

Homology Classes of 3- Δ -complexes Made up of a Small Number of Simplexes

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Abstract

By means of a computer, all the possible homogeneous compact 3- Δ -complexes made up of a small number of simplexes (from 1 to 3) have been classified in homology classes. The analysis shows that, with a small number of simplexes, it is already possible to build quite a large number of separate topological spaces.

Key Words: topology, 3-delta-complexes.

1 Introduction

Given a number T of 3-simplexes, having $n = 4 \cdot T$ faces, it is possible to build several compact topological spaces by identifying the n faces in couples.

There are:

$$(n - 1) \cdot (n - 3) \cdot \dots \cdot 1 \tag{1}$$

different combinations possible for identifying faces and, for each couple of faces, there are 6 different orientations for the identification.

In total, all the possible compact 3- Δ -complexes that is possible to build up with a number T of 3-simplexes (i.e. $4 \cdot T$ faces) are:

$$(n - 1) \cdot (n - 3) \cdot \dots \cdot 1 \cdot 6^{\frac{n}{2}} \tag{2}$$

where the above equation takes already into account the most obvious symmetries.

For example, with 1 tetrahedron is possible to build 108 different compact Δ -complexes, with 2 tetrahedra is possible to build 136080 compact Δ -complexes and so on. Note that some of the combination lead to non feasible Δ -complexes meaning that, given the instruction for identifying faces, it is simply not possible to build a space that makes sense.

By means of a computer we have evaluated the homology groups of all the possible Δ -complexes made up of T simplexes (with T going from 1 to 3) and we have classified them in classes of complexes having the same homology group.

The homology groups have been evaluated from the boundary maps ∂_i expressed as a matrix D_i of integers. In particular the rank of the homology

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group H_i has been evaluated from the ranks of D_{i+1}, D_i , and the chain space dimensions while the torsion part of the group H_i has been red out from the boundary map D_{i+1} expressed in Smith normal form.

The results are reported below.

Type of comp.	Num. of comp.	Note
Non feasible complexes	69	
Path connected complexes	39	in 8 homology classes
Non path connected complexes	0	
Total	108	

Table 1 : Complexes made of 1 simplex

Type of comp.	Num. of comp.	Note
Non feasible complexes	98991	
Path connected complexes	35568	in 50 homology classes
Non path connected complexes	1521	
Total	136080	

Table 2 : Complexes made of 2 simplexes

Type of comp.	Num. of comp.	Note
Non feasible complexes	366924249	
Path connected complexes	113844096	in 234 homology classes
Non path connected complexes	4220775	
Total	484989120	

Table 3 : Complexes made of 3 simplexes

2 Spaces made of 1 simplex - sum up table

The following table sums up the homology classes of all path connected compact homogeneous 3- Δ -complexes made up using only of 1 simplex.

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
1	\mathbb{Z}	0	0	\mathbb{Z}	0	20
2	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}	0	5
3	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	5
4	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	3
5	\mathbb{Z}	0	0	0	1	3
6	\mathbb{Z}	0	\mathbb{Z}	0	2	1
7	\mathbb{Z}	\mathbb{Z}_4	0	0	1	1
8	\mathbb{Z}	\mathbb{Z}_5	0	0	1	1

Table 4 : Homology classes (spaces made of 1 simplex)

3 Spaces made of 2 simplexes - sum up table

The following table sums up the homology classes of of all path connected compact homogeneous 3- Δ -complexes made up using only of 2 simplexes.

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
1	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^2	0	11359
2	\mathbb{Z}	0	0	\mathbb{Z}^2	-1	6473
3	\mathbb{Z}	0	0	\mathbb{Z}	0	5621
4	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	3226
5	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}	0	1765
6	\mathbb{Z}	0	0	0	1	1595
7	\mathbb{Z}	0	\mathbb{Z}	0	2	903
8	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}	0	732
9	\mathbb{Z}	\mathbb{Z}_3	0	0	1	578
10	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^2	1	440
11	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}	-1	396
12	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}	0	364
13	\mathbb{Z}	\mathbb{Z}_2	0	0	1	358
14	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	212
15	\mathbb{Z}	\mathbb{Z}_8	0	\mathbb{Z}	0	180
16	\mathbb{Z}	\mathbb{Z}_7	0	0	1	165
17	\mathbb{Z}	0	\mathbb{Z}^2	0	3	154
18	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	0	2	142
19	\mathbb{Z}	0	\mathbb{Z}_2	0	1	139
20	\mathbb{Z}	\mathbb{Z}	0	0	0	130
21	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^2	-1	114
22	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	0	2	94
23	\mathbb{Z}	\mathbb{Z}_8	0	0	1	87
24	\mathbb{Z}	\mathbb{Z}_5	0	0	1	83
25	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	0	2	40

