Should the Standard Model of particle physics have merely a conventional definition of the electric charge ?

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Abstract

It is most disconcerting that the Standard Model of particle physics defines, as fundamental a quantity as the electric charge, only conventionally and arbitrarily. We look into the nature of this conventionality and try to find an underlying structure to it. This brings in the Higgs field in a non-trivial manner, which points as to how the above arbitrariness and conventionality may be avoided. Next, the same is demonstrated as actually being part of the intrinsic mathematical structure of the Standard Model.

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The Standard Model of particle physics has been an exteremly successful model in particle physics. In spite of this success, a very disconcerting fact is that, in it, the definition of as fundamental a quantity as the electric charge, is merely **conventional** [1]. This fact is reflected by the fact that there exist two **arbitrary** definitions of the electric charge in the Standard Model

The first one was given by Glashow [2] in 1961 for the group $SU(2)_W \otimes U(1)_W$. It is used very extensively, say, by about half the particle physics community, see e.g. ref. [1,3]:

$$Q = {T_3}^W + \frac{Y_W}{2} \tag{1}$$

Here Y_W is called weak-hypercharge. One fixes the values of the weak-hypercharge Y_W to fit the various charges of say, the first generation of matter particles (ν, e) and (u,d). For example to get the correct electric charge for the left-handed electron and the corresponding ν one fixes $Y_W = -1$ and so on.

The second definition is used by the other half of the particle physics community, who define the same electric charge as (see e.g. ref. [4]):

$$Q = {T_3}^W + Y_W \tag{2}$$

Here the values of weak-hypercharges change with respect to the first Glashow definition, e.g, for (ν, e) now the value is Y_W = -1/2. Again all the values of hypercharges are fixed as per this new convention. The reader is invited to identify which convention he/she has been following.

However for a model as successful as the Standard Model of particle physics, it appears as a major weakness, that the definition of the electric charge itself is conventional. As scientists, we would like our definitions of quantities, and that too as basic as the electric charge, not to be arbitray and conventional.

So, is it that we are destined to live with this shortcoming for ever? We need not become as pessimistic as that. One should realize that if there are any arbitrary and conventional definitions in physics, it may be indicating that we are actually missing some essential and basic aspects of the mathematical and phyiscal reality relevant to the physics at hand. What possibly may we be missing here?

Note that Higgs comes in a basic and fundamenatl way in the Stadard Model. Let us now include and Higgs field also in the standard way as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{3}$$

With an unknown weak-hypercharge defined as Y_ϕ. Now to get the correct charges for the Higgs field itself, we need, for

Convention I : $Y_\phi = 1$;

Convention II: $Y_\phi = 1/2$

Since Higgs is the uniform and ubiquitous field providing all the well known properties to the Standard Model, is it possible that the above different results for the Higgs hypercharges are trying to tell us something. A moments thought tells us that indeed it is so. These arbitrary conventions, for the the electric charge can be generalised to

$$Q = T_3{}^W + bY_W \tag{4}$$

where b is fixed so that the upper and lower components of the Higgs doublet get the correct charges. Immediately we see that it is so when we take b $= \frac{1}{2Y_\phi}$. So the above electric charge becomes

$$Q = T_3{}^W + (\frac{1}{2Y_\phi})Y_W \tag{5}$$

Now we see that for $Y_W = Y_\phi$ the correct Higgs doublet charges are obtained. So clearly hidden within the above arbitarainess of these two conventions, is an exact definition of the electric charge which is not arbitrary or conventional. It is exact and gives correct quantized charges to the Higgs doublet.

How about the matter fields? No problem, as long as we take their weak hypecharges to be proportinal to the Higgs Weak-hypercharge. For example for the first generation

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3}; Q(u) = \frac{1}{2}(1 + \frac{1}{3}); Q(d) = \frac{1}{2}(-1 + \frac{1}{3})$$

$$u_R, Y_u = Y_\phi(\frac{4}{3}); Q(u_R) = \frac{1}{2}(\frac{4}{3}); d_R, Y_d = Y_\phi(\frac{-2}{3}); Q(d_R) = \frac{1}{2}(\frac{-2}{3})$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L; Y_l = -Y_\phi; Q(\nu) = 0, Q(e) = -1; e_R, Y_e = -2Y_\phi; Q(e_R) = -1 \tag{6}$$

Thus only by including the Higgs doublet in a consistent manner and by looking deeply into the the two arbitrary and conventional defintions of the electric charge in the Standard Model, we find, entirely on the basis of internal consistency and logic, that the electric charge is unambigously defined in the Standard Model as $Q = {T_3}^W + (\frac{1}{2Y_\phi})Y_W$. This gives correct charges to the Higgs doublet and also with proper definitions of the weak-hypercharges for the matter field in each generation - which are always proportional to the Higgs weak-hypercharges, give the correct and actually properly quantized electric charges in the Standard Model. Hence we state that contrary to the popular belief [1,2,3,4], there are no arbitaray conventions and that the electric charge can be exactly defined in the Standard Model. In addition, the electric charge is also consistently quantised in the Standard Model. Also we note that this quantization never fixes the Higgs weak-hypercharge.

The above amazingly follows from simple consistency arguments within the Standard Model. Next, is it possible to obtain the above electric charge expressions in the Standard Model in a more mathematical and rigorous manner? Or is it possible that we may understand the various hypercharges defined as being proportional to the Higgs hypercharge and the correct definition of the electric charge in terms of the Higgs hypercharge as $Q = {T_3}^W + (\frac{1}{2Y_\phi})Y_W$ in a mathematically consistent manner? It is heartening to note that indeed it is so. Below we do just that.

