A Proof of the Beal's Conjecture

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Introduction: The Beal's Conjecture was discovered by Andreew Beal in 1993. Later the conjecture plus the prize which solve it was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still an unsolved problem at number theory hitherto.

Abstract

In this article, we first have proven a lemma of $E^P+F^V \neq 2^M$. Successively have proven the Beal's conjecture by mathematical analyses with the aid of the lemma, such that enable the Beal's conjecture holds water.

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A Lemma and the Proof thereof

In order to lay a foundation for the proof of the conjecture, let us first prove a lemma of $E^P+F^V \neq 2^M$, where E and F are positive odd numbers without any common prime factor >1, and P, V and M are integers ≥3.

To start with, let smaller one within E^P and F^V is S_1, and greater one is W_1, so there is $E^P+F^V=2S_1+(W_1-S_1)$. Manifestly W_1-S_1 is a positive even number, and let W_1-S_1=2H_1, where H_1 is a positive integer. Then, divide 2S_1+ (W_1-S_1) by 2, and we get S_1+H_1. Also divide 2^M on the right of the expression by 2 synchronously, and we get 2^{M-1}.

If H_1 is a positive even number, then there is only $S_1+H_1 \neq 2^{M-1}$ because S_1+H_1 is an odd number, yet 2^{M-1} is an even number. So we backed up $E^P+F^V \neq 2^M$ from $S_1+H_1 \neq 2^{M-1}$.

If H_1 is a positive odd number, then let smaller one within S_1 and H_1 is S_2, and greater one is W_2, so there is $S_1+H_1=2S_2+ (W_2-S_2)$. Likewise W_2-S_2 is a positive even number, and let W_2-S_2 =2H_2, where H_2 is a positive integer. Then, divide 2S_2+ (W_2-S_2) by 2, and we get S_2+H_2. Also divide 2^{M-1} on the right of the expression by 2 synchronously, and we get 2^{M-2}.

If H_2 is a positive even number, then there is only $S_2+H_2 \neq 2^{M-2}$ because S_2+H_2 is an odd number, yet 2^{M-2} is an even number. So we backed up $E^P+F^V \neq 2^M$ from $S_2+H_2 \neq 2^{M-2}$.

If H_2 is a positive odd number, and so on and so forth …

Overall, in the process which compares S_Y+H_Y with 2^{M-Y} seriatim, where $1 \leq Y \leq M-1$, S_Y is always a smaller positive odd number, if H_Y is a positive even number, then there is only $S_Y+H_Y \neq 2^{M-Y}$ because S_Y+H_Y is an odd number, yet 2^{M-Y} is an even number. So we backed up $E^P+F^V \neq 2^M$ from $S_Y+H_Y \neq 2^{M-Y}$.

In order to make an orderly concrete comparison among E^P, F^V and 2^M, we first divide all positive odd numbers into two kinds, i.e. A and B. Or rather, the form of A is 1+4n, and the form of B is 3+4n, where n is a positive integer plus 0. Odd numbers of A plus B from small to great are respectively arranged below.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Judging from above two ranks of odd numbers, a difference between two odd numbers of A divided by 2 is an even number, and a difference between two odd numbers of B divided by 2 is an even number too. Namely operations of aforementioned H_1 are respectively $(1+4n_1)-(1+4n_2) =4(n_1-n_2)$, $H_1=4(n_1-n_2)/2=2(n_1-n_2)$, and $(3+4n_1)-(3+4n_2) =4(n_1-n_2)$, $H_1= 4(n_1-n_2)/2 =2(n_1-n_2)$, where $n_1\in n$, $n_2\in n$, and $n_1>n_2$. Because of this, if E^P and F^V belong either in A or in B together, then S_1+H_1 on the left of the expression is an odd number, yet 2^{M-1} on the right is an even number, thus $S_1+H_1\neq 2^{M-1}$. So we backed up $E^P+F^V\neq 2^M$ from $S_1+H_1\neq 2^{M-1}$.

We divide again odd numbers of A into two kinds, i.e. A_1 and A_2, and divide again odd numbers of B into two kinds, i.e. B_1 and B_2. Well then the form of A_1 is 1+8n; the form of B_1 is 3+8n; the form of A_2 is 5+8n; and the form of B_2 is 7+8n, where $n\geq 0$. Four kinds of odd numbers are all positive odd numbers. They are arranged as follows respectively.

A_1: 1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105…1+8n …

B_1: 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107…3+8n …

A_2: 5, 13, 21, 29, 37, 45, 53, 61, 69, 77, 85, 93, 101, 109…5+8n …

B_2: 7, 15, 23, 31, 39, 47, 55, 63, 71, 79, 87, 95, 103, 111…7+8n …

Thus it can seen, a difference of A_1-B_1 divided by 2 is B_1 or B_2; a difference of A_2-B_2 divided by 2 is B_1or B_2; a difference of B_1-A_1 divided by 2 is A_1or A_2; and a difference of B_2-A_2 divided by 2 is A_1or A_2. We shall further analyze them thereinafter.

(1) On the supposition that $A_1+B_1=2^M$ and $A_1>B_1$, let $H_1= (A_1-B_1)/2$, well then it has: $A_1-B_1= (1+8n_1)-(3+8n_2) =6+8(n_1-1-n_2)$ or $14+8(n_1-2-k_2)$, where $n_1 \in n$, $n_2 \in n$, and $n_1> n_2$, similarly hereinafter.

