic moment. The situation is not made easier by the fact that magnetic interaction energies are inherently very weak and deep below the thermal threshold.

It is interesting to find whether these findings could be explained by and allow a more detailed formulation of the TGD based model for water based on the notion of fractional hydrogen atom, the new view about hydrogen bond, and the notion of dark protonic strings forming atomic sized super-nuclei carrying exotic weak charges.

1. Dark matter brings in long ranged exotic weak interactions which can produce large parity breaking effects in atomic and even longer length scales. The long ranged parity breaking weak interactions of the dark protonic super nuclei assignable to amino-acids and water could explain the chiral selection.

2. The magnetic interaction energy is scaled up by r, so that magnetic interactions could indeed play a key role. Ordinary classical magnetic fields are in TGD framework always accompanied by Z^0 magnetic fields. If amino-acids possess exotic em charge implying also exotic weak charge, one can understand the chiral breaking as being induced by the Z^0 magnetic interaction of aminocids with the dark magnetic fields generated by water molecules or their clusters possessing a net magnetic moment. In heavy water these fields would be absent so that the experimental findings could be understood.

3. The experimental evidence that water behaves as $H_{1.5}O$ in atto-second time scales means that 1/4:th of protons of water are effectively dark. The notion of fractional hydrogen atom leads to a model of hydrogen bond predicting correctly $H_{1.5}O$ formula and the dropping of 1/4:th of protons at larger possibly dark space-time sheets. The model also predicts that the mass of $H-O-H_r-O-H \equiv 2H_{1.5}O$ hydrogen bonded pairs is very near to the mass of 2 water molecules since there are $r \simeq m_p/m_e$ electrons involved. The paired molecules have three protons and non-vanishing net nuclear spin and thus generate a magnetic field and make hydrogen bonded water a magnetic system. The natural identification would be as dark magnetic field accompanied by Z^0 magnetic field responsible for the chiral selection.

   In the case of $D-O-D_r-O-D$ mass would be by about one proton mass m_p lower than mass of two D_2O molecules so that this D-bonded heavy water would look like $D_{1.25}O$ as far as masses are considered and $D_{1.5}O$ as far neutron diffraction and electron scattering are considered. In this case no magnetic field is generated since the nuclear spin of D vanishes and no chiral breaking results. This picture explains the experimental findings. The model is not equivalent with the proposal of the experimentalists.

4. The model predicts that the protons liberated in the formation of hydrogen bonds drop to larger space-time sheets but does not specify their fate. A strong constraint comes from the requirement that the dropped particles have exotic weak charges acting as sources of the geometrically unavoidable classical Z^0 magnetic field at dark space-time sheets causing the large parity breaking. This constraint is satisfied if the protons form super-nuclei (scaled up variants of nuclei) consisting of protonic strings connected by color bonds involving exotic quark and antiquark at its ends and some of these bonds are charged (of type $u\overline{d}$ or $d\overline{u}$: this could also generate the em charge needed to make the protonic string stable.

10.4 TGD based model for qualia and sensory receptors

The identification of quantum number increments in quantum jump for a subsystem representing sub-self and the capacitor model of sensory receptor are already more than decade old ideas.

The concrete realization of this vision is based on several ideas that I have developed during last five years.

1. The vision about dark matter as a hierarchy of phases partially labeled by the value of Planck constant led to the model of DNA as topological quantum computer [K31] . In this model magnetic flux tubes connecting DNA nucleotides with the lipids of the cell membrane define strands of the braids defining topological quantum computations. The braid strand corresponds to so called wormhole flux tube and has quark and antiquark at its ends. u and d quarks and their antiquarks code for four DNA nucleotides in this model.

2. Zero energy ontology assigns to elementary particles so called causal diamonds (CDs). For u and d quarks and electron these time scales are (6.5, .78, 100) ms respectively, and correspond to fundamental biorhythms. Electron time scale corresponds to 10 Hz fundamental biorhythm defining also the fundamental frequency of speech organs, .78 ms to kHz cortical synchrony [J12] , and 160 Hz to cerebellar synchrony [J13] . Elementary particles therefore seem to be directly associated with neural activity, language, and presumably also hearing. One outcome was the modification of the earlier model of memetic code involving the notion of cognitive neutrino pair by replacing the sequence of cognitive neutrino pairs with that of quark sub-CDs within electron CD. Nerve pulses could induce the magnetization direction of quark coding for bit but there are also other possibilities. The detailed implications for the model of nerve pulse [K69] remain to be disentangled.

3. The understanding of the Negentropy Maximization Principle [K51] and the role of negentropic entanglement in living matter together with the vision about life as something in the intersection of real and p-adic worlds was a dramatic step forward. In particular, space-like and time-like negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) become basic aspects of conscious intelligence and are expected to be especially important for understanding the difference between speech and music.

4. One of the basic challenge has been to construct a quantitative model for cell membrane.

   (a) The first model was based on the assumption that long range weak forces however play a key role [K7]. They are made possible by the exotic ground state represented as almost vacuum extremal of Kähler action for which classical em and Z^0 fields are proportional to each other whereas for the standard ground state classical Z^0 fields are very weak. Neutrinos are present but it seems that they do not define cognitive or Boolean representations in the time scales characterizing neural activity. Electrons and quarks for which the time scales of causal diamonds correspond to fundamental biorhythms - one of the key observations during last years- take this role. The essential element is that the energies of the Josephson photons are in visible range. This would explain bio-photons and even why the frequencies assignable to visual receptors. The problem is that Weinberg angle must be assumed to be much smaller in the near vacuum extremal phase than in standard model.

   (b) Second model is based on Gerald Pollack's findings about fourth phase of water and exclusion zones [I4]. These zones inspire a model for pre-biotic cells. The outcome is a modification of the simplest model of Josephson junction. Besides resting potential also the difference between cyclotron energies between the two sides of the membrane plays a key role. This model allows to understand what happens in metabolism in terms of a quantum model replacing the thermodynamical model for cell membrane with its quantal "square root" inspired by Zero Energy Ontology. The model allows also to understand bio-photons as decay products of dark photons.

   (c) The success of the latter model does not of course mean that the weak forces could not be important in cell membrane scale and the realistic model could be a hybrid of these two models. The inclusion of Z^0 contribution to the effective magnetic field could also to the fact that the endogenous magnetic field deduced from Blackman's experiments is $B_{end} = 2B_E/5$ rather than B_E (Earth's magnetic field).

10.4.1 A general model of qualia and sensory receptor

The identification of sensory qualia in terms of quantum number increments and geometric qualia representing geometric and kinematic information in terms of moduli of CD, the assignment of sensory qualia with the membrane of sensory receptor, and capacitor model of qualia are basic ideas behind the model. The communication of sensory data to magnetic body using Josephson photons is also a key aspect of the model.

A general model of qualia

It is good to start by summarizing the general vision about sensory qualia and geometric qualia in TGD Universe.

1. The basic assumption is that sensory qualia correspond to increments of various quantum numbers in quantum jump. Standard model quantum numbers- color quantum numbers, electromagnetic charge and weak isospin, and spin are the most obvious candidates. Also cyclotron transitions changing the integer characterizing cyclotron state could corresponds to some kind of quale- perhaps 'a feeling of existence'. This could make sense for the qualia of the magnetic body.

2. Geometric qualia could correspond to the increments of zero modes characterizing the induced CP_2 Käjhler form of the partonic 2-surface and of the moduli characterizing the causal diamonds serving as geometric correlates of selves. This moduli space involves the position of CD and the relative position of tips as well as position in CP_2 and relative position of two CP_2 points assigned to the future and past boundaries of CD. There are good motivations for proposing that the relative positions are quantized. This gives as a special case the quantization of the scale of CD in powers of two. Position and orientation sense could would represent this kind of qualia. Also kinematical qualia like sensation of acceleration could correspond to geometric qualia in generalized 4-D sense. For instance, the sensation about motion could be coded by Lorentz boots of sub-CD representing mental image about the object.

3. One can in principle distinguish between qualia assignable to the biological body (sensory receptors in particular) and magnetic body. The basic question is whether sensory qualia can be assigned only with the sensory receptors or with sensory pathways or with both. Geometric qualia might be assignable to the magnetic body and could provide third person perspective as a geometric and kinematical map of the body and its state of motion represented using the moduli space assignable to causal diamonds (CD). This map could be provided also by the body in which case the magnetic body would only share various mental images. The simplest starting assumption consistent with neuro-science is that sensory qualia are assigned with the cell membrane of sensory receptor and perhaps also with the neurons receiving data from it carried by Josephson radiation coding for the qualia and possibly partially regenerating them if the receiving neuron has same value of membrane potential as the sensory receptor when active. Note that during nerve pulse also this values of membrane potential is achieved for some time.

Could some sensory qualia correspond to the sensory qualia of the magnetic body?

Concerning the understanding of a detailed model for how sensory qualia are generated, the basic guideline comes from the notion of magnetic body and the idea that sensory data are communicated to the magnetic body as Josephson radiation associated with the cell membrane. This leaves two options: either the primary a sensory qualia are generated at the level of sensory receptor and the resulting mental images negentropically entangle with the "feeling of existence" type mental images at the magnetic body or they can be also generated at the level of the magnetic body by

Josephson radiation -possibly as cyclotron transitions. The following arguments are to-be-or-not-to-be questions about whether the primary qualia must reside at the level of sensory receptors.

1. Cyclotron transitions for various cyclotron condensates of bosonic ions or Cooper pairs of fermionic ions or elementary particles are assigned with the motor actions of the magnetic body and Josephson frequencies with the communication of the sensory data. Therefore it would not be natural to assign qualia with cyclotron transitions. One the other hand, in zero energy ontology motor action can be regarded formally as a time reversed sensory perception, which suggests that cyclotron transitions correlated with the "feeling of existence" at magnetic body entangled with the sensory mental images. They could also code for the pitch of sound as will be found but this quale is strictly speaking also a geometric quale in the 4-D framework.

2. If Josephson radiation induces cyclotron transitions, the energy of Josephson radiation must correspond to that of cyclotron transition. This means very strong additional constraint not easy to satisfy except during nerve pulse when frequencies varying from about 10^{14} Hz down to kHz range are emitted the system remains Josephson contact. Cyclotron frequencies are also rather low in general, which requires that the value of $\hbar$ must be large in order to have cyclotron energy above the thermal threshold. This would however conform with the very beautiful dual interpretation of Josephson photons in terms of bio-photons and EEG. One expects that only high level qualia can correspond to a very large values of $\hbar$ needed.

   For the sake of completeness it should be noticed that one might do without large values of $\hbar$ if the carrier wave with frequency defined by the metabolic energy quantum assignable to the kicking and that the small modulation frequency corresponds to the cyclotron frequency. This would require that Josephson frequency corresponds to the frequency defined by the metabolic quantum. This is not consistent with the fact that very primitive organisms possess sensory systems.

3. If all primary qualia are assigned to the magnetic body, Josephson radiation must include also gluons and light counterparts of weak bosons are involved besides photons. This is quite a strong additional assumption and it will be found that the identification of sensory qualia in terms of quantum numbers of quark pair restricts them to the cell membrane. The coding of qualia by Josephson frequencies is however possible and makes it possible to regenerate them in nervous system. The successful model explaining the peak frequencies of photoreceptors in terms of ionic cyclotron frequencies supports this view and provides a realization for an old idea about spectroscopy of consciousness which I had already been ready to give up.

Capacitor model of sensory qualia

In capacitor model of sensory receptor the increments of quantum numbers are amplified as particles with given quantum numbers flow between the plates of capacitor like system and the second plate defines the sub-self responsible for the mental image. The generation of complementary qualia assignable to the two plates and bringing in mind complementary colors is predicted. The capacitor is at the verge of di-electric breakdown. The interior and exterior of the receptor cell are the most plausible candidates for the capacitor plates with lipid layers defining the analog of di-electric able to changes its properties. Josephson currents generating Josephson radiation could communicate the sensory percept to the magnetic body but would not generate genuine sensory qualia there (the pitch of sound would be interpreted as a geometric quale). The coding is possible if the basic qualia correspond in one-one manner to ionic Josephson currents. There are sensory receptors which themselves do not fire (this is the case for hair cells for hearing and tactile receptor cells) and in this case the neuron next to the receptor in the sensory pathway would take the role of the quantum critical system.

The notion of sensory capacitor can be generalized. In zero energy ontology the plates could be effectively replaced with positive and negative energy parts of zero energy state or with cyclotron Bose-Einstein condensates corresponding to two different energies. Plates could also correspond to a pair of space-time sheets labeled by different p-adic primes and the generation of quale would correspond in this case to a flow of particles between the space-time sheets or magnetic flux tubes connected by contacts defining Josephson junctions.

The TGD inspired model for photoreceptors [K69] relies crucially on the assumption that sensory neurons at least and probably all cell membranes correspond to nearly vacuum extremals with the value of Weinberg angle equal to $sin^2(\theta_W) = .0295$ and weak bosons having Compton length of order cell size and ordinary value of Planck constant. This also explains the large parity breaking effects in living matter. The almost vacuum extremal property conforms with the vision about cell membrane as a quantum critical system ideal for acting as a sensory receptor.

10.4.2 Detailed model for the qualia

The proposed vision about qualia requires a lot of new physics provided by TGD. What leads to a highly unique proposal is the intriguing coincidence of fundamental elementary particle time scales with basic time scales of biology and neuro science and the model of DNA as topological quantum computer [K31] .

1. Zero energy ontology brings in the size scale of CD assignable to the field body of the elementary particle. Zero energy states with negentropic time-like entanglement between positive and negative energy parts of the state might provide a key piece of the puzzle. The negentropic entanglement (see fig. `http://www.tgdtheory.fi/appfigures/cat.jpg` or fig. 21 in the appendix of this book) beween positive energy parts of the states associated with the sub-CD assignable to the cell membrane and sub-CD at the magnetic body is expected to be an important factor.

2. For the standard value of $\hbar$ the basic prediction would be 1 ms second time scale of d quark, 6.5 ms time scale of u quark, and .1 second time scale of electron as basic characterizes of sensory experience if one accept the most recent estimates $m(u) = 2$ MeV and $m(d) = 5$ MeV for the quark masses [C48] . These time scales correspond to 10 Hz, 160 Hz, and 1280 Hz frequencies, which all characterize neural activity (for the identification of 160 Hz frequency as cerebellar resonance frequency see [J13]). Hence quarks could be the most interesting particles as far as qualia are considered and the first working hypothesis would be that the fundamental quantum number increments correspond to those for quark-anti-quark pair. The identification in terms of quantum numbers of single quark is inconsistent with the model of color qualia.

3. The model of DNA as topological quantum computer led to the proposal that DNA nucleotides are connected to the lipids of the cell membrane by magnetic flux tubes having quark and antiquark at its ends such that the u and d quarks and their antiquarks code for the four nucleotides. The outer lipid layer was also assumed to be connected by flux tubes to the nucleotide in some other cell or in cell itself.

4. The model for DNA as topological quantum computer did not completely specify whether the flux tubes are ordinary flux tubes or wormhole flux tubes with possibly opposite signs of energy assigned with the members of the flux tube pair. Although it is not necessary, one could assume that the quantum numbers of the two parallel flux tubes cancel each other so that wormhole flux tube would be characterized by quantum numbers of quark pairs at its ends. It is not even necessary to assume that the net quantum numbers of the flux tubes vanish. Color confinement however suggests that the color quantum at the opposite ends of the flux tube are of opposite sign.

   (a) The absence of a flux tube between lipid layers was interpreted as an isolation from external world during the topological quantum computation. The emergence of the flux tube connection means halting of topological quantum computation. The flux tube connection with the external world corresponds to sensory perception at the level of DNA nucleotide in consistency with the idea that DNA plays the role of the brain of cell [K74] . The total color quantum numbers at the ends of the flux tubes were assumed to sum up to zero. This means that the fusion of the flux tubes ending to the interior and exterior cell membrane to single one creates a flux tube state not localized inside cell and that the interior of cell carries net quantum numbers. The attractive interpretation is that this process represents the generation of quale of single nucleotide.

   (b) The formation of the flux tube connection between lipid layers would involve the transformation of both quark-antiquark pairs to an intermediate state. There would be no kinematic constraints on the process nor to the mass scales of quarks. A possible mechanism for the separation of the two quark-antiquark pairs associated with the lipids from the system is double reconnection of flux tubes which leads to a situation in which the quark-antiquark pairs associated with the lipid layers are connected by short flux loops and separated to a disjoint state and there is a long wormhole flux tube connecting the nucleotides possibly belonging to different cells.

   (c) The state of two quark pairs need not have vanishing quantum numbers and one possibility is that the quantum numbers of this state code for qualia. If the total numbers of flux tubes are vanishing also the net quantum numbers of the resulting long flux tube connecting two different cells provide equivalent coding. A stronger condition is that this state has vanishing net quantum numbers and in this case the ends of the long flux tube would carry opposite quantum numbers. The end of flux tube at DNA nucleotide would characterize the quale.

5. Two identification of primary qualia are therefore possible.

   (a) If the flux tubes have vanishing net quantum numbers, the primary sensory quale can be assigned to single receptor cell and the flow of the quantum numbers corresponds to the extension of the system with vanishing net quantum numbers in two-cell system.

   (b) If the net quantum numbers of the flux tube need not vanish, the resulting two cell system carries non-vanishing quantum numbers as the pair of quark-antiquark pairs removes net quantum numbers out of the system.

6. If the net quantum numbers for the flux tubes vanish always, the specialization of the sensory receptor membrane to produce a specific quale would correspond to an assignment of specific quantum numbers at the DNA ends of the wormhole flux tubes attached to the lipid layers of the cell membrane. The simplest possibility that one can imagine is that the outer lipid layer is connected to the conjugate DNA nucleotide inside same cell nucleus. This option would however assign vanishing net quantum number increments to the cell as whole and is therefore unacceptable.

7. The formation of a temporary flux tube connection with another cell is necessary during the generation of quale and the question is what kind of cell is in question. The connection of the receptor to cells along the sensory pathway are expected to be present along the entire sensory pathway from DNA nucleotide to a nucleotide in the conjugate strand of second neuron to DNA nucleotide of the third neutron.... If Josephson photons are able to regenerate the quale in second neuron this would make it possible to replicate the quale along entire sensory pathway. The problem is that Josephson radiation has polarization orthogonal to axons and must propagate along the axon whereas the flux tube connection must be orthogonal to axon. Hence the temporary flux tube connection is most naturally between receptor cells and would mean horizontal integration of receptor cells to a larger structure. A holistic process in directions parallel and orthogonal to the sensory pathway would be in question. Of course, the flux tube could be also curved and connect the receptor to the next neuron along the sensory pathway.

8. The specialization of the neuron to sensory receptor would require in the framework of positive energy ontology that -as far as qualia assignable to the electro-weak quantum numbers are considered - all DNA nucleotides are identical by the corresponds of nucleotides with quarks and antiquarks. This cannot be the case. In zero energy ontology and for wormhole flux tubes it is however enough to assume that the net electroweak quantum numbers for the quark antiquark pairs assignable to the DNA wormhole contact are same for all nucleotides. This condition is easy to satisfy. It must be however emphasized that there is no reason to require that all nucleotides involved generate same quale and at the level of neurons sensory maps assigning different qualia to different nucleotides and lipids allowing DNA to sensorily perceive the external world are possible.

The model should be consistent with the assignment of the fundamental bio-rhythms with the CDs of electron and quarks.

1. Quark color should be free in long enough scales and cellular length scales are required at least. The QCD in question should therefore have long enough confinement length scales. The first possibility is provided by almost vacuum extremals with a long confinement scale also at the flux tubes. Large $\hbar$ for the cell membrane space-time sheet seems to be unavoidable and suggests that color is free in much longer length scale than cell length scale.

2. Since the length of the flux tubes connecting DNA and cell membrane is roughly 1 micrometer and by a factor of order 10^7 longer than the d quark Compton length, it seems that the value of Planck constant must be of this order for the flux tubes. This however scales up the time scale of d quark CD by a factor of 10^{14} to about 10^4 years! The millisecond and 160 ms time scales are much more attractive. This forces to ask what happens to the quark-anti-quark pairs at the ends of the tubes.

3. The only possibility seems to be that the reconnection process involves a phase transition in which the closed flux tube structure containing the two quark pairs assignable to the wormhole contacts at lipid layers is formed and leaks to the page of the Big Book with pages partially labeled by the values of Planck constant. This page would correspond to the standard value of Planck constant so that the corresponding d quark CDs would have a duration of millisecond. The reconnection leading to the ordinary situation would take place after millisecond time scale. The standard physics interpretation would be as a quantum fluctuation having this duration. This sequence of quark sub-CDs could define what might be called memetic codon representation of the nerve pulse sequence.

4. One can also consider the possibility is that near vacuum extremals give rise to a copy of hadron physics for which the quarks associated with the flux tubes are light. The Gaussian Mersennes corresponding to $k = 151, 157, 163, 167$ define excellent p-adic time scales for quarks and light variants of weak gauge bosons. Quark mass 5 MeV would with $k = 120$ would be replaced with $k = 163$ (167) one would have mass 1.77 eV (.44 eV). Small scaling of both masses gives 2 eV and .5 eV which correspond to basic metabolic quanta in TGD framework. For quark mass of 2 MeV with $k = 123$ $k = 163$ (167) one would give masses .8 eV (.05 eV). The latter scale correspond to Josephson energy assignable with the membrane potential in the ordinary phase.

   In this case a phase transition transforming almost vacuum extremal to ordinary one takes place. What this would mean that the vacuum extremal property would hold true below much shorter p-adic length scale. In zero energy ontology the scaling up of quark masses is in principle possible. This option looks however too artificial.

10.4.3 Overall view about qualia

This picture leads to the following overall view about qualia. There are two options depending on whether single quark-antiquark pair or two of them labels the qualia. In the following only the simpler option with single quark-antiquark pair is discussed.

1. All possible pairings of spin and electroweak isospin (or em charge) define 16 basic combinations if one assumes color singletness. If arbitrary color is allowed, there is a nine-fold increase of quantum numbers decomposable to color singlet and octet qualia and further into 3×15 qualia with vanishing increments of color quantum numbers and 6×16 qualia with non-vanishing increments of color quantum numbers. The qualia with vanishing increments for electroweak quantum numbers could correspond to visual colors. If electroweak quantum numbers of the quark-anti-quark pair vanish, one has 3×7 *resp.* 6×8 combinations of colorless *resp.* colored qualia.

2. There is a huge number of various combinations of these fundamental qualia if one assumes that each nucleotide defines its own quale and fundamental qualia would be analogous to constant functions and more general qualia to general functions having values in the space

with $9 \times 16 - 1$ points. Only a very small fraction of all possible qualia could be realized in living matter unless the neurons in brain provide representations of body parts or of external world in terms of qualia assignable to lipid-nucleotide pairs. The passive DNA strand would be ideal in this respect.

3. The basic classification of qualia is as color qualia, electro-weak quale, and spin quale and products of these qualia. Also combinations of color qualia and and electroweak and spin quale are possible and could define exotic sensory qualia perhaps not yet realized in the evolution. Synesthesia is usually explained in terms of sensory leakage between sensory pathways and this explanation makes sense also in TGD framework if there exists a feedback from the brain to the sensory organ. Synesthesia cannot however correspond to the product qualia: for "quantum synesthesia" cross association works in both directions and this distinguishes it from the ordinary synesthesia.

4. The idea about brain and genome as holograms encourages to ask whether neurons or equivalently DNA could correspond to sensory maps with individual lipids representing qualia combinations assignable to the points of the perceptive field. In this framework quantum synesthesia would correspond to the binding of qualia of single nucleotide (or lipid) of neuron cell membrane as a sensory representation of the external world. DNA is indeed a holographic representation of the body (gene expression of course restricts the representation to a part of organism). Perhaps it is this kind of representation also at the level of sensory experience so that all neurons could be little sensory copies of body parts as holographic quantum homunculi. In particular, in the associative areas of the cortex neurons would be quantum synesthetes experiencing the world in terms of composite qualia.

5. The number of flux tube connections generated by sensory input would code for the intensity of the quale. Josephson radiation would do the same at the level of communications to the magnetic body. Also the temporal pattern of the sequence of quale mental images matters. In the case of hearing this would code for the rhythmic aspects and pitch of the sound.

10.4.4 About detailed identification of the qualia

One can make also guesses about detailed correspondence between qualia and quantum number increments.

1. Visual colors would correspond to the increments of only color quantum numbers. Each biologically important ion would correspond to its own color increment in one-one correspondence with the three pairs of color-charged gluons and these would correspond to blue-yellow, red-green, and black white [K69] . Black-white vision would mean a restriction to the $SU(2)$ subgroup of color group. The model for the cell membrane as a nearly vacuum extremal assigns the peak frequencies corresponding to fundamental colors with biologically important ions. Josephson radiation could induce artificially the same color qualia in other neurons and this might provide an manner to communicate the qualia to the brain where they could be re-experienced at neuronal level. Some organisms are able to perceive also the polarization of light. This requires receptors sensitive to polarization. The spin of quark pair would naturally code for polarization quale.

2. Also tastes and odours define qualia with "colors". Certainly the increments of electroweak numbers are involved but since these qualia do not have any directional flavor, spin is probably not involved. This would give c 3×4 basic combinations are possible and can certainly explain the 5 or 6 basic tastes (counted as the number of different receptors). Whether there is a finite number of odours or not has been a subject of a continual debate and it might be that odours already correspond to a distribution of primary qualia for the receptor cell. That odours are coded by nerve pulse patterns for a group of neurons [J18] would conform with this picture.

3. Hearing seems to represent a rather colorless quale so that electroweak isospin suggests again itself. If we had a need to hear transversely polarized sound also spin would be involved. Cilia are involved also with hair cells acting as sensory receptors in the auditory system

and vestibular system. In the case of hearing the receptor itself does not fire but induces a firing of the higher level neuron. The temporal pattern of qualia mental images could define the pitch of the sound whereas the intensity would correspond to the number of flux tube connections generated.

The modulation of Josephson frequencies -rather than Josephson frequencies as such- would code for the pitch and the total intensity of the Josephson radiation for the intensity of the sound and in fact any quale. Pitch represents non-local information and the qualia sub-selves should be negentropically entangled in time direction. If not, the experience corresponds to a sequence of sound pulses with no well-defined pitch and responsible for the rhythmic aspects of music. Right brain sings-left brain talks metaphor would suggests that right and left brain have different kind of specializations already at the level of sensory receptors.

4. Somato-sensory system gives rise to tactile qualia like pain, touch, temperature, proprioception (body position). There are several kinds of receptors: nocireceptors, mechanoreceptors, thermoreceptors, etc... Many of these qualia have also emotional coloring and it might be that the character of entanglement involved (negentropic/entropic defines the emotional color of the quale. If this is the case, one might consider a pure quale of touch as something analogous to hearing quale. One can argue that directionality is basic aspect of some of these qualia -say sense of touch- so that spin could be involved besides electroweak quantum numbers. The distribution of these qualia for the receptor neuron might distinguish between different tactile qualia.

10.5 Could cell membrane correspond to almost vacuum extremal?

The question whether cell membrane or even cell could correspond almost vacuum extremal of Kähler action (in some cases) was the question which led to the realization that the frequencies of peak sensitivity for photoreceptors correspond to the Josephson frequencies of biologically important ions if one accepts that the value of the Weinberg angle equals to $sin^2(\theta_W) = .0295$ instead of the value .23 in the normal phase, in which the classical electromagnetic field is proportional to the induced Kähler form of CP_2 in a good approximation. Another implication made possible by the large value of Planck constant is the identification of Josephson photons as the counterparts of bio-photons one one hand and those of EEG photons on the other hand. These observation in turn led to a detailed model of sensory qualia and of sensory receptor. Therefore the core of this argument deserves to be represented also here although it has been discussed in [K69] .

10.5.1 Cell membrane as almost vacuum extremal

Although the fundamental role of vacuum extremals for quantum criticality and life has been obvious from the beginning, it took a long time to realize how one could model living cell as this kind of system.

1. Classical electric fields are in a fundamental role in biochemistry and living biosystems are typically electrets containing regions of spontaneous electric polarization. Fröhlich [I8] proposed that oriented electric dipoles form macroscopic quantum systems with polarization density serving as a macroscopic order parameter. Several theories of consciousness share this hypothesis. Experimentally this hypothesis has not been verified.

2. TGD suggests much more profound role for the unique di-electric properties of the biosystems. The presence of strong electric dipole fields is a necessary prerequisite for cognition and life and could even force the emergence of life. Strong electric fields imply also the presence of the charged wormhole BE condensates: the surface density of the charged wormholes on the boundary is essentially equal to the normal component of the electric field so that wormholes are in some sense 'square root' of the dipole condensate of Fröhlich! Wormholes make also possible pure vacuum polarization type dipole fields: in this case the magnitudes of the em field at the two space-time sheets involved are same whereas the directions of the

fields are opposite. The splitting of wormhole contacts creates fermion pairs which might be interpreted as cognitive fermion pairs. Also microtubules carry strong longitudinal electric fields. This formulation emerged much before the identification of ordinary gauge bosons and their superpartners as wormhole contacts.

Cell membrane is the basic example about electret and one of the basic mysteries of cell biology is the resting potential of the living cell. Living cell membranes carry huge electric fields: something like 10^7 Volts per meter. For neuron resting potential corresponds to about .07 eV energy gained when unit charge travels through the membrane potential. In TGD framework it is not at all clear whether the presence of strong electromagnetic field necessitates the presence of strong Kähler field. The extremely strong electric field associated with the cell membrane is not easily understood in Maxwell's theory and almost vacuum extremal property could change the situation completely in TGD framework.

1. The configuration could be a small deformation of vacuum extremal so that the system would be highly critical as one indeed expects on basis of the general visiona about living matter as a quantum critical system. For vacuum extremals classical em and Z^0 fields would be proportional to each other. The second half of Maxwell's equations is not in general satisfied in TGD Universe and one cannot exclude the presence of vacuum charge densities in which case elementary particles as the sources of the field would not be necessarily. If one assumes that this is the case approximately, the presence of Z^0 charges creating the classical Z^0 fields is implied. Neutrinos are the most candidates for the carrier of Z^0 charge. Also nuclei could feed their weak gauge fluxes to almost non-vacuum extremals but not atomic electrons since this would lead to dramatic deviations from atomic physics. This would mean that weak bosons would be light in this phase and also Weinberg angle could have a non-standard value.

2. There are also space-time surfaces for CP_2 projection belongs to homologically non-trivial geodesic sphere. In this case classical Z^0 field can vanish [L1] , [L1] and the vision has been that it is sensible to speak about two basic configurations.

   (a) Almost vacuum extremals (homologically trivial geodesic sphere).

   (b) Small deformations of non-vacuum extremals for which the gauge field has pure gauge Z^0 component (homologically non-trivial geodesic sphere).

   The latter space-time surfaces are excellent candidates for configurations identifiable as TGD counterparts of standard electroweak physics. Note however that the charged part of electroweak fields is present for them.

3. To see whether the latter configurations are really possible one must understand how the gauge fields are affected in the color rotation.

   (a) The action of color rotations in the holonomy algebra of CP_2 is non-trivial and corresponds to the action in $U(2)$ sub-group of $SU(3)$ mapped to $SU(2)_L \times U(1)$. Since the induced color gauge field is proportional to Kähler form, the holonomy is necessary Abelian so that also the representation of color rotations as a sub-group of electro-weak group must correspond to a local $U(1)$ sub-group local with respect to CP_2 point.

   (b) Kähler form remains certainly invariant under color group and the right handed part of Z^0 field reducing to $U(1)_R$ sub-algebra should experience a mere Abelian gauge transformation. Also the left handed part of weak fields should experience a local $U(1)_L$ gauge rotation acting on the neutral left handed part of Z^0 in the same manner as it acts on the right handed part. This is true if the $U(1)_L$ sub-group does not depend on point of CP_2 and corresponds to Z^0 charge. If only Z^0 part of the induced gauge field is non-vanishing as it can be for vacuum extremals then color rotations cannot change the situation. If Z^0 part vanishes and non-vacuum extremal is in question, then color rotation rotation of W components mixing them but acts as a pure $U(1)$ gauge transformation on the left handed component.

(c) It might not be without importance that for any partonic 2-surface induced electro-weak gauge fields have always U(1) holonomy, which could allow to define what neutral part of induced electroweak gauge field means locally. This does not however hold true for the 4-D tangent space distribution. In any case, the cautious conclusion is that there are two phases corresponding to nearly vacuum extremals and small deformations of extremals corresponding to homologically non-trivial geodesic spheres for which the neutral part of the classical electro-weak gauge field reduces to photon field.

4. The unavoidable presence of long range Z^0 fields would explain large parity breaking in living matter, and the fact that neutrino Compton length is of the order of cell size would suggest the possibility that within neutrino Compton electro-weak gauge fields or even longer scales could behave like massless fields. The explanation would be in terms of the different ground state characterized also by a different value of Weinberg angle. For instance, of the p-adic temperature of weak bosons corresponds to $T_p = 1/2$, the mass scale would be multiplied by a factor $\sqrt{M_{89}}$ and Compton lengths of weak bosons would be around 10^{-4} meters corresponding to the size scale of a large neuron. If the value of Planck constant is also large then the Compton length increases to astrophysical scale.

5. From the equations for classical induced gauge fields in terms of Kähler form and classical Z^0 field [L1] , [L1]

$$\gamma = 3J - \frac{p}{2}Z^0 \ , \quad Q_Z = I_L^3 - pQ_{em} \ , \quad p = sin^2(\theta_W) \tag{10.5.1}$$

it follows that for the vacuum extremals the part of the classical electro-weak force proportional to the electromagnetic charge vanishes for $p = 0$ so that only the left-handed couplings to the weak gauge bosons remain. The absence of electroweak symmetry breaking and vanishing or at least smallness of p would make sense below the Compton length of dark weak bosons. If this picture makes sense it has also implications for astrophysics and cosmology since small deformations of vacuum extremals are assumed to define the interesting extremals. Dark matter hierarchy might explain the presence of unavoidable long ranged Z^0 fields as being due to dark matter with arbitrarily large values of Planck constant so that various elementary particle Compton lengths are very long.

6. The simplest option is that the dark matter -say quarks with Compton lengths of order cell size and Planck constant of order $10^7\hbar_0$ - are responsible for dark weak fields making almost vacuum extremal property possible. The condition that Josephson photons correspond to EEG frequencies implys $\hbar \sim 10^{13}\hbar_0$ and would mean the scaling of intermediate gauge boson Compton length to that corresponding to the size scale of a larger neuron. The quarks involved with with DNA as topological quantum computer model could be in question and membrane potential might be assignable to the magnetic flux tubes. The ordinary ionic currents through cell membrane -having no coupling to classical Z^0 fields and not acting as its sourse- would be accompanied by compensating currents of dark fermions taking care that the almost vacuum extremal property is preserved. The outcome would be large parity breaking effects in cell scale from the left handed couplings of dark quarks and leptons to the classical Z^0 field. The flow of Na^+ ions during nerve pulse could take along same dark flux tube as the flow of dark quarks and leptons. This near vacuum extremal property might be fundamental property of living matter at dark space-time sheets at least.

Could nuclei and neutrinos couple to light variants of weak gauge fields in the critical phase?

One of the hard-to-kill ideas of quantum TGD inspired model of quantum biology is that neutrinos might have something do with hearing and cognition. This proposal looks however unrealistic in the recent vision. I would be more than happy to get rid of bio-neutrinos but the following intriguing finding does not allow me to have this luxury.

1. Assume that the endogenous magnetic field $B_{end} = .2$ Gauss is associated with a nearly vacuum extremal and therefore accompanied by $B_Z = 2B_{end}/p$. Assume for definiteness $m_\nu = .3$ eV and $p = sin^2(\theta_W) = .23$. The neutrino cyclotron frequency is given by the following expression

$$f_\nu = \frac{m_e}{m_\nu}\frac{1}{2sin^2(\theta_W)}f_e \ .$$

From $f_e \simeq .57\times$ MHz and $p = sin^2(\theta_W) = .23$ one obtains $E_\nu = 1.7 \times 10^{-2}$ eV, which is roughly one third to the Josephson frequency of electron assignable to cell membrane. Could Josephson frequency of cell membrane excite neutrino cyclotron transitions?

2. The model for photoreceptors to be discussed below forces to conlude that the value of Weinberg angle in the phase near vacuum extremal must be $p = .0295$ if one wants to reproduces the peak energies of photoreceptors as Josephson frequencies of basic biological ions. This would predict $E_\nu = .41$ eV, which is rather near to the metabolic energy quantum. The non-relativistic formula however fails in this case and one must use the relativistic formula giving

$$E = \sqrt{g_Z Q_Z B_Z 2\pi} \simeq .48 \ eV$$

giving the metabolic energy quantum. Does this mean that Z^0 cyclotron frequency for neutrino is related to the transfer of metabolic energy using Z^0 MEs in the phase near vacuum extremals.

3. Josephson frequency is proportional to $1/\hbar$, whereas neutrino cyclotron frequency does not depend on $\hbar$ at non-relativistic energies. For larger values of $\hbar$ the neutrino becomes relativistic so that the mass in the formula for cyclotron frequency must be replaced with energy. This gives

$$E = \sqrt{n}r^{1/2}\sqrt{g_Z Q_Z B_Z 2\pi} \simeq r^{1/2} \times .48 \ eV \ \ , \ \ r = \sqrt{\hbar/\hbar_0} \ .$$

Here n refers to the cyclotron harmonic.

These observations raise the question whether the three frequencies with maximum response assignable to the three different types of receptors of visible light in retina could correspond to the three cyclotron frequencies assignable to the three neutrinos with different mass scales? The first objection is that the dependence on mass disappears completely at the relativistic limit. The second objection is that the required value value of Planck constant is rather small and far from being enough to have electroweak boson Compton length of order cell size. One can of course ask whether the electroweak gauge bosons are actually massless inside almost vacuum extremals. If fermions -including neutrino- receive their masses from p-adic thermodynamics then massless electroweak gauge bosons would be consistent with massive fermions. Vacuum extremals are indeed analogous to the unstable extrema of Higgs potential at which the Higgs vacuum expecation vanishes so that this interpretation might make sense.

Ionic Josephson frequencies defined by the resting potential for nearly vacuum extremals

If cell membrane corresponds to an almost vacuum extremal, the membrane potential potential is replaced with an effective resting potential containing also the Z^0 contribution proportional to the ordinary resting potential. The surprising outcome is that one could understand the preferred frequencies for photo-receptors [J3] as Josephson frequencies for biologically important ions. Furthermore, most Josephson energies are in visible and UV range and the interpretation in terms of bio-photons is suggestive. If the value of Planck constant is large enough Josephson frequencies are in EEG frequency range so that bio-photons and EEG photons could be both related to Josephson photons with large $\hbar$.

1. One must assume that the interior of the cell corresponds to many fermion state -either a state filled with neutrinos up to Fermi energy or Bose-Einstein condensate of neutrino Cooper pairs creating a harmonic oscillator potential. The generalization of nuclear harmonic oscillator model so that it applies to multi-neutrino state looks natural.

2. For exact vacuum extremals elementary fermions couple only via left-handed isospin to the classical Z^0 field whereas the coupling to classical em field vanishes. Both K_+,Na_+, and Cl_- $A-Z=Z+1$ so that by p-n pairing inside nucleus they have the weak isospin of neuron (opposite to that of neutrino) whereas Ca_{++} nucleus has a vanishing weak isospin. This might relate to the very special role of Ca_{++} ions in biology. For instance, Ca_{++} defines an action potential lasting a time of order .1 seconds whereas Na_+ defines a pulse lasting for about 1 millisecond [J1] . These time scales might relate to the time scales of CDs associated with quarks and electron.

3. The basic question is whether only nuclei couple to the classical Z^0 field or whether also electrons do so. If not, then nuclei have a large effective vector coupling to em field coming from Z^0 coupling proportional to the nuclear charge increasing the value of effective membrane potential by a factor of order 100. If both electrons and nuclei couple to the classical Z^0 field, one ends up with difficulties with atomic physics. If only quarks couple to the Z^0 field and one has $Z^0=-2\gamma/p$ for vacuum extremals, and one uses average vectorial coupling $\langle I_L^3\rangle = \pm 1/4$ with + for proton and - for neutron, the resulting vector coupling is following

$$\begin{aligned} (\frac{Z-N}{4}-pZ)Z^0+q_{em}\gamma &= Q_{eff}\gamma \ , \\ Q_{eff} &= -\frac{Z-N}{2p}+2Z+q_{em} \ . \end{aligned} \tag{10.5.2}$$

Here γ denotes em gauge potential. For K^+, Cl^-, Na^+, Ca^{++} one has $Z=(19,17,11,20)$, $Z-N=(-1,-1,-1,0)$, and $q_{em}=(1,-1,1,2)$. Table 1 below gives the values of Josephson energies for some values of resting potential for $p=.23$. Rather remarkably, they are in IR or visible range. This is basically due to the large value of weak isospin for nuclei.

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$E(Ion)/eV$	$V=-40\ mV$	$V=-60\ mV$	$V=-70\ mV$
Na^+	1.01	1.51	1.76
Cl^-	1.40	2.11	2.46
K^+	1.64	2.47	2.88
Ca^{++}	1.68	2.52	2.94

Table 1. Values of the Josephson energy of cell membrane for some values of the membrane voltage for $p=.23$. The value $V=-40$ mV corresponds to the resting potential for photoreceptors and $V=-70$ mV to the resting state of a typical neuron.

10.5.2 Are photoreceptors nearly vacuum extremals?

In Hodgkin-Huxley model ionic currents are Ohmian currents. If one accepts the idea that the cell membrane acts as a Josephson junction, there are also non-dissipative oscillatory Josephson currents of ions present, which run also during flow equilibrium for the ionic parts of the currents. A more radical possibility is that that the dominating parts of the ionic currents are oscillatory Josephson currents so that no metabolic energy would be needed to take care that density gradients for ions are preserved. Also in this case both nearly vacuum extremals and extremals with nearly vanishing Z^0 field can be considered. Since sensory receptors must be highly critical the natural question is whether they could correspond to nearly vacuum extremals. The quantitative success of the following model for photoreceptors supports this idea.

Photoreceptors can be classified to three kinds of cones responsible for color vision and rods responsible for black-white vision. The peak sensitivities of cones correspond to wavelengths (405, 535, 565) nm and energies (3.06, 2.32, 2.19) eV. The maximum absorption occurs in the wave length range 420-440 nm, 534-545 nm, 564-580 nm for cones responsible for color vision and 498 nm for rods responsible black-white vision [J2, J3] . The corresponding photon energies are (2.95, 2.32, 2.20) eV for color vision and to 2.49 eV for black-white vision. For frequency distribution the maxima are shifted from these since the maximum condition becomes $dI/d\lambda + 2I/\lambda = 0$, which means a shift to a larger value of λ, which is largest for smallest λ. Hence the energies for maximum absorbance are actually lower and the downwards shift is largest for the highest energy.

From Table 1 above it is clear that the energies of Josephson photons are in visible range for reasonable values of membrane voltages, which raises the question whether Josephson currents of nuclei in the classical em and Z^0 fields of the cell membrane could relate to vision.

Consider first the construction of the model.

1. Na^+ and Ca^{++} currents are known to present during the activation of the photoreceptors. Na^+ current defines the so called dark current [J3] reducing the membrane resting potential below its normal value and might relate to the sensation of darkness as eyes are closed. Hodgkin-Huxley model predicts that also K^+ current is present. Therefore the Josephson energies of these three ion currents are the most plausible correlates for the three colors.

2. One ends up with the model in the following manner. For Ca^{++} the Josephson frequency does not depend on p and requiring that this energy corresponds to the energy 2.32 eV of maximal sensitivity for cones sensitive to green light fixes the value of the membrane potential during hyper-polarization to $V = .055$ V, which is quite reasonable value. The value of the Weinberg angle parameter can be fixed from the condition that other peak energies are reproduced optimally. The result of $p = .0295$.

The predictions of the model come as follows summarized also by the Table 2 below.

1. The resting potential for photoreceptors is $V = -40$ mV [J4] . In this case all Josephson energies are below the range of visible frequencies for $p = .23$. Also for maximal hyper-polarization Na^+ Josephson energy is below the visible range for this value of Weinberg angle.

2. For $V = -40$ mV and $p = .0295$ required by the model the energies of Cl^- and K^+ Josephson photons correspond to red light. 2 eV for Cl^- corresponds to a basic metabolic quantum. For Na^+ and Ca^{++} the wave length is below the visible range. Na^+ Josephson energy is below visible range. This conforms with the interpretation of Na^+ current as a counterpart for the sensation of darkness.

3. For $V = -55$ mV - the threshold for the nerve pulse generation- and for $p = .0295$ the Josephson energies of Na^+, Ca^{++}, and K^+ a correspond to the peak energies for cones sensitive to red, green, and blue respectively. Also Cl^- is in the blue region. Ca^{++} Josephson energy can be identified as the peak energy for rods. The increase of the hyper-polarization to $V = -59$ mV reproduces the energy of the maximal wave length response exactly. A possible interpretation is that around the criticality for the generation of the action potential ($V \simeq -55$ mV) the qualia would be generated most intensely since the Josephson currents would be strongest and induce Josephson radiation inducing the quale in other neurons of the visual pathway at the verge for the generation of action potential. This supports the earlier idea that visual pathways defines a neural window. Josephson radiation could be interpreted as giving rise to bio-photons (energy scale is correct) and to EEG photons (for large enough values of $\hbar$ the frequency scales is that of EEG).

4. In a very bright illumination the hyper-polarization is $V = -65$ mV [J4] , which the normal value of resting potential. For this voltage Josephson energies are predicted to be in UV region except in case of Ca^{++}. This would suggests that only the quale 'white' is generated at the level of sensory receptor: very intense light is indeed experienced as white.

The model reproduces basic facts about vision assuming that one accepts the small value of Weinberg angle, which is indeed a natural assumption since vacuum extremals are analogous to the unstable extrema of Higgs potential and should correspond to small Weinberg angle. It deserves to be noticed that neutrino Josephson energy is 2 eV for $V = -50$ mV, which correspond to color red. 2 eV energy defines an important metabolic quantum.

Ion	Na^+	Cl^-	K^+	Ca^{++}
$E_J(.04\ mV, p = .23)/eV$	1.01	1.40	1.51	1.76
$E_J(.065\ V, p = .23)/eV$	1.64	2.29	2.69	2.73
$E_J(40\ mV, p = .0295)/eV$	1.60	2.00	2.23	1.68
$E_J(50\ mV, p = .0295)/eV$	2.00	2.49	2.79	2.10
$E_J(55\ mV, p = .0295)/eV$	2.20	2.74	3.07	2.31
$E_J(65\ mV, p = .0295)/eV$	2.60	3.25	3.64	2.73
$E_J(70\ mV, p = .0295)/eV$	2.80	3.50	3.92	2.94
$E_J(75\ mV, p = .0295)/eV$	3.00	3.75	4.20	3.15
$E_J(80\ mV, p = .0295)/eV$	3.20	4.00	4.48	3.36
$E_J(90\ mV, p = .0295)/eV$	3.60	4.50	5.04	3.78
$E_J(95\ mV, p = .0295)/eV$	3.80	4.75	5.32	3.99
Color	R	G	B	W
E_{max}	2.19	2.32	3.06	2.49
energy-interval/eV	1.77-2.48	1.97-2.76	2.48-3.10	

Table 2. The table gives the prediction of the model of photoreceptor for the Josephson energies for typical values of the membrane potential. For comparison purposes the energies E_{max} corresponding to peak sensitivities of rods and cones, and absorption ranges for rods are also given. R,G,B,W refers to red, green, blue, white. The values of Weinberg angle parameter $p = sin^2(\theta_W)$ are assumed to be .23 and .0295. The latter value is forced by the fit of Josephson energies to the known peak energies if one allows that ions - rather than their Cooper pairs - are charge carriers.

