

Three formulas that generate easily certain types of triplets of primes

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Abstract. In this paper I present three formulas, each of them with the following property: starting from a given prime p , are obtained in many cases two other primes, q and r . I met the triplets of primes $[p, q, r]$ obtained with these formulas in the study of Carmichael numbers; the three primes mentioned are often the three prime factors of a 3-Carmichael number.

Note:

To refer to the three formulas easily I will name them the formula alpha, beta or gama and the triplets obtained the triplet alpha, beta or gama.

Formula alpha:

The formula alpha is $30 \cdot a^n - (a^p + a - 1)$. The first prime of a triplet alpha is p and the other two ones are obtained giving to n values of integers, under the condition that $a^p + a - 1$ is prime.

Examples:

- : For $p = 11$ and $a = 2$ the condition that $a^p + a - 1$ is prime is met because $2^{11} + 2 - 1 = 23$ which is prime; the formula alpha becomes $60 \cdot n - 23$; it can be seen that for $n = 1$ is obtained 47 (prime) and for $n = 2$ is obtained 97 (prime) so we have the triplet alpha $[11, 47, 97]$; also for $n = 3$ is obtained 157 (prime) so other two triplets alpha are $[11, 47, 157]$ and $[11, 97, 157]$;
- : For $p = 7$ and $a = 3$ the condition that $a^p + a - 1$ is prime is met because $3^7 + 3 - 1 = 23$ which is prime; the formula alpha becomes $90 \cdot n - 23$; it can be seen that for $n = 1$ is obtained 67 (prime) and for $n = 2$ is obtained 157 (prime) so we have the triplet alpha $[7, 67, 157]$; also for $n = 4$ is obtained 337 (prime) so other two triplets alpha are $[7, 67, 337]$ and $[7, 157, 337]$.

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula alpha.

Formula beta:

The formula beta is $30*a*n + (a*p + a - 1)$. The first prime of a triplet beta is p and the other two ones are obtained giving to n values of integers, under the condition that $a*p + a - 1$ is prime.

Examples:

- : For $p = 11$ and $a = 2$ the condition that $a*p + a - 1$ is prime is met because $2*11 + 2 - 1 = 23$ which is prime; the formula beta becomes $60*n + 23$; it can be seen that for $n = 1$ is obtained 83 (prime) and for $n = 4$ is obtained 263 (prime) so we have the triplet beta [11, 83, 263]; also for $n = 6$ is obtained 383 (prime) so other two triplets beta are [11, 83, 383] and [11, 263, 383];
- : For $p = 19$ and $a = 3$ the condition that $a*p + a - 1$ is prime is met because $3*19 + 3 - 1 = 59$ which is prime; the formula beta becomes $90*n + 59$; it can be seen that for $n = 1$ is obtained 149 (prime) and for $n = 2$ is obtained 239 (prime) so we have the triplet beta [59, 149, 239]; also for $n = 4$ is obtained 419 (prime) so other two triplets beta are [59, 149, 419] and [59, 239, 419].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula beta.

Formula gama:

The formula gama is $2*p*n - 2*n + p$. The first prime of a triplet gama is p and the other two ones are obtained giving to n values of integers, under the condition that $2*p - 1$ is prime.

Example:

- : For $p = 7$ the condition that $2*p - 1$ is prime is met; the formula gama becomes $12*n + 7$; for $n = 1$ is obtained 19 (prime) and for $n = 2$ is obtained 31 so we have the triplet gama [7, 19, 31]; also for $n = 3$ is obtained 43 so other two triplets gama are [7, 19, 43] and [7, 31, 43].

Note: see the sequence A182207 in OEIS for the connection between Carmichael numbers and formula gama.