For $H_1= [6+8(n_1-1-n_2)]/2=3+4(n_1-1-n_2)$:

If n_1-1-n_2 is an even number, then $H_1=3+4(n_1-1-n_2)=3+8[(n_1-1-n_2)/2] \in B_1$;

If n_1-1-n_2 is an odd number, then $H_1=3+4(n_1-1-n_2)=7+8[(n_1-2-n_2)/2] \in B_2$.

For $H_1= [14+8(n_1-2-n_2)]/2= 7+4(n_1-2-n_2)$:

If n_1-2-n_2 is an even number, then $H_1=7+4(n_1-2-n_2)=7+8[(n_1-2-n_2)/2] \in B_2$;

If n_1-2-n_2 is an odd number, then $H_1=7+4(n_1-2-n_2)=3+8[(n_1-1-n_2)/2] \in B_1$.

(2) On the supposition that $A_2+B_2=2^M$ and $A_2>B_2$, let $H_1= (A_2-B_2)/2$, well then it has: $A_2-B_2= (5+8n_1)-(7+8n_2) =6+8(n_1-1-n_2)$ or $14+8(n_1-2-n_2)$.

For $H_1= [6+8(n_1-1-n_2)]/2 = 3+4(n_1-1-n_2)$:

If n_1-1-n_2 is an even number, then $H_1=3+4(n_1-1-n_2)=3+8[(n_1-1-n_2)/2] \in B_1$;

If n_1-1-n_2 is an odd number, then $H_1=3+4(n_1-1-n_2)=7+8[(n_1-2-n_2)/2] \in B_2$.

For $H_1= [14+8(n_1-2-n_2)]/2 =7+4(n_1-2-n_2)$:

If n_1-2-n_2 is an even number, then $H_1=7+4(n_1-2-n_2)=7+8[(n_1-2-n_2)/2] \in B_2$;

If n_1-2-n_2 is an odd number, then H_1=7+4(n_1-2-n_2)=3+8[(n_1-1-n_2)/2] ∈B_1.

(3) On the supposition that $A_1+B_1=2^M$ and $B_1>A_1$, let H_1= (B_1-A_1)/2, well then it has: B_1-A_1 = (3+8n_1)-(1+8n_2) =2+8(n_1-n_2) or 10+8(n_1-1-n_2).

For H_1= [2+8(n_1-n_2)]/2 =1+4(n_1-n_2):

If n_1-n_2 is an even number, then H_1=1+4(n_1-n_2) = 1+8[(n_1-n_2)/2] ∈ A_1;

If n_1-n_2 is an odd number, then H_1=1+4(n_1-n_2) =5+8[(n_1-1-n_2)/2] ∈ A_2.

For H_1= [10+8(n_1-1-n_2)]/2=5+4(n_1-1-n_2):

If n_1-1-n_2 is an even number, then H_1=5+4(n_1-1-n_2)=5+8[(n_1-1-n_2)/2] ∈A_2;

If n_1-1-n_2 is an odd number, then H_1=5+4(n_1-1-n_2) = 1+8[(n_1-n_2)/2] ∈ A_1.

(4) On the supposition that $A_2+B_2=2^M$ and $B_2>A_2$, let H_1= (B_2-A_2)/2, well then it has: B_2 - A_2 = (7+8n_1)-(5+8n_2) =2+8(n_1-n_2) or 10+8(n_1-1-n_2).

For H_1= [2+8(n_1-n_2)]/2 = 1+4(n_1-n_2):

If n_1-n_2 is an even number, then H_1= 1+4(n_1-n_2) =1+8[(n_1-n_2)/2] ∈ A_1;

If n_1-n_2 is an odd number, then H_1=1+4(n_1-n_2) =5+8[(n_1-1-n_2)/2] ∈ A_2.

For H_1= [10+8(n_1-1-n_2)]/2 = 5+4(n_1-1-n_2):

If n_1-1-n_2 is an even number, then H_1=5+4(n_1-1-n_2)=5+8[(n_1-1-n_2)/2]∈A_2;

If n_1-1-n_2 is an odd number, then H_1=5+4(n_1-1-n_2) =1+8[(n_1-n_2)/2] ∈ A_1.

So we get B_1+ B_2 (or B_1) =2^{M-1}; B_2+ B_1 (or B_2) =2^{M-1}; A_1+ A_2 (or A_1) =2^{M-1} and A_2+ A_1 (or A_2) =2^{M-1}.

Followed a step after the above-mentioned four results is to operate respectively a difference between B_1 and B_2 (or B_1) divided by 2, a difference between B_2 and B_1 (or B_2) divided by 2, a difference between

A_1 and A_2 (or A_1) divided by 2, and a difference between A_2 and A_1 (or A_2) divided by 2. Obviously every result of the four operations is an even number according to the preceding result operated between either two B or two A. Each of these results is exactly aforementioned H_2, yet smaller one in each expression of the step is odd number S_2. Undoubtedly, S_2+H_2 on the left of each of the four expressions is an odd number, yet 2^{M-2} on the right of each of them is an even number, so $S_2+H_2 \neq 2^{M-2}$. Consequently we backed up $A_1+B_1\neq 2^M$ and $A_2+B_2\neq 2^M$ from $S_2+H_2 \neq 2^{M-2}$.

