A Proof of the Collatz Conjecture

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Abstract

If every positive integer is able to be operated to 1 by the set operational rule of the Collatz conjecture, then begin with 1, we can get all positive integers by operations on the contrary of the set operational rule for infinite many times. In this article, we will apply the mathematical induction with the help of certain operations by each other's- opposed operational rules to prove that the Collatz conjecture is tenable.

Keywords

Mathematical induction, classify positive integers, the bunch of integers' chains, the two-way operational rules, operational routes

Basic Concepts

The Collatz conjecture is also known variously as 3n+1 conjecture, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, Hasse's algorithm, or the Syracuse problem, etc.

The Collatz conjecture states that take any positive integer n, if n is an even number, then divide n by 2 to obtain an integer; if n is an odd number, then multiply n by 3 and add 1 to obtain an even number.

Repeat the above process indefinitely, then no matter which positive integer you start with, you will always eventually reach a result of 1.

We consider the way of aforesaid two steps as leftward operational rule for any positive integer. Also consider operations on the contrary of the leftward operational rule as rightward operational rule for any positive integer. Taken one with another, we consider such each other's- opposed operational rules as two-way operational rules.

The rightward operational rule stipulates that for any positive integer n, multiply n by 2 to obtain an even number. Additionally where n is an even number, if divide the difference of n minus 1 by 3 and obtain an odd number, then must operate the step, and proceed from here to operate uninterruptedly; if it is not such, then don't operate the step.

Begin with a positive integer to operate by either operational rule continuously, manifestly each operational result is a positive integer, then we consider a string of such consecutive positive integers on an operational direction plus arrowheaded signs inter se as an operational route. Each operational result comes only from an adjacent positive integer at an identical operational route. If any positive integer P exists at an operational route, then we can term the operational route "route of P". Two routes of P either branch or converge at P.

Begin with 1 to operate each positive integer successively got by the rightward operational rule, so such an operational course forms a bunch of operational routes spontaneously. We term such a bunch of operational routes "a bunch of integers' chains". Manifestly whole a bunch of integers' chains must consist of infinite many operational routes.

Since a direct origin of each positive integer is unique, then each positive integer except for 1 is unique at the bunch of integers' chains.

Comparatively speaking, inside greater limits, positive integers on the left are smaller, yet positive integers on the right are larger, at the bunch of integers' chains. Overall, from left to right positive integers at the bunch of integers' chains are getting both more and more absolutely, and greater and greater relatively. Please, see a beginning of the bunch of integers' chains below.

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                                 …            …    452→…
                               168↑        680↑  226↑→75→…
                             84↑        340↑→113↑     227→…
                           42↑        170↑          682↑→…
       1                  21↑       85↑          341↑
1→2→4↑→8→16↓→32→64↑→128→256↑→512→1024↑→2048→…
              5→10↓→20→40↓→80→160↓→320→…
                   3↓       13↓      53→106↓→212→…      …
                    6↓       26↓          35→70↓→……   30↑
                     12↓       52↓→104↓        23→46↑→15↑
                      24↓        17↓
                       …          34↓→11→22↓→7→14→28↓→..
                                   68↓       44→88↓→…   9↓
                                    136↓→…       29↓     18↓
                                       45→…       58↓→19↓ ...
                                                    …     …
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First Illustration

Annotation: ↓ and ↑ must rightwards tilt, but each page is narrow, thus it can only so.

No matter which positive integer, it is surely at the bunch of integers' chains so long as it is able to be operated to 1 by the leftward operational rule. Likewise, the converse proposition holds water too.

That is to say, positive integers at the bunch of integers' chains and positive integers which can operate to 1 by the leftward operational rule are one-to-one correspondence.

Thus it can seen, whether or not a positive integer suits the conjecture, need merely us to determine that whether or not it can be at the bunch of integers' chains.

If every positive integer is able to be operated to 1 by the leftward operational rule, then there are all positive integers at whole the bunch of integers' chains. Correspondingly, if we can prove that all positive integers exist at the bunch of integers' chains, then every positive integer is able to be operated to 1 by the leftward operational rule.

Because of this, we will prove that the bunch of integers' chains contains all positive integers by mathematical induction in the rear proof.

