Moments defined by Doo-Sabin and Loop Subdivision Surface Examples

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Abstract

Simple meshes such as the cube, tetrahedron, and tripod frequently appear in the literature to illustrate the concept of subdivision. The formulas for the volume, centroid, and inertia of the sets bounded by subdivision surfaces have only recently been derived. We specify simple meshes and state the moments of degree 0 and 1 defined by the corresponding limit surfaces. We consider the subdivision schemes Doo-Sabin, Loop, and Loop with sharp creases.

In case of Doo-Sabin, the moment of degree 2 is also available for certain simple meshes. The inertia is computed and visualized with respect to the centroid.

Introduction

In this article, we state the volume and centroid of the set bounded by subdivision surfaces generated from various example meshes. Each of the subdivision schemes is treated in a separate section:

- Doo-Sabin
- Loop
- Loop with sharp creases

Each example has the following structure:

• We state the vertex coordinates and topology of the mesh. The specification is omitted if the mesh was already introduced previously.

• The subdivision iteration is illustrated up to a certain level. After one or two rounds of subdivision, the mesh topology admits the application of the volume formula. In that case, the facets are colored based on their contribution to the volume.

• We state the moments of degree 0 and 1 defined by the corresponding subdivision surface. In some cases, we are unable to obtain the volume, or centroid in symbolic form, and revert to numeric precision.

The figures can help to validate alternative implementations of the volume, or centroid formula. The theory and derivation is published in [Hakenberg et al. 2014a], [Hakenberg et al. 2014b], and [Hakenberg et al. 2014c].

Doo Sabin

The Doo-Sabin subdivision scheme is published as [Doo/Sabin 1978]. The algorithm applies to meshes with *n*-gons. One round of Doo-Sabin subdivision requires the contraction of each face in the mesh. The weights to contract an *n*-gon are $\omega_0, \omega_1, \ldots, \omega_{n-1}$. Specifically,

$$\omega_0 = \frac{1}{4n}(n+5), \text{ and } \omega_k = \frac{1}{4n}\left(3 + 2\,\mathrm{Cos}\left[\frac{2\pi k}{n}\right]\right) \text{ for } k = 1, 2, \ldots, n-1.$$

The weights are applied in a rotational fashion. For instance,

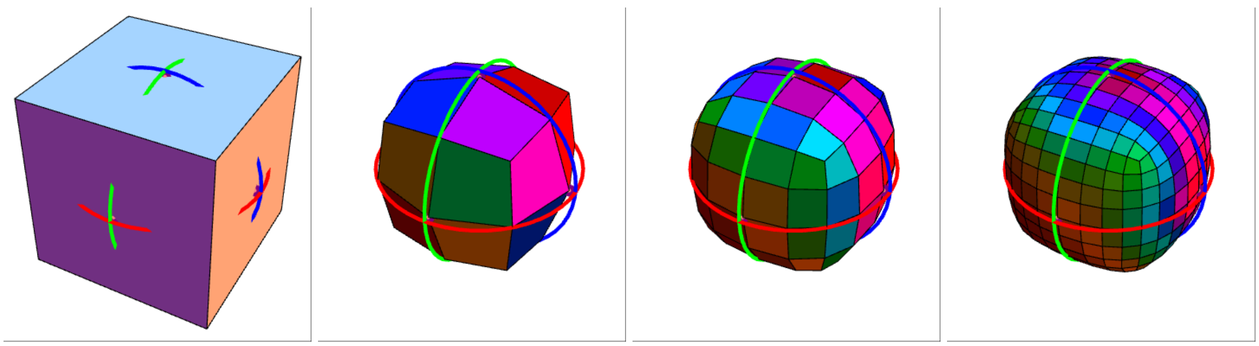
- a triangle is contracted using the mask $\frac{2}{3}, \frac{1}{6}, \frac{1}{6}$ centered at all 3 vertices;
- a quad is contractred using $\frac{9}{16}, \frac{3}{16}, \frac{1}{16}, \frac{3}{16}$;
- for a pentagon, the mask is $\frac{1}{2}, \frac{1}{40}\left(5+\sqrt{5}\right), \frac{1}{40}\left(5-\sqrt{5}\right), \frac{1}{40}\left(5-\sqrt{5}\right), \frac{1}{40}\left(5+\sqrt{5}\right)$;
- a hexagon is contracted using $\frac{11}{24}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{12}, \frac{1}{6}$.

After one round of subdivision, all newly introduced faces are quads.

Cube

Vertices ↓ Faces ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 6 & 5 \\ 3 & 4 & 2 & 1 \\ 4 & 8 & 6 & 2 \\ 5 & 6 & 8 & 7 \\ 7 & 8 & 4 & 3 \\ 7 & 3 & 1 & 5 \end{pmatrix}$$



Required valences $\tau(f) \in \{3\}$

Limit volume ↓ (≈0.629133064516129032258064516129)

$$\frac{6241}{9920}$$

Limit centroid ↓

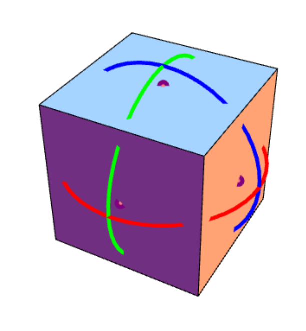
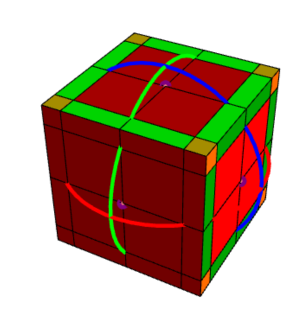
x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
z	$\frac{1}{2}$	(≈0.500000000000000000000000000000)

Limit inertia ↓

xx	$\frac{3\,003\,739\,685\,043\,074\,286\,227\,439\,869}{83\,986\,143\,313\,084\,156\,058\,923\,008\,000}$	(≈0.0357647055401235848573491360067)
yy	$\frac{3\,003\,739\,685\,043\,074\,286\,227\,439\,869}{83\,986\,143\,313\,084\,156\,058\,923\,008\,000}$	(≈0.0357647055401235848573491360067)
zz	$\frac{3\,003\,739\,685\,043\,074\,286\,227\,439\,869}{83\,986\,143\,313\,084\,156\,058\,923\,008\,000}$	(≈0.0357647055401235848573491360067)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

Calibration Unit Cube

The mesh is a degenerate quad mesh that spans the unit cube regardless of subdivision.

Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈1.0000000000000000000000000000000)

1

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
z	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)

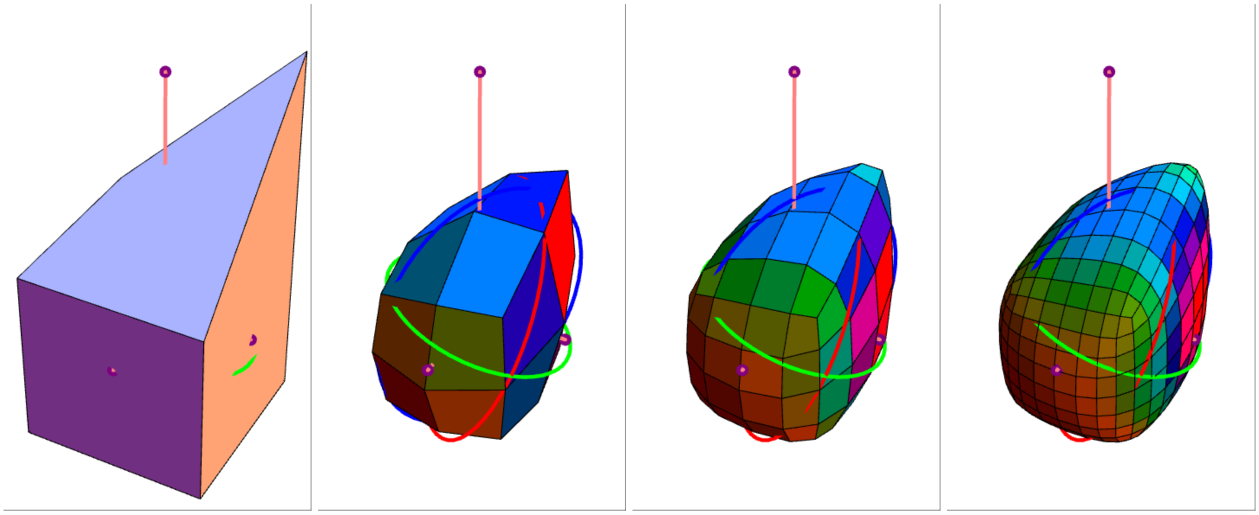
Limit inertia ↓

xx	$\frac{1}{12}$	(≈0.083333333333333333333333333333333)
yy	$\frac{1}{12}$	(≈0.083333333333333333333333333333333)
zz	$\frac{1}{12}$	(≈0.083333333333333333333333333333333)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

Cube Corner Peak

Vertices ↓ Faces ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 6 & 5 \\ 3 & 4 & 2 & 1 \\ 4 & 8 & 6 & 2 \\ 5 & 6 & 8 & 7 \\ 7 & 8 & 4 & 3 \\ 7 & 3 & 1 & 5 \end{pmatrix}$$



Required valences $\tau(f) \in \{3\}$

Limit volume ↓ (≈0.786416330645161290322580645161)

$$\frac{6241}{7936}$$

Limit centroid ↓

x	$\frac{627\,993\,399\,483\,164\,828\,677}{1\,205\,575\,753\,252\,213\,728\,000}$	(≈0.520907456697816331314557529609)
y	$\frac{627\,993\,399\,483\,164\,828\,677}{1\,205\,575\,753\,252\,213\,728\,000}$	(≈0.520907456697816331314557529609)
z	$\frac{1\,534\,164\,279\,236\,156\,416\,837}{2\,411\,151\,506\,504\,427\,456\,000}$	(≈0.636278672284809955392871776991)

Limit inertia ↓

xx	$\frac{427\,784\,612\,308\,930\,843\,724\,906\,407\,382\,835\,854\,313\,790\,629\,557}{9\,643\,015\,046\,420\,933\,761\,756\,270\,849\,140\,931\,763\,109\,888\,000\,000}$	(≈0.0443621222459572763720049539507)
yy	$\frac{427\,784\,612\,308\,930\,843\,724\,906\,407\,382\,835\,854\,313\,790\,629\,557}{9\,643\,015\,046\,420\,933\,761\,756\,270\,849\,140\,931\,763\,109\,888\,000\,000}$	(≈0.0443621222459572763720049539507)
zz	$\frac{19\,808\,587\,557\,249\,130\,167\,993\,211\,438\,400\,948\,117\,740\,841\,568\,143\,411}{250\,448\,386\,785\,644\,491\,660\,333\,866\,493\,888\,279\,751\,490\,011\,136\,000\,000}$	(≈0.0790924941121822515441558368718)
xy	$\frac{64\,008\,728\,463\,039\,337\,305\,264\,250\,059\,837\,085\,308\,480\,168\,667\,601}{62\,612\,096\,696\,411\,122\,915\,083\,466\,623\,472\,069\,937\,872\,502\,784\,000\,000}$	(≈0.00102230610122194275991575350793)
yz	$\frac{1\,463\,826\,682\,547\,567\,633\,387\,208\,877\,142\,931\,075\,009\,881\,779\,372\,181}{125\,224\,193\,392\,822\,245\,830\,166\,933\,246\,944\,139\,875\,745\,005\,568\,000\,000}$	(≈0.0116896475264617120677269079975)
zx	$\frac{1\,463\,826\,682\,547\,567\,633\,387\,208\,877\,142\,931\,075\,009\,881\,779\,372\,181}{125\,224\,193\,392\,822\,245\,830\,166\,933\,246\,944\,139\,875\,745\,005\,568\,000\,000}$	(≈0.0116896475264617120677269079975)

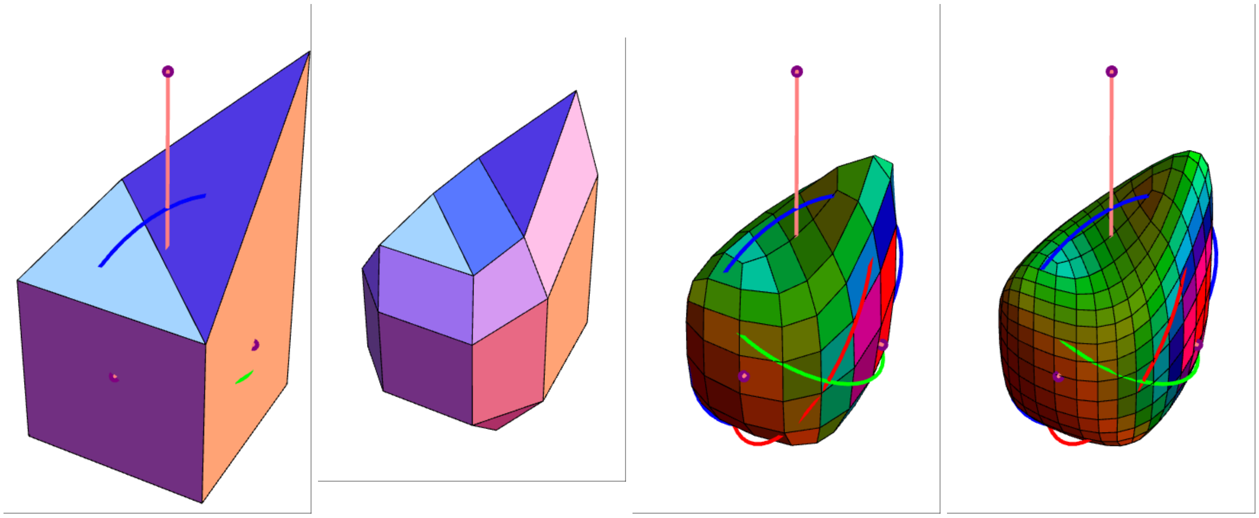
Cube Corner Peak 2

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} \{1, 2, 6, 5\} \\ \{3, 4, 2, 1\} \\ \{4, 8, 6, 2\} \\ \{7, 8, 4, 3\} \\ \{7, 3, 1, 5\} \\ \{5, 6, 7\} \\ \{7, 6, 8\} \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.758672598229580570637521950771)

$$\frac{35\,377\,977\,079\,427}{46\,631\,415\,398\,400}$$

Limit centroid ↓

x	$\frac{55\,916\,010\,814\,194\,887\,959\,473\,263}{106\,683\,757\,678\,488\,047\,763\,625\,920}$	(≈0.524128621178759977480502186022)
y	$\frac{55\,916\,010\,814\,194\,887\,959\,473\,263}{106\,683\,757\,678\,488\,047\,763\,625\,920}$	(≈0.524128621178759977480502186022)
z	$\frac{422\,589\,611\,872\,049\,272\,205\,740\,727}{682\,776\,049\,142\,323\,505\,687\,205\,888}$	(≈0.618928581930912439864044257001)

Limit inertia ↓

xx	$\frac{48\,414\,353\,733\,011\,762\,435\,628\,635\,216\,776\,391\,489\,485\,999\,621\,139\,369\,624\,738\,347}{1\,093\,594\,683\,187\,122\,500\,711\,955\,592\,607\,588\,753\,022\,105\,433\,583\,218\,600\,181\,760\,000}$	(≈0.0442708386181205422016398948742)
yy	$\frac{48\,414\,353\,733\,011\,762\,435\,628\,635\,216\,776\,391\,489\,485\,999\,621\,139\,369\,624\,738\,347}{1\,093\,594\,683\,187\,122\,500\,711\,955\,592\,607\,588\,753\,022\,105\,433\,583\,218\,600\,181\,760\,000}$	(≈0.0442708386181205422016398948742)
zz	$\frac{540\,059\,815\,465\,761\,797\,862\,903\,294\,513\,493\,730\,573\,237\,252\,624\,686\,876\,471\,977\,227}{7\,410\,712\,206\,068\,030\,122\,471\,604\,956\,964\,366\,138\,126\,267\,408\,752\,163\,690\,643\,456\,000}$	(≈0.0728755618149023104119513115424)
xy	$\frac{2\,465\,705\,535\,578\,918\,900\,519\,556\,013\,720\,681\,736\,778\,926\,968\,689\,285\,233\,890\,507}{1\,093\,594\,683\,187\,122\,500\,711\,955\,592\,607\,588\,753\,022\,105\,433\,583\,218\,600\,181\,760\,000}$	(≈0.00225467952019753705257925133187)
yz	$\frac{105\,057\,921\,969\,639\,027\,891\,292\,348\,954\,556\,174\,367\,848\,794\,554\,225\,525\,189\,197\,567}{7\,873\,881\,718\,947\,282\,005\,126\,080\,266\,774\,639\,021\,759\,159\,121\,799\,173\,921\,308\,672\,000}$	(≈0.0133425832035085491896020390529)
zx	$\frac{105\,057\,921\,969\,639\,027\,891\,292\,348\,954\,556\,174\,367\,848\,794\,554\,225\,525\,189\,197\,567}{7\,873\,881\,718\,947\,282\,005\,126\,080\,266\,774\,639\,021\,759\,159\,121\,799\,173\,921\,308\,672\,000}$	(≈0.0133425832035085491896020390529)

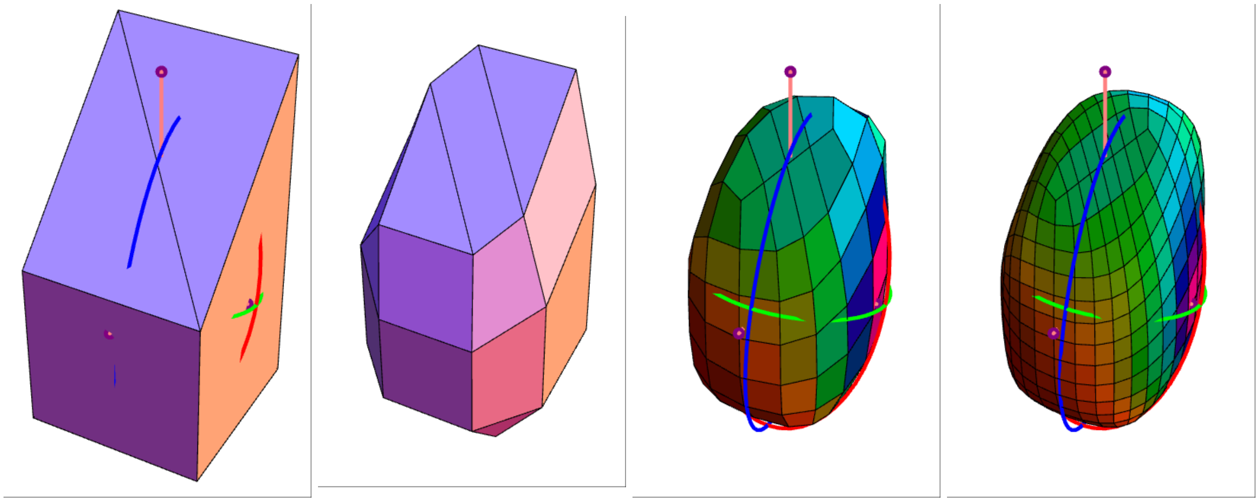
Cube Corner Peak 3

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} \{1, 2, 6, 5\} \\ \{3, 4, 2, 1\} \\ \{4, 8, 6, 2\} \\ \{7, 8, 4, 3\} \\ \{7, 3, 1, 5\} \\ \{5, 6, 7\} \\ \{7, 6, 8\} \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.983070969494803572940479214292)

$$\frac{237\,794\,329}{241\,889\,280}$$

Limit centroid ↓

x	$\frac{69\,289\,696\,314\,592\,766\,937\,088\,127}{138\,238\,425\,026\,921\,933\,909\,395\,200}$	(≈0.501233259139769555833238793341)
y	$\frac{74\,096\,992\,662\,230\,895\,127\,627\,679}{138\,238\,425\,026\,921\,933\,909\,395\,200}$	(≈0.536008657851827416769810251739)
z	$\frac{1\,401\,576\,947\,839\,272\,109\,926\,644\,251}{1\,769\,451\,840\,344\,600\,754\,040\,258\,560}$	(≈0.792096691123458320794256109509)