Thereinafter, we shall prove remainder four kinds, i.e. a difference of A_1-B_2 divided by 2 where $A_1>B_2$, a difference of B_1-A_2 divided by 2 where $B_1>A_2$, a difference of A_2-B_1 divided by 2 where $A_2>B_1$ and a difference of B_2-A_1 divided by 2 where $B_2>A_1$, because each result of the four operations is an odd number. Judging from this, we are very hard to come to conclusions, if use successively the aforementioned way of doing. Such being the case, we shall further analyze permutations of four kinds of odd numbers, so as to discover some law to prove them.

Since E and F have not any common prime factor>1, so $E^P\neq F^V$ according to the unique factorization theorem of natural number, so let $F^V>E^P$. If $E^P+F^V=2^M$, then F^V is greater than 2^{M-1}, yet E^P is smaller than 2^{M-1}. We list from small to great odd numbers and label a kind of each itself of them, well then you would discover that Permutations of seriate odd numbers just showed the law of permutations of four kinds of odd numbers:

1^k, A_1; 3, B_1; 5, A_2; 7, B_2; (2^3); 9, A_1; 11, B_1; 13, A_2; 15, B_2; (2^4);

17, A_1; 19, B_1; 21, A_2; 23, B_2; 25, A_1; 3^3, B_1; 29, A_2; 31, B_2; (2^5);

33, A_1; 35, B_1; 37, A_2; 39, B_2; 41, A_1; 43, B_1; 45, A_2; 47, B_2;

49, A_1; 51, B_1; 53, A_2; 55, B_2; 57, A_1; 59, B_1; 61, A_2; 63, B_2; (2^6);

65, A_1; 67, B_1; 69, A_2; 71, B_2; 73, A_1; 75, B_1; 77, A_2; 79, B_2;

3^4, A_1; 83, B_1; 85, A_2; 87, b_2; 89, A_1; 91, B_1; 93, A_2; 95, B_2;

97, A_1;99, B_1;101, A_2;103, B_2; 105, A_1;107, B_1;109, A_2;111, B_2;

113,A_1;115, B_1;117, A_2;119, B_2; 121, A_1;123, B_1; 5^3, A_2; 127, B_2; (2^7);

129, A_1;131, B_1;133,A_2;135,B_2; 137, A_1;139, B_1;141, A_2;143, B_2;

145, A_1;147, B_1;149,A_2;151,B_2;153, A_1;155, B_1;157, A_2;159, B_2…→

Overall, permutations of seriate odd numbers from small to great are infinitely many cycles of $A_1B_1A_2B_2$ from left to right.

If we regard 2^{M-1} as a center of symmetry, then $2^{M-1}-1 \in B_2$ on the left of 2^{M-1}, $2^{M-1}-3 \in A_2$ on the left of B_2, $2^{M-1}-5 \in B_1$ on the left of A_2, $2^{M-1}-7 \in A_1$ on the left of B_1 … Yet $2^{M-1}+1 \in A_1$ on the right of 2^{M-1}, $2^{M-1}+3 \in B_1$ on the right of A_1, $2^{M-1}+5 \in A_2$ on the right of B_1, $2^{M-1}+7 \in B_2$ on the right of A_2… But also B_2 and $A_1$where $A_1>B_2$; A_2 and B_1 where $B_1>A_2$; B_1 and A_2 where $A_2>B_1$; A_1 and B_2 where $B_2>A_1$ are respectively one-to-one bilateral symmetries whereby 2^{M-1} to act as the center of the symmetry, where $M-1\geq3$, similarly hereinafter.

In other words, there are $B_2+(1+8n)=2^{M-1}$ with $A_1-(1+8n)=2^{M-1}$, $A_2+(3+8n)=2^{M-1}$ with $B_1-(3+8n)=2^{M-1}$, $B_1+(5+8n)=2^{M-1}$ with $A_2-(5+8n)=2^{M-1}$, and $A_1+(7+8n)=2^{M-1}$ with $B_2-(7+8n)=2^{M-1}$, where $n\geq 0$. Please, see a series below.

$A_1B_1A_2B_2\ldots A_1B_1A_2B_2A_1B_1A_2B_2\,(2^{M-1})\,A_1B_1A_2B_2A_1B_1A_2B_2\ldots A_1B_1A_2B_2\rightarrow$

After regard 2^{M-1} as a symmetric center, if leave from 2^{M-1}, then there are finite cycles of $B_2A_2B_1A_1$ leftwards until $7(B_2)5(A_2)3(B_1)1(A_1)$, and there are infinitely many cycles of $A_1B_1A_2B_2$ rightwards. We consider such symmetric permutations among four kinds of odd numbers for symmetric center 2^{M-1} as the symmetric law of permutations among the four kinds of odd numbers, or the symmetric law of odd numbers for short.

Under the symmetric law of odd numbers, not only two distances from symmetric two odd numbers to 2^{M-1} are each other's equivalent, but also all odd numbers on an identical distance on the either direction belong within a kind and the same, no matter 2^{M-1} is what a great value.

So the sum of two symmetric odd numbers is equal to $2\times 2^{M-1}$, i.e. $A_1+B_2=2^M$ where $A_1>B_2$; $B_1+A_2=2^M$ where $B_1>A_2$; $A_2+B_1=2^M$ where $A_2>B_1$; and $B_2+A_1=2^M$ where $B_2>A_1$.

Provided we now return to understand proven inequalities $A_1+B_1\neq 2^M$ and $A_2+B_2\neq 2^M$ by the symmetric law of odd numbers. Why both of them are inequalities? Since A_1 and B_1 are not bilateral symmetry whereby 2^{M-1} to act as the center of the symmetry, like that A_2 and B_2 as well.