If we resolve the bunch of integers' chains into one-way operational routes according to already arisen un-operated smallest odd number except for 1 to operate in a row in proper order, so a beginning of the bunch of integers' chains is dismembered into certain operational routes as the follows.

1→2→4↓→8→16↓→32→64↓→128→256↓→512→1024↓→2048→4096↓→8192…

1 5 21 85 341 1365

5→10↓→20→40↓→80→160↓→320→640↓→1280→2560↓→5120→10240↓→…

3 13 53 213 853 3413

3→6→12→24→48→96→192→384→768→1536→3072→6144→12288→…

13→26→52↓→104→208↓→416→832↓→1664→3328↓→6656→13312↓→ …

17 69 277 1109 4437

17→34↓→68→136↓→272→544↓→1088→2176↓→4352→8704↓→17408→…

11 45 181 725 2901

11→22↓→44→88↓→176→352↓→704→1408↓→2816→5632↓→11264→…
7 29 117 469 1877

7→14→28↓→56→112↓→224→448↓→896→1792↓→3584→7168↓→14336→…
9 37 149 597 2389

9→18→36→72→144→288→576→1152→2304→4608→9216→18432→…

21→42→84→168→336→672→1344→2688→5376→10752→21504→…

29→58↓→116→232↓→464→928↓→1856→3712↓→7424→14848↓→29696→…
19 77 309 1237 4949

19→38→76↓→152→304↓→608→1216↓→2432→4864↓→9728→19456↓→ …
25 101 405 1621 6485

25→50→100↓→200→400↓→800→1600↓→3200→6400↓→12800→25600↓→…
33 133 533 2133 8533

33→66→132→264→528→1056→2112→4224→8448→16896→33792→…

37→74→148↓→296→592↓→1184→2368↓→4736→9472↓→18944→37888↓→…
49 197 789 3157 12629

45→90→180→360→720→1440→2880→5760→11520→23040→46080→…

49→98→196↓→392→784↓→1568→3136↓→6272→12544↓→25088→50176↓→…
65 261 1045 4181 16725

53→106↓→212→424↓→848→1696↓→3392→6784↓→13568→27136↓→54272…
35 141 565 2261 9035

35→70↓→140→280↓→560→1120↓→2240→4480↓→8960→17920↓→35840→…
23 93 373 1493 5973

23→46↓→92→184↓→368→736↓→1472→2944↓→5888→17776↓→23552→…
15 61 245 981 5925

……

From the above listed rows, we can see that first integer at every row is an odd number, yet others are all even numbers irrespective of rows of all odd numbers without arrowheads. On operations of the contrary, we look upon which multiply an odd number by 3 and add 1 to obtain an even number as which the operation upgrades a stair, also look upon which divide an even number by 2 to obtain an integer as which the operation goes a step leftwards, at the operational course by the leftward operational rule. Whether upgraded a stair or gone a step leftwards, enable the operation to approach further final result of 1.

Furthermore, we need also to first determine an axiom, so that after an anticipative result arises out, we just use it to give an affirmation.

Axiom* For any positive integer P, if there is a positive integer C< P at a route of P or at another operational route which directly/ indirectly links up the route of P, and C∈L, then P suits the conjecture, where L expresses limits of positive integers which suit the conjecture.

Give three examples: (1) Let $P=31+3^2\eta$ and $\eta\geq 0$, $27+2^3\eta\to 82+3*2^3\eta\to 41+3*2^2\eta\to 124+3^2*2^2\eta\to 62+3^2*2\eta\to 31+3^2\eta>27+2^3\eta$, where $27+2^3\eta\in L$, then $31+3^2\eta$ suits the conjecture.

(2) Let $P=5+12\mu$ and $\mu\geq 0$, $4+9\mu\to 8+18\mu\to 16+36\mu\to 5+12\mu > 4+9\mu$, where $4+9\mu\in L$, then $5+12\mu$ suits the conjecture.