Limit inertia ↓

xx	$\frac{58\,514\,139\,558\,123\,443\,414\,654\,366\,739\,021\,506\,094\,049\,863\,649\,425\,077\,922\,473}{1\,033\,844\,000\,320\,866\,924\,772\,770\,680\,282\,364\,140\,520\,380\,988\,869\,994\,086\,400\,000}$	(≈0.0565986159807116156388472886852)
yy	$\frac{57\,197\,872\,449\,988\,895\,469\,080\,396\,406\,323\,076\,610\,304\,436\,963\,132\,547\,527\,017}{1\,033\,844\,000\,320\,866\,924\,772\,770\,680\,282\,364\,140\,520\,380\,988\,869\,994\,086\,400\,000}$	(≈0.0553254382984635904648541884627)
zz	$\frac{5\,311\,106\,587\,090\,265\,033\,923\,137\,233\,051\,360\,539\,226\,215\,156\,304\,005\,702\,982\,929}{33\,877\,000\,202\,514\,167\,390\,954\,149\,651\,492\,508\,156\,571\,844\,243\,291\,966\,223\,155\,200}$	(≈0.156776177209932049458838358284)
xy	$\frac{1\,114\,306\,942\,031\,311\,338\,143\,516\,335\,904\,499\,615\,246\,749\,608\,543\,260\,118\,121}{1\,033\,844\,000\,320\,866\,924\,772\,770\,680\,282\,364\,140\,520\,380\,988\,869\,994\,086\,400\,000}$	(≈0.00107782890038097787813930821619)
yz	$\frac{386\,150\,879\,646\,262\,486\,950\,300\,847\,757\,126\,271\,129\,930\,687\,626\,921\,901\,118\,753}{13\,233\,203\,204\,107\,096\,637\,091\,464\,707\,614\,260\,998\,660\,876\,657\,535\,924\,305\,920\,000}$	(≈0.0291804541720039137426425955860)
zx	$\frac{19\,922\,632\,031\,414\,395\,042\,688\,410\,372\,395\,154\,504\,750\,754\,002\,412\,072\,924\,929}{13\,233\,203\,204\,107\,096\,637\,091\,464\,707\,614\,260\,998\,660\,876\,657\,535\,924\,305\,920\,000}$	(≈0.00150550337088688755349793241053)

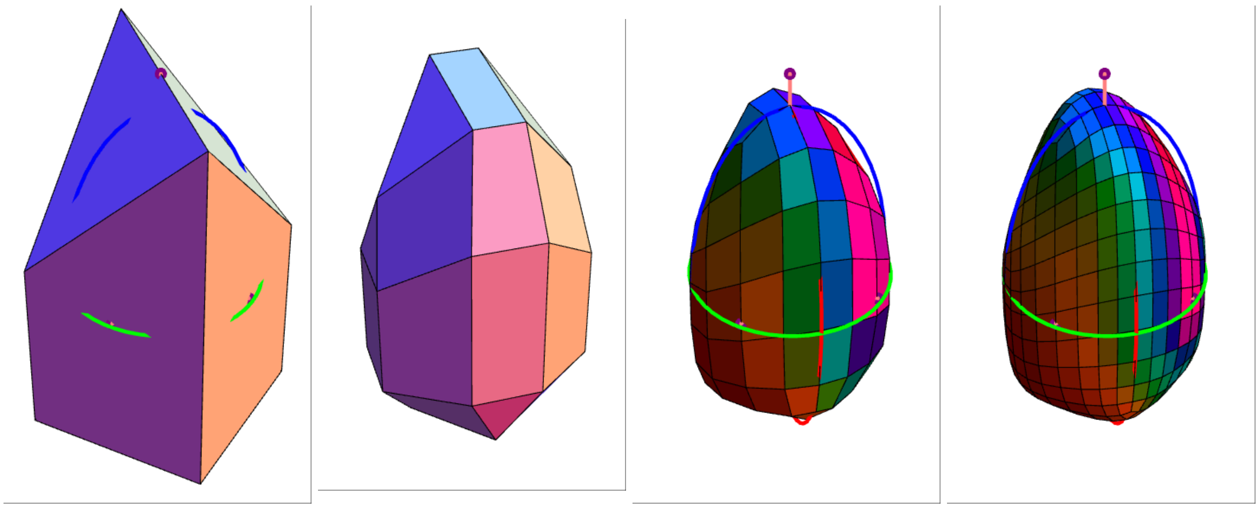
Keil

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} \{1, 2, 6, 5\} \\ \{3, 4, 2, 1\} \\ \{4, 8, 6, 2\} \\ \{7, 8, 4, 3\} \\ \{7, 3, 1, 5\} \\ \{5, 6, 7\} \\ \{7, 6, 8\} \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈1.10417738886031505323290066990)

$$\frac{25\,744\,677\,246\,733}{23\,315\,707\,699\,200}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
z	$\frac{2\,138\,627\,275\,064\,292\,184\,600\,110\,607}{2\,484\,292\,555\,437\,051\,651\,698\,165\,760}$	(≈0.860859672257099382950815016984)

Limit inertia ↓

xx	$\frac{11\,440\,018\,974\,364\,916\,964\,981\,525\,502\,186\,417\,890\,893}{185\,990\,654\,655\,654\,041\,817\,483\,697\,720\,217\,567\,232\,000}$	(≈0.0615085687802171782127473969818)
yy	$\frac{11\,440\,018\,974\,364\,916\,964\,981\,525\,502\,186\,417\,890\,893}{185\,990\,654\,655\,654\,041\,817\,483\,697\,720\,217\,567\,232\,000}$	(≈0.0615085687802171782127473969818)
zz	$\frac{46\,969\,134\,187\,647\,752\,305\,822\,638\,814\,979\,031\,887\,439\,446\,694\,163\,011\,005\,384\,744\,783}{229\,194\,046\,994\,992\,533\,512\,157\,818\,106\,112\,851\,394\,568\,883\,285\,910\,250\,607\,083\,520\,000}$	(≈0.204931737117386565942235121960)
xy	$-\frac{446\,771\,089\,041\,595\,804\,449\,414\,289\,729\,543\,563\,091}{185\,990\,654\,655\,654\,041\,817\,483\,697\,720\,217\,567\,232\,000}$	(≈−0.00240211579376799694113562274670
yz	0	(≈0)
zx	0	(≈0)

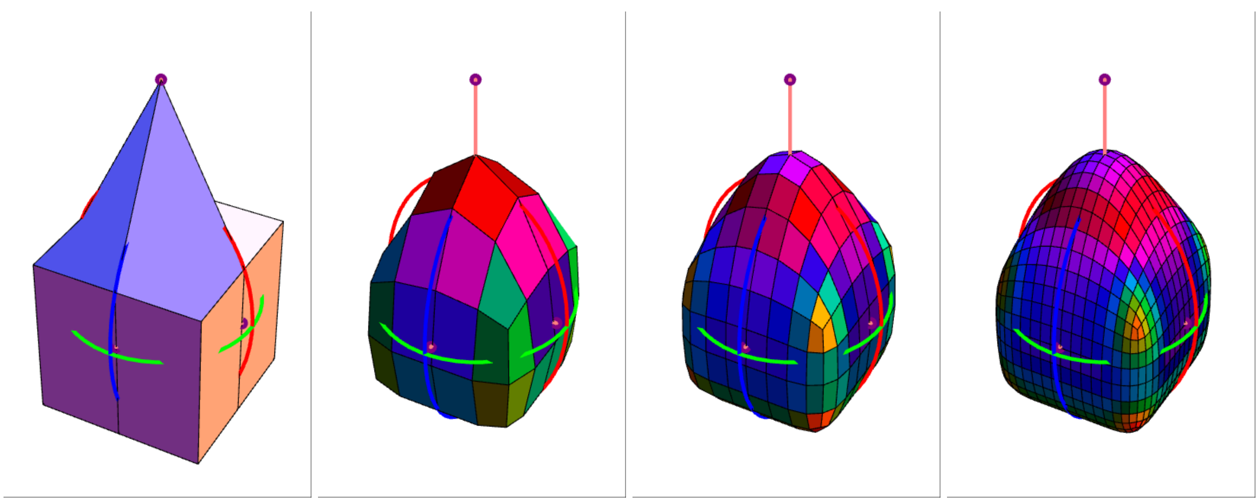
Cube Center Peak Valence 4

Vertices ↓

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 2 & 10 & 11 & 3 \\ 3 & 11 & 12 & 4 \\ 4 & 12 & 13 & 5 \\ 5 & 13 & 14 & 6 \\ 6 & 14 & 15 & 7 \\ 7 & 15 & 16 & 8 \\ 8 & 16 & 9 & 1 \\ 1 & 9 & 10 & 2 \\ 1 & 2 & 3 & 18 \\ 3 & 4 & 5 & 18 \\ 5 & 6 & 7 & 18 \\ 7 & 8 & 1 & 18 \\ 17 & 11 & 10 & 9 \\ 17 & 13 & 12 & 11 \\ 17 & 15 & 14 & 13 \\ 17 & 9 & 16 & 15 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈1.03337808709433694220122126311)

$$\frac{9\,448\,604\,479}{9\,143\,414\,784}$$

Limit centroid ↓

x	0	(≈0)
y	0	(≈0)
z	$\frac{2\,345\,677\,453\,599\,952\,916\,021}{17\,742\,681\,315\,983\,968\,713\,600}$	(≈0.132205353397560417406807776320)

Limit inertia ↓

xx	$\frac{5597547428444281210164177632001053}{83761463582492993324634177168998400}$	(≈0.0668272399864587149681012044145)
yy	$\frac{5597547428444281210164177632001053}{83761463582492993324634177168998400}$	(≈0.0668272399864587149681012044145)
zz	$\frac{589160281821832869373146207554180550650047938332154631}{4423074270549315634970016138659051153108541206691840000}$	(≈0.133201534901801046096741802987)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

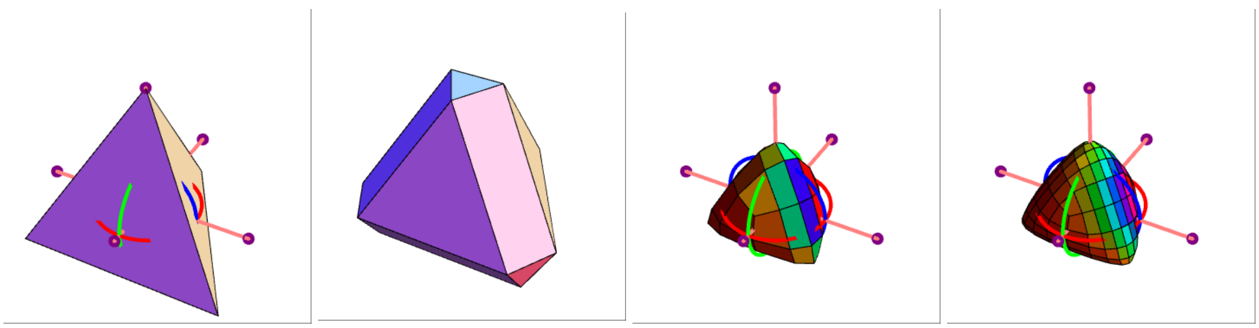
Tetrahedron

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.0615838840593582221075615350861)

$$\frac{20317}{233280\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882254574390251)
z	$\frac{1}{2\sqrt{6}}$	(≈0.204124145231931508183107006225)

Limit inertia ↓

xx	$\frac{597902807693543568419621292558811}{530053011732963473382450652397568000\sqrt{2}}$	(≈0.000797620465221646008395587629377)
yy	$\frac{597902807693543568419621292558811}{530053011732963473382450652397568000\sqrt{2}}$	(≈0.000797620465221646008395587629377)
zz	$\frac{597902807693543568419621292558811}{530053011732963473382450652397568000\sqrt{2}}$	(≈0.000797620465221646008395587629377)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

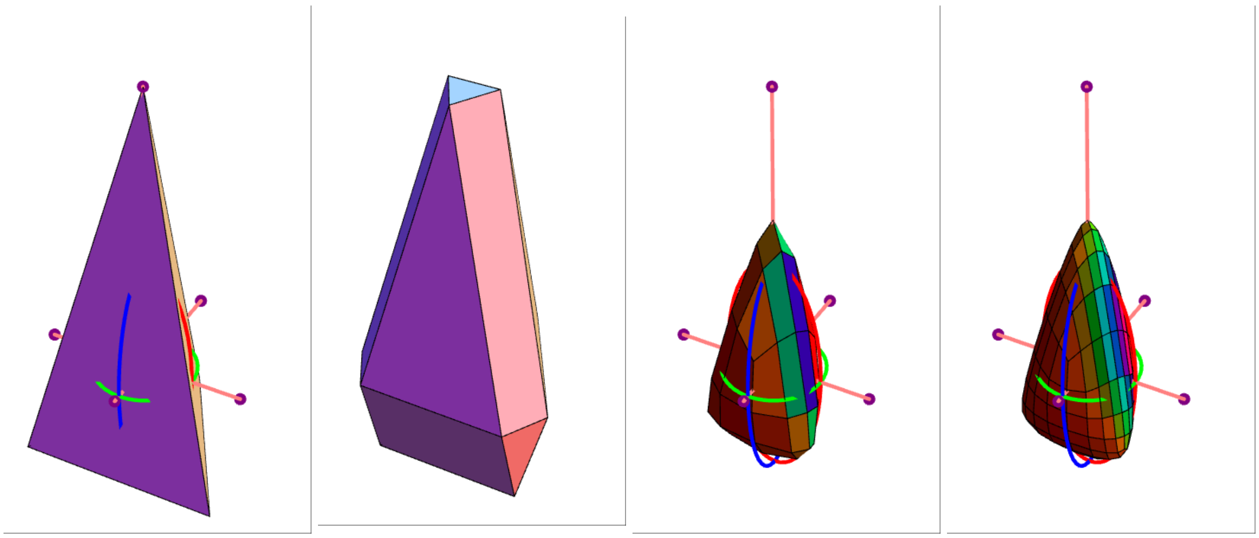
Tetrahedron Peak

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & 2 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.150849092324146433353039811901)

$$\frac{20317}{77760\sqrt{3}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882254574390251)
z	$\frac{1}{2}$	(≈0.500000000000000000000000000000)

Limit inertia ↓

xx	$\frac{597902807693543568419621292558811}{176684337244321157794150217465856000\sqrt{3}}$	(≈0.00195376314819436853295575946095)
yy	$\frac{597902807693543568419621292558811}{176684337244321157794150217465856000\sqrt{3}}$	(≈0.00195376314819436853295575946095)
zz	$\frac{597902807693543568419621292558811}{29447389540720192965691702910976000\sqrt{3}}$	(≈0.0117225788891662111977345567657)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

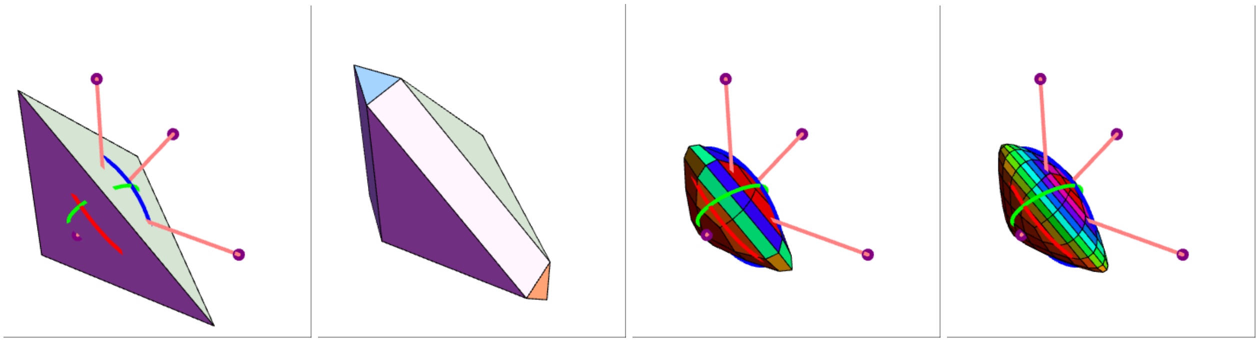
Tetrahedron Axis Aligned

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.0870927640603566529492455418381)

$$\frac{20\,317}{233\,280}$$

Limit centroid ↓

x	$\frac{1}{4}$	(≈0.2500000000000000000000000000000)
y	$\frac{1}{4}$	(≈0.2500000000000000000000000000000)
z	$\frac{1}{4}$	(≈0.2500000000000000000000000000000)

Limit inertia ↓

xx	$\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{353\,368\,674\,488\,642\,315\,588\,300\,434\,931\,712\,000}$	(≈0.00169200851931418405114995235160)
yy	$\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{353\,368\,674\,488\,642\,315\,588\,300\,434\,931\,712\,000}$	(≈0.00169200851931418405114995235160)
zz	$\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{353\,368\,674\,488\,642\,315\,588\,300\,434\,931\,712\,000}$	(≈0.00169200851931418405114995235160)
xy	$-\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{1\,060\,106\,023\,465\,926\,946\,764\,901\,304\,795\,136\,000}$	(≈−0.000564002839771394683716650783867)
yz	$-\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{1\,060\,106\,023\,465\,926\,946\,764\,901\,304\,795\,136\,000}$	(≈−0.000564002839771394683716650783867)
zx	$-\frac{597\,902\,807\,693\,543\,568\,419\,621\,292\,558\,811}{1\,060\,106\,023\,465\,926\,946\,764\,901\,304\,795\,136\,000}$	(≈−0.000564002839771394683716650783867)

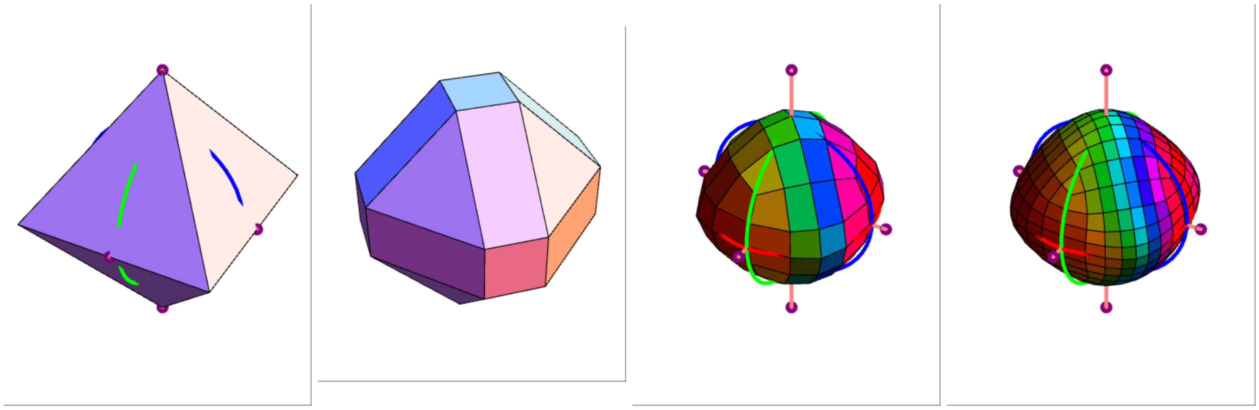
Octahedron

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.327811296353841155440449188121)

$$\frac{93127}{200880\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
z	0	(≈0)

