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TWO HUNDRED AND THIRTEEN CONJECTURES ON PRIMES

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(COLLECTED PAPERS)

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INTRODUCTION

In two of my previous published books, “Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes”, respectively “Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function”, I already expressed my passion for integer numbers, especially for primes and Fermat pseudoprimes, fascinating numbers that seem to be a little bit more willing to let themselves ordered and understood than the prime numbers.

This book brings together sixty-two papers on prime numbers, many of them supporting the author’s belief, expressed above, namely that new ordered patterns can be discovered in the “undisciplined” set of prime numbers, observing the ordered patterns in the set of Fermat pseudoprimes, especially in the set of Carmichael numbers, the absolute Fermat pseudoprimes, and in the set of Poulet (sometimes also called Sarrus) numbers, the relative Fermat pseudoprimes to base two. Other papers, which are not based on the observation of Fermat pseudoprimes, are based on the observation of Mersenne numbers, Fermat numbers, Smarandache generalized Fermat numbers, and other well known or less known classes of integers which are very much related with the study of primes.

Part One of this book of collected papers contains two hundred and thirteen conjectures on primes and Part Two of this book brings together the articles regarding primes, submitted by the author to the preprint scientific database Vixra, representing the context of the conjectures listed in Part One, papers regarding squares of primes, semiprimes, twin primes, sequences of primes, types of duplets or triplets of primes, special classes of composites, ways to write primes, formulas for generating large primes, generalizations of the twin primes and de Polignac’s conjecture, generalizations of Cunningham chains and Fermat numbers and many other classic issues regarding prime numbers. Finally, in the last eight from these collected papers, I defined a new function, the MC function, and I showed some of its possible applications (for instance, I conjectured that for any pair of twin primes p and p + 2, where $p \geq 5$, there exist a positive integer n of the form 15 + 18*k such that the value of Smarandache function for n is equal to p and the value of MC function for n is equal to p + 2, I also made a diophantine analysis of few Smarandache type sequences using the MC function).

SUMMARY

Part one. Two hundred and thirteen conjectures on primes

Part two. Sixty-two articles on primes

Part one. Two hundred and thirteen conjectures on primes

Conjecture 1: The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q and r are distinct primes, q ≥ 5 and r ≥ 5.

Conjecture 2: There exist an infinity of primes p that can be written as p = (q^2 + q + 1)/3, where q is also a prime.

Conjecture 3: The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q is a Poulet number and r a prime, r ≥ 5.

Conjecture 4: For any prime p, p ≥ 5, there exist an infinity of pairs of distinct primes [q, r] such that p = sqrt(3*q – r – 1).

Conjecture 5: For any prime p, p ≥ 5, there exist at least a pair of distinct primes [q, r] such that p = (q^2 + r + 1)/3.

Conjecture 6: For any prime p of the form p = 6*k + 1 there exist an infinity of pairs of distinct primes [q, r] such that p = 3*q – p^2 – 1.

Conjecture 7: For an infinity of odd positive integers m there is an infinite set of primes with the property that the sum of their digits is equal to m + 1.

Conjecture 8: For an infinity of primes p there is an infinite set of primes with the property that the sum of their digits is equal to p + 1.

Conjecture 9: There is an infinite number of values the sum of the digits of the numbers p + 1, where p is odd prime, may have.

Conjecture 10: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it's equal to 14.

Conjecture 11: There is an infinity of primes with the property that the sum of their digits is equal to 14.

Conjecture 12: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

Conjecture 13: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

Conjecture 14: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

Conjecture 15: Let a_i be the sequence of the lesser of twin primes whose digital root is equal to 2, b_i be the sequence of the lesser of twin primes whose digital root is equal to 5 and c_i be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms n of a_i for which the number of the terms of b_i smaller than n is equal to the number of the terms of c_i smaller than n;
: there exist an infinity of terms n of b_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of c_i smaller than n;
: there exist an infinity of terms n of c_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of b_i smaller than n.

Conjecture 16: There is an infinity of primes with the property that the sum of their digits is equal to 14.

Conjecture 17: If n and n + 6 are both primes (in other words if [n, n + 6] is a pair of sexy primes), where n ≥ 7, then the number m = n + 3 can be written at least in one way as m = p + q, where p and q are primes, q = p + 6*r and r is positive integer.

Conjecture 18: If p and p + p(n)# are both primes, where p > p(n)#, n ≥ 2 and p(n)# is a primorial number (which means the product of first n primes), then the number m = p + p(n)#/2 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + p(n)#*r and r is positive integer.

Conjecture 19: For any odd prime p there exist at least one prime q such that p + n! = q, where n is a positive integer, n < p.

Conjecture 20: For any odd prime p, p ≥ 5, there exist an infinity of primes q of the form q = (p + n!)/n^k, where n and k are positive integers and n ≥ p.

Conjecture 21: Any odd prime p can be written in an infinity of distinct ways like p = q – r + 1, where q and r are also primes; in other words, there exist an infinity of pairs of primes (q, r) such that q – r = p – 1, for any odd prime p (it can be seen that for p = 3 the conjecture states the same thing with the twin primes conjecture).

Conjecture 22: Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1 and, where h and i are positive integers.

Conjecture 23: Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 24: Any prime p of the form p = 6*k - 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 25: There exist an infinity of pairs of primes (p, q), where p is of the form 6*k – 1 and q is of the form 6*h + 1, such that q – p + 1 = 3^n, for any n non-null positive integer (it can be seen that for n = 1 the conjecture states the same thing with the twin primes conjecture).

Conjecture 26: Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1.

Conjecture 27: Any square of prime p^2, $p \geq 5$, can be written in an infinity of distinct ways like $p^2 = q - r + 1$, where q is a prime of the form $q = 6*h - 1$ and r is a prime of the form $q = 6*i - 1$.

Conjecture 28: For any r prime, $r \geq 5$, there exist an infinity of pairs of primes (p, q) such that the numbers $(q^2 - p^2 - 2*r)/2$ and $(q^2 - p^2 + 2*r)/2$ are both primes.

Conjecture 29: For any pair of primes (p, r), $p \geq 5$, $r \geq 5$, there exist an infinity of primes q such that the numbers $(q^2 - p^2 - 2*r)/2$ and $(q^2 - p^2 + 2*r)/2$ are both primes.

Conjecture 30: For any p prime, $p \geq 5$, there exist an infinity of primes q such that the numbers $(q^2 - p^2 - 2*p)/2$ and $(q^2 - p^2 + 2*p)/2$ are both primes.

Conjecture 31: If x, y and r are odd primes such that $y = x + 2*r$, where $r \geq 5$, then there exist p and q also primes such that $x = (q^2 - p^2 - 2*r)/2$ and $y = (q^2 - p^2 + 2*r)/2$.

Conjecture 32: For any p prime, $p \geq 7$, there exist a pair of smaller primes (q, r) such that the numbers $x = (p^2 - q^2 - 2*r)/2$ and $y = (p^2 - q^2 + 2*r)/2$ are both primes.

Conjecture 33: There exist an infinity of primes p such that $2*p^2 - 1 = q*r$, where q and r are also primes.

Conjecture 34: If p is prime and $2*p^2 - 1 = q*r$, where q and r are also primes, there exist an infinity of pairs of even positive integers [m, n] such that $2*(p + m)^2 - 1 = (q + n)*(r + n)$, such that p + m, q + n and r + n are also primes.

Conjecture 35: If p is prime and $2*p^2 - 1 = q^2$, where q is also prime, there exist an infinity of pairs of even positive integers [m, n] such that $2*(p + m)^2 - 1 = (q + n)^2$, such that p + m and q + n are also primes.

Conjecture 36: Any Poulet number of the form $10*n + 1$ or $10*n + 9$ can be written at least in one way as $p*q + 10*k*h$, where p and q are primes or powers of primes of the same form from the following four ones: $10*m + 1$, $10*m + 3$, $10*m + 7$ or $10*m + 9$, k and h are non-null positive integers and $q - p = 10*k$.

Conjecture 37: For any Poulet number N not divisible by 3 there exist at least a pair of numbers [p, q], where p is prime and q is prime or square of prime, such that $N = p^2 + q - 1$.

Conjecture 38: There exist an infinity of Poulet numbers of the form $n^2 + 120*n$, where n is prime or a composite positive integer.

Conjecture 39: There exist an infinity of duplets of primes [p, q] such that $p - q = 120$; there also exist an infinity of triplets of primes [p1, p2, q] such that $p1*p2 - q = 120$; there also exist an infinity of quadruplets of primes [p1, p2, p3, q] such that $p1*p2*p3 - q = 120$; generally, for any non-null positive integer i there exist i primes p1, p2, ..., pi and a prime q such that $p1*p2*...*pi - q = 120$.

Conjecture 40: For any non-null positive integer i there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that $p1*p2*...*pi - q = 2$.

Conjecture 41: For any n even positive integer and for any i non-null positive integer there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = n.

Conjecture 42: For any n even positive integer and for any i and j non-null positive integers there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi – q1*q2*...*qj = n.

Conjecture 43: For any n even positive integer and for any i, j, k, l non-null positive integers, for any k given primes a1, a2, ..., ak and for any l given primes b1, b2, ..., bl, there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi*a1*a2*...*ak – q1*q2*...*qj*b1*b2*...bl = n.

Conjecture 44: For any square of a prime p of the form p = 6*k – 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime q*r where r – q = 8*k and a number congruent to 2, 3 or 5 modulo 6;
(3) p^2 can be deconcatenated into a semiprime 3*q, where q is of the form 10*k + 7, and a number congruent to 1 modulo 6;
(4) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(5) p^2 can be deconcatenated into a number of the form 121 + 48*k and a number congruent to 0 modulo 6;
(6) p^2 is a palindromic number.

Conjecture 45: For any square of a prime p of the form p = 6*k + 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime 3^n*q and a number congruent to 1 modulo 6;
(3) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 1;
(4) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 9;
(5) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(6) p^2 can be deconcatenated into a number of the form 121 + 24*k and a number congruent to 0 modulo 6.

Conjecture 46: For any prime p there exist an infinity of positive integers n such that the number n*p – n + 1 is prime.

Conjecture 47: For any prime p there exist an infinity of positive integers n such that the number n*p + n - 1 is prime.

Conjecture 48: For any prime p there exist an infinity of positive integers n such that the number n^2*p – n + 1 is prime.

Conjecture 49: For any prime p there exist an infinity of positive integers n such that the number n^2*p + n - 1 is prime.

Conjecture 50: For any prime p there exist an infinity of positive integers n such that the number n*p – p + n is prime.

Conjecture 51: For any prime p there exist an infinity of positive integers n such that the number n*p - p - n is prime.

Conjecture 52: For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p + n is prime.

Conjecture 53: For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p - n is prime.

Conjecture 54: For any two distinct primes greater than three p and q there exist an infinity of positive integers n such that the number (p^2 – 1)*n + q^2 is prime, also an infinity of positive integers m such that the number (q^2 – 1)*n + p^2 is prime.

Conjecture 55: For any p, q distinct primes, p > 30, there exist n positive integer such that p – 30*n and q + 30*n are both primes.

Conjecture 56: For any p, q, r distinct primes there exist n positive integer such that the numbers 30*n – p, 30*n – q and 30*n – r are all three primes.

Conjecture 57: There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q + (q - p) is prime.

Conjecture 58: There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Conjecture 59: For any p prime there exist an infinity of primes q, q > p, where p and q are both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Conjecture 60: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m + 1) – n and y = q*(n + 1) – m are both primes.

Conjecture 61: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m - 1) + n and y = q*(n - 1) + m are both primes.

Conjecture 62: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p + (m + 1)*n and y = q + m*n are both primes.

Conjecture 63: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n + 2*m are both primes.

Conjecture 64: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n - 2*m are both primes.

Conjecture 65: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m + 2*n and y = q*n + 2*m are both primes.

Conjecture 66: There exist an infinity of positive integers n such that the numbers 30*n + 7, 60*n + 13 and 150*n + 31 are all three primes.

Conjecture 67: There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

Conjecture 68: There exist an infinity of positive integers n such that the numbers 30*n - 29, 60*n - 59 and 90*n - 89 are all three primes.

Conjecture 69: There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

Conjecture 70: There exist an infinity of positive integers n such that the numbers 30*n - 7, 90*n - 23 and 300*n - 79 are all three primes.

Conjecture 71: There exist an infinity of positive integers n such that the numbers 30*n - 17, 90*n - 53 and 150*n - 89 are all three primes.

Conjecture 72: There exist an infinity of positive integers n such that the numbers 60*n + 13, 180*n + 37 and 300*n + 61 are all three primes.

Conjecture 73: There exist an infinity of positive integers n such that the numbers 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four primes.

Conjecture 74: There exist an infinity of positive integers n such that the numbers 90*n + 1, 180*n + 1, 270*n + 1 and 540*n + 1 are all four primes.

Conjecture 75: There exist an infinity of pairs of primes [p, q] such that the numbers p + 30*n, q + 30*n and p*q + 30*n are all three primes.

Conjecture 76: There exist an infinity of primes p such that the numbers x = 30*n + p and y = 30*m*n + m*p – m + 1, where m, n are non-null positive integers, are both primes.

Conjecture 77: There exist an infinity of primes of the form 360*p*q + 1, where p, q are primes, both greater than or equal to 7.

Conjecture 79: There exist an infinity of primes of the form 360*p*q + r, where p, q, r are primes, all of them greater than or equal to 7.

Conjecture 80: There exist an infinity of primes of the form 2^m + n^2, where m is non-null positive integer and n odd integer.

Conjecture 81: There exist an infinity of primes of the form (2^n)^k + 2^n + 1, where n is non-null positive integer and k positive integer.

Conjecture 82: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 83: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 7.

Conjecture 84: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 7, 10*y + 7 respectively 10*z + 7.

Conjecture 85: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 9 respectively 10*z + 9.

Conjecture 86: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 87: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 9.

Conjecture 88: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 3 respectively 10*z + 7.

Conjecture 89: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 7, 10*y + 7 respectively 10*z + 9.

Conjecture 90: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 3.

Conjecture 91: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 7 respectively 10*z + 7.

Conjecture 92: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 7 respectively 10*z + 9.

Conjecture 93: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 9, 10*y + 9 respectively 10*z + 9.

Conjecture 94: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 7.

Conjecture 95: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 3 respectively 10*z + 3.

Conjecture 96: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 7 respectively 10*z + 9.

Conjecture 97: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 9 respectively 10*z + 9.

Conjecture 98: Any prime greater than or equal to 5 can be written at least in one way as (9*p^2 – q^2)/(2^n), where p and q are primes and n non-null positive integer.

Conjecture 99: Any prime p of the form 6*k + 1 greater than or equal to 13 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Conjecture 100: Any prime p of the form 6*k – 1 greater than or equal to 11 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Conjecture 101: Any odd prime p can be written at least in one way as p = (q*2^a*3^b*5^c ± 1)*2^n ± 1, where q is an odd prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Conjecture 102: Any pair of twin primes [p_1, p_2] can be written as [p_1 = (q*2^a*3^b*5^c ± 1)*2^n - 1, p_2 = (q*2^a*3^b*5^c ± 1)*2^n + 1], where q is prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Conjecture 103: For any pair of primes, greater than 3, [p_1, q_1], where q_1 – p_1 = d, there exist at least a pair of positive integers [m, n], where n – m = d, such that the numbers p_2 = p_1*q_1 – n + 1 and q_2 = p_1*q_1 – m + 1 are both primes.

Conjecture 104: For any even number d there exist an infinity of pairs of primes [p_1, q_1], where q_1 – p_1 = d, such that the numbers p_2 = p_1*q_1 – p_1 + 1 and q_2 = p_1*q_1 – q_1 + 1 are both primes.

Conjecture 105: For any n non-null positive integer there exist q, r primes such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 106: For any q odd prime there exist n non-null positive integer and r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 107: For any q, r odd primes there exist n non-null positive integer such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 108: For any n non-null positive integer and any q prime there exist r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 109: For any n non-null positive integer there exist q prime such that 120*n*q^2 + 1 = p^2, where p is prime or a power of prime.

Conjecture 110: There exist an infinity of primes p of the form p = 25^n + 906304.

Conjecture 111: There exist an infinity of infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 112: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 113: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is a multiple of p(2) - 1, the period of the rational number a(2) is a multiple of p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 114: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) divides p(2) - 1, the period of the rational number a(2) divides p(3) – 1, the period of the rational number a(n) divides a(n) – 1.

Conjecture 115: For any Poulet number P there exist a rational number r equal to a sum of unit fractions 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1), p(2), p(3)... are distinct odd primes, such that the period of r is equal to P – 1.

Conjecture 116: Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Conjecture 117: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 1.

Conjecture 118: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 3.

Conjecture 119: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 7.

Conjecture 120: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 9.

Conjecture 121: Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Conjecture 122. Any square of a prime p^2, where p is greater than or equal to 7, can be written as p^2 = 2*m + n, where m and n are distinct primes, both of the form 10k + 3 or both of the form 10k + 7.

Conjecture 123. There exist an infinity of positive integers k such that 6*k – 1 and 18*k – 5 are both primes.

Conjecture 124. There exist an infinity of positive integers k such that 6*k + 1 and 12*k + 1 are both primes.

Conjecture 125. There exist an infinity of positive integers k such that 6*k + 1 and 18*k + 1 are both primes.

Conjecture 126. There exist an infinity of positive integers k such that 6*k - 5 and 24*k - 5 are both primes.

Conjecture 127. There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 1 are all three primes.

Conjecture 128. There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 13 are all three primes.

Conjecture 129. There exist an infinity of positive integers k such that k, 2*k - 1 and 5*k - 4 are all three primes.

Conjecture 130. There exist an infinity of positive integers k such that k, 2*k - 1 and 3*k - 2 are all three primes.

Conjecture 131. There exist an infinity of positive integers k such that k, 3*k - 2 and 4*k - 3 are all three primes.

Conjecture 132. There exist an infinity of positive integers k such that 40*k + 1, 60*k + 1 and 100*k + 1 are all three primes.

Conjecture 133. There exist an infinity of positive integers k such that k, 2*k – 1, 7*k - 6 and 14*k - 13 are all four primes.

Conjecture 134. There exist an infinity of positive integers k such that k, 2*k – 1, 6*k - 5 and 12*k - 11 are all four primes.

Conjecture 135. There exist an infinity of pairs of distinct non-null positive integers m, n such that 60*m*n – 29 and 60*m*n – (60*m + 29) are both primes.

Conjecture 136. There exist an infinity of pairs of distinct non-null positive integers [m, n] such that 40*m – 10*n - 29 and 40*m – 10*n – 129 are both primes.

Conjecture 137. For any pair of twin primes [q, r] there exist an infinity of primes p of the form p = 7200*q*r*n + 1, where n is positive integer.

Conjecture 138. There exist an infinity of primes p of the form p = n*s(p) – n + 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 139. There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 140. There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 141. There exist an infinity of primes p of the form p = m*n + m - n, where m and n are distinct odd primes.

Conjecture 142. There exist an infinity of primes p of the form p = m^2 – m*n + n, where m and n are distinct odd primes.

Conjecture 143. There exist an infinity of primes p of the form p = (q + 5^k)/ 10, where q is prime and k positive integer.

Conjecture 144. There exist an infinity of primes p of the form p = (q + 5^k)/ 30, where q is prime and k positive integer.

Conjecture 145. If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of numbers which are primes or squares of primes.

Conjecture 146. If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of semiprimes q = m*n, where m < n, with the following property: the number n – m + 1 is a prime or a square of a prime.

Conjecture 147. Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of primes of the form a(i)/p as well as an infinity of primes of the form b(i)/q for any pair [p, q].

Conjecture 148. Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of pairs [p, q] such that the sequence of primes a(i)/p is the same with the sequence of primes b(i)/q.

Conjecture 149. There exist an infinity of primes, for k positive integer, of the form n*2^k + 1, for n equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

Conjecture 150. There exist an infinity of positive integers n such that the sequence n*2^k + 1, where k is positive integer, contains an infinity of primes.

Conjecture 151. Any Mersenne number 2^n – 1 with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number.

Conjecture 152. Any Mersenne-Coman number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number.

Conjecture 153. For any prime p greater than or equal to 5 the number (4^p – 1)/3 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \leq i < j \leq n$.

Conjecture 154. Any Fermat number F = 2^(2^n) + 1 is either prime either divisible by a 2-Poulet number.

Conjecture 155. Any Fermat-Coman number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number.

Conjecture 156. For any prime p greater than or equal to 7 the number (4^p + 1)/5 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \leq i < j \leq n$.

Conjecture 157. For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 158. For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a prime.

Conjecture 159. For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 160. For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a prime.

Conjecture 161. There exist an infinity of the primes of the form 30*k + n, fir any k positive integer and n equal to 1, 7, 11, 13, 17, 19, 23 or 29.

Conjecture 162. There exist an infinity of primes p such that the number 7*p^2 + 186 is prime.

Conjecture 163. There exist an infinity of primes p such that the number 7*p^2 + 186 is square of prime.

Conjecture 164. Let be F(k) = 2^(2^k) + n, where k is positive integer and n is an odd number. Then, there exist an infinity of numbers n such that F(k) is prime for k = 0, k = 1 and k = 2.

Conjecture 165. There exist an infinity of quadruplets of primes of the form [30*k + 17, 30*k + 19, 30*k + 31, 30*k + 43]. Such primes are, as can be seen above, for instance, [17, 19, 31, 43] or [137, 139, 151, 163]. There exist an infinity of quadruplets of primes of the form [30*k + 17, 30*k + 19, 30*k + 31, 30*k + 43]. Such primes are, as can be seen above, for instance, [17, 19, 31, 43] or [137, 139, 151, 163].

Conjecture 166. Let be F(k) = 4^(4^k) + 3, where k is positive integer. Then, there exist an infinity of numbers k such that F(k) is equal to 7*p, where p is prime.

Conjecture 167. Let be F(k) = m^(m^k) + n, where m is even and n is odd, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Conjecture 168. Let be F(k) = m^(m^k) + n, where m is odd and n is even, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Conjecture 169. Let be F(k) = m^(n^k) + n, where k is positive integer and m and n are coprime positive integers, not both of them odd or both of them even. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Conjecture 170. Let p and q be two odd primes; then, through the iterative operation ((((p + 1)*q + 1) + 1)*q + 1...) finally it will be obtained a prime.

Conjecture 171. Let p be an odd prime; then the formula n*p + n – 1, where n integer, n > 1, conducts to an infinity of prime numbers (note that, for n = 2, the conjecture states the infinity of Sophie Germain primes).

Conjecture 172. There exist an infinity of semiprimes p*q = N/3^m, where p < q, m is positive integer and N is the number formed concatenating to the left a Carmichael number with the number 584, with the property that q – p + 1 is a prime number.

Conjecture 173. Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 174. Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 175. Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 176. Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 177. Let p be an odd prime; then the formula ((((n*p – n + 1)*p – n + 1))*p – n + 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 178. Let p and q be distinct odd primes; then the formula ((((n*p – (n + 1)*q)*p – (n + 1)*q)*p – (n + 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 179. Let p be an odd prime; then the formula ((((n*p – n + 1)*p – n + 1))*p – n + 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 180. Let p and q be distinct odd primes; then the formula ((((n*p – (n + 1)*q)*p – (n + 1)*q)*p – (n + 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 181. There exist an infinity of odd integers k such that the number p = (2^(2^k) + 2)/6 is a prime of the form 30*n + 13, where n positive integer.

Conjecture 182. There exist an infinity of odd integers k such that the number p = (2^(2^k) - 2)/2 is a prime of the form 30*n + 7, where n positive integer.

Conjecture 183. For any given odd prime p there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the first kind and n = 30*p*q + 1 is a prime.

Conjecture 184. For any given odd prime p there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Conjecture 185. For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the first kind and n = 30*k*p*q + 1 is a prime.

Conjecture 186. For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Conjecture 187. For any given odd prime p there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the second kind and n = 30*p*q – 1 is a prime.

Conjecture 188. For any given odd prime p there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Conjecture 189. For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the second kind and n = 30*k*p*q – 1 is a prime.

Conjecture 190. For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q – 1 are both Coman semiprimes of the second kind.

Conjecture 191. For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a prime.

Conjecture 192. For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a prime.

Conjecture 193. For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a prime.

Conjecture 194. For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a prime.

Conjecture 195. For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a Coman semiprime of the first kind.

Conjecture 196. For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a Coman semiprime of the second kind.

Conjecture 197. For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a Coman semiprime of the first kind.

Conjecture 198. For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a Coman semiprime of the second kind.

Conjecture 199. There exist an infinity of primes q of the form q = 2*k*p + 1, where p is a lesser prime from a pair of twin primes, for any positive integer k, under the condition that k has not the digital root equal to 2, 5 or 8.

Conjecture 200. There exist an infinity of primes q of the form q = 2*k*p + 1, where k is a positive integer which digital root is not equal to 2, 5 or 8, for any p lesser prime from a pair of twin primes.

Conjecture 201. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of primes p such that F(p) = p. Such primes p are 13, 61 (...). Note that, up to x = 241, there is no other odd number x for which F(x) = x.

Conjecture 202. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of pairs of twin primes (p, q) such that F(p) = F(q). Such pairs are (59, 61), (239, 241) (...) with corresponding F(p) = F(q) equal to 61, 331 (...).

Conjecture 203. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of pairs of primes (p, q) such that F(p) = q. Such pairs are (5, 7), (11, 19), (23, 43), (29, 37), (43, 61), (59, 61), (101, 109), (157, 229), (167, 241), (239, 331), (241, 331) (...).

Conjecture 204. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of pairs of primes (p, q) such that F(p) = q^2. Such pairs are (31, 7), (83, 11), (103, 11) (...).

Conjecture 205. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of pairs of primes (p, q) such that F(p^2) = q. Such pairs are (13, 223), (19, 541), (29, 1129) (...).

Conjecture 206. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then, there exist an infinity of pairs of primes (p, F(p)) such that F(p) - p = 2 (in other words, p and F(p) are twin primes). Such pairs of twin primes are (5, 7), (59, 61) (...).

Conjecture 207. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number, and G(x) = F(x) – x. Then, there exist an infinity of pairs of primes (p, F(p)) such that G(p) is a multiple of 9. Such pairs of primes are (43, 61) (...) with corresponding G(p) equal to 18 (...).

Conjecture 208. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number, and G(x) = F(x) – x. Then, there exist an infinity of pairs of primes (p, F(p)) such that G(p) is a power of the number 2. Such pairs of primes are (5, 7), (11, 19), (29, 37), (101, 109) (...) with corresponding exponents (powers of 2): 1, 3, 3, 3 (...).

Conjecture 209. Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number, and G(x) = F(x) – x. Then, there exist an infinity of primes p such that G(p) is also prime. Such pairs of primes (p, G(p)) are (17, 5), (41, 11), (47, 17), (53, 23), (71, 5), (113, 47), (173, 53) (...).

Conjecture 210. Let H(x) be the sum of the digits of the number 2^x + x^2, where x is an odd positive number. Then, there exist an infinity of pairs of twin primes (x = 11 + 18*k, y = 13 + 18*k) such that H(x) = H(y). Such pairs of twin primes are: (11, 13), (29, 31), (101, 103), (191, 193), (227, 229), (569, 571) (...) with corresponding H(x) = H(y) equal to: 18, 45, 117, 243, 315, 810 (...).