It interesting to try to interpret the resting potentials of various cells in this framework in terms of the Josephson frequencies of various ions.

1. The maximum value of the action potential is +40 mV so that Josephson frequencies are same as for the resting state of photoreceptor. Note that the time scale for nerve pulse is so slow as compared to the frequency of visible photons that one can consider that the neuronal membrane is in a state analogous to that of a photoreceptor.

2. For neurons the value of the resting potential is -70 mV. Na^+ and Ca^{++} Josephson energies 2.80 eV and 2.94 eV are in the visible range in this case and correspond to blue light. This does not mean that Ca^{++} Josephson currents are present and generate sensation of blue at neuronal level: the quale possibly generated should depend on sensory pathway. During the hyper-polarization period with -75 mV the situation is not considerably different.

3. The value of the resting potential is -95 mV for skeletal muscle cells. In this case Ca^{++} Josephson frequency corresponds to 4 eV metabolic energy quantum as the table below shows.

4. For smooth muscle cells the value of resting potential is -50 mV. In this case Na^+ Josephson frequency corresponds to 2 eV metabolic energy quantum.

5. For astroglia the value of the resting potential is -80/-90 mV for astroglia. For -80 mV the resting potential for Cl^- corresponds to 4 eV metabolic energy quantum. This suggests that glial cells could also provide metabolic energy as Josephson radiation to neurons.

6. For all other neurons except photo-receptors and red blood cells Josephson photons are in visible and UV range and the natural interpretation would be as bio-photons. The bio-photons detected outside body could represent sensory leakage. An interesting question is whether the IR Josephson frequencies could make possible some kind of IR vision.

To sum up, the basic criticism against the model is that the value of Weinberg angle must be by a factor of 1/10 smaller than the standard model value, and at this moment it is difficult to say anything about its value for nearly vacuum extremals.

A possible cure could be that the voltage is not same for different ions. This is possible since at microscopic level the Josephson junctions correspond to transmembrane proteins acting as channels and pumps. The membrane potential through receptor protein is different for color receptors. For this option one would have the correspondences
$Na^+ \leftrightarrow$ 2.19 eV (R) and $eV = 86.8$ eV,
$Cl^- \leftrightarrow$ 2.32 eV (G) and $eV = 65.8$ eV,
$K^+ \leftrightarrow$ 2.49 eV (W) and $eV = 60.2$ eV,
$Ca^{++} \leftrightarrow$ 3.06 eV (B) and $eV = 67.3$ meV.
For Na^+ the value of the membrane potential is suspiciously large.

It is interesting to look what happens when the model is generalized so that Josephson energy includes the difference of cyclotron energies at the two sides of the cell membrane and Weinberg angle has its standard model value.

1. Consider first *near to vacuum extremals*. In the formula for cyclotron frequencies in the effective magnetic field the factor Z/A in the formula of is replaced with

$$\frac{\frac{N-Z}{2p} + 2Z + q_{em}}{A} ,$$

which is not far from unity so that the cyclotron frequency would be near to that for proton for all ions. Also neutral atoms would experience classical and magnetic Z^0 fields. Cyclotron frequency would be almost particle independent so that cyclotron contribution gives an almost constant shift to the generalized Josephson energy. When the difference of cyclotron energies vanishes, the model reduces to that discussed above.

The weak independence of the cyclotron frequency on particle properties does not conform with the idea that EEG bands correspond to bosonic ions or Cooper pairs of fermionic ions.

2. For *far from vacuum extremals* the proportionality of cyclotron energy to h_{eff} and B_{end} allows easy reproduction the energies for which photon absorption is maximal if one allows the cyclotron energies to differ at the two sides of the membrane for sensory receptors.

A remark about decade later: The model just discussed neglects the fact that superconductivity requires that Cooper pairs of fermionic ions are present unless one assumes that the nuclei are bosonic counterparts of fermionic nuclei with same chemical properties - TGD inspired nuclear physics indeed predicts this kind of exotic nuclei [L2]. For Cooper pairs of Na^+, Cl^-, and K^+, $p = .23$ and $E_J = .04$ eV assignable to visual receptors the Josephson energies are doubled being $2.02, 2.80, 3.02$ eV. These energies could correspond to peak energies for visible photons. The assumption of ionic Cooper pairs is rather attractive since it would allow to avoid two questionable assumptions.

For electron the Josephson energy would be scaled by a factor $-1+1/2p$ to $E_J = 1.0859 \times eV_{rest}$ for $p = .2397$. For neutrino the energy would be given by $E_J = -0.0859 \times V_{rest}$: for $p = 1/4$ it would vanish by the vanishing of vectorial part of Z^0 charge. For proton the energy would be $E_J = (3 - 1/2p)V_{rest} = .914 \times V_{rest}$ and for neutron $E_J = V_{rest}/2p = 2.086 \times V_{rest}$.

10.6 Pollack's findings about fourth phase of water and the model of cell

The discovery of negatively charged exclusion zone formed in water bounded by gel phase has led Pollack to propose the notion of gel like fourth phase of water. In this article this notion is discussed in TGD framework. The proposal is that the fourth phase corresponds to negatively charged regions - exclusion zones - with size up to 100-200 microns generated when energy is fed into the water - say as radiation, in particular solar radiation. The stoichiometry of the exclusion zone is $H_{1.5}O$ and can be understood if every fourth proton is dark proton residing at the flux tubes of the magnetic body assignable to the exclusion zone and outside it.

This leads to a model for prebiotic cell as exclusion zone. Dark protons are proposed to form dark nuclei whose states can be grouped to groups corresponding to DNA, RNA, amino-acids, and tRNA and for which vertebrate genetic code is realized in a natural manner. The voltage associated with the system defines the analog of membrane potential, and serves as a source of metabolic energy as in the case of ordinary metabolism. The energy is liberated in a reverse phase transition in which dark protons transform to ordinary ones. Dark proton strings serve as analogs of basic biopolymers and one can imagine analog of bio-catalysis with enzymes replaced with their dark analogs. The recent discovery that metabolic cycles emerge spontaneously in absence of cell support this view.

One can find a biographical sketch [I3] (`http://faculty.washington.edu/ghp/cv/`) giving a list of publications containing items related to the notions of exclusion zone and fourth phase of water discussed in the talk.

10.6.1 Pollack's findings

I list below some basic experimental findings about fourth gel like phase of water made in the laboratory led by Gerald Pollack [I4].

1. In water bounded by a gel a layer of thickness up to 100-200 microns is formed. All impurities in this layer are taken outside the layer. This motivates the term "exclusion zone". The layer consists of layers of molecular thickness and in these layers the stoichiometry is $H_{1.5}O$. The layer is negatively charged. The outside region carries compensating positive charge. This kind of blobs are formed in living matter. Also in the splitting of water producing Brown's gas negatively charged regions are reported to emerge [H9, H1].

2. The process requires energy and irradiation by visible light or thermal radiation generates the layer. Even the radiation on skin can induce the phase transition. For instance, the blood flow in narrow surface veins requires metabolic energy and irradiation forces the blood to flow.

3. The layer can serve as a battery: Pollack talks about a form of free energy deriving basically from solar radiation. The particles in the layer are taken to the outside region, and this makes possible disinfection and separation of salt from sea water. One can even understand how clouds are formed and mysteries related to the surface tension of water as being due the presence of the layer formed by $H_{1.5}O$.

4. In the splitting of water producing Brown's gas [H9, H1] having a natural identification as Pollack's fourth phase of water the needed energy can come from several alternative sources: cavitation, electric field, etc...

10.6.2 Dark nuclei and Pollack's findings

While listening the lecture of Pollack I realized that a model for dark water in term of dark proton sequences is enough to explain the properties of the exotic water according to experiments done in the laboratory of Pollack. There is no need to assume sequences of half-dark water molecules containing one dark proton each.

Model for the formation of exclusion zones

The data about formation of exclusion zones allows to construct a more detailed model for what might happen in the formation of exclusion zones.

1. The dark proton sequences with dark proton having size of order atomic nucleus would reside at the flux tubes of dark magnetic field which is dipole like field in the first approximation and defines the magnetic body of the negatively charged water blob. This explains the charge separation if the flux tubes have length considerably longer than the size scale of the blob which is given by size of small cell. In the model inspired by Moray B. King's lectures charge separation is poorly understood.

2. An interesting question is whether the magnetic body is created by the electronic currents or whether it consists of flux tubes carrying monopole flux: in the latter case no currents would be needed. This is obviously purely TGD based possibility and due to the topology of CP_2.

3. This means that in the model inspired by the lectures of Moray B. King discussed above, one just replaces the sequences of partially dark water molecules with sequences of dark protons at the magnetic body of the $H1.5O$ blob. The model for the proto-variants of photosynthesis and metabolism remain as such. Also now genetic code would be realized [K41, L2].

4. The transfer of impurities from the exclusion zone could be interpreted as a transfer of them to the magnetic flux tubes outside the exclusion zone as dark matter.

These primitive forms of photosynthesis and metabolism form could be key parts of their higher level chemical variants. Photosynthesis by irradiation would induce a phase transition generating dark magnetic flux tubes (or transforming ordinary flux tubes to dark ones) and the dark proton sequences at them. Metabolism would mean burning of the resulting blobs of dark water to ordinary water leading to the loss of charge separation. This process would be analogous to the catabolism of organic polymers liberating energy. Also organic polymers in living matter carry their metabolic energy as dark proton sequences: the layer could also prevent their hydration. That these molecules are typically negatively charged would conform with the idea that dark protons at magnetic flux tubes carry the metabolic energy.

The liberation of energy would involve increase of the p-adic prime characterizing the flux tubes and reduction of Planck constant so that the thickness of the flux tubes remains the same but the intensity of the magnetic field is reduced. The cyclotron energy of dark protons is liberated in coherent fashion and in good approximation the frequencies of the radiation corresponds to multiplies of cyclotron frequency: this prediction is consistent with that in the original model for the findings of Blackman and others [J8].

The phase transition generating dark magnetic flux tubes containing dark proton sequences would be the fundamental step transforming inanimate matter to living matter and the fundamental purpose of metabolism would be to make this possible.

Minimal metabolic energy consumption and the value of membrane potential

This picture raises a question relating to the possible problems with physiological temperature.

1. The Josephson radiation generated by cell membrane has photon energies coming as multiples of ZeV, where V is membrane potential about .06 V and $Z = 2$ is the charge of electron Cooper pair. This gives $E = .12$ eV.

2. There is a danger that thermal radiation masks Josephson radiation. The energy for photons at the maximum of the energy density of blackbody radiation as function of frequency is given as the maximum of function $x^3/(e^x - 1)$, $x = E/T$ given by $e^{-x} + x/3 - 1 = 0$. The maximum is given approximately by $x = 3$ and thus $E_{max} \simeq 3T$ (in units $c = 1, k_B = 1$). At physiological temperature $T = 310$ K (37 C) this gives .1 eV, which is slightly below Josephson energy: living matter seems to have minimized the value of Josephson energy - presumably to minimize metabolic costs. Note however that for the thermal energy density as function of *wavelength* the maximum is at $E \simeq 5T$ corresponding to 1.55 eV which is larger than Josephson energy. The situation is clearly critical.

3. One can ask whether also a local reduction of temperature around cell membrane in the fourth phase of water is needed.

   (a) "Electric expansion" of water giving rise to charge separation and presumably creating fourth phase of water is reported to occur [H9, H1].

   (b) Could the electric expansion/phase transition to dark phase be adiabatic involving therefore no heat transfer between the expanding water and environment? If so, it would transform some thermal energy of expanding water to work and reduce its temperature. The formula for the adiabatic expansion of ideal gas with f degrees of freedom for particle ($f = 3$ if there are no other than translational degrees of freedom) is

$(T/T_0) = (V/V_0)^{-\gamma}$, $\gamma = (f+2)/f$. This gives some idea about how large reduction of temperature might be involved. If p-adic scaling for water volume by a power of two takes place, the reduction of temperature can be quite large and it does not look realistic.

(c) The electric expansion of water need not however involve the increase of Planck constant for water volume. Only the Planck constant for flux tubes must increase and would allow the formation of dark proton sequences and the generation of cyclotron Bose-Einstein condensates or their dark analog in which fermions (electrons in particular) effectively behave as bosons (the anti-symmetrization of wave function would occur in dark degrees of freedom corresponding to multi-sheeted covering formed in the process).

10.6.3 Fourth phase of water and pre-biotic life in TGD Universe

If the fourth phase of water defines pre-biotic life form then the phase transition generating fourth phase of water and its reversal are expected to be fundamental elements of the ordinary metabolism, which would have developed from the pre-biotic metabolism. The following arguments conforms with this expectation.

Metabolism and fourth phase of water

1. Cell interiors, in particular the interior of the inner mitochondrial membrane are negatively charged as the regions formed in Pollack's experiments. Furthermore, the citric acid cycle, (`http://www.en.wikipedia.org/wiki/Citric_acid_cycle`), which forms the basic element of both photosynthesis (`http://www.en.wikipedia.org/wiki/Photo-synthesis`) and cellular respiration `http://www.en.wikipedia.org/wiki/Cellular_respiration`, involves electron transport chain (`http://www.en.wikipedia.org/wiki/Electron_transport_chain`) in which electron loses gradually its energy via production of NADP and proton at given step. Protons are pumped to the other side of the membrane and generates proton gradient serving as metabolic energy storage just like battery. The interpretation for the electron transport chain in terms of Pollack's experiment would be in terms of generation of dark protons at the other side of the membrane.

2. When ATP is generated from ADP three protons per ATP flow back along the channel formed by the ATP synthase molecule (`http://www.en.wikipedia.org/wiki/ATP_synthase`) (perhaps Josephson junction) and rotate the shaft of a "motor" acting as a catalyst generating three ATP molecules per turn by phosphorylating ADP. The TGD based interpretation is that dark protons are transformed back to ordinary ones and possible negentropic entanglement is lost.

3. ATP is generated also in glycolysis (`http://www.en.wikipedia.org/wiki/Glycolysis`), which is ten-step process occurring in cytosol so that membrane like structure need not be involved. Glycolysis involves also generation of two NADH molecules and protons. An open question (to me) is whether the protons are transferred through an endoplasmic reticulum or from a region of ordered water (fourth phase of water) to its exterior so that it would contribute to potential gradient and could go to magnetic flux tubes as dark proton. This would be natural since glycolysis is realized for nearly all organisms and electron transport chain is preceded by glycolysis and uses as input the output of glycolysis (two pyruvate molecules (`http://www.en.wikipedia.org/wiki/Pyruvate`)).

4. Biopolymers - including DNA and ATP - are typically negatively charged. They could thus be surrounded by fourth phase of water and neutralizing protons would reside at the magnetic bodies. This kind of picture would conform with the idea that the fourth phase (as also magnetic body) is fractal like. In phosphorylation the metabolic energy stored to a potential difference is transferred to shorter length scales (from cell membrane scale to molecular scale).

In glycolysis (`http://www.en.wikipedia.org/wiki/Glycolysis`) the net reaction $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2(g) + 6H_2O(l) + heat$ takes place. The Gibbs free energy change is $\Delta G = -2880$ kJ per mole of $C_6H_{12}O_6$ and is negative so that the process takes place spontaneously. Single glucose molecule is theoretized to produce $N = 38$ ATP molecules in optimal situation but there are various energy losses involved and the actual value is estimated to be 29-30. From $Joule = 6.84 \times 10^{18}$ eV and $mol = 6.02 \times 10^{23}$ and for $N = 38$ one would obtain the energy yield .86 eV per single ATP. The nominal value that I have used .5 eV. This is roughly 5 to 8 times higher than $E = ZeV, Z = 2$, which varies in the range .1-.16 eV so that the metabolic energy gain cannot be solely due to the electrostatic energy which would actually give only a small contribution.

In the thermodynamical approach to metabolism the additional contribution would be due to the difference of the chemical potential μ for cell exterior and interior, which is added to the membrane potential as effective potential energy. The discrepancy is however rather large and this forces the question the feasibility of the model. This forces to reconsider the model of osmosis in the light of Pollack's findings.

Pollack's findings in relation to osmosis and model for cell membrane and EEG

Osmosis (`http://en.wikipedia.org/wiki/Osmotic`) has remained to me poorly understood phenomenon. Osmosis means that solvent molecules move through a semipermeable membrane to another side of the membrane if the concentration of solute is higher at that side. Solute can be water or more general liquid, supercritical liquid, and even gas.

Osmosis is not diffusion: it can occur also towards a higher concentration of water. Water molecules are not attracted by solute molecules. A force is required and the Wikipedia explanation is that solute molecules approaching pores from outside experience repulsion and gain momentum which is transferred to the water molecules.

The findings of Pollack inspire the question whether the formation of exclusion zone could relate to osmosis and be understood in terms of the fourth phase of water using genuine quantal description.

In the thermodynamical model for ionic concentrations one adds to the membrane resting potential a contribution from the difference of chemical potentials μ_i at the two sides of the membrane. Chemical potentials for the ions parametrize the properties of the cell membrane reducing basically to the properties of the channels and pumps (free diffusion and membrane potential do not entirely determine the outcome).

If the transfer of ions - now protons - through cell membrane is quantal process and through Josephson junctions defined by transmembrane proteins, then the thermodynamical model can at best be a phenomenological parameterization of the situation. One should find the quantum counterpart of thermodynamical description, and here the identification of quantum TGD as square root of thermodynamics in Zero Energy Ontology (ZEO) suggests itself. In this approach thermodynamical distributions are replaced by probability amplitudes at single particle level such that their moduli squared give Boltzmann weights.

1. Simplest Josephson junction model for cell membrane

The first guess is that quantum description is achieved by a generalization of the Josephson junction model allowing different values of Planck constant at magnetic flux tubes carrying dark matter.

1. Josephson junctions correspond microscopically to transmembrane proteins defining channels and pumps. In rougher description entire cell membrane is described as Josephson junction.

2. The magnetic field strength at flux tube can differ at the opposite side of the membrane and even the values of h_{eff} could in principle be different. The earlier modelling attempts suggest that $h_{eff}/h = n = 2^k A$, where A is the atomic weight of ion, is a starting assumption deserving testing. This would mean that each ion resides at its own flux tubes.

   The phase transitions changing the value of h_{eff} could induce ionic flows through cell membrane, say that occurring during nerve pulse since the energy difference defining the ratio of square roots of Boltzmann weights at the two sides of the membrane would change. Also the change of the local value of the magnetic field could do the same.

Consider first the simplest model taking into account only membrane potential.

1. The simplest model for Josephson junction defined by the transmembrane protein is as a two state system (Ψ_1, Ψ_2) obeying Schrödinger equation.

$$i\hbar_1 \frac{\partial \Psi_1}{\partial t} = ZeV\Psi_1 + k_1\Psi_2 \ ,$$

$$i\hbar_2 \frac{\partial \Psi_2}{\partial t} = k_2\Psi_2 \ .$$

One can use the decomposition $\Psi_i = R_i exp(i\Phi(t)$ to express the equations in a more concrete form. The basic condition is that the total probability defined as sum of moduli squared equals to one: $R_1^2 + R_2^2 = 1$. This is guaranteed if the hermiticity condition $k_1/\hbar_1 = \overline{k_2}\hbar_2$ holds true. Equations reduce to those for an ordinary Josephson junction except that the frequency for the oscillating Josephson current is scaled down by $1/h_{eff}$.

2. One can solve for R_2 assuming $\Phi_1 = eVt/\hbar_{eff}$. This gives

$$R_2(t) = sin(\Phi_0) + \frac{k_1}{\hbar_1} sin(\frac{eVt}{\hbar_1}) \ .$$

R_2 oscillates around $sin(\Phi_0)$ and the concentration difference is coded by $\Phi 0$ taking the role of chemical potential as a phenomenological parameter.

3. The counterparts of Boltzmann weights would be apart from a phase factor square roots of ordinary Boltzmann weights defined by the exponent of Coulomb energy:

$$R = sin(\phi_0) = exp(\frac{ZeV(t)}{2T}) \ .$$

Temperature would appear as a parameter in single particle wave function and the interpretation would be that thermodynamical distribution is replaced by its square root in quantum theory. In ZEO density matrix is replaced by its hermitian square root multiplied by density matrix.

2. The counterpart of chemical potential in TGD description

This model is not as such physically realistic since the counterpart of chemical potential is lacking. The most straightforward generalization of the thermodynamical model is obtained by the addition of an ion dependent chemical potential term to the membrane potential: $ZeV \rightarrow ZeV + \mu_I$. This would however require a concrete physical interpretation.

1. The most obvious possibility is that also the chemical potential actually correspond to an interaction energy - most naturally the cyclotron energy $E_c = \hbar_{eff} ZeB_{end}/m$ of ion - in this case proton - at the magnetic flux tube. Cyclotron energy is proportional to h_{eff} and can be rather large as assumed in the model for the effects of ELF em fields on brain.

2. This model would predict the dependence of the effective chemical potential on the mass and charge of ion for a fixed value of on h_{eff} and B_{end}. The scales of ionic chemical potential and ion concentrations would also depend on value of h_{eff}.

3. The model would provide a different interpretation for the energy scale of bio-photons, which is in visible range rather than infrared as suggested by the value of membrane potential.

   The earlier proposal [K38] was that cell membrane can be in near vacuum extremal configuration in which classical Z^0 field contributes to the membrane potential and gives a large contribution for ions. The problematic aspect of the model was the necessity to assume Weinberg angle in this phase to have much smaller value than usually. This difficulty could be perhaps avoided by noticing that the membrane potentials can differ for color receptors so that the earlier assignment of specific ions to color receptors could make sense for ordinary value of Weingerg angle. Second problem is that for proton the Z^0 contribution is negligible in good approximation so that this model does not explain the high value of the metabolic energy currency.

4. The simplest model the communications to magnetic body rely on Josephson radiation whose fundamental frequency f_J is at resonance identical with the cyclotron frequency $f_c(MB)$ at particular part of the flux tube of the magnetic body:($f_c(MB) = f_J$. $f_c(MB)$ corresponds to EEG frequency in the case of brain and biophotons are produced from dark EEG photons as ordinary photons in phase transition reducing $h_{eff} = n \times h$ to h.

   In the modified model the sum $f_c + f_{J,n}$ ($f_{J,n} = E_J/n \times h$) of h_{eff}-independent cyclotron frequency and Josephson frequency proportional to $1/h_{eff}$ equals to cyclotron frequency $f_c(MB)$ at "personal" magnetic body varying slowly along the flux tube: $f_c + f_{J,n} = f_c(MB)$. If also the variation of f_J assignable to the action potential is included, the total variation of membrane potential gives rise to a frequency band with width roughly

$$\frac{\Delta f}{f} \simeq \frac{2f_{J,n}}{f_c + f_{J,n}} = \frac{2f_{J,1}}{nf_c + f_{J,1}} \quad .$$

   If dark photons correspond to biophotons the energy is of cyclotron photon is in visible and UV range one has $nf_c = E_{bio}$ and

$$\frac{\Delta f}{f} \simeq \frac{2ZeV}{E_{bio} + ZeV} \quad .$$

   The prediction is scale invariant and same for all ions and also electron unless E_{bio} depends on ion. For $eV = .05$ eV, $Z = 1$, and $E_{bio} = 2$ eV ($f \simeq 5 \times 10^{14}$ Hz) one has $\Delta f/f \sim .1$ giving 10 per cent width for EEG bands assumed in the simpler model.

If this vision is on the correct track, the fundamental description of osmosis would be in terms of a phase transition to the fourth phase of water involving generation of dark matter transferred to the magnetic flux tubes. For instance, the swelling of cell by an in-flow of water in presence of higher concentration inside cell could be interpreted as a phase transition extending exclusion zone as a process accompanied by a phase transition increasing the value of h_{eff} so that the lengths of the flux tube portions inside the cell increase and the size of the exclusion zone increases. In general case the phase transitions changing h_{eff} and B_{end} by power of two factor are possible. This description should bring magnetic body as part of bio-chemistry and allow understanding of both equilibriumion distributions, generation of nerve pulse, and basic metabolic processes leading to the generation of ATP.

One can also model sensory receptors and try to understand the maximal sensitivity of color receptors to specific wavelengths in this framework. The new degrees of freedom make this task easy if one is only interested in reproducing these frequencies. More difficult challenge is to understand the color receptors from the first principles. It is also possible to combine the new view with the assumption that sensory receptor cells are near to vacuum extremals. This would add a cyclotron contribution to the generalized Josephson frequency depending only weakly on particle and being non-vanishing also for em neutral particles.

Why would charge separation generate large h_{eff}?

The basic question is whether and how the separation of electron and proton charges generates large h_{eff}? A possible mechanism emerged from a model [K111] explaining anomalously large gravimagnetic effect claimed by Tajmar et al [E14, **?**] to explain the well-established anomaly related to the mass of Cooper pairs in rotating super-conduction. The mass is too large by fraction of order 10^{-4} and the proposal is that gravimagnetism changes slightly the effective Thomson magnetic field associated with the rotating super-conductor leading to wrong value of Cooper pairs mass when only ordinary Thomson field is assumed to be present. The needed gravimagnetic field is however gigantic: 28 orders larger than that predicted by GRT. Gravimagnetic field is proportional h_{eff}^2 in TGD and if one uses h_{gr} for electron-Earth system one obtains correct order of magnitude.

Nottale's finding that planetary orbits seem to correspond to Bohr orbits in gravitational potential with gigantic value of gravitational Planck constant is the basic input leading to the model of gravimagnetic anomaly.

1. By Equivalence Principle h_{gr} has the general form $h_{gr} = GMm/v_0$, where M and m are the interacting masses and v_0 is a parameter with dimensions of velocity. For three inner planets one has $v_0/c \simeq 2^{-11}$.

2. The notion of h_{gr} generalizes to that for other interactions. For instance, in electromagnetic case the formation of strong em fields implying charge separation leads to systems in which $h_{em} = Z_1Z_2e^2/v_0$ is large. Pollack's exclusion zone and its complement define this kind of systems and is identified as prebiotic life form.

3. Since the natural expansion parameter of perturbative expansion is the $g^2/4\pi\hbar$, one can say that transition to dark matter phase make the situation perturbative. Mother Nature is theoretician friendly.

h_{em} might be large in the exclusion zones (EZ) appearing in the water bounded by gel and their variants could play central role in living matter.

1. EZ carries very large negative charge with positive charge outside the exclusion zone.

2. TGD interpretation is in terms of $H_{1.5}O$ phase of water formed when every 4:th proton is transferred to magnetic body as dark particle with large value of h_{eff}. The proposal is that primitive life form is in question.

3. The pair formed by EZ and its complement could have large value of $h_{eff} = h_{em} = Z^2e^2/v_0$.

4. The velocity parameter v_0 should correspond to some natural rotation velocity. What comes in mind is that complement refers to Earth and v_0 is the rotation velocity at the surface of Earth. The prediction for h_{eff} would be of order $h_{em}/h = 4\pi\alpha Z^2 \times .645 \times 10^6 \simeq 5.9 \times 10^4 Z^2$.

5. Cell membrane involves also large charge separation due to very strong electric field over the cell membrane. Also now dark phases with large h_{em} or h_{gr} could be formed.

I have proposed that metabolic machinery generates large h_{eff} phase somehow. $h_{eff} = h_{em}$ hypothesis allows to develop this hypothesis in more detail.

1. I have speculated earlier [K45] that the rotating shaft of a molecular motor associated with ATP synthase plays a key role in generating dark matter phase. What comes in mind is that charge separation takes place associating exclusion zone with the shaft and the rotational velocity v_0 of the shaft appears in the formula for h_{em}. Of course, some numerical constant not far from unity could be present. The electric field over the mitochondrial membrane generates charge separation. One can imagine several identifications for the product of charges. The charge Z associated with the complement would be naturally associated with single dark flux tube containing dark nucleon consisting of dark protons. For instance, the charge associated with the exclusion zone could be the charge of the electronic Cooper pair giving $h_{em} = 2e \times Z/v_0$.

2. The value of v_0/c is expected to be of order 10^{-14} from the angular rotation rate of ADP synthase about few hundred revolutions per second. The order of magnitude for h_{em} could be same as for h_{gr} associated with Earth-particle system.

$h_{eff}(ATPsynthase) = h_{gr}(2e, Earth)$ would make possible reconnection of electromagnetic flux tubes with gravimagnetic flux tubes [K67].

Which came first: metabolism or cell membrane?

One of the basic questions of biology is whether metabolism preceded basic biopolymers or vice versa. RNA world scenario assumes that RNA and perhaps also genetic code was first.

1. The above view suggests that both approaches are correct to some degree in TGD Universe. Both metabolism and genetic code realized in terms of dark proton sequences would have emerged simultaneously and bio-chemistry self-organized around them. Dark proton sequences defining analogs of amino-acid sequences could have defined analogs of protein

catalysts and played a key role in the evolution of the metabolic pathways from the primitive pathways involving only the phase transition between ordinary water and fourth phase of water.

2. There is very interesting article [?]eporting that complex metabolic pathways are generated spontaneously in laboratory environments mimicking hot thermal vents. Glycolysis and pentose phosphate pathway were detected. The proposal is that these pathways are catalyzed by metals rather than protein catalysts.

3. In standard biology these findings would mean that these metabolic pathways emerged before basic biopolymers and that genetic code is not needed to code for the metabolic pathways during this period. In TGD framework dark genetic code [K41, L2] would be there, and could code for the dark pathways. Dark proton strings in one-one correspondence with the amino-acid sequences could be responsible for catalysts appearing in the pathways. Only later these catalysts would have transformed to their chemical counterparts and might be accompanied by their dark templates. One cannot even exclude the possibility that the chemical realization of the DNA-aminoacid correspondence involves its dark analog in an essential manner.

10.7 Could photosensitive emulsions make dark matter visible?

The article "Possible detection of tachyon monopoles in photographic emulsions" by Keith Fredericks [H4] describes in detail (`http://restframe.com/downloads/tachyon_monopoles.pdf`) very interesting observations by him and also by many other researchers about strange tracks in photographic emulsions induced by various (probably) non-biological mechanisms and also by the exposure to human hands (touching by fingertips) as in the experiments of Fredericks. That the photographic emulsion itself consists of organic matter (say gelatin) might be of significance.

10.7.1 The findings

The tracks have width between 5 μm-110 μm (horizontal) and 5 μm-460 μm (vertical). Even tracks of length up to at least 6.9 cm have been found. Tracks begin at some point and end abruptly. A given track can have both random and almost linear portions, regular periodic structures (figs 11 and 12), tracks can appear in swarms (fig. 24), bundles (fig. 25), and correlated pairs (fig. 16), tracks can also split and recombine (fig. 32) (here and below "fig." refers to a figure of the article at `http://restframe.com/downloads/tachyon_monopoles.pdf`).

Tracks differ from tracks of known particles: the constant width of track implies that electrons are not in question. No delta rays (fast electrons caused by secondary ionization appearing as branches in the track) characteristic for ions are present. Unlike alpha particle tracks the tracks are not straight. In magnetic fields tracks have parabolic portions whereas ordinary charged particle move along spiral. The magnetic field needed to cause spiral structure for baryons should be by two orders of magnitude higher than in the experiments.

For particle physicist all these features - for instance constant width - strongly suggest pre-existing structures becoming visible for some reason. The pre-existing structure could of course correspond to something completely standard structures present in the emulsion. If one is ready to accept that biology involves new physics, it could be something more interesting.

Also evidence for cold fusion is reported by the group of Urutskoev [H3]. There is evidence for cold fusion in living matter [C94, C116]: the fact that the emulsion contains gelatin might relate to this. In [L2] a dark matter based mechanism of cold fusion allowing protons to overcome the Coulomb wall is discussed. Either dark protons or dark nuclei with much larger quantum size than usually would make this possible and protons could end up to the dark nuclei along dark flux tubes. In TGD inspired biology dark protons (large h_{eff}) with scaled up Compton length of order atomic size are proposed to play key role since their states allow interpretation in terms of vertebrate genetic code [L2, K106].

10.7.2 The importance of belief system

These structures could be something quite standard or not. This readiness to consider non-standard explanations depends on belief system.

1. In the belief system of standard physics these pre-existing structures would be organic material consisting of ordinary matter so that no new physics is involved. Probably it is easy to kill this hypothesis. If this can be done, the situation becomes really interesting.

2. In my own belief system they *could* correspond to dark matter structures made visible by some mechanism. The presence of human hands could induce this phenomenon in the experiments of Fredericks. If so we might be already considering remote interactions involving dark photons and magnetic flux tubes, whose images "tracks" would be.

3. The first guess is that these structures are in the emulsion. This need not be the case! They could be structures outside- say in human hands - sending dark photon beam absorbed by the small photosensitive crystals in the emulsion. A photograph of dark matter (say in the hands of sender) would be formed! One possibility is that tracks represent a photograph of the dark matter at the flux tubes of the magnetic body of the emulsion. This would be a variant for what Gariaev perhaps managed to achieve with camera: taking a photo of dark matter [K1]!

4. Unfortunately belief system becomes important also in second manner. The reductionistic belief system tells that the tracks must be something trivial. There cannot be new physics in scale of cell as we have read in text books. Therefore these tracks are not studied by professionals who could very easily find whether there is something really interesting involved.

Dark matter in TGD based belief system corresponds to a hierarchy of phases of ordinary matter with an effective value h_{eff} of Planck constant coming as integer multiple of ordinary Planck constant. This makes possible macroscopic quantum phases consisting of dark matter. The flux tubes could carry magnetic monopole flux but the magnetic charge would be topological (made possible by the non-trivial second homology of CP_2 factor of the 8-D imbedding space containing space-times as surfaces) rather than Dirac type magnetic charge.

The TGD inspired identification of tracks could be as images of magnetic flux tubes or bundles of them containing dark matter defining one of the basic new physics elements in TGD based quantum biology. One can imagine two options for the identification of the tracks as "tracks".

1. The primary structures are in the photo-sensive emulsion.

2. The structures in photograph are photographs of dark matter in external world, say structures in human hands or human body or of dark matter at some magnetic body, say at the flux tubes of the magnetic body of the emulsion.

The fact that the tracks have been observed in experimental arrangements not involving exposure to human hands, indeed suggests that tracks represent photographs about parts of the magnetic body assignable to the emulsion. For this option the external source would serve only as the source of possibly dark photons.

This would imply a close analogy with the experiments of Peter Gariaev's group interpreted in TGD framework as photographing of the magnetic body of DNA sample [K1]. Also here one has an external source of light: the light would be transformed to dark photons in DNA sample, scatter from the dark charged particles at the flux tubes of the magnetic body of DNA sample, and return back transforming to ordinary light and generating the image in the photosensitive emulsion.

10.7.3 Why not tachyonic monopoles?

The identification of the tracks as orbits of particles proposed by author and also by other experimentalists is to my opinion problematic for the reasons which I have already explained. The article of Fredericks lists further details which do not conform with the particle interpretation. A further proposal is that the particles are tachyonic magnetic monopoles. One motivation for the monopole hypothesis is the (unsuccessful) attempt to explain the parabolic shape of the tracks in external magnetic field.

To my view the interpretation as a tachyonic monopole - a notion introduced by Recami and Mignani [H12](http://link.springer.com/journal/11546) - adopted in the article is theoretically problematic. Of course, if the tracks are actually pre-existing structures made visible by some mechanism, there is no need to postulate super-luminal propagation. To see the problem, one can start from a general formula relating energy, momentum and mass. One has

$$E^2 = p^2 + m^2 \quad . \tag{10.7.1}$$

When m is imaginary as for tachyon so that one can write $m = iM$, one obtains

$$E^2 = p^2 - M^2 \quad . \tag{10.7.2}$$

If E and p are assumed to be real as is done usually the condition $E \geq 0$ and more generally the reality of E gives $p \geq M$. Tachyon cannot therefore be at rest and one cannot assign to it kinetic energy since tachyon at rest would have imaginary energy.

This has two implications.

1. The identification as tachyon and the conclusion $p \ll M$ from experiments (see figure 34 for the relation between E, p and m in various cases) is not consistent with $p \geq M$.

2. Recami and Mignani assign a kinetic energy to tachyon (formula 14). Unfortunately, this formula does not make sense if one accepts that E and p are real since one cannot assign to tachyon kinetic energy: the analogy of kinetic energy would be "kinetic momentum" defined as the difference of the actual momentum and minimal momentum $p = M$ ($p_{kin} = \sqrt{E^2 + M^2} - M \simeq E - M - M^2/2E$). As Fredericks notices, the behavior is not actually consistent with a motion of magnetic monopole in magnetic field. Parabolic orbits are in plane orthogonal to magnetic field rather than containing its direction vector (http://restframe.com/downloads/tachyon_monopoles.pdf)!

10.7.4 Interpretation as dark matter structures becoming visible in presence of living matter

As such the observations are extremely interesting. I cannot however believe that the tracks represent particles. To my opinion tachyonic monopole interpretation fails because it does not make sense to talk about kinetic energy of tachyon.

To me the complex structures of tracks very strongly suggest pre-existing structures becoming visible for some reason. Looking the shape of tracks brings to my mind linear structure such as protein molecules. They contain regular helical portions and denatured portions. Now the longitudinal scale is of course much longer. The transversal scale is that for cells. This is perhaps not too surprising since organic materials such as gelatin are involved. The flux tubes could carry magnetic monopole fluxes and in purely formal sense would thus be analogous to magnetic monopoles with space-like momentum in their direction - that is tachyonic monopoles. They would be however actually ordinary systems with non-tachyonic momentum.

The particles possibly causing the tracks cannot be electrically charged since in this case they would not have managed to reach the emulsion. There seems however to be an interaction with magnetic fields since the tracks are parabola. Urutskoev et al [H3] propose that tracks are caused by magnetic monopoles. Unfortunately, the predicted parabolic orbit would be in the plane containing the magnetic field lines: the situation is completely analogous to the parabolic motion of projectile in the Earth's gravitational field.

"Tracks" as photographs of magnetic flux tubes?

Consider first the identification of "tracks" (for convenience I will drop the quotation marks in the sequel) as images of magnetic flux tubes.

1. The hypothesis that tracks are photographs of flux tubes explains the "track-ness". In the Earth's magnetic field the thickness of flux tubes is by flux quantization of the same order of magnitude as the thickness of thickest tracks observed for single flux quantum. Flux tube hypothesis seems to be also consistent with the other strange properties of the "tracks". In particular, the composition to random and smoothly curved portion would conform with the idea that also linear molecules are formed around templates defined by magnetic flux tubes.

2. The tracks have been observed to be created in several situations and it is not at all clear whether the exposure to hands in the experiments of Fredericks is absolutely necessary. TGD suggests that the analog of dielectric breakdown associated with nerve pulses (the electric field at cell membrane is two times higher than the electric field inducing di-electric breakdown in air) replaces the strong electric fields causing di-electric breakdown used in the experiments of Urutskoev [H3]. Dark magnetic flux tubes can accompany any kind of matter so that tracks could be also images about the dark magnetic body of an external object rather than that of emulsion. In principle, one cannot exclude the possibility that the presence of the experimenter is decisive in all cases. If so, this would be a new kind of experimenter effect.

3. To what could the abrupt ending of the track correspond in this picture? Magnetic flux tubes cannot end but they can go to another space-time sheet through wormhole contact and apparently disappear. This would indeed take place for the closed flux tubes representing elementary particles and carrying magnetic monopole flux. The flux tubes could quite generally carry a multiple of magnetic monopole flux. They would have rather large scale as compared to the CP_2 scale of 10^4 Planck lengths.

1. Explanation for parabolic portions of tracks

The presence of parabolic tracks in the plane orthogonal to the external static magnetic fields is very interesting feature to be explained. Parabolic character could be simply due to the simplest non-linear fit to the shape of the flux tube: it is however argued that parabolic character is exact. One should understand why the flux tube is orthogonal to the external magnetic field or magnetic field generated by the emulsion? Could this reflect the geometry of the experimental arrangement?

In TGD framework one can consider a very natural possibility that a constant electric field orthogonal to the external magnetic field is present.

1. In standard physics the presence of the electric field might be excluded easily. In TGD framework simplest space-time sheets representing constant Kähler magnetic fields allow a simple deformation to sheets containing orthogonal electric field. A simple situation (not necessarily a preferred extremal of Kähler action) corresponds to a space-time sheet $X^4 \subset M^4 \times S^2$, S^2 a geodesic sphere of CP_2. Using spherical coordinates $(u = cos(\Theta), \Phi)$ for S^2 and Cartesian coordinates (t, z, x, y) for M^4, one has $(u = f(x), \Phi = \omega t + ky)$ $(c = 1)$. The non-vanishing components of magnetic and electric fields are apart from a coefficient of proportionality of order unity given by $E_x \equiv J_{0x} = \partial_x u \times \omega$ and $B^z \equiv J_{xy} = \partial_x u \times k$ with $E_x/B_z = \omega/k$. Electric and magnetic fields are orthogonal and the value of the ω/k ratio fixes the electric field strength in terms of the magnetic field strength. In fact, the mere assumption that the CP_2 projection is 2-dimensional implies that electric and magnetic parts of various induced gauge fields are orthogonal.

2. This field would be represented by a space-time sheet at which the flux tubes of the external magnetic are topologically condensed (glued by wormhole contacts). The charged particles inside the flux tube would experience the presence of this electric field as a constant force trying to force them out from the flux tube. If the flux tube adopts a parabolic shape of the orbit of individual charged particle, the electric force is parallel to the flux tube and one has equilibrium situation. *All* charged particles inside flux tube must move with the same velocity at given point of flux tube: this conforms with super-conductivity implying the existence of global order parameter. Note that the dark charged particles inside flux tube would not directly interact with the emulsion or with air so that they can reach the emulsion easily.

3. For non-relativistic motion the equation for the parabolic orbit is $y = x^2/L$, where the length $L = 2mv^2/qE$ characterizes the size scale of the parabola. Parametrizing E in terms of voltage and length L as $E = V_e/L$ one has $eV_c/mc^2 = 2(v/c)^2$. For electron rest energy $m_ec^2 = .5$ MeV and $v/c = 10^{-3}$ one would have $V_c = 1$ V. For proton the electric field would be by a factor 2^{11} stronger for the same orbit parameters.

   For a given electric field the parameters of the parabola allow to distinguish between flux tubes carrying different charged particles since the kinetic energies from the are expected to be different. I have indeed proposed that magnetic flux tubes could serve as a kind of filter allowing to distill ions with different masses at their own magnetic flux tubes: the equilibrium condition would make the flux tubes filters. The cyclotron energy scale $E_c = \hbar_{eff}ZeB/m$ would give a rough guess for the order of magnitude of kinetic energy of the particle: cyclotron energy scale is proportional to h_{eff} so that quite high energies can be considered. eV as a typical atomic energy scale and also as the energy scale of bio-photons (interpreted as decay products of dark photons [K105]) is the first guess for the energy scale.

4. It should be easy to check whether the emulsion is accompanied by electric field and also to deduce bounds for its values. Living matter is electret and one could imagine that gelatin contains some kind of remnants of bio-electric fields - perhaps as dark variants.

2. The decrease of the track thickness with the increase of distance

Urutskoev et al [H3] have reported the decrease of the track thickness with the increase of the source distance. Does this mean that the flux tubes photographed are near the source and the reduction of track thickness with distance is an optical effect similar to that for ordinary photographs?

If the flux tubes belong to the magnetic body of emulsion, this explanation fails. It is however easy to invent plausible explanation also in this case. based on a simple model for the quantization of the magnetic flux.

1. The reconnection for flux tubes of the source and emulsion can take place only for flux tubes with same magnetic field strength and by flux conservation same transversal area S. Note that conservation of magnetic flux implies $B \times S = constant$ so that increasing the thickness of flux tube decreases the strength of the magnetic field.

2. If the flux tubes have a fractal structure with flux tubes containing bundles of flux tubes (bundle structure has been observed for the tracks), one can argue that the weaker the magnetic field, the smaller the number of flux tubes in the typical bundle and the smaller the radius of the bundle if the flux tubes inside bundle have constant density. For dipole field the weakening of the average field with distance could mean that flux tube bundles split to smaller bundles. A "temporary" splitting of at track to a bundle of widely separated tracks has been observed for tracks and would mean reduction of the average magnetic field strength.

3. If the number of grains corresponds to the number of flux tubes within a bundle, the number of flux tubes in the bundle would be thousands. The average size of the grain suggests a diameter of order .34 μ m for the flux tubes. If the magnetic length $L_B = \sqrt{h/eB}$ equals to $L_B = .17$ μm (scaling rule: 1 Tesla corresponds to $L_B = 64$ nm), the magnetic field strength would be 354 Gauss (the Earth's magnetic field has nominal value of .5 Gauss). The external magnetic field of 20 Gauss used by Urutskoev et al defines a good candidate for the flux tube radius. For this field single flux tube would correspond to 18-19 crystals.

If this model is on correct track, these photographs could among other things provide means for the detailed study of the quantized dynamics of magnetic fields based on decomposition to flux tubes consisting of flux tubes consisting of...

What could be the source of dark photons?

Photographic emulsion would work as usually by detecting photons. What is clear that the photons must be dark when they scatter from the magnetic flux tubes of the magnetic body of the emulsion. There are however several options for how the dark photons are produced.

1. Ordinary photons from the source could hit the emulsion, transform to dark photons and propagate to the magnetic flux tubes, reflect back, transform to ordinary photons, and interact with the micro-crystals of the emulsion and generate the visible track as the image of the flux tube. Emulsion would take the role possessed by DNA sample in Gariaev's experiments and the external source would take the role of lamps used to generate visible light [K1].

2. Dark photons could also originate from the source. They could arrive along the flux tubes of its magnetic body. In the experimental situations considered these would reconnect with the flux tubes of the magnetic body of the emulsion and scatter from dark matter at them. After this the photons would propagate to the emulsion and transform to ordinary photons and give rise to the image. Reconnection of the flux tubes is the basic mechanism of attention in TGD inspired theory of consciousness and in TGD inspired biology, and also used to explain various findings of Persinger et al [K106].

3. The emission of dark photons is expected to take place in critical systems in which large values of effective Planck constant h_{eff} making possible long range correlations can be present. The situations studied (glow discharge plasma processes, exploding wires and foils, low energy discharges in water, super-compression of solid targets using electron beams) indeed seem to be critical. Only the search of monopoles of solar origin at the north pole represents a situation in which criticality is not present in obvious manner (the measurement method might involve criticality to guarantee maximal sensitivity). This kind of situations would generate time varying magnetic fields, whose flux tubes could reconnect with the magnetic flux tubes assignable to the photographic emulsion. This in turn would make possible for dark photons to propagate from source to the emulsion. In some experiments also static magnetic fields are present.

4. What is interesting that the "cold currents" reported already by Tesla in his experiments involving di-electric breakdowns at surfaces of wires of coils could correspond to dark currents propagating along the magnetic flux tubes [L19] [L19]. Most of these experiments correspond to critical situations making possible the manifestation of otherwise hidden new physics. Whether one can see these manifestations of course depends on whether one believes on the reductionistic Bible or not.

Chapter 11

Super-Conductivity in Many-Sheeted Space-Time

11.1 Introduction

In this chapter various TGD based ideas related to high T_c super-conductivity are discussed studied.

1. Supra currents and Josephson currents provide excellent tools of bio-control allowing large space-time sheets to control the smaller space-time sheets. The predicted hierarchy of dark matter phases characterized by a large value of $\hbar$ and thus possessing scaled up Compton and de Broglie wavelengths allows to have quantum control of short scales by long scales utilizing de-coherence phase transition. Quantum criticality is the basic property of TGD Universe and quantum critical super-conductivity is therefore especially natural in TGD framework. The competing phases could be ordinary and large $\hbar$ phases and supra currents would flow along the boundary between the two phases.

2. It is possible to make a tentative identification of the quantum correlates of the sensory qualia quantum number increments associated with the quantum phase transitions of various macroscopic quantum systems [K38] and various kind of Bose-Einstein condensates and super-conductors are the most relevant ones in this respect.