Limit inertia ↓

xx	$\frac{179303402643211906495736507}{10498267914135519507365376000\sqrt{2}}$	(≈0.0120769114425174510262195418489)
yy	$\frac{179303402643211906495736507}{10498267914135519507365376000\sqrt{2}}$	(≈0.0120769114425174510262195418489)
zz	$\frac{179303402643211906495736507}{10498267914135519507365376000\sqrt{2}}$	(≈0.0120769114425174510262195418489)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

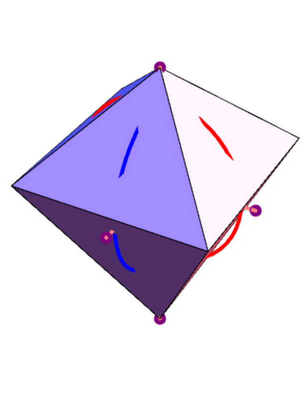
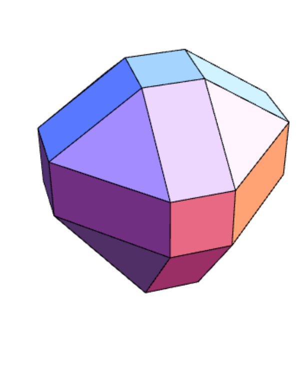
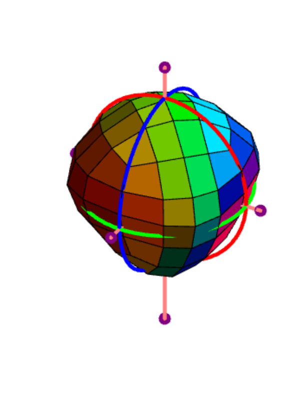
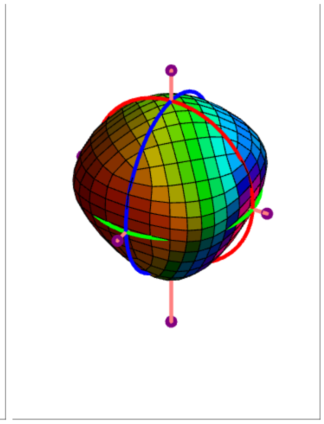
Octahedron Peak

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix}$$

Required valences τ(f) ∈ {3, 4}

`Limit volume ↓` (≈0.347696385902031063321385902031)

$$\frac{93127}{267840}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
z	$-\frac{15666722476505}{138085610373312}$	(≈−0.113456589967269532908886567766)

Limit inertia ↓

xx	$\frac{179303402643211906495736507}{13997690552180692676487168000}$	(≈0.0128094989651902488952276612727)
yy	$\frac{179303402643211906495736507}{13997690552180692676487168000}$	(≈0.0128094989651902488952276612727)
zz	$\frac{19019388286130211232243334940803776733}{1286299674166779259452712064039878656000}$	(≈0.0147861254014934875802567576793)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

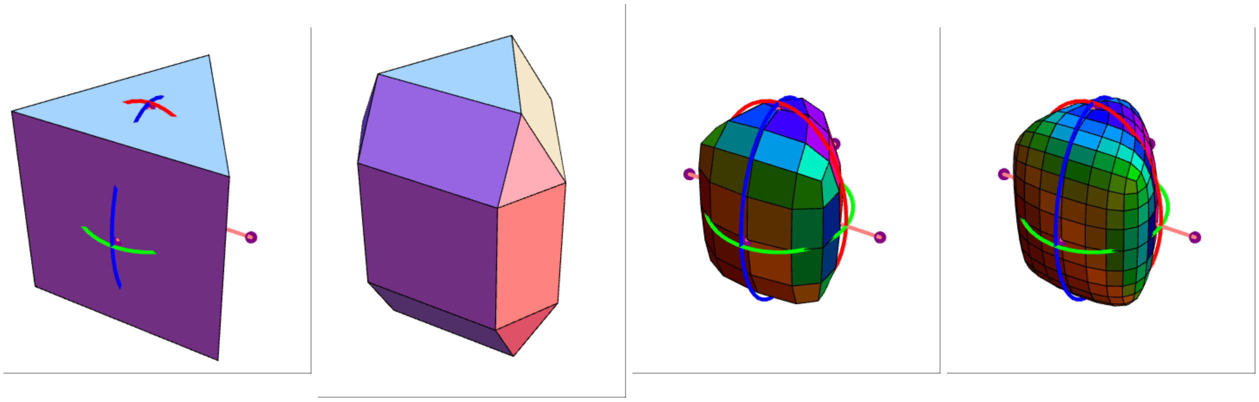
Triquad Prism

`Vertices ↓`

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{pmatrix}$$

`Faces ↓`

$$\begin{pmatrix} \{1, 3, 2\} \\ \{1, 2, 5, 4\} \\ \{4, 5, 6\} \\ \{2, 3, 6, 5\} \\ \{3, 1, 4, 6\} \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈0.248657653187791899128274787041)

$$\frac{18\,229\,\sqrt{3}}{126\,976}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882254574390251)
z	$\frac{1}{2}$	(≈0.500000000000000000000000000000)

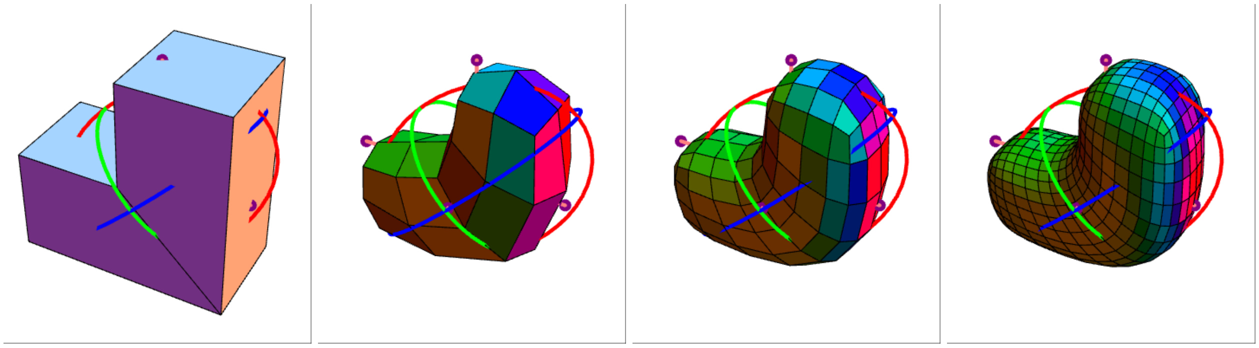
Limit inertia ↓

xx	$\frac{6\,059\,465\,642\,258\,369\,049\,554\,753\,479\,132\,889}{609\,174\,280\,599\,949\,042\,360\,975\,833\,956\,352\,000\,\sqrt{3}}$	(≈0.00574291172676841108976818320530)
yy	$\frac{6\,059\,465\,642\,258\,369\,049\,554\,753\,479\,132\,889}{609\,174\,280\,599\,949\,042\,360\,975\,833\,956\,352\,000\,\sqrt{3}}$	(≈0.00574291172676841108976818320530)
zz	$\frac{6\,923\,595\,538\,498\,090\,777\,834\,534\,458\,681\,019}{279\,204\,878\,608\,309\,977\,748\,780\,590\,563\,328\,000\,\sqrt{3}}$	(≈0.0143168692747655728054442290380)
xy	0	(≈0)
yz	0	(≈0)
zx	0	(≈0)

Corner Reduced

Vertices ↓ Faces ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 2 \\ 2 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 9 & 4 & 3 \\ 2 & 10 & 9 & 1 \\ 3 & 4 & 6 & 5 \\ 5 & 6 & 10 & 2 \\ 5 & 2 & 1 & 3 \\ 4 & 9 & 11 & 7 \\ 10 & 12 & 11 & 9 \\ 7 & 11 & 12 & 8 \\ 4 & 7 & 8 & 6 \\ 6 & 8 & 12 & 10 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4\}$

Limit volume ↓ (≈2.08953293010752688172043010753)

$$\frac{124\,369}{59\,520}$$

Limit centroid ↓

x	$\frac{25\,886\,168\,965\,117\,648\,128\,439}{22\,422\,768\,915\,100\,613\,875\,200}$	(≈1.15445907073878852965792068678)
y	$\frac{1}{2}$	(≈0.50000000000000000000000000000)
z	$\frac{18\,959\,368\,865\,083\,579\,621\,961}{22\,422\,768\,915\,100\,613\,875\,200}$	(≈0.845540929261211470342079313219)

Limit inertia ↓

xx	$\frac{6\,136\,685\,881\,466\,737\,067\,728\,348\,019\,943\,749\,810\,294\,778\,936\,330\,677}{12\,797\,100\,816\,414\,126\,509\,683\,302\,988\,910\,550\,768\,250\,100\,121\,600\,000}$	(≈0.479537199050237442563441149875)
yy	$\frac{7\,265\,536\,540\,662\,374\,467\,193\,622\,269}{55\,990\,762\,208\,722\,770\,705\,948\,672\,000}$	(≈0.129763129738756805857452275439)
zz	$\frac{6\,136\,685\,881\,466\,737\,067\,728\,348\,019\,943\,749\,810\,294\,778\,936\,330\,677}{12\,797\,100\,816\,414\,126\,509\,683\,302\,988\,910\,550\,768\,250\,100\,121\,600\,000}$	(≈0.479537199050237442563441149875)
xy	0	(≈0)
yz	0	(≈0)
zx	$\frac{36\,130\,586\,876\,675\,236\,560\,308\,632\,992\,424\,310\,386\,502\,817\,612\,790\,999}{166\,362\,310\,613\,383\,644\,625\,882\,938\,855\,837\,159\,987\,251\,301\,580\,800\,000}$	(≈0.217180121768329027949647472220)

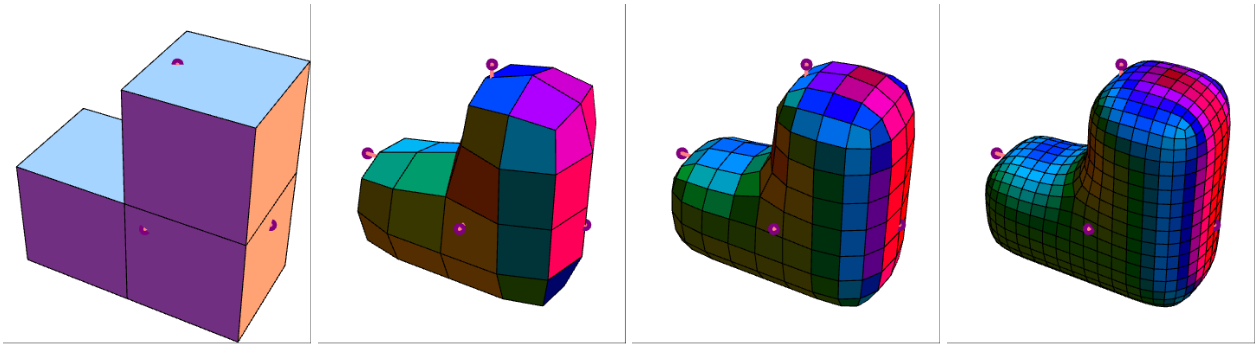
Corner

Vertices ↓

0	0	0
1	0	0
0	1	0
1	1	0
0	0	1
1	0	1
0	1	1
1	1	1
1	0	2
1	1	2
2	0	0
2	1	0
2	0	1
2	1	1
2	0	2
2	1	2

Faces ↓

1	2	6	5
3	4	2	1
5	6	8	7
7	8	4	3
7	3	1	5
2	11	13	6
6	13	15	9
4	12	11	2
12	14	13	11
14	16	15	13
9	15	16	10
6	9	10	8
4	8	14	12
8	10	16	14



Required valences $\tau(f) \in \{3, 4, 5\}$

Limit volume ↓ (≈2.32171405683749043717321019725)

$$\frac{11\,271\,914\,948\,361 + 2\,535\,566\,756\sqrt{5}}{4\,857\,439\,104\,000}$$

Limit centroid ↓

x	1.1776443186794487
y	0.5000000000000002
z	0.8223556813205511

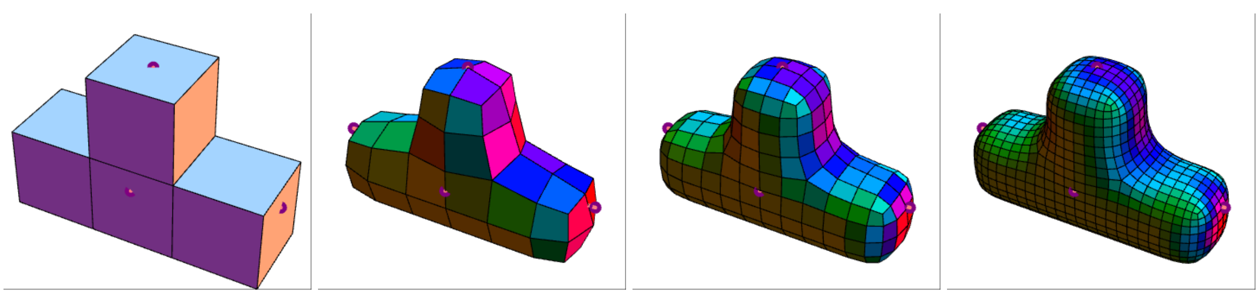
Tetris

Vertices ↓

0	0	0
1	0	0
0	1	0
1	1	0
0	0	1
1	0	1
0	1	1
1	1	1
1	0	2
1	1	2
2	0	0
2	1	0
2	0	1
2	1	1
2	0	2
2	1	2
3	0	0
3	1	0
3	0	1
3	1	1

Faces ↓

1	2	6	5
3	4	2	1
5	6	8	7
7	8	4	3
7	3	1	5
2	11	13	6
6	13	15	9
4	12	11	2
14	16	15	13
9	15	16	10
6	9	10	8
4	8	14	12
8	10	16	14
12	18	17	11
11	17	19	13
14	20	18	12
17	18	20	19
13	19	20	14



Required valences $\tau(f) \in \{3, 4, 5\}$

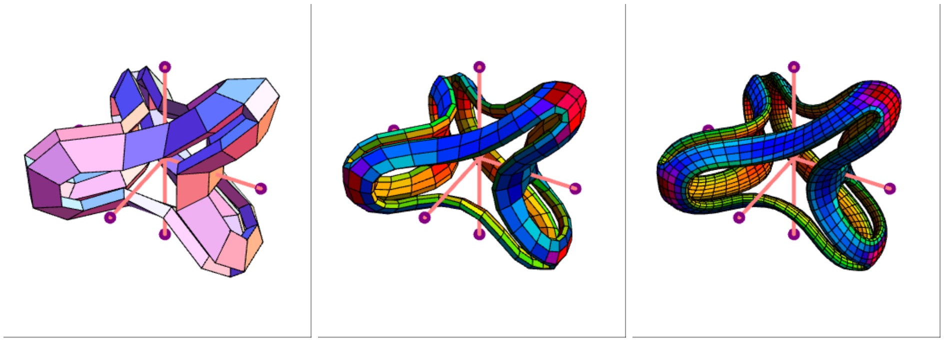
Limit volume ↓ (≈3.18096171582551850875502254504)

$$\frac{7\,719\,994\,213\,761 + 2\,535\,566\,756\,\sqrt{5}}{2\,428\,719\,552\,000}$$

Limit centroid ↓

x 1.4999999999999991
y 0.5
z 0.7406826308260923

Print 11



Required valences τ(f) ∈ {4, 5}

Limit volume ↓

2.995113713508302

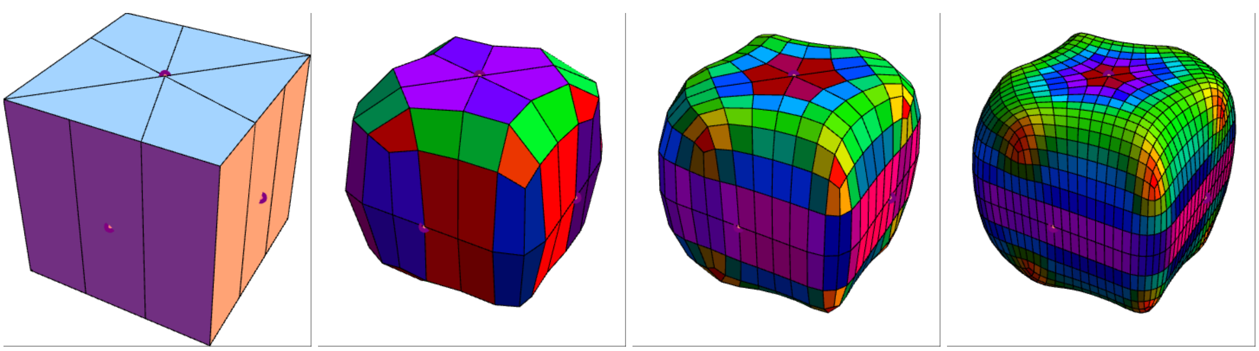
Limit centroid ↓

x −2.5947466159049234*^-16
y −6.862408436447557*^-7
z 6.862408433111454*^-7

Valence 6 Test

Vertices ↓ Faces ↓

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{6} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{6} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{6} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{6} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{6} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}
\quad
\begin{pmatrix} 2 & 14 & 15 & 3 \\ 3 & 15 & 16 & 4 \\ 4 & 16 & 17 & 5 \\ 5 & 17 & 18 & 6 \\ 6 & 18 & 19 & 7 \\ 7 & 19 & 20 & 8 \\ 8 & 20 & 21 & 9 \\ 9 & 21 & 22 & 10 \\ 10 & 22 & 23 & 11 \\ 11 & 23 & 24 & 12 \\ 12 & 24 & 13 & 1 \\ 1 & 13 & 14 & 2 \\ 1 & 2 & 3 & 26 \\ 3 & 4 & 5 & 26 \\ 5 & 6 & 7 & 26 \\ 7 & 8 & 9 & 26 \\ 9 & 10 & 11 & 26 \\ 11 & 12 & 1 & 26 \\ 25 & 15 & 14 & 13 \\ 25 & 17 & 16 & 15 \\ 25 & 19 & 18 & 17 \\ 25 & 21 & 20 & 19 \\ 25 & 23 & 22 & 21 \\ 25 & 13 & 24 & 23 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 6\}$

Limit volume ↓ (≈0.880212271536190020253840233332)

$$\frac{12\,934\,520\,191}{14\,694\,773\,760}$$

Limit centroid ↓

x	0	(≈0)
y	0	(≈0)
z	0	(≈0)