Conjecture 211. Let MC(x) be the function defined on the set of odd positive integers with values in the set of primes such that: MC(x) = 1 for x = 1; MC(x) = x, for x prime; for x composite, MC(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),..., p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then MC(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),..., q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then MC(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of MC(x). Then, there exist an infinity of primes p such that MC(p^2) = 5.

Conjecture 212. There exist an infinity of pairs of twin primes (p, p + 2) such that MC(p^2) + MC((p + 2)^2) – 1 = q^2, where q is prime.

Conjecture 213. For any pair of twin primes p and p + 2, where p ≥ 5, there exist a positive integer n of the form 15 + 18*k such that the value of Smarandache function for n is equal to p and the value of MC function for n is equal to p + 2.

Part two. Sixty-two articles on primes

1. A conjecture about a way in which the squares of primes can be written and five other related conjectures

Abstract. I was playing with randomly formed formulas based on two distinct primes and the difference of them, when I noticed that the formula p + q + 2*(q – p) – 1, where p, q primes, conducts often to a result which is prime, semiprime, square of prime or product of very few primes. Starting from here, I made a conjecture about a way in which any square of a prime seems that can be written. Following from there, I made a conjecture about a possible infinite set of primes, a conjecture regarding the squares of primes and Poulet numbers and yet three other related conjectures.

Conjecture 1:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q and r are distinct primes, q ≥ 5 and r ≥ 5.

Comment:

I really have no idea yet how it could be proved the conjecture or what implications it could have if it were true, so I'll just check it for the first few squares of primes.

Verifying the conjecture:
(for the first few squares of primes)

: 5^2 = 3*11 – 7 – 1, so [p, q, r] = [5, 11, 7];
: 7^2 = 3*19 – 7 – 1, so [p, q, r] = [7, 19, 7];
: 11^2 = 3*43 – 7 – 1, so [p, q, r] = [11, 43, 7];
: 13^2 = 3*59 – 7 – 1, so [p, q, r] = [13, 59, 7];
: 17^2 = 3*101 – 13 – 1, so [p, q, r] = [17, 101, 13];
: 19^2 = 3*127 – 19 – 1, so [p, q, r] = [19, 127, 19];
: 23^2 = 3*179 – 7 – 1, so [p, q, r] = [23, 179, 7];
: 29^2 = 3*283 – 7 – 1, so [p, q, r] = [29, 283, 7];
: 31^2 = 3*331 – 31 – 1, so [p, q, r] = [31, 331, 31];
: 37^2 = 3*461 – 13 – 1, so [p, q, r] = [37, 461, 13];
: 41^2 = 3*563 – 7 – 1, so [p, q, r] = [41, 563, 7];
: 43^2 = 3*619 – 7 – 1, so [p, q, r] = [43, 619, 7];
: 47^2 = 3*739 – 7 – 1, so [p, q, r] = [47, 739, 7].

Note:

It can be seen that in few cases from the ones above we have p = r, so we make yet another conjecture:

Conjecture 2:

There exist an infinity of primes p that can be written as p = (q^2 + q + 1)/3, where q is also a prime.

Examples of such primes:

: 19 = (7^2 + 7 + 1)/3, so [p, q] = [19, 7];
: 127 = (19^2 + 19 + 1)/3, so [p, q] = [127, 19];
: 331 = (31^2 + 31 + 1)/3, so [p, q] = [331, 31].

Conjecture 3:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q is a Poulet number and r a prime, r ≥ 5.

Verifying the conjecture:
(for the first few squares of primes)

: 5^2 = 3*341 – 997 – 1, so [p, q, r] = [5, 341, 997];
: 7^2 = 3*1387 – 4111 – 1, so [p, q, r] = [7, 1387, 4111];
: 11^2 = 3*4371 – 13921 – 1, so [p, q, r] = [11, 4371, 13921];
: 13^2 = 3*341 – 853 – 1, so [p, q, r] = [13, 341, 853].

Comment:

Considering the results from the three conjectures above, I make three other related conjectures.

Conjecture 4:

For any prime p, p ≥ 5, there exist an infinity of pairs of distinct primes [q, r] such that p = sqrt(3*q – r – 1).

Example:
(for p = 7)

: 7 = sqrt(3*19 – 7 – 1), so [q, r] = [19, 7];
: 7 = sqrt(3*23 – 19 – 1), so [q, r] = [23, 19];
: 7 = sqrt(3*29 – 37 – 1), so [q, r] = [29, 37];
: 7 = sqrt(3*31 – 43 – 1), so [q, r] = [31, 43];
: 7 = sqrt(3*37 – 61 – 1), so [q, r] = [37, 61];
: 7 = sqrt(3*41 – 73 – 1), so [q, r] = [41, 73] (...).

Conjecture 5:

For any prime p, p ≥ 5, there exist at least a pair of distinct primes [q, r] such that p = (q^2 + r + 1)/3.

Verifying the conjecture:

(for the first few primes)

: 5 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 7 = (3^2 + 11 + 1)/3, so [p, q, r] = [7, 3, 11];
: 11 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 13 = (5^2 + 13 + 1)/3, so [p, q, r] = [13, 5, 13];
: 17 = (3^2 + 41 + 1)/3, so [p, q, r] = [17, 3, 41];
: 19 = (5^2 + 31 + 1)/3, so [p, q, r] = [19, 5, 31].

Conjecture 6:

For any prime p of the form p = 6*k + 1 there exist an infinity of pairs of distinct primes [q, r] such that p = 3*q – p^2 – 1.

Example:
(for p = 37)

: 37 = 3*29 – 7^2 – 1, so [q, r] = [29, 7];
: 37 = 3*53 – 11^2 – 1, so [q, r] = [53, 11];
: 37 = 3*109 – 17^2 – 1, so [q, r] = [109, 17];
: 37 = 3*293 – 29^2 – 1, so [q, r] = [293, 29];
: 37 = 3*1693 – 71^2 – 1, so [q, r] = [1693, 71];
: 37 = 3*1789 – 73^2 – 1, so [q, r] = [1789, 73] (...).

2. A conjecture about an infinity of sets of integers, each one having an infinite number of primes

Abstract. In this paper, inspired by one of my previous papers posted on Vixra, I make, considering the sum of the digits of an odd integer, a conjecture about an infinity of sets of integers, each one having an infinite number of primes and I also make, considering the sum of the digits of a prime number, two other conjectures.

Conjecture 1:

For an infinity of odd positive integers m there is an infinite set of primes with the property that the sum of their digits is equal to m + 1.

Conjecture 2:

For an infinity of primes p there is an infinite set of primes with the property that the sum of their digits is equal to p + 1.

Comment:

Such a prime p I conjectured to be, in a previous paper posted on Vixra, the number 13.

Conjecture 3:

There is an infinite number of values the sum of the digits of the numbers p + 1, where p is odd prime, may have.

Note:

For a list with prime numbers with the property that the sum of their digits is equal to an even number see the sequence A119449 in OEIS.

Note:

We will refer hereinafter with D(m) to the set of primes with the property that the sum of their digits is equal to m + 1, where m is an odd integer.

The sequence D(1):

: 101 (...).

The sequence D(3):

: 13, 31, 103, 211, 1021, 1201 (...).

The sequence D(5):

: (...).

The sequence D(7):

: 17, 53, 71, 107, 233, 251, 431, 503, 521, 701, 1061, 1151, 1223 (...).

The sequence D(9):

: 19, 37, 73, 109, 127, 163, 181, 271, 307, 433, 523, 541, 613, 631, 811, 1009, 1063, 1117, 1153, 1171 (...).

The sequence D(11):

: (...).

The sequence D(13):

: 59, 149, 167, 239, 257, 293, 347, 419, 491, 563, 617, 653, 743, 761, 941, 1049, 1193, 1229, 1283, 1319 (...).

The sequence D(15):

: 79, 97, 277, 349, 367, 383, 439, 457, 547, 619, 673, 691, 709, 727, 853, 907, 1069, 1087, 1249 (...).

The sequence D(17):

: (...).

The sequence D(19):

: 389, 479, 569, 587, 659, 677, 839, 857, 929, 947, 983, 1289 (...).

The sequence D(21):

: 499, 769, 787, 859, 877, 967 (...).

Note:

It can easily be seen that for some values of odd integers m were obtained much more primes with the sum of the digits equal to m + 1 than for other values of m; for instance were obtained, from the first hundred of primes having the sum of digits equal to an even number, 20 such primes for which m = 9, 21 such primes for which m = 13, 19 such primes for which m = 15, but no such primes at all for which m = 5, m = 11 or m = 17.

3. A trivial but notable observation about a relation between the twin primes and the number 14

Abstract. There are known few interesting properties which distinquish twin primes from the general set of primes, like for instance that 46% of primes smaller than 19000 are Ramanujan primes while about 78% of the lesser of twin primes smaller than 19000 are Ramanujan primes. But seems that a much more trivial observation about the lesser of twin primes escaped attention: from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have the following remarkable property: the sum of their digits is equal to 14.

Note:

For a list of lesser of twin primes and also for the property mentioned in Abstract regarding Ramanujan primes see the sequence A001359 in OEIS.

Observation:

Like I mentioned in abstract, this paper is a trivial observation of a fact: from the first 100 numbers which are lesser in a pair of twin primes, 20 of them have the property that the sum of their digits is equal to 14; from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have this property; these numbers are:

: 59, 149, 239, 347, 419, 617, 1049, 1229, 1319, 1427, 1481, 1607, 2129, 2237, 2309, 2381, 3119, 3371, 3461, 3821, 4019, 4091, 4127, 4217, 4271, 4721, 5009, 5441, 6701, 7331, 8231, 9041, 10067, 10139, 10427, 11057, 12821, 13217, 13721, 13901, 14009, 14081, 16061, 18041, 18131, 18311, 19211, 20147, 20507, 21191, 21317, 22037, 22091, 22109, 22271, 22541, 23027, 24107, 25601, 29021, 30137, 31181, 31541, 31721, 32027, 32117.

Conjecture:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it's equal to 14.

Note:

The conjecture above implies the following one:

Conjecture:

There is an infinity of primes with the property that the sum of their digits is equal to 14.

4. An observation about the digital root of the twin primes, few conjectures and an open problem on primes

Abstract. Few interesting properties which distinquish twin primes from the general set of primes there are already known. I wrote myself an article regarding an interesting property of a set of (pairs of) twin primes based on the sum of the digits of the lesser (implicitly greater) prime from a pair of twin primes. This paper notes a property regarding twin primes based on their digital root.

Observation:

The digital root of the lesser prime p from a pair of twin primes [p, q], under the condition that p > 3, is, for the first such 100 primes (for a list of lesser of twin primes see the sequence A001359 in OEIS):

: 5, 2, 8, 2, 5, 5, 8, 2, 8, 2, 5, 8, 2, 8, 2, 5, 8, 2, 5, 5, 8, 2, 8, 2, 5, 5, 2, 2, 8, 2, 8, 2, 8, 2, 5, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 5, 2, 5, 8, 2, 5, 2, 5, 8, 5, 5, 5, 8, 2, 2, 8, 5, 5, 8, 5, 8, 5, 8, 5, 2, 8, 2, 5, 2, 2, 8, 2, 8, 2, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 8, 2, 2, 8, 8, 5, 5.

Note:

Obviously the digital root of a lesser from a pair of twin primes can never be equal to 3, 6 or 9 (it would be then a number divisible by 3 not a prime) or 1, 4 or 7 (the greater from the pair of twin primes would be in this case divisible by 3).

Conjecture 1:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

Conjecture 2:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

Conjecture 3:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

Note:

Is remarkable that the three subsets of the set of (lesser of) twin primes seem to have (of course, for a given term great enough) an approximately equal number of terms; for instance, from the first 100 from the lesser of twin primes, 33 of them have the digital root equal to 2, 35 have the digital root equal to 5 and 32 have the digital root equal to 7.

Conjecture 4:

Let a_i be the sequence of the lesser of twin primes whose digital root is equal to 2, b_i be the sequence of the lesser of twin primes whose digital root is equal to 5 and c_i be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms n of a_i for which the number of the terms of b_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of b_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of c_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of b_i smaller than n.

Conjecture 5:

There is an infinity of integers n for which the set of the lesser (greater) of the twin primes smaller than n is divided in three subsets whith an equal number of terms, a_i with the property that the digital root of its terms is equal to 2 (4), b_i with the property that the digital root of its terms is equal to 5 (7) and c_i with the property that the digital root of its terms is equal to 8 (1).

Open problem:

Is there any other prime p beside p = 23 with the property that the following six subsets of odd primes have an equal number of terms smaller than p? The terms of the six subsets are the primes whose digital root is equal to 1, 2, 4, 5, 7 respectively 8 (it can be seen that, for p = 23, we have the following odd primes smaller than 23 that belong to the six subsets: 5, 7, 11, 13, 17, 19, whose digital root is 5, 7, 2, 4, 8, 1).

5. A conjecture on primes involving the pairs of sexy primes

Abstract. This paper states a conjecture on primes involving two types of pairs of primes: the pairs of sexy primes, which are the two primes that differ from each other by six and the pairs of primes of the form [p, q], where q = p + 6*r, where r is positive integer.

Conjecture:

If n and n + 6 are both primes (in other words if [n, n + 6] is a pair of sexy primes), where n ≥ 7, then the number m = n + 3 can be written at least in one way as m = p + q, where p and q are primes, q = p + 6*r and r is positive integer.

Verifying the conjecture:
(for the first fifteen pairs of sexy primes)

: for [n, n + 6] = [7, 13] we have [p, q, r] = [5, 5, 0];
: for [n, n + 6] = [11, 17] we have [p, q, r] = [7, 7, 0];
: for [n, n + 6] = [13, 19] we have [p, q, r] = [5, 11, 1];
: for [n, n + 6] = [17, 23] we have [p, q, r] = [7, 13, 1];
: for [n, n + 6] = [23, 29] we have [p, q, r] = [13, 13, 0];
: for [n, n + 6] = [31, 37] we have [p, q, r] = [5, 29, 1] or [17, 17, 0];
: for [n, n + 6] = [37, 43] we have [p, q, r] = [11, 29, 3] or [17, 23, 1];
: for [n, n + 6] = [41, 47] we have [p, q, r] = [7, 37, 1] or [13, 31, 3];
: for [n, n + 6] = [47, 53] we have [p, q, r] = [7, 37, 1] or [13, 37, 4] or [19, 31, 2];
: for [n, n + 6] = [53, 59] we have [p, q, r] = [13, 43, 5] or [19, 37, 3];
: for [n, n + 6] = [61, 67] we have [p, q, r] = [17, 47, 5] or [23, 41, 3];
: for [n, n + 6] = [67, 73] we have [p, q, r] = [11, 59, 8] or [17, 53, 6] or [23, 47, 4] or [29, 41, 2];
: for [n, n + 6] = [73, 79] we have [p, q, r] = [17, 59, 7] or [23, 53, 5] or [29, 47, 3];
: for [n, n + 6] = [83, 89] we have [p, q, r] = [7, 79, 12] or [13, 73, 10] or [19, 67, 8] or [43, 43, 0];
: for [n, n + 6] = [97, 103] we have [p, q, r] = [11, 89, 13];
or [17, 83, 11] or [29, 71, 7] or [47, 53, 1].

6. A conjecture on the pairs of primes p, q, where q is equal to the sum of p and a primorial number

Abstract. In a previous paper I stated a conjecture on primes involving the pairs of sexy primes, which are the two primes that differ from each other by six. In this paper I extend that conjecture on the pairs of primes [p, q], where q is of the form p + p(n)#, where p(n)# is a primorial number, which means the product of first n primes.

Conjecture:

If p and p + p(n)# are both primes, where p > p(n)#, n ≥ 2 and p(n)# is a primorial number (which means the product of first n primes), then the number m = p + p(n)#/2 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + p(n)#*r and r is positive integer.

Note:

For a list of primorial numbers, see the sequence A002110 in OEIS; the first ten primorial numbers are: 1, 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870.

Note:

Because p(0)# is, by convention or justified through the concept of "empty product", equal to 1, then p(1)# is equal to 2, p(2)# is equal to 6, p(3)# is equal to 30, p(4)# is equal to 210 and so on.

Comment:

The conjecture it will be then formulate:

: for p(2)# = 6:

If p and p + 6 are both primes, where p > 6, then the number m = p + 3 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 6*r and r is positive integer (this is the conjecture which I made in a previous paper, where I also verified it for the first fifteen pairs of sexy primes, with the difference that in the formulation from there x and y were "primes" not "primes or squares of primes");

: for p(3)# = 30:

If p and p + 30 are both primes, where p > 30, then the number m = p + 15 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 30*r and r is positive integer;

: for p(4)# = 210:

If p and p + 210 are both primes, where p > 210, then the number m = p + 105 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 210*r and r is positive integer;

: for p(5)# = 2310:

If p and p + 2310 are both primes, where p > 2310, then the number m = p + 1155 can be written at least in one way as m = x + y, where x and y are primes, y = x + 2310*r and r is positive integer.

Verifying the conjecture:
(for the first ten pairs of primes [p, p + 30], where p > 30)

: for [p, p + 30] = [31, 61] we have [x, y, r] = [23, 23, 0];
: for [p, p + 30] = [37, 67] we have [p, q, r] = [11, 41, 1];
: for [p, p + 30] = [41, 71] we have [p, q, r] = [13, 43, 1];
: for [p, p + 30] = [43, 73] we have [p, q, r] = [29, 29, 0];
: for [p, p + 30] = [53, 83] we have [p, q, r] = [19, 49, 1];
: for [p, p + 30] = [59, 89] we have [p, q, r] = [37, 37, 0] or [7, 67, 2];
: for [p, p + 30] = [67, 97] we have [p, q, r] = [41, 41, 0] or [11, 71, 2];
: for [p, p + 30] = [71, 101] we have [p, q, r] = [43, 43, 0] or [13, 73, 2];
: for [p, p + 30] = [73, 103] we have [p, q, r] = [29, 59, 1];
: for [p, p + 30] = [79, 109] we have [p, q, r] = [47, 47, 0].

Verifying the conjecture:
(for the first eight pairs of primes [p, p + 210], where p > 210)

: for [p, p + 210] = [13, 223] we have [x, y, r] = [59, 59, 0] or [29, 89, 2];
: for [p, p + 210] = [17, 227] we have [x, y, r] = [61, 61, 0];
: for [p, p + 210] = [19, 229] we have [x, y, r] = [17, 107, 3];
: for [p, p + 210] = [23, 233] we have [x, y, r] = [19, 109, 3] or [49, 79, 1];
: for [p, p + 210] = [29, 239] we have [x, y, r] = [67, 67, 0] or [7, 127, 4] or [37, 97, 2];
: for [p, p + 210] = [31, 241] we have [x, y, r] = [23, 113, 3] or [53, 83, 1];
: for [p, p + 210] = [41, 251] we have [x, y, r] = [73, 73, 0] or [43, 103, 2];
: for [p, p + 210] = [47, 257] we have [x, y, r] = [31, 121, 3].

Verifying the conjecture:
(for the first pair of primes [p, p + 2310], where p > 2310)

: for [p, p + 2310] = [23, 2333] we have [x, y, r] = [49, 1129, 36] or [109, 1069, 32] or [139, 1039, 30] or [169, 1009, 28] or [349, 829, 16] or [469, 709, 12] or [439, 739, 10].

7. Two conjectures involving the sum of a prime and a factorial number

Abstract. In this paper I state two conjectures about the sum of a prime and a factorial.

Note:

For a list of factorial numbers see the sequence A000142 in OEIS.

Conjecture 1:

For any odd prime p there exist at least one prime q such that p + n! = q, where n is a positive integer, n < p.

Verifying the conjecture:
(for the first five odd primes p)

: 3 + 2! = 5, so [p, q, n] = [3, 5, 2];
: 5 + 2! = 7 and 5 + 3! = 11 and 5 + 4! = 29, so [p, q, n] = [5, 7, 2] or [5, 11, 3] or [5, 29, 4];
: 7 + 3! = 13 and 7 + 4! = 31 and 7 + 5! = 127 and 7 + 6! = 727 so [p, q, n] = [7, 13, 3] or [7, 31, 4] or [7, 127, 5] or [7, 727, 6];
: 11 + 2! = 13 and 11 + 6! = 17 and 11 + 5! = 131 and 11 + 7! = 5051 and 11 + 10! = 3628811 so [p, q, n] = [11, 13, 2] or [11, 17, 6] or [11, 131, 10] or [11, 5051, 7] or [11, 3628811, 10];
: 13 + 3! = 19 and 13 + 4! = 37 and 13 + 6! = 733 so [p, q, n] = [13, 19, 3] or [13, 37, 4] or [13, 733, 6].

Note:

From the primes q, q ≥ 5, q ≤ 401, just three primes can't be written as p + n!, where p is a lesser odd prime and n is a positive integer, *i.e.* the primes 41, 101, 367 (but, interesting, 367 can be written as 7^3 + 5!); indeed:

: q can be written as p + 2! for q = 5, 7, 31, 43, 61, 73, 103, 109, 139, 151, 181, 193, 199, 229, 241, 271, 283, 313, 349 [...];
: q can be written as p + 3! for q = 11, 13, 17, 23, 29, 37, 47, 53, 59, 67, 79, 89, 107, 113, 157, 163, 173, 179, 197, 239, 251, 257, 263, 269, 277, 337, 359, 373, 379, 389 [...];
: q can be written as p + 4! for q = 71, 83, 97, 127, 131, 191, 223, 251, 281, 293, 307, 317, 331, 383, 397 [...];
: q can be written as p + 5! for q = 149, 167, 227, 233, 311, 347, 353, 401 [...].

Conjecture 2:

For any odd prime p, p ≥ 5, there exist an infinity of primes q of the form q = (p + n!)/n^k, where n and k are positive integers and n ≥ p.

Examples:

: for $p = 5$, we have:
: $q = 29 = (6! + 5)/5^2$;
: $q = 1009 = (7! + 5)/5^1$;:
: $q = 1613 = (8! + 5)/5^2$;
: $q = 72577 = (9! + 5)/5^1$;
[...]
: for $p = 7$, we have:
: $q = 103 = (7! + 7)/7^2$;
: $q = 823 = (8! + 7)/7^2$;
[...]
: for $p = 11$, we have:
: $q = 329891 = (11! + 11)/11^2$;
[...]
: for $p = 13$, we have:
: $q = 2834329 = (13! + 13)/13^3$;
: $q = 515847877 = (13! + 13)/13^2$;
[...].

8. Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture

Abstract. In this paper I make few conjectures about a way to write an odd prime p, id est p = q – r + 1, where q and r are also primes; two of these conjectures can be regarded as generalizations of the twin primes conjecture, which states that there exist an infinity of pairs of twin primes.

Conjecture 1
(Which can be regarded as a generalization of the twin primes conjecture)

Any odd prime p can be written in an infinity of distinct ways like p = q – r + 1, where q and r are also primes; in other words, there exist an infinity of pairs of primes (q, r) such that q – r = p – 1, for any odd prime p (it can be seen that for p = 3 the conjecture states the same thing with the twin primes conjecture).

Conjecture 2

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 11 – 5 + 1; 17 – 11 + 1; 23 – 17 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k - 1 (sexy primes are the primes that differ by each other by six).

Conjecture 3

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 13 – 7 + 1; 19 – 13 + 1; 37 – 31 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k + 1.

Conjecture 4

Any prime p of the form p = 6*k – 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 5
(Which can be regarded as a generalization of the twin primes conjecture)

There exist an infinity of pairs of primes (p, q), where p is of the form 6*k – 1 and q is of the form 6*h + 1, such that q – p + 1 = 3^n, for any n non-null positive integer (it can be seen that for n = 1 the conjecture states the same thing with the twin primes conjecture).

Example: for n = 2 we have the pairs of primes (p, q): (11, 19); (23, 31) etc.; for n = 3 we have the pairs of primes (5, 31); (11, 37) etc.

Conjecture 6

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1.

Example: the number 49 = 7^2 can be written as 61 – 13 + 1; 67 – 19 + 1; 79 – 31 + 1 etc.

Conjecture 7

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h 1 and r is a prime of the form q = 6*i –1.

Example: the number 49 = 7^2 can be written as 53 – 5 + 1; 59 – 11 + 1; 71 – 23 + 1 etc.

9. An interesting formula for generating primes and five conjectures about a certain type of pairs of primes

Abstract. In this paper I just enunciate a formula which often leads to primes and products of very few primes and I state five conjectures about the pairs of primes of the form [(q^2 – p^2 – 2*r)/2,(q^2 – p^2 + 2*r)/2], where p, q, r are odd primes.

Conjecture 1:

For any r prime, r ≥ 5, there exist an infinity of pairs of primes (p, q) such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Conjecture 2:

For any pair of primes (p, r), p ≥ 5, r ≥ 5, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Note:

The numbers m = (q^2 – p^2 – 2*r)/2 and n = (q^2 – p^2 + 2*r)/2, where p, q, r are odd primes, seems to be often primes and generally products of very few primes.

Examples:

: For (p, q, r) = (13, 11, 5) we have (m, n) = (19, 29);
: For (p, q, r) = (17, 11, 5) we have (m, n) = (79, 89);
: For (p, q, r) = (37, 11, 5) we have (m, n) = (1039, 1049);

: For (p, q, r) = (13, 11, 7) we have (m, n) = (17, 31);
: For (p, q, r) = (19, 11, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (23, 11, 7) we have (m, n) = (197, 211);

: For (p, q, r) = (17, 13, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (19, 13, 7) we have (m, n) = (89, 103);
: For (p, q, r) = (37, 13, 7) we have (m, n) = (593, 607);

: For (p, q, r) = (19, 17, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (23, 17, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (29, 17, 7) we have (m, n) = (269, 283);
: For (p, q, r) = (11, 7, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (13, 7, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (17, 7, 7) we have (m, n) = (113, 127);

: For (p, q, r) = (19, 11, 11) we have (m, n) = (109, 131);
: For (p, q, r) = (31, 11, 11) we have (m, n) = (409, 431);
: For (p, q, r) = (61, 11, 11) we have (m, n) = (1789, 1811).

Conjecture 3:

For any p prime, $p \geq 5$, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*p)/2 and (q^2 – p^2 + 2*p)/2 are both primes.

Conjecture 4:

If x, y and r are odd primes such that y = x + 2*r, where $r \geq 5$, then there exist p and q also primes such that x = (q^2 – p^2 – 2*r)/2 and y = (q^2 – p^2 + 2*r)/2.

Examples:

: For (x, y, r) = (17, 31, 7), we have (p, q) = (11, 13);
: For (x, y, r) = (29, 43, 7), we have (p, q) = (17, 19);
: For (x, y, r) = (53, 67, 7), we have (p, q) = (13, 17).

Conjecture 5:

For any p prime, $p \geq 7$, there exist a pair of smaller primes (q, r) such that the numbers x = (p^2 – q^2 – 2*r)/2 and y = (p^2 – q^2 + 2*r)/2 are both primes.