3. The state basis for the fermionic Fock space spanned by N creation operators can be regarded as a Boolean algebra consisting of statements about N basic statements. Hence fermionic degreees of freedom could correspond to the Boolean mind whereas bosonic degrees of freedom would correspond to sensory experiencing and emotions. The integer valued magnetic quantum numbers (a purely TGD based effect) associated with the defect regions of super conductors of type I provide a very robust information storage mechanism and in defect regions fermionic Fock basis is natural. Hence not only fermionic super-conductors but also their defects are biologically interesting [K40, K68] .

11.1.1 General ideas about super-conductivity in many-sheeted space-time

The notion of many-sheeted space-time alone provides a strong motivation for developing TGD based view about superconductivity and I have developed various ideas about high T_c super-conductivity [D29] in parallel with ideas about living matter as a macroscopic quantum system. A further motivation and a hope for more quantitative modelling comes from the discovery of various non-orthodox super-conductors including high T_c superconductors [A69] , [D29, D8] , heavy fermion super-conductors and ferromagnetic superconductors [D55, D41, D37] . The standard BCS theory does not work for these super-conductors and the mechanism for the formation of Cooper pairs is not understood. There is experimental evidence that quantum criticality [D78] is a key feature of many non-orthodox super-conductors. TGD provides a conceptual framework and bundle of ideas making it possible to develop models for non-orthodox superconductors.

Quantum criticality, hierarchy of dark matters, and dynamical $\hbar$

Quantum criticality is the basic characteristic of TGD Universe and quantum critical superconductors provide an excellent test bed to develop the ideas related to quantum criticality into a more concrete form. The hypothesis that Planck constants in CD (causal diamond defined as the intersection of the future and past directed light-cones of M^4) and CP_2 degrees of freedom are dynamical possessing quantized spectrum given as rational multiples of minimum value of Planck constant [K32, K30] adds further content to the notion of quantum criticality.

The value of $\hbar$ is in the general case given by $\hbar = x_a x_b \hbar_0$ (as it became clear after few guesses). a refers to CD and b to CP_2. $x_i = n_i$ holds true for singular coverings and $x_i = 1/n_i$ for singular factor spaces. n is is the order of maximal cyclic subgroup $Z_n \subset G$, where G defines singular covering or factor space. In principle all rational values of $r = \hbar/\hbar_0$ are possible.

Phases with different values x_i behave like dark matter with respect to each other in the sense that they do not have direct interactions except at criticality corresponding to a leakage between different sectors of imbedding space glued together along CD or CP_2 factors. The scalings of CD and CP_2 covariant metrics are from anyonic arguments given by r^2 and 1 so that the value of effective $\hbar$ appearing in Schrödinger equation is given by $r = x_a x_b$ and in principle can have all positive rational values. In large $\hbar(CD)$ phases various quantum time and length scales are scaled up which means macroscopic and macro-temporal quantum coherence.

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and seems to be especially favored in living matter [K29] . There is however no reason to assume that this would be the value of Planck constant in say high T_c super-conductivity.

The only coupling constant strength of theory is Kähler coupling constant g_K^2 which appears in the definition of the Kähler function K characterizing the geometry of the configuration space of 3-surfaces (the "world of classical worlds"). The exponent of K defines vacuum functional analogous to the exponent of Hamiltonian in thermodynamics. The allowed value(s) of g_K^2, which is (are) analogous to critical temperature(s), is (are) determined by quantum criticality requirement. Contrary to the original hypothesis inspired by the requirement that gravitational coupling is renormalization group invariant, α_K does not seem to depend on p-adic prime whereas gravitational constant is proportional to L_p^2. The situation is saved by the assumption that gravitons correspond to the largest non-super-astrophysical Mersenne prime M_{127} so that gravitational coupling is effectively RG invariant in p-adic coupling constant evolution [K5] .

$\hbar(CD)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only $r = \hbar/\hbar_0 = x_a x_b$ of CD and CP_2 Planck constants appears in Kähler action and is due to the fact that the CD and CP_2 metrics of the imbedding space sector with given values of Planck constants are proportional to the corresponding Planck constants [K32] . This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of Planck constants coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors. For a fixed value of $x_a x_b$ one obtains zoomed up versions of particles with size scaled up by $x_a x_b$.

A further great idea is that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces gauge coupling strength α so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

TGD actually predicts an infinite hierarchy of phases behaving like dark or partially dark matter with respect to the ordinary matter [K37] and the value of $\hbar$ is only one characterizer of these phases. These phases, especially so large $\hbar$ phase, seem to be essential for the understanding of even ordinary hadronic, nuclear and condensed matter physics [K37, K80, K30] . This strengthens the motivations for finding whether dark matter might be involved with quantum critical super-conductivity.

Cusp catastrophe serves as a metaphor for criticality. In the recent case temperature and doping are control variables and the tip of cusp is at maximum value of T_c. Critical region correspond to the cusp catastrophe. Quantum criticality suggests the generalization of the cusp to a fractal cusp. Inside the critical lines of cusp there are further cusps which corresponds to higher levels in the hierarchy of dark matters labeled by increasing values of $\hbar$ and they correspond to a hierarchy of subtle quantum coherent dark matter phases in increasing length scales. The proposed model for high T_c super-conductivity involves only single value of Planck constant but it might be that the full description involves very many values of them.

Many-sheeted space-time concept and ideas about macroscopic quantum phases

Many-sheeted space-time leads to obvious ideas concerning the realization of macroscopic quantum phases.

1. The dropping of particles to larger space-time sheets is a highly attractive mechanism of super-conductivity. If space-time sheets are thermally isolated, the larger space-time sheets could be at extremely low temperature and super-conducting.

2. The possibility of large $\hbar$ phases allows to give up the assumption that space-time sheets characterized by different p-adic length scales are thermally isolated. The scaled up versions of a given space-time sheet corresponding to a hierarchy of values of $\hbar$ are possible such that the scale of kinetic energy and magnetic interaction energy remain same for all these space-time sheets. For the scaled up variants of space-time sheet the critical temperature for superconductivity could be higher than room temperature.

3. The idea that wormhole contacts can form macroscopic quantum phases and that the interaction of ordinary charge carriers with the wormhole contacts feeding their gauge fluxes to larger space-time sheets could be responsible for the formation of Cooper pairs, have been around for a decade [K96] . The rather recent realization that wormhole contacts can be actually regarded as space-time correlates for Higgs particles suggests also a new view about the photon massivation in super-conductivity.

4. Quantum classical correspondence has turned out be a very powerful idea generator. For instance, one can ask what are the space-time correlates for various notions of condensed matter such as phonons, BCS Cooper pairs, holes, etc...

11.1.2 TGD inspired model for high T_c superconductivity

The TGD inspired model for high T_c super-conductivity relies on the notions of quantum criticality, dynamical quantized Planck constant requiring a generalization of the 8-D imbedding space to a book like structure, and many-sheeted space-time. In particular, the notion of magnetic flux tube as a carrier of supra current of central concept.

With a sufficient amount of twisting and weaving these basic ideas one ends up to concrete models for high T_c superconductors as quantum critical superconductors consistent with the qualitative facts that I am personally aware. The following minimal model looks the most realistic option found hitherto.

1. The general idea is that magnetic flux tubes are carriers of supra currents. In anti-ferromagnetic phases these flux tube structures form small closed loops so that the system behaves as an insulator. Some mechanism leading to a formation of long flux tubes must exist. Doping

creates holes located around stripes, which become positively charged and attract electrons to the flux tubes.

2. Usually magnetic field tends to destroy Cooper pairs since it tends to flip the spins of electrons of pair to same direction. In TGD flux quantization comes in rescue and magnetic fields favor the formation of Cooper pairs. If one has two parallel flux tubes with opposite directions of magnetic fluxes with large value of $h_{eff} = nh$, $S = 0$ Cooper pairs with even $L \geq 2$ are favored. This situation is encountered in systems near antiferromagnetic phase transition in small scales leading to formation of sequences of flux loops carrying Cooper pairs. Macroscopic super-conductivity results when the loops are reconnected to two long flux tubes with opposite fluxes. If the magnetic fluxes have same sign, $S = 1$ Cooper pairs with odd $L \geq 1$ are favored.

3. The higher critical temperature T_{c1} corresponds to a formation local configurations of parallel spins assigned to the holes of stripes giving rise to a local dipole fields with size scale of the order of the length of the stripe. Conducting electrons form Cooper pairs at the magnetic flux tube structures associated with these dipole fields. The presence of magnetic field favors Cooper pairs with spin $S = 1$. It took long time to realize that pairs of large h_{eff} magnetic flux tubes with fluxes in opposite directions are ideal for carrying Cooper pairs with members of the pair at the different flux tubes. Large spin interaction energy with magnetic field proportional to $h_{eff} = nh$ stablizes the pair.

4. Stripes can be seen as 1-D metals with de-localized electrons. The interaction responsible for the energy gap corresponds to the transversal oscillations of the magnetic flux tubes inducing oscillations of the nuclei of the stripe. These transverse phonons have spin and their exchange is a good candidate for the interaction giving rise to a mass gap. This could explain the claimed BCS type aspects of high T_c super-conductivity. Another interpretation is as spin density waves now known to be important for high temperature superconductivity.

5. Above T_c supra currents are possible only in the length scale of the flux tubes of the dipoles which is of the order of stripe length. The reconnections between neighboring flux tube structures induced by the transverse fluctuations give rise to longer flux tubes structures making possible finite conductivity. These occur with certain temperature dependent probability $p(T, L)$ depending on temperature and distance L between the stripes. By criticality $p(T, L)$ depends on the dimensionless variable $x = TL/\hbar$ only: $p = p(x)$. At critical temperature T_c transverse fluctuations have large amplitude and makes $p(x_c)$ so large that very long flux tubes are created and supra currents can run. The phenomenon is completely analogous to percolation [D15] .

6. The critical temperature $T_c = x_c\hbar/L$ is predicted to be proportional to $\hbar$ and inversely proportional to L (, which is indeed to be the case). If flux tubes correspond to a large value of $\hbar$, one can understand the high value of T_c. Both Cooper pairs and magnetic flux tube structures represent dark matter in TGD sense.

7. The model allows to interpret the characteristic spectral lines in terms of the excitation energy of the transversal fluctuations and gap energy of the Cooper pair. The observed 50 meV threshold for the onset of photon absorption suggests that below T_c also $S = 0$ Cooper pairs are possible and have gap energy about 9 meV whereas $S = 1$ Cooper pairs would have gap energy about 27 meV. The flux tube model indeed predicts that $S = 0$ Cooper pairs become stable below T_c since they cannot anymore transform to $S = 1$ pairs. Their presence could explain the BCS type aspects of high T_c super-conductivity. The estimate for $\hbar/\hbar_0 = r$ from critical temperature T_{c1} is about $r = 3$ contrary to the original expectations inspired by the model of of living system as a super-conductor suggesting much higher value. An unexpected prediction is that coherence length is actually r times longer than the coherence length predicted by conventional theory so that type I super-conductor could be in question with stripes serving as duals for the defects of type I super-conductor in nearly critical magnetic field replaced now by ferromagnetic phase.

8. TGD suggests preferred values for $r = \hbar/\hbar_0$ and the applications to bio-systems favor powers of $r = 2^{11}$. $r = 2^{11}$ predicts that electron Compton length is of order atomic size scale.

Bio-superconductivity could involve electrons with $r = 2^{22}$ having size characterized by the thickness of the lipid layer of cell membrane.

At qualitative level the model explains various strange features of high T_c superconductors. One can understand the high value of T_c and ambivalent character of high T_c super conductors, the existence of pseudogap and scalings laws for observables above T_c, the role of stripes and doping and the existence of a critical doping, etc...

The model explains the observed ferromagnetic super-conductivity at quantum criticality [D55] . Since long flux tubes already exist, the overcritical transverse of fluctuations of the magnetic flux tubes inducing reconnections are now not responsible for the propagation of the super currents now. The should however provide the binding mechanism of $S = 1, L = 2$ Cooper pairs via the coupling of the fluctuations to excitation in the direction of flux tubes. I have considered effectively one-dimensional phonons in the direction of flux tubes as a candidates for this excitation. Spin density waves looks however a more realistic possibility. Also a modulated ferromagnetic phase consisting of stripes of opposite magnetization direction allows superconductivity [D55] and could be understood in terms of $S = 0$ Cooper pairs with electrons of the pair located at the neighboring stripes (flux tubes in TGD model.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Magnetic body [L31]
- Basic Mechanisms associated with magnetic body [L23]
- Pollack's observations [L39]
- Geometrization of fields [L26]
- High temperature superconductivity [L28]

11.2 General TGD based view about super-conductivity

Today super-conductivity includes besides the traditional low temperature super-conductors many other non-orthodox ones [D77] . These unorthodox super-conductors carry various attributes such as cuprate, organic, dichalcogenide, heavy fermion, bismute oxide, ruthenate, antiferromagnetic and ferromagnetic. Mario Rabinowitz has proposed a simple phenomenological theory of super-fluidity and super-conductivity which helps non-specialist to get a rough quantitative overall view about super-conductivity [D77] .

11.2.1 Basic phenomenology of super-conductivity

The following provides the first attempt by a non-professional to form an overall view about super-conductivity.

Basic phenomenology of super-conductivity

The transition to super-conductivity occurs at critical temperature T_c and involves a complete loss of electrical resistance. Super-conductors expel magnetic fields (Meissner effect) and when the external magnetic field exceeds a critical value H_c super-conductivity is lost either completely or partially. In the transition to super-conductivity specific heat has singularity. For long time magnetism and super-conductivity were regarded as mutually exclusive phenomena but the discovery of ferromagnetic super-conductors [D55, D37] has demonstrated that reality is much more subtle.

The BCS theory developed by Bardeen, Cooper, and Schrieffer in 1957 provides a satisfactory model for low T_c super-conductivity in terms of Cooper pairs. The interactions of electrons with the crystal lattice induce electron-electron interaction binding electrons to Cooper pairs at sufficiently low temperatures. The electrons of Cooper pair are at the top of Fermi sphere (otherwise they cannot interact to form bound states) and have opposite center of mass momenta and spins. The binding creates energy gap E_g determining the critical temperature T_c. The singularity of the specific heat in the transition to super-conductivity can be understood as being due to the loss of thermally excitable degrees of freedom at critical temperature so that heat capacity is reduced exponentially. BCS theory has been successful in explaining the properties of low temperature super conductors but the high temperature super-conductors discovered in 1986 and other non-orthodox superconductors discovered later remain a challenge for theorists.

The reasons why magnetic fields tend to destroy super-conductivity is easy to understand. Lorentz force induces opposite forces to the electrons of Cooper pair since the momenta are opposite. Magnetic field tends also to turn the spins in the same direction. The super-conductivity is destroyed in fields for which the interaction energy of magnetic moment of electron with field is of the same order of magnitude as gap energy $E_g \sim T_c$: $e\hbar H_c/2m \sim T_c$.

If spins are parallel, the situation changes since only Lorentz force tends to destroy the Cooper pair. In high T_c super-conductors this is indeed the case: electrons are in spin triplet state ($S = 1$) and the net orbital angular momentum of Cooper pair is $L = 2$. The fact that orbital state is not $L = 0$ state makes high T_c super-conductors much more fragile to the destructive effect of impurities than conventional super-conductors (due to the magnetic exchange force between electrons responsible for magnetism). Also the Cooper pairs of ^3He superfluid are in spin triplet state but have $S = 0$.

The observation that spin triplet Cooper pairs might be possible in ferro-magnets stimulates the question whether ferromagnetism and super-conductivity might tolerate each other after all, and the answer is affirmative [D37] . The article [D55] provides an enjoyable summary of experimental discoveries.

Basic parameters of super-conductors from universality?

Super conductors are characterized by certain basic parameters such as critical temperature T_c and critical magnetic field H_c, densities n_c and n of Cooper pairs and conduction electrons, gap energy E_g, correlation length ξ and magnetic penetration length λ. The super-conductors are highly complex systems and calculation of these parameters from BCS theory is either difficult or impossible.

It has been suggested [D77] that these parameters might be more or less universal so that they would not depend on the specific properties of the interaction responsible for the formation of Cooper pairs. The motivation comes from the fact that the properties of ordinary Bose-Einstein condensates do not depend on the details of interactions. This raises the hope that these parameters might be expressible in terms of some basic parameters such as T_c and the density of conduction electrons allowing to deduce Fermi energy E_F and Fermi momentum k_F if Fermi surface is sphere. In [D77] formulas for the basic parameters are indeed suggested based on this of argumentation assuming that Cooper pairs form a Bose-Einstein condensate.

1. The most important parameters are critical temperature T_c and critical magnetic field H_c in principle expressible in terms of gap energy. In [D77] the expression for T_c is deduced from the condition that the de Broglie wavelength λ must satisfy in supra phase the condition

$$\lambda \geq 2d = 2(\frac{n_c}{g})^{-1/D} \tag{11.2.1}$$

guaranteeing the quantum overlap of Cooper pairs. Here n_c is the density of Bose-Einstein condensate of Cooper pairs and g is the number of spin states and D the dimension of the condensate. This condition follows also from the requirement that the number of particles per energy level is larger than one (Bose-Einstein condensation).

Identifying this expression with the de Broglie wavelength $\lambda = \hbar/\sqrt{2mE}$ at thermal energy $E = (D/2)T_c$, where D is the number of degrees of freedom, one obtains

$$T_c \leq \frac{h^2}{4Dm}(\frac{n_c}{g})^{2/D} . \quad (11.2.2)$$

m denotes the effective mass of super current carrier and for electron it can be even 100 times the bare mass of electron. The reason is that the electron moves is somewhat like a person trying to move in a dense crowd of people, and is accompanied by a cloud of charge carriers increasing its effective inertia. In this equation one can consider the possibility that Planck constant is not the ordinary one. This obviously increases the critical temperature unless n_c is scaled down in same proportion in the phase transition to large $\hbar$ phase.

2. The density of n_c Cooper pairs can be estimated as the number of fermions in Fermi shell at E_F having width Δk deducible from kT_c. For $D = 3$-dimensional spherical Fermi surface one has

$$\begin{aligned} n_c &= \frac{1}{2}\frac{4\pi k_F^2 \Delta k}{\frac{4}{3}\pi k_F^3} n , \\ kT_c &= E_F - E(k_F - \Delta k) \simeq \frac{h^2 k_F \Delta k}{m} . \end{aligned} \quad (11.2.3)$$

Analogous expressions can be deduced in $D = 2$- and $D = 1$-dimensional cases and one has

$$n_c(D) = \frac{D}{2}\frac{T_c}{E_F} n(D) . \quad (11.2.4)$$

The dimensionless coefficient is expressible solely in terms of n and effective mass m. In [D77] it is demonstrated that the inequality 11.2.2 replaced with equality when combined with 11.2.4 gives a satisfactory fit for 16 super-conductors used as a sample.

Note that the Planck constant appearing in E_F and T_c in Eq. 11.2.4 must correspond to ordinary Planck constant $\hbar_0$. This implies that equations 11.2.2 and 11.2.4 are consistent within orders of magnitudes. For $D = 2$, which corresponds to high T_c superconductivity, the substitution of n_c from Eq. 11.2.4 to Eq. 11.2.2 gives a consistency condition from which n_c disappears completely. The condition reads as

$$n\lambda_F^2 = \pi = 4g .$$

Obviously the equation is not completely consistent.

3. The magnetic penetration length λ is expressible in terms of density n_c of Cooper pairs as

$$\lambda^{-2} = \frac{4\pi e^2 n_c}{m_e} . \quad (11.2.5)$$

The ratio $\kappa \equiv \frac{\lambda}{\xi}$ determines the type of the super conductor. For $\kappa < \frac{1}{\sqrt{2}}$ one has type I super conductor with defects having negative surface energy. For $\kappa \geq \frac{1}{\sqrt{2}}$ one has type II super conductor and defects have positive surface energy. Super-conductors of type I this results in complex stripe like flux patterns maximizing their area near criticality. The super-conductors of type II have $\kappa > 1/\sqrt{2}$ and the surface energy is positive so that the flux penetrates as flux quanta minimizing their area at lower critical value H_{c_1} of magnetic field and completely at higher critical value H_{c_2} of magnetic field. The flux quanta contain a core of size ξ carrying quantized magnetic flux.

4. Quantum coherence length ξ can be roughly interpreted as the size of the Cooper pair or as the size of the region where it is sensible to speak about the phase of wave function of Cooper pair. For larger separations the phases of wave functions are un-correlated. The values of ξ vary in the range $10^3 - 10^4$ Angstrom for low T_c super-conductors and in the range $5 - 20$ Angstrom for high T_c super-conductors (assuming that they correspond to ordinary $\hbar$!) the ratio of these coherence lengths varies in the range $[50 - 2000]$, with upper bound corresponding to $n_F = 2^{11}$ for $\hbar$. This would give range $1 - 2$ microns for the coherence lengths of high T_c super-conductors with lowest values of coherence lengths corresponding to the highest values of coherence lengths for low temperatures super conductors.

Uncertainty Principle $\delta E \delta t = \hbar/2$ using $\delta E = E_g \equiv 2\Delta$, $\delta t = \xi/v_F$, gives an order of magnitude estimate for ξ differing only by a numerical factor from the result of a rigorous calculation given by

$$\xi = \frac{4\hbar v_F}{E_g} . \quad (11.2.6)$$

E_g is apart from a numerical constant equal to T_c: $E_g = nT_c$. Using the expression for v_F and T_c in terms of the density of electrons, one can express also ξ in terms of density of electrons.

For instance, BCS theory predicts $n = 3.52$ for metallic super-conductors and $n = 8$ holds true for cuprates [D77] . For cuprates one obtains $\xi = 2n^{-1/3}$ [D77] . This expression can be criticized since cuprates are Mott insulators and it is not at all clear whether a description as Fermi gas makes sense. The fact that high T_c super-conductivity involves breakdown of anti-ferromagnetic order might justify the use of Fermi gas description for conducting holes resulting in the doping.

For large $\hbar$ the value of ξ would scale up dramatically if deduced theoretically from experimental data using this kind of expression. If the estimates for ξ are deduced from v_F and T_c purely calculationally as seems to be the case, the actual coherence lengths would be scaled up by a factor $\hbar/\hbar_0 = n_F$ if high T_c super-conductors correspond to large $\hbar$ phase. As also found that this would also allow to understand the high critical temperature.

11.2.2 Universality of the parameters in TGD framework

Universality idea conforms with quantum criticality of TGD Universe. The possibility to express everything in terms of density of critical temperature coding for the dynamics of Cooper pair formation and the density charge carriers would make it also easy to understand how p-adic scalings and transitions to large $\hbar$ phase affect the basic parameters. The possible problem is that the replacement of inequality of Eq. 11.2.2 with equality need not be sensible for large $\hbar$ phases. It will be found that in many-sheeted space-time T_c does not directly correspond to the gap energy and the universality of the critical temperature follows from the p-adic length scale hypothesis.

The effect of p-adic scaling on the parameters of super-conductors

p-Adic fractality expresses as $n \propto 1/L^3(k)$ would allow to deduce the behavior of the various parameters as function of the p-adic length scale and naive scaling laws would result. For instance, E_g and T_c would scale as $1/L^2(k)$ if one assumes that the density n of particles at larger space-time sheets scales p-adically as $1/L^3(k)$. The basic implication would be that the density of Cooper pairs and thus also T_c would be reduced very rapidly as a function of the p-adic length scale. Without thermal isolation between these space-time sheets and hight temperature space-time sheets there would not be much hopes about high T_c super-conductivity.

In the scaling of Planck constant basic length scales scale up and the overlap criterion for super-conductivity becomes easy to satisfy unless the density of electrons is reduced too dramatically. As found, also the critical temperature scales up so that there are excellent hopes of obtain high T_c super-conductor in this manner. The claimed short correlation lengths are not a problem since they are calculational quantities.

It is of interest to study the behavior of the various parameters in the transition to the possibly existing large $\hbar$ variant of super-conducting electrons. Also small scalings of $\hbar$ are possible and the considerations to follow generalize trivially to this case. Under what conditions the behavior of the various parameters in the transition to large $\hbar$ phase is dictated by simple scaling laws?

1. Scaling of T_c and E_g

T_c and E_g remain invariant if E_g corresponds to a purely classical interaction energy remaining invariant under the scaling of $\hbar$. This is not the case for BCS super-conductors for which the gap energy E_g has the following expression.

$$\begin{aligned} E_g &= \hbar\omega_c exp(-1/X) \ , \\ X &= n(E_F)U_0 = \frac{3}{2}N(E_F)\frac{U_0}{E_F} \ , \\ n(E_F) &= \frac{3}{2}\frac{N(E_F)}{E_F} \ . \\ \omega_c &= \omega_D = (6\pi^2)^{1/3}c_s n_n^{1/3} \ . \end{aligned} \tag{11.2.7}$$

Here ω_c is the width of energy region near E_F for which "phonon" exchange interaction is effective. n_n denotes the density of nuclei and c_s denotes sound velocity.

$N(E_F)$ is the total number of electrons at the super-conducting space-time sheet. U_0 would be the parameter characterizing the interaction strength of of electrons of Cooper pair and should not depend on $\hbar$. For a structure of size $L \sim 1$ μ m one would have $X \sim n_a 10^{12}\frac{U_0}{E_F}$, n_a being the number of exotic electrons per atom, so that rather weak interaction energy U_0 can give rise to $E_g \sim \omega_c$.

The expression of ω_c reduces to Debye frequency ω_D in BCS theory of ordinary super conductivity. If c_s is proportional to thermal velocity $\sqrt{T_c/m}$ at criticality and if n_n remains invariant in the scaling of $\hbar$, Debye energy scales up as $\hbar$. This can imply that $E_g > E_F$ condition making scaling non-sensible unless one has $E_g \ll E_F$ holding true for low T_c super-conductors. This kind of situation would *not* require large $\hbar$ phase for electrons. What would be needed that nuclei and phonon space-time sheets correspond to large $\hbar$ phase.

What one can hope is that E_g scales as $\hbar$ so that high T_c superconductor would result and the scaled up T_c would be above room temperature for $T_c > .15$ K. If electron is in ordinary phase X is automatically invariant in the scaling of $\hbar$. If not, the invariance reduces to the invariance of U_0 and E_F under the scaling of $\hbar$. If n scales like $1/\hbar^D$, E_F and thus X remain invariant. U_0 as a simplified parameterization for the interaction potential expressible as a tree level Feynman diagram is expected to be in a good approximation independent of $\hbar$.

It will be found that in high T_c super-conductors, which seem to be quantum critical, a high T_c variant of phonon mediated superconductivity and exotic superconductivity could be competing. This would suggest that the phonon mediated superconductivity corresponds to a large $\hbar$ phase for nuclei scaling ω_D and T_c by a factor $r = \hbar/\hbar_0$.

Since the total number $N(E_F)$ of electrons at larger space-time sheet behaves as $N(E_F) \propto E_F^{D/2}$, where D is the effective dimension of the system, the quantity $1/X \propto E_F/n(E_F)$ appearing in the expressions of the gap energy behaves as $1/X \propto E_F^{-D/2+1}$. This means that at the limit of vanishing electron density $D = 3$ gap energy goes exponentially to zero, for $D = 2$ it is constant, and for $D = 1$ it goes zero at the limit of small electron number so that the formula for gap energy reduces to $E_g \simeq \omega_c$. These observations suggests that the super-conductivity in question should be 2- or 1-dimensional phenomenon as in case of magnetic walls and flux tubes.

2. Scaling of ξ and λ

If n_c for high T_c super-conductor scales as $1/\hbar^D$ one would have $\lambda \propto \hbar^{D/2}$. High T_c property however suggests that the scaling is weaker. ξ would scale as $\hbar$ for given v_F and T_c. For $D = 2$ case the this would suggest that high T_c super-conductors are of type I rather than type II as they would be for ordinary $\hbar$. This conforms with the quantum criticality which would be counterpart of critical behavior of super-conductors of type I in nearly critical magnetic field.

3. Scaling of H_c and B

The critical magnetization is given by

$$H_c(T) = \frac{\Phi_0}{\sqrt{8}\pi\xi(T)\lambda(T)} , \quad (11.2.8)$$

where Φ_0 is the flux quantum of magnetic field proportional to $\hbar$. For $D = 2$ and $n_c \propto \hbar^{-2}$ $H_c(T)$ would not depend on the value of $\hbar$. For the more physical dependence $n_c \propto \hbar^{-2+\epsilon}$ one would have $H_c(T) \propto \hbar^{-\epsilon}$. Hence the strength of the critical magnetization would be reduced by a factor $2^{-11\epsilon}$ in the transition to the large $\hbar$ phase with $n_F = 2^{-11}$.

Magnetic flux quantization condition is replaced by

$$\int 2eBdS = n\hbar 2\pi . \quad (11.2.9)$$

B denotes the magnetic field inside super-conductor different from its value outside the super-conductor. By the quantization of flux for the non-super-conducting core of radius ξ in the case of super-conductors of type II $eB = \hbar/\xi^2$ holds true so that B would become very strong since the thickness of flux tube would remain unchanged in the scaling.

11.2.3 Quantum criticality and super-conductivity

The notion of quantum criticality has been already discussed in introduction. An interesting prediction of the quantum criticality of entire Universe also gives naturally rise to a hierarchy of macroscopic quantum phases since the quantum fluctuations at criticality at a given level can give rise to higher level macroscopic quantum phases at the next level. A metaphor for this is a fractal cusp catastrophe for which the lines corresponding to the boundaries of cusp region reveal new cusp catastrophes corresponding to quantum critical systems characterized by an increasing length scale of quantum fluctuations.

Dark matter hierarchy could correspond to this kind of hierarchy of phases and long ranged quantum slow fluctuations would correspond to space-time sheets with increasing values of $\hbar$ and size. Evolution as the emergence of modules from which higher structures serving as modules at the next level would correspond to this hierarchy. Mandelbrot fractal with inversion analogous to a transformation permuting the interior and exterior of sphere with zooming revealing new worlds in Mandelbrot fractal replaced with its inverse would be a good metaphor for what quantum criticality would mean in TGD framework.

How the quantum criticality of superconductors relates to TGD quantum criticality

There is empirical support that super-conductivity in high T_c super-conductors and ferromagnetic systems [D55, D41] is made possible by quantum criticality [D78] . In the experimental situation quantum criticality means that at sufficiently low temperatures quantum rather than thermal fluctuations are able to induce phase transitions. Quantum criticality manifests itself as fractality and simple scaling laws for various physical observables like resistance in a finite temperature range and also above the critical temperature. This distinguishes sharply between quantum critical super conductivity from BCS type super-conductivity. Quantum critical super-conductivity also exists in a finite temperature range and involves the competition between two phases.

The absolute quantum criticality of the TGD Universe maps to the quantum criticality of subsystems, which is broken by finite temperature effects bringing dissipation and freezing of quantum fluctuations above length and time scales determined by the temperature so that scaling laws hold true only in a finite temperature range.

Reader has probably already asked what quantum criticality precisely means. What are the phases which compete? An interesting hypothesis is that quantum criticality actually corresponds to criticality with respect to the phase transition changing the value of Planck constant so that the competing phases would correspond to different values of $\hbar$. In the case of high T_c super-conductors (anti-ferromagnets) the fluctuations can be assigned to the magnetic flux tubes of the dipole field patterns generated by rows of holes with same spin direction assignable to the stripes. Below T_c fluctuations induce reconnections of the flux tubes and a formation of very long flux tubes and make possible for the supra currents to flow in long length scales below T_c. Percolation type phenomenon is in question. The fluctuations of the flux tubes below $T_{c1} > T_c$ induce transversal phonons generating the energy gap for $S = 1$ Cooper pairs. $S = 0$ Cooper pairs are predicted to stabilize below T_c.

Scaling up of de Broglie wave lengths and criterion for quantum overlap

Compton lengths and de Broglie wavelengths are scaled up by an integer n, whose preferred values correspond to $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes. In particular, $n_F = 2^{k11}$ seem to be favored in living matter. The scaling up means that the overlap condition $\lambda \geq 2d$ for the formation of Bose-Einstein condensate can be satisfied and the formation of Cooper pairs becomes possible. Thus a hierarchy of large $\hbar$ super-conductivities would be associated with to the dark variants of ordinary particles having essentially same masses as the ordinary particles.

Unless one assumes fractionization, the invariance of $E_F \propto \hbar_{eff}^2 n^{2/3}$ in $\hbar$ increasing transition would require that the density of Cooper pairs in large $\hbar$ phase is scaled down by an appropriate factor. This means that supra current intensities, which are certainly measurable quantities, are also scaled down. Of course, it could happen that E_F is scaled up and this would conform with the scaling of the gap energy.

Quantum critical super-conductors in TGD framework

For quantum critical super-conductivity in heavy fermions systems, a small variation of pressure near quantum criticality can destroy ferromagnetic (anti-ferromagnetic) order so that Curie (Neel) temperature goes to zero. The prevailing spin fluctuation theory [D33] assumes that these transitions are induced by long ranged and slow spin fluctuations at critical pressure P_c. These fluctuations make and break Cooper pairs so that the idea of super-conductivity restricted around critical point is indeed conceivable.

Heavy fermion systems, such as cerium-indium alloy $CeIn_3$ are very sensitive to pressures and a tiny variation of density can drastically modify the low temperature properties of the systems. Also other systems of this kind, such as $CeCu_2Ge_2$, $CeIn_3$, $CePd_2Si_2$ are known [D55, D37] . In these cases super-conductivity appears around anti-ferromagnetic quantum critical point.

The last experimental breakthrough in quantum critical super-conductivity was made in Grenoble [D41] . URhGe alloy becomes super-conducting at $T_c = .280$ K, loses its super-conductivity at $H_c = 2$ Tesla, and becomes again super-conducting at $H_c = 12$ Tesla and loses its super-conductivity again at $H = 13$ Tesla. The interpretation is in terms of a phase transition changing the magnetic order inducing the long range spin fluctuations.

TGD based models of atomic nucleus [K80] and condensed matter [K30] assume that weak gauge bosons with Compton length of order atomic radius play an essential role in the nuclear and condensed matter physics. The assumption that condensed matter nuclei possess anomalous weak charges explains the repulsive core of potential in van der Waals equation and the very low compressibility of condensed matter phase as well as various anomalous properties of water phase, provide a mechanism of cold fusion and sono-fusion, etc. [K30, K28] . The pressure sensitivity of these systems would directly reflect the physics of exotic quarks and electro-weak gauge bosons. A possible mechanism behind the phase transition to super-conductivity could be the scaling up of the sizes of the space-time sheets of nuclei.

Also the electrons of Cooper pair (and only these) could make a transition to large $\hbar$ phase. This transition would induce quantum overlap having geometric overlap as a space-time correlate. The formation of join along boundaries bonds between neighboring atoms would be part of the mechanism. For instance, the criticality condition $4n^2\alpha = 1$ for BE condensate of n Cooper pairs would give $n = 6$ for the size of a higher level quantum unit possibly formed formed from Cooper pairs. If one does not assume invariance of energies obtained by fractionization of principal quantum number, this transition has dramatic effects on the spectrum of atomic binding energies scaling as $1/\hbar^2$ and practically universal spectrum of atomic energies would result [K28] not depending much on nuclear charge. It seems that this prediction is non-physical.

Quantum critical super-conductors resemble superconductors of type I with $\lambda \ll \xi$ for which defects near thermodynamical criticality are complex structures looking locally like stripes of thickness λ. These structures are however dynamical in super-conducting phase. Quite generally, long range quantum fluctuations due to the presence of two competing phases would manifest as complex dynamical structures consisting of stripes and their boundaries. These patterns are dynamical rather than static as in the case of ordinary spin glass phase so that quantum spin glass or 4-D spin glass is a more appropriate term. The breaking of classical non-determinism for vacuum extremals indeed makes possible space-time correlates for quantum non-determinism and this makes TGD Universe a 4-dimensional quantum spin glass.

Could quantum criticality make possible new kinds of high T_c super-conductors?

The transition to large $\hbar = r\hbar_0$ phase increases various length scales by r and makes possible long range correlations even at high temperatures. Hence the question is whether large $\hbar$ phase could correspond to ordinary high T_c super-conductivity. If this were the case in the case of ordinary high T_c super-conductors, the actual value of coherence length ξ would vary in the range $5-20$ Angstrom scaled up by a factor r. For effectively D-dimensional super-conductor The density of Cooper pairs would be scaled down by an immensely small factor $1/r^D$ from its value deduced from Fermi energy.

Large $\hbar$ phase for some nuclei might be involved and make possible large space-time sheets of size at least of order of ξ at which conduction electrons forming Cooper pairs would topologically condense like quarks around hadronic space-time sheets (in [K30] a model of water as a partially dark matter with one fourth of hydrogen ions in large $\hbar$ phase is developed).

Consider for a moment the science fictive possibility that super conducting electrons for some quantum critical super-conductors to be discovered or already discovered correspond to large $\hbar$ phase with $\hbar = r\hbar_0$ keeping in mind that this affects only quantum corrections in perturbative approach but not the lowest order classical predictions of quantum theory. For $r \simeq n2^{k11}$ with $(n,k) = (1,1)$ the size of magnetic body would be $L(149) = 5$ nm, the thickness of the lipid layer of cell membrane. For $(n,k) = (1,2)$ the size would be $L(171) = 10\ \mu$m, cell size. If the density of Cooper pairs is of same order of magnitude as in case of ordinary super conductors, the critical temperature is scaled up by 2^{k11}. Already for $k = 1$ the critical temperature of 1 K would be scaled up to $4n^2 \times 10^6$ K if n_c is not changed. This assumption is not consistent with the assumption that Fermi energy remains non-relativistic. For $n = 1$ $T_c = 400$ K would be achieved for $n_c \rightarrow 10^{-6}n_c$, which looks rather reasonable since Fermi energy transforms as $E_F \rightarrow 8 \times 10^3 E_F$ and remains non-relativistic. H_c would scale down as $1/\hbar$ and for $H_c = .1$ Tesla the scaled down critical field would be $H_c = .5 \times 10^{-4}$ Tesla, which corresponds to the nominal value of the Earth's magnetic field.

Quantum critical super-conductors become especially interesting if one accepts the identification of living matter as ordinary matter quantum controlled by macroscopically quantum coherent dark matter. One of the basic hypothesis of TGD inspired theory of living matter is that the magnetic flux tubes of the Earth's magnetic field carry a super-conducting phase and the spin triplet Cooper pairs of electrons in large $\hbar$ phase might realize this dream. That the value of Earth's magnetic field is near to its critical value could have also biological implications.

11.2.4 Space-time description of the mechanisms of super-conductivity

The application of ideas about dark matter to nuclear physics and condensed matter suggests that dark color and weak forces should be an essential element of the chemistry and condensed matter physics. The continual discovery of new super-conductors, in particular of quantum critical superconductors, suggests that super-conductivity is not well understood. Hence super-conductivity provides an obvious test for these ideas. In particular, the idea that wormhole contacts regarded as parton pairs living at two space-time sheets simultaneously, provides an attractive universal mechanism for the formation of Cooper pairs and is not so far-fetched as it might sound first.

Leading questions

It is good to begin with a series of leading questions. The first group of questions is inspired by experimental facts about super-conductors combined with TGD context.

1. The work of Rabinowitch [D77] suggests that that the basic parameters of super-conductors might be rather universal and depend on T_c and conduction electron density only and be to a

high degree independent of the mechanism of super-conductivity. This is in a sharp contrast to the complexity of even BCS model with its somewhat misty description of the phonon exchange mechanism.
Questions: Could there exist a simple universal description of various kinds of super-conductivities?

2. The new super-conductors possess relatively complex chemistry and lattice structure.
Questions: Could it be that complex chemistry and lattice structure makes possible something very simple describable in terms of quantum criticality. Could it be that the transversal oscillations magnetic flux tubes allow to understand the formation of Cooper pairs at T_{c1} and their reconnections generating very long flux tubes the emergence of supra currents at T_c?

3. The effective masses of electrons in ferromagnetic super-conductors are in the range of 10-100 electron masses [D55] and this forces to question the idea that ordinary Cooper pairs are current carriers.
Questions: Can one consider the possibility that the p-adic length scale of say electron can vary so that the actual mass of electron could be large in condensed matter systems? For quarks and neutrinos this seems to be the case [K57, K58] . Could it be that the Gaussian Mersennes $(1+i)^k-1$, $k = 151, 157, 163, 167$ spanning the p-adic lengthscale range 10 nm-2.5 μm very relevant from the point of view of biology correspond to p-adic length especially relevant for super-conductivity?

Second group of questions is inspired by quantum classical correspondence.

1. Quantum classical correspondence in its strongest form requires that bound state formation involves the generation of join along boundaries bonds between bound particles. The weaker form of the principle requires that the particles are topologically condensed at same space-time sheet. In the case of Cooper pairs in ordinary superconductors the length of join along boundaries bonds between electrons should be of order $10^3 - 10^4$ Angstroms. This looks rather strange and it seems that the latter option is more sensible.
Questions: Could quantum classical correspondence help to identify the mechanism giving rise to Cooper pairs?

2. Quantum classical correspondence forces to ask for the space-time correlates for the existing quantum description of phonons.
Questions: Can one assign space-time sheets with phonons or should one identify them as oscillations of say space-time sheets at which atoms are condensed? Or should the microscopic description of phonons in atomic length scales rely on the oscillations of wormhole contacts connecting atomic space-time sheets to these larger space-time sheets? The identification of phonons as wormhole contacts would be completely analogous to the similar identification of gauge bosons except that phonons would appear at higher levels of the hierarchy of space-time sheets and would be emergent in this sense. As a matter fact, even gauge bosons as pairs of fermion and anti-fermion are emergent structures in TGD framework and this plays fundamental role in the construction of QFT limit of TGD in which bosonic part of action is generated radiatively so that all coupling constants follow as predictions [K63, K34] . Could Bose-Einstein condensates of wormhole contacts be relevant for the description of super-conductors or more general macroscopic quantum phases?

The third group of questions is inspired by the new physics predicted or by TGD.

1. TGD predicts a hierarchy of macroscopic quantum phases with large Planck constant.
Questions: Could large values of Planck constant make possible exotic electronic super-conductivities? Could even nuclei possess large $\hbar$ (super-fluidity)?

2. TGD predicts that classical color force and its quantal counterpart are present in all length scales.
Questions: Could color force, say color magnetic force which play some role in the formation of Cooper pair. The simplest model of pair is as a space-time sheet with size of order ξ so that the electrons could be "outside" the background space-time. Could the Coulomb interaction

energy of electrons with positively charged wormhole throats carrying parton numbers and feeding em gauge flux to the large space-time sheet be responsible for the gap energy? Could wormhole throats carry also quark quantum numbers. In the case of single electron condensed to single space-time sheet the em flux could be indeed feeded by a pair of $u\overline{u}$ and $\overline{d}d$ type wormhole contacts to a larger space-time sheet. Could the wormhole contacts have a net color? Could the electron space-time sheets of the Cooper pair be connected by long color flux tubes to give color singlets so that dark color force would be ultimately responsible for the stability of Cooper pair?

3. Suppose that one takes seriously the ideas about the possibility of dark weak interactions with the Compton scale of weak bosons scaled up to say atomic length scale so that weak bosons are effectively massless below this length scale [K30] .
   Questions: Could the dark weak length scale which is of order atomic size replace lattice constant in the expression of sound velocity? What is the space-time correlate for sound velocity?

Photon massivation, coherent states of Cooper pairs, and wormhole contacts

The existence of wormhole contacts is one of the most stunning predictions of TGD. First I realized that wormhole contacts can be regarded as parton-antiparton pairs with parton and antiparton assignable to the light-like causal horizons accompanying wormhole contacts. Then came the idea that Higgs particle could be identified as a wormhole contact. It was soon followed by the identification all bosonic states as wormhole contacts [K48] . Finally I understood that this applies also to their super-symmetric partners, which can be also fermion [K34] . Fermions and their super-partners would in turn correspond to wormhole throats resulting in the topological condensation of small deformations of CP_2 type vacuum extremals with Euclidian signature of metric to the background space-time sheet. This framework opens the doors for more concrete models of also super-conductivity involving the effective massivation of photons as one important aspect in the case of ordinary super-conductors.

There are two types of wormhole contacts. Those of first type correspond to elementary bosons. Wormhole contacts of second kind are generated in the topological condensation of space-time sheets carrying matter and form a hierarchy. Classical radiation fields realized in TGD framework as oscillations of space-time sheets would generate wormhole contacts as the oscillating space-time sheet develops contacts with parallel space-time sheets (recall that the distance between space-time sheets is of order CP_2 size). This realizes the correspondence between fields and quanta geometrically. Phonons could also correspond to wormhole contacts of this kind since they mediate acoustic oscillations between space-time sheets and the description of the phonon mediated interaction between electrons in terms of wormhole contacts might be useful also in the case of super-conductivity. Bose-Einstein condensates of wormhole contacts might be highly relevant for the formation of macroscopic quantum phases. The formation of a coherent state of wormhole contacts would be the counterpart for the vacuum expectation value of Higgs.

The notions of coherent states of Cooper pairs and of charged Higgs challenge the conservation of electromagnetic charge. The following argument however suggests that coherent states of wormhole contacts form only a part of the description of ordinary super-conductivity. The basic observation is that wormhole contacts with vanishing fermion number define space-time correlates for Higgs type particle with fermion and anti-fermion numbers at light-like throats of the contact.

The ideas that a genuine Higgs type photon massivation is involved with super-conductivity and that coherent states of Cooper pairs really make sense are somewhat questionable since the conservation of charge and fermion number is lost for coherent states. A further questionable feature is that a quantum superposition of many-particle states with widely different masses would be in question. These interpretational problems can be resolved elegantly in zero energy ontology [K24] in which the total conserved quantum numbers of quantum state are vanishing. In this picture the energy, fermion number, and total charge of any positive energy state are compensated by opposite quantum numbers of the negative energy state in geometric future. This makes possible to speak about superpositions of Cooper pairs and charged Higgs bosons separately in positive energy sector.

If this picture is taken seriously, super-conductivity can be seen as providing a direct support for both the hierarchy of scaled variants of standard model physics and for the zero energy ontology.

Space-time correlate for quantum critical superconductivity

The explicit model for high T_c super-conductivity relies on quantum criticality involving long ranged quantum fluctuations inducing reconnection of flux tubes of local (color) magnetic fields associated with parallel spins associated with stripes to form long flux tubes serving as wires along which Cooper pairs flow. Essentially [D15] [D15] type phenomenon would be in question. The role of the doping by holes is to make room for Cooper pairs to propagate by the reconnection mechanism: otherwise Fermi statistics would prevent the propagation. Too much doping reduces the number of current carriers, too little doping leaves too little room so that there exists some optimal doping. In the case of high T_c super-conductors quantum criticality corresponds to a quite wide temperature range, which provides support for the quantum criticality of TGD Universe. The probability $p(T)$ for the formation of reconnections is what matters and exceeds the critical value at T_c.

11.2.5 Super-conductivity at magnetic flux tubes

Super-conductivity at the magnetic flux tubes of magnetic flux quanta is one the basic hypothesis of the TGD based model of living matter. There is also evidence for magnetically mediated super-conductivity in extremely pure samples [D49] . The magnetic coupling was only observed at lattice densities close to the critical density at which long-range magnetic order is suppressed. Quantum criticality that long flux tubes serve as pathways along which Cooper pairs can propagate. In anti-ferromagnetic phase these pathways are short-circuited to closed flux tubes of local magnetic fields.

Almost the same model as in the case of high T_c and quantum critical super-conductivity applies to the magnetic flux tubes. Now the flux quantum contains BE condensate of exotic Cooper pairs interacting with wormhole contacts feeding the gauge flux of Cooper pairs from the magnetic flux quantum to a larger space-time sheet. The interaction of spin 1 Cooper pairs with the magnetic field of flux quantum orients their spins in the same direction. Large value of $\hbar$ guarantees thermal stability even in the case that different space-time sheets are not thermally isolated.

