Relations of Energy Density and Its Entropic Density with Temperature concerning SBH and HDE

Yong Bao

Postbox 777, 100 Renmin South Road, Luoding 527200, Guangdong, China
E-mail: baoyong9803@163.com

We consider the relations of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE). On the basis of literature [8], we obtain the energy density of the scalar field of SBH being proportional to the square of Hawking-Unruh temperature near the outside of event horizon when the distance from the film to the horizon ε and film thickness δ are constant, and its entropic density being directly proportional to the Hawking-Unruh temperature when ε and n are constant. Basing on [16], we find the equation of gravitational energy density inside SBH; derive that the gravitational energy density is proportional to the square of effective temperature far from event horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; and their entropic density is directly proportional to effective temperature. Basing on [20-22], we gain the HDE density being proportional to the square of the Gibbons-Hawking temperature, and its entropic density being directly proportional to the Gibbons-Hawking temperature. These equations are similar and have relations with each other. We suggest that these relations are interesting and significant for SBH and HDE.

PACS numbers: 97.60.Lf, 04.70.Dy, 95.36. +x

I. Introduction

What is the relation of the energy density and its entropic density with the temperature of Schwarzschild black hole (SBH) and holographic dark energy (HDE)? It is an interesting and significant question. For the relations relate to nearby the horizon of black holes, please refer to [1-14]. In SBH interior, C. A. Egan *et al* calculate the interior entropy [15]; T. Clifton *et al* proposed the gravitational energy density formula and the effective temperature expression respectively [16], but they are irrelative; A. C. Wall proposed the generalized entropy [17]. Moreover, in order to explain the cosmic accelerated expansion [18], numerous theoretical models have been proposed [19]; thereinto M. Li *et al* proposed the HDE density equation [20-22] which has no relation with the temperature.

This paper is organized as follows. In Sec. II, we get the energy density of the scalar field being proportional to the square of the Hawking-Unruh temperature [3, 23] near the outside of the horizon of SBH, and that its entropic density is directly proportional to the Hawking-Unruh one. In Sec. III, we find that the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, and its entropic density is directly proportional to the effective temperature. In Sec. IV, we obtain the HDE density being proportional to the square of the Gibbons-Hawking temperature [24-27], and that its entropic density is directly proportional to the Gibbons-Hawking one. We conclude in Sec. V.

II. Energy Density of scalar field, Entropic Density and Hawking-Unruh Temperature near outside of Event Horizon

In this section, we review [8]; obtain the relations between the energy density of the scalar field and the entropic density with the Hawking-Unruh temperature near the outside of event horizon of SBH.

A. Energy density of scalar field and Hawking-Unruh temperature

First let us review [8] briefly. The Schwarzschild coordinates is used, and the energy density ρ_{sf} of the scalar field of SBH is (we work with ħ = c = G = k = 1 units)

$$\rho_{sf} = 6\pi^2 M^2 T_{H-U}^4 / 45\varepsilon(\varepsilon+\delta) \tag{1}$$

Where $\delta = n\varepsilon$ is the film thickness, ε is the distance from the film to the horizon of SBH, and n = 1, 2, 3….

Substituting the Hawking-Unruh temperature $T_{H-U} = 1 / 8\pi M$ into (1), we obtain

$$\rho_{sf} = T_{H-U}^2 / 480\varepsilon^2(n+1) \tag{2}$$

Therefore the energy density of the scalar field is proportional to the square of Hawking-Unruh temperature near the outside of horizon when ε and n are constant. Recovering ħ, c, G and k, we obtain

$$\rho_{sf} = \mathrm{k}^2 T_{H-U}^2 / 480\varepsilon^2(n+1) \tag{3}$$

B. Entropic density and Hawking-Unruh temperature

The entropic density s_{sf} of the scalar field near the outside of horizon is [8]

$$s_{sf} = 8\pi^2 M^2 / 45\beta^3\varepsilon(\varepsilon+\delta) \tag{4}$$

where β is [8]

$$\beta = 8\pi M \tag{5}$$

Using (5) and $T_{H-U} = 1 / 8\pi M$ to (4), we obtain

$$s_{sf} = T_{H-U} / 360\varepsilon^2(\mathrm{n}+1) \tag{6}$$

So the entropic density is directly proportional to the Hawking-Unruh temperature when ε and n are constant.