Examples:

: For p = 7, (q, r) = (5, 5) and (x, y) = (7, 17);
: For p = 11, (q, r) = (5, 5) and (x, y) = (43, 53) and also (q, r) = (7, 7) and (x, y) = (43, 53) and also (q, r) = (7, 5) and (x, y) = (31, 41);
: For p = 13, (q, r) = (7, 7) and (x, y) = (53, 67) and also (q, r) = (11, 5) and (x, y) = (19, 29) and also (q, r) = (11, 7) and (x, y) = (17, 31).

10. Few possible infinite sets of triplets of primes related in a certain way and an open problem

Abstract. In this paper I make three conjectures about a type of triplets of primes related in a certain way, i.e. the triplets of primes [p, q, r], where 2*p^2 – 1 = q*r and I raise an open problem about the primes of the form q = (2*p^2 – 1)/r, where p, r are also primes.

Conjecture 1:

There exist an infinity of primes p such that 2*p^2 – 1 = q*r, where q and r are also primes.

Examples: such primes are: 5, 19, 23, 29, 31, 47, 53, 61, 67, 71, 79, 83, 97 (...).

Conjecture 2:

If p is prime and 2*p^2 – 1 = q*r, where q and r are also primes, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)*(r + n), such that p + m, q + n and r + n are also primes.

Examples:

: for p = 5, [q, r] = [7, 7]; for [m, n] = [24, 34], [p + n, q + n, r + n] = [29, 41, 41];

: for p = 19, [q, r] = [7, 103]; for [m, n] = [34, 34], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 23, [q, r] = [7, 151]; for [m, n] = [44, 40], [p + n, q + n, r + n] = [67, 47, 191];

: for p = 31, [q, r] = [17, 113]; for [m, n] = [22, 24], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 71, [q, r] = [17, 593]; for [m, n] = [26, 13], [p + n, q + n, r + n] = [97, 31, 607];

: for p = 83, [q, r] = [23, 599]; for [m, n] = [254, 210], [p + n, q + n, r + n] = [307, 233, 809]; also for [m, n] = [198, 258], [p + n, q + n, r + n] = [347, 281, 857];

: for p = 139, [q, r] = [17, 2273]; for [m, n] = [250, 110], [p + n, q + n, r + n] = [389, 127, 2383].

Conjecture 3:

If p is prime and 2*p^2 – 1 = q^2, where q is also prime, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)^2, such that p + m and q + n are also primes.

Example: for p = 5, q = 7; for [m, n] = [24, 34], [p + n, q + n] = [29, 41].

Open problem:

Which primes q can be written as q = (2*p^2 – 1)/r, where p, r are also primes?

11. Two types of pairs of primes that could be associated to Poulet numbers

Abstract. In this paper I combine two of my objects of study, the Poulet numbers and the different types of pairs of primes and I state two conjectures about few ways in which types of Poulet numbers could be associated with types of pairs of primes.

Conjecture 1:

Any Poulet number of the form 10*n + 1 or 10*n + 9 can be written at least in one way as p*q + 10*k*h, where p and q are primes or powers of primes of the same form from the following four ones: 10*m + 1, 10*m + 3, 10*m + 7 or 10*m + 9, k and h are non-null positive integers and q – p = 10*k.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 9*(9 + 20) + 4*20 = 9*(9 + 10) + 17*10, so [p, q] = [3^2, 29] or [3^2, 19];
: 561 = 19*(29 + 10) + 1*10 = 9*(9 + 50) + 3*10, so [p, q] = [19, 29] or [3^2, 59];
: 1729 = 23*(23 + 50) + 1*50 = 17*(17 + 80) + 1*80 = 23*(23 + 30) + 17*30 = 27*(27 + 10) + 73*10 = 23*(23 + 20) + 37*20 = 13*(13 + 60) + 13*60 = 7*(7 + 120) + 7*120 = 17*(17 + 30) + 31*30 = 13*(13 + 40) + 26*40, so [p, q] = [23, 73] or [17, 97] or [23, 53] or [3^3, 37] or [23, 43] or [13, 73] or [7, 127] or [17, 47] etc.;
: 2701 = 29*(29 + 60) + 2*60, so [p, q] = [29, 89] etc.;
: 2821 = 29*(29 + 60) + 4*60, so [p, q] = [29, 89] etc.;
: 4369 = 27*(27 + 130) + 1*130, so [p, q] = [3^3, 157] etc.

Note 1:

Some such Poulet numbers can be written as p*q + (q – p), where p, q primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 23*53 + (53 – 23) = 17*97 + (97 – 17).

Note 2:

Probably this conjecture can stipulate for h to be equal to 1 or prime or power of prime (in the examples above, we found that h is equal to: 2^2 or 17; 1 or 3; 1 or 17 or 73 or 37 or 13 or 7 or 31; 2; 2^2; 1.

Conjecture 2:

For any Poulet number N not divisible by 3 there exist at least a pair of numbers [p, q], where p is prime and q is prime or square of prime, such that N = p^2 + q – 1.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 7^2 + 293 – 1 = 13^2 + 173 – 1 = 17^2 + 53 – 1, so [p, q] = [7, 293] or [13, 173] or [17, 53];

:   1105 = 13^2 + 937 – 1 = 23^2 + 577 – 1, so [p, q] = [13, 937] or [23, 577];

:   1387 = 23^2 + 859 – 1 = 29^2 + 547 – 1 = 37^2 + 19 – 1, so [p, q] = [23, 859] or [29, 547] or [37, 19];

:   1729 = 7^2 + 41^2 – 1 = 11^2 + 1609 – 1 = 19^2 + 37^2 – 1 = 23^2 + 1201 – 1 = 31^2 + 769 – 1, so [p, q] = [7, 41^2] or [41, 7^2] or [11, 1609] or [19, 37^2] or [37, 19^2] or [23, 1201] or [31, 769].

Note:

Some such Poulet numbers can be written as p^2 + q^2 – 1, where p, q are primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 7^2 + 41^2 – 1 = 19^2 + 37^2 – 1.

12. A set of Poulet numbers and generalizations of the twin primes and de Polignac's conjectures inspired by this

Abstract. In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac's conjecture.

Conjecture 1:

There exist an infinity of Poulet numbers of the form n^2 + 120*n, where n is prime or a composite positive integer.

Note:

In the first case, obviously n is a prime factor of such a Poulet number and the product of the other prime factors is equal to n + 120; for instance, the number 1729 is a part of this set of Poulet numbers because 1729 = 7*13*19 can be written as 13^2 + 13*120 and implicitly 7*19 = 13 + 120. First few such Poulet numbers are:

: 1729 = 7*13*19 = 13^2 + 13*120;
: 4681 = 31*151 = 31^2 + 31*120;
: 6601 = 7*23*41 = 41^2 + 41*120.

Note:

In the second case, obviously n is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to n + 120. Such a Poulet number is 75361 = 11*13*17*31 = 221^2 + 221*120 and implicitly 11*31 = 13*17 + 120.

Conjecture 2:

There exist an infinity of duplets of primes [p, q] such that p – q = 120; there also exist an infinity of triplets of primes [p1, p2, q] such that p1*p2 – q = 120; there also exist an infinity of quadruplets of primes [p1, p2, p3, q] such that p1*p2*p3 – q = 120; generally, for any non-null positive integer i there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = 120.

Examples:

: 151 – 31 = 120;
: 7*19 – 13 = 120;
: 7*17*37 – 4283 = 120.

Conjecture 3:
(generalization of the twin primes conjecture)

For any non-null positive integer i there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = 2.

Conjecture 4:
(generalization of de Polignac’s conjecture)

For any n even positive integer and for any i non-null positive integer there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = n.

13. A very exhaustive generalization of de Polignac's conjecture

Abstract. In a previous paper I made a generalization of de Polignac's conjecture. In this paper I extend that generalization as much as is possible.

Conjecture:

For any n even positive integer and for any i and j non-null positive integers there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi – q1*q2*...*qj = n.

Case [i, j, n] = [1, 1, 2]:

In this case we have p – q = 2, which gave us the twin primes conjecture.

Case [i, j, n] = [1, 1, n]:

In this case we have p – q = n, which gave us de Polignac's conjecture.

Case [i, j, n] = [2, 1, 2]:

In this case we have p1*p2 – q = 2.

Such triplets of primes [p1, p2, q], are: [7, 13, 89], [7, 19, 131], [7, 37, 257]...Note that the conjecture can be further extended in this case to: for any p1 odd prime there exist an infinity of pairs of primes [p2, q] such that p1*p2 – q = 2.

Case [i, j, n] = [1, 2, 2]:

In this case we have p – q1*q2 = 2.

Such triplets of primes [p, q1, q2], are: [79, 11, 7], [163, 23, 7], [331, 47, 7]...Note that the conjecture can be further extended in this case to: for any q1 odd prime there exist an infinity of pairs of primes [p, q2] such that p – q1*q2 = 2.

Conjecture:
(the most exhaustive generalization of de Polignac's conjecture)

For any n even positive integer and for any i, j, k, l non-null positive integers, for any k given primes a1, a2, ..., ak and for any l given primes b1, b2, ..., bl, there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi*a1*a2*...*ak – q1*q2*...*qj*b1*b2*...bl = n.

14. A formula which conducts to primes or to a type of composites that could form a class themselves

Abstract. In this paper I present a very simple formula which conducts often to primes or composites with very few prime factors; for instance, for the first 27 consecutive values introduced as “input” in this formula were obtained 10 primes, 4 squares of primes and 12 semiprimes; just 2 from the numbers obtained have three prime factors; but the most interesting thing is that the composites obtained have a special property that make them form a class of numbers themselves.

Observation:

The numbers C = 3^3*(3^3 + n*10) + n*10, where n is a positive integer of the form 4 + 9*k, or in other words C = 2520*k + 1849, are very often primes or numbers with very few prime factors, composites that have certain very interesting properties. Let’s see the case of the first 27 consecutive such numbers C; we will consider all 27 numbers but we will list them separatelly in three different lists: the case C is prime or square of prime, the case C is Coman semiprime and the case of the other numbers C (note that a Coman semiprime is a semprime p*q with the property that p – q + 1 is a prime or a square of prime; this is a class of numbers that I met it often in my research, for instance in the study of 2-Poulet numbers, many of these semiprimes having this property, but as well in the study of the prime factors of Carmichael numbers):

The case C is prime or square of prime:

: for k = 0 we have C = 43^2 where 43 prime;
: for k = 1 we have C = 4369 prime;
: for k = 2 we have C = 83^2 where 83 prime;
: for k = 3 we have C = 97^2 where 97 prime;
: for k = 5 we have C = 14449 prime;
: for k = 7 we have C = 19489 prime;
: for k = 11 we have C = 29569 prime;
: for k = 12 we have C = 32089 prime;
: for k = 16 we have C = 42169 prime;
: for k = 19 we have C = 223^2 where 223 prime;
: for k = 20 we have C = 52249 prime;
: for k = 23 we have C = 59809 prime;

The case C is Coman semiprime:

: for k = 4 we have C = 79*151 and 151 – 79 + 1 = 73 prime;
: for k = 6 we have C = 71*239 and 239 – 71 + 1 = 13^2, where 13 prime;
: for k = 8 we have C = 13*1693 and 1693 – 13 + 1 = 41^2, where 41 prime;
: for k = 13 we have C = 53*653 and 653 – 53 + 1 = 601 prime;
: for k = 14 we have C = 107*347 and 347 – 107 + 1 = 241 prime;
: for k = 15 we have C = 31*1279 and 1279 – 31 + 1 = 1249 prime;
: for k = 24 we have C = 157*397 and 397 – 157 + 1 = 241 prime.

: for k = 25 we have C = 64849 prime;
: for k = 26 we have C = 67369 prime.

The other numbers C:

: for k = 9 we have C = 19*1291 and 1291 – 19 + 1 = 19*67 and 67 – 19 + 1 = 7^2, where 7 prime;
: for k = 10 we have C = 11*2459 and 2459 – 11 + 1 = 31*79 and 79 – 31 + 1 = 7^2, where 7 prime;
: for k = 17 we have C = 23*29*67 and 23*29 – 67 + 1 = 601 prime, 29*67 – 23 + 1 = 17*113 where 113 – 17 + 1 = 97 prime and 23*67 – 28 = 17*89 where 89 – 17 + 1 = 73 prime;
: for k = 18 we have C = 17*2777 and 2777 – 17 + 1 = 11*251 and 251 – 11 + 1 = 241 prime;
: for k = 21 we have C = 11*13*383 and 11*13 – 383 + 1 = -239 prime in absolute value, 11*383 – 13 + 1 = 4201 prime, 13*383 – 11 + 1 = 4969 prime;
: for k = 22 we have C = 59*971 and 971 – 59 + 1 = 11*83 and 83 – 11 + 1 = 73 prime;
: for k = 27 we have C = 47*1487 and 1487 – 47 + 1 = 11*131 and 131 – 11 + 1 = 11^2, where 11 prime.

Note:

It can be seen that also “the other numbers C” have special properties; for instance, the semiprimes can be considered a kind of “extended Coman semiprimes” because of the iterative process that ends also in a prime or in a square of prime: let N = p1*q1; than p1- q1 + 1 = p2*q2 then p2 – q2 + 1 = p3*q3 and so on until is obtained a prime. On the other side, the numbers with three prime factors obtained p*q*r have the property that p*q – r + 1, p*r – q + 1 and q*r – p + 1 are primes or (extended) Coman semiprimes.

15. Four sequences of numbers obtained through concatenation, rich in primes and semiprimes

Abstract. In this paper I will define four sequences of numbers obtained through concatenation, definitions which also use the notion of "sum of the digits of a number", sequences that have the property to produce many primes, semiprimes and products of very few prime factors.

Observation 1:

Let x be a number with the sum of the digits equal to p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 2911 (semiprime), 9211 (semiprime), 4913 (cube of prime), 9413 (prime), 8917 (semiprime), 9817 (prime), 99119 (prime), 91919 (semiprime), 19919 (prime), 99523 (prime), 95923 (prime), 59923 (semiprime), 999431 (prime), 949931 (prime), 499931 (semiprime) etc.

Observation 2:

Let x be a number equal to the sum of the digits of p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 211 (prime), 413 (semiprime), 817 (semiprime), 1019 (prime), 523 (prime), 1129 (prime), 431 (prime), 1037 (semiprime), 541 (prime), 743 (prime), 1147 (semiprime), 853 (prime), 1459 (prime), 761 (prime), 1367 (prime), 871 (semiprime), 1073 (semiprime) etc.

Observation 3:

Let x be a number whose sum of the digits is equal to the sum of the digits of p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 1111 (semiprime), 10111 (prime), 2011 (prime), 20011 (prime), 200011 (semiprime), 2213 (prime), 22013 (prime), 3113 (semiprime), 4013 (prime), 40013 (prime), 400013 (semiprime), 4000013 (semiprime) etc.

Observation 4:

Let x be a number with the sum of the digits equal to p, where p is prime, let y = 6*x + 5 and z the number obtained through concatenation of y and p; then z is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: For [p, x, y] = [11, 29, 179] we have z = 17911 prime;
: For [p, x, y] = [11, 92, 557] we have z = 55711 prime;
: For [p, x, y] = [11, 902, 5417] we have z = 541711 prime;
: For [p, x, y] = [29, 9299, 55799] we have z semiprime;
: For [p, x, y] = [29, 9929, 59579] we have z semiprime;
: For [p, x, y] = [29, 9992, 59957] we have z = 5995729 prime;
: For [p, x, y] = [29, 2999, 17999] we have z = 1799929 prime;
: For [p, x, y] = [29, 9299, 55799] we have z semiprime;
: For [p, x, y] = [29, 9929, 59579] we have z semiprime;
: For [p, x, y] = [29, 9992, 59957] we have z = 5995729 prime.

Note:

In order to see wherefrom the idea of this sequence originate see my previous paper "A conjecture about a large subset of Carmichael numbers related to concatenation".

16. A conjecture on the squares of primes of the form 6k – 1

Abstract. In this paper I make a conjecture on the squares of primes of the form 6k – 1, conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

Conjecture:

For any square of a prime p of the form p = 6*k – 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime q*r where r – q = 8*k and a number congruent to 2, 3 or 5 modulo 6;
(3) p^2 can be deconcatenated into a semiprime 3*q, where q is of the form 10*k + 7, and a number congruent to 1 modulo 6;
(4) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(5) p^2 can be deconcatenated into a number of the form 121 + 48*k and a number congruent to 0 modulo 6;
(6) p^2 is a palindromic number.

Examples for case (1):

: for 5^2 = 25 we got 5 prime and 2 ≡ 2 (mod 6);
: for 17^2 = 289 we got 89 prime and 2 ≡ 2 (mod 6);
: for 23^2 = 529 we got 29 prime and 5 ≡ 5 (mod 6);
: for 29^2 = 841 we got 41 prime and 8 ≡ 2 (mod 6);
: for 53^2 = 2809 we got 809 prime and 2 ≡ 2 (mod 6);
: for 71^2 = 5041 we got 41 prime and 50 ≡ 2 (mod 6);
: for 83^2 = 6889 we got 89 prime and 68 ≡ 2 (mod 6);
: for 107^2 = 11449 we got 449 prime and 11 ≡ 5 (mod 6);
: for 167^2 = 27889 we got 89 prime and 278 ≡ 2 (mod 6);
: for 173^2 = 29929 we got 29 prime and 29 ≡ 5 (mod 6) also 929 prime and 2 ≡ 2 (mod 6);
: for 179^2 = 32041 we got 41 prime and 320 ≡ 2 (mod 6);
: for 191^2 = 36481 we got 6481 prime and 3 ≡ 3 (mod 6);
: for 197^2 = 38809 we got 809 prime and 38 ≡ 2 (mod 6);
: for 227^2 = 51529 we got 29 prime and 515 ≡ 5 (mod 6);
: for 233^2 = 54289 we got 89 prime and 542 ≡ 2 (mod 6);
: for 239^2 = 57121 we got 7121 prime and 5 ≡ 5 (mod 6);
: for 269^2 = 72361 we got 61 prime and 723 ≡ 3 (mod 6).

Examples for case (2):

: for 47^2 = 2209 we got 209 = 11*19 where 19 – 11 = 8*1 and 2 ≡ 2 (mod 6);
: for 59^2 = 3481 we got 481 = 13*37 where 37 – 13 = 8*3 and 3 ≡ 3 (mod 6);

:	for 131^2 = 17161 we got 161 = 7*23 where 23 – 7 = 8*2 and 17 ≡ 5 (mod 6);
:	for 149^2 = 22201 we got 2201 = 31*71 where 71 – 31 = 8*5 and 2 ≡ 2 (mod 6).

Examples for case (3):

:	for 41^2 = 1681 we got 681 = 3*227 and 1 ≡ 1 (mod 6);
:	for 89^2 = 7921 we got 921 = 3*307 and 7 ≡ 1 (mod 6).

Examples for case (4):

:	for 83^2 = 6889 we got 889 = 49 + 120*7 and 6 ≡ 0 (mod 6);
:	for 113^2 = 12769 we got 769 = 49 + 120*6 and 12 ≡ 0 (mod 6);
:	for 137^2 = 18769 we got 769 = 49 + 120*6 and 18 ≡ 0 (mod 6);
:	for 257^2 = 66049 we got 6049 = 49 + 120*50 and 6 ≡ 0 (mod 6);
:	for 263^2 = 69169 we got 9169 = 49 + 120*76 and 6 ≡ 0 (mod 6).

Examples for case (5):

:	for 251^2 = 63001 we got 3001 = 121 + 48*60 and 6 ≡ 0 (mod 6).

Examples for case (6):

:	11^2 = 121;
:	101^2 = 10201.

Note:

This conjecture is verified up to p = 269.

17. A conjecture on the squares of primes of the form 6k + 1

Abstract. In this paper I make a conjecture on the squares of primes of the form 6k + 1, conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

Conjecture:

For any square of a prime p of the form p = 6*k + 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime 3^n*q and a number congruent to 1 modulo 6;
(3) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 1;
(4) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 9;
(5) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(6) p^2 can be deconcatenated into a number of the form 121 + 24*k and a number congruent to 0 modulo 6.

Examples for case (1):

: for 67^2 = 4489 we got 89 prime and 44 ≡ 2 (mod 6);
: for 73^2 = 5329 we got 29 prime and 53 ≡ 5 (mod 6);
: for 79^2 = 6241 we got 41 prime and 62 ≡ 2 (mod 6);
: for 109^2 = 11881 we got 881 prime and 11 ≡ 2 (mod 6);
: for 163^2 = 26569 we got 569 prime and 26 ≡ 2 (mod 6);
: for 181^2 = 32761 we got 761 prime and 32 ≡ 5 (mod 6);
: for 199^2 = 39601 we got 601 prime and 39 ≡ 3 (mod 6).

Examples for case (2):

: for 13^2 = 169 we got 69 = 3*23 and 1 ≡ 1 (mod 6);
: for 37^2 = 1369 we got 369 = 3^2*41 and 1 ≡ 1 (mod 6);
: for 43^2 = 1849 we got 849 = 3*283 and 1 ≡ 1 (mod 6);
: for 61^2 = 3721 we got 21 = 3*7 and 37 ≡ 1 (mod 6);
: for 127^2 = 16129 we got 6129 = 3^3*227 and 1 ≡ 1 (mod 6);
: for 193^2 = 37249 we got 249 = 3*83 and 37 ≡ 1 (mod 6).

Examples for case (3):

: for 19^2 = 361 we got 36 + 1 = 37 prime;
: for 31^2 = 961 we got 96 + 1 = 97 prime;
: for 79^2 = 6241 we got 624 + 1 = 625 power of prime.

: for 139^2 = 19321 we got 1932 + 1 = 1933 prime;
: for 151^2 = 22801 we got 2280 + 1 = 2281 prime.

Examples for case (4):

: for 7^2 = 49 we got 4 + 1 = 5 prime;
: for 97^2 = 9409 we got 940 + 1 = 941 prime;
: for 103^2 = 10609 we got 1060 + 1 = 1061 prime.

Examples for case (5):

: for 157^2 = 24649 we got 649 = 49 + 120*5 and 24 ≡ 0 (mod 6).

Examples for case (6):

: for 79^2 = 6241 we got 241 = 121 + 24*5 and 6 ≡ 0 (mod 6).

Note:

This conjecture is verified up to p = 199.

Note:

I mention that this conjecture and the one from my previous paper "A conjecture on the squares of primes of the form 6k – 1" were made with the title of *jocandi causa*. It is not relevant if they are not true if they raise interesting questions about squares of primes.

18. Nine conjectures on the infinity of certain sequences of primes

Abstract. In this paper I enunciate nine conjectures on primes, all of them on the infinity of certain sequences of primes.

Conjecture 1:

For any prime p there exist an infinity of positive integers n such that the number n*p – n + 1 is prime.

Examples:

: For p = 19 we have the following primes: 2*19 – 1 = 37; 4*19 – 3 = 73; 6*19 – 5 = 109; 7*19 – 6 = 127; 9*19 – 8 = 163; 10*19 - 9 = 181 etc.

Conjecture 2:

For any prime p there exist an infinity of positive integers n such that the number n*p + n - 1 is prime.

Examples:

: For p = 11 we have the following primes: 2*11 + 1 = 23; 4*11 + 3 = 47; 5*11 + 4 = 59; 6*11 + 5 = 71; 7*11 + 6 = 83; 9*11 + 8 = 107 etc.

Conjecture 3:

For any prime p there exist an infinity of positive integers n such that the number n^2*p – n + 1 is prime.

Examples:

: For p = 7 we have the following primes: 3^2*7 – 2 = 61; 4^2*7 – 3 = 109; 7^2*7 – 6 = 337; 10^2*7 – 9 = 691; 12^2*7 – 11 = 997 etc.

Conjecture 4:

For any prime p there exist an infinity of positive integers n such that the number n^2*p + n - 1 is prime.

Examples:

: For p = 11 we have the following primes: 3^2*11 + 2 = 101; 4^2*11 + 3 = 179; 6^2*11 + 5 = 401; 10^2*11 + 9 = 1109; 13^2*11 + 12 = 1871 etc.

Conjecture 5:

For any prime p there exist an infinity of positive integers n such that the number n*p – p + n is prime.

Examples:

: For p = 5 we have the following primes: 1*5 + 2 = 7; 2*5 + 3 = 13; 3*5 + 4 = 19; 5*5 + 6 = 31; 6*5 + 7 = 37; 7*5 + 8 = 43 etc.

Conjecture 6:

For any prime p there exist an infinity of positive integers n such that the number n*p - p - n is prime.

Examples:

: For p = 5 we have the following primes: 1*5 - 2 = 3; 2*5 - 3 = 7; 5*5 - 6 = 19; 6*5 - 7 = 23; 8*5 - 9 = 31; 11*5 - 12 = 43 etc.

Conjecture 7:

For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p + n is prime.

Examples:

: For p = 7 we have the following primes: 2^2*7 + 3 = 31; 3^2*7 + 4 = 67; 5^2*7 + 4 = 179; 6^2*7 + 5 = 257; 7^2*7 + 6 = 349 etc.

Conjecture 8:

For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p - n is prime.

Examples:

: For p = 7 we have the following primes: 3^2*7 - 4 = 59; 4^2*7 - 5 = 107; 8^2*7 - 9 = 439; 9^2*7 - 10 = 557; 15^2*7 - 16 = 1559 etc.

Conjecture 9:

For any two distinct primes greater than three p and q there exist an infinity of positive integers n such that the number (p^2 – 1)*n + q^2 is prime, also an infinity of positive integers m such that the number (q^2 – 1)*n + p^2 is prime.

Examples:

: For (p, q) = (7, 11) we have the following primes of the form 48*n + 121: 313, 409, 457, 601, 937, 1033 etc. and the following primes of the form 120*n + 49: 409, 769, 1009 etc.

Note:
The idea of these sequences didn’t come to me from “nowhere”. Many from the types of primes presented in this paper are met in the study of Fermat peudoprimes.

19. Five conjectures on primes based on the observation of Poulet and Carmichael numbers

Abstract. In this paper I enunciate five conjectures on primes, based on the study of Fermat pseudoprimes and on the author's believe in the importance of multiples of 30 in the study of primes.

Conjecture 1:

For any p, q distinct primes, p > 30, there exist n positive integer such that p – 30*n and q + 30*n are both primes.

Note:

This conjecture is based on the observation of 2-Poulet numbers (see my paper "A conjecture about 2-Poulet numbers and a question about primes").

Conjecture 2:

For any p, q, r distinct primes there exist n positive integer such that the numbers 30*n – p, 30*n – q and 30*n – r are all three primes.

Note:

This enunciation is obviously equivalent to the enunciation that there exist m such that p + 30*m, q + 30*m and r + 30*m are all three primes (take x = 30*n – p, y = 30*n – q and z = 30*n – r. Then there exist k such that 30*k – 30*n + p, 30*k – 30*n + q and 30*k – 30*n + r are all three primes).

Note:

This conjecture implies of course that for any pair of twin primes (p, q) there exist a pair of primes (30*n – p, 30*n – q) so that there are infinitely many pairs of twin primes.