Let us start by looking at the first generation of quarks and leptons (u, d, e,ν) and assign them to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ representation as follows [5,6].

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (3, 2, Y_q)$$

$$u_R; (3, 1, Y_u)$$

$$d_R; (3, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}; (1, 2, Y_l)$$

$$e_R; (1, 1, Y_e) \tag{7}$$

To keep things as general as possible this brings in five unknown hypercharges.

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q' = a' I_3 + b' Y \tag{8}$$

We can always scale the electric charge once as $Q = \frac{Q'}{a'}$ and hence ($b = \frac{b'}{a'}$)

$$Q = I_3 + bY \tag{9}$$

In the Standard Model $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaniously broken through the Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. In this model the Higgs is assumed to be doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the Higgs develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in (3) and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{10}$$

Note that this is exactly the same as the eqn. (8) above. To proceed further one imposes the anomaly cancellation conditions to establish constraints on the various hypercharges above. First $[SU(3)_c]^2 U(1)_Y$ gives $2Y_q = Y_u + Y_d$ and $[SU(2)_L]^2 U(1)_Y$ gives $3Y_q = -Y_l$. Next $[U(1)_Y]^3$ does not provide any new constraints. So the anomaly conditions themselves are not sufficient to provide quantization of electric charge in the Standard Model. One has to provide new physical inputs to proceed further. Here one demands that fermions acquire masses through Yukawa coupling in the Standard Model. This brings about the following constraints:

$$Y_u = Y_q + Y_\phi; Y_d = Y_q - Y_\phi; Y_e = Y_l - Y_\phi \tag{11}$$

Note that $2Y_q = Y_u + Y_d$ from the anomaly cancellation condition for $[SU(3)_c]^2 U(1)_Y$ is automatically satisfied here from the Yukawa condition above. Now using $3Y_q = -Y_l$ from anomaly cancellation along with Yukawa terms above in $[U(1)_Y]^3$ does provide a new constrains of $Y_l = -Y_\phi$. Putting all these together one immediately gets charge quantization in the Standard Model [5,6] as follows:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3},$$

$$Q(u) = \frac{2}{3}, Q(d) = \frac{-1}{3}$$

$$u_R, Y_u = \frac{3}{4}Y_\phi, Q(u_R) = \frac{2}{3}$$

$$d_R, Y_d = \frac{-2}{3}Y_\phi, Q(d_R) = \frac{-1}{3}$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}, Y_l = -Y_\phi, Q(\nu) = 0, Q(e) = -1$$

$$e_R, Y_e = -2Y_\phi, Q(e_R) = -1 \tag{12}$$

It has also been shown [5] that for arbitrary N_c the colour dependence of the electric charge as demanded by the Standard Model is

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c})$$

$$Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{13}$$

Note that within the Standard Model group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ the electroweak sector consists of $SU(2)_L \otimes U(1)_Y$. Still the electric charge knows of the colour degree of freedom. Interestingly though the electro-magnetism does not know of the colour, still the electric charge has colour, existing within its guts, so to say. This shows that the Standard Model still is more unified than we had visualized so far.

Hence here we have shown that mathematically the structure of the Standard Model is such that the electric charge has no arbitrariness or conventionality involved in it, and that it is also fully and consistently quanitized. In adition, this structure, as obtained on the basis of our earlier phenomenological considerations, is fully supported by our mathemtical analysis here.

References

(1). T-P. Cheng and L-F. Li, "Gauge theory of elementary particle physics", Clarendon Press, Oxford, 1984, p. 346

(2). S. L. Glashow, "Partial symmetry of weak interactions", Nucl. Phys. **22**, 579 (1961)

(3). L. H. Ryder, "Quantum Field Theory", Cambridge University Press, Cambridge, 1986 (reprinted 2006), p. 309;

F. Halzen and A. D. Martin, "Quarks and leptons", John Wiley, New York 1984 (reprinted 2010), p. 294;

A. Zee, "Quantum Field Theory in a nutshell", Princeton University Press, Princeton, 2003, p. 363;

R. Mann, "An introduction to particle physics and Standard Model", CRC Press, London, 2010, p. 440;

A. Bettini, "Introduction to elementary particle physics", Cambridge University Press, Cambridge, 2008, p. 305;

C. Quigg, "spontaneous symmetry breaking as a basis of particle mass", Rep. Prog. Phys. **70**, 1019 (2007), p. 1024

(4). T. D. Lee, "Particle physics and introduction to field theory", Harwood, New York, 1981, p. 674;

S. Weinberg, "The quantum theory of fields", Vol. II, Cambridge University Press, Cambridge, 1996, p. 385;

H. Georgi, "Weak interactions and modern particle theory", Benjamin/Cummings, Menlo Park, 1984, p. 16;

K. Huang, "Quarks, leptons, and gauge fields", "World Scientific, Singapore, 1992, p. 114;

D. H. Perkins, "Introduction to High Energy Physics", Cambridge University Press, Cambridge, 2000, p. 350;

M. Srednicki, "Quantum Field Theory", Cambridge University Press, Cambridge, 2007, p. 543;

W. N. Cottingham and D. A. Greenwood, "An introduction to the Standard Model of particle physics", Cambridge University Press, Cambridge, 1998, p. 132

(5). A Abbas, "Anomalies and charge quantization in the Standard Model with arbitrary number of colours", Phys. Lett **B 238**, 344 (1990)

(6). A Abbas, "Spontaneous symmetry breaking, quantization of the electric charge and the anomalies", J. Phys. **G 16**, L163 (1990)