(3) Let $P=63+3*2^8\varphi$, and $\varphi\geq 0$, $63+3*2^8\varphi\to 190+3^2*2^8\varphi\to 95+3^2*2^7\varphi\to 286+3^3*2^7\varphi\to 143+3^3*2^6\varphi\to 430+3^4*2^6\varphi\to 215+3^4*2^5\varphi\to 646+3^5*2^5\varphi\to 323+3^5*2^4\varphi\to 970+3^6*2^4\varphi\to 485+3^6*2^3\varphi\to 1456+3^7*2^3\varphi\to 728+3^7*2^2\varphi \to 364+3^7*2\varphi\to 182+3^7\varphi\uparrow\to \ldots$

$\uparrow 121+3^6*2\varphi\leftarrow 242+3^6*2^2\varphi\leftarrow 484+3^6*2^3\varphi\leftarrow 161+3^5*2^3\varphi\leftarrow 322+3^5*2^4\varphi$

$\leftarrow 107+3^4*2^4\varphi \leftarrow 214+3^4*2^5\varphi \leftarrow 71+3^3*2^5\varphi \leftarrow 142+3^3*2^6\varphi \leftarrow 47+3^2*2^6\varphi <$ $63+3*2^8\varphi$, where $47+3^2*2^6\varphi \in L$, then $63+3*2^8\varphi$ suits the conjecture.

Actually, every positive integer at directly/ indirectly linked operational routes suits the conjecture provided they contain a positive integer $C \in L$, where L expresses limits of positive integers which suit the conjecture.

The Proof

Let us set about the proof that the bunch of integers' chains contains all positive integers by mathematical induction hereinafter.

1. From preceding first illustration, we can directly find consecutive positive integers from 1 to 24 within positive integers already got at the bunch of integers' chains.

2. Suppose that after further operate positive integers successively got by the rightward operational rule, there are consecutive positive integers $\leq$ n within all positive integers successively got at a bunch of integers' chains, where n $\geq$24.

3. Prove that after continue to operate positive integers successively got by the rightward operational rule, we can get consecutive positive integers $\leq$ 2n within all positive integers successively got at a bunch of integers' chains.

Let us divide limits of consecutive positive integers at the number axis into segments, according to greatest positive integer $2^X n$ per segment, where $X \geq 0$ and $n \geq 24$, so as to accord with the proof. A simple illustration follows.

1———n———2n——————4n————————8n——→

Second Illustration

Proof * Since there are consecutive positive integers ≤ n within all positive integers successively got at a bunch of integers' chains, thus multiply each positive integer ≤ n by 2 by the rightward operational rule, so we get all even numbers between n and 2n+1 at the bunch of integers' chains, irrespective of repeated even numbers ≤ n.

After that, we must seek an origin of each kind of odd numbers between n and 2n+1 by the two-way operational rules, whether or not each kind of odd numbers has an origin, it is smaller than the kind of odd numbers.

First, let us divide all odd numbers between n and 2n+1 into two kinds, i.e. 5+4k and 7+4k, where k is a natural number ≥ 5, then any odd number between n and 2n+1 belongs in one of the two kinds. By now we list the two kinds of odd numbers in correspondence with k below.

k:	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16…
5+4k:	25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69 …
7+4k:	27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71 …

From 5+4k→16+12k→8+6k→4+3k<5+4k, 5+4k suit the conjecture according to the axiom, so 5+4k are at the bunch of integers' chains.

For 7+4k, let us again divide them into three kinds, i.e. 11+12c, 15+12c and 19+12c, where c ≥ 1.

From 7+8c→22+24c→11+12c > 7+8c, 11+12c suit the conjecture according to the axiom, so 11+12c are at the bunch of integers' chains.

Likewise, we list remainder two kinds of odd numbers in correspondence with c below.

c:	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12, …
15+12c:	27,	39,	51,	63,	75,	87,	99,	111,	123,	135,	147,	159 …
19+12c:	31,	43,	55,	67,	79,	91,	103,	115,	127,	139,	151,	163 …

Hereinafter, we will operate respectively 15+12c and 19+12c by the leftward operational rule, also discover and affirm satisfactory results at certain operational branches. Firstly, let us operate 15+12c below.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   **…**
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**
g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**
F=2g: 304+1458g→152+729g ↓→**…**
…

…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**   **…**
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**
f=2g: 20+243g↓→g=2h: 10+243h (**6**)   **…**
g=2h+1:880+1458h→440+729h↑→ **…**

Annotation:
Each letter c, d, e, f, g … in the above-listed operational routes expresses respectively each of natural numbers plus 0, similarly hereinafter.
Also there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

We conclude several partial satisfactory results from above-listed a bunch of operational routes of 15+12c, as the follows.