III. Gravitational Energy Density, Entropic Density and effective temperature far from Event Horizon inside SBH

In this section, we review [16] briefly; find the relations between the gravitational energy density and its entropic density with effective temperature far from the event horizon inside SBH.

A. Relations for Coulomb-like gravitational fields

In [16], the gravitational fields can be classified two types: Coulomb-like gravitational fields and wave-like ones. In general they are mixed. For the Coulomb-like gravitational fields

$$8\pi\rho_{grav} = 2\alpha\sqrt{2W/3} \text{ and } p_{grav} = 0 \tag{7}$$

where ρ_{grav} is the gravitational energy density, α is a constant, $W = T_{abcd}u^a u^b u^c u^d$, T_{abcd} is the Weyl tensor, u^a, u^b, u^c, u^d are the timelike unit vectors, and p_{grav} is the isotropic pressure. For SBH, the Schwarzschild geometry can be written in Gullstrand–Painlevé coordinates as

$$\mathrm{d}s^2 = -[1-(2m / r)]\mathrm{d}t^2 - 2\sqrt{2m/r}\,\mathrm{d}r\mathrm{d}t + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{8}$$

where m is the constant mass parameter. The gravitational energy density and temperature is given at each point in the region $r < 2m$ by

$$\rho_{grav} = 2\alpha m / 8\pi r^3 \tag{9}$$

$$T_{grav} = m / 2\pi r^2\sqrt{|1-(2m/\mathrm{r})|} \tag{10}$$

where T_{grav} is the effective temperature. Taking (10) to (9), we find

$$\rho_{grav} = \alpha\pi[2-(r / m)]T_{grav}^2 \tag{11}$$

It is the equation concerning the gravitational energy density and the effective temperature in the region r < $2m$. Note that the isotropic pressure is zero. When $r << 2m$, we derive

$$\rho_{grav} = 2\alpha\pi T_{grav}^2 \tag{12}$$

Therefore the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH. There are two regions far from horizon inside SBH: Singularity and vacuum. In (11) when $r \to 0$, we gain (12) also, that is the gravitational energy density being proportional to the square of the effective temperature in the singularity and nearby the one.

B. Relations for the wave-like gravitational fields

In [16], for the wave-like gravitational fields

$$8\pi\rho_{grav} = \beta\sqrt{4W} \text{ and } p_{grav} = \rho_{grav} / 3 \tag{13}$$

For SBH, the gravitational energy density is given at each point in region r < $2m$ by

$$\rho_{grav} = \sqrt{6}\beta m / r^3 \tag{14}$$

Using (10) to (14), we find

$$\rho_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav}^2 \tag{15}$$

When $r << 2m$ and $r \rightarrow 0$, we obtain

$$\rho_{grav} = 2\sqrt{6}\beta\pi T_{grav}^2 \tag{16}$$

It is very similar to (12). So the gravitational energy density is proportional to the square of the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

C. Relations of entropic density

For the Coulomb-like gravitational fields, its entropic density s_{grav} is [16]

$$\delta s_{grav} = \delta (\rho_{grav} v) / T_{grav} \tag{17}$$

where $v = z^a \eta_{abcd} \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d$ and we can set an arbitrary constant to zero. Substituting (11) into (17) and integral, we get

$$s_{grav} = \alpha\pi[2-(r / m)]T_{grav} \tag{18}$$

This is the equation concerning the entropic density and the effective temperature in the region r < $2m$. When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\alpha\pi T_{grav} \tag{19}$$

So the entropic density is directly proportional to the effective temperature far from the horizon inside SBH.

For the wave-like gravitational fields, substituting (15) into (17) and integral, we obtain

$$s_{grav} = \sqrt{6}\beta\pi[2-(r / m)]T_{grav} \tag{20}$$

When $r << 2m$ and $r \rightarrow 0$, we derive

$$s_{grav} = 2\sqrt{6}\beta\pi T_{grav} \tag{21}$$

Therefore the entropic density is directly proportional to the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

IV. HDE Density, Entropic Density and Gibbons-Hawking Temperature

In this section, we review [20-22]; obtain the relations between the HDE density and its entropic density with Gibbons-Hawking temperature.