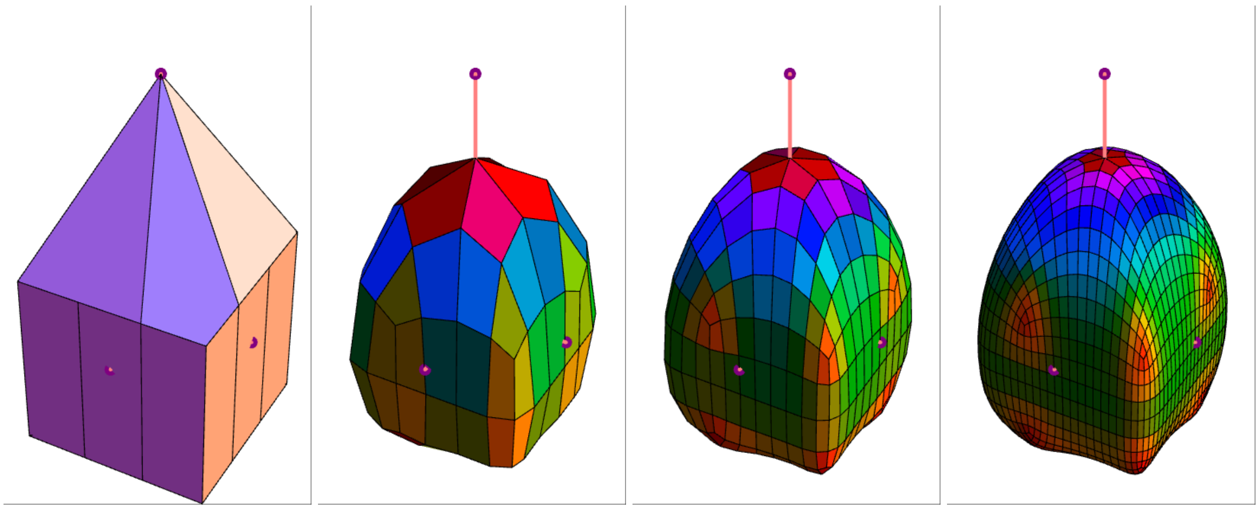
Cube Center Peak Valence 6

Vertices ↓

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{6} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{6} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{6} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{6} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{6} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{6} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 2 & 14 & 15 & 3 \\ 3 & 15 & 16 & 4 \\ 4 & 16 & 17 & 5 \\ 5 & 17 & 18 & 6 \\ 6 & 18 & 19 & 7 \\ 7 & 19 & 20 & 8 \\ 8 & 20 & 21 & 9 \\ 9 & 21 & 22 & 10 \\ 10 & 22 & 23 & 11 \\ 11 & 23 & 24 & 12 \\ 12 & 24 & 13 & 1 \\ 1 & 13 & 14 & 2 \\ 1 & 2 & 3 & 26 \\ 3 & 4 & 5 & 26 \\ 5 & 6 & 7 & 26 \\ 7 & 8 & 9 & 26 \\ 9 & 10 & 11 & 26 \\ 11 & 12 & 1 & 26 \\ 25 & 15 & 14 & 13 \\ 25 & 17 & 16 & 15 \\ 25 & 19 & 18 & 17 \\ 25 & 21 & 20 & 19 \\ 25 & 23 & 22 & 21 \\ 25 & 13 & 24 & 23 \end{pmatrix}$$



Required valences $\tau(\mathrm{f}) \in \{3, 4, 6\}$

Limit volume ↓ (≈1.14470422828010906638520865427)

$$\frac{20\,017\,191\,891\,463}{17\,486\,780\,774\,400}$$

Limit centroid ↓

x	0	(≈0)
y	0	(≈0)
z	$\frac{450\,407\,919\,300\,636\,127\,377\,007}{2\,874\,412\,982\,913\,351\,130\,130\,880}$	(≈0.156695618193363004861888707717)

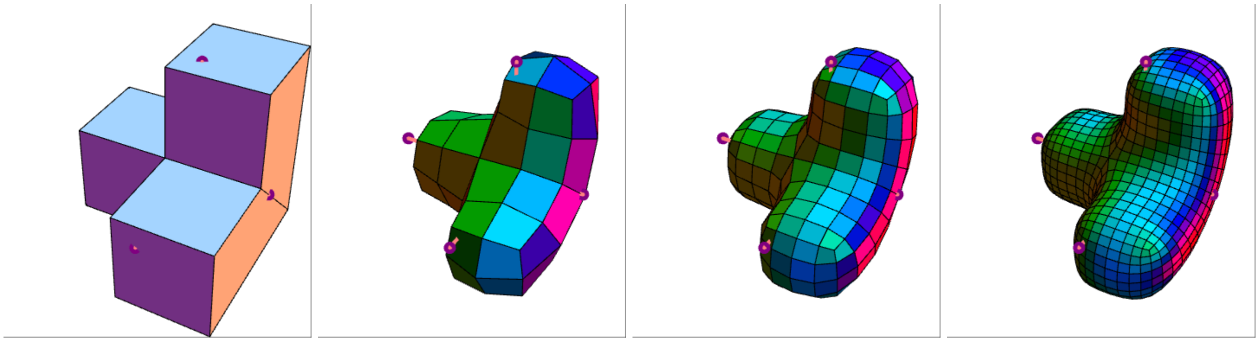
Tripod 346

Vertices ↓

0	0	0
1	0	0
0	1	0
0	0	1
1	0	1
0	1	1
1	1	1
1	0	2
1	1	2
2	1	0
2	0	1
2	0	2
2	1	2
1	−1	0
2	−1	0
1	−1	1
2	−1	1

Faces ↓

1	2	5	4
3	10	2	1
4	5	7	6
6	7	10	3
6	3	1	4
5	11	12	8
10	13	12	11
8	12	13	9
5	8	9	7
7	9	13	10
14	15	17	16
16	17	11	5
10	11	17	15
10	15	14	2
14	16	5	2



Required valences $\tau(f) \in \{3, 4, 6\}$

Limit volume ↓ (≈3.01127602411213680336818545055)

$$\frac{1\,032\,659\,545\,961\,971\,813}{342\,930\,883\,018\,752\,000}$$

Limit centroid ↓

x	$\frac{289\,659\,596\,693\,831\,389\,450\,591\,610\,918\,172\,289}{232\,340\,825\,596\,133\,315\,440\,945\,477\,782\,862\,992}$	(≈1.24670124568349640015667580018)
y	$\frac{57\,318\,771\,097\,698\,074\,009\,646\,133\,135\,309\,297}{232\,340\,825\,596\,133\,315\,440\,945\,477\,782\,862\,992}$	(≈0.246701245683496400156675800182)
z	$\frac{175\,022\,054\,498\,435\,241\,431\,299\,344\,647\,553\,695}{232\,340\,825\,596\,133\,315\,440\,945\,477\,782\,862\,992}$	(≈0.753298754316503599843324199818)

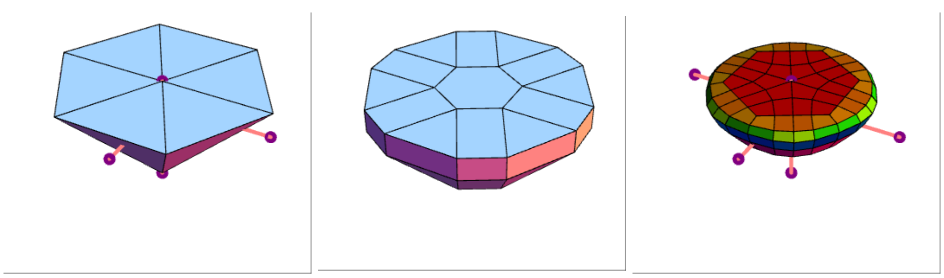
Diamond 6

Vertices ↓

$$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 8 \\ 2 & 3 & 8 \\ 3 & 4 & 8 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \\ 6 & 1 & 8 \\ 2 & 1 & 7 \\ 3 & 2 & 7 \\ 4 & 3 & 7 \\ 5 & 4 & 7 \\ 6 & 5 & 7 \\ 1 & 6 & 7 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 6\}$

Limit volume ↓ (≈0.662250171635437626307901384715)

$$\frac{2\,457\,809}{2\,142\,720\,\sqrt{3}}$$

Limit centroid ↓

x	0	(≈0)
y	0	(≈0)
z	$-\frac{4672633358307091}{20043964776091872}$	(≈−0.233119216208189263052640341360)

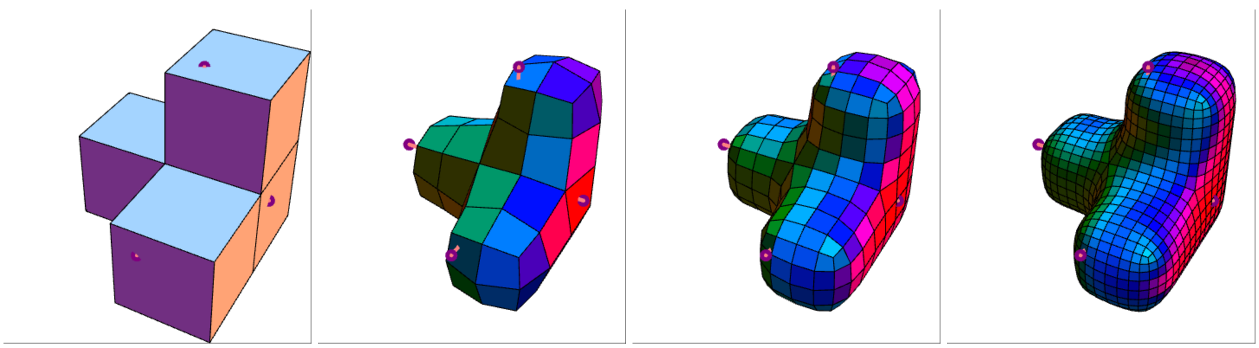
Tripod

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & -1 & 0 \\ 1 & -1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 6 & 5 \\ 3 & 4 & 2 & 1 \\ 5 & 6 & 8 & 7 \\ 7 & 8 & 4 & 3 \\ 7 & 3 & 1 & 5 \\ 6 & 13 & 15 & 9 \\ 4 & 12 & 11 & 2 \\ 12 & 14 & 13 & 11 \\ 14 & 16 & 15 & 13 \\ 9 & 15 & 16 & 10 \\ 6 & 9 & 10 & 8 \\ 4 & 8 & 14 & 12 \\ 8 & 10 & 16 & 14 \\ 17 & 18 & 20 & 19 \\ 19 & 20 & 13 & 6 \\ 11 & 13 & 20 & 18 \\ 11 & 18 & 17 & 2 \\ 17 & 19 & 6 & 2 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 5, 6\}$

Limit volume ↓ (≈3.20028781912720339769529916684)

$$\frac{10357799098161 + 2535566756\sqrt{5}}{3238292736000}$$

Limit centroid ↓

x	1.2614812793758137
y	0.26148127937581384
z	0.7385187206241858

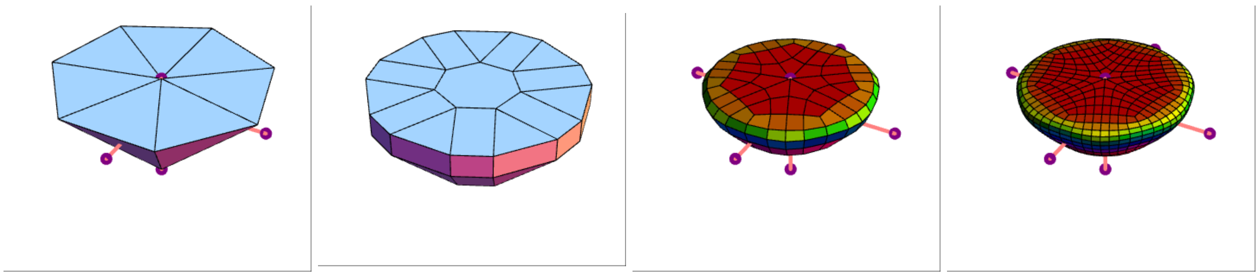
Diamond 7

Vertices ↓

-0.433884	-0.900969	0.
0.433884	-0.900969	0.
0.974928	-0.222521	0.
0.781831	0.62349	0.
0.	1.	0.
-0.781831	0.62349	0.
-0.974928	-0.222521	0.
0.	0.	-1.
0.	0.	0.

Faces ↓

1	2	9
2	3	9
3	4	9
4	5	9
5	6	9
6	7	9
7	1	9
2	1	8
3	2	8
4	3	8
5	4	8
6	5	8
7	6	8
1	7	8



Required valences $\tau(f) \in \{3, 4, 7\}$

Limit volume ↓

0.7132255589220029

Limit centroid ↓

x	-1.8241671105408113*^-18
y	-7.783113004974128*^-17
z	-0.23452863898999224

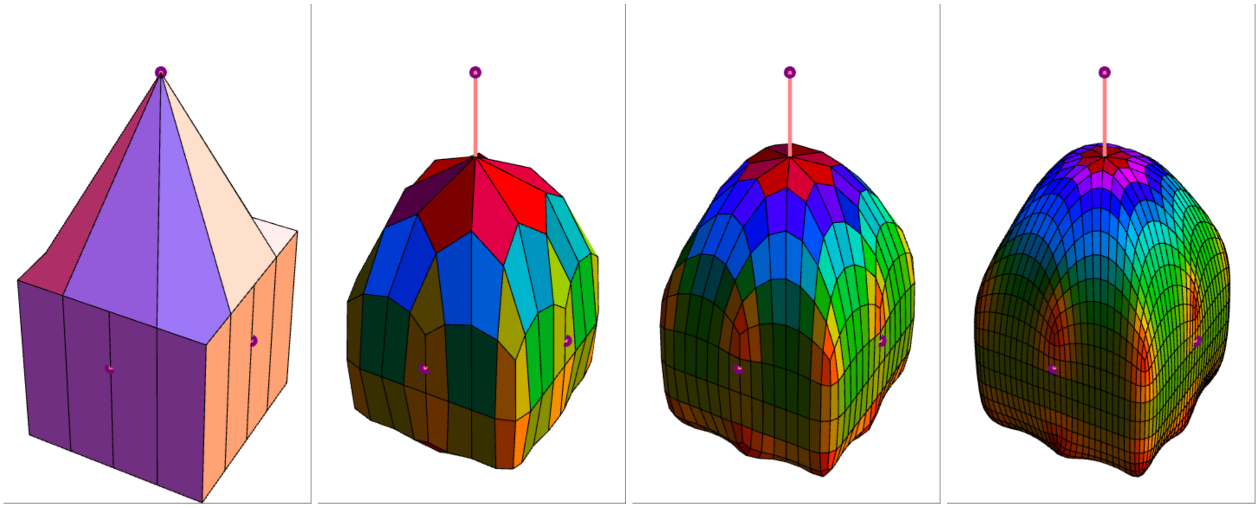
Cube Center Peak Valence 8

Vertices ↓

$$\begin{pmatrix}
\frac{1}{2} & -\frac{1}{4} & \frac{1}{2} \\
\frac{1}{2} & 0 & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & \frac{1}{2} \\
-\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\
-\frac{1}{2} & 0 & \frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{4} & \frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
-\frac{1}{4} & -\frac{1}{2} & \frac{1}{2} \\
0 & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{4} & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \\
\frac{1}{2} & 0 & -\frac{1}{2} \\
\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} \\
0 & \frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} \\
-\frac{1}{2} & 0 & -\frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} \\
0 & -\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{4} & -\frac{1}{2} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
0 & 0 & -\frac{1}{2} \\
0 & 0 & \frac{3}{2}
\end{pmatrix}$$

Faces ↓

$$\begin{pmatrix}
2 & 18 & 19 & 3 \\
3 & 19 & 20 & 4 \\
4 & 20 & 21 & 5 \\
5 & 21 & 22 & 6 \\
6 & 22 & 23 & 7 \\
7 & 23 & 24 & 8 \\
8 & 24 & 25 & 9 \\
9 & 25 & 26 & 10 \\
10 & 26 & 27 & 11 \\
11 & 27 & 28 & 12 \\
12 & 28 & 29 & 13 \\
13 & 29 & 30 & 14 \\
14 & 30 & 31 & 15 \\
15 & 31 & 32 & 16 \\
16 & 32 & 17 & 1 \\
1 & 17 & 18 & 2 \\
1 & 2 & 3 & 34 \\
3 & 4 & 5 & 34 \\
5 & 6 & 7 & 34 \\
7 & 8 & 9 & 34 \\
9 & 10 & 11 & 34 \\
11 & 12 & 13 & 34 \\
13 & 14 & 15 & 34 \\
15 & 16 & 1 & 34 \\
33 & 19 & 18 & 17 \\
33 & 21 & 20 & 19 \\
33 & 23 & 22 & 21 \\
33 & 25 & 24 & 23 \\
33 & 27 & 26 & 25 \\
33 & 29 & 28 & 27 \\
33 & 31 & 30 & 29 \\
33 & 17 & 32 & 31
\end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 8\}$

Limit volume ↓ (≈1.15960211095749528415279495124)

$$\frac{7\,148\,771\,762\,558\,972\,189 + 6\,483\,464\,155\,703\,889\,\sqrt{2}}{6\,172\,755\,894\,337\,536\,000}$$

Limit centroid ↓

x	0.
y	0.
z	0.15916545290853934

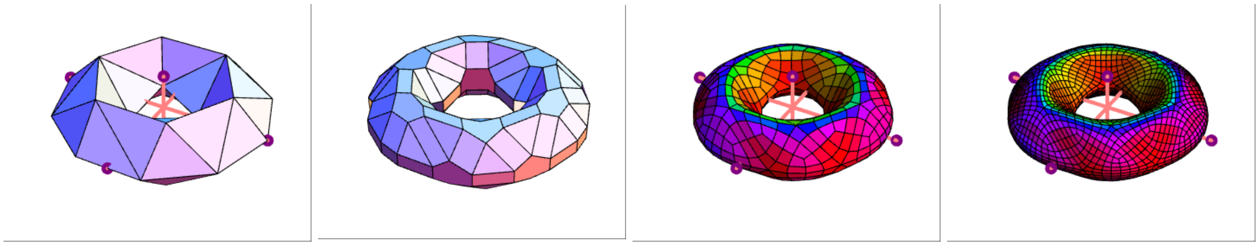
Torus

Vertices ↓

$\frac{3}{2}$	0	0
1.21353	0.881678	0
0.463525	1.42658	0
−0.463525	1.42658	0
−1.21353	0.881678	0
$-\frac{3}{2}$	0	0
−1.21353	−0.881678	0
−0.463525	−1.42658	0
0.463525	−1.42658	0
1.21353	−0.881678	0
1	0	$\frac{1}{2}$
$\frac{1}{2}$	0.866025	$\frac{1}{2}$
$-\frac{1}{2}$	0.866025	$\frac{1}{2}$
−1	0	$\frac{1}{2}$
$-\frac{1}{2}$	−0.866025	$\frac{1}{2}$
$\frac{1}{2}$	−0.866025	$\frac{1}{2}$
$\frac{1}{2}$	0	0
$-\frac{1}{4}$	0.433013	0
$-\frac{1}{4}$	−0.433013	0
1	0	$-\frac{1}{2}$
$\frac{1}{2}$	0.866025	$-\frac{1}{2}$
$-\frac{1}{2}$	0.866025	$-\frac{1}{2}$
−1	0	$-\frac{1}{2}$
$-\frac{1}{2}$	−0.866025	$-\frac{1}{2}$
$\frac{1}{2}$	−0.866025	$-\frac{1}{2}$