Note:

This conjecture is based on the observation of 3-Carmichael numbers (see my paper "A conjecture about primes based on heuristic arguments involving Carmichael numbers).

Conjecture 3:

There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q + (q - p) is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

: 31*151 + (151 – 31) = 4801 prime;
: 37*127 + (127 – 37) = 4789 prime;
: 41*101 + (101 – 41) = 4201 prime;
: 13*103 + (103 – 13) = 1429 prime;
: 17*47 + (47 – 17) = 829 prime;
: 19*109 + (109 – 19) = 2161 prime;
: 23*53 + (53 – 23) = 1249 prime.

Conjecture 4:

There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

: 31*61 – (61 – 31) = 1861 prime;
: 7*37 – (37 – 7) = 229 prime;
: 11*41 – (41 – 11) = 421 prime;
: 13*73 – (73 – 13) = 919 prime;
: 17*47 – (47 – 17) = 769 prime;
: 19*139 - (139 – 19) = 2521 prime;
: 23*293 - (293 – 23) = 6469 prime.

Conjecture 5:

For any p prime there exist an infinity of primes q, q > p, where p and q are both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

20. Six conjectures on primes based on the study of 3-Carmichael numbers and a question about primes

Abstract. In this paper are stated six conjectures on primes, more precisely on the infinity of some types of pairs of primes, all of them met in the study of 3-Carmichael numbers.

Conjecture 1:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m + 1) – n and y = q*(n + 1) – m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [5, 13] for [m, n] = [2, 4];
: for [p, q] = [7, 11] we have [x, y] = [29, 73] for [m, n] = [4, 6].

Conjecture 2:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m - 1) + n and y = q*(n - 1) + m are both primes.

Examples:
: for [p, q] = [7, 7] we have [x, y] = [11, 23] for [m, n] = [2, 4];
: for [p, q] = [5, 13] we have [x, y] = [11, 67] for [m, n] = [2, 6].

Conjecture 3:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p + (m + 1)*n and y = q + m*n are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [17, 13] for [m, n] = [2, 4];
: for [p, q] = [5, 7] we have [x, y] = [29, 23] for [m, n] = [2, 8].

Conjecture 4:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [11, 11] we have [x, y] = [23, 61] for [m, n] = [3, 5];
: for [p, q] = [11, 13] we have [x, y] = [23, 71] for [m, n] = [3, 5].

Conjecture 5:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n - 2*m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [7, 17] for [m, n] = [11, 13];
: for [p, q] = [3, 5] we have [x, y] = [13, 61] for [m, n] = [17, 19].

Conjecture 6:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m + 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [29, 41] for [m, n] = [3, 7];
: for [p, q] = [5, 11] we have [x, y] = [19, 79] for [m, n] = [1, 7].

Question:

Are there an infinity of primes with the property that can be written as p*m + n – q as well as q*n + m – p, where p, q are distinct primes and m, n are distinct positive integers? But under the condition that m, n, p, q are all four primes? Such number is, for instance, 397 = 13*31 + 7 – 13 = 61*7 + 31 – 61.

Note:

Like I already said in Abstract, I met these pairs of primes in the study of 3-Carmichael numbers: see my previous paper “Connections between the three prime factors of a 3-Carmichael number”.

21. Ten conjectures on primes based on the study of Carmichael numbers, involving the multiples of 30

Abstract. In this paper are stated ten conjectures on primes, more precisely on the infinity of some types of triplets and quadruplets of primes, all of them using the multiples of the number 30 and also all of them met on the study of Carmichael numbers.

Conjecture 1:

There exist an infinity of positive integers n such that the numbers 30*n + 7, 60*n + 13 and 150*n + 31 are all three primes.

The sequence of these numbers n is: 0, 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 31], [37, 73, 181], [67, 73, 181], [97, 193, 481]...

Conjecture 2:

There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

The sequence of these numbers n is: 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 19], [37, 73, 109], [67, 133, 199]...

Conjecture 3:

There exist an infinity of positive integers n such that the numbers 30*n - 29, 60*n - 59 and 90*n - 89 are all three primes.

The sequence of these numbers n is: 8, 10 (...), corresponding to the triplets of primes [211, 421, 691], [271, 541, 811]...

Conjecture 4:

There exist an infinity of positive integers n such that the numbers 30*n - 7, 90*n - 23 and 300*n - 79 are all three primes.

The sequence of these numbers n is: 2, 9 (...), corresponding to the triplets of primes [53, 157, 521], [263, 787, 2621]...

Conjecture 5:

There exist an infinity of positive integers n such that the numbers 30*n - 17, 90*n - 53 and 150*n - 89 are all three primes.

The sequence of these numbers n is: 1, 2 (...), corresponding to the triplets of primes [13, 37, 61], [43, 127, 211]...

Conjecture 6:

There exist an infinity of positive integers n such that the numbers 60*n + 13, 180*n + 37 and 300*n + 61 are all three primes.

The sequence of these numbers n is: 2, 6 (...), corresponding to the triplets of primes [133, 397, 661], [373, 1117, 1861]...

Conjecture 7:

There exist an infinity of positive integers n such that the numbers 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four primes.

The sequence of these numbers n is: 1 (...), corresponding to the quadruplets of primes [133, 397, 661]...

Conjecture 8:

There exist an infinity of positive integers n such that the numbers 90*n + 1, 180*n + 1, 270*n + 1 and 540*n + 1 are all four primes.

The sequence of these numbers n is: 3 (...), corresponding to the quadruplets of primes [271, 541, 811, 1621]...

Conjecture 9:

There exist an infinity of pairs of primes [p, q] such that the numbers p + 30*n, q + 30*n and p*q + 30*n are all three primes.

Examples: [p, q] = [7, 7], [7, 11], [11, 7] etc. corresponding to the triplets [37, 67, 137], [37, 71, 167], [41, 67, 167] etc.

Conjecture 10:

There exist an infinity of primes p such that the numbers x = 30*n + p and y = 30*m*n + m*p – m + 1, where m, n are non-null positive integers, are both primes.

Examples:

: for p = 7 we have [x, y] = [30*n + 7, 30*m*n + 6*m + 1]; for [m, n] = [2, 1] we have [x, y] = [37, 73];

: for p = 11 we have [x, y] = [30*n + 11, 30*m*n + 10*m + 1]; for [m, n] = [2, 1] we have [x, y] = [41, 101];

Note:

Like I already said in Abstract, I met these triplets and quadruplets of primes in the study of Carmichael numbers: see my previous paper “A list of 13 sequences of Carmichael numbers based on the multiples of the number 30”.

22. Two sequences of primes whose formulas contain the number 360

Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the number 360.

Conjecture 1:

There exist an infinity of primes of the form 360*p*q + 1, where p, q are primes, both greater than or equal to 7.

The first few such primes:
: 360*7*17 + 1 = 42841;
: 360*7*19 + 1 = 47881;
: 360*11*13 + 1 = 51481;
: 360*13*17 + 1 = 79561;
: 360*11*23 + 1 = 91081;
: 360*13*23 + 1 = 107641.

Conjecture 2:

There exist an infinity of primes of the form 360*p*q + r, where p, q, r are primes, all of them greater than or equal to 7.

The first few such primes for p = q = 7:
: 360*7*7 + 17 = 17657;
: 360*7*7 + 19 = 17659;
: 360*7*7 + 29 = 17669;
: 360*7*7 + 41 = 17681;
: 360*7*7 + 41 = 17683.

The first few such primes for p = 7, q = 11:
: 360*7*11 + 13 = 27733;
: 360*7*11 + 17 = 27737;
: 360*7*11 + 19 = 27739;
: 360*7*11 + 23 = 27743;
: 360*7*11 + 29 = 27749;
: 360*7*11 + 31 = 27751.

Note the six consecutive primes obtained above!

23. Two sequences of primes whose formulas contain the powers of the number 2

Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the powers of the number 2.

Conjecture 1:

There exist an infinity of primes of the form 2^m + n^2, where m is non-null positive integer and n odd integer.

The first few such primes for [m, n] = [2, n]:
: 3^2 + 4 = 13 for n = 3;
: 5^2 + 4 = 29 for n = 5;
: 7^2 + 4 = 53 for n = 7;
: 13^2 + 4 = 173 for n = 13;
: 17^2 + 4 = 293 for n = 17.

The first few such primes for [m, n] = [4, n]:
: 5^2 + 16 = 41 for n = 5;
: 11^2 + 16 = 137 for n = 11;
: 29^2 + 16 = 857 for n = 29;
: 31^2 + 16 = 977 for n = 31;
: 41^2 + 16 = 1697 for n = 41.

The first few such primes for [m, n] = [8, n]:
: 5^2 + 256 = 281 for n = 5;
: 19^2 + 256 = 617 for n = 19;
: 29^2 + 256 = 1097 for n = 29;
: 31^2 + 256 = 1217 for n = 31;
: 71^2 + 256 = 5297 for n = 71.

The first few such primes for [m, n] = [m, 1]:
: 2^1 + 1 = 3 for m = 1;
: 2^2 + 1 = 5 for m = 2;
: 2^4 + 1 = 17 for m = 4;
: 2^8 + 1 = 257 for m = 8;
: 2^16 + 1 = 65537 for m = 16.

The first few such primes for [m, n] = [m, 3]:
: 2^1 + 9 = 11 for m = 1;
: 2^2 + 9 = 13 for m = 2;
: 2^3 + 9 = 17 for m = 3;
: 2^5 + 9 = 41 for m = 5;
: 2^6 + 9 = 73 for m = 6.

Conjecture 2:

There exist an infinity of primes of the form (2^n)^k + 2^n + 1, where n is non-null positive integer and k positive integer.

The first few such primes for [n, k] = [n, 1]:
: 5 for n = 1;
: 17 for n = 3;
: 257 for n = 7.

The first few such primes for [n, k] = [n, 2]:
: 7 for n = 1;
: 73 for n = 3;
: 262657 for n = 9.

The first few such primes for [n, k] = [n, 3]:
: 11 for n = 1;
: 521 for n = 3;
: 32801 for n = 5.

The first few such primes for [n, k] = [1, k]:
: 5 for k = 1;
: 7 for k = 2;
: 11 for k = 3.

The first few such primes for [n, k] = [3, k]:
: 17 for k = 1;
: 73 for k = 2;
: 521 for k = 3.

The first few such primes for [n, k] = [5, k]:
: 32801 for k = 3;
: 1048609 for k = 4;
: 1073741857 for k = 6.

24. Conjectures about a way to express a prime as a sum of three other primes of a certain type

Abstract. These conjectures state that any prime p greater than 60 can be written as a sum of three primes of a certain type from the following four ones: 10k + 1, 10k + 3, 10k + 7 and 10k + 9.

Conjecture 1a:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Examples:
: 61 = 11 + 31 + 19;
: 71 = 11 + 31 + 29 = 11 + 41 + 19.

Conjecture 1b:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 7.

Examples:
: 61 = 41 + 13 + 7 = 31 + 23 + 7 = 31 + 13 + 17;
: 71 = 41 + 23 + 7 = 41 + 13 + 17 = 31 + 23 + 7.

Conjecture 1c:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 7, 10*y + 7 respectively 10*z + 7.

Examples:
: 61 = 7 + 17 + 37 = 7 + 7 + 47;
: 71 = 17 + 17 + 37 = 7 + 17 + 47.

Conjecture 1d:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 9 respectively 10*z + 9.

Examples:
: 61 = 13 + 19 + 29 = 23 + 19 + 19;
: 71 = 23 + 19 + 29 = 13 + 29 + 29.

Conjecture 2a:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 2b:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 9.

Conjecture 2c:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 3 respectively 10*z + 7.

Conjecture 2d:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 7, 10*y + 7 respectively 10*z + 9.

Conjecture 3a:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 3.

Conjecture 3b:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 7 respectively 10*z + 7.

Conjecture 3c:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 7 respectively 10*z + 9.

Conjecture 3d:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 9, 10*y + 9 respectively 10*z + 9.

Conjecture 4a:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 7.

Conjecture 4b:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 3 respectively 10*z + 3.

Conjecture 4c:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 7 respectively 10*z + 9.

Conjecture 4d:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 9 respectively 10*z + 9.

Addenda

In one of my previous papers, "Two conjectures that relates any Poulet number by a type of triplets respectively of duplets of primes" I made the following two conjectures:

Conjecture:

Any square of a prime of the form p^2 = 10*k + 1 can be written as p^2 = x + y + z, where x, y, z are primes, not necessarily all three distinct, of the form 10*k + 7.
Examples:
: 11^2 = 121 = 37 + 37 + 47;
: 19^2 = 361 = 7 + 37 + 317.

Conjecture:

Any square of a prime of the form p^2 = 10*k + 9 can be written as p^2 = x + y + z, where x, y, z are primes, not necessarily all three distinct, of the form 10*k + 3.
Examples:
: 7^2 = 49 = 13 + 13 + 23;
: 19^2 = 169 = 13 + 43 + 113.

25. A bold conjecture about a way in which any prime can be written

Abstract. In this paper I make a conjecture which states that any prime greater than or equal to 5 can be written in a certain way, in other words that any such prime can be expressed using just two other primes and a power of the number 2.

Conjecture:

Any prime greater than or equal to 5 can be written at least in one way as (9*p^2 – q^2)/(2^n), where p and q are primes and n non-null positive integer.

Verifying the conjecture:
(For the first nine such primes)

: 5 = (9*7^2 – 11^2)/64, so [p, q, n] = [7, 11, 6] but also 5 = (9*7^2 – 19^2)/16 so [p, q, n] = [7, 19, 4];
: 7 = (9*5^2 – 13^2)/64, so [p, q, n] = [5, 13, 3];
: 11 = (9*5^2 – 7^2)/16, so [p, q, n] = [5, 7, 4];
: 13 = (9*5^2 – 11^2)/8, so [p, q, n] = [5, 11, 3] but also 13 = (9*7^2 – 5^2)/32 so [p, q, n] = [7, 5, 5];
: 17 = (9*7^2 – 13^2)/16, so [p, q, n] = [7, 13, 4];
: 19 = (9*7^2 – 17^2)/8, so [p, q, n] = [7, 17, 3] but also 19 = (9*13^2 – 37^2)/8 so [p, q, n] = [13, 37, 3] but also 19 = (9*17^2 – 13^2)/128 so [p, q, n] = [17, 11, 7];
: 23 = (9*13^2 – 7^2)/64, so [p, q, n] = [13, 7, 6];
: 31 = (9*11^2 – 29^2)/8, so [p, q, n] = [11, 29, 3] but also 31 = (9*13^2 – 23^2)/32 so [p, q, n] = [13, 23, 5];
: 37 = (9*23^2 – 5^2)/128, so [p, q, n] = [23, 5, 7].

Note:

For the prime 29 I didn't find primes solution [p, q] up to the denominator 2^12, but surely I conjecture that there exist such solutions.

Note:

For some of the primes we found that they verify also the formula (9*p^2 – q^4)/(2^n).

26. Two conjectures, on the primes of the form 6k + 1 respectively of the form 6k – 1

Abstract. In this paper I make two conjectures, one about how could be expressed a prime of the form 6k + 1 and one about how could be expressed a prime of the form 6k – 1.

Conjecture 1:

Any prime p of the form 6*k + 1 greater than or equal to 13 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 229)

: 13*3 – 20 = 19, prime, so [p, q, r] = [13, 5, 19];
: 19*3 – 20 = 37, prime, so [p, q, r] = [19, 5, 37];
: 31*3 – 20 = 73, prime, so [p, q, r] = [31, 5, 73];
: 37*3 – 110 = 1, so [p, q, r] = [37, 11, 1];
: 43*3 – 20 = 109, prime, so [p, q, r] = [43, 5, 109] and also 43*3 – 110 = 29, prime, so [p, q, r] = [43, 11, 29];
: 61*3 – 20 = 163, prime, so [p, q, r] = [61, 5, 163] and also 61*3 – 110 = 73, prime, so [p, q, r] = [61, 11, 73];

We also found the following triplets [p, q, r]: [67, 5, 181], [73, 5, 199], [73, 11, 109], [79, 11, 127], [97, 5, 271], [97, 11, 181], [97, 17, 19], [103, 11, 199], [109, 5, 307], [127, 11, 271], [127, 17, 109], [139, 5, 397], [139, 11, 307], [151, 5, 433], [151, 17, 71], [157, 17, 199], [163, 1, 379], [181, 5, 523], [181, 11, 433], [181, 17, 271], [181, 23, 37], [193, 17, 307], [193, 23, 73], [199,5, 577], [199, 11, 487], [211, 5, 613], [211, 11, 523], [211, 17, 19^2], [211, 23, 127], [223, 17, 397], [223, 506, 163], [229, 11, 577], [229, 23, 181], so the conjecture is verified up to p = 229.

Conjecture 2:

Any prime p of the form 6*k – 1 greater than or equal to 11 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 179)

: 11*3 – 20 = 13, prime, so [p, q, r] = [11, 5, 13];
: 17*3 – 20 = 31, prime, so [p, q, r] = [17, 5, 31];
We also found the following triplets [p, q, r]: [29, 5, 67], [41, 5, 103}, [47, 11, 31], [53, 5, 129], [59, 5, 157], [59, 11, 67], [71, 5, 193], [71, 11, 103], [83, 5, 229], [83, 11, 139], [891, 11, 157], [101, 5, 283], [101, 11, 193], [101, 17, 31], [107, 11, 211], [113, 11, 229], [113, 17, 67], [131, 5, 373], [131, 11, 283], [137, 17, 139], [149, 11, 337], [167, 17, 229], [173, 5, 449], [173, 11, 409], [173, 23, 13], [179, 23, 31] so the conjecture is verified up to p = 179.

Comment:

In the case that the conjectures are invalidated, still remain two open problems:
(1) Which are the smallest primes that don't satisfy each from the two conjectures?;
(2) Which is the maximum length of a chain formed in the following way: p_2 = 3*p_1 – (q^2 – q), p_3 = 3*p_2 – (q^2 – q), ..., p_n = 3*p_{n-1} – (q^2 – q)? For instance, such a chain of length 3 is [43, 109, 307] for q = 5.

27. A possible way to write any prime, using just another prime and the powers of the numbers 2, 3 and 5

Abstract. In this paper I make a conjecture which states that any odd prime can be written in a certain way, in other words that any such prime can be expressed using just another prime and the powers of the numbers 2, 3 and 5. I also make a related conjecture about twin primes.

Conjecture:

Any odd prime p can be written at least in one way as p = (q*2^a*3^b*5^c ± 1)*2^n ± 1, where q is an odd prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Verifying the conjecture:
(For the first five odd primes)

: 3 = (1*2^1*3^0*5^0 – 1)*2^1 + 1, but also 3 = (1*2^0*3^1*5^0 – 1)*2^1 - 1;

: 5 = (1*2^1*3^0*5^0 + 1)*2^1 - 1, but also 5 = (1*2^0*3^1*5^0 – 1)*2^1 + 1, also 5 = (1*2^2*3^0*5^0 - 1)*2^1 - 1;

: 7 = (1*2^0*3^1*5^0 + 1)*2^1 - 1, but also 7 = (1*2^1*3^0*5^0 + 1)*2^1 + 1, also 7 = (3*2^0*3^0*5^0 + 1)*2^1 – 1, also 7 = (5*2^0*3^0*5^0 - 1)*2^1 – 1, also 7 = (1*2^2*3^0*5^0 - 1)*2^1 + 1;

: 11 = (1*2^1*3^1*5^0 - 1)*2^1 + 1, but also 11 = (1*2^0*3^0*5^1 + 1)*2^1 - 1, also 11 = (3*2^1*3^0*5^0 - 1)*2^1 + 1, also 11 = (5*2^0*3^0*5^0 + 1)*2^1 – 1, also 11 = (7*2^0*3^0*5^0 - 1)*2^1 – 1, also 11 = (1*2^2*3^0*5^0 + 1)*2^1 + 1;

: 13 = (1*2^1*3^1*5^0 + 1)*2^1 + 1, but also 13 = (1*2^0*3^0*5^1 + 1)*2^1 + 1, also 13 = (3*2^1*3^0*5^0 + 1)*2^1 + 1, also 13 = (5*2^0*3^0*5^0 + 1)*2^1 + 1, also 13 = (7*2^0*3^0*5^0 - 1)*2^1 + 1.

Conjecture:

Any pair of twin primes [p_1, p_2] can be written as [p_1 = (q*2^a*3^b*5^c ± 1)*2^n - 1, p_2 = (q*2^a*3^b*5^c ± 1)*2^n + 1], where q is prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Verifying the conjecture:
(For the first three pairs of twin primes)

: 3 = (1*2^0*3^1*5^0 – 1)*2^1 - 1 and
5 = (1*2^0*3^1*5^0 – 1)*2^1 + 1;

: 5 = (1*2^1*3^0*5^0 + 1)*2^1 - 1 and
7 = (1*2^1*3^0*5^0 + 1)*2^1 + 1, also

5 = (1*2^2*3^0*5^0 - 1)*2^1 – 1 and
7 = (1*2^2*3^0*5^0 - 1)*2^1 + 1;

: 11 = (1*2^0*3^0*5^1 + 1)*2^1 – 1 and
13 = (1*2^1*3^1*5^0 + 1)*2^1 + 1, also

11 = (5*2^0*3^0*5^0 + 1)*2^1 – 1 and
13 = (5*2^0*3^0*5^0 + 1)*2^1 + 1.

28. Two conjectures about the pairs of primes separated by a certain distance

Abstract. In this paper I make two conjectures abut the pairs of primes [p1, q1], where the difference between p1 and q1 is a certain even number d. I state that any such pair has at least one other corresponding, in a specified manner, pair of primes [p2, q2], such that the difference between p2 and q2 is also equal to d.

Conjecture 1:

For any pair of primes, greater than 3, $[p_1, q_1]$, where $q_1 – p_1 = d$, there exist at least a pair of positive integers [m, n], where n – m = d, such that the numbers $p_2 = p_1*q_1 – n + 1$ and $q_2 = p_1*q_1 – m + 1$ are both primes.

Examples:

: For $[p_1, q_1] = [5, 7]$ there exist the pair [m, n] = [5, 7] such that $p_2 = 5*7 – 7 + 1 = 29$ and $q_2 = 5*7 – 5 + 1 = 31$ are both primes;

: For $[p_1, q_1] = [5, 11]$ there exist the pair [m, n] = [3, 9] such that $p_2 = 5*11 – 9 + 1 = 47$ and $q_2 = 5*11 – 3 + 1 = 53$ are both primes;

: For $[p_1, q_1] = [5, 13]$ there exist the pair [m, n] = [5, 13] such that $p_2 = 5*13 – 13 + 1 = 53$ and $q_2 = 5*13 – 5 + 1 = 61$ are both primes;

: For $[p_1, q_1] = [7, 11]$ there exist the pair [m, n] = [7, 11] such that $p_2 = 7*11 – 11 + 1 = 67$ and $q_2 = 7*11 – 7 + 1 = 71$ are both primes;

: For $[p_1, q_1] = [7, 13]$ there exist the pair [m, n] = [7, 11] such that $p_2 = 7*11 – 11 + 1 = 67$ and $q_2 = 7*11 – 7 + 1 = 71$ are both primes;

: For $[p_1, q_1] = [11, 13]$ there exist the pair [m, n] = [5, 7] such that $p_2 = 11*13 – 5 + 1 = 137$ and $q_2 = 11*13 – 7 + 1 = 139$ are both primes.

Conjecture 2:

For any even number d there exist an infinity of pairs of primes $[p_1, q_1]$, where $q_1 – p_1 = d$, such that the numbers $p_2 = p_1*q_1 – p_1 + 1$ and $q_2 = p_1*q_1 – q_1 + 1$ are both primes.

Note: See, for instance, from the examples to the Conjecture 1 from above, the pair [5, 7] for d = 2, the pair [7, 11] for d = 4, the pair [5, 13] for d = 8.

29. Five conjectures on a diophantine equation involving two primes and a square of prime

Abstract. In this paper I make five conjectures about the primes q, r and the square of prime p^2, which appears as solutions in the diophantine equation 120*n*q*r + 1 = p^2, where n is non-null positive integer.

Conjecture 1:

For any n non-null positive integer there exist q, r primes such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 2:

For any q odd prime there exist n non-null positive integer and r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 3:

For any q, r odd primes there exist n non-null positive integer such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 4:

For any n non-null positive integer and any q prime there exist r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Examples:

: For [n, q] = [1, 5] there exist r = 17 such that p = 101 prime; also r = 37 such that p = 149 prime;
: For [n, q] = [1, 7] there exist r = 23 such that p = 139 prime; also r = 53 such that p = 211 prime;
: For [n, q] = [1, 11] there exist r = 13 such that p = 131 prime; also r = 83 such that p = 331 prime;

: For [n, q] = [2, 5] there exist r = 19 such that p = 151 prime;
: For [n, q] = [2, 7] there exist r = 3 such that p = 71 prime; also r = 17 such that p = 169 square of prime;
: For [n, q] = [2, 11] there exist r = 3 such that p = 89 prime;

: For [n, q] = [3, 7] there exist r = 13 such that p = 181 prime;
: For [n, q] = [3, 11] there exist r = 3 such that p = 109 prime;

: For [n, q] = [4, 5] there exist r = 67 such that p = 401 prime;
: For [n, q] = [4, 7] there exist r = 17 such that p = 239 prime;
: For [n, q] = [4, 11] there exist r = 11 such that p = 241 prime.

Conjecture 5:

For any n non-null positive integer there exist q prime such that 120*n*q^2 + 1 = p^2, where p is prime or a power of prime.
Note, for instance, the case from the examples below: 480*11^2 + 1 = 241^2.

30. An amazing formula for producing big primes based on the numbers 25 and 906304

Abstract. In this paper I present a formula for generating big primes and products of very few primes, based on the numbers 25 and 906304, formula equally extremely interesting and extremely simple, id est 25^n + 906304. This formula produces for n from 1 to 30 (and for n = 30 is obtained a number p with not less than 42 digits) only primes or products of maximum four prime factors.

Observation:

The number p = 25^n + 906304 is often a prime or a product of very few primes.

Note:

I came to this formula more or less by chance, but the number 906304 has at least one other special property: 906304 = 952^2 = 1105^2 – 561^2, where 561 and 1105 are the first and the second Carmichael numbers.

Examples:

: p = 25^1 + 906304 = 906329 prime;
: p = 25^2 + 906304 = 906929 prime;
: p = 25^3 + 906304 = 921929 = 37*24917;
: p = 25^4 + 906304 = 1296929 prime;
: p = 25^5 + 906304 = 10671929 = 421*25349;
: p = 25^6 + 906304 = 245046929 = 97*2526257;
: p = 25^7 + 906304 = 245046929 = 113*2957*18269;
: p = 25^8 + 906304 = 152588796929 = 36269*4207141;
: p = 25^9 + 906304 = 3814698171929 prime;
: p = 25^10 + 906304 = 95367432546929 = 41*2326034940169;
: p = 25^11 + 906304 = 2384185791921929 = 5573*427810118773;
: p = 25^12 + 906304 = 59604644776296929 = 61*139361*7011468949;
: p = 25^13 + 906304 = 1490116119385671929 = 1097*84389*16096358813;:
: p = 25^14 + 906304 = 37252902984620046929 prime;
: p = 25^15 + 906304 = 931322574615479421929 = 671477*1386976135616677;
: p = 25^16 + 906304 = 23283064365386963796929
= 1609*1830341*7905914013541;
: p = 25^17 + 906304 = 582076609134674073171929 prime;
: p = 25^18 + 906304 = 14551915228366851807546929
= 53^2*5180461099454201426681;
: p = 25^19 + 906304 = 363797880709171295166921929 prime;
: p = 25^20 + 906304 = 9094947017729282379151296929
= 41*237776289649*932927233281481;

Notes:

For n from 1 to 20, were obtained for p seven values which are primes, seven values which are semiprimes and six values which are products of three prime factors! Note also that the larger prime obtained in the examples above, p = 25^19 + 906304 = 363797880709171295166921929, has 27 digits!