From c =2d+1 and d =2e+1, get c=2d+1=2(2e+1) +1=4e+3, and 15+12c = 15+12(4e+3) = 51+48e > 29+27e where mark (1), so 15+12c where c=4e+3 suit the conjecture according to the axiom.

From c=2d+1, d=2e, and e=2f+1, get c =2d+1=4e+1=4(2f+1) +1=8f+5, and 15+12c=15+12(8f+5) = 75+96f > 64+81f where mark (2), so 15+12c where c=8f+5 suit the conjecture according to the axiom.

From c=2d, d=2e+1 and e=2f+1, get c=2d= 4e+2= 4(2f+1) +2=8f+6, and 15+12c=15+12(8f+6) = 87+96f >74+81f where mark (3), so 15+12c where c=8f+6 suit the conjecture according to the axiom.

From c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, get c=2d+1=4e+1=8f+1 = 8(2g+1)+1=16g+9=16(2h+1)+9=32h+25, and 15+12c=15+12(32h+25) = 315+384h >200+243h where mark (4), so 15+12c where c=32h+25 suit the conjecture according to the axiom.

From c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, get c=2d=2(2e+1)=4e+2= 8f+2 = 8(2g+1)+2 = 16g+10 = 32h+10, and 15+12c = 15+12(32h+10) = 135+384h > 86+243h where mark (5), so 15+12c where c=32h+10 suit the conjecture according to the axiom.

From c=2d, d=2e, e=2f, f=2g and g=2h, get c=2d =32h, and 15+12c = 15+12(32h) = 15+384h > 10+243h where mark (6), so 15+12c where c=32h suit the conjecture according to the axiom.

Secondly we operate 19+12c by the leftward operational rule below.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**) e=2f:37+81f (**β**)

♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥

c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠

d=2e+1:94+81e↓→e=2f:47+81f (γ)

e=2f+1:516+486f ♦

g=2h: 129+243h (**δ**) **…**

f=2g+1:258+243g↑→g=2h+1:1504+1458h→752+729h↑→**…**

♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓

g=2h: 175+729h↓→**…** **…**

…

g=2h+1:172+243h (**ε**)

f=2g: 101+243g↑→g=2h: 304+1458h→**…**

e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**

♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦516+486f→258+243f↓→f=2g+1:1504+1458g→**…**

f=2g: 129+243g↓→g=2h: 388+1458h→**…**

g=2h+1:186+243h (**ζ**)

Annotation:
Each letter c, d, e, f, g, h **…** in the above-listed operational routes expresses respectively each of natural numbers plus 0, similarly hereinafter.
Also there are ♣↔♣, ♥↔♥, ♠↔♠, ♦↔♦.

We conclude too several partial satisfactory results from above-listed a bunch of operational routes of 19+12c, as the follows.

From c=2d, d=2e, get c=2d=4e, and 19+12c = 19+12(4e) = 19+48e > 11+27e where mark (α), so 19+12c where c=4e suit the conjecture according to the axiom.

From c=2d, d=2e+1 and e=2f, get c=2d = 2(2e+1) = 4e+2 = 8f+2, and 19+12c=19+12(8f+2) = 43+96f >37+81f where mark (β), so 19+12c where c=8f+2 suit the conjecture according to the axiom.

From c=2d+1, d=2e, and e=2f, get c=2d+1= 4e+1= 8f+1, and 19+12c =

$19+12(8f+1) = 31+96f > 47+81f$ where mark (γ), so $19+12c$ where $c=8f+1$ suit the conjecture according to the axiom.

From $c=2d$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h$, get $c=2d=2(2e+1)= 4e+2= 4(2f+1)+2 = 8f+6= 8(2g+1)+6 = 16g+14 = 32h+14$, and $19+12c = 19+12(32h+14) = 187+384h > 129+243h$ where mark (δ), so $19+12c$ where $c=32h+14$ suit the conjecture according to the axiom.