The understanding of gap energy is not obvious. The transversal oscillations of magnetic flux tubes generated by spin flips of electrons define the most plausible candidate for the counterpart of phonons. In this framework phonon like states identified as wormhole contacts would be created by the oscillations of flux tubes and would be a secondary phenomenon.

Large values of $\hbar$ allow to consider not only the Cooper pairs of electrons but also of protons and fermionic ions. Since the critical temperature for the formation of Cooper pairs is inversely proportional to the mass of the charge carrier, the replacement of electron with proton or ion would require a scaling of $\hbar$. If T_{c1} is proportional to $\hbar^2$, this requires scaling by $(m_p/m_e)^{1/2}$. For $T_{c1} \propto \hbar$ scaling by $m_p/m_e \simeq 2^{11}$ is required. This inspired idea that powers of 2^{11} could define favored values of $\hbar/\hbar_0$. This hypothesis is however rather ad hoc and turned out to be too restrictive.

Besides Cooper pairs also Bose-Einstein condensates of bosonic ions are possible in large $\hbar$ phase and would give rise to super-conductivity. TGD inspired nuclear physics predicts the existence of exotic bosonic counterparts of fermionic nuclei with given (A, Z) [L2] , [L2] .

Superconductors at the flux quanta of the Earth's magnetic field

Magnetic flux tubes and magnetic walls are the most natural candidates for super-conducting structures with spin triplet Cooper pairs. Indeed, experimental evidence relating to the interaction of ELF em radiation with living matter suggests that bio-super-conductors are effectively 1- or 2-dimensional. $D \leq 2$-dimensionality is guaranteed by the presence of the flux tubes or flux walls of, say, the magnetic field of Earth in which charge carries form bound states and the system is equivalent with a harmonic oscillator in transversal degrees of freedom.

The effect of Earth's magnetic field is completely negligible at the atomic space-time sheets and cannot make super conductor 1-dimensional. At cellular sized space-time sheets magnetic field makes possible transversal the confinement of the electron Cooper pairs in harmonic oscillator states but does not explain energy gap which should be at the top of 1-D Fermi surface. The critical temperature extremely low for ordinary value of $\hbar$ and either thermal isolation between space-time sheets or large value of $\hbar$ can save the situation.

An essential element of the picture is that topological quantization of the magnetic flux tubes occurs. In fact, the flux tubes of Earth's magnetic field have thickness of order cell size from the quantization of magnetic flux. The observations about the effects of ELF em fields on bio-matter [J21] suggest that similar mechanism is at work also for ions and in fact give very strong support for bio-super conductivity based on the proposed mechanism.

Energy gaps for superconducting magnetic flux tubes and walls

Besides the formation of Cooper pairs also the Bose-Einstein condensation of charge carriers to the ground state is needed in order to have a supra current. The stability of Bose-Einstein condensate requires an energy gap $E_{g,BE}$ which must be larger than the temperature at the magnetic flux tube.

Several energies must be considered in order to understand $E_{g,BE}$.

1. The Coulombic binding energy of Cooper pairs with the wormhole contacts feeding the em flux from magnetic flux tube to a larger space-time sheet defines an energy gap which is expected to be of order $E_{g,BE} = \alpha/L(k)$ giving $E_g \sim 10^{-3}$ eV for $L(167) = 2.5$ μm giving a rough estimate for the thickness of the magnetic flux tube of the Earth's magnetic field $B = .5 \times 10^{-4}$ Tesla.

2. In longitudinal degrees of freedom of the flux tube Cooper pairs can be described as particles in a one-dimensional box and the gap is characterized by the length L of the magnetic flux tube and the value of $\hbar$. In longitudinal degrees of freedom the difference between $n = 2$ and $n = 1$ states is given by $E_0(k_2) = 3h^2/4m_eL^2(k_2)$. Translational energy gap $E_g = 3E_0(k_2) = 3h^2/4m_eL^2(k_2)$ is smaller than the effective energy gap $E_0(k_1) - E_0(k_2) = h^2/4m_eL^2(k_1) - h^2/4m_eL^2(k_2)$ for $k_1 > k_2+2$ and identical with it for $k_1 = k_2+2$. For $L(k_2 = 151)$ the zero point kinetic energy is given by $E_0(151) = 20.8$ meV so that $E_{g,BE}$ corresponds roughly to a temperature of 180 K. For magnetic walls the corresponding temperature would be scaled by a factor of two to 360 K and is above room temperature.

3. Second troublesome energy gap relates to the interaction energy with the magnetic field. The magnetic interaction energy E_m of Cooper pair with the magnetic field consists of cyclotron term $E_c = n\hbar eB/m_e$ and spin-interaction term which is present only for spin triplet case and is given by $E_s = \pm\hbar eB/m_e$ depending on the orientation of the net spin with magnetic field. In the magnetic field $B_{end} = 2B_E/5 = .2$ Gauss ($B_E = .5$ Gauss is the nominal value of the Earth's magnetic field) explaining the effects of ELF em fields on vertebrate brain, this energy scale is $\sim 10^{-9}$ eV for $\hbar_0$ and $\sim 1.6 \times 10^{-5}$ eV for $\hbar = 2^{14} \times \hbar_0$.

The smallness of translational and magnetic energy gaps in the case of Cooper pairs at Earth's magnetic field could be seen as a serious obstacle.

1. Thermal isolation between different space-time sheets provides one possible resolution of the problem. The stability of the Bose-Einstein condensation is guaranteed by the thermal isolation of space-time if the temperature at the magnetic flux tube is below E_m. This can be achieved in all length scales if the temperature scales as the zero point kinetic energy in transversal degrees of freedom since it scales in the same manner as magnetic interaction energy.

2. The transition to large $\hbar$ phase could provide a more elegant way out of the difficulty. The criterion for a sequence of transitions to a large $\hbar$ phase could be easily satisfied if there is a large number of charge Cooper pairs at the magnetic flux tube. Kinetic energy gap remains invariant if the length of the flux tube scales as $\hbar$. If the magnetic flux is quantized as a multiple of $\hbar$ and flux tube thickness scales as $\hbar^2$, B must scale as $1/\hbar$ so that also magnetic energy remains invariant under the scaling. This would allow to have stability without assuming low temperature at magnetic flux tubes.

11.3 TGD based model for high T_c super conductors

High T_c superconductors are quantum critical and involve in an essential magnetic structures, they provide an attractive application of the general vision for the model of super-conductivity based on magnetic flux tubes.

11.3.1 Some properties of high T_c super conductors

Quite generally, high T_c super-conductors are cuprates with CuO layers carrying the supra current. The highest known critical temperature for high T_c superconductors is 164 K and is achieved under huge pressure of 3.1×10^5 atm for LaBaCuO. High T_c super-conductors are known to be super conductors of type II.

This is however a theoretical deduction following from the assumption that the value of Planck constant is ordinary. For $\hbar = 2^{14}\hbar_0$ (say) ξ would be scaled up accordingly and type I super-conductor would be in question. These super-conductors are characterized by very complex patterns of penetrating magnetic field near criticality since the surface area of the magnetic defects is maximized. For high T_c super-conductors the ferromagnetic phase could be regarded as an analogous to defect and would indeed have very complex structure. Since quantum criticality would be in question the stripe structure would fluctuate with time too in accordance with 4-D spin glass character.

The mechanism of high T_c super conductivity is still poorly understood [D76, D83] .

1. It is agreed that electronic Cooper pairs are charge carriers. It is widely accepted that electrons are in relative d-wave state rather than in s-wave (see [D36] and the references mentioned in [D76]). Cooper pairs are believed to be in spin triplet state and electrons combine to form $L = 2$ angular momentum state. The usual phonon exchange mechanism does not generate the attractive interaction between the members of the Cooper pair having spin. There is also a considerable evidence for BCS type Cooper pairs and two kinds of Cooper pairs could be present.

2. High T_c super conductors have spin glass like character [D74] . High T_c superconductors have anomalous properties also above T_c suggesting quantum criticality implying fractal scaling of various observable quantities such as resistivity. At high temperatures cuprates are anti-ferromagnets and Mott insulators meaning freezing of the electrons. Superconductivity and conductivity are believed to occur along dynamical stripes which are antiferromagnetic defects.

3. These findings encourage to consider the interpretation in terms of quantum criticality in which some new form of super conductivity which is not based on quasiparticles is involved. This super-conductivity would be assignable with the quantum fluctuations destroying antiferromagnetic order and replacing it with magnetically disordered phase possibly allowing phonon induced super-conductivity.

4. The doping of the super-conductor with electron holes is essential for high T_c superconductivity, and there is a critical doping fraction $p = .14$ at which T_c is highest. The interpretation is that holes make possible for the Cooper pairs to propagate. There is considerable evidence that holes gather on one-dimensional stripes with thickness of order few atom sizes and lengths in the range 1-10 nm [D83] , which are fluctuating in time scale of 10^{-12} seconds. These stripes are also present in non-superconducting state but in this case they do not fluctuate appreciably. The most plausible TGD based interpretation is in terms of fluctuations of magnetic flux tubes allowing for the formation of long connected flux tubes making super-conductivity possible. The fact that the fluctuations would be oscillations analogous to acoustic wave and might explain the BCS type aspects of high T_c super-conductivity.

5. T_c is inversely proportional to the distance L between the stripes. A possible interpretation would be that full super-conductivity requires de-localization of electrons also with respect to stripes so that T_c would be proportional to the hopping probability of electron between neighboring stripes expected to be proportional to $1/L$ [D83] .

From free fermion gas to Fermi liquids to quantum critical systems

The article of Jan Zaanen [D81] gives an excellent non-technical discussion of various features of high T_c super-conductors distinguishing them from BCS super-conductors. After having constructed a color flux tube model of Cooper pairs I found it especially amusing to learn that the analogy of high T_c super-conductivity as a quantum critical phenomenon involving formation of dynamical stripes to QCD in the vicinity of the transition to the confined phase leading to the generation of string like hadronic objects was emphasized also by Zaanen.

BCS super-conductor behaves in a good approximation like quantum gas of non-interacting electrons. This approximation works well for long ranged interactions and the reason is Fermi statistics plus the fact that Fermi energy is much larger than Coulomb interaction energy at atomic length scales.

For strongly interacting fermions the description as Fermi liquid (a notion introduced by Landau) has been dominating phenomenological approach. ^{3}He provides a basic example of Fermi liquid and already here a paradox is encountered since low temperature collective physics is that of Fermi gas without interactions with effective masses of atoms about 6 times heavier than those of real atoms whereas short distance physics is that of a classical fluid at high temperatures meaning a highly correlated collective behavior.

It should be noticed that many-sheeted space-time provides a possible explanation of the paradox. Space-time sheets containing join along boundaries blocks of ^3He atoms behave like gas whereas the ^3He atoms inside these blocks form a liquid. An interesting question is whether the ^3He atoms combine to form larger units with same spin as ^3He atom or whether the increase of effective mass by a factor of order six means that $\hbar$ as a unit of spin is increased by this factor forcing the basic units to consist of Bose-Einstein condensate of 3 Cooper pairs.

High T_c super conductors are neither Fermi gases nor Fermi liquids. Cuprate superconductors correspond at high temperatures to doped Mott insulators for which Coulomb interactions dominate meaning that electrons are localized and frozen. Electron spin can however move and the system can be regarded as an anti-ferromagnet. CuO planes are separated by highly oxidic layers and become super-conducting when doped. The charge transfer between the two kinds of layers is what controls the degree of doping. Doping induces somehow a de-localization of charge carriers accompanied by a local melting of anti-ferromagnet.

Collective behavior emerges for high enough doping. Highest T_c results with 15 per cent doping by holes. Current flows along electron stripes. Stripes themselves are dynamical and this is essential for both conductivity and superconductivity. For completely static stripes super-conductivity disappears and quasi-insulating electron crystal results.

Dynamical stripes appear in mesoscopic time and length scales corresponding to 1-10 nm length scale and picosecond time scale. The stripes are in a well-defined sense dual to the magnetized stripe like structures in type I super-conductor near criticality, which suggests analog of type I super-conductivity. The stripes are anti-ferromagnetic defects at which neighboring spins fail to be antiparallel. It has been found that stripes are a very general phenomenon appearing in insulators, metals, and super-conducting compounds [D80] .

Quantum criticality is present also above T_c

Also the physics of Mott insulators above T_c reflects quantum criticality. Typically scaling laws hold true for observables. In particular, resistivity increases linearly rather than transforming from T^2 behavior to constant as would be implied by quasi-particles as current carriers. The appearance of so called pseudo-gap [D28] at $T_{c1} > T_c$ conforms with this interpretation. In particular, the pseudo-gap is non-vanishing already at T_{c_1} and stays constant rather than starting from zero as for quasi-particles.

Results from optical measurements and neutron scattering

Optical measurements and neutron scattering have provided especially valuable microscopic information about high T_c superconductors allowing to fix the details of TGD based quantitative model.

Optical measurements of copper oxides in non-super-conducting state have demonstrated that optical conductivity $\sigma(\omega)$ is surprisingly featureless as a function of photon frequency. Below the critical temperature there is however a sharp absorption onset at energy of about 50 meV [D46] . The origin of this special feature has been a longstanding puzzle. It has been proposed that this absorption onset corresponds to a direct generation of an electron-hole pair. Momentum conservation implies that the threshold for this process is $E_g + E$, where E is the energy of the 'gluon' which binds electrons of Cooper pair together. In the case of ordinary super-conductivity E would be phonon energy.

Soon after measurements, it was proposed that in absence of lattice excitations photon must generate two electron-hole pairs such that electrons possess opposite momenta [D46] . Hence the energy of the photon would be $2E_g$. Calculations however predicted soft rather than sharp onset of absorption since pairs of electron-hole pairs have continuous energy spectrum. There is something wrong with this picture.

Second peculiar characteristic [D47, D44, D43] of high T_c super conductors is resonant neutron scattering at excitation energy $E_w = 41$ meV of super conductor. This scattering occurs only below the critical temperature, in spin-flip channel and for a favored momentum exchange $(\pi/a, \pi/a))$, where a denotes the size of the lattice cube [D47, D44, D43] . The transferred energy is concentrated in a remarkably narrow range around E_w rather than forming a continuum.

In [D64] is is suggested that e-e resonance with spin one gives rise to this excitation. This resonance is assumed to play the same role as phonon in the ordinary super conductivity and e-e resonance is treated like phonon. It is found that one can understand the dependence of the second derivative of the photon conductivity $\sigma(\omega)$ on frequency and that consistency with neutron scattering data is achieved. The second derivative of $\sigma(\omega)$ peaks near 68 meV and assuming $E = E_g + E_w$ they found nearly perfect match using $E_g = 27$ meV. This would suggest that the energy of the excitations generating the binding between the members of the Cooper pair is indeed 41 meV, that two electron-hole pairs and excitation of the super conductor are generated in photon absorption above threshold, and that the gap energy of the Cooper pair is 27 meV. Of course, the theory of Carbotte *et al* does not force the 'gluon' to be triplet excitation of electron pair. Also other possibilities can be considered. What comes in mind are spin flip waves of the spin lattice associated with stripe behaving as spin 1 waves.

In TGD framework more exotic options become possible. The transversal fluctuations of stripes- or rather of the magnetic flux tubes associated with the stripes- could define spin 1 excitations analogous to the excitations of a string like objects. Gauge bosons are identified as wormhole contacts in quantum TGD and massive gauge boson like state containing electron-positron pair or quark-antiquark pair could be considered.

11.3.2 TGD inspired vision about high T_c superconductivity

The following general view about high T_c super-conductivity as quantum critical phenomenon suggests itself. It must be emphasized that this option is one of the many that one can imagine and distinguished only by the fact that it is the minimal option.

The interpretation of critical temperatures

The two critical temperatures T_c and $T_{c_1} > T_c$ are interpreted as critical temperatures. The recent observation that there exists a spectroscopic signature of high T_c super-conductivity, which prevails up to T_{c_1} [D22] , supports the interpretation that Cooper pairs exist already below T_{c1} but that for some reason they cannot form a coherent super-conducting state.

One can imagine several alternative TGD based models but for the minimal option is the following one.

1. T_{c_1} would be the temperature for the formation of two-phase system consisting of ordinary electrons and of Cooper pairs with a large value of Planck constant explaining the high critical temperature.

2. Magnetic flux tubes are assumed to be carriers of supra currents. These flux tubes are very short in in anti-ferromagnetic phase. The holes form stripes making them positively charged so that they attract electrons. If the spins of holes tend to form parallel sequences along stripes, they generate dipole magnetic fields in scales of order stripe length at least. The corresponding magnetic flux tubes are assumed to be carriers of electrons and Cooper pairs.

The flux tube structures would be closed so that the supra currents associated with these flux tubes would be trapped in closed loops above T_c.

3. Below T_{c1} transversal fluctuations of the flux tubes structures occur and can induce reconnections giving rise to longer flux tubes. Reconnection can occur in two manners. Recall that upwards going outer flux tubes of the dipole field turn downwards and eventually fuse with the dipole core. If the two dipoles have opposite directions the outer flux tube of the first (second) dipole can reconnect with the inward going part of the flux tube of second (first) dipole. If the dipoles have same direction, the outer flux tubes of the dipoles reconnect with each other. Same applies to the inwards going parts of the flux tubes and the dipoles fuse to a single deformed dipole if all flux tubes reconnect. This alternative looks more plausible. The reconnection process is in general only partial since dipole field consists of several flux tubes.

4. The reconnections for the flux tubes of neighboring almost dipole fields occur with some probability $p(T)$ and make possible finite conductivity. At T_c the system the fluctuations of the flux tubes become large and also $p(T,L)$, where L is the distance between stripes, becomes large and the reconnection leads to a formation of long flux tubes of length of order coherence length at least and macroscopic supra currents can flow. One also expects that the reconnection occurs for practically all flux tubes of the dipole field. Essentially a percolation type phenomenon [D15] would be in question. Scaling invariance suggests $p_c(T,L) = p_c(TL/\hbar)$, where L is the distance between stripes, and would predict the observed $T_c \propto \hbar/L$ behavior. Large value of $\hbar$ would explain the high value of T_c.

This model relates in an interesting manner to the vision of Zaanen [D84] expressed in terms of the highway metaphor visualizing stripes as quantum highways along which Cooper pairs can move. In antiferromagnetic phase the traffic is completely jammed. The doping inducing electron holes allows to circumvent traffic jam due to the Fermi statistics generates stripes along which the traffic flows in the sense of ordinary conductivity. In TGD framework highways are replaced with flux tubes and the topology of the network of highways fluctuates due to the possibility of reconnections. At quantum criticality the reconnections create long flux tubes making possible the flow of supra currents.

The interpretation of fluctuating stripes in terms of 1-D phonons

In TGD framework the phase transition to high T_c super-conductivity would have as a correlate fluctuating stripes to which supra currents are assigned. Note that the fluctuations occur also for $T > T_c$ but their amplitude is smaller. Stripes would be parallel to the dark magnetic flux tubes along which dark electron current flows above T_c. The fluctuations of magnetic flux tubes whose amplitude increases as T_c is approached induce transverse oscillations of the atoms of stripes representing 1-D transverse phonons.

The transverse fluctuations of stripes have naturally spin one character in accordance with the experimental facts. They allow identification as the excitations having 41 meV energy and would propagate in the preferred diagonal direction $(\pi/a, \pi/a)$. Dark Cooper pairs would have a gap energy of 27 meV. Neutron scattering resonance could be understood as a generation of these 1-D phonons and photon absorption a creation of this kind of phonon and breaking of dark Cooper pair. The transverse oscillations could give rise to the gap energy of the Cooper pair below T_{c1} and for the formation of long flux tubes below T_c but one can consider also other mechanisms based on the new physics predicted by TGD.

Various lattice effects such as superconductivity-induced phonon shifts and broadenings, possible isotope effects in T_c (questionable), the penetration depth, infrared and photoemission spectra have been observed in the cuprates [D8]. A possible interpretation is that ordinary phonons are replaced by 1-D phonons defined by the transversal excitations of stripes but do not give rise to the binding of the electrons of the Cooper pair but to to reconnection of flux tubes. An alternative proposal which seems to gain experimental support is that spin waves appearing near antiferromagnetic phase transitions replace phonons.

More precise view about high T_c superconductivity taking into account recent experimental results

There are more recent results allowing to formulate more precisely the idea about transition to high T_c super-conductivity as a percolation type phenomenon. Let us first summarize the recent picture about high T_c superconductors.

1. 2-dimensional phenomenon is in question. Supra current flows along preferred lattice planes and type II super-conductivity in question. Proper sizes of Cooper pairs (coherence lengths) are ξ =1-3 nm. Magnetic length λ is longer than $\xi/\sqrt{2}$.

2. Mechanism for the formation of Cooper pairs is the same water bed effect as in the case of ordinary superconductivity. Phonons are only replaced with spin-density waves for electrons with periodicity in general not that of the underlying lattice. Spin density waves relate closely to the underlying antiferromagnetic order. Spin density waves appear near phase transition to antiferromagnetism.

3. The relative orbital angular mentum of Cooper pair is L=2 ($x^2 - y^2$ wave), and vanishes at origin unlike for ordinary s wave SCs. The spin of the Cooper pair vanishes.

Consider now the translation of this picture to TGD language. Basic notions are following.

1. Magnetic flux tubes and possibly also dark electrons forming Cooper pairs.

2. The appearence of spin waves means sequences of electrons with opposite spins. The magnetic field associated with them can form closed flux tube containing both spins. Assume that spins-are orthogonal to the lattice plane in which supracurrent flows. Assume that the flux tube branches associated with electron with given spin branches so that it is shared with both neighboring electrons.

3. Electrons of opposite spins at the two portions of the closed flux tube have magnetic interaction energy. The total energy is minimal when the spins are in opposite directions. Thus the closed flux tube tends to favor formation of Cooper pairs.

4. Since magnetic interaction energy is proportional to $h_{eff} = n \times h$, it is expected stabilize the Cooper pairs at high temperatures. For ordinary super-conducti- vity magnetic fields tends to de-stabilize the pairs by trying to force the spins of spin singlet pair to the same direction.

5. This does not yet give super-conductivity. The closed flux tubes associated with paired spins can however reconnect so that longer flux closed flux tubes are formed. If this occurs for entire sequences, one obtains two flux tubes containing electrons with opposite spins forming Cooper pairs: this would be the "highway" and percolation would corresponds to this process. The pairs would form supracurrents in longer scales.

6. The phase phase transitions generating the reconnections could be percolation type phase transition.

This picture might apply also in TGD based model of bio-superconductivity.

1. The stability of dark Cooper pairs assume to reside at magnetic flux tubes is a problem also now. Fermi statistics favors opposite spins but this means that magnetic field tends to spit the pairs if the members of the pair are at the same flux tube.

2. If the members of the pair are at different flux tubes, the situation changes. One can have $L = 1$ and $S = 1$ with parallel spins (ferromagnetism like situation) or $L = 2$ and $S = 0$ state (anti-ferromagnetism like situation). $L > 0$ is necessary since electrons must reside at separate flux tubes.

Explanation for the spectral signatures of high T_c superconductor

The model should explain various spectral signatures of high T_c super-conductors. It seems that this is possible at qualitative level at least.

1. Below the critical temperature there is a sharp absorption onset at energy of about $E_a = 50$ meV.

2. Second characteristic [D47, D44, D43] of high T_c super conductors is resonant neutron scattering at excitation energy $E_w = 41$ meV of super conductor also visible only below the critical temperature.

3. The second derivative of $\sigma(\omega)$ peaks near 68 meV and assuming $E = E_g + E_w$ they found nearly perfect match using $E_g = 27$ meV for the energy gap.

$E_g = 27$ meV has a natural interpretation as energy gap of spin 1 Cooper pair. $E_w = 41$ meV can be assigned to the transversal oscillations of magnetic flux tubes inducing 1-D transversal photons which possibly give rise to the energy gap. $E_a = 50$ meV can be understood if also $S = 0$ Cooper pair for which electrons of the pair reside dominantly at the "outer" dipole flux tube and inner dipole core. The presence of this pair might explain the BCS type aspects of high T_c super-conductivity. This identification would predict the gap energy of $S = 0$ Cooper pair to be $E_g(S = 0) = 9$ meV. Since the critical absorption onset is observed only below T_c these Cooper pairs would become thermally stable at T_c and the formation of long flux tubes should somehow stabilize them. For very long flux tubes the distance of a point of "outer" flux tube from the nearby point "inner" flux tube becomes very long along dipole flux tube. Hence the transformation of $S = 0$ pairs to $S = 1$ pairs is not possible anymore and $S = 0$ pairs are stabilized.

Model for Cooper pairs

The TGD inspired model for Cooper pairs of high T_c super-conductor involves several new physics aspects: large $\hbar$ phases, the notion of magnetic flux tubes. One can also consider the possibility that color force predicted by TGD to be present in all length scales is present.

1. One can consider two options for the topological quantization of the dipole field. It could decompose to a flux tube pattern with a discrete rotational symmetry Z_n around dipole axis or to flux sheets identified as walls of finite thickness invariant under rotations around dipole axis. Besides this there is also inner the flux tube corresponding to the dipole core. For the flux sheet option one can speak about eigenstates of L_z. For flux tube option the representations of Z_n define the counterparts of the angular momentum eigenstates with a cutoff in L_z analogous to a momentum cutoff in lattice. The discretized counterparts of spherical harmonics make sense. The counterparts of the relative angular momentum eigenstates for Cooper pair must be defined in terms of tensor products of these rather than using spherical harmonics assignable with the relative coordinate $r_1 - r_2$. The reconnection mechanism makes sense only for the flux tube option so that it is the only possibility in the recent context.

2. Exotic Cooper pair is modeled as a pair of large $\hbar$ electrons with zoomed up size at space-time representing the dipole field pattern associated with a sequence of holes with same spin. If the members of the pair are at diametrically opposite flux tubes or at the "inner" flux tube (dipole core) magnetic fluxes flow in same direction for electrons and spin 1 Cooper pair is favored. If they reside at the "inner" flux tube and outer flux tube, spin zero state is favored. This raises the question whether also $S = 0$ variant of the Cooper pair could be present.

3. Large $\hbar$ is needed to explain high critical temperature. By the general argument the transition to large $\hbar$ phase occurs in order to reduce the value of the gauge coupling strength - now fine structure constant- and thus guarantee the convergence of the perturbation theory. The generation or positive net charge along stripes indeed means strong electromagnetic interactions at stripe.

Color force in condensed matter length scales is a new physics aspect which cannot be excluded in the case that transverse oscillations of flux tubes do not bind the electrons to form a Cooper pair. Classically color forces accompany any non-vacuum extremal of Kähler action since a non-vanishing induced Kähler field is accompanied by a classical color gauge field with Abelian holonomy. Induced Kähler field is always non-vanishing when the dimension of the CP_2 projection of the space-time surface is higher than 2. One can imagine too alternative scenarios.

1. Electromagnetic flux tubes for which induced Kähler field is non-vanishing carry also classical color fields. Cooper pairs could be color singlet bound states of color octet excitations of electrons (more generally leptons) predicted by TGD and explaining quite impressive number of anomalies [K88] . These states are necessarily dark since the decay widths of gauge bosons do not allow new light fermions coupling to them. The size of these states is of order electron size scale $L(127)$ for the standard value of Planck constant. For the non-standard value of Planck constant it would be scaled up correspondingly. For $r = \hbar/\hbar_0 = 2^{14}$ the size would be around 3.3 Angströms and for $r = 2^{24}$ of order 10 nm. Color binding could be responsible for the formation of the energy gap in this case and would distinguish between ordinary two-electron states and Cooper pair. The state with minimum color magnetic energy corresponds to spin triplet state for two color octed fermions whereas for colored fermion and anti-fermion it corresponds to spin singlet (pion like state in hadron physics).

2. A more complex variant of this picture served as the original model for Cooper pairs. Electrons at given space-time sheet feed their gauge flux to large space-time sheet via wormhole contacts. If the wormhole throats carry quantum numbers of quark and antiquark one can say that in the simplest situation the electron space-time sheet is color singlet state formed by quark and antiquark associated with the upper throats of the wormhole contacts carrying quantum numbers of u quark and $\overline{d}$ quark. It can also happen that the electronic space-time sheets are not color singlet but color octet in which case the situation is analogous to that above. Color force would bind the two electronic space-time sheets to form a Cooper pair. The neighboring electrons in stripe possess parallel spins and could form a pair transforming to a large $\hbar$ Cooper pair bound by color force. The Coulombic binding energy of the charged particles with the quarks and antiquarks assignable to the two wormhole throats feeding the em gauge flux to Y^4 and color interaction would be responsible for the energy gap.

Estimate for the gap energy

If transverse oscillations are responsible for the binding of the Cooper pairs, one expects similar expression for the gap energy as in the case of BCS type super conductors. The 3-D formula for the gap energy reads as

$$\begin{aligned}
E_g &= \hbar\omega_D exp(-1/X) \ , \\
\omega_D &= (6\pi^2)^{1/3} c_s n^{1/3} \\
X &= n(E_F)U_0 = \frac{3}{2}N(E_F)\frac{U_0}{E_F} \ , \\
n(E_F) &= \frac{3}{2}\frac{N(E_F)}{E_F} \ .
\end{aligned} \tag{11.3.1}$$

X depends on the details of the binding mechanism for Cooper pairs and U_0 parameterizes these details.

Since only stripes contribute to high T_c super-conductivity it is natural to replace 3-dimensional formula for Debye frequency in 1-dimensional case with

$$\begin{aligned}
E_g &= \hbar\omega exp(-1/X) \ , \\
\omega &= kc_s n \ .
\end{aligned} \tag{11.3.2}$$

where n is the 1-dimensional density of Cooper pairs and k a numerical constant. X would now correspond to the binding dynamics at the surface of 1-D counterpart of Fermi sphere associated with the stripe.

There is objection against this formula. The large number of holes for stripes suggests that the counterpart of Fermi sphere need not make sense, and one can wonder whether it could be more advantageous to talk about the counterpart of Fermi sphere for holes and treat Cooper pair as a pair of vacancies for this "Fermi sphere". High T_c super conductivity would be 1-D conventional super-conductivity for bound states of vacancies. This would require the replacement of n with the linear density of holes along stripes, which is essentially that of nuclei.

From the known data one can make a rough estimate for the parameter X. If $E_w = hf = 41$ meV is assigned with transverse oscillations the standard value of Planck constant would give $f = f_0 = 9.8 \times 10^{12}$ Hz. In the general case one has $f = f_0/r$. If one takes the 10^{-12} second length scale of the transversal fluctuations at a face value one obtains $r = 10$ as a first guess. $E_g = 27$ meV gives the estimate

$$exp(-1/X) = \frac{E_g}{E_w} \quad (11.3.3)$$

giving $X = 2.39$.

The interpretation in terms of transversal oscillations suggests the dispersion relation

$$f = \frac{c_s}{L} .$$

L is the length of the approximately straight portion of the flux tube. The length of the "outer" flux tube of the dipole field is expected be longer than that of stripe. For $L = x$ nm and $f_D \sim 10^{12}$ Hz one would obtain $c_s = 10^3 x$ m/s.

Estimate for the critical temperatures and for $\hbar$

One can obtain a rough estimate for the critical temperature T_{c1} by following simple argument.

1. The formula for the critical temperature proposed in the previous section generalize in 1-dimensional case to the following formula

$$T_{c1} \leq \frac{h^2}{8m_e}(\frac{n_c}{g})^2 . \quad (11.3.4)$$

   g is the number of spin degrees of freedom for Cooper pair and n_c the 1-D density of Cooper pairs. The effective one-dimensionality allows only single $L = 2$ state localized along the stripe.The $g = 3$ holds true for $S = 1$.

2. By parameterizing n_c as $n_c = (1 - p_h)/a$, $a = x$ Angstrom, and substituting the values of various parameters, one obtains

$$T_{c1} \simeq \frac{r^2(1-p_h)^2}{9x^2} \times 6.3\ meV . \quad (11.3.5)$$

3. An estimate for p_h follows from the doping fraction p_d and the fraction p_s of parallel atomic rows giving rise to stripes one can deduce the fraction of holes for a given stripe as

$$p_h = \frac{p_d}{p_s} . \quad (11.3.6)$$

One must of course have $p_d \leq p_s$. For instance, for $p_s = 1/5$ and $p_d = 15$ per cent one obtains $p_h = 75$ per cent so that a length of four atomic units along row contains one Cooper pair on the average. For $T_{c1} = 23$ meV (230 K) this would give the rough estimate $r = 23.3$: $r = 24$ satisfies the Fermat polygon constraint. Contrary to the first guess inspired by the model of bio-superconductivity the value of $\hbar$ would not be very much higher than its standard value. Notice however that the proportionality $T_c \propto r^2$ makes it difficult to explain T_{c1} using the standard value of $\hbar$.

4. One $p_h \propto 1/L$ whereas scale invariance for reconnection probability ($p = p(x = TL/\hbar)$) predicts $T_c = x_c\hbar/L = x_c p_s \hbar/a$. This implies

$$\frac{T_c}{T_{c1}} = 32\pi^2 \frac{m_e a}{\hbar_0} x^2 g^2 \frac{p_s}{(1-(p_d/p_s)^2)^2} \frac{x_c}{r} \ . \tag{11.3.7}$$

   This prediction allows to test the proposed admittedly somewhat ad hoc formula. For $p_d << p_s$ T_c/T_{c1} does behaves as $1/L$. One can deduce the value of x_c from the empirical data.

5. Note that if the reconnection probability p is a universal function of x as quantum criticality suggests and thus also x_c is universal, a rather modest increase of $\hbar$ could allow to raise T_c to room temperature range.

The value of $\hbar$ is predicted to be inversely proportional to the density of the Cooper pairs at the flux tube. The large value of $\hbar$ needed in the modelling of living system as magnetic flux tube super-conductor could be interpreted in terms of phase transitions which scale up both the length of flux tubes and the distance between the Cooper pairs so that the ratio rn_c remains unchanged.

Coherence lengths

The coherence length for high T_c super conductors is reported to be 5-20 Angstroms. The naive interpretation would be as the size of Cooper pair. There is however a loophole involved. The estimate for coherence length in terms of gap energy is given by $\xi = \frac{4\hbar v_F}{E_g}$. If the coherence length is estimated from the gap energy, as it seems to be the case, then the scaling up of the Planck constant would increase coherence length by a factor $r = \hbar/\hbar_0$. $r = 24$ would give coherence lengths in the range $12 - 48$ nm.

The interpretation of the coherence length would be in terms of the length of the connected flux tube structure associated with the row of holes with the same spin direction which can be considerably longer than the row itself. As a matter fact r would characterize the ratio of size scales of the "magnetic body" of the row and of row itself. The coherence lengths could relate to the p-adic length scales $L(k)$ in the range $k = 151, 152, .., 155$ varying in the range $(10, 40])$ nm. $k = 151$ correspond to thickness cell membrane.

Why copper and what about other elements?

The properties of copper are somehow crucial for high T_c superconductivity since cuprates are the only known high T_c superconductors. Copper corresponds to $3d^{10}4s$ ground state configuration with one valence electron. This encourages the question whether the doping by holes needed to achieve superconductivity induces the phase transition transforming the electrons to dark Cooper pairs.

More generally, elements having one electron in s state plus full electronic shells are good candidates for doped high T_c superconductors. If the atom in question is also a boson the formation of atomic Bose-Einstein condensates at Cooper pair space-time sheets is favored. Superfluid would be in question. Thus elements with odd value of A and Z possessing full shells plus single s wave valence electron are of special interest. The six stable elements satisfying these conditions are ^{5}Li, ^{39}K, ^{63}Cu, ^{85}Rb, ^{133}Cs, and ^{197}Au.

A new phase of matter in the temperature range between pseudo gap temperature and T_c?

Kram sent a link to a Science Daily popular article titled " High-Temperature Superconductor Spills Secret: A New Phase of Matter?" (see alsothis). For more details see the article in Science [D50] .

Zhi-Xun Shen of the Stanford Institute for Materials and Energy Science (SIMES), a joint institute of the Department of Energy's SLAC National Accelerator Laboratory and Stanford University, led the team of researchers, which discovered that in the temperature region between the pseudo gap temperature and genuine temperature for the transition to super-conducting phase there exists a new phase of matter. The new phase would not be super-conducting but would be characterized by an order of its own which remains to be understood. This phase would be present also in the super-conducting phase.

The announcement does not come as a complete surprise for me. A new phase of matter is what TGD inspired model of high T_c superconductivity indeed predicts. This phase would consist of Cooper pairs of electrons with a large value of Planck constant but associated with magnetic flux tubes with short length so that no macroscopic supra currents would be possible.

The transition to super-conducting phase involves long range fluctuations at quantum criticality and the analog of a phenomenon known as percolation [D15] . For instance, the phenomenon occurs for the filtering of fluids through porous materials. At critical threshold the entire filter suddenly wets as fluid gets through the filter. Now this phenomenon would occur for magnetic flux tubes carrying the Cooper pairs. At criticality the short magnetic flux tubes fuse by reconnection to form long ones so that supra currents in macroscopic scales become possible.

It is not clear whether this prediction is consistent with the finding of Shen and others. The simultaneous presence of short and long flux tubes in macroscopically super-conducting phase is certainly consistent with TGD prediction. The situation depends on what one means with super-conductivity. Is super-conductivity super-conductivity in macroscopic scales only or should one call also short scale super-conductivity not giving rise to macroscopic super currents as super-conductivity. In other words: do the findings of Shen's team prove that the electrons above gap temperature do not form Cooper pairs or only that there are no macroscopic supra currents?

Whether the model works as such or not is not a life and death question for the TGD based model. One can quite well imagine that the first phase transition increasing $\hbar$ does not yet produce electron Compton lengths long enough to guarantee that the overlap criterion for the formation of Cooper pairs is satisfied. The second phase transition increasing $\hbar$ would do this and also scale up the lengths of magnetic flux tubes making possible the flow of supra currents as such even without reconnections. Also reconnections making possible the formation of very long flux tubes could be involved and would be made possible by the increase in the length of flux tubes.

11.3.3 Speculations

21-Micrometer mystery

21 micrometer radiation from certain red giant stars have perplexed astronomers for more than a decade [D58] . Emission forms a wide band (with width about 4 micrometers) in the infrared spectrum, which suggests that it comes form a large complex molecule or a solid or simple molecules found around starts. Small molecules are ruled out since they produce narrow emission lines. The feature can be only observed in very precise evolutionary state, in the transition between red giant phase and planetary nebular state, in which star blows off dust that is rich in carbon compounds. There is no generally accepted explanation for 21-micrometer radiation.

One can consider several explanations based on p-adic length scale hypothesis and some explanations might relate to the wormhole based super-conductivity.

1. 21 micrometers corresponds to the photon energy of 59 meV which is quite near to the zero point point kinetic energy 61.5 meV of proton Cooper pair at $k = 139$ space-time sheet estimated from the formula

$$\Delta E(2m_p, 139) = \frac{1}{2}\frac{\pi^2}{(2m_p)L_e(139)^2} = \frac{1}{8}\Delta E(m_p, 137) \simeq 61.5 \ meV \ .$$

Here the binding energy of the Cooper pair tending to reduce this estimate is neglected, and this estimate makes sense only apart from a numerical factor of order unity. This energy is liberated when a Cooper pair of protons at $k = 139$ space-time sheet drops to the magnetic flux tube of Earth's magnetic field (or some other sufficiently large space-time sheet). This energy is rather near to the threshold value about 55 meV of the membrane potential.

2. 21 micrometer radiation could also result when electrons at $k = 151$ space-time sheet drop to a large enough space-time sheet and liberate their zero point kinetic energy. Scaling argument gives for the zero point kinetic energy of electron at $k = 151$ space-time sheet the value $\Delta(e, 151) \simeq 57.5$ meV which is also quite near to the observed value. If electron is bound to wormhole with quantum numbers of $\overline{d}$ Coulombic binding energy changes the situation.

3. A possible explanation is as a radiation associated with the transition to high T_c super conducting phase. There are two sources of photons. Radiation could perhaps result from the de-excitations of wormhole BE condensate by photon emission. $\lambda = 20.5$ micrometers is precisely what one expects if the space-time sheet corresponds to $p \simeq 2^k$, $k = 173$ and assumes that excitation energies are given as multiples of $E_w(k) = 2\pi/L_e(k)$. This predicts excitation energy $E_w(173) \simeq 61.5$ meV. Unfortunately, this radiation should correspond to a sharp emission line and cannot explain the wide spectrum.

Are living systems high T_c superconductors?

The idea about cells and axons as superconductors has been one of the main driving forces in development of the vision about many-sheeted space-time. Despite this the realization that the supra currents in high T_c superconductors flow along structure similar to axon and having same crucial length scales came as a surprise. Axonal radius which is typically of order $r = .5$ μm. $r = 151 - 127 = 24$ favored by Mersenne hypothesis would predict $r = .4$ μm. The fact that water is liquid could explain why the radius differs from that predicted in case of high T_c superconductors.

Interestingly, Cu is one of the biologically most important trace elements [D2] . For instance, copper is found in a variety of enzymes, including the copper centers of cytochrome c-oxidase, the Cu-Zn containing enzyme superoxide dismutase, and copper is the central metal in the oxygen carrying pigment hemocyanin. The blood of the horseshoe crab, Limulus polyphemus uses copper rather than iron for oxygen transport. Hence there are excellent reasons to ask whether living matter might be able to build high T_c superconductors based on copper oxide.

Neuronal axon as a geometric model for current carrying "rivers"

Neuronal axons, which are bounded by cell membranes of thickness $L_e(151)$ consisting of two lipid layers of thickness $L_e(149)$ are good candidates for high T_c superconductors in living matter.

These flux tubes with radius .4 μm would define "rivers" along which conduction electrons and various kinds of Cooper pairs flow. Scaled up electrons have size $L_e(k_{eff} = 151)$ corresponding to 10 nm, the thickness of the lipid layer of cell membrane. Also the quantum fluctuating stripes of length 1-10 nm observed in high T_c super conductors might relate to the scaled up electrons with Compton length 10 nm, perhaps actually representing zoomed up electrons!

The original assumption that exotic *resp.* BCS type Cooper pairs reside at boundaries *resp.* interior of the super-conducting rivulet. It would however seem that the most natural option is that the hollow cylindrical shells carry all supra currents and there are no Cooper pairs in the interior. If exotic Cooper pairs reside only at the boundary of the rivulet or the Cooper pairs at boundary remain critical against exotic-BCS transition also below T_c, the time dependent fluctuations of the shapes of stripes accompanying high T_c super-conductivity can be understood as being induced by the fluctuations of membrane like structures. Quantum criticality at some part of the boundary is necessary in order to transform ordinary electron currents to super currents at the ends of rivulets. In biology this quantum criticality would correspond to that of cell membrane.

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Chapter 12

Quantum Hall effect and Hierarchy of Planck Constants

12.1 Introduction

Quantum Hall effect [D71, D69, D72] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of rational filling fractions of form $\nu = k/m$ was observed.

The observed fractions are not arbitrary but obey quite precise selection rules. This led to the notion of composite fermion as a bound state of electron and magnetic vortices carrying electron at their boundaries and FQHE reduces to QHE for these effective particles in an effective magnetic field than the original field and reduced by the binding of the magnetic vortices with the electron. Besides quasiparticles also corresponding holes contribute to FQHE.

What I see as a general problem of the composite model is the nature of the bound states. How both the number of vortices per electron and the number of electrons per vortex can be larger than one?

The composite model fails to explain only the observations for $\nu = 5/2$ and $\nu = 7/2$ (these values of ν belong to the spectrum but the these phases do not behave as predicted). The conjecture is that in these cases electron carrying magnetic vortices form Cooper pair like bound states. Non-Abelian braid statistics assigned with these phases would be essential for topological quantum computation but it has not been established convincingly yet [D53].

Also the well-established charge fractionization should be understood. Whether fractional braid statistics is realized is still an open question. In these issues TGD might provide some new insights.

The phenomenology of FQHE is described in an extremely lucid manner in the Nobel lecture of Horst L. Stormer [D79]. As a matter fact, I regret that I did not read it for years ago!

12.1.1 Abelian and non-Abelian anyons

The model explaining FQHE is based on pseudo particles known as anyons identifiable as magnetic vortices [A36] , [D71]. According to the general argument of [D56] anyons have a fractional charge νe. The braid statistics of anyon is believed to be fractional so that in the general case anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [A36] each of them defining an incompressible hydrodynamical flow. According to [B9] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = e/4$ rather than being $Q = \nu e$ [D48] . This finding favors non-Abelian models [D72] .

Non-Abelian anyons [D71, D73] are always created in pairs since they carry a conserved topological charge. In the model of [B9] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be shown to possess 2^{n-1}-dimensional vacuum degeneracy [D75]. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

12.1.2 TGD based view about FQHE

In this chapter I try to formulate more precisely the recent TGD based view about fractional quantum Hall effect (FQHE). This view is much more realistic than the original rough scenario, which neglected the existing rather detailed understanding. The spectrum of ν, and the mechanism producing it is the same as in composite fermion approach. The new elements relate to the not so well-understood aspects of FQHE, namely charge fractionization, the emergence of braid statistics, and non-abelianity of braid statistics.

1. The starting point is composite fermion model so that the basic predictions are same. Now magnetic vortices correspond to (Kähler) magnetic flux tubes carrying unit of magnetic flux. The magnetic field inside flux tube would be created by de-localized electron at the boundary of the vortex. One can raise two questions.

   Could the boundary of the macroscopic system carrying anyonic phase have identification as a macroscopic analog of partonic 2-surface serving as a boundary between Minkowskian and Euclidian regions of space-time sheet? If so, the space-time sheet assignable to the macroscopic system in question would have Euclidian signature, and would be analogous to blackhole or to a line of generalized Feynman diagram.

   Could the boundary of the vortex be identifiable a light-like boundary separating Minkowskian magnetic flux tube from the Euclidian interior of the macroscopic system and be also analogous to wormhole throat? If so, both macroscopic objects and magnetic vortices would be rather exotic geometric objects not possible in general relativity framework.

2. Taking composite model as a starting point one obtains standard predictions for the filling fractions. One should also understand charge fractionalization and fractional braiding statistics. Here the vacuum degeneracy of Kähler action suggests the explanation. Vacuum degeneracy implies that the correspondence between the normal component of the canonical momentum current and normal derivatives of imbedding space coordinates is 1- to-n. These kind of branchings result in multi-furcations induced by variations of the system parameters and the scaling of external magnetic field represents one such variation.

3. At the orbits of wormhole throats, which can have even macroscopic M^4 projections, one has $1 \rightarrow n_a$ correspondence and at the space-like ends of the space-time surface at light-like boundaries of causal diamond one has $1 \rightarrow n_b$ correspondence. This implies that at partonic 2-surfaces defined as the intersections of these two kinds of 3-surfaces one has $1 \rightarrow$

$n_a \times n_b$ correspondence. This correspondence can be described by using a local singular n-fold covering of the imbedding space. Unlike in the original approach, the covering space is only a convenient auxiliary tool rather than fundamental notion.

4. The fractionalization of charge can be understood as follows. A de-localization of electron charge to the n sheets of the multi-furcation takes place and single sheet is analogous to a sheet of Riemann surface of function $z^{1/n}$ and carries fractional charge $q = e/n$, $n = n_a n_b$. Fractionalization applies also to other quantum numbers. One can have also many-electron stats of these states with several de-localized electrons: in this case one obtains more general charge fractionalization: $q = \nu e$.

5. Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the braid rotation by $\Delta\Phi = 2\pi$, where Φ corresponds to M^4 angle, leads to a second branch of multi-furcation and one can give up the usual quantization condition for angular momentum. For the natural angle coordinate Φ of the n-branched covering $\Delta\Phi = 2/pi$ corresponds to $\Delta\Phi = n \times 2\pi$. If one identifies the sheets of multi-furcation and therefore uses Φ as angle coordinate, single valued angular momentum eigenstates become in general n-valued, angular momentum in braid statistics becomes fractional and one obtains fractional braid statistics for angular momentum.