For n from 21 to 30 were also obtained products of maximum four primes; these are the following values of p:

: n = 21, p = 2273736754432320594787606719 29;
: n = 22, p = 56843418860808014869689950 46929;
: n = 23, p = 1421085471520200371742248544 21929;
: n = 24, p = 35527136788005009293556213387 96929;
: n = 25, p = 888178419700125232338905334481 71929;
: n = 26, p = 2220446049250313080847263336182 546929;
: n = 27, p = 5551115123125782702118158340454 1921929;
: n = 28, p = 13877787807814456755295395851135 26296929;
: n = 29, p = 34694469519536141888238489627838 135671929;
: n = 30, p = 86736173798840354720596224069595 3370046929.

For n from 31 to 37 were also obtained products of maximum five primes; these are the following values of p:

: n = 31, p = 216840434497100886801490560173988342294 21929
: n = 32, p = 542101086242752217003726400434970855713 796929
: n = 33, p = 1355252715606880542509316001087427139282 3171929
: n = 34, p = 3388131789017201356273290002718567848205 57546929
: n = 35, p = 8470329472543003390683225006796419620513 916921929
: n = 36, p = 2117582368135750847670806251699104905128 47901296929
: n = 37, p = 5293955920339377119177015629247762262821 197510671929

Note that the number 25^34 + 906304 is a prime with 48 digits!

Conjecture:

There exist an infinity of primes p of the form p = 25^n + 906304.

31. Four unusual conjectures on primes involving Egyptian fractions

Abstract. In this paper I make four conjectures on primes, conjectures which involve the sums of distinct unit fractions such as 1/p(1) + 1/p(2) + (...), where p(1), p(2), (...) are distinct primes, more specifically the periods of the rational numbers which are the results of the sums mentioned above.

Conjecture 1:

There exist an infinity of infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Examples:

: the period of a(1) = 1/3 + 1/7 is equal to 6;
: the period of a(2) = 1/3 + 1/7 + 1/19 is equal to 18;
: the period of a(3) = 1/3 + 1/7 + 1/19 + 73 is equal to 72.
(...)
The sequence of p(1), p(2), p(3)... is 3, 7, 19, 72...
: the period of a(1) = 1/5 + 1/29 is equal to 28;
: the period of a(2) = 1/5 + 1/29 + 1/113 is equal to 112;
: the period of a(3) = 1/5 + 1/29 + 1/113 + 1/337 is equal to 336.
(...)
The sequence of p(1), p(2), p(3)... is 5, 29, 113, 337...

Conjecture 2:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 3:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is a multiple of p(2) - 1, the period of the rational number a(2) is a multiple of p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Example:

: the period of a(1) = 1/7 + 1/17 is equal to 48 which is a multiple of 16;
: the period of a(2) = 1/7 + 1/17 + 1/19 is equal to 144 which is a multiple of 18;
: the period of a(3) = 1/7 + 1/17 + 1/19 + 1/23 is equal to 1584 a multiple of 22.
(...)
The sequence of p(1), p(2), p(3)... is 7, 17, 19, 23...

Conjecture 4:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where $p(1) < p(2) < p(3)$..., such that the period of the rational number a(1) divides p(2) - 1, the period of the rational number a(2) divides p(3) – 1, the period of the rational number a(n) divides a(n) – 1.

Example:

: the period of a(1) = 1/3 + 1/11 is equal to 2 which divides 10;
: the period of a(2) = 1/3 + 1/11 + 1/13 is equal to 6 which divides 12;
: the period of a(3) = 1/3 + 1/11 + 1/13 + 1/37 is equal to 6 which divides 36.
(...)

The sequence of p(1), p(2), p(3)... is 3, 11, 13, 37...

Conjecture 5:

For any Poulet number P there exist a rational number r equal to a sum of unit fractions 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1), p(2), p(3)... are distinct odd primes, such that the period of r is equal to P – 1.

Example: the period of r = 1/5 + 1/29 + 1/113 + 1/271 is equal to 560, while 561 the second Poulet number and the first Carmichael number.

32. Three formulas that generate easily certain types of triplets of primes

Abstract. In this paper I present three formulas, each of them with the following property: starting from a given prime p, are obtained in many cases two other primes, q and r. I met the triplets of primes [p, q, r] obtained with these formulas in the study of Carmichael numbers; the three primes mentioned are often the three prime factors of a 3-Carmichael number.

Note:

To refer to the three formulas easily I will name them the formula alpha, beta or gama and the triplets obtained the triplet alpha, beta or gama.

Formula alpha:

The formula alpha is 30*a*n – (a*p + a – 1). The first prime of a triplet alpha is p and the other two ones are obtained giving to n values of integers, under the condition that a*p + a – 1 is prime.

Examples:

: For p = 11 and a = 2 the condition that a*p + a – 1 is prime is met because 2*11 + 2 – 1 = 23 which is prime; the formula alpha becomes 60*n – 23; it can be seen that for n = 1 is obtained 47 (prime) and for n = 2 is obtained 97 (prime) so we have the triplet alpha [11, 47, 97]; also for n = 3 is obtained 157 (prime) so other two triplets alpha are [11, 47, 157] and [11, 97, 157];

: For p = 7 and a = 3 the condition that a*p + a – 1 is prime is met because 3*7 + 3 – 1 = 23 which is prime; the formula alpha becomes 90*n – 23; it can be seen that for n = 1 is obtained 67 (prime) and for n = 2 is obtained 157 (prime) so we have the triplet alpha [7, 67, 157]; also for n = 4 is obtained 337 (prime) so other two triplets alpha are [7, 67, 337] and [7, 157, 337].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula alpha.

Formula beta:

The formula beta is 30*a*n + (a*p + a – 1). The first prime of a triplet beta is p and the other two ones are obtained giving to n values of integers, under the condition that a*p + a – 1 is prime.

Examples:

: For p = 11 and a = 2 the condition that a*p + a – 1 is prime is met because 2*11 + 2 – 1 = 23 which is prime; the formula beta becomes 60*n + 23; it can be seen that for n = 1 is obtained 83 (prime) and for n = 4 is obtained 263 (prime) so we have the triplet beta [11, 83, 263]; also for n = 6 is obtained 383 (prime) so other two triplets beta are [11, 83, 383] and [11, 263, 383];

: For p = 19 and a = 3 the condition that a*p + a – 1 is prime is met because 3*19 + 3 – 1 = 59 which is prime; the formula beta becomes 90*n + 59; it can be seen that for n = 1 is obtained 149 (prime) and for n = 2 is obtained 239 (prime) so we have the triplet beta [59, 149, 239]; also for n = 4 is obtained 419 (prime) so other two triplets beta are [59, 149, 419] and [59, 239, 419].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula beta.

Formula gama:

The formula gama is 2*p*n – 2*n + p. The first prime of a triplet gama is p and the other two ones are obtained giving to n values of integers, under the condition that 2*p – 1 is prime.

Example:

: For p = 7 the condition that 2*p – 1 is prime is met; the formula gama becomes 12*n + 7; for n = 1 is obtained 19 (prime) and for n = 2 is obtained 31 so we have the triplet gama [7, 19, 31]; also for n = 3 is obtained 43 so other two triplets gama are [7, 19, 43] and [7, 31, 43].

Note: see the sequence A182207 in OEIS for the connection between Carmichael numbers and formula gama.

33. A new bold conjecture about a way in which any prime can be written

Abstract. In this paper I make a conjecture which states that any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Conjecture:

Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Verifying the conjecture:
(For the first few primes greater than or equal to 53)

(Note that we will not show all ways in which a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 53 = 11 + 11 + 31;
: 59 = 13 + 23 + 23;
: 61 = 7 + 17 + 37;
: 67 = 19 + 19 + 29;
: 71 = 17 + 17 + 37;
: 73 = 11 + 31 + 31;
: 79 = 13 + 23 + 43;
: 83 = 11 + 31 + 41;
: 89 = 23 + 23 + 23;
: 97 = 19 + 19 + 59;
: 101 = 17 + 17 + 67;
: 103 = 11 + 31 + 61;
: 107 = 19 + 29 + 59;
: 109 = 13 + 13 + 83;
: 113 = 11 + 31 + 71;
: 127 = 19 + 29 + 79;
: 131 = 7 + 17 + 107;
: 137 = 19 + 29 + 89;
: 139 = 13 + 13 + 113;
: 149 = 13 + 23 + 113;
: 151 = 7 + 7 + 137.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 1.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 3.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 7.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 9.

34. A bold conjecture about a way in which any square of prime can be written

Abstract. In this paper I make a conjecture which states that any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Conjecture:

Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Verifying the conjecture:
(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 7^2 = 49 = 13 + 13 + 23;
: 11^2 = 121 = 37 + 37 + 47;
: 13^2 = 169 = 13 + 43 + 113;
: 17^2 = 289 = 13 + 13 + 263;
: 19^2 = 361 = 7 + 17 + 337;
: 23^ = 529 = 13 + 53 + 563.

Conjecture:

Any square of a prime p^2, where p is greater than or equal to 7, can be written as p^2 = 2*m + n, where m and n are distinct primes, both of the form 10k + 3 or both of the form 10k + 7.

Verifying the conjecture:
(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 7^2 = 49 = 2*13 + 23;
: 11^2 = 121 = 2*37 + 47;
: 13^2 = 169 = 2*43 + 83;
: 17^2 = 289 = 2*13 + 263;
: 19^2 = 361 = 2*7 + 347;
: 23^2 = 529 = 2*13 + 503.

35. Statements on the infinity of few sequences or types of duplets or triplets of primes

Abstract. In this paper I make few statements on the infinity of few sequences or types of duplets and triplets of primes which, though could appear heterogenous, are all based on the observation of the prime factors of absolute Fermat pseudoprimes, Carmichael numbers, or of relative Fermat pseudoprimes to base two, Poulet numbers.

Note:

See, in my book "Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes", Part Four, "One hundred and fifty open problems regarding Fermat pseudoprimes".

Conjecture 1:

There exist an infinity of positive integers k such that 6*k – 1 and 18*k – 5 are both primes.

Conjecture 2:

There exist an infinity of positive integers k such that 6*k + 1 and 12*k + 1 are both primes.

Conjecture 3:

There exist an infinity of positive integers k such that 6*k + 1 and 18*k + 1 are both primes.

Conjecture 4:

There exist an infinity of positive integers k such that 6*k - 5 and 24*k - 5 are both primes.

Conjecture 5:

There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 1 are all three primes.

Conjecture 6:

There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 13 are all three primes.

Conjecture 7:

There exist an infinity of positive integers k such that k, 2*k - 1 and 5*k - 4 are all three primes.

Conjecture 8:

There exist an infinity of positive integers k such that k, 2*k - 1 and 3*k - 2 are all three primes.

Conjecture 9:

There exist an infinity of positive integers k such that k, 3*k - 2 and 4*k - 3 are all three primes.

Conjecture 10:

There exist an infinity of positive integers k such that 40*k + 1, 60*k + 1 and 100*k + 1 are all three primes.

Conjecture 11:

There exist an infinity of positive integers k such that k, 2*k – 1, 7*k - 6 and 14*k - 13 are all four primes.

Conjecture 12:

There exist an infinity of positive integers k such that k, 2*k – 1, 6*k - 5 and 12*k - 11 are all four primes.

Conjecture 13:

There exist an infinity of pairs of distinct non-null positive integers m, n such that 60*m*n – 29 and 60*m*n – (60*m + 29) are both primes.

Conjecture 14:

There exist an infinity of pairs of distinct non-null positive integers [m, n] such that 40*m – 10*n - 29 and 40*m – 10*n – 129 are both primes.

Conjecture 15:

For any pair of twin primes [q, r] there exist an infinity of primes p of the form p = 7200*q*r*n + 1, where n is positive integer.

Examples:

: for [q, r] = [5, 7], p is prime for n = 1, 2, 4, 7 (...)

: for [q, r] = [11, 13], p is prime for n = 1, 3, 8 (...)

Conjecture 16:

There exist an infinity of primes p of the form p = n*s(p) – n + 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 17:

There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 18:

There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 19:

There exist an infinity of primes p of the form p = m*n + m - n, where m and n are distinct odd primes.

Conjecture 20:

There exist an infinity of primes p of the form p = m^2 – m*n + n, where m and n are distinct odd primes.

Conjecture 21:

There exist an infinity of primes p of the form p = (q + 5^k)/ 10, where q is prime and k positive integer.

Conjecture 22:

There exist an infinity of primes p of the form p = (q + 5^k)/ 30, where q is prime and k positive integer.

Conjecture 23:

There exist an infinity of primes p of the form p = q^3 + 60, where q is prime.

Conjecture 24:

There exist an infinity of primes p, for n positive integer, of the following forms:

: 20*n^2 + 12*n + 1;
: 1800*n^2 + 840*n + 1;
: 3*n^2 + 6*n + 4;
: 4*n^2 + 172*n + 529;
: 20*n^2 + 364*n + 177;
: n^2 + 81*n + 39;
: n^2 + 10*n + 10.

36. An interesting relation between the squares of primes and the number 96 and two conjectures

Abstract. In this paper I make two conjectures based on the observation of an interesting relation between the squares of primes and the number 96.

Conjecture 1:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of numbers which are primes or squares of primes.

Example:

: for p = 5 are obtained the primes q = 313, 409, 601 (...) for k = 3, 4, 6 (...) and the squares of primes q = 11^2, 37^2 (...) for k = 1, 14 (...).

Conjecture 2:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of semiprimes q = m*n, where m < n, with the following property: the number n – m + 1 is a prime or a square of a prime.

Example:

: for p = 5 are obtained the semiprimes q = 217 = 7*31 (and 31 – 7 + 1 = 5^2) for k = 2, q = 505 = 5*101 (and 101 – 5 + 1 = 97, prime) for k = 5, q = 697 = 17*41 (and 41 – 17 + 1 = 5^2) for k = 7, q = 793 = 13*61 (and 61 – 13 + 1 = 7^2) for k = 8, q = 889 = 7*127 (and 127 – 7 + 1 = 11^2) for k = 9, q = 985 = 5*197 (and 197 – 5 + 1 = 193, prime) for k = 10, q = 1081 = 23*47 (and 47 – 23 + 1 = 5^2) for k = 11, q = 1177 = 11*107 (and 107 – 11 + 1 = 97, prime) for k = 12, q = 1273 = 19*67 (and 67 – 19 + 1 = 7^2) for k = 13, q = 1465 = 5*293 (and 293 – 5 + 1 = 17^2) for k = 15.

Note that, for p = 5, were obtained for $1 \le k \le 15$ only primes, squares of primes and semiprimes with the property mention above.

Taking randomly a prime, id est 233, is obtained:
: for k = 1, the semiprime q = 329 = 7*47 (47 – 7 + 1 = 41);
: for k = 3, the prime q = 521;
: for k = 4, the prime q = 617;
: for k = 5, the semiprime q = 713 = 23*31 (31 – 23 + 1 = 3^2);
: for k = 6, the prime q = 809.

Taking randomly another prime, id est 769, is obtained:
: for k = 1, the semiprime q = 865 = 5*173 (173 – 5 + 1 = 13^2);
: for k = 2, the square of prime q = 31^2;
: for k = 4, the prime q = 1153;

: for k = 5, the prime q = 1249;
: for k = 7, the semiprime q = 1441 = 11*131 (131 – 11 + 1 = 11^2).

Conclusion:

It is clear from these examples that the formula p^2 + 96*k, where p is prime and k is positive integer, has the property to generate primes, squares of primes and semiprimes with the property shown.

37. A formula that seems to generate easily big numbers that are primes or products of very few primes

Abstract. The formula N = (p^4 – 2*p^2 + m)/(m – 1), where p is an odd prime and m is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

Observation:

The formula N = (p^4 – 2*p^2 + m)/(m – 1), where p is an odd prime and m is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

Examples:

For m = 2 the formula becomes N = p^4 – 2*p^2 + 2 and were obtained the following results for the sequence of the first five consecutive primes of the form 10*k + 1:

: for p = 11, N = 14401 prime;
: for p = 31, N = 921601 prime;
: for p = 41, N = 2822401 = 113*24977;
: for p = 61, N = 13838401 = 3313*4177;
: for p = 71, N = 25401601 = 101*251501.

For a larger prime of the same form, p = 961752931, is obtained N = 855855567096510789934200845104477377601, a semiprime with 39 digits.

For m = 3 the formula becomes N = (p^4 – 2*p^2 + 3)/2 and were obtained the following results for the sequence of the first five consecutive primes of the form 20*k + 9:

: for p = 29, N = 352801 = 17*20753;
: for p = 89, N = 31363201 prime;
: for p = 109, N = 70567201 = 2659*26539;
: for p = 149, N = 246420001 prime;
: for p = 229, N = 1374976801 = 11*124997891.

For m = 4 the formula becomes N = (p^4 – 2*p^2 + 4)/3 and were obtained the following results for the sequence of the first eight consecutive primes of the form 30*k + 13:

: for p = 13, N = 9409 = 97^2;
: for p = 43, N = 1138369 prime;
: for p = 73, N = 9462529 = 1609*5881;
: for p = 103, N = 37509889 = 43*872323;
: for p = 163, N = 235286209 prime;
: for p = 193, N = 462471169 prime;
: for p = 223, N = 824291329 prime;
: for p = 283, N = 2138029249 prime.

For two larger primes of the same form is obtained:

: for p = 1299763, N = 951339271160353903881409 prime;
: for p = 1299853, N = 951602794365121103901889 prime.

Taking randomly a prime, id est p = 29, are obtained the following results:

: for m = 2, N = 705601, a semiprime;
: for m = 3, N = 352801, a semiprime;
: for m = 4, N = 235201, a semiprime;
: for m = 5, N = 176401, a prime;
: for m = 6, N = 141121, a prime;
: for m = 7, N = 117601, a semiprime;
: for m = 8, N = 100801, a prime;
: for m = 9, N = 88201, a semiprime;
: for m = 10, N = 78401, a prime;
: for m = 11, N = 70561, a semiprime;
: for m = 12, N is not integer;
: for m = 13, N = 58801, a semiprime;
: for m = 14, N is not integer;
: for m = 15, N = 50401, a semiprime;
: for m = 16, N = 47041, a prime;
: for m = 17, N = 44101, a prime (...)

38. Four conjectures based on the observation of a type of recurrent sequences involving semiprimes

Abstract. In this paper I make four conjectures starting from the observation of the following recurrent relations: (((p*q – p)*2 – p)*2 – p)...), respectively (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes.

Observation:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes, p < q. Very interesting patterns can be observed between a(i) and b(i) in the case of the same semiprime p*q or between the terms of this recurrence relation for different semiprimes:

Let p*q = 7*13 = 91; then:

: a(1) = 2*91 – 7 = 175;
: a(2) = 2*175 – 7 = 343;
: a(3) = 2*343 – 7 = 679 = 7*97;
: a(4) = 2*679 – 7 = 1351 = 7*193;
: a(5) = 2*1351 – 7 = 2695;
: a(6) = 2*2695 – 7 = 5383 = 7*769
(...)
: b(1) = 2*91 – 13 = 169;
: b(2) = 2*169 – 13 = 325;
: b(3) = 2*325 – 13 = 637;
: b(4) = 2*637 – 13 = 1261 = 13*97;
: b(5) = 2*1261 – 13 = 2509 = 13*193;
: b(6) = 2*2509 – 13 = 5005;
: b(7) = 2*5005 – 13 = 9997 = 13*769
(...)

Note that a(3)/p = b(4)/q = 97, a(4)/p = b(5)/q = 193 and a(6)/p = a(7)/q = 769.

Let p*q = 11*13 = 143; then:

: a(1) = 2*143 – 11 = 275;
: a(2) = 2*275 – 11 = 539;
: a(3) = 2*539 – 11 = 1067 = 11*97;
: a(4) = 2*1067 – 11 = 2123 = 11*193;
: a(5) = 2*2123 – 11 = 4235;
: a(6) = 2*4235 – 11 = 8459 = 11*769;
: a(7) = 2*8459 – 11 = 16907;
: a(8) = 2*16907 – 11 = 33803;
: a(9) = 2*33803 – 11 = 67595;
: a(10)= 2*8459 – 11 = 135179 = 11*12289
(...)
: b(1) = 2*143 – 13 = 273;
: b(2) = 2*273 – 13 = 533 = 13*41;

: b(3) = 2*533 – 13 = 1053;
: b(4) = 2*1053 – 13 = 2093;
: b(5) = 2*2093 – 13 = 4173;
: b(6) = 2*4173 – 13 = 8333 = 13*641;
: b(7) = 2*8333 – 13 = 16653;
: b(8) = 2*16653 – 13 = 33293;
: b(9) = 2*33293 – 13 = 66573;
: b(10) = 2*66573 – 13 = 133133;
: b(11) = 2*66573 – 13 = 266253;
: b(12) = 2*266253 – 13 = 532493 = 13*40961
(...)

Note that, in the case of this semiprime, were obtained for a(i)/p the primes obtained for the first semiprime, id est 97, 193, 769, 12289 (which are primes of the form 6*2^n + 1, see the sequence A039687 in OEIS) but for b(i)/q other primes, id est 41, 641, 40961 ((which are primes of the form 5*2^n + 1, see the sequence A050526 in OEIS).

Let p*q = 7*11 = 77; then:
: a(1) = 2*77 – 7 = 147;
: a(2) = 2*147 – 7 = 287 = 7*41;
: a(3) = 2*287 – 7 = 567;
: a(4) = 2*567 – 7 = 1127;
: a(5) = 2*1127 – 7 = 2247;
: a(6) = 2*2247 – 7 = 4487 = 7*641.
(...)
: b(1) = 2*77 – 11 = 143 then for the following terms see a(i) in the first example of p*q = 11*13.

Let p*q = 193*199; then we obtain, as b(i)/q, the primes 769, 12289 (which are primes of the form 6*2^n + 1, obtained above) but for a(i)/p other set of primes not met before: 397, 3169, 6337 (...). To make things even more complicated, for p*q = 197*199 we obtain, for a(i)/p, the set of primes 397, 3169, 6337 mentioned above but for b(i)/q other set of primes not met before: 3137, 50177 (...), which are primes of the form 49*2^n + 1 (see the sequence A077498 in OEIS). Note also the interesting thing that 397, 3169 and 6337 are all three primes of the form 99*2^n + 1.

Let p*q = 13*233; then we obtain, as a(i)/p, the primes 929, 59393, which are primes of the form 29*2^n + 1. Seems amazing how many possible infinite sequences of primes can be obtained starting from a simple recurrence relation and a randomly chosen pair of distinct odd primes.

Conjecture 1:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of primes of the form a(i)/p as well as an infinity of primes of the form b(i)/q for any pair [p, q].

Conjecture 2:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of pairs [p, q] such that the sequence of primes a(i)/p is the same with the sequence of primes b(i)/q.

Conjecture 3:

There exist an infinity of primes, for k positive integer, of the form n*2^k + 1, for n equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

Conjecture 4:

There exist an infinity of positive integers n such that the sequence n*2^k + 1, where k is positive integer, contains an infinity of primes.

39. Conjecture that states that a Mersenne number with odd exponent is either prime either divisible by a 2-Poulet number

Abstract. In this paper I make a conjecture which states that any Mersenne number (number of the form 2^n – 1, where n is natural) with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Mersenne-Coman numbers) and I finally enunciate yet another related conjecture.

Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Mersenne numbers see the sequence A000225 in OEIS.

Conjecture 1:

Any Mersenne number 2^n – 1 with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number.

Verifying the conjecture:
(For the first thirteen such n)

: 2^3 – 1 = 7, prime;
: 2^5 – 1 = 31, prime;
: 2^7 – 1 = 127, prime;
: 2^11 – 1 = 2047 = 23*89, a 2-Poulet number;
: 2^13 – 1 = 8191, prime;
: 2^15 – 1 = 32767 = 7*31*151, which is divisibe by 4681 = 31*151, a 2-Poulet number;
: 2^17 – 1 = 131071, prime;
: 2^19 – 1 = 524287, prime;
: 2^21 – 1 = 2097151 = 7^2*127*337, which is divisibe by 42799 = 127*337, a 2-Poulet number;
: 2^23 – 1 = 8388607 = 47*178481, a 2-Poulet number;
: 2^25 – 1 = 33554431 = 31*601*1801, which which is divisibe by 1082401 = 601*1801, a 2-Poulet number;
: 2^29 – 1 = 536870911 = 233*1103*2089, which which is divisibe by 256999 = 233*1103, a 2-Poulet number;
: 2^31 – 1 = 2147483647, prime.

Conjecture 2:

Any Mersenne-Coman number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number.

Verifying the conjecture:
(For m = 2 and the first twelve such n)

: (4^5 – 1)/3 = 341 = 11*31, a 2-Poulet number;
: (4^7 – 1)/3 = 5461 = 43*127, a 2-Poulet number;
: (4^11 – 1)/3 = 1398101 = 23*89*683, which is divisibe by:
    : 2047 = 23*89, a 2-Poulet number;
    : 15709 = 23*683, a 2-Poulet number;
    : 60787 = 89*683, a 2-Poulet number.
: (4^13 – 1)/3 = 22369621 = 2731*8191, a 2-Poulet number;
: (4^15 – 1)/3^2 is divisibe by:
    : 341 = 11*31, a 2-Poulet number;
    : 4681 = 31*151, a 2-Poulet number;
    : 10261 = 31*331, a 2-Poulet number;
    : 49981 = 151*331, a 2-Poulet number.
: (4^17 – 1)/3 = 5726623061 = 43691*131071, a 2-Poulet number;
: (4^19 – 1)/3 = 91625968981 = 174763*52487, a 2-Poulet number;
: (4^21 – 1)/3^2 divides 5461, 14491, 233017, 42799, 688213 and 1826203, all of them 2-Poulet numbers;
: (4^23 – 1)/3 = 23456248059221 = 47*178481*2796203, which is divisibe by**:**
    : 8388607 = 47*178481, a 2-Poulet number;
    : 131421541 = 47*2796203, a 2-Poulet number;
    : 499069107643 = 178481*2796203, a 2-Poulet number.
: (4^29 – 1)/3 = 96076792050570581 = 59*233*1103*2089*3033169, which is divisibe by**:**
    : 13747 = 59*233, a 2-Poulet number;
    : 65077 = 59*1103, a 2-Poulet number;
    : 123251 = 59*2089, a 2-Poulet number;
    : 178956971 = 59*3033169, a 2-Poulet number;
    : 256999 = 233*1103, a 2-Poulet number;
    : 486737 = 233*2089, a 2-Poulet number;
    : 706728377 = 233*3033169, a 2-Poulet number;
    : 2304167 = 1103*2089, a 2-Poulet number;
    : 3345585407 = 1103*3033169, a 2-Poulet number;
    : 6336290041 = 2089*3033169, a 2-Poulet number.
: (4^31 – 1)/3 = 1537228672809129301= 715827883*2147483647, a 2-Poulet number;
: (4^37 – 1)/3 = 6296488643826193618261= 223*1777*25781083*616318177, which is divisibe by 396271 = 223*1777 and other 2-Poulet numbers.