A. HDE density and Gibbons-Hawking temperature

In [20-22], the equation of HDE model can be rewritten as

$$\rho_{de} = 3c_L^2 \mathrm{M}_{\mathrm{pl}}^2 L^{-2} \tag{22}$$

where ρ_{de} is the HDE density, $c_L \geq 0$ is a dimensionless model parameter, $\mathrm{M}_{\mathrm{pl}} \equiv 1 / \sqrt{8\pi \mathrm{G}} = 1 / \sqrt{8\pi}$ is the reduced Planck mass and L is the cosmic cutoff.

Substituting the Gibbons-Hawking temperature $T_{G-H} = 1 / 2\pi L$ into (22), we gain

$$\rho_{de} = 12\pi^2 c_L^2 \mathrm{M}_{\mathrm{pl}}^2 T_{G-H}^2 = 3\pi c_L^2 T_{G-H}^2 / 2 \tag{23}$$

where $\mathrm{M}_{\mathrm{P}} \equiv 1 / \sqrt{\mathrm{G}} = 1$ is the Planck mass. So the holographic dark energy density is proportionate to the square of the Gibbons-Hawking temperature.

B. HDE entropic density and Gibbons-Hawking temperature

In [22], the HDE entropy S_{de} is

$$S_{de} = \pi \mathrm{M}_{\mathrm{pl}}^2 L^2 \tag{24}$$

The HDE entropic density s_{de} is

$$s_{de} = \pi \mathrm{M}_{\mathrm{pl}}^2 L^2 / L^3 = \pi \mathrm{M}_{\mathrm{pl}}^2 / L = \pi T_{G-H} / 4 \tag{25}$$

Therefore the HDE entropic density is directly proportional to the Gibbons-Hawking temperature.

-

V. Conclusion

In this paper, we have obtained the energy density of the scalar field [8] of SBH being proportional to the square of the Hawking-Unruh temperature near the outside of horizon when the distance ε from the film to the horizon and the film thickness δ are constant, and that the entropic density is directly proportional to the Hawking-Unruh temperature when ε and n are constant; found the equation concerning the gravitational energy density and the effective temperature in the region $r < 2m$ [16] inside SBH; derived that the gravitational energy density is proportional to the square of the effective temperature far from the horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; obtained that their entropic density are directly proportional to the effective one. These equations are true in the singularity and nearby the one of SBH also. Also we have got that the HDE density [20] is proportional to the square of the Gibbons-Hawking temperature, and its entropic density is directly proportional to the Gibbons-Hawking one.

Eq. (2), (12), (16), and (23) are similar. Eq. (2) belongs to the Hawking radiation; (12) and (16) belong to the quantum gravity. It is well-known that the Hawking radiation is produced by the quantum gravity, so Eq. (2) has the relation with (12) and (16). Eq. (12), (16) and (23) are similar also. Eq. (23) belongs to the dark energy which can produce the repulsion; therefore it has the relation with (12) and (16). Their entropic density is also. We suggest that these relations are interesting and significant for SBH and HDE.

References

[1] J. D. Bekenstein, Phys. Rev. D., 1993, **7** (2); 333; S. W. Hawking, Commun Math. Phys., 1975, 43, 199.

[2] Zhao Z, Zhang J H, Acta Phys. Sin., 1981, **31** (1): 508; Zhang J H, Meng Q M, Li C A, Acta Phys. Sin. 1996, **45** (9): 177.

[3] S. W. Hawking, Nature, 1974 (248): 30-31.

[4] ’t Hoot G. Nucl. Phys. B, 1985 (256): 727-745.

[5] Liu W-B, Zhao Z, Chin. Phys. Lett, 2001, **18** (2): 310-312; Li X. Zhao Z, Chin. Phys. Lett, 2001, **18** (3): 463-465.

[6] M. K. Parikh, F, Wilczek, Phys. Rev. Lett, 2000, 85 (4): 5042-5047.

