Four conjectures based on the observation of a type of recurrent sequences involving semiprimes

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Abstract. In this paper I make four conjectures starting from the observation of the following recurrent relations: (((p*q – p)*2 – p)*2 – p)...), respectively (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes.

Observation:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes, p < q. Very interesting patterns can be observed between a(i) and b(i) in the case of the same semiprime p*q or between the terms of this recurrence relation for different semiprimes:

Let p*q = 7*13 = 91; then:

: a(1) = 2*91 – 7 = 175;
: a(2) = 2*175 – 7 = 343;
: a(3) = 2*343 – 7 = 679 = 7*97;
: a(4) = 2*679 – 7 = 1351 = 7*193;
: a(5) = 2*1351 – 7 = 2695;
: a(6) = 2*2695 – 7 = 5383 = 7*769
(...)
: b(1) = 2*91 – 13 = 169;
: b(2) = 2*169 – 13 = 325;
: b(3) = 2*325 – 13 = 637;
: b(4) = 2*637 – 13 = 1261 = 13*97;
: b(5) = 2*1261 – 13 = 2509 = 13*193;
: b(6) = 2*2509 – 13 = 5005;
: b(7) = 2*5005 – 13 = 9997 = 13*769
(...)

Note that a(3)/p = b(4)/q = 97, a(4)/p = b(5)/q = 193 and a(6)/p = a(7)/q = 769.

Let p*q = 11*13 = 143; then:

: a(1) = 2*143 – 11 = 275;
: a(2) = 2*275 – 11 = 539;

: a(3) = 2*539 - 11 = 1067 = 11*97;
: a(4) = 2*1067 - 11 = 2123 = 11*193;
: a(5) = 2*2123 - 11 = 4235;
: a(6) = 2*4235 - 11 = 8459 = 11*769;
: a(7) = 2*8459 - 11 = 16907;
: a(8) = 2*16907 - 11 = 33803;
: a(9) = 2*33803 - 11 = 67595;
: a(10)= 2*8459 - 11 = 135179 = 11*12289
(...)
: b(1) = 2*143 - 13 = 273;
: b(2) = 2*273 - 13 = 533 = 13*41;
: b(3) = 2*533 - 13 = 1053;
: b(4) = 2*1053 - 13 = 2093;
: b(5) = 2*2093 - 13 = 4173;
: b(6) = 2*4173 - 13 = 8333 = 13*641;
: b(7) = 2*8333 - 13 = 16653;
: b(8) = 2*16653 - 13 = 33293;
: b(9) = 2*33293 - 13 = 66573;
: b(10) = 2*66573 - 13 = 133133;
: b(11) = 2*66573 - 13 = 266253;
: b(12) = 2*266253 - 13 = 532493 = 13*40961
(...)

Note that, in the case of this semiprime, were obtained for a(i)/p the primes obtained for the first semiprime, id est 97, 193, 769, 12289 (which are primes of the form 6*2^n + 1, see the sequence A039687 in OEIS) but for b(i)/q other primes, id est 41, 641, 40961 ((which are primes of the form 5*2^n + 1, see the sequence A050526 in OEIS).

Let p*q = 7*11 = 77; then:
: a(1) = 2*77 - 7 = 147;
: a(2) = 2*147 - 7 = 287 = 7*41;
: a(3) = 2*287 - 7 = 567;
: a(4) = 2*567 - 7 = 1127;
: a(5) = 2*1127 - 7 = 2247;
: a(6) = 2*2247 - 7 = 4487 = 7*641.
(...)
: b(1) = 2*77 - 11 = 143 then for the following terms see a(i) in the first example of p*q = 11*13.

Let p*q = 193*199; then we obtain, as b(i)/q, the primes 769, 12289 (which are primes of the form 6*2^n + 1, obtained above) but for a(i)/p other set of primes not met before: 397, 3169, 6337 (...). To make things even more complicated, for p*q = 197*199 we obtain, for a(i)/p, the set of primes 397, 3169, 6337 mentioned above but for b(i)/q other set of primes not met before: 3137, 50177 (...), which are primes of the form 49*2^n + 1 (see the sequence A077498 in OEIS). Note also the interesting thing that 397, 3169 and 6337 are all three primes of the form 99*2^n + 1.

Let p*q = 13*233; then we obtain, as a(i)/p, the primes 929, 59393, which are primes of the form 29*2^n + 1. Seems amazing how many possible infinite sequences of primes can be obtained starting from a simple recurrence relation and a randomly chosen pair of distinct odd primes.

Conjecture 1:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of primes of the form a(i)/p as well as an infinity of primes of the form b(i)/q for any pair [p, q].

Conjecture 2:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of pairs [p, q] such that the sequence of primes a(i)/p is the same with the sequence of primes b(i)/q.

Conjecture 3:

There exist an infinity of primes, for k positive integer, of the form n*2^k + 1, for n equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

Conjecture 4:

There exist an infinity of positive integers n such that the sequence n*2^k + 1, where k is positive integer, contains an infinity of primes.