By now, we analyze odd numbers which have a common base number, and label the belongingness of each of them.

1^1, A_1; $3^1=3$, B_1; $5^1=5$, A_2; $7^1=7$, B_2; (2^3);

1^2, A_1; $3^2=9$, A_1; $5^2=25$, A_1; $7^2=49$, A_1;

1^3, A_1; $3^3=27$, B_1; $5^3=125$, A_2; $7^3=343$, B_2;

1^4, A_1; $3^4=81$, A_1; $5^4=625$, A_1; $7^4=2481$, A_1;

1^5, A_1; $3^5=243$, B_1; $5^5=3125$, A_2; $7^5=16807$, B_2;

1^6, A_1; $3^6=729$, A_1; $5^6=15625$, A_1; $7^6=117609$, A_1;

… … … …

$9^1=9$, A_1; $11^1=11$, B_1; $13^1=13$, A_2; $15^1=15$, B_2; (2^4);

$9^2=81$, A_1; $11^2=121$, A_1; $13^2=169$, A_1; $15^2=225$, A_1;

$9^3=729$, A_1; $11^3=1331$, B_1; $13^3=2197$, A_2; $15^3=3375$, B_2;

$9^4=6561$, A_1; $11^4=14641$, A_1; $13^4=28561$, A_1; $15^4=50625$, A_1;

$9^5=59049$, A_1; $11^5=161051$, B_1; $13^5=371293$, A_2; $15^5=759375$, B_2;

$9^6=531441$, A_1; $11^6=1771561$, A_1; $13^6=4826809$, A_1; $15^6=11390625$, A_1;

… … … …

$17^1=17$, A_1; $19^1=19$, B_1; $21^1=21$, A_2; $23^1=23$; B_2…

$17^2=289$, A_1; $19^2=361$, A_1; $21^2=441$, A_1; $23^2=529$; A_1…

$17^3=4193$, A_1; $19^3=6859$, B_1; $21^3=9261$, A_2; $23^3=12167$; B_2…

$17^4=83521$, A_1; $19^4=130321$, A_1; $21^4=194481$, A_1; $23^4=279841$; A_1…

$17^5=1419857$, A_1; $19^5=2476099$, B_1; $21^5=4084101$, A_2; $23^5=6436343$, B_2…

$17^6=24137569$,A_1;$19^6=47045881$,A_1;$21^6=85766121$,A_1;$23^6=148035889$,A_1..

… … … …

From the above listed odd numbers, we are not difficult to sum up, all odd numbers whereby A_1 to act as a base number belong still within A_1; all odd numbers whereby B_1 to act as a base number belong within B_1 plus A_1, and one B_1 alternates with one A_1; all odd numbers whereby A_2 to act as a base number belong within A_2 plus A_1, and one A_2 alternates with one A_1; and all odd numbers whereby B_2 to act as a base number belong within B_2 plus A_1, and one B_2 alternates with one A_1. Moreover, classify them into four kinds according to respective belongingness, namely all odd numbers of even exponents and odd numbers 1+8n of odd exponents belong within A_1; odd numbers 3+8n of odd exponents belong within B_1; odd numbers 5+8n of odd exponents belong within A_2; and odd numbers 7+8n of odd exponents belong within B_2, where $n \geq 0$.

Excepting odd number 1, two adjacent odd numbers which have a common base number are an even number ≥ 6 apart, but also such even numbers are getting greater and greater along which exponents of them are getting greater and greater.

At all events, whether odd numbers of odd exponents or odd numbers of even exponents, all of them are included and dispersed within aforementioned four kinds of odd numbers, thus they entirely conform to the symmetric law of odd numbers.

First we need to get rid of these circumstances, namely E^P and F^V in $E^P+F^V \neq 2^M$ can not be two such odd numbers which have a common base number because the prerequisite stipulates that E^P and F^V have not any common prime factor >1. After that, we start to prove $E^P+F^V\neq2^M$ by mathematical induction under these circumstances that $E^P\in B_2$, $F^V\in A_1$, $A_1+B_2=2^M$; $E^P\in A_1$, $F^V\in B_2$, $B_2+A_1=2^M$; $E^P\in A_2$, $F^V\in B_1$, $A_2+B_1=2^M$; and $E^P\in B_1$, $F^V\in A_2$, $B_1+A_2=2^M$, where A_1, B_2, A_2, and B_1 under respective definiendum are one another's- different odd numbers.

(1)*When M=3, symmetric odd numbers on two sides of 2^3 are listed as the follows.

1^3, 3, 5, 7, (2^3), 9, 11, 13, 15…→

To wit: $A_1B_1A_2\ B_2\ (2^3)\ A_1\ B_1\ A_2\ B_2$ …→

It is clear at a glance, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^3 to act as the center of the symmetry, where higher exponents ≥3, similarly hereinafter.

When M=4, symmetric odd numbers on two sides of 2^4 are listed as the follows.

1^4, 3, 5, 7, 9, 11, 13, 15, (2^4) 17, 19, 21, 23, 25, 27, 29, 31…→

To wit: $A_1B_1A_2\ B_2\ A_1B_1A_2\ B_2\ (2^4)\ A_1B_1A_2\ B_2\ A_1B_1A_2\ B_2$…→

Evidently, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^4 to act as the center of the symmetry.