[7] Meng Q M, Jiang J J, Liu J L, *et al*, Physical Journal, 2009, **58** (1): 78-82; Meng Q M, Acta Phys. Sin. 2005, 54 (1): 471-474; Meng Q M, Jiang J J, Chinese Physics G, 2008, 38 (2): 171-177; Meng Q M, Wang S, Jiang J J and Deng D L, Chinese Physics B, 2008, 17 (12): 2811-2816; Meng Q M, Jiang J J, Wang S, Acta Phys. Sin. 2009, 58 (15): 7486-7489; Jiang J-J, Meng Q-M and Wang S, Chin. Phys. B, 18 (2009), 457; Deng D L, Journal of Guizhou University (Natural Sciences), 2010, 27 (2): 119-121; Meng Q M, Jiang J J, Li C A, Acta Phys. Sin. 2010, 59 (2): 778-782.

[8] DENG De-Li, Journal of Guizhou University (Natural Sciences), 27, **4** (2010), 1000-5269(2010)04-0022-03.

[9] Y. Wang et al., arXiv: 1311.6707 [astro-ph.SR].

[10] He T-M, Yang J-B and Zhang J-Y, Chin. Phys. Lett. **31** (2014), 080401.

[11] A. I. Fisenko, V. Lemberg, arXiv: 1403.6868.

[12] J. Stefan, Wien. Ber., **79** (1879), 391; L. E. Boltzmann, Wied. Ann. Phys., **22** (1884), 291.

[13] A. Dutta, S. Gangopadhyay, arXiv: 1408.3928 [gr-qc].

[14] Y-G Miao, Y-Y Zhao, and S-J Zhang, arXiv: 1410.4115 [hep-th].

[15] C. A. Egan and C. H. Lineweaver, The Astrophysical Journal, 2010, 710: 1825-1834.

[16] T. Clifton, G. F. R. Ellis and R. Tavakol, Class. Quantum Grav.**30** (2013) 125009 (15pp).

[17] A. C. Wall, Class. Quantum Grav. **30** (2013), 165003 (35pp).

[18] A. G. Riess *et al.*, Astron. J, **116** (1998) 1009, Astrophys. J, 607 (2004) 665; S. Perlmutter *et al.*, Astrophys. J, **517** (1999) 565; R. A. Knop *et al.*, Astrophys. J, **598** (2003) 102; C. L. Bennett *et al.*, Astrophys. J. Suppl, **148** (2003) 1, arXiv: 0302207 [astro-ph]; D. N. Spergel *et al.*, Astrophys.J.Suppl, **148** (2003) 175, arXiv: 0302209 [astro-ph]; H. V. P. Peiris *et al.*, arXiv: 0302225 [astro-ph]; M. Tegmark *et al.*, Phys. Rev. D, **69** (2004) 103501; M. Tegmark *et al.*, Astrophys. J, **606** (2004) 702; S. W. Allen *et al.*, Mon. Not. Roy. Astron. Soc. J, **353** (2004) 457.

[19] H. Wei, R-g. Cai, Science & Technology Review, **23**, 12 (2005) 28-32; M. Li, X-D. Li, Sh. Wang and Y. Wang, Commun. Theor. Phys. **56** (2011) 525-604; arXiv: 1103.5870 [astro-ph]; M. Li *et al.*, arXiv: 1209.0922 [astro- ph]; X. Lei *et al.*, Commun. Theor. Phys. 2013, **59** (02) 249-252; M. Sharif, A. Jawad, Commun. Theor. Phys. 2013, **60** (02), 183-188.

[20] M. Li, Phys. Lett. B, **603** (2004) 1; arXiv: 0403127 [hep-th].

[21] K.Ke, M. Li, Phys. Lett. B **606** (2005) 173-176.
[22] Q-G. Huang and M. Li, arXiv: 0404229 [astro-ph].
[23] W.G. Unruh, Phys.Rev.D**10**, (1974) 3194.
[24] G.W. Gibbons and S.W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[25] J.W. Lee, J. Lee, and H.C. Kim, J. Cosmol. Astropart. Phys. **0708** (2007) 005.
[26]LI Miao, LI Xiao-Dong, WANG Shuang, and WANG Yi, Commun. Theor. Phys. **56** (2011) 525–604.
[27] S. Mohanty and A. Nautiyal, arXiv: 1404.2222 [hep-ph].