6. How to understand the exceptional values $\nu = 5/2, 7/2$ of the filling fraction? The non-abelian braid group representations [D73] can be interpreted as higher-dimensional projective representations of permutation group: for ordinary statistics only Abelian representations are possible. It seems that the minimum number of braids is $n > 2$ from the condition of non-abelianity of braid group representations. The condition that ordinary statistics is fermionic, gives $n > 3$. The minimum value is $n = 4$ consistent with the fractional charge $e/4$.

   The model introduces Z_4 valued topological quantum number characterizing flux tubes. This also makes possible non-Abelian braid statistics. The interpretation of this quantum number as a Z_4 valued momentum characterizing the four de-localized states of the flux tube at the sheets of the 4-furcation suggests itself strongly. Topology would corresponds to that of 4-fold covering space of imbedding space serving as a convenient auxiliary tool. The more standard explanation is that $Z_4 = Z_2 \times Z_2$ such that Z_2:s correspond to the presence or absence of neutral Majorana fermion in the two Cooper pair like states formed by flux tubes.

   What remains to be understood is the emergence of non-abelian gauge group realizing non-Abelian fractional statistics in gauge theory framework. TGD predicts the possibility of dynamical gauge groups [K103] and maybe this kind of gauge group indeed emerges. Dynamical gauge groups emerge also for stacks of N branes and the n sheets of multi-furcation are analogous to the N sheets in the stack for many-electron states.

The genuinely new element to the existing theory of FQHE are multi-furcations of partonic 2-surfaces and their second quantization. This notion leads to an explanation of the fractional charges, fractional braid statistics, and existence of Z_n valued topological quantum number in terms of many-sheeted space-time and mulfi-furcations of preferred extremals of Kähler action. One ends up also to a concrete geometric realization for the bound states of electron and flux quanta and geometric understanding of how n flux quanta "split" out from the magnetic field experienced by the electron. The rather radical "almost prediction" is that partonic 2-surfaces and their light-like orbits serving as boundaries between Euclidian and Minkowskian regions of space-time sheet would be realized even in macroscopic scales. Anyonic system would be in well-defined sense an elementary particle like object.

The first two sections of the chapter give brief summaries about FQHE and existing theories of FQHE. The third section represents a view about the effective hierarchy of Planck constants assignable to multi-furcations associated with Kähler action and the recent simplifications of this picture. The last section summarizes the TGD inspired model of FQHE, a model for flux tubes, a microscopic description for the 2-D surface representing the boundary of the anyonic system and with electrons attached to this surface. Here the TGD based view about elementary particles is in active role.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hierarchy of Planck constants [L27]
- Quantum Hall effect and TGD [L41]

12.2 Fractional Quantum Hall effect

12.2.1 Basic facts about FQHE

12.2.2 A simple model for fractional quantum Hall effect

Recall first the basic facts. Quantum Hall effect (QHE) [D71, D6, D69] is an essentially 2-dimensional phenomenon and occurs at the end of current carrying region for the current flowing transversally along the end of the wire in external magnetic field along the wire. For quantum Hall effect transversal Hall conductance characterizing the 2-dimensional current flow is dimensionless and quantized and given by

$$\sigma_{xy} = 2\nu\alpha_{em} \ ,$$

ν is so called filling factor telling the number of filled Landau levels in the magnetic field. In the case of integer quantum Hall effect (IQHE) ν is integer valued. For fractional quantum Hall effect (FQHE) ν is rational number.

The formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{12.2.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15...$, $2/3, 3/5, 4/7, 5/9, 6/11, 7/13...$, $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9...$, $1/5, 2/9, 3/13...$, $2/7, 3/11...$, $1/7....$ with odd denominator have been observed [D6]. Only fractions smaller than 1 are listed because the integer part of ν should not matter since it represents full Landau levels. Also $\nu = 1/2$, $\nu = 5/2$, $7/2$ states with even denominator have been observed. $\nu = 1/2$ can be understood easily in the existing theory. One might think that $\nu = 5/2 = 2 + 1/2$ and $\nu = 7/2 = 3 + 1/2$ would reduce to $\nu = 1/2$. This not however the case experimentally and these values of ν represent an unsolved problem of anyon physics.

The following gives a brief summary about the evolution of the understanding of FQHE.

1. Laughlin introduced his many-electron wave wave function predicting fractional quantum Hall effect for filling fractions $\nu = 1/m$ [D69] . The model of Laughlin [D69] cannot explain all observed filling fractions.

2. The best existing model proposed originally by Jain [D65] is based on the notion of composite fermion. These would result as bound states of electron and even number of magnetic flux quanta [D65]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

3. The description of the magnetic flux tubes led to the notion of anyon introduced by Wilzeck [D71] . Anyon has been compared to a vortex like excitation of a dense 2-D electron plasma formed by the current carriers. ν is inversely proportional to the magnetic flux and the fractional filling factor can be also understood in terms of fractional magnetic flux.

4. The starting point of the quantum field theoretical models is the effective 2-dimensionality of the system implying that the projective representations for the permutation group of n objects are representations of braid group allowing fractional statistics. This is due to the non-trivial first homotopy group of 2-dimensional manifold containing punctures. Quantum field theoretical models allow to assign to the anyon-like states also magnetic charge, fractional spin, and fractional electric charge.

Topological quantum computation [K94, K31] , [B9] , [C106] is one of the most fascinating applications of FQHE. It relies on the notion of braids with strands representing the orbits of of anyons. The unitary time evolution operator coding for topological computation is a representation of the element of the element of braid group represented by the time evolution of the braid. It is essential that the group involved is non-Abelian so that the system remembers the order of elementary braiding operations (exchange of neighboring strands). There is experimental evidence that $\nu = 5/2$ anyons possessing fractional charge $Q = e/4$ are non-Abelian [D72, D48] .

Before continuing, it is good to represent both classical view about QHE effect and simple quantum explanation for IQHE effect.

1. Consider first the classical explanation. Electrons are assumed to drift in the orthogonal electric and magnetic fields with drift velocity $v = E \times B/B^2$ having magnitude $v = E/B$ It is easy to see that this solves Newtons equation of motion identically. Here the 2-D current transversal Hall current can be written as $j = e\rho v$, where ρ is 2-D electron density obtained by averaging in the direction of the electric field. This can be expressed as $j = e(N/S)(E/B)$, where one concludes that the Hall conductivity is given by

$$\sigma_{xy} = \frac{\rho}{B} = e^2\frac{N}{\Phi} \ , \ \Phi = eBS = e\int BdS \ .$$

Using elementary flux quantum as a unit of magnetic flux, this says that Hall conductivity equals to the ratio of electrons per elementary flux quantum. To proceed further one must use quantization of electron's states in the magnetic field to concluded that N equals to integer multiple of $h\Phi$.

2. Consider next a quantum explanation. Choose the coordinates of the current currying slab so that x varies in the direction of Hall current and y in the direction of the main current. For IQHE the value of Hall conductivity is given by $\sigma = j_y/E_x = n_eev/vB = n_ee/B = Ne^2/heBS = Ne^2/mh$, were m characterizes the value of magnetized flux and N is the total number of electrons in the current. In the Landau gauge $A_y = xB$ one can assume that energy eigenstates are momentum eigenstates in the direction of current and harmonic oscillator Gaussians in x-direction in which Hall current runs. This gives

$$\Psi \propto exp(iky)H_n(x+kl^2)exp(-\tfrac{(x+kl^2)^2}{2l^2}) \ , \ l^2 = \tfrac{\hbar}{eB} \ . \qquad (12.2.2)$$

Only the states for which the oscillator Gaussian differs considerably from zero inside slab are important so that the momentum eigenvalues are in good approximation in the range $0 \leq k \leq k_{max} = L_x/l^2$. Using $N = (L_y/2\pi)\int_0^{k_{max}} dk$ one obtains that the total number of momentum eigenstates associated with the given value of n is $N = eBdL_xL_y/h = n$. If ν Landau states are filled, the value of σ is $\sigma = \nu e^2/h$, where ν is the integer valued filling fraction.

The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires such a low temperature.

12.2.3 The model of FQHE based on composite fermions

12.2.4 The model of FQHE based on composite fermions

The model of FQHE based on composite fermions produces FQHE as integer QHE for effective particles - composite fermions. This phenomenological picture is described with enjoyable clarity in the Nobel lecture of Nobel lecture of Horst L. Stormer [D79].

The empirical inspiration for the model is the observation that in strong enough magnetic fields electrons behave in an unexpected manner. For instance, they seem to respond only to effective magnetic field much weaker than the actual field. The difference in field strengths corresponds to an integer number of magnetic flux quanta multiplied by 2-D electron number density. It would seem that these flux quanta somehow separate from the external magnetic field and somehow combine with the electrons to form bound states, which become the basic dynamical units interacting with the external magnetic field. They of course have different mass.

From the experimental data one can conclude that the flux quanta behave like fermions. Most naturally they would carry a rotating electron current concentrated near their boundaries and serving as a source of a magnetic field concentrated around flux quantum or better to say, separating a magnetic field from incoming magnetic field outside the flux quantum. By the conservation of magnetic flux the external magnetic field is reduced correspondingly. The number of flux quanta per electron is integer valued. Since flux quanta behave like fermions, the number of the flux quanta per electron is even for fractional quantum Hall effect. For odd values of flux quanta one obtains composite bosons and something totally different.

The basic formula for the filling fraction is easy to deduce by using the assumption that FQHE is IQHE in effective magnetic field B_{eff} [D65] with even number $2p$ flux quanta subtracted. B_{eff} is given by

$$B_{eff} = B - 2pB_1 \ , \ B_1 = \rho\Phi_0 \ , \ \Phi_0 = h/e \ . \tag{12.2.3}$$

Here B is the external magnetic field, $2p$ the even number of flux quanta per electron, and Φ_0 the elementary flux quantum - twice the flux quantum in super-conductors because the charge carriers are now electrons rather than Cooper pairs.

The integer QHE for B_{eff} gives $\nu_{eff} = \rho\phi_0/B_{eff} = B_1/B_{eff} = n$ saying that n Landau levels are filled. This translates FQHE for B with ν given by completely analogous formula $\nu = B_1/B$. From $\nu_{eff} = 1/[(B/B_1) - 2p] = n = 1/(1/\nu) - 2p$ one obtains

$$\nu = \frac{\nu_{eff}}{1+2p\nu_{eff}} = \frac{n}{1+2pn} \ . \tag{12.2.4}$$

The formula is amazingly simple and consistent with all experimental findings hitherto. Note that at the limit $n \rightarrow \infty$ the formula gives filling fractions $1/2p$.

My understanding is that charge fractionalization is motivated by a paradox created by the flux quantum picture. Classically the number of electrons per flux quantum is higher than one since the generation of flux quantum requires at least one electron per flux quantum. How it is then possible that the number of flux quanta per electron given by ν is higher than one?

What this fractionalization actually means geometrically is not easy to visualize and might require new physics. The solution of the paradox might also require a de-localization of some kind. The number of electrons in the center and at the boundary of flux quantum is fractional. This could might be understood in terms of de-localization of electron wave functions at several flux quanta. In TGD framework electron is string like object defined by Kähler magnetic flux tube with wormhole contacts at its ends and could have rather long length: could it be that also electron charge is de-localized and shared between the two wormhole ends?

12.3 About theories of quantum Hall effect

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

12.3.1 Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [A36] , [D71] . This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \to H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i \oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [D71] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \to H$ defines a representation of the braid group. The construction of these representations is discussed in [D71] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

12.3.2 Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A\wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi}Tr\left(A\wedge(dA+\frac{2}{3}A\wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [A55, A72] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links [K11] .

Witten's article [A84] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k\rightarrow\infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3=\Sigma^2\times R^1$: in the Coulomb gauge $A_3=0$ the action reduces to a sum of $n=dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The WCW consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.

2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical physics [A85]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.

3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G=SU(N)$, in terms of which more complex invariants are expressible.

4. For $SU(N)$ the invariants are expressible as functions of the phase $q = exp(i2\pi/(k + N))$ associated with quantum groups [K11] . Note that for $SU(2)$ and $k = 3$, the invariants are expressible in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k + 1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [B23] .

12.3.3 Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [D69] . Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [A36] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow.

12.3.4 Topological quantum computation using braids and anyons

By the general mathematical results braids are able to code all quantum logic operations [B19] . In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [D75] . The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1 - 1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

   Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with

$n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [B10] . There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [B19] . From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [B9] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [B11] . If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

   In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap} dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

12.4 Quantum Hall effect, charge fractionalization, and hierarchy of Planck constants

The proportionality $\sigma_{xy} \propto \alpha_{em} \propto 1/\hbar$ suggests an explanation of FQHE [D71, D6, D69] in terms of the hierarchy of Planck constants. The idea was that perhaps filling factors and magnetic fluxes are actually integer valued but the value of Planck constant defining the unit of magnetic flux is changed from its standard value - to its rational multiple in the most general case. This naive guess turned out be incorrect.

A careful study of what was known about FQHE much before 2005 (see for instance [D79]) - in particular understanding of the notion of composite fermion - would have demonstrated that FQHE is basically IQHE for composite fermions so that fractionization cannot be due to the integer values of Planck constant or of effective Planck constant. In fact, accepting that composite fermion description one has only to explain what really happens in charge fractionization and how braid statistics emerges. One should of course also have a concrete description for the bound states of electron and flux tubes.

In the picture using multi-sheeted covering of imbedding space as an auxiliary tool, the phase transition corresponds to the leakage of 3-surface from a given 8-D page to another one in the Big

Book having local singular coverings of $CD \times CP_2$ as pages. This auxiliary tool is not absolutely necessary since multi-furcations of preferred extremals of Kähler action is the fundamental notion and one can see FQHE as a function of external magnetic field as a hierarchy of multi-furcations of preferred extremals. In the following this view is adopted since this minimizes the number un-necessary assumptions.

One particular assumption of this kind in the previous approach was that the singular coverings are products of those for CD and CP_2. The coverings has product structure in the sense that the number of sheets is product of two integers but this does not require that these integers could be assigned with singular coverings of CD and CP_2.

The proposed general principle governing the transition to large $\hbar$ phase states that Nature loves lazy theoreticians: if perturbation theory fails to converge, a phase transition increasing the effective value of Planck constant occurs and guarantees the convergence. The killer test for the hypothesis is to find whether higher order perturbative QED corrections in powers of α_{em} are reduced from those predicted by QED in QHE phase.

At the level of preferred extremals of Kähler action these phase transitions corresponds to multi-furcations and their presence is unavoidable due to the enormous vacuum degeneracy of Kähler action which makes also ordinary path integral quantization impossible and also implies 4-D spin glass degeneracy as a basic aspect of the dynamics.

In this section the most recent view about the relationship between dark matter hierarchy, effective hierarchy of Planck constants, and FQHE is discussed. Besides explanations for charge fractionization and fractional exchange statistics also a models for the magnetic flux quanta and the macroscopic 2-surface carrying the anyonic phase are proposed. All these models rely on the notion of many-sheeted space-time and the notion of multi-furcations for a preferred extremal of Kähler action implying also the effective hierarchy of Planck constants.

12.4.1 General description of the anyonic phase

It is appropriate to start with a general description of the anyonic phase in TGD framework. This involves two highly non-trivial new physics elements.

1. The first element corresponds to the description of electrons as pairs of Kähler magnetic flux tubes connecting two wormhole contacts (see fig. `http://www.tgdtheory.fi/appfigures/wormholecontact.jpg` or fig. 10 in the appendix of this book) such that one obtains closed flux tube carrying monopole flux. This description applies to all elementary particles. The "upper" wormhole throat of the second end of this flux tube structure by definition contains electron's quantum numbers and they are assignable to the end of braid strand. This strand continues along the light-like end of the wormhole throat as well as along space-like braid strand assignable to the end of space-time at either end of causal diamond (CD).

   One can imagine a de-localization of electron's quantum numbers in the sense that the state superposition of flux tubes with electron's quantum numbers at either end. This might allow to understand the paradoxical aspects of FQHE in composite fermion description (number of flux quanta per electron large than one and number of electrons per flux quanta larger than one).

2. Second element corresponds to the assumption that wormhole contacts, which have induced metric with Euclidian signature can have M^4 projection which has macroscopic size. All macroscopic objects could correspond to macroscopic wormhole contacts and be analogous to black-holes.

3. Also the nanoscopic magnetic flux quanta with Minkowskian signature of metric and appearing in the composite fermion model of FQHE would have as their boundaries wormhole contacts, now with cylindrical M^4 projection.

4. The natural interpretation is that the generation of flux tubes changes the topology of the macroscopic boundary. It would describe the leakage of a Minkowskian region with magnetic field to Euclidian region occurring also in super-conductivity. Depending on the character of super-conductivity the penetration can take as flux tubes or as complex flux sheets. Flux quanta are long Minkowskian flux tubes connecting opposite sides of the boundary. Single

flux tube boundary is a mesoscopic wormhole throat with tubular geometry - like a cave eaten by an worm in apple - and changes its topology by adding a handle.

5. This leads to the vision that macroscopic objects are obtained simply by somehow gluing elementary particles to the two throats of macroscopic wormhole contact along their second end. One can also imagine that Minkowskian flux tube like regions get branched and that there are Minkowskian islands connected by the flux tube Minkowskian flux quanta.

6. The flux tubes define space-like braids with effectively 1-D strands whereas the braids associated with electrons at the light-like orbit of the partonic 2-surface representing macroscopic boundary define time-like braids with literally 1-D braid strands. The space-like braids defined by magnetic flux tubes are in key role in TGD inspired quantum biology [K31].

Geometric description of the condensation of electrons to the anyonic 2-surface

There is a strong temptation to interpret the macroscopic 2-surface at which the anyonic phase resides as a partonic 2-surface or rather pair of parallel partonic 2-surfaces within distance of CP_2 size associated with macroscopic wormhole contact connecting two space-time sheets. The first space-time sheet would carry the external magnetic field with flux quanta subtracted and the other one the flux quanta.

The rather radical conceptual implication would be that the interior of this boundary surface - more precisely the space-time sheet corresponding to the interior of the entire macroscopic system - has Euclidian signature of metric, and is in several aspects analogous to blackhole interior and indeed proposed to replace blackhole in TGD Universe. In many-sheeted space-time this does not lead to any obvious problems and would say only that entire macroscopic system in this length scale behaves as a line of a generalized Feynman diagram.

Electrons can in some sense condense at this pair of space-time sheets. The simplest view would be that electronic flux tube pair attaches to this surface along its second wormhole contact. Another wormhole contact remains at the Minkowskian side. The two wormhole throats at the second end of electron attach to the macroscopic wormhole throats and the flux turns back through the macroscopic wormhole contact. This allows to have ordinary many-electron state - or rather, boundary state.

If one tries to add electrons as braid strands to the light-like orbit of the upper macroscopic partonic 2-surface, one obtains quite different state. This state has nothing to do with ordinary many-electron state but is more like super-conformal excitation of a primary state containing only single fermionic braid strand and its propagator as particle would be of form $1/p^N$, N large. In conformal theory conformal descendant of a primary field would be the analogy.

I have proposed that this kind of macroscopic and even astroscopic structures emerge naturally in TGD framework. so tht anyons could be important even in astrophysics.

Possible solution of the paradox

It has been already noticed that FQHE leads to what looks like a paradox - at least for an outsider to condensed matter physics like me. The number of flux tubes per electron is larger than 1 on one hand and the number of electrons per flux tube is larger than 1 on one hand.

The bi-locality of the electrons might solve the paradox. If the charge of free electron is delocalized to its both ends, electron can be said to reside at the both ends of its monopole flux tube.

Consider what the following two statements could mean. *Electron current generates the magnetic field inside flux quantum. Electron resides at the center of the flux quantum.*

1. Suppose first that electron is associated with either wormhole end of its monopole flux tube, call it E. If the electronic charge is always at the Minkowskian end of E, then two statements could be special cases of a more general statements E would connect second electron wormhole in the Minkowskian interior of the mesoscopic magnetic flux quantum - call it M - to electron wormhole fused to the boundary of M. The location of interior wormhole would be center of flux quantum or a point near to its boundary in the two cases respectively. It seems that the paradox remains unsolved in this picture.

2. Suppose that electron corresponds to a superposition of states for which charge is associated with either upper end of the flux tube perhaps having length of order Compton length. If the electronic charge is de-localized and shared between ends of E, one cannot anymore say that the electron is either at the center or at the boundary. Paradox would disappear since quantum logic would not allow its formulation.

What happens to electron in external magnetic fields in FQHE?

What happens in external magnetic field when $2p$ flux quanta are formed? The first challenge is to construct a concrete model for what happens to electron as a geometric object in this process.

1. Assume that electron's "upper" space-time sheet by definition containing its quantum numbers suffers a $2p$-furcation. Each sheet of multi-furcation corresponds to flux quantum Φ_0. Electron near the center of flux quantum is de-localized at the sheets of the multi-furcation to "plane wave" like state. The corresponding conserved momentum defined modulo $2p$ defines a topological quantum number making in turn possible non-Abelian braid group representations. Intuitively it is clear that if one identifies electron's charge as that associated with single branch of the covering, charge fractionalization takes place.

2. Conservation of the magnetic flux requires that the lower sheet carries a reduced magnetic field $B_{eff} = B - \rho 2p\Phi_0$. Since electron experiences only this field one obtains IQHE in B_{eff} so that the basic formula for ν follows.

3. The flux of the magnetic flux quanta at the upper sheet must return back along the lower sheet and this leads to the replacement of B with B_{eff}. Also the fluxes assignable to electrons must return back to the lower sheet and this would take place at the boundaries of flux sheets representing second wormhole throat end of electron.

12.4.2 Basic aspects of FQHE

The following gives a brief summary about how one might understand basic aspects of FQHE in TGD framework

The identification of composite fermions

The basic aspects of FQHE can be understood in terms of composite fermions identified as bound states of electron and $2p$ magnetic flux tubes with magnetic field generated by electrons flowing around its boundary. The electrons are at the center of flux tube to minimize Coulomb repulsion. This picture is however somewhat problematic since it seem to be in conflict with $\nu < 1$ stating that the number of electrons per flux tube is smaller than 1. It has been already proposed that the TGD inspired identification of electron as a bi-local object consisting of two wormhole contacts attaching along its neutral wormhole contact to the cylindrical flux tube representing the magnetic flux quantum resolves the problem. Note that this requires that electron's geometric size is given by flux tube radius and can be large.

The formation of anyons - that is flux tubes would mean a topological transition changing the spherical (say) topology of the space-time sheet representing the macroscopic system- to sphere with handles with handle addition representing as drilling of wormhole connecting the opposite sides of the surface. In this process electrons de-localized at the spherical surfaces would be de-localized so that they would be de-localized also at the flux tube boundaries representing part of the macroscopic wormhole contact.

Charge fractionalization

Since the system is extremely non-linear, the increase of the external magnetic field is expected to lead to a series of multi-furcations meaning that the upper space-time sheet associated with electrons and attached to the upper anyonic 2-surface suffers a multi-furcation. The natural reason for the multi-furcation is that it allows to keep the local magnetic field strength at the flux quantum below critical value. Without multi-furcation this field strength would be proportional to $2p$. Once the first multi-furcation has taken place leading to the generation of the flux tubes, the subsequent multi-furcations only add the number of branches of the multi-furcation of the flux tube. Electron de-localizes to this n-branched structure and single branch carries fractional charge e/n. Also other quantum numbers are fractional.

One should demonstrate convincingly that the fractional charges identified in this sense correspond to measured fractional charges.

Fractional exchange statistics

Also the fractional braid statistics can be understood. For ordinary statistics rotations of M^4 rotate entire partonic 2-surfaces. For braid statistics rotations of M^4 (and particle exchange) induce a flow braid ends along partonic 2-surface. If the singular local covering is analogous to the Riemann surface of $z^{1/n}$, the rotation of 2π leads to a second branch of multi-furcation. For the natural angle coordinate of the n-branched covering its variation of $2/pi$ corresponds to a variation $n2\pi$ of M^4 angle coordinate, and a rotation by 2π in M^4 to a rotation of $2\pi/n$ in at space-time level and phase factor $exp(i2\pi)$ is mapped to $exp(i2\pi/n)$: one has fractional exchange statistics for angular momentum.

Quantum groups relate closely to the fractional statistics and the quantum phase $q = exp(i2\pi/n)$ characterizes the statistics. Quantum groups realize particles exchange as braiding and one can formulate statistics in terms of braid group representations. What is remarkable that also genuinely non-Abelian higher-dimensional braid group representations are possible and these representations are conjectured to be associated with the anomalously behaving filling fractions $\nu = 5/2, 7/2$ allowed also by the standard rules when the entire external magnetic field is transformed to flux quanta. Also the limit $n \to \infty$ gives $\nu = 1/2p$, given $\nu = 1/2$ for $p = 1$.

How non-Abelian gauge group is generated?

The emergence of Abelian braid statistics is explained in terms of the velocity field of electrons defining effectively Abelian gauge potential giving rise to Chern-Simons term defining a topological QFT. This requires that the electron flow is incompressible.

In the case of non-Abelian braid statistics a non-Abelian gauge group is needed to define Chern-Simons action. The challenge is to understand the physical origin of this gauge symmetry and to my best knowledge this problem is not well-understood.

In TGD framework Kähler action reduces to Abelian Chern-Simons terms for preferred extremals so that non-Abelian Chern-Simons term and corresponding gauge group should be generated dynamically. The study of the preferred extremals of Kähler action and solutions of modified Dirac action indeed leads to a mechanism generating not only electro-weak gauge symmetries dynamically but also a larger gauge group [K103]. What might happen is follows. The core part of the dynamical gauge group would be $U(n)$ acting in the space of modes of the modified Dirac operator. Its action commutes with electroweak and other quantum numbers. By taking the n^2 generators of $U(n)$ and the 4 generators of electroweak $U(2)$, and forming their tensor products, one would obtain $4n^2$ generators having interpretation as generators of $U(2n)$. The non-Abelian Chern-Simons term would be associated with $U(n)$.

The stack of N branes very near to each other gives rise to a dynamical gauge group $U(N)$ in M-theory context. This encourages to think that the n-furcation giving rise to n space-time branches gives rise to a dynamical gauge group $U(2)$ for $n = 4$: $SU(2)$ is the minimal requirement for non-trivial braid statistics.

Some problems of composite model

Composite model as at least the following not so well understood aspects.

1. The flux quanta must be assumed to behave like fermions. What gives them the fermionic statistics and maybe also fermion number?

2. How both the number of flux quanta per electron and the number of electrons per flux quantum can be larger than one?

3. How to understand charge fractionization. How general phenomenon the fractionization is?

Could TGD based model provide deeper justification for the com-posite model?

1. Composite model as starting point. Flux quanta now realized as magnetic flux tubes. An interesting possibility is that flux quanta correspond to mono- pole fluxes for which the transversal section of the flux tube is closed 2-surface rather than disk or annulus.

2. Interesting possibility is that the underlying 2-D system corresponds to a partonic 2-surface of macroscopic size at which electrons and accompanying flux tubes are attached. This kind of surfaces are proposed to appear even in astrophysical scales in TGD Universe and carry dark matter. This would give first a principle justification for braid statistics.

3. Could the braid statistics has justification in terms of hierachy of Planck constants? The proposal is that h_{eff}= nh corresponds to a formation of effective n-sheeted covering of imbedding space. M^4 (CP_2) could be covered n_1 (n_2) times. This means that ordinary rotations and color rotations in Cartan algebra induceonly a phase correspond to $2\pi/n_1$ ($2\pi/n_2$). This would bring in various kinds of fractionizations.

4. A delocalization of em charge to n sheets implies $1/n$ fractionization.

5. Non-abelian braid statistics is pos- sible only for $n > 4$ and $n = 4$ would be minimal value of n.

6. What gives for the flux tube fermionic statistics? One possibility is based on the fact that a magnetic flux tube carrying Khler magnetic flux equal to Khler electric flux at its end is dyon with minimal magnetic charge and odd electric charge. By a well-known argument dyons obey fermionic statistics (`http://quantumfrontiers.com/tag/magnetic-monopole/`) [B12]. The objection is that in TGD physical fermions are obtained by adding "ur-fermions" at dyonic wormhole throats. Does this mean that fermions behave as bosons in scales longer than flux tube length and as fermions only at wormhole throats? This need not be the case since the two flux tube portions both would behave like fermion so that spin statistics would be correct since flux tubes are necessarily closed albeit in sense of many-sheeted space-time.

7. A solution of the problem consistent with basic TGD could be that the dyonic flux tube assignable to elementary particle defines only a classical space-time correlate for fermion.

8. An alternative explanation would be that the flux tube contains two parts located at parallel space-time sheets connected by wormhole contacts at the ends of the flux tubes so that a closed flux tube results (no Dirac magnetic monopoles in TGD). One would have two flux tube portions of this kind and statistics would be bosonic. The appearence of monopole fluxes as pairs in FQHE would conform with this picture about electron.

9. Another possibility is that covariantly constant right handed neutrino assignable to the flux tube gives rise to fermion number without contributing to four-momentum. The first geometric explanation would only define space-time correlate for spin 1/2. Note that this is consistent with the assumption that neutrino pairs neutralizes the weak isospin of electron. For larger h_{eff} the fractionization of weak isospin would be required.

Chapter 13

A Possible Explanation of Shnoll Effect

13.1 Introduction

Usually one is not interested in detailed patterns of the fluctuations of physical variables, and assumes that possible deviations from the predicted spectrum are due to the random character of the phenomena studied. Shnoll and his collaborators have however studied during last four decades the patterns associated with random fluctuations and have discovered a strange effect described in detail in [E15] , [E15, E23, E22, E16, E26, E17] . The examples of [E15] , [E15] give the reader a clear picture about what is involved.

1. Some examples studied by Shnoll and collaborators are fluctuations of chemical and nuclear decay rates, of particle velocity in external electric field, of discharge time delay in a neon lamp RC oscillator, of relaxation time of water protons using the spin echo technique, of amplitude of concentration fluctuations in the Belousov-Zhabotinsky reaction. Shnoll effect appears also in financial time series [E28] which gives additional support for its universality. Often the measurement reduces to a measurement of a number of events in a given time interval τ. More generally, it is plausible that in all measurement situations one divides the value range of the studied observable to intervals of fixed length and counts the number of events in each interval to get a histogram representing the distribution $N(n)$, where n is the number of events in a given interval and $N(n)$ is the number of intervals with n events. These histograms allow to estimate the probability distribution $P(n)$, which can be compared with theoretical predictions for the spectrum of fluctuations of n. Typical theoretical expectations for the fluctuation spectrum are characterized by Gaussian and Poisson distributions.

2. Contrary to the expectations, the histograms describing the distribution of $N(n)$ has a distribution having several maxima and minima (see the figures in the article of Shnoll and collaborators). Typically -say for Poisson distribution - one expects single peak. As the duration of the measurement period increases, this structure becomes gets more pronounced: standard intuition would suggest just the opposite to take place. The peaks also tend to be located periodically. According to [E15] , [E15] the smoothed out distribution is consistent with the expected distribution in the case that it can be predicted reliably.

3. There are also other strange features involved with the effect. The anomalous distribution for the number n of events per fixed time interval (or more general value interval of measured observable) seems to be universal as the experiments carried out with biological, chemical, and nuclear physics systems demonstrate. The distribution seems also to be same at laboratories located far away from each other. The comparison of consecutive histograms shows that the histogram shape is likely to be similar to the shape of its nearest temporal neighbors. The shapes of histograms tend to recur with periods of 24 hours, 27 days, or 365 days. The regular time variation of consecutive histograms, the similarity of histograms for simultaneous independent processes of different nature and occurring in different geographical positions,

and the above mentioned periods, suggest a common reason for the phenomenon possibility related to gravitational interactions in Sun-Earth and Earth-Moon system.

In the case that the observable is number n of events per given time interval, theoretical considerations predict a distribution characterized by some parameters. For instance, for Poisson distribution the probabilities $P(n)$ are given by the expression

$$P(n|\lambda) = exp(-\lambda)\frac{\lambda^n}{n!} . \quad (13.1.1)$$

The mean value of n is $\lambda > 0$ and also variance equals to λ. The replacement of distribution with a many-peaked one means that the probabilities $P(n|\lambda)$ are modified so that several maxima and minima result. This can occur of course by the randomness of the events but for large enough samples the effect should disappear.

The universality and position independence of the patterns suggest that the modification changes slowly as a function of geographic position and time. The interpretation of the periodicities as periods assignable to gravitational interactions in Sun-Earth system is highly suggestive. It is however very difficult to imagine any concrete physical models for the effect since distributions look the same even for processes of different nature. It would seem that the very notion of probability somehow differs from the ordinary probability based on real numbers and that this deformation of the notion of probability concept somehow relates to gravitation.

In the following the possibility that direct p-adic variants of real distribution functions such as Poisson distribution might allow to understand the findings is discussed. It turns out that this is not the case but that the replacement of integers with quantum integers [A17] n_q identified as the product of quantum integers associated with their prime factors with quantum phase $q = exp(i\pi/m)$, where $m \geq 2$ is not of form $m = p$, p prime, leads to a well-defined correspondence between p-adic probabilities $P(n)$ and real probabilities conserving the sum of probabilities.

There is however a difficulty, which was not fully realized in the original version of this article. Quantum primes l_q are non-negative only for $l < m$ and this could lead to non-negative probabilities (consider for instance the counterparts of $n!$ in Poisson distribution). The solution of the problem is provided by what I call quantum arithmetics [K98, K100] providing a more rigorous formulation of quantum integers. The recipe is following. To define quantum integer n_q decompose first n to its prime factors l. For $l < m$ one has $l_q > 0$ but not necessarily for $l > m$. Express $l > m$ as a q-adic expansion in powers of m with coefficients smaller than m and thus expressible as products of quantum primes l_q for $l < m$ so that the resulting quantum q-adic integers for $q = m$ is non-negative. For $m = p$ one obtains what one might call quantum p-adics. For quantum p-adics one can

Usually quantum groups are assigned with exotic phenomena in Planck length scale. In TGD they are assignable to a finite measurement resolution [K95] . TGD inspired quantum measurement theory describes finite measurement resolution in terms of inclusions of hyper-finite factors of type II_1 (HFFs) and quantum groups related closely to the inclusions and appear also in the models of topological quantum computation [B18] based on topological quantum field theories [A72] .

The universal modification of probability distributions $P(n|\lambda_i)$ characterized by rational numbers predicts patterns analogous to the ones observed by Shnoll. The parameters P and m characterize the deformation of the probability distribution and the periodic slow variation of the p-adic prime P and explain the periodically occurring peaks of the histograms for $N(n)$ as function of n. Also the dependence of the distribution of $N(n)$ on the direction of the momentum of alpha particle [E16, E26] can be understood in terms of the effect of the measurement apparatus on many-sheeted space-time topology and geometry.

The p-adic primes P in question are small. This makes sense in TGD framework only if one accepts that a very large value of Planck constant is involved. TGD indeed predicts a hierarchy of Planck constants and identifies dark matter as phases with a large value of Planck constant. The Planck constant associated with the space-time sheets mediating gravitational interaction is predicted to be gigantic meaning macroscopic quantum coherence in astrophysical scales. This modification allows also to formulate a general correspondence principle between real and p-adic physics as a rule stating that all primes p except the p-adic prime P itself appearing in various formulas are replaced with their quantum counterparts and P is mapped to its inverse in the modified distribution.

For the reader not familiar with TGD the article series in Prespace-time journal [L7, L8, L12, L13, L10, L6, L11, L17] and the two articles about TGD inspired theory of consciousness and of quantum biology in Journal of Consciousness Research and Exploration [L16, L14, L15] are recommended. Also the online books at my homepage provide the needed background.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- p-Adic number fields [L36]
- p-Adic physics [L37]
- p-Adic length scale hypothesis [L38]
- p-Adic manifold [L34]
- Quantum gravity and biology [L40]

13.2 p-Adic topology and the notion of canonical identification

p-Adic physics has become gradually a central part of quantum TGD [K83] and the notion of p-adic probability has already demonstrated its explanatory power in the understanding of elementary particles masses using p-adic thermodynamics [K48] . This encourages the attempt to understand Shnoll effect in terms of an appropriate modification of probability concept based on p-adic numbers.

p-Adic topology [A23] is characterized by p-adic norm given by $|x|_p = p^{-k}$ for $x = p^k(x_0 + \sum_{k>0} x_k p^k)$, $x_0 > 0$. This notion of nearness differs radically from its real counterpart. For instance, numbers differing bya large power of p are p-adically near to each other. Therefore p-adic continuity means short range chaos and long range correlations in real sense. One might hope that p-adic notion of nearness allow the existence of p-adic variants of standard probability distributions characterized by rational valued parameters and transcendental numbers existing also p-adically such that these distributions can be mapped to their real counterparts by canonical identification mapping sum of probabilities to the sum of the images of the probabilities.

13.2.1 Canonical identification

In the case of p-adic thermodynamics [K48] the map of real integers to p-adic integers and vice versa relies on canonical identification and its various generalizations and canonical identification is also now a natural starting point.

1. The basic formula for the canonical identification for given prime p characterizing p-adic number field Q_p is obtained by using for a real number x pinary expansion $x = \sum x_n p^{-n}$, $x_n \in \{0, p-1\}$ analogous to decimal expansion. The map is very simple and given by

$$\sum_n x_n p^{-n} \rightarrow I(x) = \sum_n x_n p^n \quad . \tag{13.2.1}$$

The map from reals to p-adics is two-valued in the case of real numbers since pinary expansion itself is non-unique ($p = (p-1)\sum_{k\geq 0} p^{-k}$ as the analog of 1=.99999.. for decimal expansion). The inverse of the canonical identification has exactly the same form. Canonical identification maps p-adic numbers to reals in a continuous manner and also the inverse map is continuous apart from the 2-valuedness eliminated if one introduces pinary cutoff which is indeed natural when finite measurement resolution is assumed.

2. The first modification of canonical identification replaces pinary expansion of real number in powers of p with expansion in powers of p^k: $x = \sum x_n p^{-nk}$, $x_n \in \{0, p^k - 1\}$ and reads as

$$\sum_n x_n p^{-nk} \rightarrow I_k(x) = \sum_n x_n p^{nk} \quad . \tag{13.2.2}$$

3. A further variant applies to rational numbers. By using the unique representation $q = r/s$ of given rational number as ratio of co-prime integers one has

$$I_k(q = \frac{r}{s}) = \frac{I_k(r)}{I_k(s)} \quad . \tag{13.2.3}$$

13.2.2 Estimate for the p-adic norm of factorial

In the p-adic variant of Poisson distribution canonical images of the factorial $n!$ appear and the the basic properties of $I(n!)$ as function of n will be needed in the sequel.

1. Given integer n can be written as $n = p^{k(n)} m(n)$ such that $m(n)$ has unit norm p-adically. $n!$ in turn can be written as

$$n! = \prod_{r=1}^{n} p^{k(r)} m(r) = p^{K(n)} \times \prod_r m(r) \quad , \quad K(n) = \sum_r k(r) \quad . \tag{13.2.4}$$

2. The p-adic norm of $n!$ is given by

$$N_p(n!) \quad = \quad p^{-K(n)} \quad . \tag{13.2.5}$$

$\prod_r m(r)$ has unit norm p-adically and its p-adic canonical image satisfies the upper bound

$$I_k(\prod_r m(r)) \leq p^k \quad . \tag{13.2.6}$$

3. $N_p(n!)$ is reduced by the power $p^{k(r)}$ in the step $n = r - 1 \rightarrow r$. Therefore $I(n!) \equiv I_{k=1}(n!)$ is a decreasing function with discontinuous drops of the value which are especially large when n is proportional to a large power of p. The peaks corresponding to given value k of $k(r)$ occur periodically and one has fractal pattern with periodicities define by powers of p. Similar consideration applies to $I_k(n!)$: now the periodicities correspond to powers of p^k rather than p. In both cases one has local chaos and long range correlations due to the fact that in p-adic topology nearby points differing by a large power p^n are far away in real sense. The natural question is whether the periodicity of peaks in histograms of [E15] , [E15] could represent a special case of of these periodicities.

In the sequel an estimate for the maximal power of p dividing $n!$ defining the norm $N_p(n!)$ is needed. The following estimate gives $N_p(n!) \simeq p^{-n}$ for $n \gg p$.

1. What is needed is an estimate for the number $N(n,k)$ of for the number of integers $k(r)$ with given value of $k \geq 1$. If this estimate is available for large values of n, one obtains for the exponent defined associated with the p-adic norm of $n!$ the formula

$$K(n) \equiv \sum(k_r) = \sum N(k)k \quad . \tag{13.2.7}$$

2. By studying the 2-adic numbers one finds that the formula

$$K(n=2^m) = \sum N(k)k \ , \ N(k) = \frac{2^m}{2^k} = 2^{m-k} \tag{13.2.8}$$

holds true.

3. The generalization of the this formula to for $p > 2$ reads as

$$K(n=p^m) = \sum N(k)k \ , \ N(k) = (p-1)\frac{p^m}{p^{k+1}} = p^{m-k} \quad . \tag{13.2.9}$$

This would give at the limit $n \to \infty$

$$K(n=p^m) = \frac{p^{m+1}}{p-1} \simeq p^m = n \quad . \tag{13.2.10}$$

There one has $K(n) = n$ in this special case.

4. For a general value of n the approximate formula would be

$$K(n) \leq \sum N(k)k \ , \ N(k) \simeq (p-1)\frac{n}{p^k} \quad . \tag{13.2.11}$$

Also now one would have $K(n) \simeq n$ so that the p-adic norm of $n!$ would be approximately p^{-n} . The justification for this formula comes by noticing that the number of integers smaller than n with p-adic norm p^k is roughly $(p-1)n/p^k$ since the numbers kp^k+X with $N_p(X) \leq p^{-k-1}$ and k running from $1,...,p-1$ satisfy the required conditions.

13.3 Arguments leading to the identification of the deformed Poisson distribution

The following argument represents a trial and error procedure to a unique identificaiton of deformed Poisson distribution $P(n|\lambda)$ with a rational value of λ and more generally, to a modication of any distribution $P(n,\lambda_i)$ characterized by rational parameters λ_i.

13.3.1 The naive modification of Poisson distribution based on canonical identification fails

To gain some intuition it is instructive to study the possible variants of Poisson distribution based on canonical identification. The discussion generalizes to more general distributions for probabilities of integer valued observables provided the parameters of the distribution exist p-adically. The idea is to start from a p-adic variant of probability theory [A57] , assume that the p-adic valued probability distributions are mappable to their real counterparts using canonical identification, and to look whether this procedure yields something consistent with the findings of Shnoll.

To begin with, assume that the notion of p-adic valued probability makes sense. This requires that the probabilities exist as p-adic numbers. This is true if probabilities are rational numbers which can be regarded as being common to reals and padic numbers. Also the sum of probabilities must make sense p-adically so that it can be normalized to to unity. In absence of cutoff to the values of N this condition is highly non-trivial.

The condition that the canonical identification commutes with the summation of probabilities is especially strong and would state

$$\sum(P(n))_R \quad = \quad (\sum P_n)_R \ . \tag{13.3.1}$$

Here x_R denotes the image of x under canonical identification. For ordinary p-adic numbers this condition requires that the probabilities are just powers of p. If one allows algebraic extensions of p-adic numbers defined by quantum phases defined by roots of unity mapped to real numbers as such, the probabilities can be of form Xp^n where X is function of these phases. This condition excludes automatically the naivest attempts to define canonical image of p-adic variant of Poisson distribution. This is due to the presence of $1/n!$ and possible rational appearing in λ.

Optimist could give up the normalization condition and consider instead of probabilities rational numbers. There are problems also now.

1. The first problem is that normalization factor is defined only up to a multiplication with a rational and each choice of the normalization factor gives different real counterpart of the p-adic distribution irrespective of the manner how the real probabilities are defined.

2. The normalization factor $exp(-\lambda)$ is p-adic number only if λ is proportional to a positive power of p. This condition also implies that the powers $\lambda^k/k!$ approach to zero with respect to p-adic norm since the p-adic norm of λ^k is always small than that of $k!$. The naive guess for the canonical identification map of p-adic probabilities to their real counterparts is given by the formula

$$\lambda^n \rightarrow I(\lambda^n)/I(n!)$$

   One can consider also other other variants but for the purposes of argument one can restrict the consideration to this one. The problem is that $I(\lambda^n)$ does not increase but decreases like p^{-n} so that $\lambda_R < 1$ would hold true. The decrease of the factor $1/n!$ guarantees the convergence of probabilities for Poisson distribution. The canonical image $I(1/n!) = 1/I(n!)$ however increases. The same result is obtained irrespective of the detailed definition of canonical identification. Therefore the first guess for the canonical image of the proposed p-adic variant of Poisson distribution has very little to do with ordinary Poisson distribution. The attempts to cure the situation by modifying the map from p-adics to reals fail. This suggests that one must modify the p-adic variant of the Poisson distribution itself.

13.3.2 Quantum integers as a solution of the problems

The problems associated with the naive generalization of the Poisson distribution relate to the behavior of canonical identification when applied to integers other than powers of p. This suggests that one should replace the integers systematically with some of kind of deformations of integers guaranteeing also that canonical identification maps sum of probabilities the sum of their images. The notion of quantum integer [A17] is what comes first in mind.

TGD based motivation for the notion of quantum integer comes from the fact that the so called hyper-finite factors of type II_1 (HFFs) play a key role in quantum TGD and allow to formulate the notion of finite measurement resolution in terms of inclusions of HFFs [K95] to which the quantum groups assignable to roots of unity are closely related. The findings of Shnoll would therefore relate to the delicacies of quantum measurement theory with finite measurement resolution.

The quantum groups based on quantum phases

$$q = U_m = exp(i\phi_m) \ , \ \phi_m = \frac{\pi}{m} \ . \ m \geq 3 \tag{13.3.2}$$

appear in TGD framework and the long standing intuitive expectation has been that there might exist a deep connection between p-adic length scale hypothesis and quantum phases defined by roots of unity defining algebraic extensions of p-adic numbers.

The standard definition of quantum integer does not help

The first thing to do is to see whether the standard notions of quantum integer and quantum factorial [A17] could allow to get rid of the problems.

1. Quantum integers for $q = U_m$ are given by

$$n_{U_m} = \frac{U_m^n - \overline{U}_m^n}{U_m - \overline{U}_m} = \frac{sin(n\phi_m)}{sin(\phi_m)} \ . \tag{13.3.3}$$

   For $n \ll m$ one has

$$n_{U_m} \simeq n \ . \tag{13.3.4}$$

   This property makes quantum integers a good candidate if one wants to generalize the notion of Poisson distribution and more generally, any probability distribution $P(n|\lambda_i)$ parametrized by rationals. The rule would be very simple: replace all integers by their quantum counterparts: $n \rightarrow n_q$.

This proposal has however some problematic features.

1. n_q is negative for $n \ mod \ 2m > m$ so that in the case of Poisson distribution one would have negative probabilities in real context. In the p-adic context there is no well-defined notion of negative number so that one might avoid this difficulty if one can map p-adic probabilities to positive real probabilities. Quantum integers have unit norm p-adically so that p-adic Poisson distribution makes sense for $N_p(\lambda) < 1$.

2. n_{U_m} vanishes for $n = m$ always. Therefore $n_q!$ defined as a product of quantum integers smaller than n vanishes for all $n > m$. One way out is to restrict the values of n to satisfy $n < m$. This number theoretic cutoff would mean in the p-adic case that the sum of p-adic probabilities is finite without the condition $N_p(\lambda) < 1$.