When M=5 and M=6, symmetric odd numbers on two sides of 2^6 including 2^5 are listed as the follows.

1^6, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127…→

To wit: $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$ (2^5) $A_1B_1A_2B_2A_1B_1A_2B_2$ $A_1B_1A_2B_2A_1B_1A_2B_2$ (2^6) $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2$ B_2 $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$…→

Likewise, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^6 including 2^5 to act as the center of the symmetry.

From above several initial cases, we can get $E^P+F^V\neq 2^4$, $E^P+F^V\neq 2^5$, $E^P+F^V\neq 2^6$ and $E^P+F^V\neq 2^7$ such being the case $P\geq 3$ and $V\geq 3$.

(2)*Suppose that when M=X and X≥6, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^X to act as the center of the symmetry. Namely, there is $E^P+F^V\neq 2^{X+1}$ such being the case $P\geq 3$ and $V\geq 3$.

(3)*Prove that when M=X+1, there are not two odd numbers of higher exponents either on two places of every bilateral symmetry whereby 2^{X+1} to act as the center of the symmetry. That is to say, this needs us to prove $E^P+F^V\neq 2^{X+2}$ such being the case $P\geq 3$ and $V\geq 3$.

Proof * We know that permutations of odd numbers on two sides of 2^M conform to the symmetric law of odd numbers, including odd numbers on two sides of 2^X and of 2^{X+1}, where M≥3 and X≥6. Please, see symmetric permutations of odd numbers on two sides of 2^X and of 2^{X+1} below.

$A_1B_1A_2B_2\ldots B_1A_2B_2A_1B_1A_2B_2\,(2^X)\,A_1B_1A_2B_2A_1B_1A_2\ldots A_1B_1A_2B_2 \rightarrow$

$A_1B_1A_2B_2\ldots B_1A_2B_2A_1B_1A_2B_2\,(2^{X+1})\,A_1B_1A_2B_2A_1B_1A_2\ldots A_1B_1A_2B_2 \rightarrow$

From $E^P \neq F^V$, let $F^V > E^P$, get $F^V > 2^M$ and $E^P < 2^M$ in $E^P+F^V=2^{M+1}$, then each of $A_1B_1A_2B_2\ldots B_1A_2B_2A_1B_1A_2B_2$ on the left of 2^M expresses E^P, and each of symmetry with E^P expresses F^V, where E and F are positive odd numbers without any common prime factor>1, and P, V and M are positive integers, and M≥3. Special emphasis is to regard not P and V here as integers which must be not less than 3.

We also know that all odd numbers on the left of 2^{M+1} are exactly all odd numbers of bilateral symmetry whereby 2^M to act as the center of the symmetry. Thereby, we divide all odd numbers of bilateral symmetry whereby 2^{M+1} to act as the center of the symmetry into four equivalent segments per 2^{M-1} odd numbers by 2^M, 2^{M+1} and 3×2^M. And number the ordinal of each segment from left to right as №1, №2, №3 and №4. Well then odd numbers at №1 segment and odd numbers at №4 segment are one-to-one bilateral symmetry whereby 2^{M+1} to act as the center of the symmetry; also odd numbers at №2 segment and odd numbers at №3 segment as well.

When M≤X, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^M to act as the center of the symmetry. Under these circumstances there are four kinds of symmetric odd numbers, i.e. A_1 and B_2, where $A_1>B_2$; B_1 and A_2, where $B_1>A_2$; A_2 and B_1, where $A_2>B_1$; and B_2 and A_1, where $B_2>A_1$.

A_1 and B_2 away from 2^M is respectively 1+8n, where $A_1>B_2$, and $n \geq 0$ and the same below; B_1 and A_2 away from 2^M is respectively 3+8n, where $B_1>A_2$; A_2 and B_1 away from 2^M is respectively 5+8n, where $A_2>B_1$; B_2 and A_1 away from 2^M is respectively 7+8n, where $B_2>A_1$.

Since when $M\leq X$, there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^X to act as the center of the symmetry, i.e. there is $E^P+F^V\neq 2^{X+1}$ such being the case $P\geq 3$ and $V\geq 3$.

When M=X+1, likewise there are such four kinds of symmetric odd numbers. In addition, all odd numbers of bilateral symmetries whereby 2^X to act as the center of the symmetry are turned into all odd numbers on the left of 2^{X+1}, yet the right odd numbers of symmetries with left odd numbers are formed from 2^{X+1} plus each and every odd number of bilateral symmetries whereby 2^X to act as the center of these symmetries.

Thus, odd numbers of bilateral symmetries from whereby 2^X to whereby 2^{X+1} to act as a center of symmetries, a half of them retained still original places, and the half lies on the left of 2^{X+1}, yet another half is formed from 2^{X+1} plus each and every odd number of bilateral symmetries whereby 2^X to act as the center of symmetries.

If any odd number E^P on the left of 2^X and an odd number F^V on the right of 2^X are bilateral symmetry whereby 2^X to act as the center of the symmetry, then F^V plus 2^{X+1} is F^V+2^{X+1}, moreover E^P and F^V+2^{X+1} are bilateral symmetry whereby 2^{X+1} to act as the center of the symmetry. Besides, 0 and 2^{X+2} are bilateral symmetry for symmetric center 2^{X+1}, so there is $F^V+2^{X+1}=2^{X+2}-E^P$, and from this to get $E^P+F^V=2^{X+1}$.