Faces ↓

3	4	13
12	2	3
18	13	14
23	18	19
21	20	17
22	4	3
24	19	25
8	24	9
7	6	23
6	14	5
14	13	5
11	1	2
12	3	13
13	4	5
18	12	13
11	2	12
10	16	9
19	18	14
17	11	12
18	17	12
2	20	21
23	5	22
2	1	20
3	2	21
22	3	21
22	5	4
24	23	19
21	17	18
18	22	21
23	6	5
18	23	22
24	7	23
7	24	8
24	25	9
25	10	9
25	19	17
25	20	10
10	20	1
25	17	20
16	10	11
19	16	17
16	15	9
19	14	15
17	16	11
19	15	16
7	14	6
15	14	7
15	8	9
15	7	8
10	1	11



Required valences $\tau(f) \in \{3, 4, 6, 7, 8\}$

```
Limit volume ↓
```

```
2.7972060102776015
```

Limit centroid ↓

```
x    0.000016815364736807547
y    -3.9690427538977556*^-17
z    9.922606884744389*^-18
```

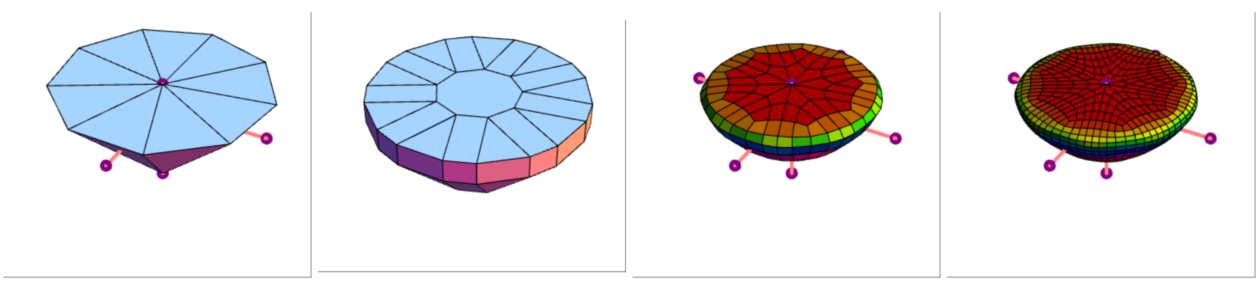
Diamond 9

Vertices ↓

$$\begin{pmatrix} -0.34202 & -0.939693 & 0. \\ 0.34202 & -0.939693 & 0. \\ 0.866025 & -0.5 & 0. \\ 0.984808 & 0.173648 & 0. \\ 0.642788 & 0.766044 & 0. \\ 0. & 1. & 0. \\ -0.642788 & 0.766044 & 0. \\ -0.984808 & 0.173648 & 0. \\ -0.866025 & -0.5 & 0. \\ 0. & 0. & -1. \\ 0. & 0. & 0. \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 11 \\ 2 & 3 & 11 \\ 3 & 4 & 11 \\ 4 & 5 & 11 \\ 5 & 6 & 11 \\ 6 & 7 & 11 \\ 7 & 8 & 11 \\ 8 & 9 & 11 \\ 9 & 1 & 11 \\ 2 & 1 & 10 \\ 3 & 2 & 10 \\ 4 & 3 & 10 \\ 5 & 4 & 10 \\ 6 & 5 & 10 \\ 7 & 6 & 10 \\ 8 & 7 & 10 \\ 9 & 8 & 10 \\ 1 & 9 & 10 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 9\}$

Limit volume ↓

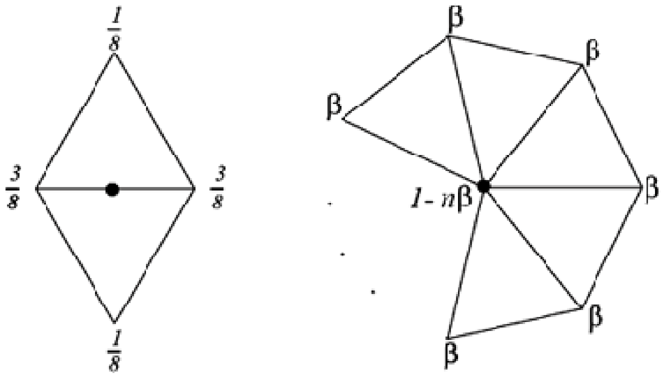
```
0.7731481089612686
```

Limit centroid ↓

```
x    1.1218571550976328*^-17
y    0.
z    -0.2361023748509849
```

Loop

The Loop subdivision scheme is published as [Loop 1987]. The algorithm applies to meshes with triangles. The weights for the insertion of an edge midpoint, as well as the repositioning of a vertex that already existed in the input mesh are

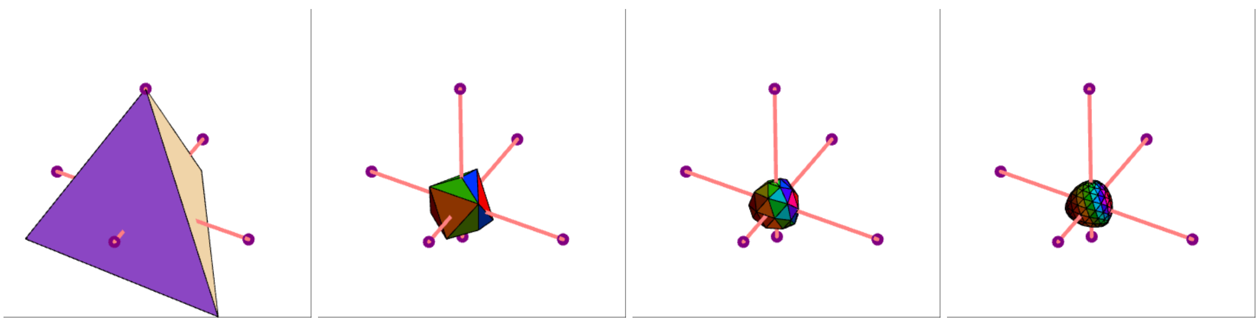


where $\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\,\mathrm{Cos}\left[\frac{2\pi}{n}\right]\right)^2\right)$.

Tetrahedron

Vertices ↓ Faces ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 6\}$

Limit volume ↓ (≈0.00455169559584284472638894206836)

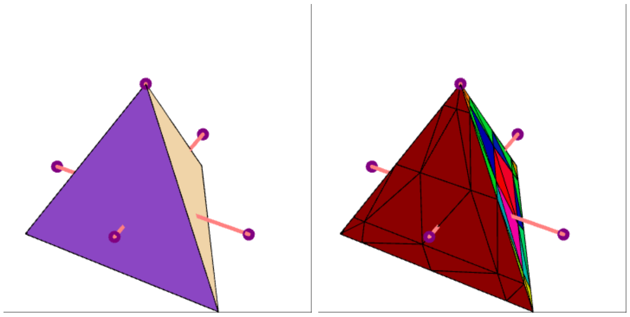
$$\frac{44\,192\,429\,513\,855\,101}{6\,865\,302\,375\,425\,894\,400\,\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882254574390251)
z	$\frac{1}{2\sqrt{6}}$	(≈0.204124145231931508183107006225)

Calibration Tetrahedron

The mesh is a degenerate triangle mesh that spans the tetrahedron regardless of subdivision.



Required valences $\tau(f) \in \{3, 6\}$

Limit volume ↓

0.117851

Limit centroid ↓

x	0.49999999999999967
y	0.28867513459481275
z	0.20412414523193134

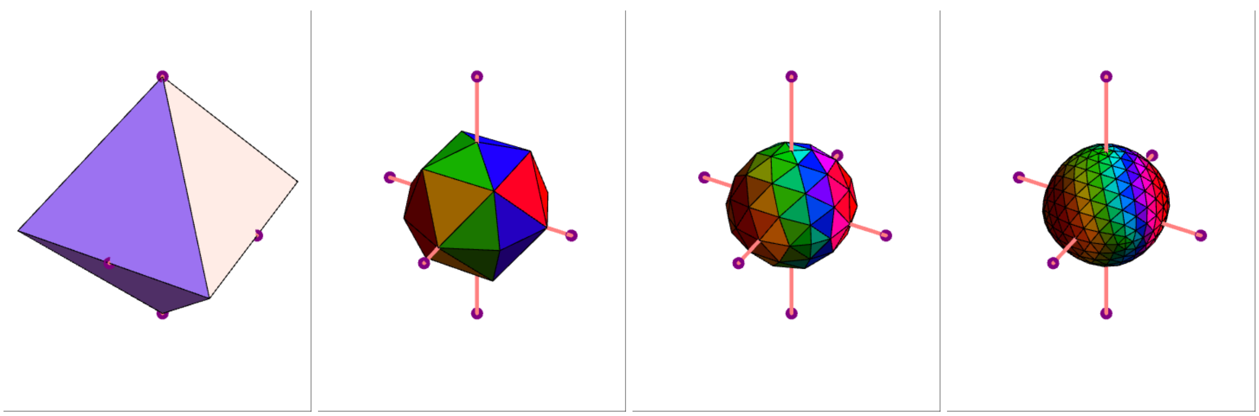
Octahedron

Vertices ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.107428933423629243357597485668)

$$\frac{3\,969\,077\,707\,781\,314\,018\,093\,365\,433\,145\,909\,318\,003\,197}{26\,124\,821\,989\,633\,711\,204\,270\,304\,840\,270\,381\,056\,000\,000\,\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
z	0	(≈0)

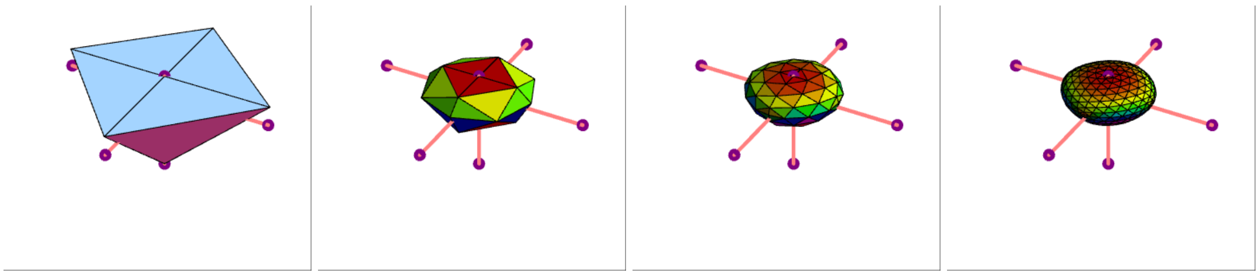
Diamond 4

Vertices ↓

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 6 \\ 2 & 3 & 6 \\ 3 & 4 & 6 \\ 4 & 1 & 6 \\ 2 & 1 & 5 \\ 3 & 2 & 5 \\ 4 & 3 & 5 \\ 1 & 4 & 5 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.151927454638972770374515101434)

$$\frac{3\,969\,077\,707\,781\,314\,018\,093\,365\,433\,145\,909\,318\,003\,197}{26\,124\,821\,989\,633\,711\,204\,270\,304\,840\,270\,381\,056\,000\,000}$$

Limit centroid ↓

x	0
y	0
z	$-\frac{7\,639\,240\,833\,283\,641\,128\,717\,986\,237\,678\,777\,708\,279\,139\,594\,159\,474\,664\,939\,328\,915\,149\,494\,970\,666\,444\,652\,367\,707\,489\,459\,113\,364\,407\,948\,084\,418\,082\,30}{41\,373\,652\,822\,585\,638\,565\,574\,251\,441\,180\,144\,184\,368\,245\,224\,036\,588\,211\,651\,523\,904\,268\,488\,663\,362\,337\,484\,835\,200\,004\,585\,995\,623\,769\,522\,959\,137\,221\,1}$

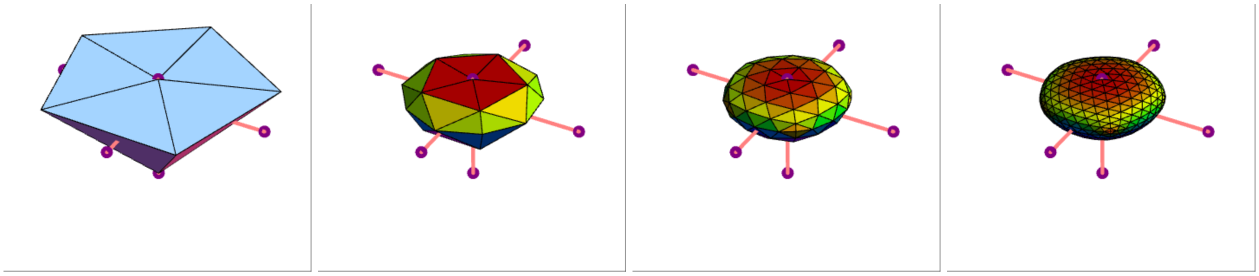
Diamond 5

Vertices ↓

$$\begin{pmatrix} -0.587785 & -0.809017 & 0. \\ 0.587785 & -0.809017 & 0. \\ 0.951057 & 0.309017 & 0. \\ 0. & 1. & 0. \\ -0.951057 & 0.309017 & 0. \\ 0. & 0. & -1. \\ 0. & 0. & 0. \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 7 \\ 2 & 3 & 7 \\ 3 & 4 & 7 \\ 4 & 5 & 7 \\ 5 & 1 & 7 \\ 2 & 1 & 6 \\ 3 & 2 & 6 \\ 4 & 3 & 6 \\ 5 & 4 & 6 \\ 1 & 5 & 6 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 5, 6\}$

Limit volume ↓

0.24625

Limit centroid ↓

x	3.5222777147868545*^-18
y	1.7611388573934272*^-18
z	-0.1952785972016765

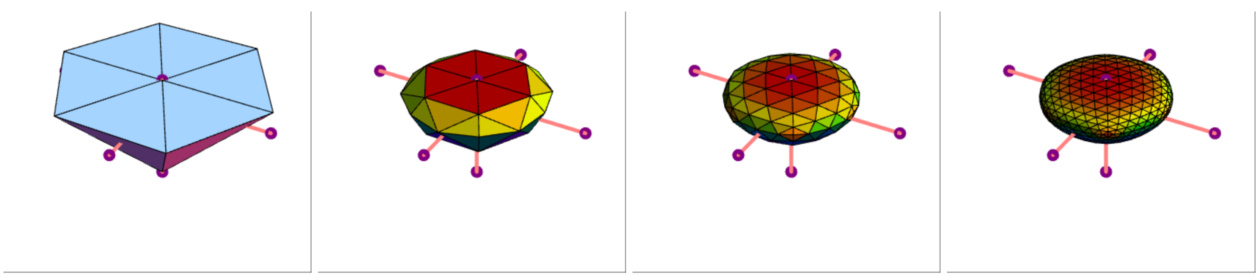
Diamond 6

Vertices ↓

$$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 8 \\ 2 & 3 & 8 \\ 3 & 4 & 8 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \\ 6 & 1 & 8 \\ 2 & 1 & 7 \\ 3 & 2 & 7 \\ 4 & 3 & 7 \\ 5 & 4 & 7 \\ 6 & 5 & 7 \\ 1 & 6 & 7 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.323804981286794907149245864859)

$$\frac{703\,296\,943\,730\,008\,018\,721\,488\,321\,382\,314\,600\,186\,877\,113}{1\,253\,991\,455\,502\,418\,137\,804\,974\,632\,332\,978\,290\,688\,000\,000\,\sqrt{3}}$$

Limit centroid ↓

x	0
y	0
z	$-\frac{427\,423\,997\,068\,630\,041\,137\,903\,657\,882\,039\,466\,134\,799\,443\,342\,477\,612\,130\,762\,421\,858\,247\,865\,816\,540\,982\,993\,370\,012\,952\,473\,741\,960\,618\,838\,154\,368\,457}{2\,094\,618\,571\,968\,769\,058\,650\,593\,366\,613\,827\,773\,497\,176\,494\,939\,906\,293\,775\,479\,732\,310\,445\,654\,018\,883\,996\,044\,402\,774\,381\,804\,965\,212\,818\,064\,339\,379\,41}$

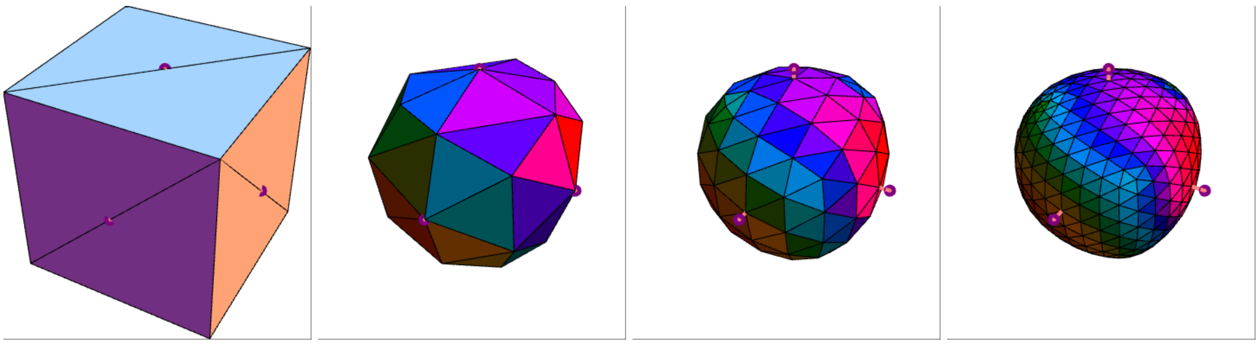
Triangulated Cube

Vertices ↓

0.	0.	0.
1.	0.	0.
0.	1.	0.
1.	1.	0.
0.	0.	1.
1.	0.	1.
0.	1.	1.
1.	1.	1.

Faces ↓

1	2	6
1	6	5
3	4	2
3	2	1
4	8	6
4	6	2
5	6	8
5	8	7
7	8	4
7	4	3
7	3	1
7	1	5



Required valences $\tau(f) \in \{4, 5, 6\}$

Limit volume ↓

0.370924

Limit centroid ↓

x	0.5000000000000026
y	0.5000000000000027
z	0.5000000000000019

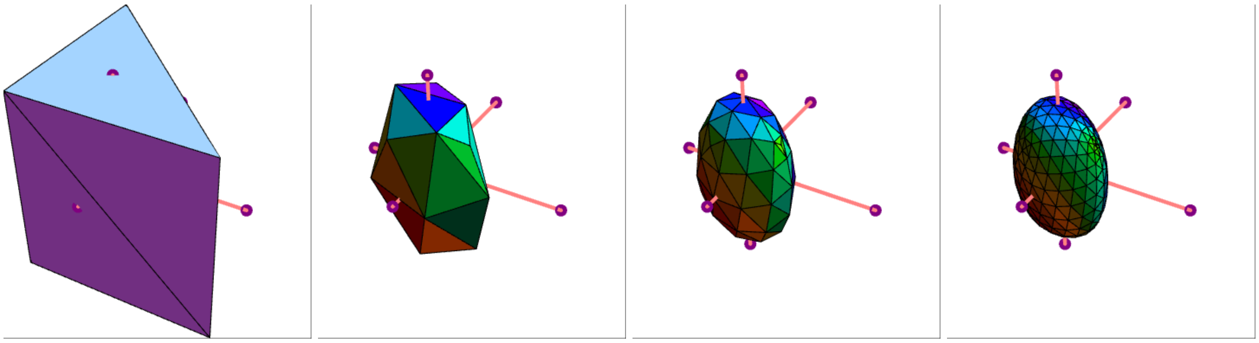
Twisted Prism

Vertices ↓

0.	0.	0.
1.	0.	0.
0.	1.	0.
0.	0.	1.
1.	0.	1.
0.	1.	1.

