Four unusual conjectures on primes involving Egyptian fractions

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Abstract. In this paper I make four conjectures on primes, conjectures which involve the sums of distinct unit fractions such as 1/p(1) + 1/p(2) + (...), where p(1), p(2), (...) are distinct primes, more specifically the periods of the rational numbers which are the results of the sums mentioned above.

Conjecture 1:

There exist an infinity of infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Examples:

: the period of a(1) = 1/3 + 1/7 is equal to 6;
: the period of a(2) = 1/3 + 1/7 + 1/19 is equal to 18;
: the period of a(3) = 1/3 + 1/7 + 1/19 + 73 is equal to 72.
(...)
The sequence of p(1), p(2), p(3)... is 3, 7, 19, 72...

: the period of a(1) = 1/5 + 1/29 is equal to 28;
: the period of a(2) = 1/5 + 1/29 + 1/113 is equal to 112;
: the period of a(3) = 1/5 + 1/29 + 1/113 + 1/337 is equal to 336.
(...)
The sequence of p(1), p(2), p(3)... is 5, 29, 113, 337...

Conjecture 2:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 3:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is a multiple of p(2) - 1, the period of the rational number a(2) is a multiple of p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Example:

: the period of a(1) = 1/7 + 1/17 is equal to 48 which is a multiple of 16;
: the period of a(2) = 1/7 + 1/17 + 1/19 is equal to 144 which is a multiple of 18;
: the period of a(3) = 1/7 + 1/17 + 1/19 + 1/23 is equal to 1584 which is a multiple of 22.
(...)

The sequence of p(1), p(2), p(3)... is 7, 17, 19, 23...

Conjecture 4:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) divides p(2) - 1, the period of the rational number a(2) divides p(3) – 1, the period of the rational number a(n) divides a(n) – 1.

Example:

: the period of a(1) = 1/3 + 1/11 is equal to 2 which divides 10;
: the period of a(2) = 1/3 + 1/11 + 1/13 is equal to 6 which divides 12;
: the period of a(3) = 1/3 + 1/11 + 1/13 + 1/37 is equal to 6 which divides 36.
(...)

The sequence of p(1), p(2), p(3)... is 3, 11, 13, 37...

Conjecture 5:

For any Poulet number P there exist a rational number r equal to a sum of unit fractions 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1), p(2), p(3)... are distinct odd primes, such that the period of r is equal to P – 1.

Example: the period of r = 1/5 + 1/29 + 1/113 + 1/271 is equal to 560, while 561 the second Poulet number and the first Carmichael number.