First and Second Least Action Principles: de Broglie frequency and Neutron Decay

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Abstract – We propose two kinds of least action principles. The first one is defined in a periodic time, and when applied to creation and annihilation of particle pairs, leads to the formula for the de Broglie frequency. The second one is defined in a double-time's metric, namely: the longitudinal and transverse (related to the discreteness of the space) times. If applied to a problem dealing with the fluctuations of the metric, this second principle permit us to infer a coherence time. We interpret this as the neutron decay time, where we take the fluctuation in the kinetic energy as being the difference between the mass-energy of the neutron minus the sum of the mass-energies of the proton and electron. The neutron decay time evaluated in this way, does not make any explicit reference to the weak interactions.

1 – Introduction

The least action (or Hamilton's) principle [1,2] states that the variation of the action A gives null result, namely

$$\delta A = \delta \int L dt = 0. \qquad (1)$$

In (1) L is the Lagrangian function, which depends on the coordinates and velocities and sometimes also on the time. Performing the variation of the action A we consider the various paths , all of them starting in the initial time t_1 and ending in the final time t_2.

2 – The de Broglie frequency

In this section we are giving somewhat more general character to the Lagrangian L, as being associated to some kind of field which is able to create or to destroy virtual particles pairs from the vacuum. Let us take the difference between the initial and final times, as being a time interval of period T. We have

$$t_2 - t_1 = T. \tag{2}$$

Now we write

$$\delta A = \delta \oint L dt = \oint \delta L \; dt = 0. \tag{3}$$

In (3) we used the closed-line-integral symbol, but here it means that the difference in time is a periodic time interval. Pursuing further we get

$$\oint \delta L \; dt = \int_0^{½T} \delta L \; dt + \int_{½T}^{T} \delta L \; dt = 0. \tag{4}$$

Indeed the creation and annihilation of particles pairs is a stochastic process, but we are going to consider a “regularized” form of it and we write

$$\int_0^{½T} \delta L \; dt = -h, \qquad \text{and} \qquad \int_{½T}^{T} \delta L \; dt = +h. \tag{5}$$

According to (5), in the first half period a quantum of action is destroyed, and in the second half one a quantum of action is created. The sum of the two contributions gives null result, recovering the classical case of the least action principle.

Now we take the first integral of (5) as being the process of creation of a virtual particle-antiparticle pair. We have

$$\int_0^{\frac{1}{2}T} \delta L \, dt = \langle \delta L \rangle \int_0^{\frac{1}{2}T} dt = -h. \qquad (6)$$

In (6) $\langle \delta L \rangle$ corresponds to a time average of the quantity δL. Next we interpret it as the energy decreasing of the vacuum as a means to create a particle-antiparticle pair. Therefore we have

$$\langle \delta L \rangle|_{\text{first half-period}} = -\, 2mc^2. \qquad (7)$$

Inserting the result of (7) into (6), we get

$$2mc^2 \, (T/2) = h. \qquad (8)$$

Finally we obtain

$$mc^2 = h\upsilon. \qquad (9)$$

We observe that (9) is the relation for the de Broglie's frequency, where we took $\upsilon \equiv 1/T$.

3 – The second action and the time of coherence

Inspired in the spirit of the string theory [3], we are going to define a second action $A^{(2)}$, where the integration of the Lagrangian function will be also done along a "transverse time" t', besides the integration which is usually performed along the "longitudinal time" t. We write

$$A^{(2)} = \iint L \, dt \, dt'. \tag{10}$$

Now let us consider, as in reference [4], a fluctuating contribution for the Lagrangiian such that

$$\delta L = (p_i \, \chi_{ij} \, p_j) / (2m). \tag{11}$$

In (11) χ_{ij} is a tensor connecting the fluctuating momenta of a particle of mass m. By taking the variation of the second action (10), we have

$$\delta A^{(2)} = <\delta L> \iint \, dt \, dt' = 0. \tag{12}$$

We observe that in this case the average quantity $<\delta L>$ is equal to zero, due to the fluctuating character of δL, namely

$$< \delta L> = <(p_i \, \chi_{ij} \, p_j) / (2m)> = 0. \tag{13}$$

In (12) we have extracted from the double integral, the “first momentum” or the time-average of the function δL.