Verifying the conjecture:
(For m = 3 and the first four such n)

: 8^5 – 1 = 32767 = 7*31*151, which is divisibe by 4681 = 31*151, a 2-Poulet number;
: 8^7 – 1 = 2097151 = 7^2*127*337, which is divisibe by 42799 = 127*337, a 2-Poulet number;

: 8^11 – 1 = 8589934591 = 7*23*89*599479, which is divisibe by 2047 = 23*89, a 2-Poulet number;

: 8^13 – 1 = 549755813887 = 7*79*8191*121369, which is divisibe by 647089 = 79*8191, a 2-Poulet number.

Verifying the conjecture:
(For m = 4 and the first four such n)

: (16^5 – 1)/3 divides 341 = 11*31, a 2-Poulet number;
: (16^7 – 1)/3 divides 5461 = 43*127, a 2-Poulet number;
: (16^11 – 1)/3 divides 2047 = 23*89, a 2-Poulet number;
: (16^13 – 1)/3 divides 8321 = 53*157, a 2-Poulet number.

Note:

The Mersenne-Coman primes (Mersenne-Coman numbers which are primes) seems to be very rare. For m = 2 (*i.e.* 4^n – 1, where n is odd, n ≥ 5) there is no such a prime up to n = 107.

Conjecture 3:

For any prime p greater than or equal to 5 the number (4^p – 1)/3 is either prime either a product of primes $p_1 * p_2 * ... p_n$ such that all the numbers $p_i * p_j$ are 2-Poulet numbers for $1 \le i < j \le n$.

Note:

This Conjecture is verified for p up to 31 (see the Conjecture 2 above).

40. Conjecture that states that a Fermat number is either prime either divisible by a 2-Poulet number

Abstract. In this paper I make a conjecture which states that any Fermat number (number of the form 2^(2^n) + 1, where n is natural) is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Fermat-Coman numbers) and I finally enunciate yet another related conjecture.

Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Fermat numbers see the sequence A000215 in OEIS.

Conjecture 1:

Any Fermat number F = 2^(2^n) + 1 is either prime either divisible by a 2-Poulet number.

Note:

It is known that the first 5 Fermat numbers (3, 5, 17, 257, 65537) are primes. Also, for n = 5 is obtained F = 4294967297 = 641*6700417, which is, indeed, a 2-Poulet number (for the next two (composite) Fermat numbers, 18446744073709551617 340282366920938463463374607431768211457, semiprimes, I couldn't verify if they are 2-Poulet numbers).

Conjecture 2:

Any Fermat-Coman number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number.

Verifying the conjecture:
(For m = 1 and the first eight such p)

: (2^7 + 1)/3 = 43, prime;
: (2^11 + 1)/3 = 683, prime;
: (2^13 + 1)/3 = 2731, prime;
: (2^17 + 1)/3 = 43691, prime;
: (2^19 + 1)/3 = 174763, prime;
: (2^23 + 1)/3 = 2796203, prime;
: (2^29 + 1)/3 = 178956971 = 59*3033169, a 2-Poulet number;
: (2^31 + 1)/3 = 715827883, prime;

Verifying the conjecture:
(For m = 2 and the first four such p)

: 4^7 + 1 = 16385 = 5*29*113, which is divisibe by 3277 = 29*113, a 2-Poulet number;
: 4^11 + 1 = 4194305 = 5*397*2113, which is divisibe by 838861 = 397*2113, a 2-Poulet number;
: 4^13 + 1 = 67108865 = 5*53*157*1613, which is divisibe by:
  : 8321 = 53*157, a 2-Poulet number;
  : 85489 = 53*1613, a 2-Poulet number;
  : 253241 = 157*1613, a 2-Poulet number;
: 4^17 + 1 = 17179869185 = 5*137*953*26317, which is divisibe by:
  : 130561 = 137*953, a 2-Poulet number;
  : 3605429 = 137*26317, a 2-Poulet number;
  : 25080101 = 953*26317, a 2-Poulet number.

Verifying the conjecture:
(For m = 3 and the first two such p)

: (8^7 + 1)/3^2 = 233017 = 43*5419, a 2-Poulet number;
: (8^11 + 1)/3^2 = 954437177 = 67*683*20857, which is divisibe by 1397419 = 67*20857, a 2-Poulet number.

Note:

The Fermat-Coman primes (Fermat-Coman numbers which are primes) seems to be very rare.

Conjecture 3:

For any prime p greater than or equal to 7 the number (4^p + 1)/5 is either prime either a product of primes p_1*p_2*...p_n such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \leq i < j \leq n$ (this Conjecture is verified for p up to 17 (see the Conjecture 2 above).

41. Two exciting classes of odd composites defined by a relation between their prime factors

Abstract. In this paper I will define two interesting classes of odd composites often met (by the author of this paper) in the study of Fermat pseudoprimes, which might also have applications in the study of big semiprimes or in other fields. This two classes of composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n are defined in the following way: p(j) – p(i) + 1 is a prime or a power of a prime, respectively p(i) + p(j) – 1 is a prime or a power of prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k).

Definition 1:

We name the odd composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n, with the property that p(j) – p(i) + 1 is a prime or a power of a prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k), Coman composites of the first kind. If n = p*q is a squarefree semiprime, p < q, with the property that q – p + 1 is a prime or a power of a prime, then n it will be a Coman semiprime of the first kind.

Examples:

: 2047 = 23*89 is a Coman semiprime of the first kind because 89 – 23 + 1 = 67, a prime;
: 4681 = 31*151 is a Coman semiprime of the first kind because 151 – 31 + 1 = 121, a power of a prime;
: 1729 = 7*13*19 is a Coman composite of the first kind because 19 – 7 + 1 = 13, a prime, 19 – 13 + 1 = 7, a prime, and 13 – 7 + 1 = 7, a prime.
: 2821 = 7*13*31 is a Coman composite of the first kind because 13 – 7 + 1 = 7, a prime, 31 – 13 + 1 = 19, a prime, and 31 – 7 + 1 = 25, a power of a prime.

Note that not incidentally I chose Fermat pseudoprimes to base two with two prime factors (2-Poulet numbers) and absolute Fermat pseudoprimes as examples: they are often Coman composites.

Definition 2:

We name the odd semiprimes $n_1 = p_1*q_1$, $p_1 < q_1$, with the property that $n_2 = q_1 – p_1 + 1 = p_2*q_2$, $p_2 < q_2$, is a Coman semiprime of the first kind, a Coman semiprime of the first kind of the second degree, also the odd semiprimes n_2 with the property that $n_3 = q_2 – p_2 + 1$ is a Coman semiprime of the first kind of the second degree, a Coman semiprime of the first kind of the third degree and so on.

Examples:

: 679 = 7*97 is a Coman semiprime of the first kind of the second degree because 97 – 7 + 1 = 91, a Coman semiprime of the first kind because 91 = 7*13 and 13 – 7 + 1 = 7, a prime;

: 8983 = 13*691 is a Coman semiprime of the first kind of the third degree because 691 – 13 + 1 = 679, which is a Coman semiprime of the first kind of the second degree.

Definition 3:

We name the odd composites n = p(1)*...*p(k), where p(1), ..., p(k) are the prime factors of n, with the property that p(j) + p(i) - 1 is a prime or a power of a prime for any p(i), p(j) prime factors of n such that p(1) ≤ p(i) < p(j) ≤ p(k), Coman composites of the second kind. If n = p*q is a squarefree semiprime, p < q, with the property that q + p - 1 is a prime or a power of a prime, then n it will be a Coman semiprime of the second kind.

Examples:

: 341 = 11*31 is a Coman semiprime of the second kind because 11 + 31 - 1 = 41, a prime;
: 1729 = 7*13*19 is a Coman composite of the second kind because 7 + 13 – 1 = 19, a prime, 13 + 19 – 1 = 31, a prime, and 7 + 19 – 1 = 25, a power of a prime.

Definition 4:

We name the odd semiprimes $n_1 = p_1*q_1$, $p_1 < q_1$, with the property that $n_2 = q_1 + p_1 - 1 = p_2*q_2$, $p_1 < q_1$, is a Coman semiprime of the second kind, a Coman semiprime of the second kind of the second degree, also the odd semiprimes n_2 with the property that $n_3 = q_2 + p_2 - 1$ is a Coman semiprime of the second kind of second degree, a Coman semiprime of the second kind of the third degree and so on.

Notes:

: The odd semiprimes of the type n = p*q, p < q, where abs{p – q + 1} or q^2 – p + 1 or abs{q – p^2 + 1} or q^2 – p^2 + 1 or abs{p^2 – q^2 + 1} or, respectively, p^2 + q – 1 or p + q^2 – 1 or p^2 + q^2 – 1 is also prime, seems also to be interesting to be studied;

: The numbers of the type n = p^2 + q^2 – 1 respectively n = q^2 – p^2 + 1, where p, q primes, p < q, are often, if not primes, Coman composites;

: The seventh Fermat number, 18446744073709551617 = 274177*67280421310721 is a Coman semiprime of the second kind because the number 67280421584897 = 67280421310721 + 274177 – 1 is a prime;

: Many Mersenne numbers are Coman composites: 2047 = 23*89 is a Coman semiprime of the first kind because 89 – 23 + 1 = 67, a prime; 32767 = 7*31*151 is a Coman composite of the second kind because 7 + 31 – 1 = 37, a prime, 7 + 151 – 1 = 157, a prime, and 31 + 151 – 1 = 181, a prime; 33554431 = 31*601*1801 because 31 + 601 – 1 = 631, a prime, 31 + 1801 – 1 = 1831, a prime, and 601 + 1801 – 1 = 2401 = 7^4, a power of a prime;

: In the papers from the references given below there are few conjectures about Coman composites.

References:

: A formula which conducts to primes or to a type of composites that could form a class themselves, Marius Coman;

: An elementary formula which seems to conduct often to primes, Marius Coman;

: Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture, Marius Coman;

: Ten conjectures about certain types of pairs of primes arising in the study of 2-Poulet numbers, Marius Coman;

: The notion of chameleonic numbers, a set of composites that "hide" in their inner structure an easy way to obtain primes, Marius Coman;

: Twenty-four conjectures about "the eight essential subsets of primes", Marius Coman;

: Two types of pairs of primes that could be associated to Poulet numbers, Marius Coman.

42. A formula for generating a certain kind of semiprimes based on the two known Wieferich primes

Abstract. In one of my previous papers, "A possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes" I pointed an interesting relation between Poulet numbers and the two known Wieferich primes (not the known fact that the squares of these two primes are Poulet numbers themselves but a way to relate an entire set of Poulet numbers by a Wieferich prime). Exploring further that formula I found a way to generate primes, respectively semiprimes of the form q1*q2, where q2 – q1 is equal to a multiple of 30.

Note:

In the paper I was talking about in Abstract I conjectured that there exist, for every Wieferich prime W, an infinity of Poulet numbers which are equal to n*W – n + 1, where n is integer, n > 1. Examples of such Poulet numbers are 3277 = 1093*3 – 2, 4369 = 1093*4 – 3, 5461 = 1093*5 – 4, respectively 49141 = 1093*14 – 13. In other words, I conjectured that there exist an infinity of pairs of Poulet numbers (P1, P2) such that P2 – P1 + 1 = 1093, respectively an infinity of pairs of Poulet numbers (P1, P2) such that P2 – P1 + 1 = 3511. Examples of such pairs of Poulet numbers are (1729, 2821), (3277, 4369), (4369, 5461). Playing with this formula I noted that in many cases the number P + W – 1, where P is a Poulet number and W a Wieferich prime, is equal to a semiprime q1*q2, where q2 – q1 = 30 (examples of such semiprimes are 37*67 = 1387 + 1093 – 1 and 43*73 = 2047 + 1093 – 1). But, more than that, I noticed that often the numbers of the type q1*q2 – W + 1 (and implicitely, as we will see further, of the type q1*q2 + W – 1), where q1 and q2 are primes such that q2 – q1 = 30*k, where k positive integer, are often equal to q3*q4, where q3 and q4 are primes such that q4 – q3 = 30*h, where h positive integer.

Conjecture 1:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 2:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 1092 is equal to a prime.

The first three such semiprimes corresponding to p = 17:

: 17*47 + 1092 = 31*61;
: 17*107 + 1092 = 41*71;
: 17*137 + 1092 = 11*311.

The first three such primes corresponding to p = 17:

: 17*167 + 1092 = 3931, prime;
: 17*197 + 1092 = 4441, prime;
: 17*137 + 1092 = 4951, prime.

The first three such semiprimes corresponding to p = 23:

: 23*173 + 1092 = 11*461;
: 23*353 + 1092 = 61*151;
: 23*443 + 1092 = 29*389.

The first three such primes corresponding to p = 23:

: 23*53 + 1092 = 2311, prime;
: 23*83 + 1092 = 3001, prime;
: 23*113 + 1092 = 3691, prime.

Conjecture 3:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a semiprime pi*qi, where qi – pi = 30*m, where m positive integer.

Conjecture 4:

For every prime p, p > 5, there exist an infinity of primes q, q = p + 30*n, where n positive integer, such that the number p*q + 3510 is equal to a prime.

The first three such semiprimes corresponding to p = 17:

: 17*107 + 3510 = 73*73;
: 17*167 + 3510 = 7*907;
: 17*347 + 3510 = 97*97.

The first three such primes corresponding to p = 17:

: 17*137 + 3510 = 5839, prime;
: 17*227 + 3510 = 7369, prime;
: 17*257 + 3510 = 7879, prime.

The first three such semiprimes corresponding to p = 23:

: 23*293 + 3510 = 37*277;
: 23*383 + 3510 = 97*127;
: 23*503 + 3510 = 17*887.

The first three such primes corresponding to p = 23:

: 23*53 + 3510 = 4729, prime;
: 23*83 + 3510 = 5419, prime;
: 23*173 + 3510 = 7489, prime.

43. Formula that uses primes as input values for obtaining larger primes as output, based on the numbers 7 and 186

Abstract. In this paper I present a formula, based on the numbers 7 and 186, that, using primes as input values, often leads, as output values, to larger primes, also to squares of primes and semiprimes. I found this formula by chance, playing with two of my favourite numbers, 13 and 31, and observing that 7*13^2 + 6*31 = 37^2 (to be noted, without necessarily connection with this paper, that the difference between the two known Wieferich primes, 1093 and 3511, is equal to 6*13*31).

Observation:

The formula 7*p^2 + 186, where p is prime, often conducts to primes, squares of primes and semiprimes.

Exemplification:
(taking as input values p the first 27 primes; note that were obtained 13 primes, 6 squares of primes and 8 semiprimes)

: 7*3^2 + 186 = 3*83, semiprime;
: 7*5^2 + 186 = 19^2, square of prime;
: 7*7^2 + 186 = 23^2, square of prime;
: 7*11^2 + 186 = 1033, prime;
: 7*13^2 + 186 = 37^2, square of prime;
: 7*17^2 + 186 = 47^2, square of prime;
: 7*19^2 + 186 = 2713, prime;
: 7*23^2 + 186 = 3889, prime;
: 7*29^2 + 186 = 6073, prime;
: 7*31^2 + 186 = 31*223, semiprime;
: 7*37^2 + 186 = 9769, prime;
: 7*41^2 + 186 = 11953, prime;
: 7*43^2 + 186 = 19*691, semiprime;
: 7*47^2 + 186 = 15649, prime;
: 7*53^2 + 186 = 23*863, semiprime;
: 7*59^2 + 186 = 43*571, semiprime;
: 7*61^2 + 186 = 37*709, semiprime;
: 7*67^2 + 186 = 73*433, semiprime;
: 7*71^2 + 186 = 43^2, square of prime;
: 7*73^2 + 186 = 37489, prime;
: 7*79^2 + 186 = 73*601, semiprime
: 7*83^2 + 186 = 48409, prime;
: 7*89^2 + 186 = 55633, prime;
: 7*97^2 + 186 = 257^2, square of prime;
: 7*101^2 + 186 = 71593, prime;
: 7*103^2 + 186 = 74449, prime;
: 7*107^2 + 186 = 80329, prime.

Exemplification:

(taking as input values 10 from the 17 larger consecutive primes); note that were obtained 8 semiprimes and 2 primes, and that for the other 7 primes were obtained numbers with maximum four prime factors)

: 7*941083907^2 + 186 = 2205707657*2810650097, semiprime;
: 7*941083921^2 + 186 = 6199472624553139873, prime.
: 7*941083951^2 + 186 = 19*326288053674125947, semiprime;
: 7*941083967^2 + 186 = 743*8343840148871063, semiprime;
: 7*941083987^2 + 186 = 37*167553337678776037, semiprime;
: 7*941084021^2 + 186 = 880691281*7039327033, semiprime;
: 7*941084047^2 + 186 = 23*269542360201099463, semiprime;
: 7*941084083^2 + 186 = 17137*361759628810857, semiprime;
: 7*941084167^2 + 186 = 30631*202392212648839, semiprime;
: 7*941084173^2 + 186 = 6199475944697657689, prime.

Conjecture 1:

There exist an infinity of primes p such that the number 7*p^2 + 186 is prime.

Conjecture 2:

There exist an infinity of primes p such that the number 7*p^2 + 186 is square of prime.

Conjecture 3:

There exist an infinity of primes p such that the number 7*p^2 + 186 is semiprime.

44. Conjectures on Smarandache generalized Fermat numbers

Abstract. In this paper I make few conjectures on few classes of generalized Fermat numbers, i.e. the numbers of the form F(k) = 2^(2^k) + n, where k is positive integer and n is an odd number, the numbers of the form F(k) = 4^(4^k) + 3 and the numbers of the form F(k) = m^(m^k) + n, where m + n = p, where p is prime, all subclasses of Smarandache generalized Fermat numbers, i.e. the numbers of the form F(k) = a^(b^k) + c, where a, b are integers greater than or equal to 2 and c is integer such that (a, c) = 1.

Conjecture 1:

Let be F(k) = 2^(2^k) + n, where k is positive integer and n is an odd number. Then, there exist an infinity of numbers n such that F(k) is prime for k = 0, k = 1 and k = 2.

Note:
For n = 1, the numbers F(k) are the Fermat numbers and it is known that the first five such numbers are primes (it is conjectured that there are not Fermat numbers that are primes for k ≥ 5). For n = 3 the three primes obtained are [5, 17, 19]. For n ≥ 3 obviously n can be only of the forms 30*m + 15 or 30*m + 27, otherwise one from the three numbers F(0), F(1) and F(3) would be divisible with 3 or 5.

Examples:
(Of such numbers n)

: For n = 15 are obtained the primes [17, 19, 31];
: For n = 27 are obtained the primes [29, 31, 43];
: For n = 57 are obtained the primes [59, 61, 73];
: For n = 135 are obtained the primes [137, 139, 151];
: For n = 147 are obtained the primes [149, 151, 163].

Note:
Obviously this conjecture implies the conjecture that there exist an infinity of primes of the forms 30*k + 1, 30*k + 13, 30*k + 17, 30*k + 19 respectively 30*k + 29, but I already conjectured in a previous paper (namely "Twenty-four conjectures about the eight essential subsets of primes") that there exist an infinity of primes of the form 30*k + n, for any n equal to 1, 7, 11, 13, 17, 19, 23 or 29.

Conjecture 2:

There exist an infinity of quadruplets of primes of the form [30*k + 17, 30*k + 19, 30*k + 31, 30*k + 43]. Such primes are, as can be seen above, for instance, [17, 19, 31, 43] or [137, 139, 151, 163].

Comment:

I obviously could put the Conjecture 1 in a simpler form (i.e. Conjecture: there exist an infinity of odd numbers n such that the numbers n + 2, n + 4 and n + 16 are all three primes, but in this case it would appear like an arbitrary statement, which is not, but one from the many possible interesting related cojectures on Smarandache generalized Fermat numbers like the following ones:

Conjecture 3:

Let be F(k) = 4^(4^k) + 3, where k is positive integer. Then, there exist an infinity of numbers k such that F(k) is equal to 7*p, where p is prime.

Examples:
(Of such numbers F(k))

: F(1) = 259 = 7*37;
: F(2) = 4294967299 = 7*613566757;
: F(3) = 340282366920938463463374607431768211459 = 7*48611766702991209066196372490252601637

Conjecture 4:

Let be F(k) = m^(m^k) + n, where m is even and n is odd, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Conjecture 5:

Let be F(k) = m^(m^k) + n, where m is odd and n is even, such that m + n = p, where p is prime. Then, there exist at least a k, beside of course k = 0, for which F(k) has as a prime factor the number p.

Reference:

Florentin Smarandache, ***Conjecture (General Fermat numbers)***, in Collected Papers, vol. II, Kishinev University Press, Kishinev, 1997.

45. An interesting class of Smarandache generalized Fermat numbers

Abstract. In this paper I make few observations on a class of Smarandache generalized Fermat numbers, which are the numbers of the form F(k) = a^(b^k) + c, where a, b are integers greater than or equal to 2 and c is integer such that (a, c) = 1. The class that is observed in this paper includes the numbers of the form F(k) = m^(n^k) + n, where k is positive integer and m and n are coprime positive integers, not both of them odd or both of them even.

Observation:

The numbers of the form F(k) = m^(n^k) + n, where k is positive integer and m and n are coprime positive integers, not both of them odd or both of them even are very difficult to be factorized, not just because their obviously large size but because seem to have very few prime factors. This is a known characteristic of Fermat numbers (for instance, the 8-th such number, having 78 digits, is a semiprime) and seems that this class of Smarandache generalized Fermat numbers share this feature also.

Examples:

For F(k) = 4^(3^k) + 3 are obtained the following values:

: F(1) = 67 prime;
: F(2) = 262147 prime;
: F(3) = 18014398509481987*1422061*12667809967 semiprime;
: F(4) = 5846006549323611672814739330865132078623730171907 = 103*56757345139064190998201352726845942510910001669 semiprime;
(the number F(5) has 147 digits)

For F(k) = 2^(7^k) + 7 are obtained the following values:

: F(1) = 16387 = 7*2341 semiprime;
: F(2) = 316912650057057350374175801351 prime;
(the number F(3) has 207 digits)

For F(k) = 2^(9^k) + 9 are obtained the following values:

: F(1) = 521 prime;
: F(2) = 2417851639229258349412361 = 11*219804694475387122673851 semiprime;
(the number F(3) has 220 digits)

For F(k) = 2^(11^k) + 11 are obtained the following values:

: F(1) = 2059 = 29*71 semiprime;
: F(2) = 2658455991569831745807614120560689163 = 13*131*1.561042860581228271172997134797821;
(the number F(3) has 401 digits)

Comment:

Of course, not for every pair of (m, n) that satisfies the conditions described above are obtained relevant results, but I haven't find yet a convincing pattern for a special relation between m and n.

Reference:

Florentin Smarandache, ***Conjecture (General Fermat numbers)***, in Collected Papers, vol. II, Kishinev University Press, Kishinev, 1997.

46. Conjecture which states that an iterative operation of any pair of two odd primes conducts to a larger prime

Abstract. In this paper I make a conjecture which states that from any two odd primes p1 and p2 can be obtained, through an iterative and very simple operation, a prime p3 larger than p1 and also larger than p2.

Conjecture:

Let p and q be two odd primes; then, through the iterative operation ((((p + 1)*q + 1) + 1)*q + 1...) finally it will be obtained a prime.

Verifying the conjecture:
(For (p, q) = (1299709, 5)

: 1299709 + 1 = 1299710;
: 1299710*5 + 1 = 6498551;
: 6498551 + 1 = 6498552;
: 6498552*5 + 1 = 32492761;
: 32492761 + 1 = 32492762;
: 32492762*5 + 1 = 162463811;
: 162463811 + 1 = 162463812;
: 162463812*5 + 1 = 812319061;
: 812319061 + 1 = 812319062;
: 812319062*5 + 1 = 4061595311 which is prime.

Verifying the conjecture:
(For (p, q) = (1299721, 7)

: 1299721 + 1 = 1299722;
: 1299722*7 + 1 = 9098055;
: 9098055 + 1 = 9098056;
: 9098056*7 + 1 = 63686393;
: 63686393 + 1 = 63686394;
: 63686394*7 + 1 = 445804759;
: 445804759 + 1 = 445804760;
: 445804760*7 + 1 = 3120633321;
: 3120633321 + 1 = 3120633322;
: 3120633322*7 + 1 = 21844433255;
: 21844433255 + 1 = 21844433256;
: 21844433256*7 + 1 = 152911032793;
: 152911032793 + 1 = 152911032794;
: 152911032794*7 + 1 = 1070377229559;
: 1070377229559 + 1 = 1070377229560;
: 1070377229560*7 + 1 = 7492640606921;
: 7492640606921 + 1 = 7492640606922;
: 7492640606922*7 + 1 = 52448484248455;
: 52448484248455 + 1 = 52448484248456;

: 524484842248456*7 + 1 = 3671393897739193;
: 3671393897739194 + 1 = 3671393897739194;
: 3671393897739194*7 + 1 = 25699757284174359;
: 25699757284174359 + 1 = 25699757284174360;
: 25699757284174360*7 + 1 = 179898300989220521;
: 179898300989220521 + 1 = 179898300989220522;
: 179898300989220522*7 + 1 = 1259288106924543655;
: 1259288106924543655 + 1 = 1259288106924543656;
: 1259288106924543656*7 + 1 = 8815016748471805593;
: 8815016748471805593 + 1 = 8815016748471805594;
: 8815016748471805594*7 + 1 = 61705117239302639159, which is prime.

47. A probably infinite sequence of primes formed using Carmichael numbers, the number 584 and concatenation

Abstract. In this paper I make a conjecture which states that there exist an infinity of primes of the form N/3^m, where m is positive integer and N is the number formed concatenating to the left a Carmichael number with the number 584. Such primes are 649081 = 5841729/3^2, 1947607 = 5842821/3, 1948867 = 5846601/3 etc. I also make few comments about a certain kind of semiprimes.

Conjecture 1:

Let p be an odd prime; then the formula n*p + n – 1, where n integer, n > 1, conducts to an infinity of prime numbers (note that, for n = 2, the conjecture states the infinity of Sophie Germain primes).

Examples:
(Of such primes)

: 649081 = 5841729/3^2;
: 1947607 = 5842821/3^1;
: 1948867 = 5846601/3^1;
: 649879 = 5848911/3^2;
: 2164483 = 58441041/3^3;
: 6495997 = 58463973/3^2;
: 64909829 = 584188461/3^2;
: 194799667 = 584399001/3^1;
: 64972073 = 584748657/3^2
: 194946067 = 584838201/3^1.