Table 5 : Homology classes (spaces made of 2 simplexes) n:1-25

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
26	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	1	32
27	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}	1	25
28	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	\mathbb{Z}	0	22
29	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	0	1	21
30	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	0	2	20
31	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	0	2	14
32	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	12
33	\mathbb{Z}	0	\mathbb{Z}^3	0	4	11
34	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	0	3	11
35	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}	1	8
36	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}	1	5
37	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}	2	4
38	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	1	4
39	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	0	2	4
40	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	\mathbb{Z}	1	4
41	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	0	2	4
42	\mathbb{Z}	0	\mathbb{Z}^4	0	5	3
43	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^2	0	3	3
44	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	0	3	2
45	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}	1	2
46	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	0	3	2
47	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	0	0	2
48	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	3	1
49	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	0	4	1
50	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	0	3	1

Table 5 : Homology classes (spaces made of 2 simplexes) n:26-50

4 Spaces made of 3 simplexes - sum up table

The following table sums up the homology classes of of all path connected compact homogeneous 3- Δ -complexes made up using only of 3 simplexes.

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
1	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^3	-1	27794856
2	\mathbb{Z}	0	0	\mathbb{Z}^2	-1	21297916
3	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}^2	0	14583893
4	\mathbb{Z}	0	0	\mathbb{Z}^3	-2	13190400
5	\mathbb{Z}	0	0	\mathbb{Z}	0	8256514
6	\mathbb{Z}	0	\mathbb{Z}	\mathbb{Z}	1	7289062
7	\mathbb{Z}	0	\mathbb{Z}	0	2	3337122
8	\mathbb{Z}	0	0	0	1	1952888
9	\mathbb{Z}	0	\mathbb{Z}^2	0	3	1664641
10	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}	2	1573652
11	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^3	0	1464514
12	\mathbb{Z}	0	\mathbb{Z}^2	\mathbb{Z}^2	1	1350393
13	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}	0	1039055
14	\mathbb{Z}	0	\mathbb{Z}_2	0	1	777592
15	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}	1	648812
16	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}^2	0	612947
17	\mathbb{Z}	\mathbb{Z}_2	0	0	1	591869
18	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}	0	492526
19	\mathbb{Z}	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	462966
20	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}^2	-1	453676
21	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	0	2	445639
22	\mathbb{Z}	0	\mathbb{Z}^3	0	4	417638
23	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}^2	-1	369919
24	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}^2	-1	338930
25	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^2	-1	295738
26	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}	0	236229
27	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}	0	226886
28	\mathbb{Z}	\mathbb{Z}_3	0	0	1	226760
29	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}	0	163893
30	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	0	2	144341
31	\mathbb{Z}	\mathbb{Z}_4	0	\mathbb{Z}^3	-2	141960
32	\mathbb{Z}	\mathbb{Z}_5	0	\mathbb{Z}^3	-2	141960
33	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}	3	125687
34	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	0	3	115262
35	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}	1	86689
36	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	0	1	86288
37	\mathbb{Z}	\mathbb{Z}_5	0	0	1	81232
38	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}	2	74244
39	\mathbb{Z}	\mathbb{Z}_4	0	0	1	69779
40	\mathbb{Z}	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	68910

Table 6 : Homology classes (spaces made of 3 simplexes) n:1-40

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
41	\mathbb{Z}	0	\mathbb{Z}^4	0	5	68511
42	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}	-1	64013
43	\mathbb{Z}	\mathbb{Z}	0	\mathbb{Z}^2	-2	43511
44	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	0	3	39050
45	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	\mathbb{Z}	1	39043
46	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}	1	38081
47	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}	0	37748
48	\mathbb{Z}	\mathbb{Z}	0	0	0	37237
49	\mathbb{Z}	0	\mathbb{Z}_3	0	1	35734
50	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}^2	0	34861
51	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_2	0	1	30964
52	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	0	2	29680
53	\mathbb{Z}	\mathbb{Z}_7	0	0	1	28507
54	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}^2	1	27856
55	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	0	2	26523
56	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	1	24640
57	\mathbb{Z}	\mathbb{Z}_6	0	\mathbb{Z}	0	23764
58	\mathbb{Z}	\mathbb{Z}_6	0	0	1	21663
59	\mathbb{Z}	\mathbb{Z}_9	0	\mathbb{Z}	0	19200
60	\mathbb{Z}	\mathbb{Z}_{11}	0	\mathbb{Z}	0	19180
61	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	0	4	18905
62	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	0	2	17551
63	\mathbb{Z}	\mathbb{Z}_8	0	\mathbb{Z}	0	16746
64	\mathbb{Z}	\mathbb{Z}_8	0	0	1	16650
65	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}^2	2	15037
66	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	0	1	13680
67	\mathbb{Z}	\mathbb{Z}_{11}	0	0	1	13321
68	\mathbb{Z}	\mathbb{Z}_9	0	0	1	13235
69	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	0	11151
70	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	0	2	10450
71	\mathbb{Z}	\mathbb{Z}_{10}	0	\mathbb{Z}	0	9654
72	\mathbb{Z}	\mathbb{Z}_{12}	0	\mathbb{Z}	0	9536
73	\mathbb{Z}	\mathbb{Z}_{13}	0	\mathbb{Z}	0	9528
74	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}_2	0	1	9410
75	\mathbb{Z}	0	\mathbb{Z}^4	\mathbb{Z}	4	8774
76	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	0	0	8684
77	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}_2	0	0	8441
78	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	0	1	8176
79	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	0	0	7947
80	\mathbb{Z}	0	\mathbb{Z}^5	0	6	7444