Faces ↓

1	2	4
4	2	5
4	5	6
2	1	3
5	2	6
6	2	3
1	4	6
1	6	3



Required valences τ(f) ∈ {3, 4, 5, 6}

Limit volume ↓

0.111102

Limit centroid ↓

x	0.33263000384684377
y	0.33263000384684377
z	0.5000000000000023

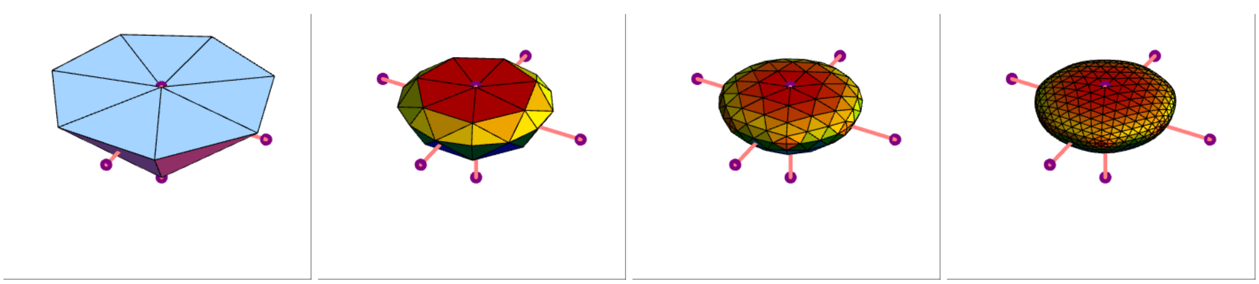
Diamond 7

Vertices ↓

-0.433884	-0.900969	0.
0.433884	-0.900969	0.
0.974928	-0.222521	0.
0.781831	0.62349	0.
0.	1.	0.
-0.781831	0.62349	0.
-0.974928	-0.222521	0.
0.	0.	-1.
0.	0.	0.

Faces ↓

1	2	9
2	3	9
3	4	9
4	5	9
5	6	9
6	7	9
7	1	9
2	1	8
3	2	8
4	3	8
5	4	8
6	5	8
7	6	8
1	7	8



Required valences τ(f) ∈ {4, 6, 7}

Limit volume ↓

0.38414

Limit centroid ↓

x	4.515862576277088*^-18
y	-4.177172883056306*^-17
z	-0.21093550467542566

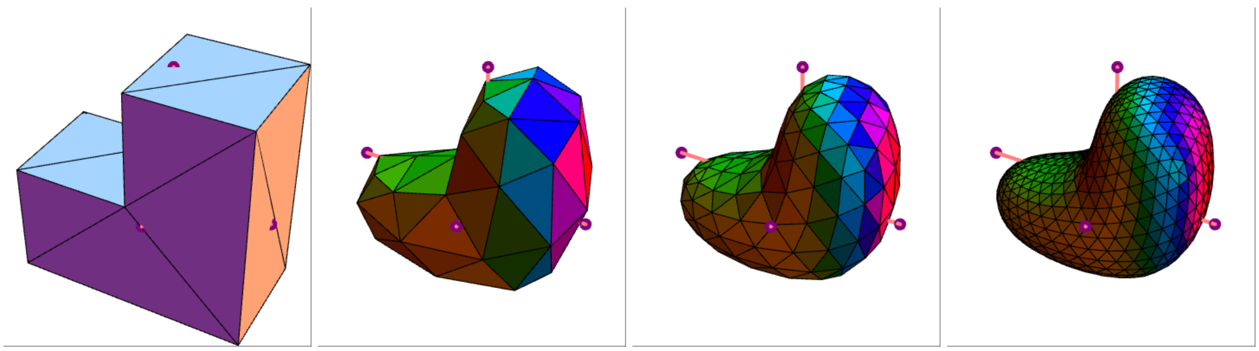
Triangulated Reduced Corner

Vertices ↓

0.	0.	0.
0.	1.	0.
0.	0.	1.
1.	0.	1.
0.	1.	1.
1.	1.	1.
1.	0.	2.
1.	1.	2.
2.	0.	0.
2.	1.	0.
2.	0.	2.
2.	1.	2.

Faces ↓

1	9	4
1	4	3
2	10	9
2	9	1
3	4	6
3	6	5
5	6	10
5	10	2
5	2	1
5	1	3
4	9	11
4	11	7
10	12	11
10	11	9
7	11	12
7	12	8
4	7	8
4	8	6
6	8	12
6	12	10



Required valences $\tau(f) \in \{4, 5, 6, 7\}$

Limit volume ↓

1.46818

Limit centroid ↓

x	1.1475201184891974
y	0.49282426915716343
z	0.867004058134731

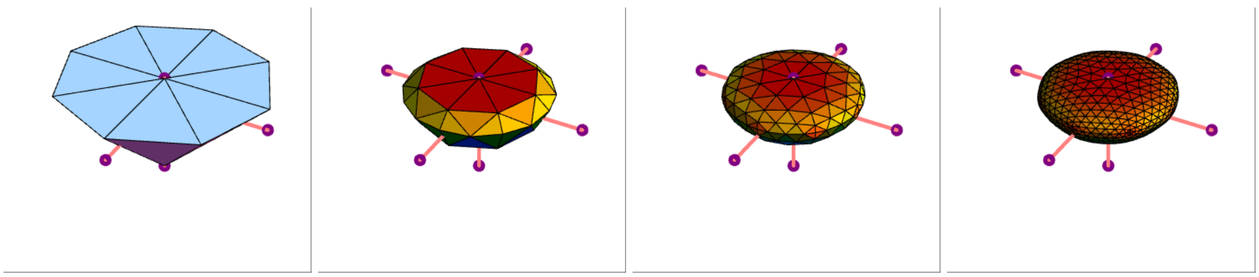
Diamond 8

Vertices ↓

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & -1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 10 \\ 2 & 3 & 10 \\ 3 & 4 & 10 \\ 4 & 5 & 10 \\ 5 & 6 & 10 \\ 6 & 7 & 10 \\ 7 & 8 & 10 \\ 8 & 1 & 10 \\ 2 & 1 & 9 \\ 3 & 2 & 9 \\ 4 & 3 & 9 \\ 5 & 4 & 9 \\ 6 & 5 & 9 \\ 7 & 6 & 9 \\ 8 & 7 & 9 \\ 1 & 8 & 9 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6, 8\}$

Limit volume ↓ (≈0.430462358866708867053800501181)

$$\left(166\,163\,048\,203\,060\,923\,883\,987\,754\,293\,681\,056\,532\,672\,613\,228\,888\,162\,118\,980\,231\,361\,004\,286\,226\,875\,154\,811\,914\,728\,436\,020\,581\,591\,117\,600\,492 + 180\,811\,905\,954\,853\,307\,640\,810\,640\,027\,730\,049\,657\,901\,648\,406\,565\,868\,193\,155\,946\,019\,174\,782\,858\,022\,555\,141\,668\,178\,289\,554\,942\,673\,873\,780\,281\sqrt{2}\right) \Big/ 980\,038\,531\,019\,557\,750\,174\,524\,937\,924\,869\,304\,658\,299\,750\,594\,177\,525\,458\,674\,818\,596\,619\,866\,454\,523\,464\,090\,074\,435\,477\,910\,114\,271\,232\,000\,000$$

Limit centroid ↓

x	2.014953735494859*^-18
y	2.014953735494859*^-18
z	−0.21624706379384373

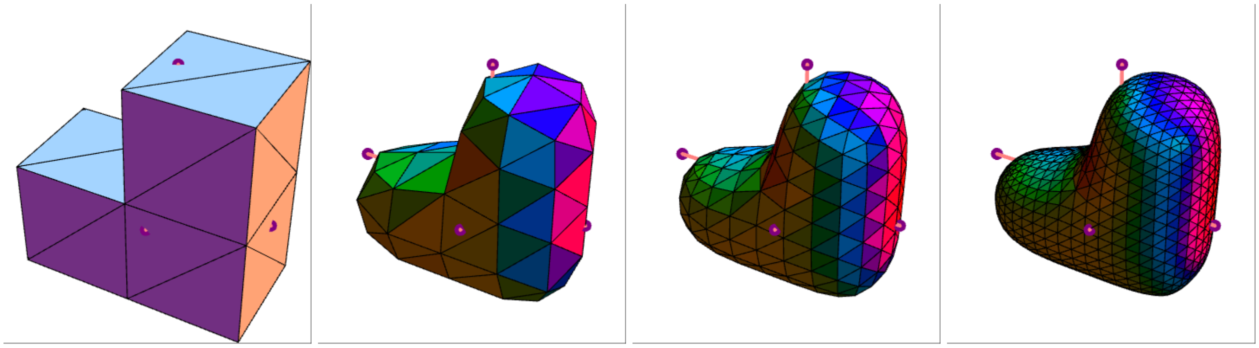
Triangulated Corner

Vertices ↓

0.	0.	0.
1.	0.	0.
0.	1.	0.
1.	1.	0.
0.	0.	1.
1.	0.	1.
0.	1.	1.
1.	1.	1.
1.	0.	2.
1.	1.	2.
2.	0.	0.
2.	1.	0.
2.	0.	1.
2.	1.	1.
2.	0.	2.
2.	1.	2.

Faces ↓

1	2	6
1	6	5
3	4	2
3	2	1
5	6	8
5	8	7
7	8	4
7	4	3
7	3	1
7	1	5
2	11	13
2	13	6
6	13	15
6	15	9
4	12	11
4	11	2
12	14	13
12	13	11
14	16	15
14	15	13
9	15	16
9	16	10
6	9	10
6	10	8
4	8	14
4	14	12
8	10	16
8	16	14



Required valences $\tau(f) \in \{4, 5, 6, 7, 8\}$

Limit volume ↓

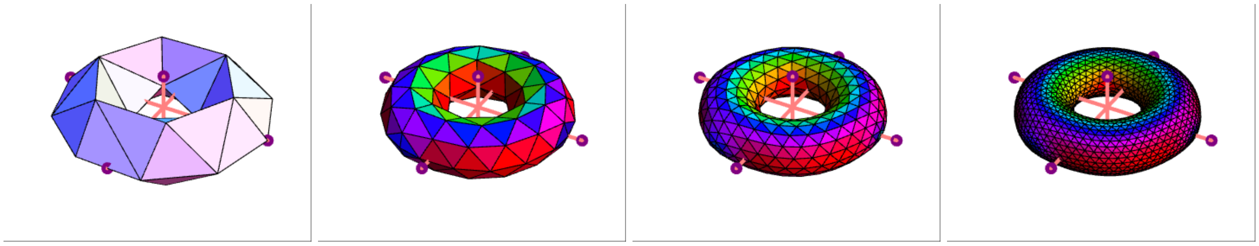
1.81521

Limit centroid ↓

x	1.182090707183006
y	0.4988317609738642
z	0.8168689634387752

Torus

The mesh was already introduced in a previous section.



Required valences $\tau(f) \in \{4, 6, 7, 8\}$

Limit volume ↓

```
2.16461
```

Limit centroid ↓

```
x    -0.0012944413545412671
y    0.
z    3.2056042637221553*^-18
```

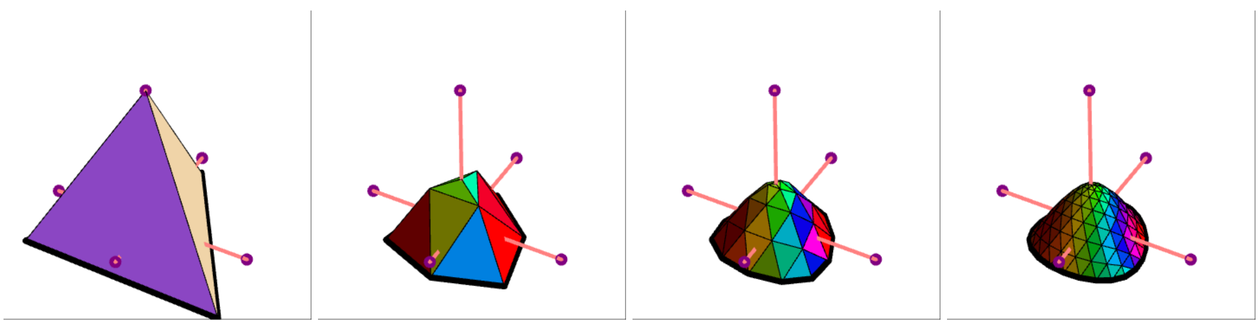
Loop with sharp creases

Loop’s subdivision scheme was extended by [Hoppe et al. 1994]. Along the cycles that are defined as sharp creases, cubic B-spline subdivision rules apply.

Tetrahedron Flat

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 6\}$

Limit volume ↓ (≈0.0304570998481925162554945923683)

$$\frac{9\,835\,279\,661\,079\,132\,863\,588\,159}{228\,340\,616\,075\,693\,288\,629\,862\,400\,\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882
z	$\frac{116\,464\,586\,675\,802\,749\,348\,124\,498\,153\,059\,161\,704\,797\,271\,340\,413\,299\,613\,987\,866\,660\,264\,765\,854\,259\,591}{497\,473\,279\,255\,564\,893\,012\,933\,703\,357\,711\,974\,914\,837\,178\,667\,766\,904\,963\,855\,589\,876\,479\,178\,038\,977\,120\,\sqrt{6}}$	(≈0.095575924160443239

Tetrahedron Circuit

Vertices ↓

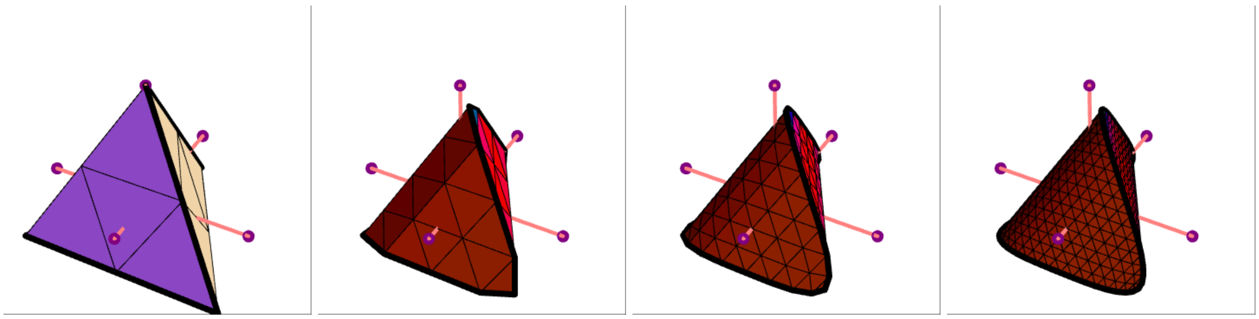
$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & 0 \\ \frac{3}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 6 & 1 & 5 \\ 5 & 2 & 9 \\ 9 & 4 & 6 \\ 5 & 9 & 6 \\ 10 & 4 & 9 \\ 9 & 2 & 8 \\ 8 & 3 & 10 \\ 9 & 8 & 10 \\ 7 & 1 & 6 \\ 6 & 4 & 10 \\ 10 & 3 & 7 \\ 6 & 10 & 7 \\ 8 & 2 & 5 \\ 5 & 1 & 7 \\ 7 & 3 & 8 \\ 5 & 7 & 8 \end{pmatrix}$$

Cycles ↓

$$\begin{pmatrix} 1 & 5 & 2 & 9 & 4 & 10 & 3 & 7 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 6\}$

Limit volume ↓ (≈0.0927376155484737923866980403475)

$$\frac{661}{5040\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2\sqrt{3}}$	(≈0.288675134594812882254574390251)
z	$\frac{1}{2\sqrt{6}}$	(≈0.204124145231931508183107006225)

Octahedron Side

Vertices ↓

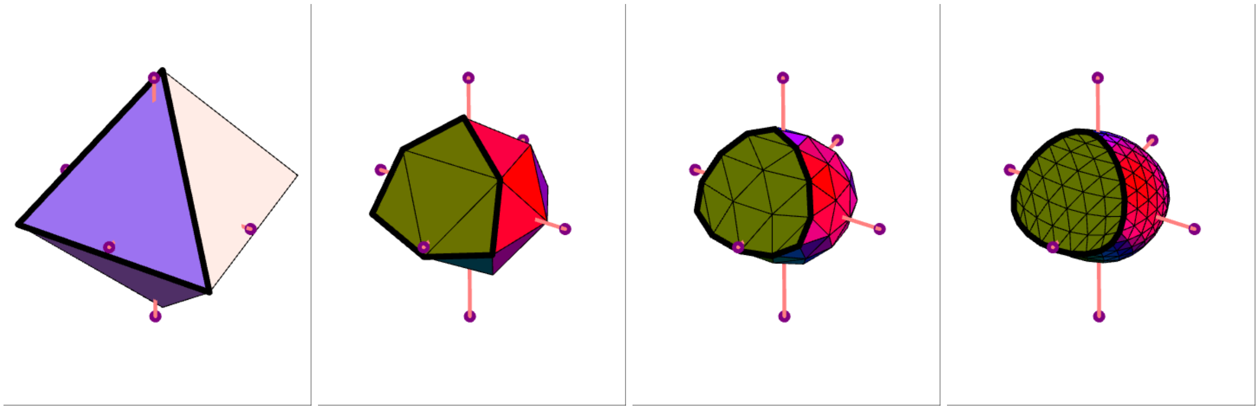
$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix}$$

Cycles ↓

$$\begin{pmatrix} 1 & 2 & 5 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.165826195153351469018892298809)

$$\frac{63\,016\,739\,687\,919\,830\,173\,887\,904\,272\,662\,896\,458\,427\,151}{268\,712\,454\,750\,518\,172\,386\,780\,278\,357\,066\,776\,576\,000\,000\,\sqrt{2}}$$

Limit centroid ↓

x $\frac{1}{2}$

y $\frac{26\,184\,604\,422\,754\,049\,520\,366\,854\,458\,375\,936\,088\,167\,385\,978\,379\,674\,063\,681\,487\,534\,620\,705\,851\,388\,713\,605\,276\,004\,073\,791\,048\,843\,121\,303\,344\,867\,784\,420}{62\,137\,891\,812\,847\,971\,397\,068\,461\,795\,003\,790\,358\,595\,449\,510\,949\,122\,585\,721\,032\,611\,515\,189\,542\,072\,879\,538\,321\,513\,882\,988\,574\,999\,672\,449\,729\,405\,596\,659}$

z $\frac{4\,884\,341\,483\,669\,936\,178\,167\,376\,439\,125\,959\,091\,130\,338\,777\,094\,887\,229\,179\,028\,771\,136\,888\,919\,647\,726\,163\,884\,752\,867\,703\,238\,656\,714\,921\,519\,835\,013\,9}{62\,137\,891\,812\,847\,971\,397\,068\,461\,795\,003\,790\,358\,595\,449\,510\,949\,122\,585\,721\,032\,611\,515\,189\,542\,072\,879\,538\,321\,513\,882\,988\,574\,999\,672\,449\,729\,405\,596\,659}$