Like that, E^P plus 2^{X+1} is E^P+2^{X+1}, also F^V and E^P+2^{X+1} are bilateral symmetry whereby 2^{X+1} to act as the center of the symmetry. In addition, 0 and 2^{X+2} are bilateral symmetry for symmetric center 2^{X+1}, so there is $E^P+2^{X+1}=2^{X+2}-F^V$, and from this to get $E^P+F^V=2^{X+1}$. Please, see a general simple illustration as the follows.

					E^P+2^{X+1}		F^V+2^{X+1}	
1, 3...	E^P	2^X	F^V	2^{X+1}	$2^{X+2}-F^V$	3×2^X	$2^{X+2}-E^P$	2^{X+2}

However, there is only $E^P+F^V\neq 2^{X+1}$ such being the case $P\geq 3$ and $V\geq 3$ in line with the known prerequisite that there are not two odd numbers of higher exponents on two places of every bilateral symmetry whereby 2^X to act as the center of the symmetry.

Now that conclusively exist to $E^P+F^V\neq 2^{X+1}$ such being the case $P\geq 3$ and $V\geq 3$, so deduce $F^V+2^{X+1}\neq 2^{X+2}-E^P$ and $E^P+2^{X+1}\neq 2^{X+2}-F^V$ from $E^P+F^V\neq 2^{X+1}$.

Since exist to $F^V+2^{X+1}\neq 2^{X+2}-E^P$ such being the case $P\geq 3$ and $V\geq 3$, if let $F^V+2^{X+1}=2^{X+2}-E^P$, then precisely speak that at least one in "F^V+2^{X+1}" and "$2^{X+2}-E^P$" is not an odd numbers of higher exponent according to the successive inference. But F^V+2^{X+1} and $2^{X+2}-E^P$ share a place and the same, so both of them are an identical odd number in reality. Consequently

F^V+2^{X+1} and $2^{X+2}-E^P$ all are not odd numbers of higher exponent.

Like that, we deduce that E^P+2^{X+1} and $2^{X+2}-F^V$ all are not odd numbers of higher exponent either from the existence of $E^P+2^{X+1}=2^{X+2}-F^V$ on the basis of $E^P+2^{X+1}\neq 2^{X+2}-F^V$ where $P\geq 3$ and $V\geq 3$.

Thus it can seen, if E^P is an odd number of higher exponent, then F^V of symmetry with E^P is an odd number of no high exponent, additionally, whether F^V+2^{X+1} or $2^{X+2}-E^P$ is not an odd number of higher exponent, therefore there are $E^P+ (F^V+2^{X+1}) \neq 2^{X+2}$ or $E^P+ (2^{X+2}-E^P) \neq 2^{X+2}$ such being the case $P\geq 3$ and $V\geq 3$.

If F^V is an odd number of higher exponent, then E^P is an odd number of no high exponent, additionally, whether E^P+2^{X+1} or $2^{X+2}-F^V$ is not an odd number of higher exponent, therefore there are $F^V+ (E^P+2^{X+1}) \neq 2^{X+2}$ or $F^V+ (2^{X+2}-F^V) \neq 2^{X+2}$ such being the case $P\geq 3$ and $V\geq 3$.

To sum up, on the one hand, odd numbers of higher exponents on the left of 2^{X+1} and odd numbers of no high exponents on the right of 2^{X+1} are one-to-one bilateral symmetry whereby 2^{X+1} to act as the center of the symmetry. On the other hand, for odd numbers of no high exponents on the left of 2^{X+1}, no matter each of them and what odd number on the right of 2^{X+1} are bilateral symmetry, there are not two odd numbers of higher exponents on two places of the bilateral symmetry whereby 2^{X+1} to act as the center of the symmetry. Consequently when $M=X+1$, there is only $E^P+F^V\neq 2^{X+2}$ such being the case $P\geq 3$ and $V\geq 3$.

Apply the above-mentioned way of doing, we can continue to prove that when M=X+2, M=X+3…up to M=every positive integer≥3, there are all $E^P+F^V\neq 2^{X+3}$, $E^P+F^V\neq 2^{X+4}$…$E^P+F^V\neq 2^W$ such being the case P≥3 and V≥3.

A Proof of the Conjecture

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are all positive integers, and X, Y and Z are greater than 2, then A, B and C must have a common prime factor.

We consider limits of values of above-mentioned A, B, C, X, Y and Z as known requirements hereinafter.

First, we must remove following two kinds from $A^X+B^Y=C^Z$ under the known requirements.

1. If A, B and C all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, evidently there is only $A^X+B^Y\neq C^Z$ under the known requirements according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number, yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ under the known requirements according to an odd number ≠ an even number.

Thus, we merely continue to have two kinds of $A^X+B^Y=C^Z$ under the known requirements as listed below.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number. For indefinite equation $A^X+B^Y=C^Z$ under the known requirements plus aforementioned either qualification, in fact, it is able to have many sets of solutions of positive integers. Let us instance following four concrete equations to prove such a viewpoint.

When A, B and C all are positive even numbers, if let A=B=C=2, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $2^3+2^3=2^4$. Evidently $A^X+B^Y=C^Z$ has a set of solutions of positive integers (2, 2, 2) here, and A, B and C have common even prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $162^3+162^3=54^4$. Evidently $A^X+B^Y=C^Z$ has a set of solutions of positive integers (162, 162, 54) here, and A, B and C have two common prime factors, i.e. even 2 and odd 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3, and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $3^3+6^3=3^5$. Manifestly $A^X+B^Y=C^Z$ has a set of solutions of positive integers (3, 6, 3) here, and A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7, and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $7^6+7^7=98^3$. Manifestly $A^X+B^Y=C^Z$ has a set of solutions of positive integers (7, 7, 98) here, and A, B and C have common prime factor 7.

