An interesting relation between the squares of primes and the number 96 and two conjectures

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Abstract. In this paper I make two conjectures based on the observation of an interesting relation between the squares of primes and the number 96.

Conjecture 1:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of numbers which are primes or squares of primes.

Example:

: for p = 5 are obtained the primes q = 313, 409, 601 (...) for k = 3, 4, 6 (...) and the squares of primes q = 11^2, 37^2 (...) for k = 1, 14 (...).

Conjecture 2:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of semiprimes q = m*n, where m < n, with the following property: the number n – m + 1 is a prime or a square of a prime.

Example:

: for p = 5 are obtained the semiprimes q = 217 = 7*31 (and 31 – 7 + 1 = 5^2) for k = 2, q = 505 = 5*101 (and 101 – 5 + 1 = 97, prime) for k = 5, q = 697 = 17*41 (and 41 – 17 + 1 = 5^2) for k = 7, q = 793 = 13*61 (and 61 – 13 + 1 = 7^2) for k = 8, q = 889 = 7*127 (and 127 – 7 + 1 = 11^2) for k = 9, q = 985 = 5*197 (and 197 – 5 + 1 = 193, prime) for k = 10, q = 1081 = 23*47 (and 47 – 23 + 1 = 5^2) for k = 11, q = 1177 = 11*107 (and 107 – 11 + 1 = 97, prime) for k = 12, q = 1273 = 19*67 (and 67 – 19 + 1 = 7^2) for k = 13, q = 1465 = 5*293 (and 293 – 5 + 1 = 17^2) for k = 15.

Note that, for $p = 5$, were obtained for $1 \leq k \leq 15$ only primes, squares of primes and semiprimes with the property mention above.

Taking randomly a prime, id est 233, is obtained:
: for k = 1, the semiprime q = 329 = 7*47 (47 – 7 + 1 = 41);
: for k = 3, the prime q = 521;
: for k = 4, the prime q = 617;
: for k = 5, the semiprime q = 713 = 23*31 (31 – 23 + 1 = 3^2);
: for k = 6, the prime q = 809.

Taking randomly another prime, id est 769, is obtained:
: for k = 1, the semiprime q = 865 = 5*173 (173 – 5 + 1 = 13^2);
: for k = 2, the square of prime q = 31^2;
: for k = 4, the prime q = 1153;
: for k = 5, the prime q = 1249;
: for k = 7, the semiprime q = 1441 = 11*131 (131 – 11 + 1 = 11^2).

Conclusion:

It is clear from these examples that the formula p^2 + 96*k, where p is prime and k is positive integer, has the property to generate primes, squares of primes and semiprimes with the property shown.