Conjecture 2:

There exist an infinity of semiprimes p*q = N/3^m, where p < q, m is positive integer and N is the number formed concatenating to the left a Carmichael number with the number 584, with the property that q – p + 1 is a prime number.

Examples:
(Of such semiprimes)

: 58429341/3^2 = 109*59561 and 59561 – 109 + 1 = 59453, which is prime;
: 58452633/3^2 = 37*88969 and 88969 – 73 + 1 = 88897, which is prime;
: 584162401/3^0 = 37*15788173 and 15788173 – 37 + 1 = 15788137, which is prime.

Note:
For more about this type of semiprimes see my previous paper “Two exciting classes of odd composites defined by a relation between their prime factors”. In the paper mentioned I observed the semiprimes with the propery from above (p*q, p < q, such that q – p + 1 is prime) but also the sequences of semiprimes p1*q1, such that q1 – p1 =

p2*q2, such that q2 – p2 = p3*q3 and so on; taking for instance the Carmichael number 1713045574801 concatenated to the left with 584 we have a chain of such consecutive semiprimes, finally the iterative operation ending with a prime:

: 5841713045574801/9 = 73*8891496264193 semiprime;
: 8891496264193 - 73 + 1 = 239*37202913239 semiprime;
: 37202913239 – 239 + 1 = 29*1282859069 semiprime;
: 1282859069 – 29 + 1 = 109*11769349 semiprime;
: 11769349 – 109 + 1 = 11*1069931 semiprime;
: 1069931 – 11 + 1 = 1069921 prime.

Comment:

There exist yet another interesting relation between Carmichael numbers and the number 584; the numbers 561 and 1729, the first and the third Carmichael numbers, are both of the form 584*n – 23 (they are obtained for n = 1, respectively n = 3). So we have the relation (1729 + 23)/(561 + 23) = 3, an integer, an interesting relation between the numbers 561, 1729 and 23, all three “cult numbers”. For other interesting relations between the number 561 and the Hardy-Ramanujan number, 1729, see my previous paper “Special properties of the first absolute Fermat pseudoprime, the number 561”.

48. Recurrent formulas which conduct to probably infinite sequences of primes and a generalization of a Cunningham chain

Abstract. In this paper I define few formulas which conduct from any odd prime respectively from any pair of distinct odd primes to an infinity of probably infinite sequences of primes, also to such sequences of a certain kind of semiprimes, and I also make a generalization of a Cunningham chain of primes of the first kind, respectively of the second kind.

Conjecture 1:

Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Examples:

First such prime for [p, n] = [104729, 2]

: 104729*2 + 1 = 209459 prime.

First such prime for [p, n] = [104729, 3]

: 104729*3 + 2 = 314189 prime.

First such prime for [p, n] = [104729, 4]

: 104729*4 + 3 = 314189 prime.

First such prime for [p, n] = [104729, 5]

: 104729*5 + 4 = 523649;
: 523649*5 + 4 = 2618249 prime.

Conjecture 2:

Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Examples:

First such prime for [p, q, n] = [104729, 3, 2]

: 104729*2 + 3 = 209461;
: 209461*2 + 3 = 418925;
: 418925*2 + 3 = 837853 prime.

First such prime for [p, q, n] = [104729, 7, 3]

:   104729*3 + 7*2 = 314201;
:   314201*3 + 7*2 = 942617;
:   942617*3 + 7*2 = 2827865;
:   2827865*3 + 7*2 = 8483609 prime.

First such prime for [p, q, n] = [104729, 13, 4]

:   104729*4 + 13*3 = 418955;
:   418955*4 + 13*3 = 1675859 prime.

First such prime for [p, q, n] = [104729, 17, 7]

:   104729*5 + 17*4 = 523713;
:   523713*5 + 17*4 = 2618633 prime.

Definition:
(generalization of a Cunningham chain of the first kind)

We name a Cunningham-Coman chain of primes of the first kind the primes p1, p2, ..., pk obtained through the recurrent formula: p2 = 2*p1 + q, p3 = 4*p1 + 3*q, p4 = 8*p1 + 7*q, ...,pi = 2^(i – 1)*p1 + 2^(i – 1) – 1, where q is an odd prime.

Example:

For q = 5, we have the following Cunningham-Coman chain of length 6 starting with p1 = 13: 13, 31 (= 2*13 + 5), 67 (= 4*13 + 3*5), 139 (= 8*13 + 7*5), 283 (= 16*13 + 15*5), 571 (= 32*13 + 31*5).

Conjecture 3:

Let p be an odd prime; then the formula ((((n*p + n – 1)*p + n – 1))*p + n – 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Note: In a previous paper, “Two exciting classes of odd composites defined by a relation between their prime factors” I defined Coman semiprimes of the first kind the semiprimes p*q with the property that q1 – p1 + 1 = p2*q2, where the semiprime p2*q2 has also the property that q2 – p2 + 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk – qk + 1 is a prime. I also defined Coman semiprimes of the second kind the semiprimes p*q with the property that q1 + p1 - 1 = p2*q2, where the semiprime p2*q2 has also the property that q2 + p2 - 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk + qk - 1 is a prime.

Examples:

First such semiprime for [p, n] = [104729, 2]

:   104729*2 + 1 = 209459;
:   209459*2 + 1 = 418919;
:   418919*2 + 1 = 837839;
:   837839*2 + 1 = 1675679;

: 1675679*2 + 1 = 3351359;
: 3351359*2 + 1 = 6702719 = 139*48221, which is a Coman semiprime because 48221 – 139 + 1 = 7*6869 and 6869 – 7 + 1 = 6863 which is prime.

Conjecture 4:

Let p and q be distinct odd primes; then the formula ((((n*p + (n – 1)*q)*p + (n – 1)*q)*p + (n – 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Examples:

First such semiprime for [p, q, n] = [104729, 7, 3]

: 104729*3 + 7*2 = 43*7307, which is a Coman semiprime of the second kind because 7307 + 43 – 1 = 7349, which is prime.

First such semiprime for [p, q, n] = [104729, 11, 5]

: 104729*5 + 11*4 = 523689;
: 523689*5 + 11*4 = 2618489 = 547*4787, which is at the same time a Coman semiprime of the first kind because 4787 – 547 + 1 = 4241, which is prime, and a Coman semiprime of the second kind, because 4787 + 547 – 1 = 5333, which is also prime.

Definition:

We name *an absolute Coman semiprime* a semiprime which is at the same time Coman semiprime of the first kind and Coman semiprime of the second kind.

Conjecture 5:

Let p be an odd prime; then the formula ((((n*p – n + 1)*p – n + 1))*p – n + 1)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Conjecture 6:

Let p and q be distinct odd primes; then the formula ((((n*p – (n + 1)*q)*p – (n + 1)*q)*p – (n + 1)*q)...) conducts to an infinity of prime numbers for any n integer, n > 1.

Definition:
(generalization of a Cunningham chain of the second kind)

We name a Cunningham-Coman chain of primes of the first kind the primes p1, p2, ..., pk obtained through the recurrent formula: p2 = 2*p1 – q, p3 = 4*p1 – 3*q, p4 = 8*p1 – 7*q, ..., pi = 2^(i – 1)*p1 – 2^(i – 1) + 1, where q is an odd prime.

Conjecture 7:

Let p be an odd prime; then the formula ((((n*p – n + 1)*p – n + 1))*p – n + 1)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

Conjecture 8:

Let p and q be distinct odd primes; then the formula ((((n*p – (n + 1)*q)*p – (n + 1)*q)*p – (n + 1)*q)...) conducts, for any n integer, n > 1, to an infinity of Coman semiprimes.

49. Two formulas of generalized Fermat numbers which seems to generate large primes

Abstract. There exist few distinct generalizations of Fermat numbers, like for instance numbers of the form F(k) = a^(2^k) + 1, where a > 2, or F(k) = a^(2^k) + b^(2^k) or Smarandache generalized Fermat numbers, which are the numbers of the form F(k) = a^(b^k) + c, where a, b are integers greater than or equal to 2 and c is integer such that (a, c) = 1. In this paper I observe two formulas based on a new type of generalized Fermat numbers, which are the numbers of the form F(k) = (a^(b^k) ± c)/d, where a, b are integers greater than or equal to 2 and c, d are positive non-null integers such that F(k) is integer.

Conjecture 1:

There exist an infinity of odd integers k such that the number p = (2^(2^k) + 2)/6 is a prime of the form 30*n + 13, where n positive integer.

Examples:

First three such primes p, obtained for k = 3, 5 and 7:

: p = 43 = 1*30 + 13 for k = 3;
: p = 715827883 = 23860929*30 + 13 for k = 5;
: p = 56713727820156410577229101238628035243 =
189045759400521368590763670795426784 1*30 + 13 for k = 7.

Note that for k = 9 is obtained a number p with 154 digits!

Conjecture 2:

There exist an infinity of odd integers k such that the number p = (2^(2^k) – 2)/2 is a prime of the form 30*n + 7, where n positive integer.

Examples:

First three such primes p, obtained for k = 3, 5 and 7:

: p = 127 = 4*30 + 7 for k = 3;
: p = 2147483647 = 71582788*30 + 7 for k = 5;
: p = 170141183460469231731687303715884105727 =
5671372782015641057722910123862803524*30 + 7 for k = 7.

Note that for k = 9 is obtained a number p with 154 digits!

50. Eight conjectures on a certain type of semiprimes involving a formula based on the multiples of 30

Abstract. In this paper I make eight conjectures about a certain type of semiprimes which I defined in a previous paper, "Two exciting classes of odd composites defined by a relation between their prime factors", in the following way: Coman semiprimes of the first kind are the semiprimes p*q with the property that q1 – p1 + 1 = p2*q2, where the semiprime p2*q2 has also the property that q2 – p2 + 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk – qk + 1 is a prime. I also defined Coman semiprimes of the second kind the semiprimes p*q with the property that q1 + p1 – 1 = p2*q2, where the semiprime p2*q2 has also the property that q2 + p2 – 1 = p3*q3, also a semiprime, and the operation is iterate until eventually pk + qk – 1 is a prime.

Conjecture 1:

For any given odd prime p there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the first kind and n = 30*p*q + 1 is a prime.

Examples:
(Of such primes, for p = 7)

: 7*137*30 + 1 = 28771, which is prime;
: 7*157*30 + 1 = 32971, which is prime;
: 7*163*30 + 1 = 34231, which is prime;
: 7*179*30 + 1 = 37591, which is prime.

(It can be seen that 7*137, 7*157, 7*163 and 7*179 are Coman semiprimes of the first kind because 137 – 7 + 1 = 131, prime, 157 – 7 + 1 = 151, prime, 163 – 7 + 1 = 157, prime and 179 – 7 + 1 = 173, prime)

Conjecture 2:

For any given odd prime p there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Examples:
(Of such primes, for p = 7)

: 7*173*30 + 1 = 36331 = 47*773, which is Coman semiprime of the first kind because 773 – 47 + 1 = 727, prime;
: 7*257*30 + 1 = 53971 = 31*1741, which is Coman semiprime of the first kind because 1741 – 31 + 1 = 1711 = 29*59 and 59 – 29 + 1 = 31, prime;
: 7*263*30 + 1 = 55231 = 11*5021, which is Coman semiprime of the first kind because 5021 – 11 + 1 = 5011, prime;
: 7*269*30 + 1 = 56491 = 17*3323, which is Coman semiprime of the first kind because 3323 – 17 + 1 = 3307, prime.

(It can be seen that 7*173, 7*257, 7*263 and 7*269 are Coman semiprimes of the first kind because 173 – 7 + 1 = 167, prime, 257 – 7 + 1 = 251, prime, 263 – 7 + 1 = 257, prime and 269 – 7 + 1 = 263, prime)

Conjecture 3:

For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the first kind and n = 30*k*p*q + 1 is a prime.

Examples:
(Of such primes, for (p, k) = (13, 2))

: 13*19*60 + 1 = 14821, which is prime;
: 13*29*60 + 1 = 22621, which is prime;
: 13*31*60 + 1 = 24181, which is prime.

Conjecture 4:

For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Examples:
(Of such primes, for (p, k) = (13, 2))

: 13*17*60 + 1 = 13261 = 89*149, which is Coman semiprime of the first kind because 149 – 89 + 1 = 61, prime;
: 13*17*60 + 1 = 13261 = 89*149, which is Coman semiprime of the first kind because 149 – 89 + 1 = 61, prime.

Conjecture 5:

For any given odd prime p there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the second kind and n = 30*p*q – 1 is a prime.

Examples:
(Of such primes, for p = 7)

: 7*167*30 – 1 = 35069, which is prime;
: 7*251*30 – 1 = 52709, which is prime;
: 7*263*30 – 1 = 55229, which is prime;
: 7*271*30 – 1 = 56909, which is prime;

(It can be seen that 7*167, 7*251, 7*163 and 7*179 are Coman semiprimes of the second kind because 167 + 7 – 1 = 173, prime, 251 + 7 – 1 = 257, prime, 263 + 7 – 1 = 269, prime and 271 + 7 – 1 = 277, prime)

Conjecture 6:

For any given odd prime p there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q + 1 are both Coman semiprimes of the first kind.

Examples:
(Of such primes, for p = 7)

: 7*257*30 – 1 = 53969 = 29*1861, which is Coman semiprime of the second kind because 1861 + 29 – 1 = 1889, prime;
: 7*433*30 – 1 = 90929 = 79*1151, which is Coman semiprime of the second kind because 1151 + 79 – 1 = 1229, prime;
: 7*461*30 – 1 = 96809 = 131*739, which is Coman semiprime of the second kind because 739 + 131 – 1 = 869 = 11*79 and 79 + 11 – 1 = 89, prime;
: 7*503*30 – 1 = 105629 = 53*1993, which is Coman semiprime of the second kind because 1993 + 53 – 1 = 2045 = 5*409 and 409 + 4 – 1 = 413 = 7*59 and 59 + 7 – 1 = 65 = 5*13 and 13 + 5 – 1 = 17, prime.

(It can be seen that 7*257, 7*433, 7*263 and 7*269 are Coman semiprimes of the second kind because 257 + 7 – 1 = 263, prime, 433 + 7 – 1 = 439, prime, 461 + 7 – 1 = 467, prime and 503 + 7 – 1 = 509, prime)

Conjecture 7:

For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the number m = p*q is a Coman semiprime of the second kind and n = 30*k*p*q – 1 is a prime.

Examples:
(Of such primes, for (p, k) = (11, 3))

: 11*31*90 – 1 = 30689, which is prime;
: 11*37*90 – 1 = 36629, which is prime.

(It can be seen that 11*31 and 11* are Coman semiprimes of the second kind because 11 + 31 – 1 = 41, prime, and 11 + 37 – 1 = 47, prime)

Conjecture 8:

For any given odd prime p and any k non-null positive integer there exist an infinity of odd primes q such that the numbers m = p*q and n = 30*p*q – 1 are both Coman semiprimes of the second kind.

Examples:
(Of such primes, for (p, k) = (11, 3))

: 11*13*90 – 1 = 12869 = 17*757, which is Coman semiprime of the second kind because 757 + 17 – 1 = 773, prime;
: 11*19*90 – 1 = 18809 = 7*2687, which is Coman semiprime of the second kind because 2687 + 7 – 1 = 2693, prime.

(It can be seen that 11*13 and 11*19 are Coman semiprimes of the second kind because 11 + 13 – 1 = 23, prime, and 11 + 19 – 1 = 29, prime)

51. Eight conjectures on chameleonic numbers involving a formula based on the multiples of 30

Abstract. In this paper I make eight conjectures about a type of numbers which I defined in a previous paper, “The notion of chameleonic numbers, a set of composites that «hide» in their inner structure an easy way to obtain primes”, in the following way: the non-null positive composite squarefree integer C not divisible by 2, 3 or 5 is such a number if the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d.

Definition 1:

The non-null positive composite squarefree integer C not divisible by 2, 3 or 5 is a *chameleonic number of first kind* if the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d.

Note: A *Coman semiprime of first kind* (see my previous paper “Eight conjectures on certain type of semiprimes involving a formula based on the multiples of 30”) is a chameleonic number of first kind with two prime factors.

Definition 2:

The non-null positive composite squarefree integer C not divisible by 2, 3 or 5 is a *chameleonic number of second kind* if the absolute value of the number P + d – 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d.

Note: A *Coman semiprime of second kind* (see my previous paper “Eight conjectures on certain type of semiprimes involving a formula based on the multiples of 30”) is a chameleonic number of second kind with two prime factors.

Conjecture 1:

For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a prime.

Example:
(Of such prime, for p = 7, k = 1)

: 1729 = 7*13*19 is a chameleonic number of the first kind because 7*13 – 19 + 1 = 73, prime, 7*19 – 13 + 1 = 121 = 11^2, square of prime and 13*19 – 7 + 1 = 241, prime; also, for m = 1729 and k = 1, n = 30*1729 + 1 = 51871 is a prime.

Conjecture 2:

For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a prime.

Example:
(Of such prime, for p = 7, k = 1)

: 8911 = 7*19*67 is a chameleonic number of the second kind because 7*19 + 67 – 1 = 199, prime, 7*67 + 19 – 1 = 487, prime and 19*67 + 7 – 1 = 1279, prime; also, for m = 8911 and k = 2, n = 60*8911 – 1 = 534659 is a prime.

Conjecture 3:

For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a prime.

Conjecture 4:

For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a prime.

Conjecture 5:

For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a Coman semiprime of the first kind.

Example:
(Of such prime, for p = 7, k = 3)

: 1729 = 7*13*19 is a chameleonic number of the first kind (see above); also, for m = 1729 and k = 3, n = 90*1729 + 1 = 155611 = 61*2551 is a Coman semiprime of the first kind because 2551 – 61 + 1 = 2491 = 47*53 and 53 – 47 + 1 = 7, which is prime.

Conjecture 6:

For any given odd prime p and any k non-null positive integer there exist an infinity of pairs of odd primes [q, r] such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a Coman semiprime of the second kind.

Example:
(Of such prime, for p = 7, k = 1)

: 8911 = 7*19*67 is a chameleonic number of the first kind (see above); also, for m = 8911 and k = 7, n = 240*8911 – 1 = 2138639 = 397*5387 is a Coman semiprime of the second kind because 5387 + 397 – 1 = 5783, which is prime.

Conjecture 7:

For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the first kind and n = 30*k*p*q*r + 1 is a Coman semiprime of the first kind.

Conjecture 8:

For any given pair of distinct odd primes [p, q] and any k non-null positive integer there exist an infinity of odd primes r such that the number m = p*q*r is a chameleonic number of the second kind and n = 30*k*p*q*r – 1 is a Coman semiprime of the second kind.

52. Two conjectures on sequences of primes obtained from the lesser term from a pair of twin primes

Abstract. In this paper I make two conjectures about two types of possible infinite sequences of primes obtained starting from any given prime which is the lesser term from a pair of twin primes for a possible infinite of positive integers which are not of the form 3*k – 1 respectively starting from any given positive integer which is not of the form 3*k – 1 for a possible infinite of lesser terms from pairs of twin primes.

Conjecture 1:

There exist an infinity of primes q of the form q = 2*k*p + 1, where p is a lesser prime from a pair of twin primes, for any positive integer k, under the condition that k has not the digital root equal to 2, 5 or 8.

Examples:
(Of such primes q, for k = 1)

: for p = 18406781, q = 36813563 prime;
: for p = 18407771, q = 36815543 prime;
: for p = 18408749, q = 36817499 prime.

Examples:
(Of such primes q, for k = 3)

: for p = 18403277, q = 110419663 prime;
: for p = 18408287, q = 110449723 prime;
: for p = 18408581, q = 110451487 prime.

Examples:
(Of such primes q, for k = 4)

: for p = 18408287, q = 147250553 prime;
: for p = 18406319, q = 147266969 prime.

Examples:
(Of such primes q, for k = 6)

: for p = 18405719, q = 220868629 prime;
: for p = 18405731, q = 220868773 prime.

Examples:
(Of such primes q, for k = 7)

: for p = 18406979, q = 257697707 prime;
: for p = 18408107, q = 257713499 prime.

Examples:
(Of such primes q, for k = 9)

: for p = 18408749, q = 331357483 prime;
: for p = 18408989, q = 331361803 prime.

Conjecture 2:

There exist an infinity of primes q of the form q = 2*k*p + 1, where k is a positive integer which digital root is not equal to 2, 5 or 8, for any p lesser prime from a pair of twin primes.

Examples:
(Of such primes q, for p = 11)

: for k = 1, q = 23 prime;
: for k = 3, q = 67 prime;
: for k = 4, q = 89 prime;
: for k = 9, q = 199 prime;
: for k = 15, q = 331 prime;
: for k = 16, q = 353 prime;
: for k = 19, q = 419 prime;
: for k = 21, q = 463 prime;
: for k = 28, q = 617 prime;
: for k = 30, q = 661 prime;
: for k = 31, q = 683 prime;
: for k = 33, q = 727 prime;
: for k = 39, q = 859 prime;
: for k = 40, q = 881 prime;
: for k = 43, q = 947 prime;
: for k = 45, q = 991 prime;
: for k = 46, q = 1013 prime;
: for k = 51, q = 1123 prime.
(...)

Examples:
(Of such primes q, for p = 18408749)

: for k = 1, q = 36817499 prime;
: for k = 9, q = 331357483 prime;
: for k = 25, q = 920437451 prime.
(...)

53. A formula based on the lesser prime p from a pair of twin primes that produces semiprimes q*r such that p is equal to r – q + 1

Abstract. In this paper I make an observation about an interesting formula based on the lesser prime p from a pair of twin primes, id est N = p^3 + 3*p^2 + 4*p + 1, that conducts sometimes to the result N = q*r, where q, r are primes such that r – q + 1 = p and sometimes to the result N = q*r, where at least one from q, r or both are composites such that r – q + 1 = p.

Observation:

Let p be the lesser of a pair of twin primes, p > 3; the formula N = p^3 + 3*p^2 + 4*p + 1 conducts sometimes to the result N = q*r, where q, r are primes such that r – q + 1 = p and sometimes to the result N = q*r, where at least one from q, r or both are composites such that r – q + 1 = p.

Verifying the observation:

(For nine from the first thirty such p, the formula conducts to the result mentioned above)

: for p = 5, N = 221 = 13*17
[17 – 13 + 1 = 5 = p];
: for p = 11, N = 1739 = 37*47
[47 – 37 + 1 = 11 = p];
: for p = 29, N = 27029 = 151*179
[179 – 151 + 1 = 29 = p];
: for p = 41, N = 74129 = 11*23*293
[293 – 11*23 + 1 = 41 = p];
: for p = 71, N = 373319 = 577*647
[647 – 577 + 1 = 71 = p];
: for p = 239, N = 13824239 = 11*13*277*349
[11*349 – 13*277 + 1 = 239 = p];
: for p = 419, N = 74088419 = 31*271*8819
[8819 – 31*271 + 1 = 419 = p];
: for p = 461, N = 98611589 = 31*313*10163
[10163 – 31*33 + 1 = 461 = p];
: for p = 599, N = 216000599 = 53*283*14401
[53*283 – 14401 + 1 = 599 = p].

Note that sometimes N is itself a prime:

: for p = 17, N = 5849 prime;
: for p = 149, N = 3375149 prime;
: for p = 191, N = 7078079 prime;
: for p = 197, N = 7762589 prime;
: for p = 227, N = 11852579 prime;
: for p = 347, N = 42144539 prime;
: for p = 431, N = 80621999 prime;
: for p = 521, N = 142237169 prime;
: for p = 641, N = 264609929 prime;
: for p = 659, N = 287496659 prime.

54. Three functions based on the digital sum of a number and ten conjectures

Abstract. In this paper I present three functions based on the digital sum of a number which might be interesting to study and ten conjectures. These functions are: (I) F(x) defined as the digital sum of the number 2^x – x^2; (II) G(x) equal to F(x) – x and (III) H(x) defined as the digital sum of the number 2^x + x^2.

(I)

Let F(x) be the sum of the digits of the number 2^x – x^2, where x is an odd positive number. Then:

Conjecture 1:

There exist an infinity of primes p such that F(p) = p. Such primes p are 13, 61 (...). Note that, up to x = 241, there is no other odd number x for which F(x) = x.

Conjecture 2:

There exist an infinity of pairs of twin primes (p, q) such that F(p) = F(q). Such pairs are (59, 61), (239, 241) (...) with corresponding F(p) = F(q) equal to 61, 331 (...).

Conjecture 3:

There exist an infinity of pairs of primes (p, q) such that F(p) = q. Such pairs are (5, 7), (11, 19), (23, 43), (29, 37), (43, 61), (59, 61), (101, 109), (157, 229), (167, 241), (239, 331), (241, 331) (...).

Conjecture 4:

There exist an infinity of pairs of primes (p, q) such that F(p) = q^2. Such pairs are (31, 7), (83, 11), (103, 11) (...).

Conjecture 5:

There exist an infinity of pairs of primes (p, q) such that F(p^2) = q. Such pairs are (13, 223), (19, 541), (29, 1129) (...).

Conjecture 6:

There exist an infinity of pairs of primes (p, F(p)) such that F(p) - p = 2 (in other words, p and F(p) are twin primes). Such pairs of twin primes are (5, 7), (59, 61) (...).

(II)

Let G(x) = F(x) – x, where x and F(x) are those defined above. Then:

Conjecture 7:

There exist an infinity of pairs of primes (p, F(p)) such that G(p) is a multiple of 9. Such pairs of primes are (43, 61) (...) with corresponding G(p) equal to 18 (...).

Conjecture 8:

There exist an infinity of pairs of primes (p, F(p)) such that G(p) is a power of the number 2. Such pairs of primes are (5, 7), (11, 19), (29, 37), (101, 109) (...) with corresponding exponents (powers of 2): 1, 3, 3, 3 (...).

Conjecture 9:

There exist an infinity of primes p such that G(p) is also prime. Such pairs of primes (p, G(p)) are (17, 5), (41, 11), (47, 17), (53, 23), (71, 5), (113, 47), (173, 53) (...).

Problem 1:

Which is the longest possible sequence of ordered odd numbers n such that F(n) has the same value for all of them? The longest sequence I met is: 75, 81, 87, 93, 99, for all of them F(n) having the value 116.

Problem 2:

Which have in common the odd numbers n for that F(n) is equal to a power of two (such number is the prime 179 for which F(p) = 256)?

(III)

Let H(x) be the sum of the digits of the number 2^x + x^2, where x is an odd positive number. Then:

Conjecture 10:

There exist an infinity of pairs of twin primes (x = 11 + 18*k, y = 13 + 18*k) such that H(x) = H(y). Such pairs of twin primes are: (11, 13), (29, 31), (101, 103), (191, 193), (227, 229), (569, 571) (...) with corresponding H(x) = H(y) equal to: 18, 45, 117, 243, 315, 810 (...).

55. An interesting property of the primes congruent to 1 mod 45 and an ideea for a function

Abstract. In this paper I show a certain property of the primes congruent to 1 mod 45 related to concatenation, namely the following one: concatenating two or three or more of these primes are often obtaied a certain kind of composites, id est composites of the form m*n, where m and n are not necessarily primes, having the property that m + n - 1 is a prime number. Plus, I present an ideea for a function which be interesting to study.