Thus it can seen, indefinite equation $A^X+B^Y=C^Z$ under the known requirements plus aforementioned either qualification can hold water according to above-mentioned four concrete examples, but A, B and C have at least one common prime factor >1.

If we can prove that there is only $A^X+B^Y\neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor, then we completely proved that there is only $A^X+B^Y=C^Z$ under the known requirements plus the qualification that A, B and C must have a common prime factor >1.

Since A, B and C have common prime factor 2 where A, B and C all are positive even numbers, so these circumstances that A, B and C have not any common prime factor can only occur under the prerequisite that A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not any common prime factor, then any two of them have not any common prime factor either. Because on the supposition that any two of them have a common prime factor, namely A^X+B^Y or C^Z-A^X or C^Z-B^Y have a common prime factor, yet another has not it, then from this lead to $A^X+B^Y\neq C^Z$ or $C^Z-A^X\neq B^Y$ or $C^Z-B^Y\neq A^X$ according to the unique factorization theorem of natural number.

Since it is so, if we can prove that there is only inequality $A^X+B^Y\neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor, then the Beal's conjecture is surely tenable, otherwise it will be negated.

Unquestionably, let following two inequalities add together, they can completely replace $A^X+B^Y\neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor.

1. $A^X+B^Y\neq 2^ZG^Z$ under the known requirements plus the qualification that A, B and 2G have not any common prime factor, where 2G=C.

2. $A^X+2^YD^Y\neq C^Z$ under the known requirements plus the qualification that A, 2D and C have not any common prime factor, where 2D=B.

We again divide $A^X+B^Y\neq 2^ZG^Z$ into two kinds, i.e. (1) $A^X+B^Y\neq 2^Z$, when G=1, and (2) $A^X+B^Y\neq 2^ZG^Z$, where G has at least an odd prime factor >1.

Likewise divide $A^X+2^YD^Y\neq C^Z$ into two kinds, i.e. (3) $A^X+2^Y\neq C^Z$, when D=1, and (4) $A^X+2^YD^Y\neq C^Z$, where D has at least an odd prime factor >1.

On the basis of proven $E^P+F^V\neq 2^M$ at the preceding chapter, we shall set to prove aforesaid four inequalities, one by one, thereinafter.

Firstly, let $A^X=E^P$, $B^Y=F^V$, and $2^Z=2^M$ for $E^P+F^V\neq 2^M$, we get $A^X+B^Y\neq 2^Z$ under the known requirements plus the qualification that A and B are two positive odd numbers without any common prime factor >1.

Secondly, let us successively prove $A^X+B^Y\neq 2^ZG^Z$ under the known requirements plus the qualifications that A and B are two positive odd numbers, and G has at least an odd prime factor >1, and A, B and 2G have not any common prime factor >1.

To begin with, multiply each term of proven $E^P+F^V\neq 2^M$ by G^M, then we obtain $E^PG^M+F^VG^M\neq 2^MG^M$.

For any positive even number, either it is able to be expressed as A^X+B^Y, or it is unable. Undoubtedly $E^PG^M+F^VG^M$ is a positive even number.

If $E^PG^M+F^VG^M$ is able to be expressed as A^X+B^Y, then there is $A^X+B^Y\neq 2^MG^M$.

If $E^PG^M+F^VG^M$ is unable to be expressed as A^X+B^Y, then the even number has nothing to do with proving $A^X+B^Y\neq 2^MG^M$.

Under these circumstances, there are still $E^PG^M+F^VG^M\neq A^X+B^Y$ and $E^PG^M+F^VG^M\neq 2^MG^M$, so let $E^PG^M+F^VG^M$ equals A^X+B^Y+2b or A^X+B^Y-2b, where b is a positive integer. Also use sign "±" to denote signs "+" and "-" hereinafter, then obtain $A^X+B^Y\pm 2b\neq 2^MG^M$, i.e. $A^X+B^Y\neq 2^MG^M\pm 2b$.

Since 2b can express every positive even number, then $2^MG^M\pm 2b$ can express all positive even numbers except for 2^MG^M.

For a positive even number, either it is able to be expressed as 2^KN^K, or it is unable, where K is an integer >2, and N is a positive integer which has at least an odd prime factor >1.

On the one hand, there is $A^X+B^Y\neq 2^KN^K$ where $2^MG^M\pm 2b=2^KN^K$. On the other hand, $2^MG^M\pm 2b$ have nothing to do with proving $A^X+B^Y\neq 2^KN^K$ where $2^MG^M\pm 2b\neq 2^KN^K$.

That is to say, for $E^PG^M+F^VG^M\neq 2^MG^M$, if $E^PG^M+F^VG^M$ is unable to be expressed as A^X+B^Y, we can deduce $A^X+B^Y\neq 2^KN^K$ elsewhere too.

Hereto, we have proven $A^X+B^Y \neq 2^M G^M$ or $A^X+B^Y \neq 2^K N^K$ on the existence. Since either M or K is to express an integer >2, also either G or N is a positive integer which has at least an odd prime factor >1, therefore $A^X+B^Y \neq 2^M G^M$ and $A^X+B^Y \neq 2^K N^K$ are of the same meaning.