Octahedron Short Loop

Vertices ↓

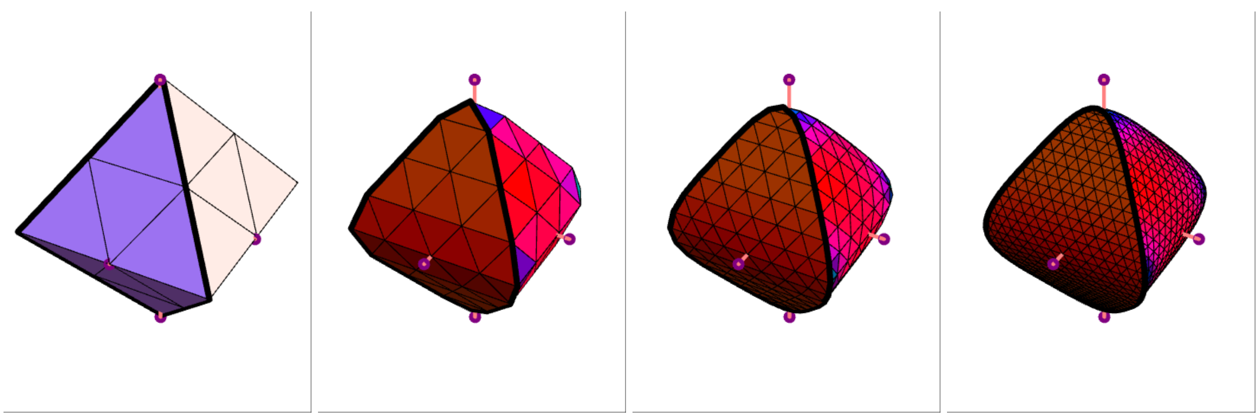
$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2\sqrt{2}} \\ 1 & \frac{1}{2} & 0 \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2\sqrt{2}} \\ \frac{3}{4} & \frac{1}{4} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2} & 1 & 0 \\ \frac{3}{4} & \frac{3}{4} & \frac{1}{2\sqrt{2}} \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{2\sqrt{2}} \\ \frac{1}{4} & \frac{3}{4} & -\frac{1}{2\sqrt{2}} \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 8 & 1 & 7 \\ 7 & 2 & 12 \\ 12 & 5 & 8 \\ 7 & 12 & 8 \\ 12 & 2 & 11 \\ 11 & 3 & 15 \\ 15 & 5 & 12 \\ 11 & 15 & 12 \\ 15 & 3 & 14 \\ 14 & 4 & 17 \\ 17 & 5 & 15 \\ 14 & 17 & 15 \\ 17 & 4 & 9 \\ 9 & 1 & 8 \\ 8 & 5 & 17 \\ 9 & 8 & 17 \\ 10 & 6 & 13 \\ 13 & 2 & 7 \\ 7 & 1 & 10 \\ 13 & 7 & 10 \\ 13 & 6 & 16 \\ 16 & 3 & 11 \\ 11 & 2 & 13 \\ 16 & 11 & 13 \\ 16 & 6 & 18 \\ 18 & 4 & 14 \\ 14 & 3 & 16 \\ 18 & 14 & 16 \\ 18 & 6 & 10 \\ 10 & 1 & 9 \\ 9 & 4 & 18 \\ 10 & 9 & 18 \end{pmatrix}$$

Cycles ↓

$$\begin{pmatrix} 1 & 10 & 6 & 13 & 2 & 12 & 5 & 8 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.37633928823869976066505236628 8)

$$\frac{274\,962\,183\,466\,592\,197\,331\,396\,286\,238\,960\,674\,452\,153}{516\,628\,559\,853\,596\,339\,732\,884\,446\,304\,175\,016\,000\,000\,\sqrt{2}}$$

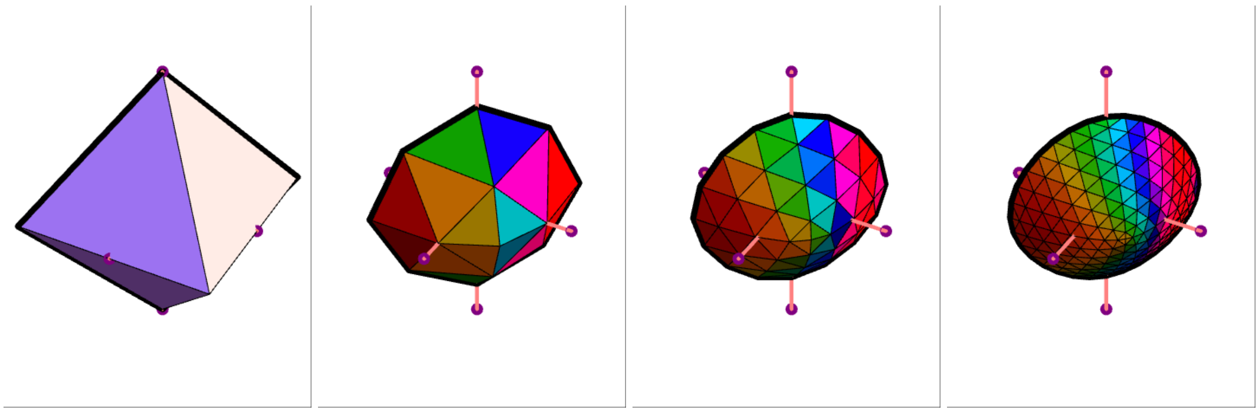
Limit centroid ↓

x	$\frac{1}{2}$
y	$\frac{254\,804\,499\,605\,772\,294\,189\,412\,796\,682\,554\,208\,152\,634\,154\,598\,554\,272\,271\,128\,783\,965\,038\,391\,856\,360\,337\,623\,504\,330\,812\,009\,326\,577\,714\,761\,585\,688\,951}{533\,148\,230\,241\,984\,055\,451\,468\,792\,256\,199\,424\,048\,229\,028\,364\,672\,896\,438\,073\,722\,726\,157\,534\,556\,047\,547\,900\,615\,060\,538\,515\,490\,732\,836\,298\,012\,580\,700}$
z	0

Octahedron Split

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 5 & 3 & 6 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.186117527071030358172669303980)

$$\frac{644\,654\,305\,568\,923\,817\,326\,730\,671\,735\,648\,899\,901\,901}{2\,449\,202\,061\,528\,160\,425\,400\,341\,078\,775\,348\,224\,000\,000\,\sqrt{2}}$$

Limit centroid ↓

x	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.500000000000000000000000000000)
z	0	(≈0)

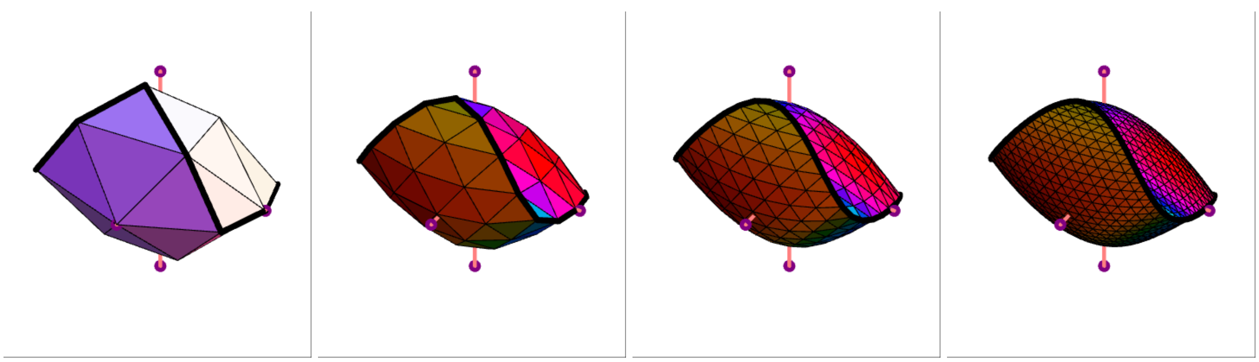
Octahedron ZigZag

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} \frac{1}{16} & \frac{3}{16} & \frac{1}{8\sqrt{2}} \\ \frac{15}{16} & \frac{3}{16} & \frac{1}{8\sqrt{2}} \\ \frac{15}{16} & \frac{13}{16} & -\frac{1}{8\sqrt{2}} \\ \frac{1}{16} & \frac{13}{16} & -\frac{1}{8\sqrt{2}} \\ \frac{1}{2} & \frac{3}{8} & \frac{3}{4\sqrt{2}} \\ \frac{1}{2} & \frac{5}{8} & -\frac{3}{4\sqrt{2}} \\ \frac{1}{2} & \frac{1}{8} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{2} & 0 \\ \frac{5}{16} & \frac{5}{16} & -\frac{3}{8\sqrt{2}} \\ 1 & \frac{1}{2} & 0 \\ \frac{3}{4} & \frac{1}{4} & \frac{1}{2\sqrt{2}} \\ \frac{11}{16} & \frac{5}{16} & -\frac{3}{8\sqrt{2}} \\ \frac{1}{2} & \frac{7}{8} & 0 \\ \frac{11}{16} & \frac{11}{16} & \frac{3}{8\sqrt{2}} \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{2\sqrt{2}} \\ \frac{5}{16} & \frac{11}{16} & \frac{3}{8\sqrt{2}} \\ \frac{1}{4} & \frac{3}{4} & -\frac{1}{2\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 8 & 1 & 7 \\ 7 & 2 & 12 \\ 12 & 5 & 8 \\ 7 & 12 & 8 \\ 12 & 2 & 11 \\ 11 & 3 & 15 \\ 15 & 5 & 12 \\ 11 & 15 & 12 \\ 15 & 3 & 14 \\ 14 & 4 & 17 \\ 17 & 5 & 15 \\ 14 & 17 & 15 \\ 17 & 4 & 9 \\ 9 & 1 & 8 \\ 8 & 5 & 17 \\ 9 & 8 & 17 \\ 10 & 6 & 13 \\ 13 & 2 & 7 \\ 7 & 1 & 10 \\ 13 & 7 & 10 \\ 13 & 6 & 16 \\ 16 & 3 & 11 \\ 11 & 2 & 13 \\ 16 & 11 & 13 \\ 16 & 6 & 18 \\ 18 & 4 & 14 \\ 14 & 3 & 16 \\ 18 & 14 & 16 \\ 18 & 6 & 10 \\ 10 & 1 & 9 \\ 9 & 4 & 18 \\ 10 & 9 & 18 \end{pmatrix}$$

$$(1 \; 8 \; 5 \; 12 \; 2 \; 11 \; 3 \; 16 \; 6 \; 18 \; 4 \; 9)$$



Required valences $\tau(f) \in \{4, 6\}$

Limit volume ↓ (≈0.225920304813625220400776827994)

$$\frac{5797}{18144\sqrt{2}}$$

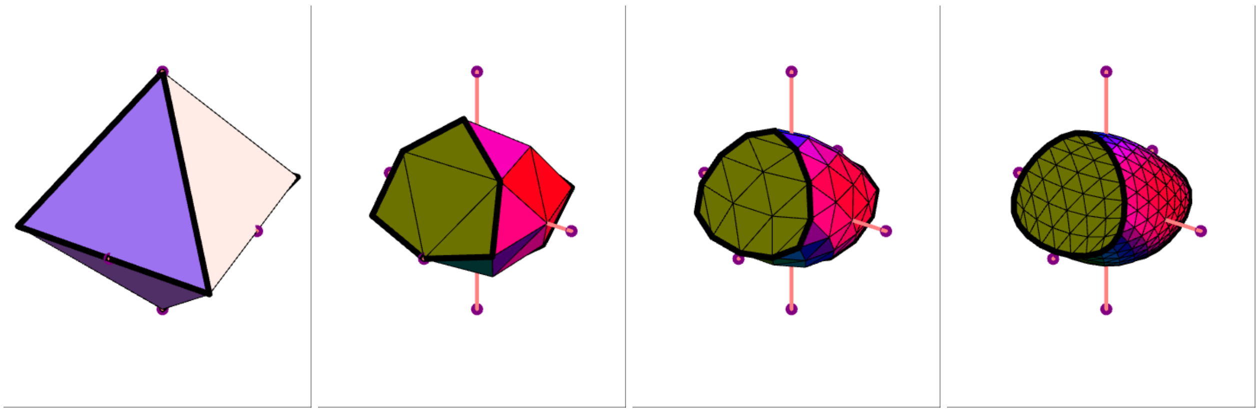
Limit centroid ↓

x	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
z	0	(≈0)

Octahedron Plates

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 5 \\ 3 & 4 & 5 \\ 4 & 1 & 5 \\ 6 & 2 & 1 \\ 6 & 3 & 2 \\ 6 & 4 & 3 \\ 6 & 1 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 5 & 2 \\ 6 & 3 & 4 \end{pmatrix}$$



Required valences τ(f) ∈ {4, 6}

Limit volume ↓ (≈0.227284322524247418557414259248)

$$\frac{9}{28\sqrt{2}}$$

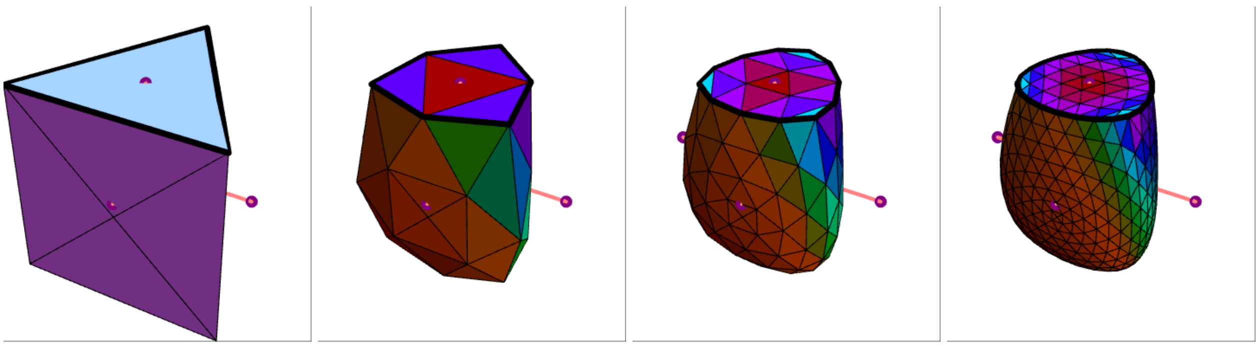
Limit centroid ↓

x	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
y	$\frac{1}{2}$	(≈0.50000000000000000000000000000000)
z	0	(≈0)

Prism Drum

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 2 \\ 7 & 4 & 1 \\ 7 & 1 & 2 \\ 7 & 2 & 5 \\ 7 & 5 & 4 \\ 4 & 5 & 6 \\ 8 & 5 & 2 \\ 8 & 2 & 3 \\ 8 & 3 & 6 \\ 8 & 6 & 5 \\ 9 & 6 & 3 \\ 9 & 3 & 1 \\ 9 & 1 & 4 \\ 9 & 4 & 6 \end{pmatrix} \quad \begin{pmatrix} 4 & 5 & 6 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 5, 6\}$

Limit volume ↓ (≈0.259001674143006938229606944794)

$$\left(2\,475\,127\,520\,074\,130\,635\,042\,526\,525\,648\,170\,787\,587\,610\,971\,841\,164\,961\,867\,261\,759\,908\,826\,701\,638\,882\,964\,258 - 697\,279\,690\,038\,962\,881\,214\,188\,256\,954\,998\,018\,842\,883\,399\,094\,620\,010\,352\,214\,129\,738\,035\,141 + 30\,320\,052\,482\,851\,631\,698\,356\,650\,363\,271\,519\,513\,947\,201\,984\,475\,344\,855\,812\,220\,893\,976\,748\,720\,547\,743\,867 - 931\,418\,893\,946\,368\,226\,832\,802\,073\,883\,760\,825\,190\,704\,394\,847\,695\,401\,546\,499\,693\,403\,747\,651\sqrt{5}\right) \Big/ \left(5\,668\,529\,225\,890\,450\,802\,551\,075\,312\,322\,624\,561\,263\,481\,125\,139\,134\,477\,943\,244\,232\,305\,177\,047\,323\,992\,798 - 823\,222\,375\,450\,991\,537\,102\,676\,617\,594\,682\,623\,204\,779\,780\,083\,289\,119\,801\,729\,713\,103\,175\,680\,000\sqrt{3}\right)$$

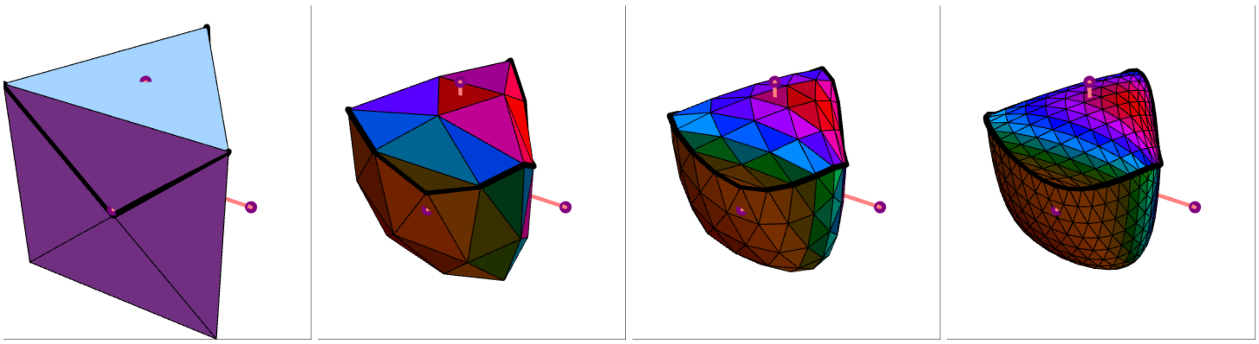
Limit centroid ↓

x	0.5000000000000007
y	0.28867513459481314
z	0.5716176817260487

Prism ZigZag

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 2 \\ 7 & 4 & 1 \\ 7 & 1 & 2 \\ 7 & 2 & 5 \\ 7 & 5 & 4 \\ 4 & 5 & 6 \\ 8 & 5 & 2 \\ 8 & 2 & 3 \\ 8 & 3 & 6 \\ 8 & 6 & 5 \\ 9 & 6 & 3 \\ 9 & 3 & 1 \\ 9 & 1 & 4 \\ 9 & 4 & 6 \end{pmatrix} \quad \begin{pmatrix} 4 & 7 & 5 & 8 & 6 & 9 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 5, 6\}$