3. Quantum integers defined in the standard manner are periodic with period m so that quantum factorial obtained by dropping the vanishing terms would behave like a product of factorial associated with $m - 1$ times quantum factorial of $k \leq m - 1$. Ordinary factorial $n!$ increases much faster. It seems that the standard definition of quantum integer is not correct.

Quantum integers must allow factorization to quantum primes

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Physics as a generalized number theory vision [K83] suggests a manner to circumvent above described problems.

1. Quantum integers defined in the standard manner do not respect the decomposition of integers to a product of factors- that is one does not have

$$(mn)_q = m_q n_q \ . \tag{13.3.5}$$

The preferred nature of the quantum phases associated with primes in TGD context however suggests that one should guarantee this property by hand by simply defining the quantum integer as a product of quantum integers associated with its prime factors:

$$n_q \equiv \prod (p_i)_q^{n_i} \ \text{for} \ n = \prod p_i^{n_i} \ . \tag{13.3.6}$$

This would guarantee that the notion of primeness and related notions crucial for p-adic physics would make sense also for quantum integers. Note that this deformation would not be made for the exponents of integers for which sum is the natural operation.

2. If $q = U_m$ is such that m is not prime, the quantum phases associated with primes are always non-vanishing and quantum integers and therefore also quantum factorials $n_q!$ defined using the proposed definition of quantum integers are non-vanishing for all values of n. In p-adic context this would mean that the probabilities associated with Poisson distribution are finite and for $N_p(\lambda_p) < 1$ sum up to a finite value.

The number theoretic definition of quantum integers does not automatically solve the problem of negative quantum integers obtained when integer contains prime factors $p > m$ and vanishing problem when integer is divisible by p.

1. If the number N_- of prime factors of n satisfying $p \ mod \ 2m > m$ is odd, the product of minus signs coming from them is odd and the over all quantum integer is negative. Since the p-adic probabilities are well defined in p-adic context, one could consider the mapping of these probabilities to real probabilities by the basic form of canonical identification. If also λ is expressed in terms of quantum primes only the real image of overall minus sign must be determined. p-Adically -1 corresponds to a positive p-adic integer $(p-1)(1+p+p^2+...)$ for which one has $I(-1) = p$ from the basic definition of canonical identification. Hence the p-adic and real quantum variants of Poisson distribution would be unique.

   This prescription would predict peaks of Poisson distribution for $n = n_+ n_-$, such that $(n_+)_q$ is positive and has only prime factors $p_+ \ mod \ 2m < m$ and $(n_-)_q$ is having therefore odd number of negative prime factors $(p_-)_q$ satisfying $p_- \ mod \ 2m > m$. These peaks would occur periodically with period n_-. Large number of this kind of periods would be present. It might be possible to identify the periodicities of the peaks of the histograms of Shnoll in this manner.

2. Second manner to solve the sign problem has been already mentioned and relies on the notion of quantum arithmetics [K98, K100]. The construction recipe for quantum integers is following. To define quantum integer n_q decompose first n into its prime factors l. This guarantees the quantum integers respect prime factorization for ordinary integers. For $l < m$ one has $l_q > 0$ but not necessarily for $l > m$. Express $l > m$ as a q-adic expansion $l = \sum l_k m^k$, with $l_k < m$ and thus expressible as products of quantum primes l_q for $l < m$ so that the resulting quantum q-adic integers for $q = exp(i\pi/m$ are non-negative.

For $m = p$ one obtains what one might call quantum p-adics and in this case $p_q = 0$ holds true so that one must assume cutoff $n < p$ or exclude integers n divisible by p. Note that q-adicity is consistent with p-adicity for prime factors of m.

One can consider also a more general recipe for quantum p-adic integers (and also quantum m-adic integers) [K100]. One allows all expansions $l = \sum l_n p^n$ of primes $l > p$ in powers of p with coefficients l_n also now having only prime factors $l < p$ but giving up the constraint $l_n < p$ so that given p-adic integer corresponds to several quantum p-adic integers. This gives quantum q-adic integers for $q = m$ which in well-defined sense forms a covering of q-adic integers and one can assign to it what might be called quantum Galois group.

The most general choice of λ

Consider next the most general choice of λ consistent with the constraint that canonical identification conserves probabilities. Denote by P the p-adic prime characterizing the deformed Poisson distribution and by p a generic prime.

1. If one assumes the following product representation

$$\lambda_q = P^n Q_{U_m} \ , \tag{13.3.7}$$

where P is the p-adic prime and Q_{U_m} is quantum rational in the proposed sense, p-adic probabilities $P(n)$ are finite for positive values of n and m satisfying the proposed constraints. The expression for the real counterpart $\lambda_R$of λ_q is given by

$$\lambda_R = \frac{Q_{U_m}}{P^{-n}} \ . \tag{13.3.8}$$

With a proper choice of Q_{U_m} arbitrary large values are possible for λ_R and standard form of of canonical identification for a well-defined p-adic probability distribution produces a real variant of quantum Poisson distribution which is in a well-defined sense a small deformation of the Poisson distribution.

2. The value of the parameter λ assignable to the ordinary Poisson distribution giving rise to q-Poisson does not correspond to λ_R as such. For given λ_q the value of λ can be determined from the condition that the average values of n are same for the two distributions:

$$\lambda = \langle n \rangle_P = \langle n \rangle_{qP} \ . \tag{13.3.9}$$

3. For $m = P$ the vanishing of P_{U_P} would require a cutoff $n < P$ in Poisson distribution. One could however argue that all values of m must be allowed. The manner to circumvent the difficulty is to to treat prime $p = P$ as an exception and *define* in the most general case

$$P_q \equiv P \ . \tag{13.3.10}$$

A stronger condition would be that P appears as a factor of m and it might well be that there could exist a number theoretical justification for this. Canonical identification would introduce to $P(n)$ a factor $P^{K(n)}$ defined by the largest power $P^{K(n)}$ dividing $n!$. By the rough estimate $n!$ of Eq. 13.2.11 one has $K(n) \sim n$. This would introduce additional peaks to the distribution coming with periodicities defined by p^m besides those coming with periodicities defined by integers n_-, which involve odd number of integers $p \ mod \ m > m/2$. This requires

$$\lambda_q = P^n Q_{U_m} \ , \ n > 1 \tag{13.3.11}$$

in order that the sum of p-adic probabilities is well-defined. The sum of real probabilities converges due to the properties of quantum factorial defined in the manner respecting the decomposition of integer to a product of primes.

4. This definition of quantum Poisson satisfies also the strongest possible constraint on the map of p-adic probabilities to real ones. One can indeed include the p-adic normalization factor to the distribution and rational canonical identification commutes with the normalization factor in the sense that one has $\sum (P(n))_R = (\sum P_n)_R$. This is due to the fact that the canonical image of the sum of probabilities is by definition a sum of images of probabilities since only numbers expressible in terms of roots of unity and not allowing expression as ordinary p-adic number multiplied by powers of p and p-adic -1 appear in the sum.

5. Fig. 13.1 represents a comparison of q-Poisson distribution characterized by $(p = 7, m = 300, \lambda_0 = 100, k = 1)$ giving $\lambda_q = p^k \times \lambda_0 = 700$ and $\lambda_R = 14.229$ with the corresponding ordinary Poisson distribution characterized by $\lambda = 25.256$ which is almost twice the value of λ_R. The presence of peaks with periodicity $p = 7$ due to the identification $p_q = p$ for the prime defining p-adicity and mapped to $1/p$ in canonical identification is clearly visible in the distribution.

These considerations are for Poisson distribution but they generalize in an obvious manner to any distribution $P(n|\lambda_i)$ for which parameters $\lambda_i)$ are rational numbers.

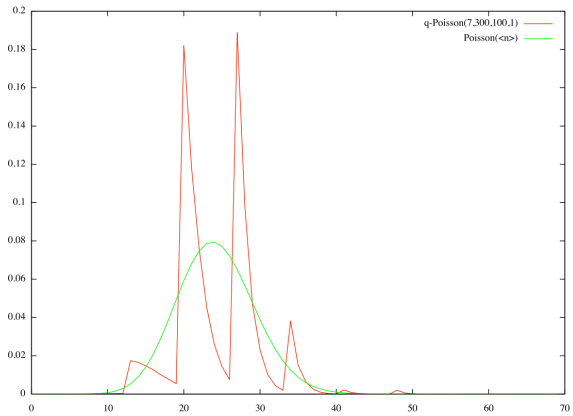

Figure 13.1: A comparison of q-Poisson distribution with Poisson distribution with the same mean value of n assuming $p_q = p$ and that p is mapped to $1/p$ and -1 in numerator is mapped to p in canonical identification. The values of quantum parameters are $(p = 7, m = 300, k = 1, \lambda_0 = 100)$ giving $\lambda_q = p^k \times \lambda_0 = 700$ and $\lambda_R = 14.229$. The mean value of Poisson distribution turns out to be $\lambda = \langle n \rangle_q = 25.256$.

Quantum integers and correspondence between real and p-adic physics

The understanding of the relationship between real and p-adic physics has been plagued by the fact that canonical identification and its variants do not make sense when applied to say energy levels characterized by integers. In this case the correspondence via common rationals is assumed or I_k for large enough k is used.

The replacement of ordinary integers with their q-counterparts using the proposed rules provides much more general correspondence principle relating p-adic and real quantum physics to each other in the case that the formulas of real physics involve only rationals. For instance, in p-adic mass calculations [K48] the integers characterizing conformal weights would be replaced by their quantum counterparts defined in the proposed manner mapping products to products. This does not affect p-adic mass calculations if the exceptional prime corresponds to p-adic prime and m which is equal to p or contains p as a factor. One can also define p-adic harmonic oscillator and p-adic hydrogen atom and for $n > m$ is large exotic effects become possible. For large values of p-adic prime P and for $m \gg P$ these effects are not detectable.

For the p-adic variants of the wave functions the natural space-time coordinates would be discretized to integers to guarantee that the wave functions exist p-adically for $p = P$. For hydrogen atom (/harmonic oscillator) one would obtain the formal analog of q-Poisson (/q-Gaussian) in the radial coordinate discretized to integer. In angle degrees of freedom the form of discretized wave functions would be same as in real context obtained by replacing $exp(i\phi)$ and $cos(\theta)$ and $sin(\theta)$ with their discretized versions in an algebraic extension of p-adic numbers containing appropriate roots of unity for $p = P$. If the integer m defines the algebraic extension it should be divisible by the integers defining the angular momentum projections M up to some cutoff.

This correspondence might apply even at space-time level and imbedding space-level when preferred coordinates are introduced for imbedding space. This would allow to map the rational imbedding space points of a real space-time surface to their p-adic counterparts by canonical identification. For $(p, m) \rightarrow (\infty, \infty)$ this map would effectively reduce to the identification along common rationals but with respect to p-adic norm it would have totally different behavior (see fig. `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also in the appendix of this `http://www.tgdtheory.fi/appfigures/book.jpg`, which is also).

One can also generalize the notion of p-adic manifolds [K112] defined in terms of canonical identification characterized by two cutoff by replacing integers with quantum integers (see fig. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg` or fig. 15 in the appendix of this book).

13.4 Explanation for the findings of Shnoll

One should be able to undertand both the many-peaked character of the distributions as well as their spatial and temporal variation involving correlations with the gravitational physics of Sun-Earth and Earth-Moon systems.

13.4.1 The basic characteristics of the distributions

The properties of the deformed distributions might allow to explain the findings of Shnoll at least qualitatively. The testing of numerical predictions would require detailed numerical data. It is assumed that the p-adic probabilities can be formally negative with $-1 = (P-1)/(1-P) = (P-1)\sum P^k$ mapped to real number by canonical identification to give a positive number. There are also other options to overcome negativity problem not considered here. The integer m characterizing quantum phase m is not prime but can be assumed to be proportional to P to avoid vanishing quantum integers. This corresponds to m-adicity consistent with P-adicity.

1. The presence of maxima and minima due to canonical identification mapping p-adic distribution function to its p-adic counterpart is consistent with the basic property of the fluctuation distributions as expressed by the histograms for the number $N(n)$, where n is the fluctuating number n of events per fixed time unit or discretization interval for the values of some observable.

2. The basic predictions are following. Modified distributions are characterized by a relatively small prime defining the p-adicity - call it P- and integer m which is not prime but could be divisible by P. The peaks in histogram for $N(n)$ should appear with periods in n giving rise to short range chaos and long range order in variable n. Periods of first kind come as powers of P. A small change of P corresponds to a small change of periodicities. The periods for second kind correspond to integers n_- which contain an odd number of primes l in the ranges $((2r+1)m, (2r+2)m)$, $r = 0, 1, 2, ..$ (quantum phase and thus l_q is same for l and $l + 2rm$). The spectrum of integers $n_\pm$ changes as m changes but if the change is

small, the new spectrum contains integers in old spectrum. For instance, if n_- corresponds to single prime which is in the middle region of interval $(m, 2m)$ a change $|\Delta m| < m/2$ does not remove n_- from spectrum.

3. For instance, in one of the experiments (Fig.1 of [E15] , [E15]) the histogram for $N(n)$ has peaks, which seem to occur periodically with a separation Δn of about 100 units. If these periods correspond to P, its value must be smaller than 100. The nearest primes are $P = 89, 97, 101, 113$. In Fig. 2 of same reference one has also periodicity and P must be near 10. Hence there are good hopes that the proposed model might be able to explain the findings.

4. According to the earlier proposal the selection of p-adic prime is outcome of a process analogous to quantum measurement. This interpretation would suggests that there is a sequence of quantum measurements in which various p-adic primes are selected with some probability each and that the probability distribution for the primes depends on external astrophysical parameters varying periodically. One can also consider the possibility that P and m behave as classical variables.

13.4.2 The temporal and spatial dependence of the distributions

One should also understand the variation of the shape of the distribution with time and its spatial variation.

1. The situation is sensitive to the values of P and m. The changes should be such that the parameters of the smoothed out real probability distribution are not affected much. For instance, in the case of q-Poisson distribution the values of P and m should change in such a manner that $\langle n \rangle = \lambda$ is not unaffected much. The change of P would affect the positions of the peaks but small changes of P would not mean too dramatic changes. Periodic time dependence of these parameters would explain the findings of Shnoll. Gravitational interactions in Sun-Earth-Moon system and therefore the periodic variations of Sun-Earth and Earth-Moon distances is the first guess for the cause of the periodic variations.

2. The correlation of the fluctuation periods with astrophysical periods assignable to Earth-Sun system (diurnal period and period of Earth's orbit) suggests that the gravitational interaction of the measurement apparatus with Sun is involved. Also the period 27.28 days which corresponds to sidereal period of Moon measured in the system defined by distant star. In [E15] , [E15] this period is somewhat confusingly referred to as synodic period of Sun with respect to Earth (recall that synodic period corresponds to a period for the appearance of third object (say Moon) in the same position relative to two other objects (say Earth and Moon)). Therefore also Moon-Earth gravitational force seems to be involved. Moon-Earth and Earth-Sun gravitational accelerations indeed have roughly the same order of magnitude. That gravitational accelerations would determine the effect conforms with Equivalence Principle. The most natural dimensionless parameter characterizing the situation is $|\Delta \mathbf{a_{gr}}|/\mathbf{a_{gr}}$ expressible in terms of $\Delta R/R$ and $\Delta r/r$, where R *resp* r denotes the distance between Earth and Sun *resp.* Earth and Moon, and the ratio R/r and cosine for the angle θ between the direction vectors for the positions of Moon and Sun from Earth. The observed palindrome effect [E17] is consistent with the assumed dependence of the effect on the distances of Earth from Sun and Moon. Also the smallness of the effect as one approaches North Pole conforms with the fact that the variations of distances fro Sun and Moon become small at this limit .

3. In 24 hour time scale it is enough to take into account only the Earth-Sun gravitational interaction. One could perform experiments at different positions at Earth's surface to see whether the the variation of distributions correlates with the variation of the gravitational potential. The maximal amplitude of $\Delta R/R$ is $2R_E/R \simeq .04$ so that for $\Delta p/p = k\Delta R/R$ one would have $\Delta p/p = .04k$. Already for $p \sim 100$ the variation range would be rather small. For $\Delta m/m$ one expects that analogous estimate holds true.

4. One observes in alpha decay rates periodicities which correspond to both sidereal and solar day [E16] . The periodicity with respect to solar day can be understood in terms of the

periodic variation of Sun-Earth distance. The periodicity with respect to sidereal day would be due to the diurnal variation of the Earth-Moon distance. Similar doubling of periodicities are predicted in other relevant time scales.

In the case of alpha decay the effect reveals intricacies not explained by the simplest model [E16, E26] . In this case one studies random fluctuations random fluctuations for the numbers of alpha particles emitted in a fixed direction. Collimators are used to select the alpha particles in a given direction and this is important for what follows. Two especially interesting situations correspond to a detector which is located to North, East, or West from the sample. What is observed that the effect is different for East and West directions and there is a phase shift of 12 hours between East and West. In Northern direction the effect vanishes. Also other experiments reveal East-West asymmetry called local time effect by the authors [E23, E22] .

1. What the findings mean is that P and m characterizing the distribution for the counts of alpha particles in a given angle depend on time and and the time dependence sensitive to the direction angle of the alpha particle. This might be however only apparent since collimators are used to select alpha particles in given direction. The authors speak about anisotropy of space-time and Finsler geometry [A5] could be considered as a possible model. In this approach the geometry of space-time would be something totally independent of measurement apparatus.

   In TGD framework the space-time is topologically non-trivial in macroscopic scales and the presence of collimators making possible to select alpha particles in a given direction affect the geometry of many-sheeted space-time sheets describing the measurement apparatus and therefore the details of the interaction with the gravitational fields of Earth, Sun, and Moon. As a consequence, the values of P and m should reflect the geometry of the measurement apparatus and depend only apparently on the direction of v_α. If this interpretation is correct, a selection of events from a sample without collimators should yield distributions without any dependence on the direction of v_α.

2. At quantitative level the distribution for counts in a given direction can depend on angles defined by the vectors formed from relevant quantities. These include at least the tangential velocity $v = \omega \times r$ of the laboratory, the direction of the velocity v_α of alpha particle with respect to sample actually reflecting the geometry of collimators, the net gravitational acceleration a_{net}, and the direction of Earth's gravitational acceleration g.

3. The first task is to construct from these vectors a scalar or a pseudo-scalar (if one is ready to allow large parity breaking effects), which vanishes for North-East direction, has opposite signs for East and West direction and has at least approximately a behavior consistent with the phase shift of 12 hours between East and West. The constraints are satisfied by the scalar

$$X = E \cdot a_{net} \ , \ E = \frac{(v \times g) \times v_\alpha}{|(v \times g) \times v_\alpha|} \ . \tag{13.4.1}$$

   Unit vector E changes sign in East-West permutation and also with a period of 12 hours meaning the change of the roles of East and West with this period in the approximation that the net acceleration vector is same at the opposite sides of Earth. The approximation makes sense if the change of sign induces much large variation than the change of the Earth-Sun and Earth-Moon distances. Unless P and m are even functions of X, the predicted effect can be consistent with the experimental findings in the approximation that a_{net} is constant in 24 hour time scale.

13.5 Hierarchy of Planck constants allows small-p p-adicity

In particle physics applications of p-adic physics [K48] the values of p-adic primes are very large and favor p-adic primes near powers of two. For instance, electron is characterized by a p-adic prime $M_{127} = 2^{127} - 1$. Small p-adic primes correspond to very short time and length scales, which are not plausible in the recent situation. Biological systems however suggest the possibility of small values of p. This is consistent with p-adic length scale hypothesis if one accepts the hypothesis that dark matter corresponds to a hierarchy of Planck constants coming as integer multiples of the ordinary Planck constant $\hbar_0$: $\hbar/\hbar_0 = r$, r integer.

13.5.1 Estimate for the value of Planck constant

In the recent formulation of quantum TGD the hierarchy of Planck constants there is an argument reducing the hierarchy of Planck constants to the basic quantum TGD and one can say that scaled up values of Planck constant are effective values of Planck constant. The scaling of the p-adic prime scales up the secondary time scale assignable with the particle characterized by prime p as $T_k = 2^k T_{CP_2} \rightarrow rT_k$. Here T_{CP_2} denotes CP_2 time expressible as $T_{CP_2} = 2^{-127}T(2,127) \simeq 5.877 \times 10^{-40}$ seconds. There $T(2,127) \simeq .1$ seconds is secondary p-adic time scale assignable to Mersenne prime M_{127} characterizing electron. T_{CP_2} is 1.0902×10^4 times Planck time $T_{Pl} = 5.391 \times 10^{-44}$ s.

To obtain small-p p-adicity one must have very large value of r. The proposed quantum model for dark matter in astrophysical scales indeed predicts gigantic values of gravitational Planck constant of order GMm for a system of two masses. This would suggests that gravitational interaction allows large values of Planck constant and small-p p-adicity in macroscopic time scales.

In the experiments described in [E15] , [E15] one studies the number of events per fixed time interval τ. This time interval is macroscopic in the measurements studied. One has $\tau = 36$ seconds ($\tau = 6$ seconds) in the experiment whose histogram is represented by Fig. 1 (Fig. 2) of [E15] , [E15] . One could argue that the secondary p-adic time scale $T_P(2) = rPT_{CP_2}$ for scaled up Planck constant $\hbar = r\hbar_0$ should of the same order of magnitude as τ. This gives the condition

$$r \sim \frac{\tau}{PT_{CP_2}} < \frac{\tau}{T_{CP_2}} \quad .$$

For $\tau = 36$ seconds one has $\frac{\tau}{T_{CP_2}} \simeq 360 \times M_{127}$. For $r = 2^{127}$ this would give $P \sim 360$. The value of P estimated from the distribution of Fig.1 of [E15] , [E15] is about $P \sim 100$ which is about 3.5 times smaller than the upper bound. This suggests that one p-adic time scale must be shorter than τ but of same order of magnitude. For the second experiment (Fig. 2 of [E15] , [E15]) one would obtain $P \leq 50$ which is 5 times larger than the estimate for $P \sim 10$ from periodicity.

$r = 2^{127}$ might make sense since M_{127} defines the secondary p-adic length scale of electron which is .1 seconds, a fundamental bio-rhythm, and corresponds to photon wavelength which is of order of circumference of Earth. This would also suggest that the modification of distributions could correspond to same value of P and m for laboratories at different sides of globe. Whether this is the case is easy to test in principle.

The notion of causal diamond (intersection of future and past directed light-cones central for the notion of zero energy ontology. The proper time distance between its tips is given by $2^k T_{CP_2}$ and assign to each elementary particle a macroscopic time scale identifiable as secondary p-adic time scale characterizing the particle. $T(127) = 2^{127}T_{CP_2}$ characterizes the causal diamond of electron, which in turn corresponds to the length scale assigned with $P = 2$ and $r = 2^{126}$. Could $r = 2^{126}$ be in preferred role that the findings of Shnoll would reflect new physics associated with electron, possibly with its gravitational interactions?

13.5.2 Is dark matter at the space-time sheets mediating gravitational interaction involved?

The periodic variation of the distributions in time scales assignable to gravitation encourages to ask whether the gigantic value of Planck constant could correspond to gravitational Planck constant introduced originally by Nottale [E25] and assumed in TGD Universe to characterize space-time sheets mediating gravitational interaction and carrying dark manner -at least gravitons- with gigantic value of Planck constant implying quantum coherence in astrophysical scales [K75, K62] .

The formula proposed by Nottale [E25] for the gravitational Planck constant is dictated by Equivalence Principle and reads as

$$r_{gr} = \frac{\hbar_{gr}}{\hbar_0} = \frac{GMm}{v_0} \quad . \tag{13.5.1}$$

Here v_0 is a parameter with dimensions of velocity and one has $v_0/c \simeq 2^{-11}$ for the inner planets in the model of Nottale and 5 times smaller for outer planets. As a matter fact, the order of magnitude of the rotation velocity of planet around Sun is related to v_0 by numerical constant of order unity by Bohr rules, which in TGD Universe are an exact part of quantum theory.

If the large value of $\hbar_{gr}$ is associated with the gravitational interaction of smaller system with Earth with mass $M_E = 5.9737 \times 10^{24}$ kg, the mass of the system in question should be estimated from the condition

$$r = M^{127} = \frac{GM_E m}{v_0 \hbar_0} \quad . \tag{13.5.2}$$

This gives $m \simeq 135 \times \frac{v_0}{c}$ kg. For $v_0 = 2^{-11}$ this would give mass about $m = .05$ g which might represent mass for some part of measurement apparatus. The mass of Sun is $M_{Sun} \simeq .333 \times 10^6 M_E$ and similar estimate gives a mass $m = .15 \times 10^{-9}$ kg to be compared with Planck mass $m_{Pl} = 4.3 \times 10^{-9}$ kg. For $c/v_0 = 70$ the estimate would give Planck mass. Note however that it is difficult to relate this value of v_0 to any velocity in Earth-Sun system. For the density of water Planck mass corresponds to a size scale 10^{-4} m assignable to a large cell.

Maybe dark matter systems representing the quanta of gravitational flux equal to Planck mass analogous to quanta of electric flux are involved and are important also for biological systems. The interaction of Planck mass with Earth's gravitational field would correspond to $r = 3 \times 2^{107}$: M_{107} defines the p-adic length scale assignable to hadrons.

13.6 Conclusions

The proposed model has the potential of explaining the findings of Shnoll but detailed numerical work is required to find whether the model works also at the level of details.

1. The universality of the modified distributions would reduce to the replacement of various rational numbers characterizing the probability distribution with their quantum variants defined in a manner respecting the decomposition of integers to primes. p-Adic counterparts of probability distributions are essential for understanding how to avoid the difficulties resulting from negative values of quantum integers. The model makes very detailed predictions about the periodically occurring positions of the peaks of the probability distribution as function of P and m based on number theoretical considerations and in principle allows to determined these parameters for a given distribution.

2. If the value of P is outcome of state function process, it is not determined by deterministic dynamics but should have a distribution. If this distribution is peaked around one particular value, one can understand the findings of Shnoll.

3. The slow variation of the p-adic prime P and integer m characterizing quantum integers would explain the slow variation of the distributions with position and time. The periodic variations occurings withboth solar and sidereal periods can be understood if the values of P and m are characterized by the sum of gravitational accelerations assignable to Earth-Sun and Earth-Moon systems.

4. Various effects such as the dependence of the probability distributions on the direction of alpha particles selected using collimators and 12 hour phase shift between the directions associated with East and West direction can be understood as direct evidence for the effects of measurement apparatus on the many-sheeted space-time affecting the values of P and m.

5. The small value of p-adic prime P involved can be understood in TGD framework in terms of hierarchy of Planck constants [K33] . The value of Planck constant could correspond to

Mersenne prime M_{127} characterizing electron but this is not required by any deep principle. Gravitational Planck constant can indeed have gigantic values and for the interaction of a system with mass of order Planck mass with Sun the gravitational Planck constant is of the required order of magnitude.

Acknowledgements: I am grateful for Dainis Deps for references related to Shnoll effect.

Chapter 1

Appendix

A-1 Introduction

The great dream of a physicist believing in reductionism and TGD would be a formalism generalizing Feynman diagrams allowing any graduate student to compute the predictions of the theory. TGD has forced myself to give up naive reductionism but I believe that TGD allows generalization of Feynman diagram in such a manner that one gets rid of the infinities plaguing practically all existing theories. The purpose of this chapter is to develop general vision about how this might be achieved. The vision is based on generalization of mathematical structures discovered in the construction of topological quantum field theories (TQFT), conformal field theories (CQFT). In particular, the notions of Hopf algebras and quantum groups, and categories are central. The following gives a very concise summary of the basic ideas.

In introduced the original version of this chapter in extended form long time ago having also remarkably long title. I ended up with diagrams which had category theoretical meaning and had huge symmetries generalizing the duality symmetry of the hadronic string model and allowing to reduce their number dramatically leaving only tree diagrams still having symmetries. These diagrams were however not Feynman diagrams and this made me scared. After the emergence of twistor Grassmann approach and twistor diagrams the situation changed completely and it might be that I was on the correct track after all. This chapter is what was left from my adventure.

1. Feynman diagrams as generalized braid diagrams

The first key idea is that generalized Feynman diagrams with diagrams analogous to knot and link diagrams in the sense that diagrams involving loops are equivalent with tree diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of preferred extremal of Kähler action (perhaps absolute minimum in Eucdlian space-time regions) as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. S-matrix element as a representation of a path defining continuation of configuration space spinor field between different sectors of it corresponding different 3-topologies leads also to the conclusion that all continuations and corresponding Feynman diagrams are equivalent. Universe as a compute metaphor idea allowing quite concrete realization by generalization of what is meant by space-time point leads to the view that generalized Feynman diagrams characterize equivalent computations.

2. Coupling constant evolution from infinite number of critical values of Kähler coupling strength

The basic objection that this vision does not allow to understand coupling constant evolution involving loops in an essential manner can be circumvented.

Quantum criticality requires that Kähler coupling constant α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that gauge couplings have an infinite number of possible values labelled by p-adic length scales and probably also by the fractal dimensions of effective tensor factors defined hierarchy of II_1 factors (so called Beraha numbers).

The dependence on p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to angular resolution and finite-dimensional extensions of p-adic number fields R_p. Finite resolution and renormalization group evolution are forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically.

TGD suggests discrete p-adic coupling constant evolution in which coupling constants are renormalization group invariants for the evolution associated with given p-adic prime p. This would mean vanishing of loops obtained also in $\mathcal{N} = 4$ SUSY allowing twistorialization. Gauge couplings could depend on prime p characterizing the p-adic length scale. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

The discrete p-adic coupling constant evolution should relate to the continuous RG evolution of QFTs. This requires understanding of how this space-time corresponds to the many-sheeted space-time of TGD. GRT space-time as effective space-time obtained by replacing the many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Also gauge potentials of standard model would correspond classically to superpositions of induced gauge potentials over space-time sheets. Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Ordinary continuous coupling constant evolution would follow only at GRT-QFT limit.

3. Equivalence of loop diagrams with tree diagrams from the axioms of generalized ribbon category

A further was that Hopf algebra related structures and appropriately generalized ribbon categories could provide a concrete realization of this picture. Generalized Feynman diagrams which are identified as braid diagrams with strands running in both directions of time and containing besides braid operations also boxes representing algebra morphisms with more than one incoming and outgoing strands describing particle reactions (3-particle vertex should be enough). In particular, fusion of 2-particles and decay of particle to two would correspond to generalizations of algebra product μ and co-product Δ to morphisms of the category defined by the super-symplectic algebras associated with 3-surfaces with various topologies and conformal structures. The basic axioms for this structure generalizing Hopf algebra axioms state that diagrams with self energy loops, vertex corrections, and box diagrams are equivalent with tree diagrams.

To sum up, I could not develop this project further - mainly by the lack of needed mathematical knowhow, and this chapter is more like an Appendix. It might however be that twistorial approach could allow to concretize the idea about equivalence of loop diagrams with tree diagrams for generalization of Feynman diagrams. What is certain, that the notion of coupling constant evolution aat the level of many-sheeted space-time, should be very simple in TGD- maybe trivial for a given algebraic extension of given p-adic number field. Only at the QFT limit the lumping of sheets to single one is expected to induce complexities.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://www.tgdtheory.fi/cmaphtml.html` [L20]. Pdf representation of same files serving as a kind of glossary can be found at `http://www.tgdtheory.fi/tgdglossary.pdf` [L21]. The topics relevant to this chapter are given by the following list.

- Hyperfinite factors and TGD [L29]

A-2 Hopf algebras and ribbon categories as basic structures

In this section the basic notions related to Hopf algebras and categories are discussed from TGD point of view. Examples are left to appendix. The new element is the graphical representation of the axioms leading to the idea about the equivalent of loop diagrams and tree diagrams based on general algebraic axioms.

A-2.1 Hopf algebras and ribbon categories very briefly

An algebraic formulation generalizing braided Hopf algebras and related structures to what might be called quantum category would involve the replacement of the co-product of Hopf algebras with morphism of quantum category having as its objects the Clifford algebras associated with WCW spinor structure for various 3-topologies. The corresponding Fock spaces would would define algebra modules and the objects of the category would consists of pairs of algebras and corresponding modules. The underlying primary structure would be second quantized free induced spinor fields associated with 3-surfaces with various 3-topologies and generalized conformal structures.

1. Bi-algebras

Bi-algebras have two algebraic operations. Besides ordinary multiplication $\mu : H \otimes H \rightarrow H$ there is also co-multiplication $\Delta : H \rightarrow H \otimes H$. Algebra satisfies the associativity axiom (Ass): $a(bc) = (ab)c$, or more formally, $\mu(id \otimes \mu) = \mu(\mu \otimes id)$, and the unit axiom (Un) stating that there is morphism $\eta : k \rightarrow A$ mapping the unit of A to the unit of field k. Commutativity axiom (Co) $ab = ba$ translates to $\mu \otimes \tau \equiv \mu^{op} = \mu$, where τ permutes factors in tensor product $A \otimes A$.

Δ satisfies mirror images of these axioms. Co-associativity axiom (Coass) reads as $(\Delta \otimes id)\Delta = (id \otimes \Delta)\Delta$, co-unit axiom (Coun) states existence of morphism $\epsilon : k \rightarrow C$ mapping the unit of A to that of k, and co-commutativity (Coco) reads as $\tau \circ \Delta \equiv \Delta^{op} = \Delta$. For a bi-algebra H also additional axioms are satisfied: in particular, Δ (μ) acts as algebra (bi-algebra) morphism. When represented graphically, this constraint states that a box diagram is equivalent to a tree diagram as will be found and served as the stimulus for the idea that loop diagrams might be equivalent with tree diagrams.

Left and right algebra modules and algebra representations are defined in an obvious manner and satisfy associativity and unit axioms. A left co-module corresponds a pair (V, Δ_V) where the co-action $\Delta_N : V \rightarrow A \otimes V$ satisfies co-associativity and co-unit axioms. Right co-module is defined in an analogous manner.

Particle fusion $A \otimes B \rightarrow C$ corresponds to μ: $A \otimes B \rightarrow C = AB$. Co-multiplication Δ corresponds time reversal $C \rightarrow A \otimes B$ of this process, which is kind of a time-reversal for multiplication. The generalization would mean that μ and Δ become morphisms $\mu : B \otimes C \rightarrow A$ and $\Delta : A \rightarrow B \otimes C$, where A, B, C are objects of the quantum category. They could be either representations of same algebra or even different algebras.

2. Drinfeld's quantum double

Drinfeld's quantum double [A55] is a braided Hopf algebra obtained by combining Hopf algebra $(H, \mu, \Delta, \eta, \epsilon, S, R)$ and its dual $H^\star$ to a larger Hopf algebra known as quasi-triangular Hopf algebra satisfying $\Delta = R\Delta^{op}R^{-1}$, where $\Delta^{op}(a)$ is obtained by permuting the two tensor factors. Duality means existence of a scalar product and the two algebras correspond to Hermitian conjugates of each other.

In TGD framework the physical states associated with these algebras have opposite energies since in TGD framework antimatter (or matter depending on the phase of matter) corresponds to negative energy states. The states of the Universe would correspond to states with vanishing conserved quantum numbers, and in concordance with crossing symmetry, particle reactions could be interpreted as transitions generating zero energy states from vacuum.

The notion of duality [A55] is needed to define an inner product and S-matrix. Essentially Dirac's bra-ket formalism is in question. The so called evaluation map $ev : V \otimes V^\star \rightarrow k$ defined as $ev(v^i \otimes v_j) = \langle v^i, v_j\rangle = \delta_{ij}$ defines an inner product in any Hopf algebra module. The inverse of this map is the linear map $k \rightarrow V$ defined by $\delta_v(1) = v_i \otimes v^i$. For a tensor category with unit I, field k is replaced with unit I, and left duality these maps are replaced with maps $b_V : I \rightarrow V \otimes V^\star$ and $d_V = V \otimes V^\star \rightarrow I$. Right duality is defined in an analogous manner. The map d_V assigns to a given zero energy state S-matrix element. Algebra morphism property $b_V(ab) = b_V(a)b_V(b)$ would mean that the outcome is essentially the counterpart of free field theory Feynman diagram. This diagram is convoluted with the S-matrix element coded to the entanglement coefficients between positive and negative energy particles of zero energy state.

3. Ribbon algebras and ribbon categories

The so called ribbon algebra [A55] is obtained by replacing one-dimensional strands with ribbons and adding to the algebra the so called twist operation θ acting as a morphism in algebra and in any algebra module. Twist allows to introduce the notion of trace, in particular quantum trace.

The thickening of one-dimensional strands to 2-dimensional ribbons is especially natural in TGD framework, and corresponds to a replacement of points of time=constant section of 4-surface with one-dimensional curves along which the S-matrix defined by R-matrix is constant. Ribbon category is defined in an obvious manner. There is also a more general definition of ribbon category with objects identified as representations of a given algebra and allowing morphisms with arbitrary number of incoming and outgoing strands having interpretation as many-particle vertices in TGD framework. The notion of quantum category defined as a generalization of a ribbon category involving the generalization of algebra product and co-product as morphisms between different objects of the category and allowing objects to correspond different algebras might catch the essentials of the physics of TGD Universe.

A-2.2 Algebras, co-algebras, bi-algebras, and related structures

It is useful to formulate the notions of algebra, co-algebra, bi-algebra, and Hopf algebra in order to understand how they might help in attempt to formulate more precisely the view about what generalized Feynman diagrams could mean. Since I am a novice in the field of quantum groups, the definitions to be represented are more or less as such from the book "Quantum Groups" of Christian Kassel [A55] with some material (such as the construction of Drinfeld double) taken from [A27]. What is new is a graphical representation of algebra axioms and the proposal that algebra and co-algebra operations have interpretation in terms of generalized Feynman diagrams.

In the following considerations the notation id_k for the isomorphism $k \rightarrow k \otimes k$ defined by $x \rightarrow x \otimes x$ and its inverse will be used.

Algebras

Algebra can be defined as a triple (A, μ, η), where A is a vector space over field k and $\mu : A \otimes A \rightarrow A$ and $\eta : k \rightarrow A$ are linear maps satisfying the following axioms (Ass) and (Un).

(Ass): The square

$$\begin{array}{ccc} A\otimes A\otimes A & \xrightarrow{\mu\otimes id} & A\otimes A \\ \downarrow id\otimes\mu & & \downarrow\mu \\ A\otimes A & \xrightarrow{\mu} & A \end{array} \qquad \text{(A-2.1)}$$

commutes.

(Un): The diagram

$$\begin{array}{ccccc} k\otimes A & \xrightarrow{\eta\otimes id} & A\otimes A & \xleftarrow[id\otimes\eta]{} & A\otimes k \\ & \searrow^{\cong} & \downarrow\mu & \swarrow_{\cong} & \\ & & A & & \end{array} \qquad \text{(A-2.2)}$$

commutes. Note that η imbeds field k to A.

(Comm) If algebra is commutative, the triangle

$$\begin{array}{ccc} A\otimes A & \xrightarrow{\tau_{A,A}} & A\otimes A \\ & \searrow^{\mu} \quad \swarrow_{\mu} & \\ & A & \end{array} \qquad \text{(A-2.3)}$$

commutes. Here $\tau_{A,A}$ is the flip switching the factors: $\tau_{A,A}(a \otimes a') = a' \otimes a$.

A morphism of algebras $f : (A, \mu, \eta) \to (A', \mu', \eta')$ is a linear map $A \to A'$ such that

$$\mu' \circ (f \otimes f) = f \circ \mu, \quad \text{and} \quad f \circ \eta = \eta' \ .$$

A graphical representation of the algebra axioms is obtained by assigning to the field k a dashed line to be referred as a vacuum line in the sequel and to A a full line, to η a vertex $\times$ at which k-line changes to A-line. The product μ can be represented as 3-particle vertex in which algebra lines fuse together. The three axioms (Ass), (Un) and (Comm) can are expressed graphically in figure A-2.2.

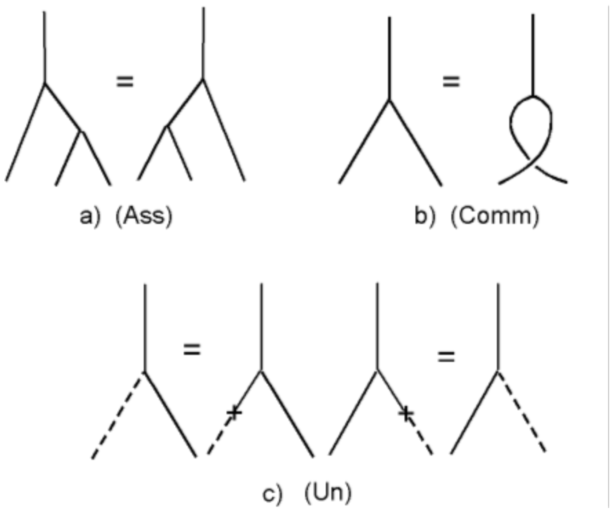

Figure 1: Graphical representation for the axioms of algebra. a) $a(bc) = (ab)c$, b) $ab = ba$, c) $ka = \mu(\eta(k), a)$ and $ak = \mu(a, \eta(k))$.

Note that associativity axiom implies that two tree diagrams not equivalent as Feynman diagrams are equivalent in the algebraic sense.

Co-algebras

The definition of co-algebra is obtained by systematically reversing the directions of arrows in the previous diagrams.

A co-algebra is a triple(C, Δ, ϵ), where C is a vector space over field k and $\Delta : C \to C \otimes C$ and $\epsilon : C \to k$ are linear maps satisfying the following axioms (Coass) and (Coun).

(Coass): The square

$$\begin{array}{ccc} C & \xrightarrow{\Delta} & C \otimes C \\ \downarrow\Delta & & \downarrow id \otimes \Delta \\ C \otimes C & \xrightarrow{\Delta \otimes id} & C \otimes C \otimes C \end{array} \tag{A-2.4}$$

commutes.

(Coun): The diagram

$$\begin{array}{ccccc} k \otimes C & \xleftarrow[\epsilon \otimes id]{} & C \otimes C & \xrightarrow{id \otimes \epsilon} & C \otimes k \\ & \nwarrow^{\cong} & \Delta\uparrow & \nearrow^{\cong} & \\ & & C & & \end{array} \tag{A-2.5}$$

commutes. The map Δ is called co-product or co-multiplication whereas ϵ is called the counit. The commutative diagram state that the co-product is co-associative and that co-unit commutes with co-product.

(Cocomm) If co-algebra is commutative, the triangle

$$\begin{array}{ccc} & C & \\ \swarrow^{\Delta} & & \searrow^{\Delta} \\ C \otimes C & \xrightarrow{\tau_{C,C}} & C \otimes C \end{array} \tag{A-2.6}$$

commutes. Here $\tau_{C,C}$ is the flip switching the factors: $\tau_{C,C}(c \otimes c') = c' \otimes c$.

A morphism of co-algebras $f : (C, \Delta, \epsilon) \to (C', \Delta', \epsilon')$ is a linear map $C \to C'$ such that

$$(f \otimes f) \circ \Delta = \Delta' \circ f \ \ , \ \ \text{and} \ \ \epsilon = \epsilon' \circ f \ \ .$$

It is straightforward to define notions like co-ideal and co-factor algebra by starting from the notions of ideal and factor algebra. A very useful notation is Sweedler's sigma notation for $\Delta(x)$, $x \in C$ as element of $C \otimes C$:

$$\Delta(x) = \sum_i x_i' \otimes x_i'' \equiv \sum_{\{x\}} x' \otimes x'' \ \ .$$

Also co-algebra axioms allow graphical representation. One assigns to ϵ a vertex $\times$ at which C-line changes to k-line: the interpretation is as an absorption of a particle by vacuum. The co-product Δ can be represented as 3-particle vertex in which C-line decays to two C-lines. The graphical representation of the three axioms (Coass), (Coun), and (Cocomm) is related to the representation of algebra axioms by "time reversal", that is turning the diagrams for the algebra axioms upside down (see figure A-2.2).

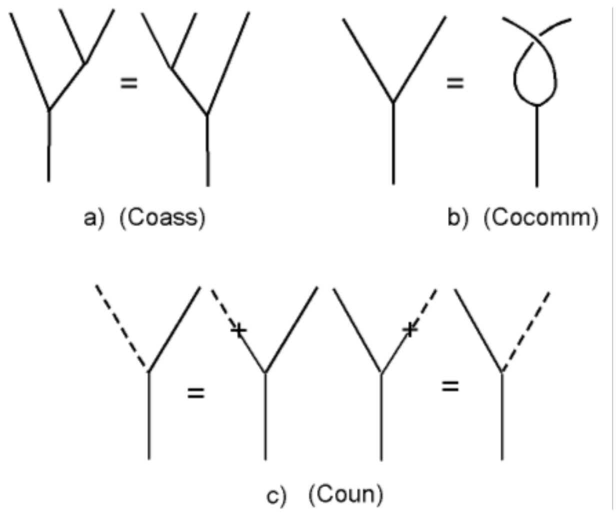

Figure 2: Graphical representation for the axioms of co-algebra is obtained by turning the representation for algebra axioms upside down. a) $(id \otimes \Delta)\Delta = (\Delta \otimes id)\Delta$, b) $\Delta = \Delta^{op}$, c) $(\epsilon \otimes id) \circ \Delta = (id \otimes \epsilon) \circ \Delta = id$.

Bi-algebras

Consider next a vector space H equipped simultaneously with an algebra structure (H, μ, η) and a co-algebra structure (H, Δ, ϵ). There are some compatibility conditions between these two structures. $H \otimes H$ can be given the induced structures of a tensor product of algebras and of co-algebras.

The following two statements are equivalent.

1. The maps μ and η are morphisms of co-algebras. For μ this means that the diagrams

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\mu} & H \\ \downarrow{\scriptstyle (id\otimes\tau\otimes id)\otimes(\Delta\otimes\Delta)} & & \downarrow{\scriptstyle \Delta} \\ (H\otimes H)\otimes(H\otimes H) & \xrightarrow{\mu\otimes\mu} & H\otimes H \end{array} \tag{A-2.7}$$

and

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\epsilon\otimes\epsilon} & k\otimes k \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle id} \\ H & \xrightarrow{\epsilon} & k \end{array} \tag{A-2.8}$$

commute. For η this means that the diagrams

$$\begin{array}{ccc} k & \xrightarrow{\eta} & H \\ \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle \Delta} \\ k\otimes k & \xrightarrow{\eta\otimes\eta} & H\otimes H \end{array} \qquad \begin{array}{ccc} k & \xrightarrow{\eta} & H \\ & {\scriptstyle id}\searrow \quad \swarrow{\scriptstyle \epsilon} & \\ & k & \end{array} \tag{A-2.9}$$

commute.

2. The maps Δ and ϵ are morphisms of algebras.
For Δ this means that diagrams

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\Delta\otimes\Delta} & (H\otimes H)\otimes(H\otimes H) \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle (\mu\otimes\mu)(id\otimes\tau\otimes id)} \\ H & \xrightarrow{\Delta} & H\otimes H \end{array} \tag{A-2.10}$$

and

$$\begin{array}{ccc} k & \xrightarrow{\eta} & H \\ \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle \Delta} \\ k\otimes k & \xrightarrow{\eta\otimes\eta} & H\otimes H \end{array} \tag{A-2.11}$$

commute.

For ϵ this means that the diagrams

$$\begin{array}{ccccccc} H\otimes H & \xrightarrow{\epsilon\otimes\epsilon} & k\otimes k & \quad & k & \xrightarrow{\eta} & H \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle id} & & & {\scriptstyle id}\searrow \quad \swarrow{\scriptstyle \epsilon} & \\ H & \xrightarrow{\epsilon} & k & & & k & \end{array} \tag{A-2.12}$$

commute. The proof of the theorem involves the comparison of the commutative diagrams expressing both statements to see that they are equivalent.

The theorem inspires the following definition.