Thirdly, we proceed to prove $A^X+2^Y \neq C^Z$ under the known requirements plus the qualification that A and C are two positive odd numbers without any common prime factor >1.

In the former chapter, we have proven $E^P+F^V \neq 2^M$, and supposed $F^V>E^P$, so let $F^V=C^Z$, then there is $E^P+C^Z \neq 2^M$.

Moreover, let $2^M>2^3$, then there is $2^M=2^{M-1}+2^{M-1}$.

So there is $E^P+C^Z > 2^{M-1}+2^{M-1}$ or $E^P+C^Z < 2^{M-1}+2^{M-1}$.

Namely there is $C^Z-2^{M-1}>2^{M-1}-E^P$ or $C^Z-2^{M-1}<2^{M-1}-E^P$.

In addition, there is $A^X+E^P \neq 2^{M-1}$ according to the same reason of proven $E^P+F^V \neq 2^M$.

Then, we deduce $2^{M-1}-E^P>A^X$ or $2^{M-1}-E^P<A^X$ from $A^X+E^P \neq 2^{M-1}$.

Therefore, there is $C^Z-2^{M-1}>2^{M-1}-E^P>A^X$ or $C^Z-2^{M-1}<2^{M-1}-E^P<A^X$.

Consequently, there is $C^Z-2^{M-1}>A^X$ or $C^Z-2^{M-1} < A^X$.

In a word, there is $C^Z-2^{M-1} \neq A^X$, i.e. $A^X+2^{M-1} \neq C^Z$.

For $A^X+2^{M-1} \neq C^Z$, let $2^{M-1}=2^Y$, we obtain $A^X+2^Y \neq C^Z$.

Fourthly, let us last prove $A^X+2^Y D^Y \neq C^Z$ under the known requirements plus the qualifications that A and C are two positive odd numbers, and D has at least an odd prime factor >1, and A, 2D and C have not any common prime factor >1.

In order to distinguish between two differing cases, let us use another inequality $H^U+2^Y \neq K^T$ according to the same reason of proven $A^X+2^Y \neq C^Z$, where H and K are two positive odd numbers without any common prime factor >1, and U, Y and T are integers >2.

We obtain $K^T-H^U \neq 2^Y$ from $H^U+2^Y \neq K^T$. Like that, multiply each term of $K^T-H^U \neq 2^Y$ by D^Y, then obtain $K^TD^Y-H^UD^Y \neq 2^YD^Y$.

For any positive even number, either it is able to be expressed as C^Z-A^X, or it is unable. Unquestionably $K^TD^Y-H^UD^Y$ is a positive even number.

If $K^TD^Y-H^UD^Y$ is able to be expressed as C^Z-A^X, then there is $C^Z-A^X \neq 2^YD^Y$, i.e. $A^X+2^YD^Y \neq C^Z$.

If $K^TD^Y-H^UD^Y$ is unable to be expressed as C^Z-A^X, then the even number has nothing to do with proving $A^X+2^YD^Y \neq C^Z$. Under these circumstances, there are still $K^TD^Y-H^UD^Y \neq C^Z-A^X$ and $K^TD^Y-H^UD^Y \neq 2^YD^Y$.

Let $K^TD^Y-H^UD^Y$ equals $C^Z-A^X \pm 2d$, where d is a positive integer.

Well then, there is $C^Z-A^X \pm 2d \neq 2^YD^Y$, i.e. $C^Z-A^X \neq 2^YD^Y \pm 2d$.

Since 2d can express every positive even number, then $2^YD^Y \pm 2d$ can express all positive even numbers except for 2^YD^Y.

For a positive even number, either it is able to be expressed as 2^SR^S, or it is unable, where S is an integer>2, and R is a positive integer which has at least an odd prime factor >1.

On the one hand, there is $C^Z-A^X \neq 2^S R^S$ where $2^Y D^Y \pm 2d = 2^S R^S$, i.e. $A^X+2^S R^S \neq C^Z$. On the other hand, $2^Y D^Y \pm 2d$ have nothing to do with proving $A^X+2^S R^S \neq C^Z$ where $2^Y D^Y \pm 2d \neq 2^S R^S$.

That is to say, for $K^T D^Y - H^U D^Y \neq 2^Y D^Y$, if $K^T D^Y - H^U D^Y$ is unable to be expressed as C^Z-A^X, we can deduce $A^X+2^S R^S \neq C^Z$ elsewhere too.

Thus far, we have proven $A^X+2^Y D^Y \neq C^Z$ or $A^X+2^S R^S \neq C^Z$ on the existence.

Since either Y or S is to express an integer >2, also either D or R is a positive integer which has at least an odd prime factor >1, therefore $A^X+2^Y D^Y \neq C^Z$ and $A^X+2^S R^S \neq C^Z$ are of the same meaning.

To sun up, we have proven every kind of $A^X+B^Y \neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor.

Previous, we have proven that $A^X+B^Y=C^Z$ has certain sets of solutions of positive integers under the known requirements plus the qualification that A, B and C have at least a common prime factor.

After the compare between $A^X+B^Y=C^Z$ under the known requirements and $A^X+B^Y \neq C^Z$ under the known requirements, we have reached such a conclusion inevitably, namely an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the known requirements is that A, B and C must have a common prime factor. The proof was thus brought to a close. As a consequence, the Beal conjecture is tenable.