Table 6 : Homology classes (spaces made of 3 simplexes) n:41-80

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
81	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	0	2	7416
82	\mathbb{Z}	\mathbb{Z}_3	0	\mathbb{Z}^3	-2	7360
83	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	0	2	7113
84	\mathbb{Z}	\mathbb{Z}_{13}	0	0	1	6729
85	\mathbb{Z}	\mathbb{Z}_{12}	0	0	1	6683
86	\mathbb{Z}	\mathbb{Z}_{10}	0	0	1	6593
87	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	0	4	6509
88	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}	0	2	6474
89	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	-1	6414
90	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}	\mathbb{Z}^3	-1	6256
91	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	2	6067
92	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	\mathbb{Z}	0	5606
93	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}	\mathbb{Z}^2	0	5550
94	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}^3	-1	5364
95	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	0	2	5252
96	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	5225
97	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	5220
98	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	0	2	5140
99	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^2	0	3	5058
100	\mathbb{Z}	\mathbb{Z}_2	0	\mathbb{Z}^3	-2	5052
101	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	0	3	5029
102	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^2	0	3	4826
103	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}	\mathbb{Z}^2	0	4673
104	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	\mathbb{Z}	3	4504
105	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	\mathbb{Z}	2	4097
106	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_2	\mathbb{Z}	0	4048
107	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	\mathbb{Z}	1	3489
108	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_3	0	1	3046
109	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	0	2	2655
110	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	0	3	2624
111	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}_2	0	1	2597
112	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	0	2	2524
113	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	0	2	2494
114	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^2	\mathbb{Z}	2	2289
115	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^2	\mathbb{Z}	2	2252
116	\mathbb{Z}	\mathbb{Z}_7	0	\mathbb{Z}^2	-1	2210
117	\mathbb{Z}	0	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	0	3	2026
118	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	\mathbb{Z}	1	2023
119	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	\mathbb{Z}	1	2021
120	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	0	3	1905

Table 6 : Homology classes (spaces made of 3 simplexes) n:81-120

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
121	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	\mathbb{Z}	-1	1744
122	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	0	5	1654
123	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	\mathbb{Z}	0	1616
124	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}	1	1607
125	\mathbb{Z}	0	\mathbb{Z}_4	0	1	1524
126	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^2	0	3	1458
127	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_3	\mathbb{Z}	0	1440
128	\mathbb{Z}	0	\mathbb{Z}_3	\mathbb{Z}	0	1332
129	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_3	\mathbb{Z}	0	1332
130	\mathbb{Z}	0	\mathbb{Z}^3	\mathbb{Z}^3	1	1312
131	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	0	3	1301
132	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^3	0	3	1236
133	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}	1	1186
134	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	\mathbb{Z}	1	1129
135	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^2	0	3	1091
136	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	\mathbb{Z}	1	1083
137	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^2	0	3	1034
138	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	-2	1028
139	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	\mathbb{Z}	1	919
140	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}_3	0	1	864
141	\mathbb{Z}	0	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	2	858
142	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^2	\mathbb{Z}^2	1	820
143	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	804
144	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^3	0	4	714
145	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^4	0	5	621
146	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^3	0	4	589
147	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^3	0	4	584
148	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	-1	576
149	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^2	0	3	568
150	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^2	\mathbb{Z}^3	0	544
151	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^2	\mathbb{Z}	2	543
152	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^2	0	3	532
153	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^3	\mathbb{Z}^2	2	512
154	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^2	0	3	502
155	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^3	0	4	502
156	\mathbb{Z}	\mathbb{Z}_6	0	\mathbb{Z}^2	-1	498
157	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}_2	0	1	488
158	\mathbb{Z}	0	\mathbb{Z}^6	0	7	479
159	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}	\mathbb{Z}	1	479
160	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}_2	\mathbb{Z}	0	448