By analogy with the previous section where we have obtained the frequency of de Broglie, let us evaluate the second momentum (the variance) of δL. We write

$$\iint (\delta L)^2 \, dt \, dt' = <(\delta L)^2> \int_0^{½\tau} dt \; \int_0^{\lambda/c} dt' = h^2. \tag{14}$$

In (14), τ is the coherence time and $\lambda$is the Planck length. Meanwhile we have

$$<(\delta L)^2> = p^4/(4m^2). \tag{15}$$

Inserting the result of (15) into (14) and performing the indicated integrations, we have

$$[p^4/(4m^2)]\,(½)\,\tau\,\lambda/c = h^2, \tag{16}$$

and solving for the coherence time we get

$$\tau = (8\,m^2\,c\,h^2)/(p^4\,\lambda). \tag{17}$$

4 – Neutron decay and the discreteness of the space-time

In the previous section, the quantum fluctuations on the metric [4] were related to the discreteness of the space-time. There a transverse time (λ/c) was considered, by taking a string of width equal to the Planck length (λ).

On the other hand, in the neutron-decay's reaction, we have the available maximum kinetic energy (K) given by

$$K = (m_n - m_p - m_e)\,c^2 = p^2/(2m). \tag{18}$$

Inserting the result of (18) into (17) we get

$$\tau = (2\,c\,h^2)/(K^2\,\lambda). \tag{19}$$

Numerical evaluation of (19) gives for the coherence time (τ), the magnitude

$$\tau = 1.04 \times 10^3 \text{ s.} \qquad (20)$$

This value for the coherence time must be compared with the calculated and measured times of the neutron decay, both of approximately 900s (please see references [5] and [6]). This result suggests that the neutron decay, besides being a process governed by the weak interactions, can also be related to the fluctuations of the metric and to the discreteness of the space-time.

5 – Concluding remarks

Besides to be essentially a quantum object, due to its size and its mass-energy content, neutron also is a composed particle with its three constituent quarks of two down and one upper flavor. Proton also is a composed particle, but some conservation laws seem to forbid its decay.

We can imagine that in the decay process of the neutron, there is an intermediate step where we have a fluctuation between the wave function describing the integer neutron and the total wave function describing the reaction's products.

It seems that the fluctuating kinetic energy introduced in section 4, nicely accounts for this feature of the neutron decay. Jointly with the here introduced concept of second action, which also considers the discreteness of the space-time, we were able to estimate the neutron decay time without explicit reference to the weak interactions [5,6,7].

Finally a paper entitled: "Improved Determination of the Neutron Lifetime", was recently published in the Physical Review Letters [8] (please see the discussions and the references cited therein.

References

[1] R. C. Tolman, The Principles of Statistical mechanics, Oxford Univ. Press, 1967, pp. 19.

[2] L. Landau and E. Lifchitz, Mécanique, Editions Mir, Moscou, 1966.

[3] D. McMahon, String Theory Demystified, McGraw Hill, 2009.

[4] A. S. Chaves, J. M. Figueiredo, M. C. Nemes, Annals of Physics (N. York) **231**,174(1994).

[5] P. R. Silva, Weak Interactions Made Simple, viXra:1210.0014 v1

[6] Particle Data Group, Phys. Lett. **204B**, 1(1988)

[7] D. J. Griffiths, Introduction to Elementary Particles, Wiley, N. York, 1987.

[8] A. T. Yue, M. S. Dewey, D. M. Gilliam, G. L. Greene, A. B. Laptev, J. S. Nico, W.M. Snow and F. E. Wietfeldt, Phys. Rev. Lett.,**111**, 222501(Nov. 2013).