Definition: A bi-algebra is a quintuple $(H, \mu, \eta, \Delta, \epsilon)$, where (H, μ, η) is an algebra and (H, Δ, ϵ) is co-algebra satisfying the mutually equivalent conditions of the previous theorem. A morphisms of bi-algebras is a morphism for the underlying algebra and bi-algebra structures.

An element $x \in H$ is known as primitive if one has $\Delta(x) = 1\otimes x + x\otimes 1$ and have $\epsilon(x) = 0$. The subspace of primitive elements is closed with respect to the commutator $[x, y] = xy - yx$. Note that for primitive elements $\mu \circ \Delta = 2id_H$ holds true so that $\mu/2$ acts as the left inverse of Δ.

Given a vector space V, there exists a unique bi-algebra structure on the tensor algebra $T(V)$ such that $\Delta(v) = 1\otimes v + v\otimes 1$ and $\epsilon(v) = 0$ for any element v of V. By the symmetry of Δ this bi-algebra structure is co-commutative and corresponds to the "classical limit". Also the Grassmann algebra associated with V allows bi-algebra structure defined in the same manner.

Figure A-2.2 provides a representation for the axioms of bi-algebra stating that Δ and ϵ act as algebra morphisms of algebra and or equivalent that μ and η act as co-algebra morphisms. The axiom stating that Δ (μ) is algebra (co-algebra) morphism implies that scattering diagrams differing by a box loop are equivalent. The statement that μ is co-algebra morphism reads $(id\otimes\mu\otimes id)(\Delta\otimes\Delta) = \Delta\circ\mu$ whereas the mirror statement $\Delta(ab) = \Delta(a)\Delta(b)$ for Δ reads as $\Delta\circ\mu = \mu(\Delta\otimes\Delta)$ and gives rise to the same graph.

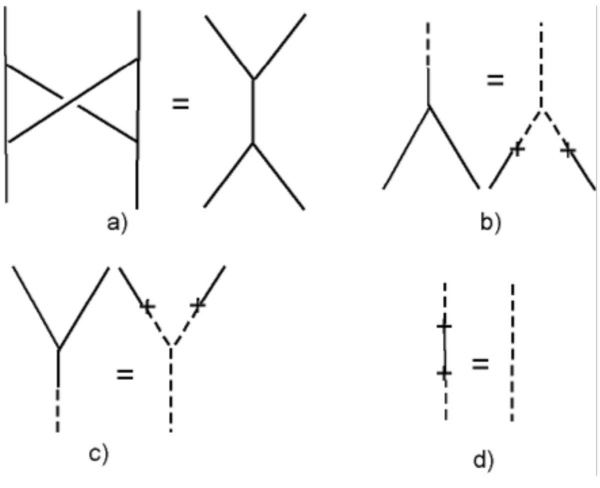

Figure 3: Graphical representation for the conditions guaranteeing that μ and η (Δ and ϵ) act as homomorphisms of co-algebra (algebra). a)$(id\otimes\mu\otimes id)(\Delta\otimes\Delta) = \Delta\circ\mu$, b) $\epsilon\circ\mu = id\circ(\epsilon\otimes\epsilon)$, c) $\Delta\circ\eta = \mu\otimes\circ id_k$, d) $\epsilon\circ\eta = id_k$.

Hopf algebras

Given an algebra (A, μ, η) and co-algebra (C, Δ, ϵ), one can define a bilinear map, the convolution on the vector space $Hom(C, A)$ of linear maps from C to A. By definition, if f and g are such linear maps, then the convolution $f \star g$ is the composition of the maps

$$C \underset{\Delta}{\rightarrow} C \otimes C \underset{f \otimes g}{\rightarrow} A \otimes A \underset{\mu}{\rightarrow} A \quad \text{(A-2.13)}$$

Using Sweedler's sigma notion one has

$$f \star g(x) = \sum_{\{x\}} f(x')g(x'') \quad . \quad \text{(A-2.14)}$$

It can be shown that the triple $(Hom(C,A), \star, \Delta, \eta \circ \epsilon)$ is an algebra and that the map $\Lambda_{C,A}$: $A \otimes C^\star \rightarrow Hom(C,A)$ defined as

$$\Lambda_{C,A}(a \otimes \gamma)(c) = \gamma(c)a$$

is a morphism of algebras, where $C^\star$ is the dual of the finite-dimensional co-algebra C.

For $A = C$ the result gives a mathematical justification for the crossing symmetry inspired re-interpretation of the unitary S-matrix interpreted usually as an element of $Hom(A,A)$ as a state generated by element of $A \otimes A^\star$ from the vacuum $|vac\rangle = |vac_A\rangle \otimes |vac_{A^\star}\rangle$. This corresponds to the interpretation of the reaction $a_i|vac_A\rangle \rightarrow a_f|vac_A\rangle$ as a transition creating state $a_i \otimes a_f^\star|vac\rangle$ with vanishing conserved quantum numbers from vacuum.

With these prerequisites one can introduce the notion of Hopf algebra. Let $(H, \mu, \eta, \Delta, \epsilon)$ be a bi-algebra. An endomorphism S of H is called an antipode for the bi-algebra H if

$$S \star id_H = id_H \star S = \eta \circ \epsilon \quad .$$

A Hopf algebra is a bi-algebra with an antipode. A morphism of a Hopf algebra is a morphism between the underlying bi-algebras commuting with the antipodes.

The graphical representation of the antipode axiom is given in the figure below.

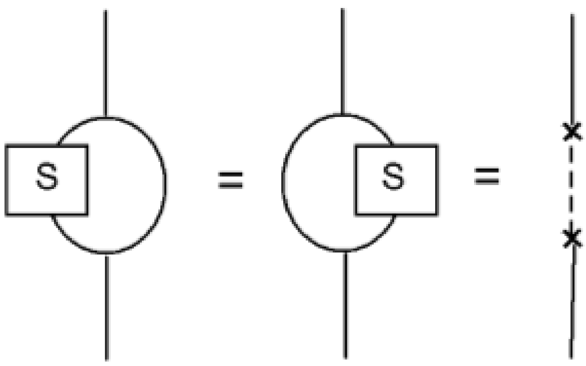

Figure 4: Graphical representation of antipode axiom $S \star id_H = id_H \star S = \eta \circ \epsilon$.

The notion of scalar product central for physical applications boils down to the notion of duality. Duality between Hopf algebras U and H means the existence of a morphism $x \rightarrow \Psi(x)$: $H \rightarrow U^\star$ defined by a bilinear form $\langle u, x\rangle = \Psi(x)(u)$ on $U \times H$, which is a bi-algebra morphism. This means that the conditions

$$\begin{aligned} &\langle uv, x\rangle = \langle u \otimes v, \Delta(x)\rangle \ , \quad \langle u, xy\rangle = \langle \Delta(u), x \otimes y\rangle \ , \\ &\langle 1, x\rangle = \epsilon(x) \ , \qquad\qquad \langle u, 1\rangle = \epsilon(u) \ , \\ &\langle S(u), x\rangle = \langle u, S(x)\rangle \end{aligned} \quad \text{(A-2.15)}$$

are satisfied. The first condition on multiplication and co-multiplication, when expressed graphically, states that the decay $x \to u \otimes v$ can be regarded as time reversal for the fusion of $u \otimes v \to x$. Second condition has analogous interpretation.

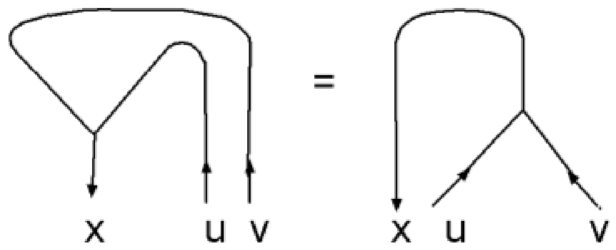

Figure 5: Graphical representation of the duality condition $\langle uv, x\rangle = \langle u \otimes v, \Delta(x)\rangle$.

Modules and comodules

Left and right algebra modules and algebra representations are defined in an obvious manner and satisfy associativity and unit axioms having diagrammatic representation similar to that for corresponding algebra axioms.

A left co-module corresponds a pair (V, Δ_V), where the co-action Δ_N: $V \to C \otimes V$ satisfies co-associativity axiom $(id_C \otimes \Delta_N) \circ \Delta_N = (\Delta \otimes id_N) \circ \Delta_N$ and co-unit axiom $(\epsilon \otimes id) \circ \Delta_N = id_N$. A right co-module is defined in an analogous manner. It is convenient to introduce Sweedlers's notation for Δ_N as $\Delta_N = \sum_{\{c\}} x_C \otimes x_N$.

One can define module and comodule morphisms and tensor product of modules and co-modules in a rather obvious manner. The module N could be also algebra, call it A, in which case μ_A and η_A are assumed to act as H-comodule morphisms.

The standard example is quantum plane $A = M(2)_q$ is the free algebra generated variables x, y subject to to relations $yx = qxy$ and having coefficients in k. The action of Δ_A reads as

$$\Delta_A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} x \\ y \end{pmatrix} \quad .$$

Δ_A defines algebra morphism from A to $SL_e(2)_q \otimes A$: $\Delta_a(yx) = \Delta_A(y)\Delta_A(x) = q\Delta_A(x)\Delta_A(y) = \Delta(qxy)$.

Braided bi-algebras

$\Delta^{op} = \tau_{H,H} \circ \Delta$ defines the opposite co-algebra H^{op} of H. A braided bi-algebra $(H, \mu, \eta, \Delta, \epsilon)$ is called quasi-co-commutative (or quasi-triangular) if there exists an element R of algebra $H \otimes H$ such that for all $x \in H$ one has

$$\Delta^{op} = R\Delta R^{-1} \quad .$$

One can express R in the form

$$R = \sum_i s_i \otimes t_i \quad .$$

It is convenient to denote by R_{ij} the R matrix acting in i^{th} and j^{th} tensor factors of n^{th} tensor power of H. More precisely, R_{ij} can be defined as an operator acting in an n-fold tensor power of H by the formula $R_{ij} = y^{(1)} \otimes y^{(2)} \otimes ... \otimes y^{(p)}$, $p \leq n$, $y^{(k_i)} = s_i$ and $y^{(k_j)} = t_j$, $y^{(k)} = 1$ otherwise. For instance, one has $R_{13} = \sum_i s_i \otimes 1 \otimes t_i$.

With these prerequisites one can define a braided bi-algebra as a quasi-commutative bi-algebra $(H, \mu, \eta, \Delta, \epsilon, S, S^{-1}, R)$ as an algebra with a preferred element $R \in H \otimes H$ satisfying the two relations

$$
\begin{aligned}
(\Delta \otimes id_H)(R) &= R_{13}R_{23} \ , \\
(id_H \otimes \Delta)(R) &= R_{13}R_{12} \ .
\end{aligned}
\tag{A-2.16}
$$

Braided bi-algebras, known also as quasi-triangular bi-algebras, are central in the theory of quantum groups, R-matrices, and braid groups. By a direct calculations one can verify the following relations.

1. Yang-Baxter equations

$$
R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \ , \tag{A-2.17}
$$

   and the relation

$$
(\epsilon \otimes id_H)(R) = 1 \tag{A-2.18}
$$

   hold true.

2. Since H has an invertible antipode S, one has

$$
\begin{aligned}
(S \otimes id_H)(R) &= R^{-1} = (id_H \otimes S^{-1})(R) \ , \\
(S \otimes S)(R) &= R \ .
\end{aligned}
\tag{A-2.19}
$$

The graphical representation of the Yang-Baxter equation in terms of the relations of braid group generators is given in the figure A-2.2.

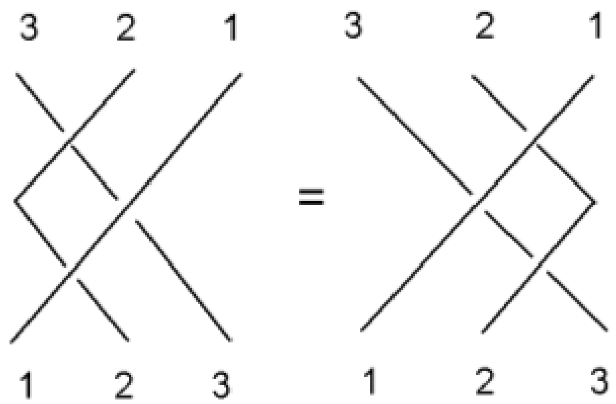

Figure 6: Graphical representation of Yang-Baxter equation $R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$.

Ribbon algebras

Let H be a braided Hopf algebra with a universal matrix $R = \sum_i s_i \otimes t_i$ and set $u = \sum_i S(t_i)s_i$. It can be shown that u is invertible with the inverse $u^{-1} = \sum s_i S^2(t_i)$ and that $uS(u) = S(u)u$ is central element in H. Furthermore, one has $\epsilon(u) = 1$ and $\Delta(u) = (R_{21}R)^{-1}(u \otimes u)$, and the antipode is given for any $x \in H$ by $S^2(x) = uxu^{-1}$.

Ribbon algebra has besides $R \in H \otimes H$ also a second preferred element called θ. A braided Hopf algebra is called ribbon algebra if there exists a central element θ of H satisfying the relations

$$\Delta(\theta) = (R_{21}R)^{-1}(\theta \otimes \theta) \ , \quad \epsilon(\theta) = 1 \ , \quad S(\theta) = \theta \ . \tag{A-2.20}$$

It can be shown that θ^2 acts like $S(u)u$ on any finite-dimensional module [A55] .

Drinfeld's quantum double

Drinfeld's quantum double construction allows to build a quasi-triangular Hopf algebra by starting from any Hopf algebra H and its dual $H^\star$, which exists in a finite-dimensional case always, and as a vector space is isomorphic with H. Besides duality normal ordering is second ingredient of the construction. Physically the generators of the algebra and its dual correspond to creation and annihilation operator type operators. Drinfeld's quantum double construction is represented in a very general manner in [A55] . A construction easier to understand by a physicist is discussed in [A27]. For this reason this representation is summarized here although the style differs from the representation of [A55] followed in the other parts of appendices.

Consider first what is known.

1. Duality means the existence of basis $\{e_a\}$ for H and $\{e^a\}$ for $H^\star$ and inner product (or evaluation as it is called in [A55]) $ev : H^\star \otimes H \rightarrow k$ defined as $ev(e^a e_b) \equiv \langle e^a, e_b\rangle = \delta^a_b$ and its inverse $\delta : k \rightarrow H^\star \otimes H$ defined by $\delta(1) = e^a e_a$. One can extend the inner product to an inner product in the tensor product $(H^\star \otimes H^\star) \otimes (H \otimes H)$ in an obvious manner.

2. The product (co-product) in H ($H^\star$) coincides with the co-product (product) in $H^\star$ (H) in the sense that one has

$$\begin{aligned} \langle e^c, e_a e_b\rangle = m^c_{ab} = \langle \Delta(e^c), e_b \otimes e_a\rangle \ , \\ \langle e^a e^b, e_c\rangle = \mu^{ab}_c = \langle e^a \otimes e^b, \Delta(e_c)\rangle \ , \end{aligned} \tag{A-2.21}$$

   These equations are quite general expressions for the duality expressed graphically in figure A-2.2.

3. The antipodes S for H and $H^\star$ can be represented as matrices

$$S_H(e_a) = S_a{}^b e_b \ , \quad S_{H^\star}(e^a) = (S^{-1})^a{}_b e^b \ . \tag{A-2.22}$$

The task is to construct algebra product μ and co-algebra product Δ, unit η and co-unit ϵ, antipode, and R-matrix R for for $H \otimes H^\star$. The natural basis for $H \otimes H^\star$ consists of $e_a \otimes e^b$.

1. Co-product Δ is simply the product of co-products

$$\Delta(e_a e^b) = \Delta(e_a)\Delta(e^b) = m^b_{vu}\mu^{cd}_a e_c e^u \otimes e_d e^v \ . \tag{A-2.23}$$

2. Product μ involves normal ordering prescription allowing to transform products $e^a e_b$ (elements of $H^\star \otimes H$) to combinations of basis elements $e_a e^b$ (elements of $H \otimes H^\star$. This map must be consistent with the requirement that co-product acts as an algebra morphism. Drinfeld's normal ordering prescription, or rather a map $c_{H^\star,H}$: $H^\star \otimes H \rightarrow H \otimes H^\star$ is given by

$$c_{H^\star,H}(e^a e_b) = R^{ac}_{bd} e_c e^d \quad , \quad R^{ac}_{bd} = m^x_{kd} m^a_{xu} \mu^{vy}_b \mu^{ck}_y (S^{-1})^u_v e_c e^d \quad . \tag{A-2.24}$$

The details of the formula are far from being obvious: the axioms of tensor category with duality to be discussed later might allow to relate $R_{H^\star,H}$ to $R_{H,H}$ and this might help to understand the origin of the expression. Normal ordering map can be interpreted as braid operation exchanging H and $H^\star$ and the matrix defining the map could be regarded as R-matrix $R_{H\otimes H^\star}$.

3. The universal R-matrix is given by

$$R \quad = \quad (e_a \otimes id_{H^\star}) \otimes (id_H \otimes e^a) \quad , \tag{A-2.25}$$

where the summation convention is applied. One can show that $R\Delta = \Delta^{op} R$ by a direct calculation.

4. The antipode $S_{H\otimes H^\star}$ follows from the product of antipodes for H and $H^\star$ using the fact that antipode is antihomomorphism using the normal ordering prescription

$$S_{H\otimes H^\star}(e_a e^b) = c_{H^\star,H}(S(e^b) S(e_a)) \quad . \tag{A-2.26}$$

Quasi-Hopf algebras and Drinfeld associator

Braided Hopf algebras are quasi-commutative in the sense that one has $\Delta^{op} = R\Delta R^{-1}$. Also the strict co-associativity can be given up and this means that one has

$$(\Delta \otimes id)\Delta = \Phi (id \otimes \Delta) \Phi^{-1} \quad , \tag{A-2.27}$$

where $\Phi \in H \otimes H \otimes H$ is known as Drinfeld's associator and appears in the of conformal fields theories. If the resulting structure satisfies also the so called Pentagon Axiom (to be discussed later, see Eq. A-2.36 and figure A-2.3), it is called quasi-Hopf algebra. Pentagon Axiom boils down to the condition

$$(id \otimes id \otimes \Delta)(\Phi)(\Delta \otimes id \otimes id)(\Phi) = (id \otimes \Phi)(id \otimes id \otimes \Delta)(\Phi)(\Phi \otimes id) \quad . \tag{A-2.28}$$

The Yang-Baxter equation for quasi-Hopf algebra reads as

$$R_{12}\Phi_{312}R_{13}\Phi^{-1}_{1322}R_{23}\Phi_{123} = \Phi_{321}R_{23}\Phi^{-1}_{231}R_{13}\Phi_{213}R_{12}\Phi_{123} \quad . \tag{A-2.29}$$

The left-hand side arises from a sequence of transformations

$$(12)3 \xrightarrow{\Phi_{123}} 1(23) \xrightarrow{R_{23}} 1(32) \xrightarrow{\Phi^{-1}_{132}} (31)2 \xrightarrow{R_{13}} 3(12) \xrightarrow{\Phi_{312}} 3(12) \xrightarrow{R_{12}} 3(21) \quad .$$

The right-hand side arises from the sequence

$$(12)3 \xrightarrow{R_{12}} (21)3 \xrightarrow{\Phi_{213}} 2(13) \xrightarrow{R_{13}} 2(31) \xrightarrow{\Phi^{-1}_{231}} (23)1 \xrightarrow{R_{23}} (32)1 \xrightarrow{\Phi_{321}} 3(21) \quad .$$

One can produce new quasi-Hopf algebras by gauge (or twist) transformations using invertible element $\Omega \in H \otimes H$ called twist operator

$$\begin{aligned} \Delta(a) &\rightarrow \Omega\Delta(a)\Omega^{-1} \ , \\ \Phi &\rightarrow \Omega_{23}(id \otimes \Delta)(\Omega)\Phi(\Delta \otimes id)(\Omega^{-1})\Omega_{12}^{-1} \ , \\ R &\rightarrow \Omega R \Omega^{-1} \ . \end{aligned} \quad \text{(A-2.30)}$$

Quasi-Hopf algebras appear in conformal field theories and correspond quantum universal enveloping algebras divided by their centralizer. Consider as an example the R-matrix R^{j_1,j_2} relating $j_1 \otimes j_2$ and $j_2 \otimes j_1$ representations $\Delta^{j_1,j_2}(a)$ and $\Delta^{j_2,j_1}(a)$ of the co-product Δ of $U(sl(2))_q$. $\Delta^{j,j}(a)$ commutes with R^{jj} for all elements of the quantum group. The action of $g_i = qR^{jj}$ acting in i^{th} and $(i+1)^{th}$ tensor factors extends to the representation $(V_j)^{\times n}$ in an obvious manner. From the Yang-Baxter equation it follows that the operators g_i define a representation of braid group B_n:

$$\begin{aligned} g_i g_{i+1} g_i &= g_{i+1} g_i g_{i+1} \ , \\ g_i g_j &= g_j g_i \ , \ \text{for} \ |j-k| \geq 2 \ . \end{aligned} \quad \text{(A-2.31)}$$

Under certain conditions the braid group generators generate the whole centralizer C_q^n for the representation of quantum group. For instance, this occurs for $j = 1/2$. In this case the additional condition

$$g_i^2 = (q^2-1)g_i + q^2 \times 1 \ , \quad \text{(A-2.32)}$$

so that the centralizer is isomorphic with the Hecke algebra $H_n(q)$, which can be regarded as a q-deformation of permutation group S_n.

The result generalizes. In Wess-Zumino-Witten model based on group G the relevant algebraic structure is $U(G_q)/C^n(q)$. This is quasi-Hopf algebra and the so called Drinfeld associator characterizes the quasi-associativity.

A-2.3 Tensor categories

Hopf algebras and related structures do not seem to be quite enough in order to formulate elegantly the construction of S-matrix in TGD framework. A more general structure known as a braided tensor category with left duality and twist operation making the category to a ribbon category is needed. The algebra product μ and co-product Δ must be generalized so that they appear as morphisms $\mu_{A\otimes B\rightarrow C}$ and $\Delta_{A\rightarrow B\otimes C}$: this gives hopes of describing 3-vertices algebraically. It is not clear whether one can assume single underlying algebra so that objects would correspond to different representations of this algebra or whether one allow even non-isomorphic algebras.

In the tensor category the tensor products of objects and corresponding morphisms belong to the category. In a braided category the objects $U\otimes V$ and $V\otimes U$ are related by a braiding morphism. The notion of braided tensor category appears naturally in topological and conformal quantum field theories and seems to be an appropriate tool also in TGD context. The basic category theoretical notions are discussed in [A55] and I have already earlier considered category theory as a possible tool in the construction of quantum TGD and TGD inspired theory of consciousness [K19] .

In braided tensor categories one introduces the braiding morphism $c_{V,W} : V \otimes W \rightarrow W \otimes V$, which is closely related to R-matrix. In categories allowing duality arrows with both directions are allowed ad diagrams analogous to pair creation from vacuum are possible. In ribbon categories one introduces also the twist operation θ_V as a morphism of object and the Θ_W satisfies the axiom: $\theta_{V\otimes W} = (\theta_V \otimes \theta_W)c_{W,V}c_{V,W}$. One can also introduce morphisms with arbitrary number of incoming lines and outgoing lines and visualize them as boxes, coupons. Isotopy principle, originally related to link and knot diagrams provides a powerful tool allowing to interpret the basic axioms of ribbon categories in terms of isotopy invariance of the diagrams and to invent theorems by just isotoping.

Categories, functors, natural transformations

Categories [A55, A61, A73, A50] are roughly collections of objects A, B, C... and morphisms $f(A \rightarrow B)$ between objects A and B such that decomposition of two morphisms is always defined. Identity morphisms map objects to objects. Examples of categories are open sets of some topological spaces with continuous maps between them acting as morphisms, linear spaces with linear maps between them acting as morphisms, groups with group homomorphisms taking the role of morphisms. Practically any collection of mathematical structures can be regarded as a category. Morphisms can be very general: for instance, partial ordering $a \leq b$ can define a morphism $f(A \rightarrow B)$.

Functors between categories map objects to objects and morphisms to morphisms so that a product of morphisms is mapped to the product of the images and identity morphism is mapped to identity morphism. Functor $F : \mathcal{C} \rightarrow \mathcal{D}$ commutes also with the maps s and b assigning to a morphism $f : V \rightarrow W$ its source $s(f) = V$ and target $b(f) = W$.

A natural transformation between functors F and G from $\mathcal{C} \rightarrow \mathcal{C}'$ is a family of morphisms $\eta(V) : F(V) \rightarrow G(V)$ in $\mathcal{C}'$ indexed by objects V of $\mathcal{C}$ such that for any morphisms $f : V \rightarrow W$ in $\mathcal{C}$, the square

$$\begin{array}{ccc} F(V) & \xrightarrow{\eta(V)} & G(V) \\ \downarrow F(f) & & \downarrow G(f) \\ F(W) & \xrightarrow{\eta(W)} & G(W) \end{array} \tag{A-2.33}$$

commutes.

The functor $F : \mathcal{C} \rightarrow \mathcal{D}$ is said to be equivalence of categories if there exists a functor $G : \mathcal{D} \rightarrow \mathcal{C}$ such and natural isomorphisms

$$\eta : id_{\mathcal{D}} \rightarrow FG \;\; \text{and} \;\; \theta : GF \rightarrow id_{\mathcal{C}}FG \;\; .$$

The notion of adjoint functor is a more general notion than equivalence of categories. In this case η and θ are natural transformations but not necessary natural isomorphisms in such a manner that the composite maps

$$F(V) \xrightarrow{\eta(F(V))} (FGF)(V) \xrightarrow{F(\theta(V))} F(V)$$

$$G(W) \xrightarrow{G(\eta(W))} (GFG)(W) \xrightarrow{\theta(G(W)))} G(W) \tag{A-2.34}$$

are identify morphisms for all objects V in $\mathcal{C}$ and W in $\mathcal{D}$.

The product $C = AB$ for objects of categories is defined by the requirement that there exist projection morphisms π_A and π_B from C to A and B and that for any object D and pair of morphisms $f(D \rightarrow A)$ and $g(D \rightarrow B)$ there exist morphism $h(D \rightarrow C)$ such that one has $f = \pi_A h$ and $g = \pi_B h$. Graphically this corresponds to a square diagram in which pairs A,B and C,D correspond to the pairs formed by opposite vertices of the square and arrows DA and DB correspond to morphisms f and g, arrows CA and CB to the morphisms π_A and π_B and the arrow h to the diagonal DC. Examples of product categories are Cartesian products of topological spaces, linear spaces, differentiable manifolds, groups, etc. The tensor products of linear spaces and algebras provides an especially interesting example of product in the recent case. One can define also more advanced concepts such as limits and inverse limits. Also the notions of sheafs, presheafs, and topos are important.

Tensor categories

Let $\mathcal{C}$ be a category. Tensor product $\otimes$ is a functor from $\mathcal{C} \times \mathcal{C}$ to $\mathcal{C}$ if

1. there is an object $V \otimes W$ associated with any pair (V, W) of objects of $\mathcal{C}$

2. there is an morphism $f \otimes g$ associated with any pair (f, g) of morphisms of $\mathcal{C}$ such that $s(f \otimes g) = s(f) \otimes s(g)$ and $b(f \otimes g) = b(f) \otimes b(g)$,
3. if f' and g' are morphisms such that $s(f') = b(f)$ and $s(g') = b(g)$ then $(f' \otimes g') \circ (f \otimes g) = (f' \circ f) \otimes (g' \circ g)$,
4. $id_{V \otimes W} = id_{W \otimes V}$.

   Any functor with these properties is called tensor product. The tensor product of vector spaces provides the most familiar example of a tensor product functor.

   In figure 4 the general rules for graphical representations of morphisms are given.

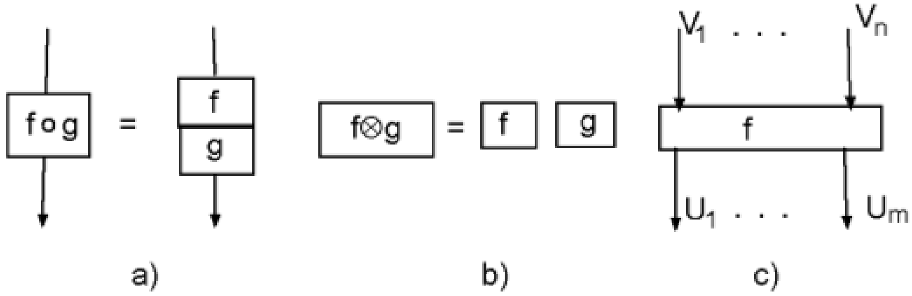

Figure 7: The graphical representation of morphisms. a) $g \circ f$: $V \rightarrow W$, b) $f \otimes g$, c) $f : U_1 \otimes ... \otimes U_m \rightarrow V_1 \otimes .... \otimes V_n$.

An associativity constraint for the tensor product is a natural isomorphism

$$a : \otimes(\otimes \times id) \rightarrow \otimes(id \times \otimes) \quad .$$

On basis of general definition of natural isomorphisms (see Eq. A-2.33) one can conclude that for any triple (U, V, W) of objects of $\mathcal{C}$ there exists an isomorphism

$$\begin{array}{ccc}
(U \otimes V) \otimes W & \overset{a_{U,V,W}}{\rightarrow} & U \otimes (V \otimes W) \\
\Big\downarrow (f \otimes g) \otimes h & & \Big\downarrow f \otimes (g \otimes h) \\
(U' \otimes V') \otimes W' & \xrightarrow{a_{U',V\ prime,W'}} & U' \otimes (V' \otimes W')
\end{array} \qquad \text{(A-2.35)}$$

Associativity constraints satisfies Pentagon Axiom [A55] if the following diagrams commutes.

$$\begin{array}{ccc}
U \otimes (V \otimes W) \otimes X & \overset{\longleftarrow}{a_{U,V,W} \otimes id_X} & ((U \otimes V) \otimes W)) \otimes X \\
 & & \Big\downarrow a_{U \otimes V,W,X} \\
\Big\downarrow a_{U,V \otimes W,X} & & (U \otimes V) \otimes (W \otimes X) \\
 & & \Big\downarrow a_{U,V,W \otimes X} \\
U \otimes ((V \otimes W) \otimes X) & \xrightarrow{id_U \otimes a_{V,W,X}} & U \otimes (V \otimes (W \otimes X))
\end{array} \qquad \text{(A-2.36)}$$

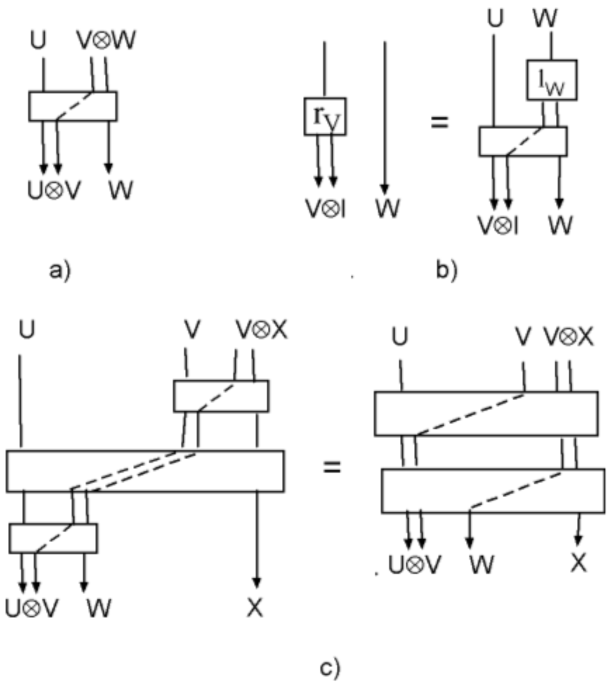

Figure 8: Graphical representations of a) the associativity isomorphism $a_{U,V,W}$, b) Triangle Axiom, c) Pentagon Axiom.

Pentagon axiom has been already mentioned while discussing the definition of quasi-Hopf algebras. In figure A-2.3 are graphical illustrations of associativity morphism $a(U, V, W)$, Triangle Axiom, and Pentagon Axiom are given.

Assume that an object I is fixed in the category. A left unit constraint with respect to I is a natural isomorphism

$$l : \otimes(I \times id) \rightarrow id$$

By Eq. A-2.33 this means that for any object V of C there exists an isomorphism

$$l_V : I \otimes V \rightarrow V \tag{A-2.37}$$

such that

$$\begin{array}{ccc} I \otimes V & \xrightarrow{l_V} & V \\ \downarrow id_I \otimes f & & \downarrow f \\ I \otimes V' & \xrightarrow{l_{V'}} & V' \end{array} \tag{A-2.38}$$

The right unit constraint $r : \otimes(id \times I) \rightarrow id$ can be defined in a completely analogous manner.

Given an associativity constraint a, and left and right unit constraints l, r with respect to an object I, one can say that the Triangle Axiom is satisfies if the triangle

$$\begin{array}{ccc} (V \otimes I) \otimes W & \xrightarrow{a_{V,I,W}} & V \otimes (I \otimes W) \\ & \searrow r_V \otimes id_W \quad id_W \otimes l_W \swarrow & \\ & V \otimes W & \end{array} \tag{A-2.39}$$

commutes (see figure A-2.3).

These ingredients lead allow to define tensor category $(\mathcal{C}, I, a, l, r)$ as a category $\mathcal{C}$ which is equipped with a tensor product $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ satisfying associativity constraint a, left unit constraint l and right unit constraint r with respect to I, such that Pentagon Axiom and Triangle Axiom are satisfied.

The definition of a tensor functor $F : \mathcal{C} \to \mathcal{D}$ involves also additional isomorphisms. $\phi_0 : I \to F(I)$ satisfies commutative diagrams involving right and left unit constraints l and r. The family of isomorphisms

$$\phi_2(U, V) : F(U) \otimes F(V) \to F(U \otimes V)$$

satisfies a commutative diagram stating that ϕ_2 commutes with associativity constraints. The interested reader can consult [A55] for details. One can also define the notions of natural tensor transformation, natural tensor isomorphism, and tensor equivalence between tensor categories by applying the general category theoretical tools.

Keeping track of associativity isomorphisms is obviously a rather heavy burden. Fortunately, it can be shown that one can assign to a tensor category $\mathcal{C}$ a strictly associative (or briefly, strict) tensor category which is tensor equivalent of $\mathcal{C}$.

Braided tensor categories

Braided tensor categories satisfy also commutativity constraint c besides associativity constraint a. Denote by $\tau : \mathcal{C} \times \mathcal{C} \to \mathcal{C} \times \mathcal{C}$ the flip functor defined by $\tau(V, W) = (W, V)$. Commutativity constraint is a natural isomorphism

$$c : \otimes \to \otimes\tau \ .$$

This means that for any pair (V, W) of objects there exists isomorphism

$$c_{V,W} : V \otimes W \to W \otimes V$$

such that the square

$$\begin{array}{ccc} V \otimes W & \xrightarrow{c_{V,W}} & W \otimes V \\ \downarrow f \otimes g & & \downarrow g \otimes f \\ V' \otimes W' & \xrightarrow{c_{V',W'}} & W' \otimes V' \end{array} \qquad \text{(A-2.40)}$$

commutes.

The commutativity constraint satisfies Hexagon Axiom if the two hexagonal diagrams

(H1)

$$\begin{array}{ccc} U \otimes (V \otimes W) & \xrightarrow{c_{U,V\otimes W}} & (V \otimes W) \otimes U \\ \uparrow a_{U,V,W} & & \downarrow a_{V,W,U} \\ (U \otimes V) \otimes W & & V \otimes (W \otimes U) \\ \downarrow c_{U,V} \otimes id_W & & \uparrow id_V \otimes c_{U,W} \\ (V \otimes U) \otimes W & \xrightarrow{a_{V,U,W}} & V \otimes (U \otimes W) \end{array} \qquad \text{(A-2.41)}$$

and (H2)

$$\begin{array}{ccc} (U\otimes V)\otimes W & \xrightarrow{\;c_{U\otimes V,W}\;} & W\otimes (U\otimes V) \\ \uparrow a^{-1}_{U,V,W} & & \downarrow a^{-1}_{W,U,V} \\ U\otimes (V\otimes W) & & (W\otimes U)\otimes V \\ \downarrow id_U\otimes c_{V,W} & & \uparrow c_{U,W}\otimes id_V \\ U\otimes (W\otimes V) & \xrightarrow{\;a^{-1}_{U,W,V}\;} & (U\otimes W)\otimes V \end{array} \qquad \text{(A-2.42)}$$

commute.

The braiding operation $c_{V,W}$ and the association operation $a(U,V,W)$, and pentagon and hexagon axioms are illustrated in the figure A-2.3 below.

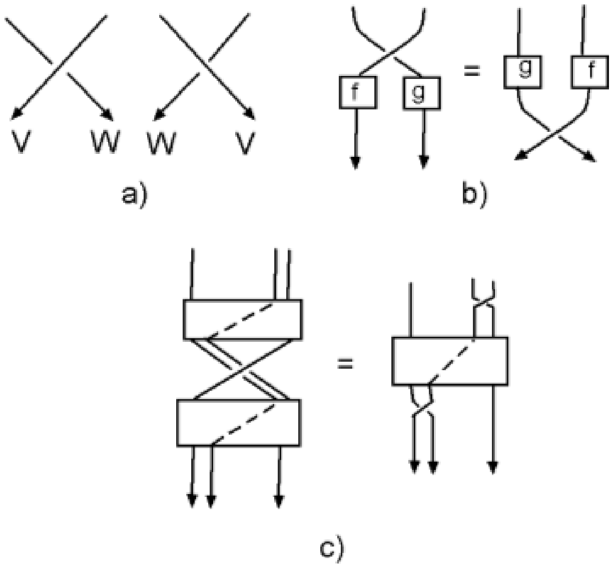

Figure 9: Graphical representations a) of the braiding morphism $c_{V,W}$ and its inverse $c^{-1}_{V,W}$, b) of naturality of $c_{V,W}$, c) of First Hexagon Axiom.

Duality and tensor categories

The notion of a dual of the finite-dimensional vector space as a space of linear maps from V to field k can lifted to a concept applying for tensor category. A strict (strictly associative) tensor category $(\mathcal{C},\otimes,I)$ with unit object I is said to possess left duality if for each object V of $\mathcal{C}$ there exists an object $V^\star$ and morphisms

$$b_V : I \to V\otimes V^\star \;\text{ and }\; d_V : V^\star\otimes V \to I$$

such that

$$(id\otimes d_V)(b_V\otimes id_V) = id_V \;\; and \;\; (d_V\otimes id_{V^\star})(id_{V^\star}\otimes b_V) = id_{V^\star} \quad . \qquad \text{(A-2.43)}$$

One can define the transpose of f in terms of b_V and d_V. The idea how this is achieved is obvious from figure A-2.3.

$$f^\star = (d_V \otimes id_{U^\star})(id_{V^\star} \otimes f \otimes id_{U^\star})(id_{V^\star} \otimes b_U) \ . \tag{A-2.44}$$

Also the braiding operation $c_{V^\star,W}$ can be expressed in terms of $c^{-1}_{V,W}$, b_V and d_V by using the isotopy of Fig. A-2.3:

$$c_{V^\star,W} = (d_V \otimes id_{W\otimes V^\star})(id_{V^\star} \otimes c^{-1}_{V,W} \otimes id_{V^\star})(id_{V^\star \otimes W} \otimes b_V) \ . \tag{A-2.45}$$

Drinfeld quantum double can be regarded as a tensor product of Hopf algebra and its dual and in this case one can introduce morphisms $ev_H : H \otimes H^\star \to k$ defined as $e^i \otimes e_j \to \delta^i_j$ defining inner product and its inverse $\delta : k \to H \otimes H$ defined as $1 \to e^i e_i$, where summation over i is understood. For categories these morphisms are generalized to morphism d_V from objects V of category to unit object I and b_V from I to object of category. The elements of H and $H^\star$ are described as strands with opposite directions, whereas d_V and b_V correspond to annihilation and creation of strand–anti-strand pair as show in figure A-2.3.

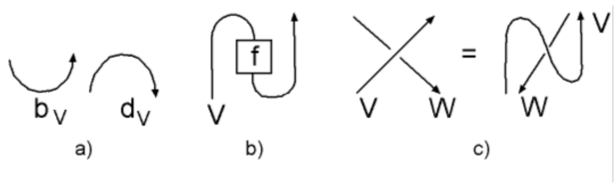

Figure 10: Graphical representations a) of the morphisms b_V and d_V, b) of the transpose $f^\star$, c) of braiding operation $c_{V^\star,W}$ expressed in terms of $c_{V,W}$.

Ribbon categories

According to the definition of [A55] ribbon category is a strict braided tensor category $(\mathcal{C}, \otimes, I)$ with a left duality with a family of natural morphisms $\theta_V : V \to V$ indexed by the objects V of $\mathcal{C}$ satisfying the conditions

$$\begin{aligned} \theta_{V\otimes W} &= \theta_V \otimes \theta_W c_{W,V} c_{V,W} \ , \\ \theta_{V^\star} &= (\theta_V)^\star \end{aligned} \tag{A-2.46}$$

for all objects V, W of $\mathcal{C}$. The naturality of twist means for for any morphisms $f : V \to W$ one has $\theta_W f = f \theta_V$. The graphical representation for the axioms and is in Fig. A-2.3.
The existence of the twist operation provides $\mathcal{C}$ with right duality necessary in order to define trace (see Fig. A-2.3).

$$\begin{aligned} d'_V &= (id_{V^\star} \otimes \theta_V) c_{V,V^\star} b_V \ , \\ b'_V &= d_V c_{V,V^\star}(\theta_V \otimes id_{V^\star}) \ . \end{aligned} \tag{A-2.47}$$

One can define quantum trace for any endomorphisms f of ribbon category:

$$tr_q(f) = d'_V(f \otimes id_{V^\star}) b_V = d_V c_{V,V^\star}(\theta_V f \otimes id_{V^\star}) b_V \ . \tag{A-2.48}$$

Again the graphical representation is the best manner to understand the definition, see figure A-2.3. Quantum trace has the basic properties of trace: $tr_q(fg) = tr_q(gf)$, $tr_q(f \otimes g) = tr_q(f) tr_q(g)$, $tr_q(f) = tr_q(f^\star)$. The proof of these properties is easiest using isotropy principle.

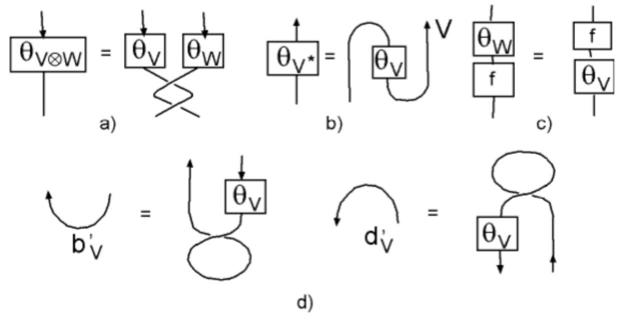

Figure 11: Graphical representations a) of $\theta_{V\otimes W} = \theta_V \otimes \theta_W c_{W,V} c_{V,W}$, b) of $\theta_{V^\star} = (\theta_V)^\star$, c) of $\theta_W f = f\theta_V$, d) of right duality for a ribbon category.

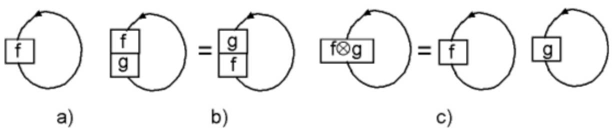

Figure 12: Graphical representations of a) $tr_q(f)$, b) of $tr_q(fg) = tr_q(gf)$, c) of $tr(f \otimes g) = tr(f)tr(g)$.

The quantum dimension of an object V of ribbon category can be defined as the quantum trace for the identity morphism of V: $dim_q(V) = tr_q(id_V) = d'_V b_V$. Quantum dimension is represented as a vacuum bubble. Quantum dimension satisfies the conditions $dim_q(V\otimes W) = dim_q(V)dim_q(W)$ and $dim_q(V) = dim_q(V^\star)$.

A more general definition of ribbon category inspired by the considerations of [A27] is obtained by allowing the generalization of morphisms μ and Δ so that they become morphisms $\mu_{A\otimes B\rightarrow C}$ and $\Delta_{C\rightarrow A\otimes B}$ of ribbon category. Graphically the general morphism with arbitrary number of incoming outgoing strands can be represented as a box or "coupon". An important special case of ribbon categories consists of modules over braided Hopf algebras allowing ribbon algebra structure.

A-3 Axiomatic approach to S-matrix based on the notion of quantum category

This section can be regarded as an attempt of a physicists with some good intuitions and intentions but rather poor algebraic skills to formulate basic axioms about S-matrix in terms of what might be called quantum category. The basic result is an interpretation for the equivalence of loop diagrams with tree diagrams as a consequence of basic algebra and co-algebra axioms generalized to the level of tensor category. The notion of quantum category emerges naturally as a generalization of ribbon category, when algebra product and co-algebra product are interpreted as morphisms between different objects of the ribbon category.

The general picture suggest that the operations Δ and μ generalized to algebra homomorphisms $A \rightarrow B\otimes C$ and $A\otimes B \rightarrow C$ in a tensor category whose objects are either representations of an algebra or even algebras might provide an appropriate mathematical tool for saying something interesting about S-matrix in TGD Universe. These algebras need not necessarily be bi-algebras. In the following it is demonstrated that the equivalence of loop diagrams to tree diagrams follows from suitably generalized bi-algebra axioms. Also the interpretation of various morphisms involved with Hopf algebra structure is discussed.

A-3.1 Δ and μ and the axioms eliminating loops

The first task is to find a physical interpretation for the basic algebraic operations and how the basic algebra axioms might allow to eliminate loops. The physical interpretation of morphisms Δ and μ as algebra or category morphisms has been already discussed. As already found, the condition that Δ (μ) acts as an algebra (co-algebra) morphism leads to a condition stating that a box graph for 2-particle scattering is equivalent with tree graph. It is interesting to identify the corresponding conditions in the case of self energy loops and vertex corrections.

The condition

$$\mu_{B\otimes C\rightarrow A}\circ\Delta_{A\rightarrow B\otimes C} \quad = \quad K\times id_A \ , \tag{A-3.1}$$

where K is a numerical factor, is a natural additional condition stating that a line with a self energy loop is equivalent with a line without the loop. The condition is illustrate in figure A-3.1. For the co-commutative tensor algebra $T(V)$ of vector space with $\Delta(x)=1\otimes x+x\otimes 1$ one would have $K=2$ for the generators of $T(V)$. For a product of n generators one has $K=2^n$.

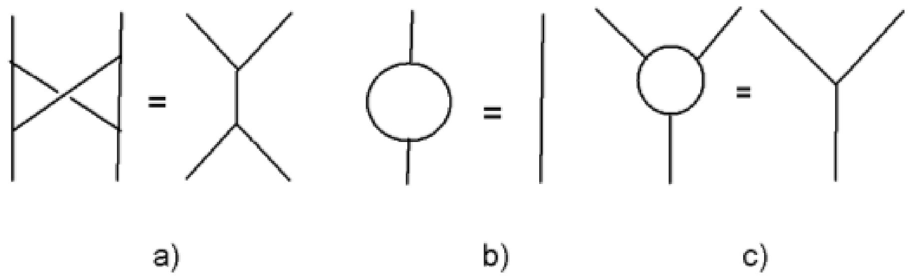

Figure 13: Graphical representations for the conditions a) $(id\otimes\mu\otimes id)(\Delta\otimes\Delta)=\Delta\circ\mu$, b) $\mu_{B\otimes C\rightarrow A}\circ\Delta_{A\rightarrow B\otimes C}=K\times id_A$, and c) $(\mu\otimes id)\circ(\Delta\otimes id)\circ\Delta=K\times\Delta$.