From $c=2d+1$, $d=2e$, $e=2f+1$, $f=2g$ and $g=2h+1$, get $c=2d+1=4e+1 = 4(2f+1)+1 = 8f+5 = 16g+5 = 16(2h+1)+5 = 32h+21$, and $19+12c = 19+ 12(32h+21)= 271+384h > 172+243h$ where mark (ε), so $19+12c$ where $c=32h+21$ suit the conjecture according to the axiom.

From $c=2d+1$, $d=2e+1$, $e=2f+1$, $f=2g$ and $g=2h+1$, get $c=2d+1=2(2e+1) +1 = 4e+3=4(2f+1)+3=8f+7=16g+7=16(2h+1)+7=32h+23$, and $19+12c = 19+12(32h+23) = 295+384h > 186+243h$ where mark (ζ), so $19+12c$ where $c=32h+23$ suit the conjecture according to the axiom.

Let χ=d, e, f, g, h … etc, then the odevity of integer's expressions which contain a variable sign χ is indeterminate, so for any such integer's expression, both consider it as an even number to operate, and consider it as an odd number to operate. Let us label such integer's expressions "odd-even integer's expressions" thereinafter.

For any odd-even integer's expression at operational routes of $15+12c$/ $19+12c$, two operations synchronize according as χ expresses both an odd number and an even number. Further, begin with a greater result thereof, it will continue to operate. If the smaller result is not smaller than a kind of 15+12c/ 19+12c, then it must too continue to operate. If the smaller result is smaller than a kind of 15+12c/ 19+12c, then the kind of 15+12c/ 19+12c has suited the conjecture according to the axiom, so operations of the branch may stop here.

In other words, on the one hand, begin with any odd-even integer's expression, two kinds' operations are always endlessly progress and branch according as χ expresses both an odd number and an even number, up to arise infinitely more progress and branch. Of course, odd-even integer's expressions successively got via orderly operations are getting more and more, and the more rear arisen odd-even integer's expressions, the greater are their values, up to engender infinitely many infinities theoretically. On the other hand, partial branches stop uninterruptedly operations in the infinite many operational routes, because each such branch has operated to a result which is smaller than a kind of 15+12c/ 19+12c, but also there are infinitely many such results.

For an odd-even integer's expression, both operate it as an even number into a half itself, and operate it as an odd number into threefold itself and add 1. Thus confront an incremental result and a reductive result after operations of each odd-even integer's expression, there is only possibly the reductive result to suit the conjecture. Consequently operate 15+12c/

19+12c will proceed infinitely.

Judging from this, 15+12c and 19+12c must be divided respectively into infinite many kinds, just enable every kind is operated to suit the conjecture by the leftward operational rule for infinite many times.

This notwithstanding, what we need is to prove merely that odd numbers of 15+12c plus 19+12c between n and 2n+1 suit the conjecture, yet it is not all of 15+12c plus 19+12c. Clearly these odd numbers between n and 2n+1 are smaller odd numbers within unproved kindred odd numbers.

After operate 15+12c/ 19+12c for finite times by the leftward operational rule, the number of kinds of arisen 15+12c/ 19+12c is finite still, though the number of odd numbers of each kind is infinite.

We know that consecutive positive integers from 1 to 24 are concrete positive integers undoubtedly, also known n ≥24.

For positive integer n which we supposed on second step of the mathematical induction, if n is the infinity, then this means that every positive integer ≥ 24 suits the conjecture, so we need not to prove it. If n is a concrete positive integer inside finite limits, then 2n is a concrete positive integer inside finite limits, of course odd numbers of 15+12c plus 19+12c between n and 2n+1 are concrete odd numbers, and the number of them is finite too, so the number of their kinds is finite.

From the preceding analysis, we can get that for each integer's expression which is smaller than a kind of 15+12c/ 19+12c at operational routes of 15+12c/ 19+12c, its constant term and coefficient of χ are throughout smaller than the constant term and coefficient of χ of another of the twin integer's expressions. Yet another will continue to be operated by us. On balance, inside some greater limits, constant terms and coefficients of χ of integer's expressions which are smaller than some kinds of 15+12c/ 19+12c are smaller than constant terms and coefficients of χ of integer's expressions which need us continue to operate.