The primes congruent to 1 mod 45 seem to have the following interesting property: concatenating two or three or more of these primes are often obtaied a certain kind of composites, id est composites of the form m*n, where m and n are not necessarily primes, having the property that m + n - 1 is a prime number or a composite which conducts through the same operation to a prime). Let's take the first 11 primes congruent to 1 mod 45 (sequence 142312 in OEIS):

181, 271, 541, 631, 811, 991, 1171, 1531, 1621, 1801, 2161.

We obtain, concatenating to the left the prime 181 with the following ten ones from the sequence above:

: 181271 =
17*10663 and 10663 + 17 - 1 = 59*181 and 181 + 59 - 1 = 239, prime;
: 181541 =
379*479 and 479 + 379 – 1 = 857, prime;
: 181631 =
1219*149 and 1219 + 149 – 1 = 1367, prime; also 181631 = 23*7897 and 23 + 7897 – 1 = 7919, prime;
: 181811 =
133*1367 and 133 + 1367 – 1 = 1499, prime;
: 181991 =
127*1433 and 127 + 1433 – 1 = 1559, prime;
: 1811171 =
563*3217 and 3217 + 562 = 3779, prime;
: 1811531 =
509*3559 and 3559 + 509 – 1 = 4067 = 49*83 and 49 + 83 – 1 = 131, prime;
: 1811621 =
7*258803 and 258803 + 7 – 1 = 258809, prime;
: 1811801 =
661*2741 and 661 + 2741 – 1 = 19*179 and 19 + 179 – 1 = 197, prime;
: 1812161
= 13*139397 and 13 + 139397 – 1 = 139409, prime.

We obtain, concatenating to the right the prime 181 with the following ten ones from the sequence above:

: 271181 prime;
: 541181 prime;
: 631181 prime;

: 811181 =
7*115883 and 7 + 115883 – 1 = 17*6817 and 17 + 6817 – 1 = 6833, prime;
: 991181 prime;
: 1171181 =
17*68893 and 17 + 68893 – 1 = 68909, prime; also 1171181 = 187*6263 and 187 + 6263 – 1 = 6449, prime;
: 1531181 prime;
: 1621181 =
41*39541 and 41 + 39541 – 1 = 39581, prime;
: 1801181 =
893*2017 and 893 + 2017 – 1 = 2909, prime;
: 2161181 = 53*40777 and 53 + 40777 – 1 = 40829, prime.

We obtain, concatenating to the left three consecutive primes from the sequence (first three cases):

: 181271541 =
11*16479231 and 11 + 16479231 – 1 = 16479241, prime; also 181271541 = 33*5493077 and 33 + 5493077 – 1 = 19*289111 and 19 + 289111 – 1 = 289129, prime;
: 271541631 =
53*5123427 and 53 + 5123427 – 1 = 5123479, prime;
: 541631811 =
2897*186963 and 2897 + 186963 – 1 = 189859, prime.

We obtain, concatenating to the left first four consecutive primes from the sequence:

: 181271541631 =
39*6630529 and 39 + 6630529 – 1 = 3*2210189 and 2210189 + 3 – 1 = 181*12211 and 181 + 12211 – 1 = 12391, prime.

We obtain, concatenating to the left first five consecutive primes from the sequence:

: 181271541631811 =
41*4421257112971 and 41 + 4421257112971 – 1 = 947* 4668698113 and 947 + 4668698113 – 1 = 4668699059, prime.

We obtain, concatenating to the left first six consecutive primes from the sequence:

: 181271541631811991 =
3*60423847210603997 and 3 + 60423847210603997 – 1 = 229*263859594806131 and 229 + 263859594806131 – 1 = 263859594806359, prime.

An ideea for a function

Let mc(x) be the function defined on the set of odd positive integers with values in the set of primes in the following way:

: mc(x) = 1 for x = 1;
: mc(x) = x, for x prime;
: for x composite, mc(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),...,p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then mc(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),...,q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then mc(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of mc(x).

Example: let's calculate the value of mc(x) for a randomly selected set of five consecutive odd numbers:

: let x = 181811 = 7*19*1367;
7 + 19 + 1367 – 2 = 1391 = 13*107;
13 + 107 – 1 = 119 = 7*17;
7 + 17 – 1 = 23, prime, so mc(x) = 23;
: let x = 181813; this is prime, so mc(x) = x = 181813;
: let x = 181815 = 3*5*17*23*31;
3 + 5 + 17 + 23 + 31 – 4 = 75 = 3*5*5;
3 + 5 + 5 – 2 = 11, prime, so mc(x) = 11;
: let x = 181817 = 113*1609;
113 + 1609 – 1 = 1721, prime, so mc(x) = 1721;
: let x = 181819 = 11*16529;
11 + 16529 – 1 = 16539 = 3*37*149;
3 + 37 + 149 – 3 = 187 = 11*17;
11 + 17 – 1 = 27 = 3*3*3;
3 + 3 + 3 – 2 = 7, prime, so mc(x) = 7.

Note: it would be interesting to construct a much larger such sequence and study, for instance, the possible relations between the odd integers who share the same value for mc(x) or between those who share the same numbers of iterative steps until the value of mc(x) is found.

56. On the sum of three consecutive values of the MC function

Abstract. In a previous paper I defined the MC(x) function in the following way: Let MC(x) be the function defined on the set of odd positive integers with values in the set of primes such that: MC(x) = 1 for x = 1; MC(x) = x, for x prime; for x composite, MC(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),..., p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then MC(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),..., q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then MC(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of MC(x). In this paper I present a property of this function.

The sequence of the first 100 values of MC(x), for $1 \leq x \leq 199$:

1, 3, 5, 7, 5, 11, 13, 7, 17, 19, 5, 23, 5, 7, 29, 31, 13, 11, 37, 7, 41, 43, 5, 47, 13, 19, 53, 7, 5, 59, 61, 11, 17, 67, 5, 71, 73, 11, 17, 79, 5, 83, 5, 31, 89, 19, 13, 23, 97, 7, 101, 103, 13, 107, 109, 7, 113, 7, 7, 23, 5, 43, 13, 127, 5, 131, 5, 11, 137, 139, 13, 23, 13, 7, 149, 151, 5, 11, 157, 7, 29, 163, 17, 167, 5, 23, 173, 7, 61, 179, 181, 11, 41, 7, 13, 191, 193, 19, 197, 199.

As I mentioned in abstract, in my previous paper “An interesting property of the primes congruent to 1 mod 45 and an ideea for a function” I defined the MC function, without having already found applications for it in diophantine analysis but sensing that for sure such applications do exist.

Calculating the value of MC function for several sets of relatively large, consecutive, odd numbers, I found out that the value of MC function is obtained in fewer steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of the values of MC function for three consecutive numbers (odd, of course, the function is defined only on odd numbers). To exemplify, MC(x), where x = MC(193) + MC(195) + MC(197) is found immediately (in one step), because MC(x) = x = 409, a prime number. In other words, MC(MC(n) + MC(n + 1) + MC(n + 2)) appear to be obtained easier than the average MC(m), where m are the numbers comparable as lenght (number of digits) to MC(n) + MC(n + 1) + MC(n + 2).

Examples:

Let’s consider the consecutive odd numbers 181811, 181813, 181815, 181817, 181819 with the following corresponding values for MC: 23, 181813, 11, 1721, 7.

Let’s calculate MC(23 + 181813 + 11) = MC(181847)
: 181847 = 43*4229;
: 43 + 4229 – 1 = 4271, prime, so is the value of MC(x), obtained in two steps.
Let’s calculate MC(181813 + 11 + 1721) = MC(183545)
: 183545 = 5*36709;
: 5 + 36709 – 1 = 36713, prime, so is the value of MC(x), obtained in two steps.
Let’s calculate MC(11 + 1721 + 7) = MC(1739)
: 1739 = 37*47;
: 37 + 47 – 1 = 83, prime, so is the value of MC(x), obtained in two steps.

Let's consider the consecutive odd numbers 982451651, 982451653, 982451655, 982451657, 982451657 with the following corresponding values for MC: 59, 982451653, 14251, 7873, 787.
Let's calculate MC(59 + 982451653 + 14251) = MC(982465963)

: 982465963 is prime, so is the value of MC(x), obtained in one step.

Let's calculate MC(982451653 + 14251 + 7873) = MC(982473777)

: 982473777 = 3*3*109163753;
: 3 + 3 + 109163753 – 2 = 109163757 = 3*1223*29753;
: 3 + 1223 + 29753 – 2 = 30977, prime, so is the value of MC(x), obtained in three steps.

Let's calculate MC(14251 + 7873 + 787) = MC(22911)

: 22911 = 3*7*1091;
: 3 + 7 + 1091 – 2 = 1099 = 7*157;
: 7 + 157 – 1 = 163, prime, so is the value of MC(x), obtained in three steps.

57. The MC function and three Smarandache type sequences, diophantine analysis

Abstract. In two of my previous papers, namely "An interesting property of the primes congruent to 1 mod 45 and an ideea for a function" respectively "On the sum of three consecutive values of the MC function", I defined the MC function. In this paper I present new interesting properties of three Smarandache type sequences analyzed through the MC function.

As I mentioned in abstract, I already defined the MC function in previous papers, but, in order to be, this paper, self-contained, I shall define the MC function again, as the function defined on the set of odd positive integers with values in the set of primes such that: MC(x) = 1 for x = 1; MC(x) = x, for x prime; for x composite, MC(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),..., p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then MC(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),..., q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then MC(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of MC(x).

1. The concatenated odd sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n – 1).

The first ten terms of the sequence (A019519 in OEIS):

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

Notes:

Florentin Smarandache conjectured that there exist an infinity of prime terms of this sequence. The terms of this sequence are primes for the following values of n: 2, 10, 16, 34, 49, 2570 (the term corresponding to n = 2570 is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.

Analysis through MC function:

Another interesting property of the terms of the concatenated odd sequence could be the following one: seems that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of three consecutive terms of this sequence. To exemplify, MC(x), where x = 1 + 13 + 135, is found immediately (in one step), because MC(x) = x = 149, a prime number.

Examples:

Let's calculate MC(135791113 + 13579111315 + 1357911131517) = MC(1371626033945):

: 1371626033945 = 5*31*33769*262051;

: 5 + 31 + 33769 + 262051 – 1 = 295853, prime, so is the value of MC(x), obtained in just two steps.

Let's calculate MC(13579111315 + 1357911131517 + 135791113151719) = MC(137162603394551):

: 137162603394551 = 7*23*1171*727533421;

: 7 + 23 + 1171 + 727533421 – 3 = 727534619 = 7*103933517;

: 7 + 103933517 – 1 = 103933523, prime, so is the value of MC(x), obtained in just three steps.

2. The concatenated prime sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n primes.

The first ten terms of the sequence (A019518 in OEIS):

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

Notes:

The terms of this sequence are known as Smarandache-Wellin numbers. Also, the Smarandache-Wellin numbers which are primes are named Smarandache-Wellin primes. The first three such numbers are 2, 23 şi 2357; the fourth is a number with 355 digits and there are known only 8 such primes. The 8 known values of n for which through the concatenation of the first n primes we obtain a prime number are 1, 2, 4, 128, 174, 342, 435, 1429. The computer programs not yet found, until n = 10^4, another such a prime. F.S. conjectured that there exist an infinity of prime terms of this sequence.

Analysis through MC function:

Another interesting property of the terms of the concatenated prime sequence could be the following one: seems that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of three consecutive terms of this sequence.

Example:

Let's calculate MC(235711131719 + 23571113171923 + 2357111317192329) = MC(2380918141495971):

: 2380918141495971 = 3*3*7*7*11*11*11*149*4337*6227;

: 3 + 3 + 7 + 7 + 11 + 11 + 11 + 149 + 4337 + 6227 – 9 = 10807 = 101*107;

: 101 + 107 – 1 = 207 = 3*3*23;

: 3 + 3 + 23 – 2 = 27 = 3*3*3;

: 3 + 3 + 3 – 2 = 7, prime, so is the value of MC(x), obtained in just five steps.

3. The pierced chain sequence

Definition:

The sequence obtained in the following way: the first term of the sequence is 101 and every next term is obtained through concatenation of the previous term with the group of digits 0101.

The first seven terms of the sequence (A031982 in OEIS):

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, 10101010101010101010101, 101010101010101010101010101.

Notes:

Kenichiro Kashihara proved that there are no primes obtained through the division of the terms of the sequence by 101 (because, of course, all of them are divisible by 101).

Analysis through MC function:

Another interesting property of the terms of the pierced chain sequence could be the following one: probably that the value of MC function is obtained in just few steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the terms of this sequence, because of the fact that these terms are products of many distinct prime factors (indeed, 1010101 is a product of 3 distinct prime factors, 10101010101 of 6, 101010101010101 of 5 such factors, 1010101010101010101 of 6 such factors, 10101010101010101010101 of 9 and so on). This fact will increase the chances to find easier the value of the function MC(x), usually a prime much smaller than the number x.

58. On the sum of three consecutive values of the MC function

Abstract. In a previous paper I defined the MC(x) function in the following way: Let MC(x) be the function defined on the set of odd positive integers with values in the set of primes such that: MC(x) = 1 for x = 1; MC(x) = x, for x prime; for x composite, MC(x) has the value of the prime which results from the following iterative operation: let x = p(1)*p(2)*...*p(n), where p(1),..., p(n) are its prime factors; let y = p(1) + p(2) +...+ p(n) – (n – 1); if y is a prime, then MC(x) = y; if not, then y = q(1)*q(2)*...*q(m), where q(1),..., q(m) are its prime factors; let z = q(1) + q(2) +...+ q(m) – (m – 1); if z is a prime, then MC(x) = z; if not, it is iterated the operation until a prime is obtained and this is the value of MC(x). In this paper I present a property of this function.

The sequence of the first 100 values of MC(x), for $1 \leq x \leq 199$:

1, 3, 5, 7, 5, 11, 13, 7, 17, 19, 5, 23, 5, 7, 29, 31, 13, 11, 37, 7, 41, 43, 5, 47, 13, 19, 53, 7, 5, 59, 61, 11, 17, 67, 5, 71, 73, 11, 17, 79, 5, 83, 5, 31, 89, 19, 13, 23, 97, 7, 101, 103, 13, 107, 109, 7, 113, 7, 7, 23, 5, 43, 13, 127, 5, 131, 5, 11, 137, 139, 13, 23, 13, 7, 149, 151, 5, 11, 157, 7, 29, 163, 17, 167, 5, 23, 173, 7, 61, 179, 181, 11, 41, 7, 13, 191, 193, 19, 197, 199.

As I mentioned in abstract, in my previous paper "An interesting property of the primes congruent to 1 mod 45 and an ideea for a function" I defined the MC function, without having already found applications for it in diophantine analysis but sensing that for sure such applications do exist.

Calculating the value of MC function for several sets of relatively large, consecutive, odd numbers, I found out that the value of MC function is obtained in fewer steps (in other words, a prime is obtained easier through the iterative operation that defines the function) for the numbers which are equal to the sum of the values of MC function for three consecutive numbers (odd, of course, the function is defined only on odd numbers). To exemplify, MC(x), where x = MC(193) + MC(195) + MC(197) is found immediately (in one step), because MC(x) = x = 409, a prime number. In other words, MC(MC(n) + MC(n + 1) + MC(n + 2)) appear to be obtained easier than the average MC(m), where m are the numbers comparable as lenght (number of digits) to MC(n) + MC(n + 1) + MC(n + 2).

Examples:

Let's consider the consecutive odd numbers 181811, 181813, 181815, 181817, 181819 with the following corresponding values for MC: 23, 181813, 11, 1721, 7.

Let's calculate MC(23 + 181813 + 11) = MC(181847)
: 181847 = 43*4229;
: 43 + 4229 – 1 = 4271, prime, so is the value of MC(x), obtained in two steps.
Let's calculate MC(181813 + 11 + 1721) = MC(183545)
: 183545 = 5*36709;
: 5 + 36709 – 1 = 36713, prime, so is the value of MC(x), obtained in two steps.
Let's calculate MC(11 + 1721 + 7) = MC(1739)
: 1739 = 37*47;
: 37 + 47 – 1 = 83, prime, so is the value of MC(x), obtained in two steps.

Let’s consider the consecutive odd numbers 982451651, 982451653, 982451655, 982451657, 982451657 with the following corresponding values for MC: 59, 982451653, 14251, 7873, 787.

Let’s calculate MC(59 + 982451653 + 14251) = MC(982465963)
: 982465963 is prime, so is the value of MC(x), obtained in one step.

Let’s calculate MC(982451653 + 14251 + 7873) = MC(982473777)
: 982473777 = 3*3*109163753;
: 3 + 3 + 109163753 – 2 = 109163757 = 3*1223*29753;
: 3 + 1223 + 29753 – 2 = 30977, prime, so is the value of MC(x), obtained in three steps.

Let’s calculate MC(14251 + 7873 + 787) = MC(22911)
: 22911 = 3*7*1091;
: 3 + 7 + 1091 – 2 = 1099 = 7*157;
: 7 + 157 – 1 = 163, prime, so is the value of MC(x), obtained in three steps.

59. On the MC function, the squares of primes and the pairs of twin primes

Abstract. In few of my previous papers I defined the MC function. In this paper I mak two conjectures, involving this function, the squares of primes and the pairs of twin primes.

Conjecture 1:

There exist an infinity of primes p such that MC(p^2) = 5.

Examples:

: 3^2:
3 + 3 – 1 = 5 (prime) so MC(3^2)= 5;
: 5^2:
5 + 5 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(5^2);
: 11^2:
11 + 11 – 1 = 21 = 3*7; 3 + 7 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(11^2) ;
: 13^2:
13 + 13 – 1 = 25 = 5*5; 5 + 5 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(13^2);
: 23^2:
23 + 23 – 1 = 45 = 3*3*5; 3 + 3 + 5 – 2 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(23^2);
: 29^2:
29 + 29 – 1 = 57 = 3*19; 3 + 19 – 1 = 21 = 3*7; 3 + 7 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(29^2);
: 41^2:
41 + 41 – 1 = 81 = 3*3*3*3; 3 + 3 + 3 + 3 – 3 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(41^2);
: 43^2:
43 + 43 – 1 = 85 = 5*17; 5 + 17 – 1 = 21 = 3*7; 3 + 7 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(43^2);
: 61^2:
61 + 61 – 1 = 121 = 11*11; 11 + 11 – 1 = 21 = 3*7; 3 + 7 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(61^2);
: 67^2:
67 + 67 – 1 = 133 = 7*19; 7 + 19 – 1 = 25 = 5*5; 5 + 5 – 1 = 9 = 3*3; 3 + 3 – 1 = 5 = MC(67^2).

Note that for 10 from the first 18 odd primes p the value of the MC(p^2) is equal to 5! For the other 8 primes p, id est 7, 17, 19, 31, 37, 47, 53 and 59, the value of MC(p^2) is equal to 13, 13, 37, 61, 73, 13, 13 and 17.

Conjecture 2:

There exist an infinity of pairs of twin primes (p, p + 2) such that MC(p^2) + MC((p + 2)^2) – 1 = q^2, where q is prime.

Examples:

: MC(3^2) + MC(5^2) – 1 = 3^2;
: MC(5^2) + MC(7^2) – 1 = 3^2;
: MC(11^2) + MC(13^2) – 1 = 3^2;
: MC(17^2) + MC(19^2) – 1 = 7^2;
: MC(41^2) + MC(43^2) – 1 = 3^2;
: MC(71^2) + MC(73^2) – 1 = 5^2.

60. A classification of primes in four classes using the MC function

Abstract. In few of my previous papers I defined the MC function. In this paper I make a classification of primes in four classes using a formula involving this function, id est the formula p + MC(p + 2) – 5, where p is prime. The classification is strict, a prime can not belong simultaneously to two classes.

Introduction: In this paper I classify the primes in four classes; the criterion of classification is the value of the number N = p + MC(p + 2) – 5, where p is the namely prime.

Class I: primes p for which p + MC(p + 2) – 5 = p.

: 3 + MC(5) – 5 = 3;
: 7 + MC(9) – 5 = 7;
: 19 + MC(19) – 5 = 19;
: 23 + MC(23) – 5 = 23;
: 43 + MC(45) – 5 = 43;
: 61 + MC(63) – 5 = 67;
: 79 + MC(81) – 5 = 79;
: 83 + MC(85) – 5 = 83;
: 127 + MC(129) – 5 = 127;
: 131 + MC(133) – 5 = 131;
: 151 + MC(153) – 5 = 151;
: 167 + MC(169) – 5 = 167.

Class II: Primes p for which p + MC(p + 2) – 5 = N, where N is prime.

: 5 + MC(7) – 5 = 7;
: 11 + MC(13) – 5 = 19;
: 17 + MC(19) – 5 = 31;
: 41 + MC(43) – 5 = 79;
: 61 + MC(63) – 5 = 67;
: 71 + MC(73) – 5 = 139;
: 73 + MC(75) – 5 = 79;
: 89 + MC(91) – 5 = 103;
: 101 + MC(103) – 5 = 199;
: 107 + MC(109) – 5 = 211;
: 137 + MC(139) – 5 = 271;
: 191 + MC(193) – 5 = 379.

Class III: Primes p for which p + MC(p + 2) – 5 = p + 2, where p + 2 is not prime.

: 13 + MC(15) – 5 = 15;
: 37 + MC(39) – 5 = 39;
: 53 + MC(55) – 5 = 55;
: 97 + MC(99) – 5 = 99;
: 109 + MC(111) – 5 = 111;
: 113 + MC(115) – 5 = 115;

: 157 + MC(159) – 5 = 159;
: 173 + MC(175) – 5 = 175.

Class IV: Primes p for which p + MC(p + 2) – 5 = N, where N is composite and does not belong to the class III.

: 29 + MC(31) – 5 = 55;
: 31 + MC(33) – 5 = 39;
: 47 + MC(13) – 5 = 55;
: 59 + MC(61) – 5 = 115;
: 103 + MC(105) – 5 = 121;
: 139 + MC(141) – 5 = 147;
: 149 + MC(151) – 5 = 295;
: 163 + MC(165) – 5 = 175;
: 179 + MC(181) – 5 = 355;
: 181 + MC(183) – 5 = 187.

Note: From the first 42 odd primes, 32 belong to one of the first three classes, the “regular ones” because they are defined by a formula, and just 10 belong to the fourth class, the “irregular” one.

61. Conjecture that relates both the lesser and the larger term of a pair of twin primes to the same number through two different functions

Abstract. In this paper I conjecture that for any pair of twin primes p and p + 2 there exist an odd positive integer n such that the value of Smarandache function for n is equal to p and the value of MC function for n is equal to p + 2.

Conjecture: For any pair of twin primes p and p + 2, where p ≥ 5, there exist a positive integer n of the form 15 + 18*k such that the value of Smarandache function for n is equal to p and the value of MC function for n is equal to p + 2.

Verifying the conjecture:
(for the first 11 pairs of twin primes):

: For n = 15 the value of Smarandache function is equal to 5 and the value of MC function is equal to 7;
: For n = 33 the value of Smarandache function is equal to 11 and the value of MC function is equal to 13;
: For n = 51 the value of Smarandache function is equal to 17 and the value of MC function is equal to 19;
: For n = 87 the value of Smarandache function is equal to 29 and the value of MC function is equal to 31;
For n = 123 the value of Smarandache function is equal to 41 and the value of MC function is equal to 43;
: For n = 177 the value of Smarandache function is equal to 59 and the value of MC function is equal to 61;
: For n = 213 the value of Smarandache function is equal to 71 and the value of MC function is equal to 73;
: For n = 303 the value of Smarandache function is equal to 101 and the value of MC function is equal to 103;
: For n = 321 the value of Smarandache function is equal to 107 and the value of MC function is equal to 109;
: For n = 411 the value of Smarandache function is equal to 137 and the value of MC function is equal to 139;
: For n = 447 the value of Smarandache function is equal to 149 and the value of MC function is equal to 151.

62. The MC function and other three Smarandache type sequences, diophantine analysis

Abstract. In seven of my previous papers, I defined the MC function and I showed some of its possible applications. In this paper I present new interesting properties of other three Smarandache type sequences analyzed through the MC function.

1. The reverse sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n positive integers, in reverse order.

The first ten terms of the sequence (A000422 in OEIS):

1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321.

Notes:

The primes appear very rare among the terms of this sequence: until now there are only two known, corresponding to n = 82 (a number having 155 digits) şi n = 37765 (a number having 177719 digits).

Analysis through MC function:

Another interesting property of the terms of the reverse sequence could be the following one: seems that the value of MC function for many of these terms is equal to a small prime; for 10 from the first 13 terms the value of MC function is equal to 3 (for 21 and 4321), to 13 (for 321, 54321 and 7654321), to 17 (for 987654321, 1110987654321 and 13121110987654321) and to 37 (for 87654321 and 141312111098765 4321).

2. The “n concatenated n times” sequence

Definition:

S_n is defined as the sequence of the numbers obtained concatenating n times the number n.

The first ten terms of the sequence (A000461 in OEIS):

1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010.

Notes:

The terms of this sequence can’t obviously be primes, all terms of this sequence being repdigit numbers, therefore multiples of repunit numbers.

Analysis through MC function:

Another interesting property of the terms of the “n concatenated n times” sequence could be the following one: seems that the value of MC function for many of the odd terms (the MC function is defined only on odd numbers) is equal to a small prime (41 for 333, 7 for

55555, 241 for 7777777, 19 for 999999999, 107 for 1111111111111111111111, 97 for 13131313131313131313131313 and 13 for 1717171717171717171717171717171717).

3. The back concatenated odd sequence

Definition:

The sequence obtained concatenating the odd numbers in reverse order.

The first seven terms of the sequence (A038395 in OEIS):

1, 31, 531, 7531, 97531, 1197531, 131197531, 15131197531, 1715131197531, 191715131197531.

Analysis through MC function:

Another interesting property of the terms of the back concatenated odd sequence could be the following one: seems that the value of MC function for many of these terms is equal to a small prime; for the first 11 terms the value of MC function is equal to 1 (for 1), to 7 (for 211917151311975531), to 11 (for 531 and 1197531), to 17 (for 97531), to 19 (for 131197531, 1715131197531 and 191715131197531) and to 23 (for 7531 and 15131197531).

This book brings together sixty-two articles regarding primes submitted by the author to the preprint scientific database Vixra, papers on squares of primes, semiprimes, twin primes, sequences of primes, ways to write primes, special classes of composites, formulas for generating large primes, formulas for generating different duplets and triplets of primes, generalizations of the twin primes and de Polignac's conjectures, generalizations of Cunningham chains, Smarandache generalized Fermat numbers, and on many other issues very much related with the study of primes. Finally, in the last eight from these collected papers, I defined a new function, the MC function, and showed some of its possible applications: for instance, I conjectured that for any pair of twin primes (p, p + 2) there exist a positive integer n of the form 15 + 18*k such that the value of Smarandache function for n is equal to p and the value of MC function for n is equal to p+2 and I also made a Diophantine analysis of few Smarandache types sequences using the MC function.

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