Table 6 : Homology classes (spaces made of 3 simplexes) n:121-160

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
161	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}_2	\mathbb{Z}	0	440
162	\mathbb{Z}	0	\mathbb{Z}^5	\mathbb{Z}	5	393
163	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}	\mathbb{Z}^2	0	325
164	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}	\mathbb{Z}^2	0	310
165	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}	\mathbb{Z}^2	0	306
166	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^2	\mathbb{Z}	2	296
167	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^2	\mathbb{Z}	2	292
168	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^2	\mathbb{Z}	1	244
169	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	\mathbb{Z}^2	-2	236
170	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}_2	0	1	235
171	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	\mathbb{Z}	3	216
172	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^4	0	4	212
173	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}^2	0	176
174	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^2	\mathbb{Z}	2	165
175	\mathbb{Z}	\mathbb{Z}_3	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	2	164
176	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^3	0	4	162
177	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}	\mathbb{Z}^2	0	158
178	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}	\mathbb{Z}^2	0	152
179	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}	\mathbb{Z}^2	0	150
180	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^4	0	4	144
181	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^2	\mathbb{Z}	2	134
182	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^3	0	4	132
183	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^2	\mathbb{Z}	2	116
184	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^3	0	4	114
185	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^3	0	4	102
186	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^3	0	4	90
187	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^5	0	6	90
188	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^2	\mathbb{Z}	2	80
189	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}_2	0	1	78
190	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^2	\mathbb{Z}	2	77
191	\mathbb{Z}	\mathbb{Z}_3	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	1	72
192	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^4	0	5	70
193	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	\mathbb{Z}	2	60
194	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^3	\mathbb{Z}^2	1	60
195	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	\mathbb{Z}	4	59
196	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^3	0	4	54
197	\mathbb{Z}	0	\mathbb{Z}^4	\mathbb{Z}^2	3	48
198	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^3	\mathbb{Z}	2	37
199	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^2	\mathbb{Z}^3	-1	36
200	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^3	\mathbb{Z}	3	34

Table 6 : Homology classes (spaces made of 3 simplexes) n:161-200

n	H_0	H_1	H_2	H_3	χ	Num. of Compl.
201	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^4	0	5	34
202	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}	\mathbb{Z}^2	0	31
203	\mathbb{Z}	\mathbb{Z}_{10}	\mathbb{Z}^3	0	4	30
204	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^4	0	5	30
205	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^3	\mathbb{Z}^2	2	24
206	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	0	3	22
207	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^4	0	5	21
208	\mathbb{Z}	\mathbb{Z}_2	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	2	20
209	\mathbb{Z}	\mathbb{Z}_5	\mathbb{Z}^3	\mathbb{Z}	3	19
210	\mathbb{Z}	0	\mathbb{Z}^7	0	8	18
211	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}^5	0	5	14
212	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^3	\mathbb{Z}	3	12
213	\mathbb{Z}	\mathbb{Z}_{13}	\mathbb{Z}^4	0	5	12
214	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}^4	\mathbb{Z}^2	3	12
215	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^3	\mathbb{Z}	3	12
216	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^4	\mathbb{Z}	4	12
217	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^5	0	6	12
218	\mathbb{Z}	\mathbb{Z}_7	\mathbb{Z}^4	0	5	12
219	\mathbb{Z}	\mathbb{Z}_8	\mathbb{Z}^4	0	5	12
220	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}^6	0	6	12
221	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^3	\mathbb{Z}	3	9
222	\mathbb{Z}	\mathbb{Z}_6	\mathbb{Z}^3	\mathbb{Z}	3	8
223	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	\mathbb{Z}	0	8
224	\mathbb{Z}	\mathbb{Z}_{11}	\mathbb{Z}^4	0	5	6
225	\mathbb{Z}	\mathbb{Z}_{12}	\mathbb{Z}^3	\mathbb{Z}	3	6
226	\mathbb{Z}	\mathbb{Z}_3	\mathbb{Z}^5	0	6	6
227	\mathbb{Z}	\mathbb{Z}_4	\mathbb{Z}^5	0	6	6
228	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^3	\mathbb{Z}	3	6
229	\mathbb{Z}	\mathbb{Z}_9	\mathbb{Z}^4	0	5	6
230	\mathbb{Z}	0	$\mathbb{Z}^3 \oplus \mathbb{Z}_2$	0	4	5
231	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z}^2 \oplus \mathbb{Z}_2$	\mathbb{Z}	1	4
232	\mathbb{Z}	\mathbb{Z}	$\mathbb{Z} \oplus \mathbb{Z}_2$	0	1	4
233	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^5	0	6	2
234	\mathbb{Z}	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	\mathbb{Z}^6	0	7	2

Table 6 : Homology classes (spaces made of 3 simplexes) n:201-234