Limit volume ↓ (≈0.240139559624273058574994148236)

$$\left(151\,725\,977\,678\,673\,852\,030\,013\,438\,036\,633\,083\,453\,857\,787\,382\,423\,814\,834\,287\,323\,591\,314\,199\,601\,650\,121\,225\,000\,758\,262\,155\,279\,680\,507\,473 + 2\,052\,306\,403\,079\,314\,090\,142\,667\,294\,316\,424\,839\,690\,517\,380\,494\,902\,496\,955\,290\,667\,284\,768\,439\,789\,893\,582\,975\,172\,486\,824\,638\,604\,105\,475\sqrt{5}\right) \Big/ \left(375\,817\,088\,906\,660\,972\,864\,094\,276\,610\,628\,205\,301\,403\,995\,393\,174\,024\,628\,197\,304\,049\,563\,453\,643\,356\,669\,187 - 943\,487\,392\,608\,776\,355\,840\,000\sqrt{3}\right)$$

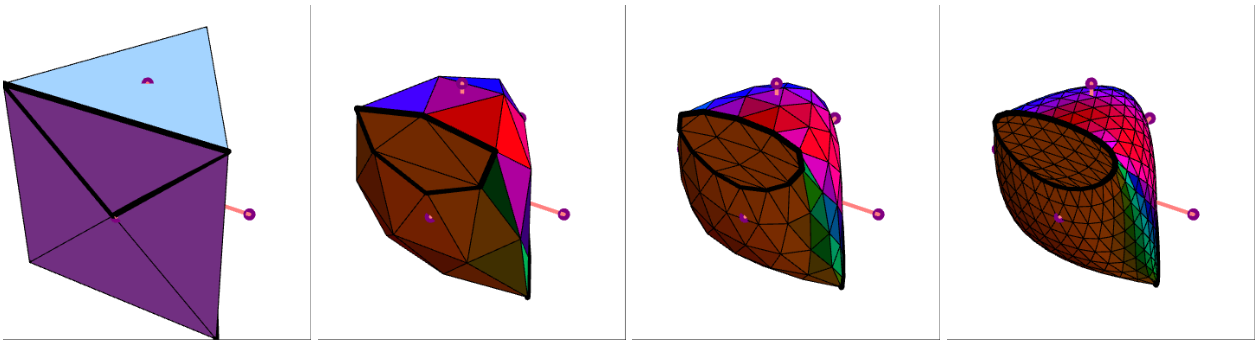
Limit centroid ↓

x	0.5000000000000007
y	0.2886751345948132
z	0.5330157917421132

Prism Patched

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 2 \\ 7 & 4 & 1 \\ 7 & 1 & 2 \\ 7 & 2 & 5 \\ 7 & 5 & 4 \\ 4 & 5 & 6 \\ 8 & 5 & 2 \\ 8 & 2 & 3 \\ 8 & 3 & 6 \\ 8 & 6 & 5 \\ 9 & 6 & 3 \\ 9 & 3 & 1 \\ 9 & 1 & 4 \\ 9 & 4 & 6 \end{pmatrix} \quad \begin{pmatrix} 4 & 7 & 5 \\ 2 & 8 & 3 \end{pmatrix}$$



Required valences $\tau(f) \in \{4, 5, 6\}$

Limit volume ↓ (≈0.2528623983955379995822860712 83)

$$\left(852\,625\,316\,956\,093\,739\,416\,011\,744\,388\,915\,443\,611\,874\,508\,574\,699\,995\,384\,354\,674\,328\,437\,567\,769\,927\,876\,804 - 243\,089\,708\,956\,980\,185\,761\,266\,362\,957\,266\,716\,307\,430\,695\,371\,709\,659\,315\,543\,811\,309\,939 + 7\,687\,992\,390\,492\,913\,983\,674\,502\,840\,778\,737\,284\,979\,584\,854\,108\,757\,756\,749\,706\,629\,207\,059\,152\,851\,782\,711 - 893\,553\,541\,211\,879\,339\,358\,018\,767\,623\,004\,620\,541\,737\,751\,805\,367\,212\,933\,178\,563\,400\,401\sqrt{5}\right) \Big/ \left(1\,986\,015\,378\,114\,841\,841\,441\,905\,863\,084\,071\,202\,560\,391\,238\,175\,498\,688\,161\,367\,101\,010\,066\,701\,413\,731\,516 - 249\,182\,551\,349\,226\,247\,038\,465\,024\,850\,208\,019\,652\,873\,638\,303\,861\,806\,221\,409\,507\,082\,240\,000\sqrt{3}\right)$$

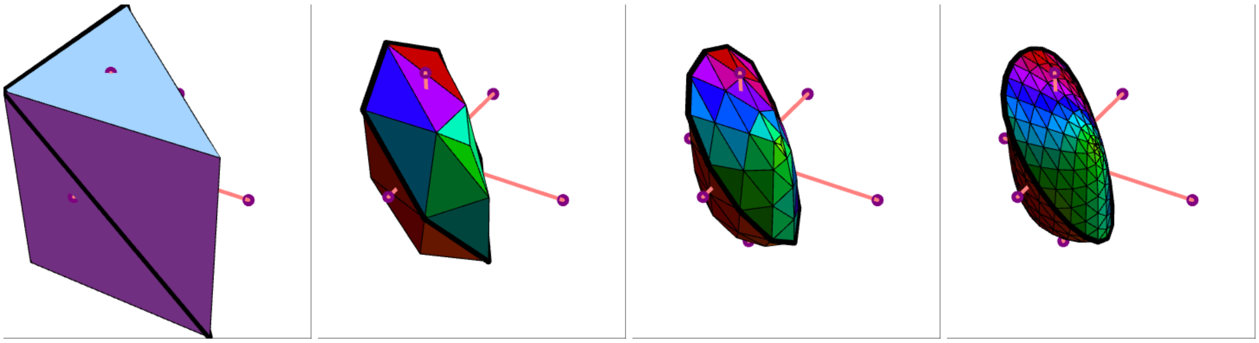
Limit centroid ↓

x	0.5149084433399879
y	0.28006774081927355
z	0.500000000000001

Wedged Prism

Vertices ↓ Faces ↓ Cycles ↓

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ 4 & 2 & 5 \\ 4 & 5 & 6 \\ 2 & 1 & 3 \\ 5 & 2 & 6 \\ 6 & 2 & 3 \\ 1 & 4 & 6 \\ 1 & 6 & 3 \end{pmatrix} \quad \begin{pmatrix} 4 & 2 & 6 \end{pmatrix}$$



Required valences $\tau(\mathrm{f}) \in \{3, 4, 5, 6\}$

`Limit volume ↓` (≈0.14790464770911802863529029983 7)

$$\frac{22\,019\,733\,629\,566\,009\,632\,050\,792\,507\,567\,438\,044\,342\,955\,396\,113}{148\,877\,901\,882\,244\,479\,728\,587\,648\,112\,669\,004\,444\,352\,512\,000\,000}$$

Limit centroid ↓

$$x \quad \frac{117\,419\,357\,705\,466\,691\,538\,832\,147\,987\,223\,039\,351\,990\,856\,138\,083\,004\,056\,177\,525\,691\,243\,891\,645\,765\,897\,192\,410\,659\,181\,686\,856\,773\,619\,097\,684\,439\,831\,74}{373\,660\,963\,415\,059\,998\,132\,571\,303\,573\,177\,330\,689\,460\,104\,768\,590\,423\,533\,208\,473\,608\,855\,892\,445\,266\,718\,291\,490\,808\,488\,935\,038\,196\,904\,535\,740\,941\,859\,17}$$

$$y \quad \frac{2\,525\,552\,207\,899\,661\,585\,386\,313\,366\,100\,429\,307\,417\,430\,422\,825\,854\,677\,109\,477\,359\,671\,169\,120\,469\,310\,756\,548\,002\,259\,070\,818\,201\,081\,347\,479\,783\,354\,623\,3}{7\,286\,388\,786\,593\,669\,963\,585\,140\,419\,676\,957\,948\,444\,472\,042\,987\,513\,258\,897\,565\,235\,372\,689\,902\,682\,701\,006\,684\,070\,765\,534\,233\,244\,839\,638\,446\,948\,366\,253\,8}$$

$$z \quad \frac{48\,728\,682\,611\,498\,124\,614\,453\,985\,060\,436\,663\,674\,671\,027\,230\,923\,394\,552\,435\,780\,923\,309\,641\,181\,196\,577\,658\,487\,775\,559\,540\,612\,918\,000\,001\,124\,020\,034\,279}{88\,858\,399\,836\,508\,170\,287\,623\,663\,654\,597\,048\,151\,761\,854\,182\,774\,551\,937\,775\,185\,797\,227\,925\,642\,471\,963\,496\,147\,204\,457\,734\,551\,766\,337\,054\,231\,077\,637\,242}$$

Twisted ZigZag

Vertices ↓

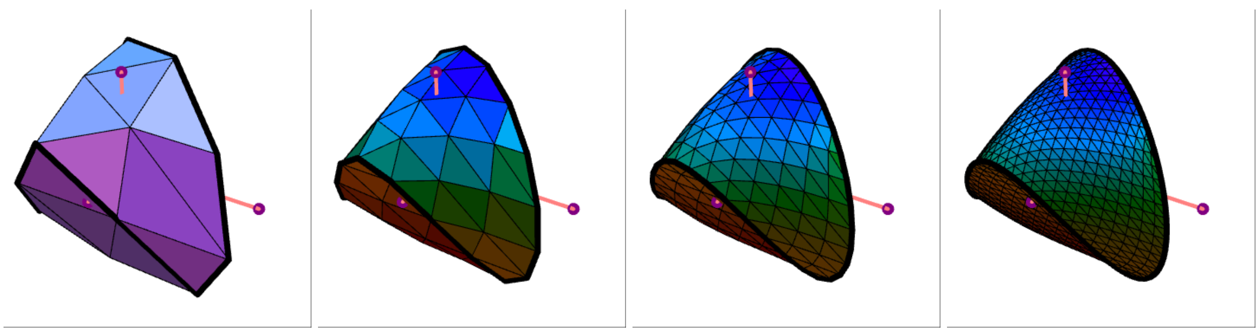
$$\begin{pmatrix} 0 & \frac{1}{8} & \frac{1}{4} \\ \frac{7}{8} & 0 & \frac{1}{4} \\ \frac{3}{16} & \frac{5}{8} & \frac{3}{16} \\ \frac{1}{8} & 0 & \frac{3}{4} \\ \frac{7}{8} & \frac{1}{8} & \frac{7}{8} \\ \frac{1}{8} & \frac{3}{4} & \frac{7}{8} \\ \frac{3}{8} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{8} & \frac{1}{2} & \frac{1}{8} \\ \frac{3}{8} & \frac{1}{2} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{8} & \frac{3}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{8} & \frac{7}{8} \\ \frac{1}{8} & \frac{3}{8} & \frac{7}{8} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

Faces ↓

$$\begin{pmatrix} 8 & 1 & 7 \\ 7 & 2 & 14 \\ 14 & 4 & 8 \\ 7 & 14 & 8 \\ 16 & 4 & 14 \\ 14 & 2 & 13 \\ 13 & 5 & 16 \\ 14 & 13 & 16 \\ 17 & 4 & 16 \\ 16 & 5 & 18 \\ 18 & 6 & 17 \\ 16 & 18 & 17 \\ 11 & 2 & 7 \\ 7 & 1 & 10 \\ 10 & 3 & 11 \\ 7 & 10 & 11 \\ 18 & 5 & 13 \\ 13 & 2 & 12 \\ 12 & 6 & 18 \\ 13 & 12 & 18 \\ 15 & 6 & 12 \\ 12 & 2 & 11 \\ 11 & 3 & 15 \\ 12 & 11 & 15 \\ 9 & 1 & 8 \\ 8 & 4 & 17 \\ 17 & 6 & 9 \\ 8 & 17 & 9 \\ 10 & 1 & 9 \\ 9 & 6 & 15 \\ 15 & 3 & 10 \\ 9 & 15 & 10 \end{pmatrix}$$

Cycles ↓

$$\begin{pmatrix} 6 & 9 & 1 & 8 & 4 & 14 & 2 & 13 & 5 & 18 \end{pmatrix}$$



Required valences $\tau(f) \in \{3, 4, 5, 6\}$

Limit volume ↓ (≈0.207642328556229883797226287710)

18 419 838 425 496 441 675 729 228 523 308 561 088 835 786 228 213 485 043 170 450 060 897 /
88 709 458 006 816 366 251 118 298 141 221 567 374 489 199 038 073 786 811 938 652 160 000

Limit centroid ↓

x: 11 273 684 330 246 900 946 531 370 019 494 371 356 988 163 497 537 510 826 158 702 013 376 487 081 611 354 069 687 490 675 330 315 858 622 043 558 502 825 089 / 30 864 230 262 267 032 966 842 288 881 423 510 081 421 128 538 398 519 501 121 984 251 794 197 138 946 177 730 383 150 692 104 372 614 040 172 614 368 320 214

y: 118 056 869 362 103 912 270 285 049 421 041 945 316 938 008 365 003 765 658 256 355 258 166 983 120 268 927 511 799 718 605 212 462 969 812 177 761 932 957 88 / 403 077 634 022 144 087 253 537 354 794 710 019 720 052 051 807 443 351 693 757 257 019 700 335 769 819 485 284 854 579 934 198 896 078 882 851 307 048 958 02

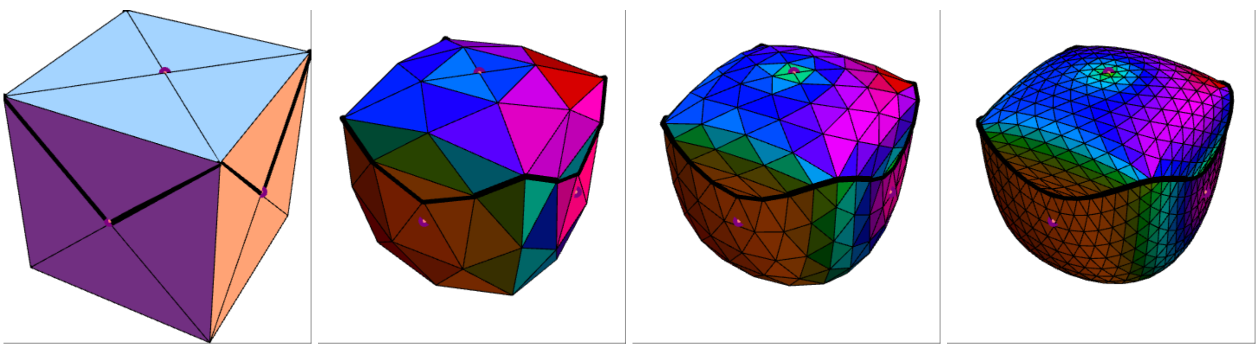
z: 4 297 573 845 555 205 398 716 174 818 329 491 768 490 112 864 638 501 528 494 760 343 736 067 846 963 225 449 001 207 696 747 681 126 390 881 884 929 680 066 / 8 001 837 475 402 564 102 514 667 487 776 465 576 664 737 028 473 690 241 031 625 546 761 458 517 504 564 596 766 002 031 286 318 825 862 266 974 095 490 425

Cube ZigZag

Vertices ↓ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Faces ↓ $\begin{pmatrix} 9 & 5 & 1 \\ 9 & 1 & 2 \\ 9 & 2 & 6 \\ 9 & 6 & 5 \\ 10 & 1 & 3 \\ 10 & 3 & 4 \\ 10 & 4 & 2 \\ 10 & 2 & 1 \\ 11 & 2 & 4 \\ 11 & 4 & 8 \\ 11 & 8 & 6 \\ 11 & 6 & 2 \\ 12 & 7 & 5 \\ 12 & 5 & 6 \\ 12 & 6 & 8 \\ 12 & 8 & 7 \\ 13 & 3 & 7 \\ 13 & 7 & 8 \\ 13 & 8 & 4 \\ 13 & 4 & 3 \\ 14 & 5 & 7 \\ 14 & 7 & 3 \\ 14 & 3 & 1 \\ 14 & 1 & 5 \end{pmatrix}$

Cycles ↓ $\begin{pmatrix} 5 & 9 & 6 & 11 & 8 & 13 & 7 & 14 \end{pmatrix}$



Required valences τ(f) ∈ {4, 6}

Limit volume ↓ (≈0.719587549369929675887134096181)

18 302 899 293 318 629 518 466 729 731 126 356 334 826 796 187 797 121 113 000 675 471 677 089 129 /
25 435 264 005 532 939 743 508 876 618 068 557 876 581 936 313 069 304 458 059 484 543 938 560 000

Limit centroid ↓

x $\frac{1}{2}$

y $\frac{1}{2}$

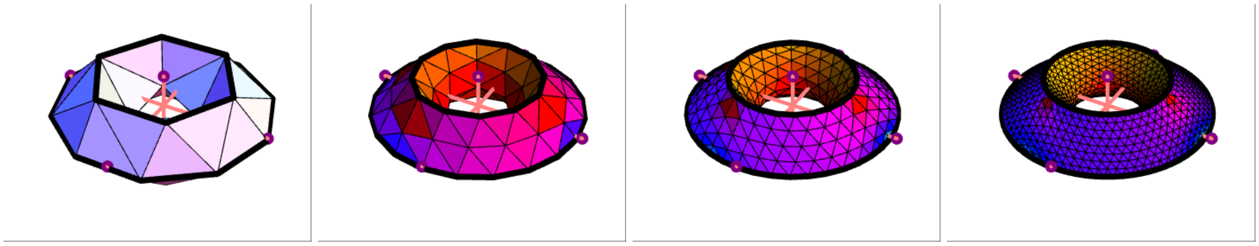
z $\frac{4\,707\,089\,773\,889\,054\,026\,478\,102\,060\,519\,800\,398\,498\,680\,571\,575\,205\,106\,360\,370\,391\,539\,282\,473\,966\,628\,092\,837\,792\,786\,229\,998\,713\,620\,177\,023\,586\,245\,220\ldots}{9\,112\,485\,403\,488\,127\,658\,030\,058\,762\,777\,226\,438\,655\,880\,226\,985\,572\,342\,081\,285\,088\,060\,670\,976\,922\,063\,610\,293\,975\,783\,440\,747\,818\,956\,524\,038\,643\,004\,313\ldots}$

Torus Loops

Vertices ↓ (see above)

Faces ↓ (see above)

Cycles ↓ $\begin{pmatrix} \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \\ \{11, 12, 13, 14, 15, 16\} \end{pmatrix}$



Required valences τ(f) ∈ {4, 6, 7, 8}

```
Limit volume ↓
```

```
2.66612
```

Limit centroid ↓

```
x    0.0009862079812430022
y    -4.164185476072622*^-17
z    0.03077614317653665
```

References

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[Hakenberg et al. 2014b] Hakenberg J., Reif U., Schaefer S., Warren J.: *Volume Enclosed by Subdivision Surfaces with Sharp Creases*, http://vixra.org/abs/1406.0060, 2014

[Hakenberg et al. 2014c] Hakenberg J., Reif U., Schaefer S., Warren J.: *On Moments of Sets Bounded by Subdivision Surfaces*, http://vixra.org/abs/1408.0070, 2014

[Hoppe et al. 1994] Hoppe H., DeRose T., Duchamp T., Halstead M., Jin H., McDonald J., Schweitzer J., Stuetzle W.: *Piecewise smooth surface reconstruction*, Computer Graphics, 28(3):295-302, July 1994

[Loop 1987] Loop C.: *Smooth subdivision surfaces based on triangles*, Master’s thesis, University of Utah, 1987