The condition $\Delta_{A\rightarrow B\otimes C}\circ\mu_{B\otimes C\rightarrow A}=K\times id_A$ cannot hold true since multiplication is not an irreversible process. If this were the case one could reduce tree diagrams to collections of free propagator lines.

In quantum field theories also vertex corrections are a source of divergences. The requirement that the graph representing a vertex correction is equivalent with a simple tree graph representing a decay gives an additional algebraic condition. For bi-algebras the condition would read

$$(\mu\otimes id)\circ(\Delta\otimes id)\circ\Delta=K\Delta \ , \tag{A-3.2}$$

where K is a simple multiplicative factor. In fact, for the co-commutative tensor algebra $T(V)$ of vector space the left hand side would be $3\times\Delta(x)$ giving $K=3$ for generators $T(V)$. The condition is illustrated in figure A-3.1.

Using the standard formulas of appendix for quantum groups one finds that in the case of $U_q(sl(2))$ the condition $\mu\circ\Delta(X)=K_XX$, K_X constant, is not true in general. Rather, one has $\mu\circ\Delta(X)=XK_X(q^{H/2}+q^{-H/2},q^{1/2},q^{-1/2})$. The action on the vacuum state is however proportional to that of X, being given by $K_X(2,1,1)X$. The function K_X for a given X can be deduced from $\mu\circ\Delta(X_\pm)=q^{H/2}X_\pm+X_\pm q^{-H/2}=X_\pm(q^{\pm 1/2}+q^{H/2}+q^{-H/2})$. The eigen states of Cartan algebra generators are expected to be eigen states of $\mu\circ\Delta$ also in the case of a general quantum group. $\mu\circ\Delta$ is analogous to a single particle operator like kinetic energy and its action on multi-particle state is a sum over all tensor factors with $\mu\circ\Delta$ applied to each of them. For eigen states of $\mu\circ\Delta$ the projective equivalence of loop diagrams with tree diagrams would make sense.

Since self energy loops, vertex corrections, and box diagrams represent the basic divergences of renormalizable quantum field theories, these axioms raise the hope that the basic infinities of quantum field theories could be eliminated by the basic axioms for the morphisms of quantum category.

There are also morphisms related to the topology changes in which the 3-surface remains connected. For instance, processes in which the number of boundary components can change could be of special relevance if the family replication phenomenon reduces to the boundary topology. Also 3-topology can change. The experience with topological quantum field theories [A84] , stimulates the hope that the braid group representations of the topological invariants of 3-topology might be of help in the construction of S-matrix.

The equivalence of loop diagrams with tree diagrams must have algebraic formulation using the language of standard quantum field theory. In the third section it was indeed found that thanks to the presence of the emission of vacuons, the equivalence of loop diagrams with tree diagrams corresponds to the vanishing of loop corrections in the standard quantum field theory framework. Furthermore, the non-cocommutative Hopf algebra of Feynman diagrams discussed in [A34] becomes co-commutative when the loop corrections vanish so that TGD program indeed has an elegant algebraic formulation also in the standard framework.

A-3.2 The physical interpretation of non-trivial braiding and quasi-associativity

The exchange of the tensor factors by braiding could also correspond to a physically non-trivial but unitary operation as it indeed does in anyon physics [D56, D71] . What would differentiate between elementary particles and anyons would be the non-triviality of the super-canonical and Super Kac-Moody conformal central extensions which have the same origin (addition of a multiplication by a multiple of the Hamiltonian of a canonical transformation to the action of isometry generator). The proposed interpretation of braiding acting in the complex plane in which the conformal weights of the elements of the super-canonical algebra represent punctures justifies the non-triviality. Hexagon Axioms would state that two generalized Feynman diagrams involving exchanges, dissociations and re-associations are equivalent.

An interesting question is whether the association $(A, B) \rightarrow (A \otimes B)$ could be interpreted as a formation of bound state entanglement between A and B. A possible space-time correlate for association is topological condensation of A and B to the same space-time sheet. Association would be trivial if all particles are at same space-time sheet X^4 but non-trivial if some subset of particles condense at an intermediate space-time sheet Y^4 condensing in turn at X^4.

Be as it may, association isomorphisms $a_{A,B,C}$ would state that the state space obtained by binding A with bound bound states $(B \otimes C)$ is unitarily related with the state space obtained by binding $(A \otimes B)$ bound states with C. With this interpretation Pentagon axiom would state that two generalized Feynman diagrams depicted in figure A-2.3 leading from initial to final to final state by dissociation and re-association are equivalent.

A-3.3 Generalizing the notion of bi-algebra structures at the level of WCW

WCW of 3-surfaces decomposes into sectors corresponding to different 3-topologies. Also other signatures might be involved and I have proposed that the sectors are characterized by the collection of p-adic primes labelling space-time sheets of the 3-surface and that a given space-time surface could be characterized by an infinite prime or integer. The general problem is to continue various geometric structures from a given sector A of WCW to other sector B.

An especially interesting special case corresponds to a continuation from 1-particle sector to two-particle sector or vice versa and corresponds to TGD variant of 3-vertex. All these continuations involve the imbedding of a structure associated with the sector A to a structure associated with sector B. For the continuation from 1-particle sector to 2-particle sector the map is analogous to co-algebra homomorphism Δ. For the reverse continuation it is analogous to the algebra product μ. Now however one does not have maps $\Delta : A \rightarrow A \otimes A$ and $\mu : A \otimes A \rightarrow A$ but $\Delta : A \rightarrow B \otimes C$ and $\mu : B \otimes C \rightarrow A$ unless the algebras are isomorphic. $\mu \circ \Delta = id$ should hold true as an additional condition but $\Delta \circ \mu = id$ cannot hold true since product maps many pairs to the same element.

Continuation of the WCW spinor structure

The basic example of a structure to be continued is configuration space spinor structure. WCW spinor fields in different sectors should be related to each other. The isometry generators and gamma matrices of WCW span a super-canonical algebra. The continuation requires that the super algebra basis of different sectors are related. Also vacua must be related. Isometry generators correspond to bosonic generators of the super-canonical algebra. There is also a natural extension of the super-canonical algebra defined by the Poisson structure of WCW.

This view suggests that in the first approximation one could see the construction of S-matrix as following process.

1. Incoming/outgoing states correspond to positive/negative energy states localized to the sectors of WCW with fixed 3-topologies.

2. In order to construct an S-matrix matrix element between two states localized in sectors A and B, one must continue the state localized in A to B or vice versa and calculate overlap. The continuation involves a sequence of morphisms mapping various structures between sectors. In particular, topological transformations describing particle decay and fusion are possible so that the analogs of product μ and co-product Δ are involved. The construction of three-manifold topological invariants [A84] in topological quantum field theories provides concrete ideas about how to proceed.

3. The S-matrix element describing a particular transition can be expressed as any path leading from the sector A to B or vice versa. There is a huge symmetry very much analogous to the independence of the final result of the analytic continuation on the path chosen since generalized Feynman graphs allow all moves changing intermediate topologies so that initial and final 3-topologies are same. Generalized conformal invariance probably also poses restrictions on possible paths of continuation. In the path integral approach one would have simply sum over all these equivalent paths and thus encounter the fundamental difficulties related to the infinite-dimensional integration.

4. Quantum classical correspondence suggests that the continuation operation has a space-time correlate. That is, the absolute minimum of Kähler action going through the initial and final 3-sheets defines a sequence of transitions changing the topology of 3-sheet. The localization to a particular sector of course selects particular absolute minimum. There are two possible interpretations. Either the continuation from A is not possible to all possible sectors but only to those with 3-topologies appearing in X^4, or the absolute minimum represents some kind of minimal continuation involving minimum amount of calculational labor.

5. Quantum classical correspondence and the possibility to represent the rows of S-matrix as zero energy quantum states suggests that the paths for continuation can be also represented at the space-time level, perhaps in terms of braided join along boundaries bonds connecting two light like 3-surfaces representing the initial and final states of particle reaction. Since light like 3-surfaces are metrically two-dimensional and allow conformal invariance, this suggests a connection with braid diagrams in the sense that it should be possible to regard the paths connecting sectors of WCW consisting of unions of disjoint 3-surfaces (corresponding interacting 4-surfaces are connected) as generalized braids for which also decay and fusion for the strands of braid are possible. Quantum algebra structure and effective metric 2-dimensionality of the light like 3-surfaces suggests different braidings for join along boundaries bonds connecting boundaries of 3-surfaces define non-equivalent 3-surfaces.

Co-multiplication and second quantized induced spinor fields

At the microscopic level the construction of S-matrix reduces to understanding what happens for the classical spinor fields in a vertex, which corresponds to an incoming 3-surface A decaying to two outgoing 3-surfaces B and C. At the classical level incoming spinor field A develops into a spinor fields B and C expressible as linear combinations of appropriate spinor basis. At quantum level one must understand how the Fock space defined by the incoming spinor fields of A is mapped to the tensor product of Fock spaces of B and C. The idea about the possible importance of co-algebras came with the realization that this mapping is obviously is very much like a co-product. Co-algebras and bi-algebras possessing both algebra and co-algebra structure indeed suggest a general approach giving hopes of understanding how Feynman diagrammatics generalizes to TGD framework.

The first guess is that fermionic oscillator operators are mapped by the imbedding Δ to a superposition of operators $a^{\dagger}_{Bn}\otimes Id_C$ and $Id_B\otimes a^{\dagger}_{Cn}$ with obvious formulas for Hermitian conjugates. Δ induces the mapping of higher Fock states and the construction of S-matrix should reduce to the construction of this map.

Δ is analogous to the definition for co-product operation although there is also an obvious difference due to the fact that Δ imbeds algebra A to $B\otimes C$ rather than to $A\otimes A$. Only in the case that the algebras are isomorphic, the situation reduces to that for Hopf algebras. Category theoretical approach however allows to consider a more general situation in which Δ is a morphism in the category of Fock algebras associated with 3-surfaces.

Δ preserves fermion number and should respect Fock algebra structure, in particular commute with the anti-commutation relations of fermionic oscillator operators. The basis of fermionic oscillator operators would naturally correspond to fermionic super-canonical generators in turn defining WCW gamma matrices.

Since any leg can be regarded as incoming leg, strong consistency conditions result on the coefficients in the expression

$$\Delta(a^{\dagger}_{An}) \quad = \quad C(A,B)_n{}^m a^{\dagger}_{Bm}\otimes Id_C + C(A,C)_n{}^m Id_B\otimes a^{\dagger}_{Cm} \qquad \text{(A-3.3)}$$

by forming the cyclic permutations in A,B,C. This option corresponds to the co-commutative situation and quantum group structure. If identity matrices are replaced with something more general, co-product becomes non-cocommutative.

A-3.4 Ribbon category as a fundamental structure?

There exists a generalization of the braided tensor category inspired by the axiomatic approach to topological quantum field theories which seems to almost catch the proposed mathematical requirements. This category is also called ribbon [A3] [A72] but in more general sense than it is defined in [A55] .

One adds to the tangle diagrams (braid diagrams with both directions of strands and possibility of strand–anti-strand annihilation) also "coupons", which are boxes representing morphisms with arbitrary numbers of incoming and outgoing strands. As a special case 3-particle vertices are obtained. The strands correspond to representations of a fixed Hopf algebra H.

In the recent case it would seem safest to postulate that strands correspond to algebras, which can be different because of the potential dependence of the details of Fock algebra on 3-topology and other properties of 3-surface. For instance, WCW metric defined by anti-commutators of the gamma matrices is degenerate for vacuum extremals so that the infinite Clifford algebra is definitely "smaller" than for surfaces with $D\geq 3$-dimensional CP_2 projection.

One might feel that the full ribbon algebra is an un-necessary luxury since only 3-particle vertices are needed since higher vertices describing decays of 3-surfaces can be decomposed to 3-vertices in the generic case. On the other hand, many-sheeted space-time and p-adic fractality suggest that coupons with arbitrary number of incoming and outgoing strands are needed in order to obtain the p-adic hierarchy of length scale dependent theories.

The situation would be the same as in the effective quantum field theories involving arbitrarily high vertices and would require what might be called universal algebra allowing n-ary multiplications and co-multiplications rather than only binary ones. Also strands within strands hierarchy is strongly suggestive and would require a fractal generalization of the ribbon algebra. Note that associativity and commutativity conditions for morphisms which more than three incoming and outgoing lines would force to generalize the notion of R-matrix and would bring in conditions stating that more complex loop diagrams are equivalent with tree diagrams.

A-3.5 Minimal models and TGD

Quaternion conformal invariance with non-vanishing c and k for anyons is highly attractive option and minimal super-conformal field theories attractive candidate since they describe critical systems and TGD Universe is indeed a quantum critical system.

Rational conformal field theories and TGD

The highest weight representations of Virasoro algebra are known as Verma modules containing besides the ground state with conformal weight Δ the states generated by Virasoro generators L_n, $n \geq 0$. For some values of Δ Verma module contains states with conformal weight $\Delta + l$ annihilated by Virasoro generators L_n, $n \geq 1$. In this case the number of primary fields is reduced since Virasoro algebra acts as a gauge algebra. The conformal weights Δ of the Verma modules allowing null states are given by the Kac formula

$$\Delta_{mm'} = \Delta_0 + \frac{1}{4}(\alpha_+ m + \alpha_- m')^2 \ , \ m, m' \in \{1, 2...\} \ , \tag{A-3.4}$$

$$\begin{aligned} \Delta_0 &= \frac{1}{24}(c-1) \ , \\ \alpha_\pm &= \frac{\sqrt{1-c} \pm \sqrt{25-c}}{\sqrt{24}} \ . \end{aligned} \tag{A-3.5}$$

The descendants $\prod_{n\geq 1} L_n^{k_n}|\Delta\rangle$ annihilated by L_n, $n > 0$, have conformal weights at level $l = \sum_n nk_n = mm'$.

In the general case the operator products of primary fields satisfying these conditions form an algebra spanned by infinitely many primary fields. The situation changes if the central charge c satisfies the condition

$$c = 1 - \frac{6(p'-p)^2}{pp'} \ , \tag{A-3.6}$$

where p and p' are mutually prime positive integers satisfying $p < p'$. In this case the Kac weights are rational

$$\Delta_{m,m'} = \frac{(mp' - m'p)^2 - (p'-p)^2)}{4pp'} \ , \ 0 < m < p \ , \ 0 < m' < p' \ . \tag{A-3.7}$$

Obviously, the number of primary fields is finite. This option does not seem to be realistic in TGD framework were super-conformal invariance is realized.

For $N = 1$ super-conformal invariance the unitary representations have central extension and conformal weights given by

$$\begin{aligned} c &= \frac{3}{2}(1 - \frac{8}{m(m+2)}) \ , \\ \Delta_{p,q}(NS) &= \frac{[(m+2)p) - mq)]^2 - 4}{8m(m+2)} \ , \ 0 \leq p \leq m \ , \ 1 \leq q \leq m+2 \ . \end{aligned} \tag{A-3.8}$$

For Ramond representations the conformal weights are

$$\Delta_{p,q}(R) = \Delta(NS) + \frac{1}{16} \ . \tag{A-3.9}$$

The states with vanishing conformal weights correspond to light elementary particles and the states with $p = q$ have vanishing conformal weight in NS sector. Also this option is non-realistic since in TGD framework super-generators carry fermion number so that G cannot be a Hermitian operator.

$N = 2$ super-conformal algebra is the most interesting one from TGD point of view since it involves also a bosonic $U(1)$ charge identifiable as fermion number and $G^{\pm}(z)$ indeed carry $U(1)$ charge[1]. Hence one has $N = 2$ super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$. $U(1)$ current would correspond to fermion number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called chiral ring defining a topological quantum field theory [A27], lend further support for the belief that $N = 2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ \ l = 0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ \ m = -l,-l+2,....,l-2,l \ . \end{aligned} \tag{A-3.10}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k = 1$ one would have $q = 0, 1/3, -1/3$, which brings in mind anyons. $\Delta_{l=0,m=0} = 0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [A27].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2) - m^2\right]$ (note that k must be even and that $(k, l, m) = (4, 1, 1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators.

Quaternion conformal invariance [K20] encourages to consider the possibility of super-symmetrizing also spin and electro-weak spin of fermions. In this case the conformal algebra would extend to a direct sum of Ramond and NS $N = 8$ algebras associated with quarks and leptons. This algebra in turn extends to a larger algebra if lepto-quark generators acting as half odd-integer Virasoro generators are allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on WCW Hamiltonians expressible in terms of Hamiltonians of $X^3_l \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-anti-fermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2 \times (3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

In TGD framework both quark and lepton numbers correspond to NS and Ramond type representations, which in conformal field theories can be assigned to the topologies of complex plane and cylinder. This would suggest that a given three-surface allows either NS or Ramond representation and is either leptonic or quark like but one must be very cautious with this kind of conclusion. Interestingly, NS and Ramond type representations allow a symmetry acting as a spectral flow in the indices of the generators and transforming NS and Ramond type representations continuously to each other [A27]. The flow acts as

[1] I realized that TGD super-conformal algebra corresponds to $N = 2$ algebra while writing this and proposed it earlier as a generalization of super-conformal algebra!

$$L_n \to L_n + \alpha J_n + \frac{c}{6}\alpha^2\delta_{n,0}$$
$$J_n \to J_n + \frac{c}{3}\alpha\delta_{n,0} \ ,$$
$$G_n^{\pm} \to G_{n\pm\alpha}^{\pm} \ . \tag{A-3.11}$$

The choice $\alpha = \pm 1/2$ transforms NS representation to Ramond representation. The idea that leptons could be transformed to quarks in a continuous manner does not sound attractive in TGD framework. Note that the action of Super Kac-Moody Virasoro algebra in the space of super-canonical conformal weights can be interpreted as a spectral flow.

Co-product for Super Kac-Moody and Super Virasoro algebras

By the previous considerations the quantized conformal weights z_1, z_2, z_3 of super-canonical generators defining punctures of 2-surface should correspond to line punctures of 3-surface. One cannot avoid the thought that these line punctures should meet at single point so that three-vertex would have also quantum field theoretical interpretation.

Each point z_k corresponds to its own Virasoro algebra $V_k = \{L_n^{z_k)}\}$ and Kac-Moody algebra $J_k = \{J_n^{z_k)}\}$ defined by Laurent series of $T(z)$ and $J(z)$ at z_k. Also super-generators are involved. To minimize notational labor denote by $X_n^{z_k)}$, $k = 1, 2, 3$ the generators in question.

The co-algebra product for Super-Virasoro and Super-Kac-Moody involves in the case of fusion $A_1 \otimes A_2 \to A_3$ a co-algebra product assigning to the generators $X_n^{z_3)}$ direct sum of generators of $X_k^{z_1)}$ and $X_l^{z_2)}$. The most straightforward approach is to express the generators $X_n^{z_3)}$ in terms of generators $X_k^{z_1)}$ and $X_l^{z_2)}$. This is achieved by using the expressions for generators as residy integrals of energy momentum tensor and Kac Moody currents. For Virasoro generators this is carried out explicitly in [A27]. The resulting co-product conserves the value of central extension whereas for the naive co-product this would not be the case. Obviously, the geometric co-product does not conserve conformal weight.

A-4 Some examples of bi-algebras and quantum groups

The appendix summarizes briefly the simplest bi- and Hopf algebras and some basic constructions related to quantum groups.

A-4.1 Hecke algebra and Temperley-Lieb algebra

Braid group is accompanied by several algebras. For Hecke algebra, which is particular case of braid algebra, one has

$$\begin{aligned} e_{n+1}e_ne_{n+1} &= e_ne_{n+1}e_n \ , \\ e_n^2 &= (t-1)e_n + t \ . \end{aligned} \tag{A-4.1}$$

The algebra reduces to that for symmetric group for $t = 1$.

Hecke algebra can be regarded as a discrete analog of Kac Moody algebra or loop algebra with G replaced by S_n. This suggests a connection with Kac-Moody algebras and imbedding of Galois groups to Kac-Moody group. $t = p^n$ corresponds to a finite field. Fractal dimension $t = \mathcal{M} : \mathcal{N}$ relates naturally to braid group representations: fractal dimension of quantum quaternions might be appropriate interpretation. t=1 gives symmetric group. Infinite braid group could be seen as a quantum variant of Galois group for algebraic closure of rationals.

Temperley-Lieb algebra assignable with Jones inclusions of hyper-finite factors of type II_1 with $\mathcal{M} : \mathcal{N} < 4$ is given by the relations

$$\begin{aligned} e_{n+1}e_nen+1 &= e_{n+1} \\ e_ne_{n+1}e_n &= e_n \ , \\ e_n^2 &= te_n \ , \ \ , t=-\sqrt{\mathcal{M}:\mathcal{N}}=-2cos(\pi/n) \ \ , n=3,4,... \end{aligned} \qquad \text{(A-4.2)}$$

The conditions involving three generators differ from those for braid group algebra since e_n are now proportional to projection operators. An alternative form of this algebra is given by

$$\begin{aligned} e_{n+1}e_nen+1 &= te_{n+1} \\ e_ne_{n+1}e_n &= te_n \ , \\ e_n^2 &= e_n = e_n^* \ , \ \ , t=-\sqrt{\mathcal{M}:\mathcal{N}}=-2cos(\pi/n) \ \ , n=3,4,... \end{aligned} \qquad \text{(A-4.3)}$$

This representation reduces to that for Temperley-Lieb algebra with obvious normalization of projection operators. These algebras are somewhat analogous to function fields but the value of coordinate is fixed to some particular values. An analogous discretization for function fields corresponds to a formation of number theoretical braids.

A-4.2 Simplest bi-algebras

Let $k(x_1,..,x_n)$ denote the free algebra of polynomials in variables x_i with coefficients in field k. x_i can be regarded as points of a set. The algebra $Hom(k(x_1,...,x_n),A)$ of algebra homomorphisms $k(x_1,...,x_n)\rightarrow A$ can be identified as A^n since by the homomorphism property the images $f(x_i)$ of the generators $x_1,...x_n$ determined the homomorphism completely. Any commutative algebra A can be identified as the $Hom(k[x],A)$ with a particular homomorphism corresponding to a line in A determined uniquely by an element of A.

The matrix algebra $M(2)$ can be defined as the polynomial algebra $k(a,b,c,d)$. Matrix multiplication can be represented universally as an algebra morphism Δ from from $M_2=k(a,b,c,d)$ to $M_2^{\otimes 2}=k(a',a'',b',b'',c',c'',d',d'')$ to $k(a,b,c,d)$ in matrix form as

$$\Delta\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}\begin{pmatrix} a'' & b'' \\ c'' & d'' \end{pmatrix} \ .$$

This morphism induces algebra multiplication in the matrix algebra $M_2(A)$ for any commutative algebra A.

$M(2)$, $GL_e(2)$ and $SL_e(2)$ provide standard examples about bi-algebras. $SL_e(2)$ can be defined as a commutative algebra by dividing free polynomial algebra $k(a,b,c,d)$ spanned by the generators a,b,c,d by the ideal $det-1=ad-bc-1=0$ expressing that the determinant of the matrix is one. In the matrix representation μ and η are defined in obvious manner and μ gives powers of the matrix

$$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \ .$$

Δ, counit ϵ, and antipode S can be written in case of $SL_e(2)$ as

$$\begin{pmatrix} \Delta(a) & \Delta(b) \\ \Delta(c) & \Delta(d) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\otimes\begin{pmatrix} a & b \\ c & d \end{pmatrix} \ ,$$

$$\begin{pmatrix} \epsilon(a) & \epsilon(b) \\ \epsilon(c) & \epsilon(d) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ .$$

$$S\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ad-bc)^{-1}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \ .$$

Note that matrix representation is only an economical manner to summarize the action of Δ on the generators a,b,c,d of the algebra. For instance, one has $\Delta(a)=a\rightarrow a\otimes a+b\otimes c$. The resulting algebra is both commutative and co-commutative.

$SL_e(2)_q$ can be defined as a Hopf algebra by dividing the free algebra generated by elements a, b, c, d by the relations

$$\begin{array}{ll} ba = qab \ , & db = qbd \ , \\ ca = qac \ , & dc = qcd \ , \\ bc = cb \ , & ad - da = (q^{-1} - 1)bc \ , \end{array}$$

and the relation

$$det_q = ad - q^{-1}bc = 1$$

stating that the quantum determinant of $SL_e(2)_q$ matrix is one.

$\mu, \eta, \Delta, \epsilon$ are defined as in the case of $SL_e(2)$. Antipode S is defined by

$$S\begin{pmatrix} a & b \\ c & d \end{pmatrix} = det_q^{-1}\begin{pmatrix} d & -qb \\ -q^{-1}c & a \end{pmatrix} \ .$$

The relations above guarantee that it defines quantum inverse of A. For q an n^{th} root of unity, $S^{2n} = id$ holds true which signals that these parameter values are somehow exceptional. This result is completely general.

Given an algebra, the R point of $SL_q(2)$ is defined as a four-tuple (A, B, C, D) in R^4 satisfying the relations defining the point of $SL_q(2)$. One can say that R-points provide representations of the universal quantum algebra $SL_q(2)$.

A-4.3 Quantum group $U_q(sl(2))$

Quantum group $U_q(sl(2))$ or rather, quantum enveloping algebra of $sl(2)$, can be constructed by applying Drinfeld's quantum double construction (to avoid confusion note that the quantum Hopf algebra associated with $SL_e(2)$ is the quantum analog of a commutative algebra generated by powers of a 2×2 matrix of unit determinant).

The commutation relations of $sl(2)$ read as

$$[X_+, X_-] = H \ , \quad [H, X_\pm] = \pm 2X_\pm \ . \tag{A-4.4}$$

$U_q(sl(2))$ allows co-algebra structure given by

$$\begin{array}{ll} \Delta(J) = J\otimes 1 + 1\otimes J \ , \quad S(J) = -J \ , \quad \epsilon(J) = 0 \ , \quad J = X_\pm, H \ , \\ S(1) = 1 \ , \qquad\qquad \epsilon(1) = 1 \ . \end{array} \tag{A-4.5}$$

The enveloping algebras of Borel algebras $U(B_\pm)$ generated by $\{1, X_+, H\}$ $\{1, X_-, hH\}$ define the Hopf algebra H and its dual $H^\star$ in Drinfeld's construction. h could be called Planck's constant vanishes at the classical limit. Note that $H^\star$ reduces to $\{1, X_-\}$ at this limit. Quantum deformation parameter q is given by $exp(2h)$. The duality map $\star : H \to H^\star$ reads as

$$\begin{array}{l} a \to a^\star \ , \quad ab = (ab)^\star = b^\star a^\star \ , \\ 1 \to 1 \ , \quad H \to H^\star = hH \ , \quad X_+ \to (X_+)^\star = hX_- \ . \end{array} \tag{A-4.6}$$

The commutation relations of $U_q(sl(2)$ read as

$$[X_+, X_-] = \frac{q^H - q^{-H}}{q - q^{-1}} \ , \quad [H, X_\pm] = \pm 2X_\pm \ . \tag{A-4.7}$$

Co-product Δ, antipode S, and co-unit ϵ differ from those $U(sl(2))$ only in the case of $X_\pm$:

$$\begin{array}{l} \Delta(X_\pm) = X_\pm \otimes q^{H/2} + q^{-H/2}\otimes X_\pm \ , \\ S(X_\pm) = -q^{\pm 1}X_\pm \ . \end{array} \tag{A-4.8}$$

When q is not a root of unity, the universal R-matrix is given by

$$R = q^{\frac{H\otimes H}{2}} \sum_{n=0}^{\infty} \frac{(1-q^{-2})^n}{[n]_q!} q^{\frac{n(1-n)}{2}} q^{\frac{nH}{2}} X_+^n \otimes q^{-\frac{nH}{2}} X_-^n \quad . \tag{A-4.9}$$

When q is m:th root of unity the q-factorial $[n]_q!$ vanishes for $n \geq m$ and the expansion does not make sense.

For q not a root of unity the representation theory of quantum groups is essentially the same as of ordinary groups. When q is m^{th} root of unity, the situation changes. For $l = m = 2n$ n^{th} powers of generators span together with the Casimir operator a sub-algebra commuting with the whole algebra providing additional numbers characterizing the representations. For $l = m = 2n+1$ same happens for m^{th} powers of Lie-algebra generators. The generic representations are not fully reducible anymore. In the case of $U_q(sl(2))$ irreducibility occurs for spins $n < l$ only. Under certain conditions on q it is possible to decouple the higher representations from the theory. Physically the reduction of the number of representations to a finite number means a symmetry analogous to a gauge symmetry. The phenomenon resembles the occurrence of null vectors in the case of Virasoro and Kac Moody representations and there indeed is a deep connection between quantum groups and Kac-Moody algebras [A27].

One can wonder what is the precise relationship between $U_q(sl(2)$ and $SL_q(2)$ which both are quantum groups using loose terminology. The relationship is duality. This means the existence of a morphism $x \rightarrow \Psi(x)$ $M_q(2) \rightarrow U_q^\star$ defined by a bilinear form $\langle u, x\rangle = \Psi(x)(u)$ on $U_q \times M_q(2)$, which is bi-algebra morphism. This means that the conditions

$$\langle uv, x\rangle = \langle u \otimes v, \Delta(x)\rangle \ , \quad \langle u, xy\rangle = \langle \Delta(u), x \otimes y\rangle \ ,$$

$$\langle 1, x\rangle = \epsilon(x) \ , \qquad \langle u, 1\rangle = \epsilon(u)$$

are satisfied. It is enough to find $\Psi(x)$ for the generators $x = A, B, C, D$ of $M_q(2)$ and show that the duality conditions are satisfied. The representation

$$\rho(E) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \ , \quad \rho(F) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \ , \quad \rho(K = q^H) = \begin{pmatrix} q & 0 \\ 0 & q^{-1} \end{pmatrix} \ ,$$

extended to a representation

$$\rho(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

of arbitrary element u of $U_q(sl(2)$ defines for elements in $U_q^\star$. It is easy to guess that $A(u), B(u), C(u), D(u)$, which can be regarded as elements of $U_q^\star$, can be regarded also as R points that is images of the generators a, b, c, d of $SL_q(2)$ under an algebra morphism $SL_q(2) \rightarrow U_q^\star$.

A-4.4 General semisimple quantum group

The Drinfeld's construction of quantum groups applies to arbitrary semi-simple Lie algebra and is discussed in detail in [A27]. The construction relies on the use of Cartan matrix.

Quite generally, Cartan matrix $A = \{a_{ij}\}$ is $n \times n$ matrix satisfying the following conditions:
i) A is indecomposable, that is does not reduce to a direct sum of matrices.
ii) $a_{ij} \leq 0$ holds true for $i < j$.
iii) $a_{ij} = 0$ is equivalent with $a_{ij} = 0$.
A can be normalized so that the diagonal components satisfy $a_{ii} = 2$.
The generators e_i, f_i, k_i satisfying the commutations relations

$$\begin{aligned} k_i k_j = k_j k_i \ , \qquad & k_i e_j = q_i^{a_{ij}} e_j k_i \ , \\ k_i f_j = q_i^{-a_{ij}} e_j k_i \ , \quad & e_i f_j - f_j e_i = \delta_{ij} \frac{k_i - k_i^{-1}}{q_i - q_i^{-1}} \ , \end{aligned} \tag{A-4.10}$$

and so called Serre relations

$$\begin{array}{l} \sum_{l=0}^{1-a_{ij}} (-1)^l \left[\begin{array}{c} 1-a_{ij} \\ l \end{array} \right]_{q_i} e_i^{1-a_{ij}-l} e_j e_i^l = 0, \quad i \neq j \ , \\ \sum_{l=0}^{1-a_{ij}} (-1)^l \left[\begin{array}{c} 1-a_{ij} \\ l \end{array} \right]_{q_i} f_i^{1-a_{ij}-l} f_j f_i^l = 0 \ , \quad i \neq j \ . \end{array} \quad \text{(A-4.11)}$$

Here $q_i = q^{D_i}$ where one has $D_i a_{ij} = a_{ij} D_i$. $D_i = 1$ is the simplest choice in this case.

Comultiplication is given by

$$\Delta(k_i) = k_i \otimes k_i \ , \quad \text{(A-4.12)}$$
$$\Delta(e_i) = e_i \otimes k_i + 1 \otimes e_i \ , \quad \text{(A-4.13)}$$
$$\Delta(f_i) = f_i \otimes 1 + k_i^{-1} \otimes 1 \ . \quad \text{(A-4.14)}$$
$$\text{(A-4.15)}$$

The action of antipode S is defined as

$$S(e_i) = -e_i k_i^{-1} \ , \quad S(f_i) = -k_i f_i \ , \quad S(k_i) = -k_i^{-1} \ . \quad \text{(A-4.16)}$$

A-4.5 Quantum affine algebras

The construction of Drinfeld and Jimbo generalizes also to the case of untwisted affine Lie algebras, which are in one-one correspondence with semisimple Lie algebras. The representations of quantum deformed affine algebras define corresponding deformations of Kac-Moody algebras. In the following only the basic formulas are summarized and the reader not familiar with the formalism can consult a more detailed treatment can be found in [A27].

1. Affine algebras

The Cartan matrix A is said to be of affine type if the conditions $det(A) = 0$ and $a_{ij}a_{ji} \geq 4$ (no summation) hold true. There always exists a diagonal matrix D such that $B = DA$ is symmetric and defines symmetric bilinear degenerate metric on the affine Lie algebra.

The Dynkin diagrams of affine algebra of rank l have $l+1$ vertices (so that Cartan matrix has one null eigenvector). The diagrams of semisimple Lie-algebras are sub-diagrams of affine algebras. From the $(l+1) \times (l+1)$ Cartan matrix of an untwisted affine algebra $\hat{A}$ one can recover the $l \times l$ Cartan matrix of A by dropping away 0:th row and column.

For instance, the algebra A_1^1, which is affine counterpart of $SL_e(2)$, has Cartan matrix a_{ij}

$$A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

with a vanishing determinant.

Quite generally, in untwisted case quantum algebra $U_q(\hat{G}_l)$ as $3(l+1)$ generators e_i, f_i, k_i $(i = 0, 1, .., l)$ satisfying the relations of Eq. A-4.11 for Cartan matrix of $\mathcal{G}^{(1)}$. Affine quantum group is obtained by adding to $U_q(\hat{\mathcal{G}}_l)$ a derivation d satisfying the relations

$$[d, e_i] = \delta_{i0} e_i \ , \quad [d, f_i] = \delta_{i0} f_i, \quad [d, k_i] = 0 \ . \quad \text{(A-4.17)}$$

with comultiplication $\Delta(d) = d \otimes 1 + 1 \otimes d$.

2. Kac Moody algebras

The undeformed extension $\hat{\mathcal{G}}_l$ associated with the affine Cartan matrix $\mathcal{G}_l^{(1)}$ is the Kac Moody algebra associated with the group G obtained as the central extension of the corresponding loop algebra. The loop algebra is defined as

$$L_e(\mathcal{G}) = \mathcal{G} \otimes C\left[t, t^{-1}\right] \ , \quad \text{(A-4.18)}$$

where $C\left[t,t^{-1}\right]$ is the algebra of Laurent polynomials with complex coefficients. The Lie bracket is

$$[x \times P, y \otimes Q] = [x,y] \otimes PQ \ . \qquad \text{(A-4.19)}$$

The non-degenerate bilinear symmetric form $(,)$ in $\mathcal{G}_l$ induces corresponding form in $L_e(\mathcal{G}_l)$ as $(x \otimes P, y \otimes Q) = (x,y)PQ$.

A two-cocycle on $L_e(\mathcal{G}_l)$ is defined as

$$\Psi(a,b) = Res(\frac{da}{dt}, b) \ , \qquad \text{(A-4.20)}$$

where the residue of a Laurent is defined as $Res(\sum_n a_n t^n) = a_{-1}$. The two-cocycle satisfies the conditions

$$\begin{aligned} \Psi(a,b) &= -\Psi(b,a) \ , \\ \Psi([a,b],c) + \Psi([b,c],a) + \Psi([c,a],b) &= 0 \ . \end{aligned} \qquad \text{(A-4.21)}$$

The two-cocycle defines the central extension of loop algebra $L_e(\mathcal{G}_l)$ to Kac Moody algebra $L_e(\mathcal{G}_l) \otimes Cc$, where c is a new central element commuting with the loop algebra. The new bracket is defined as $[,]+\Psi(,)c$. The algebra $\tilde{L}(\mathcal{G}_l)$ is defined by adding the derivation d which acts as td/dt measuring the conformal weight.

The standard basis for Kac Moody algebra and corresponding commutation relations are given by

$$\begin{aligned} J_n^x &= x \otimes t^n \ , \\ [J_n^x, J_m^y] &= J_{n+m}^{[x,y]} + n\delta_{m+n,0}c \ . \end{aligned} \qquad \text{(A-4.22)}$$

The finite dimensional irreducible representations of G defined representations of Kac Moody algebra with a vanishing central extension $c = 0$. The highest weight representations are characterized by highest weight vector $|v\rangle$ such that

$$\begin{aligned} J_n^x|v\rangle &= 0, \ \ n > 0 \ , \\ c|v\rangle &= k|v\rangle \ . \end{aligned} \qquad \text{(A-4.23)}$$

3. Quantum affine algebras

Drinfeld has constructed the quantum affine extension $U_q(\mathcal{G}_l)$ using quantum double construction. The construction of generators uses almost the same basic formulas as the construction of semi-simple algebras. The construction involves the automorphism $D_t : U_q(\tilde{\mathcal{G}}_l) \otimes C\left[t,t^{-1}\right] \rightarrow U_q(\tilde{\mathcal{G}}_l) \otimes C\left[t,t^{-1}\right]$ given by

$$\begin{array}{ll} D_t(e_i) = t^{\delta_{i0}} e_i \ , & D_t(f_i) = t^{\delta_{i0}} f_i \ , \\ D_t(k_i) = k_i & D_t(d) = d \ , \end{array} \qquad \text{(A-4.24)}$$

and the co-product

$$\Delta_t(a) = (D_t \otimes 1)\Delta(a) \ , \quad \Delta_t^{op}(a) = (D_t \otimes 1)\Delta^{op}(a) \ , \qquad \text{(A-4.25)}$$

where the $\Delta(a)$ is the co-product defined by the same general formula as applying in the case of semi-simple Lie algebras. The universal R-matrix is given by

$$\mathcal{R}(t) = (D_t \otimes 1)\mathcal{R} \ , \quad \text{(A-4.26)}$$

and satisfies the equations

$$\begin{aligned}
&\mathcal{R}(t)\Delta_t(a) = \Delta_t^{op}(a)\mathcal{R} \ , \\
&(\Delta_z \otimes id)\mathcal{R}(u) = \mathcal{R}_{13}(zu)\mathcal{R}_{23}(u) \ , \\
&(id \otimes \Delta_u)\mathcal{R}(zu) = \mathcal{R}_{13}(z)\mathcal{R}_{12}(zu) \ , \\
&\mathcal{R}_{12}(t)\mathcal{R}_{13}(tw)\mathcal{R}_{23}(w) = \mathcal{R}_{23}(w)\mathcal{R}_{13}(tw)\mathcal{R}_{12}(t) \ .
\end{aligned} \quad \text{(A-4.27)}$$

The infinite-dimensional representations of affine algebra give representations of Kac-Moody algebra when one restricts the consideration to generations e_i, f_i, k_i, $i > 0$.

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of CP_2 to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

A-5 Imbedding space $M^4 \times CP_2$ and related notions

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that imbedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Imbedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. http://www.tgdtheory.fi/appfigures/Hoo.jpg

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. http://www.tgdtheory.fi/appfigures/futurepast.jpg

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. http://www.tgdtheory.fi/appfigures/penrose.jpg

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-6 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

A-6.1 CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-6.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A76] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-6.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi+\Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi-\Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-6.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://www.tgdtheory.fi/appfigures/cp2.jpg`

A-6.2 Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-6.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-6.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-6.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r,\phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-6.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-6.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-6.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , & e^3 &= \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-6.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-6.11}$$

The explicit representation of the line element is given by the expression

$$
ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-6.12}
$$

The vierbein connection satisfying the defining relation

$$
de^A = -V^A_B \wedge e^B \ , \tag{A-6.13}
$$

is given by

$$
\begin{array}{lllllll}
V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\
V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\
V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ .
\end{array} \tag{A-6.14}
$$

The representation of the covariantly constant curvature tensor is given by

$$
\begin{array}{lllllll}
R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\
R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\
R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ .
\end{array} \tag{A-6.15}
$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$
J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-6.16}
$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$
J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-6.17}
$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$
J = dB \ , \tag{A-6.18}
$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$
\begin{array}{lll}
B & = & 2re^3 \ , \\
J & = & 2(e^0\wedge e^3 + e^1\wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ .
\end{array} \tag{A-6.19}
$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1,1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$
\begin{aligned}
B &= \sum_{k=1,2} P_k dQ_k \ , \\
J &= \sum_{k=1,2} dP_k \wedge dQ_k \ .
\end{aligned}
\tag{A-6.20}
$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$
\begin{aligned}
P_1 &= -\frac{1}{1+r^2} \ , \\
P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\
Q_1 &= \Psi \ , \\
Q_2 &= \Phi \ .
\end{aligned}
\tag{A-6.21}
$$

A-6.3 Spinors in CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A65] . However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1- factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-6.4 Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A47] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-6.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-7 CP_2 geometry and standard model symmetries

A-7.1 Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B15] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-7.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-7.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{rclrcl} V_{01} &=& -\frac{e^1}{r} \ , & V_{23} &=& \frac{e^1}{r} \ , \\ V_{02} &=& -\frac{e^2}{r} \ , & V_{31} &=& \frac{e^2}{r} \ , \\ V_{03} &=& (r-\frac{1}{r})e^3 \ , & V_{12} &=& (2r+\frac{1}{r})e^3 \ , \end{array} \tag{A-7.3}$$

and

$$B = 2re^3 \ , \tag{A-7.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-7.5}$$

where one have defined

$$\begin{array}{rcl} I_L^1 &=& \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\ I_L^2 &=& \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ . \end{array} \tag{A-7.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-7.7}$$

where $W^\pm$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{array}{rcl} X &=& re^3 \ , \\ Y &=& \frac{e^3}{r} \ , \end{array} \tag{A-7.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{array}{rcl} \bar{\gamma} &=& aX+bY \ , \\ \bar{Z}^0 &=& cX+dY \ , \end{array} \tag{A-7.9}$$

where the normalization condition

$$ad-bc=1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{aligned}
A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\
&+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ .
\end{aligned} \tag{A-7.10}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$
c = -d \ . \tag{A-7.11}
$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$
A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-7.12}
$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{aligned}
Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\
I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{aligned} \tag{A-7.13}
$$

The fields γ and Z^0 are defined via the relations

$$
\begin{aligned}
\gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\
Z^0 &= 4(a+b)\bar{Z}^0 = 4(X-Y) \ .
\end{aligned} \tag{A-7.14}
$$

The value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-7.15}
$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$
F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-7.16}
$$

where one has

$$
\begin{aligned}
R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\
J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ ,
\end{aligned} \tag{A-7.17}
$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-7.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-7.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-7.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-7.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-7.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-7.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} \ . \tag{A-7.24}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-7.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B28] .

A-7.2 Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B5] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-7.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-7.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-7.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-8 The relationship of TGD to QFT and string models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

Fig. 5. TGD replaces point-like particles with 3-surfaces. http://www.tgdtheory.fi/appfigures/particletgd.jpg

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situtation in Minkowskian regions of space-time surface. http://www.tgdtheory.fi/appfigures/fermistring.jpg

TGD based view about elementary particles has two aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacs carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompnied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. http://www.tgdtheory.fi/appfigures/elparticletgd.jpg

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats beheave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. http://www.tgdtheory.fi/appfigures/tgdgraphs.jpg

A-9 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

Induction procedure for gauge fields

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as base space the imbedded manifold. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induce gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface. Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://www.tgdtheory.fi/appfigures/induct.jpg`

Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP$_2$ projection, only vacuum extremals and space-time surfaces for which CP$_2$ projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r=\infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP$_2$ projection is $r=\infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-9.1 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://www.tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. http://www.tgdtheory.fi/appfigures/wormholecontact.jpg

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)) as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. http://www.tgdtheory.fi/appfigures/field.jpg

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-9.2 Imbedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of imbedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the imbedding space spinor connection carries W gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-9.3 Space-time surfaces with vanishing em, Z^0, or Kähler fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-9.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-9.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-9.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-9.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \quad \text{(A-9.5)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2} Z^0 \ . \end{aligned} \quad \text{(A-9.6)}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2k s_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \quad \text{(A-9.7)}$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \quad \text{(A-9.8)}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-9.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-10 p-Adic numbers and TGD

A-10.1 p-Adic number fields

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A24] . p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \quad \text{(A-10.1)}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \quad \text{(A-10.2)}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \quad \text{(A-10.3)}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \quad \text{(A-10.4)}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \quad \text{(A-10.5)}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.
2. Distances of points x and y inside single class are smaller than distances between different classes.
3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B25] . The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-10.2 Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \quad \text{(A-10.6)}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \quad \text{(A-10.7)}$$

The p-adic images associated with these expansions are different

$$
\begin{aligned}
y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\
y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\
&= y_1 + (x_N - 1)p^N - p^{N+1} \ ,
\end{aligned} \quad \text{(A-10.8)}
$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see Fig. **??**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://www.tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$
\begin{aligned}
(x+y)_R &\leq x_R + y_R \ , \\
|x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ ,
\end{aligned} \quad \text{(A-10.9)}
$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$
\begin{aligned}
(x+y)_R &\leq x_R + y_R \ , \\
|\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ ,
\end{aligned} \quad \text{(A-10.10)}
$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-10.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-10.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://www.tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

A-10.3 The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles" with reverse map interpreted as a transformation of intention to action and would be realized in terms of canonical identification or some of its variants.

Fig. 16. The basic idea between p-adic manifold. `http://www.tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-11 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://www.tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-12 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-12.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://www.tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://www.tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://www.tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://www.tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-12.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://www.tgdtheory.fi/appfigures/cat.jpg`

A-12.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. `http://www.tgdtheory.fi/appfigures/padictoreal.jpg`

A-12.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. `http://www.tgdtheory.fi/appfigures/sharing.jpg`

A-12.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see fig. http://www.tgdtheory.fi/appfigures/timemirror.jpg or fig. 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://www.tgdtheory.fi/appfigures/timemirror.jpg

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Index

HYPER-FINITE FACTORS, P-ADIC LENGTH SCALE HYPOTHESIS, AND DARK MATTER HIERARCHY

Topological Geometrodynamics (TGD) is a modification of general relativity inspired by the problems related to the definition of inertial and gravitational energies in general relativity. TGD is also a generalization of super string models. Physical space-times are seen as four-dimensional surfaces in certain 8-dimensional space H. The choice of H is fixed by symmetries of standard model and leads to a geometrization of known classical fields and elementary particle numbers. In fermionic sector strings indeed emerge.

Many-sheeted space-time replaces Einsteinian space-time, which follows as a long length scale approximation in which sheets of the many-sheeted space-time are lumped together. The extension of number concept based on the fusion of real numbers and p-adic number fields implies a further generalisation of the space-time concept allowing to identify space-time correlates of cognition and intentionality.

Zero energy ontology forces an extension of quantum measurement theory to a theory of consciousness and a hierarchy of phases identified as dark matter is predicted with far reaching implications for the understanding of consciousness and living systems. This all implies an elegant theoretical projection of our reality honoring the work by renowned scientists (such as Wheeler, Feynman, Penrose, Einstein, Josephson to name a few) and creating a solid foundation for modeling our Universe in terms of geometry.

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Matti Pitkänen started to work with the basic idea of TGD at 1977, published his thesis work about TGD at 1982, and has since then worked to transform the basic vision to a consistent predictive mathematical framework, to solve various interpretational issues, and understand the relationship of TGD with existing theories.

TGD Web Pages: http://www.tgdtheory.com
TGD Diary and Blog: http://matpitka.blogspot.com