Therefore after χ is bestowed with 0, 1, 2, 3…, we can get some smaller concrete positive odd numbers. For example, there are 51+48e, 75+96f, 87+96f, 315+384h, 135+384h, 15+384h, 19+48e, 43+96f, 31+96f, 187+384h, 271+384h and 295+384h at the above-listed two bunches of operational routes of 15+12c plus 19+12c. After e, f and h are bestowed with 0, 1, 2, 3…, we get odd numbers which are greater than 24, they are: 51, 75, 87, 315, 135, 43, 31, 187, 271, 295; 99, 171, 183, 699, 519, 399, 67, 139, 127, 571, 655, 679; 147, 267, 279, 1083, 903, 783, 115, 235, 223, 955, 1039, 1063; 195, 363, 375, 1467, 1287, 1167, 163, 331, 319, 1339, 1423, 1447; ...

Without doubt, these odd numbers belong within aforementioned twelve kinds of 15+12c plus 19+12c still. Nevertheless they are respectively smallest or smaller odd numbers within the twelve kinds.

As operations go on, front-end integer's expressions at some branches within extended and increasing operational routes are smaller than some kinds of 15+12c/ 19+12c, and enable these kinds of 15+12c/ 19+12c suit the conjecture. However, after continue to operate integer's expressions at retained branches, like that, there are both kinds of 15+12c/ 19+12c which suit the conjecture, and kinds of 15+12c/ 19+12c which need us continue to operate. After operations go beyond some limits, for every integer's expression successively got, even if let χ is bestowed with 0, it is not smaller than 2n+1 either. Namely where operations are out of the limits, the sum of constant term and coefficient of χ of every integer's expression successively got is greater than 2n always.
Thus is can seen, every kind of 15+12c/ 19+12c between n and 2n+1 is able to be operated into an integer's expression which is smaller than the kind itself by the leftward operational rule after operate finite times.

Altogether, after operate 15+12c/ 19+12c for finite times by the leftward operational rule, are completely able to get the very satisfying result that all kinds of 15+12c/ 19+12c between n and 2n+1 suit all the conjecture. Afterwards, χ of an integer's expression of each kind of 15+12c/ 19+12c is bestowed with beginning's 0, 1, 2, 3… we can find each positive integer of 15+12c/ 19+12c between n and 2n+1 in more positive integers after bestow values of χ got. So all positive integers of 15+12c/ 19+12c between n and 2n+1 suit the conjecture, justly, all of them are at the bunch of integers' chains.

To sum up, we have proven that all even numbers and all odd numbers between n and 2n+1 are at the bunch of integers' chains by two-way operational rules. Consequently, all positive integers between n and 2n+1 are proven to suit the conjecture.

So far, we have proven that positive integers $\leq 2^1n$ suit the conjecture by consecutive positive integers $\leq n$, likewise we can too prove that positive integers $\leq 2^2n$ suit the conjecture by consecutive positive integers $\leq 2^1n$ according to the foregoing way of doing.

At the beginning of the proof, we spoken that divide limits of all consecutive positive integers into segments according to greatest positive integer 2^Xn per segment, where $X\geq0$, and $n \geq 24$.

After we proven that positive integers between $2^{X-1}n$ and $2^X n$ suit the conjecture by consecutive proven positive integers $\leq 2^{X-1}n$, in the same old way, we are too able to prove that positive integers between $2^X n$ and $2^{X+1}n$ suit the conjecture by consecutive proven positive integers$\leq 2^Xn$.

For up-end 2^Xn of each segment of integers, X begins with 0, next it is orderly endowed with 1, 2, 3… In pace with which values of X are getting greater and greater, consecutive positive integers$\leq 2^Xn$ are getting more and more, and new positive integers are getting greater and greater.

Suppose X equals every natural number plus 0, then all positive integers are proven to suit the conjecture, namely every positive integer is proven to suit the conjecture.

Heretofore, the Collatz conjecture is proven at long last by us. The proof was thus brought to a close. As a consequence, the Collatz conjecture holds water.