Bootstrapping generations

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Abstract

A supersymmetric version of Chew's old democratic bootstrap argument predicts the existence of three generations of quarks and leptons with one quark, of type "up", more massive that the other five.

The origin of the now popular "dual models" can be traced to advocacy, by Geoffrey Chew in the early sixties, of the study of the S-matrix, eventually looking for an unique consistent solution that "bootstraps" itself. A guiding principle was "nuclear democracy"; no particle should be claimed to be elementary. Each particle was providing the force needed to bind the others [1, 2] and from the view of S-Matrix polologists, each particle was a composite of others.

At the start of the seventies, it was noticed that dual models with fermions [3, 4] seem to require a special symmetry, supersymmetry, relating bosonic and fermionic degrees of freedom. Schwarz [5] has proposed to interpret this need as a "quark-gluon dual model".

Given the stringy character of these models, it seems reasonable to consider that the gluon part, bosonic, is constituted by a string terminated in a pair of quarks. With this imaginery, we can try a version of the democratic bootstrap. We require that scalar leptons and scalar quarks are composed of pairs of quarks, bound by some gluonic string. Still, we keep agnostic about if quarks and leptons are composed of themselves or other particles, or if they are truly elementary.

Each possible pair of "string terminations" will produce one kind of scalar, as a triplet of colour in the case of pairs quark-quark, and as a colour singlet in the case of quark-antiquark; i.e., we are taking the triplet in $3 \times 3 = 3 + 6$ and the singlet in $3 \times \bar{3} = 1 + 8$ when combining the colour charges of the constituents. As for the electric charge, it is simply the sum of the charges of the two elements of the pair.

Following an emergent tradition, we will call squarks and sleptons to the scalar particles, and we will refer to the requirement of matching between the number of composite scalar particles and the number of degrees of freedom of the standard model fermions as the "sBootstrap". This matching, of course, must be fulfilled in each charge sector, for each value of electric and colour charge, but we do not need to consider the later in the calculations, as simply we have triplets coinciding with triplets and singlets with singlets.

So, lets first taste the sBootstrap condition at cask strength: N generations of quarks of types u, d should produce N^2 scalars of charge $+1/3$, but we have $2N$ fermionic degrees of

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freedom with such charge, so $N = 2$ for the matching in the "down antiquark" sector. And they also produce, from pairs of two down-type quarks, a total of $N(N+1)/2$ scalars of charge $-2/3$, again to be matched to $2N$ degrees of freedom in the "up antiquark" sector. So $N = 3$ for the matching in this sector. The mechanism fails, which is no surprise.

We need thus to dilute our condition for it to be palatable. We do it in the following simple way: requesting that not every quark can terminate the string. An argument for this is to imagine that some of the families have very high masses, perhaps of the order of the electroweak scale, so that on one hand they can not reach the relativistic speed at the extremes of the string, and on the other hand they would even disintegrate and decay before hadronization, not being able to form a durable bound state.

We postulate thus that only some subset of "light quarks" are in the terminations of the string and then able to form composites, and we will calculate the size of this subset. Given that we could even have "mixed" generations, with one light and a heavy quark, we have now at our disposal three parameters: the number of generations N, the number of light quarks of the down kind, say r, and the number of light quarks of the up kind, say s. The matching of scalar and fermionic degrees of freedom is now

$$rs = 2N \tag{1}$$

$$\frac{r(r+1)}{2} = 2N \tag{2}$$

respectively for $+1/3$ and $-2/3$ particles. The integer solution of the system is that N must be half of an hexagonal number

$$2N = \cancel{1}, 6, \cancel{15}, 28, \cancel{45}, 66, \cancel{91}, 120, ... \tag{3}$$

So the smallest admissible solution is $N = 3$. We could consider the requirement of asymptotic freedom in the beta function of QCD [6, 7] to put an upper limit to the number of solutions, asking the number of flavours n_f to be such that $2n_f < 33$. If this limitation applies to all the flavours, light or heavy, then the solution is unique.

We obtain N=3 generations of quarks with r=3 light down quarks but only s=2 light up quarks.

Perhaps it could be argued that the beta function should be built only with the light quarks. In this case we have still a very short list of solutions

N	s	r	Total light flavours
3	2	3	5
14	4	7	11
33	6	11	17

Where we have included the third solution only because 17 flavours are very near of the theoretical bound of 16.5. Of course all the solutions beyond $N = 3$ already contain a lot of heavy generations. Note that we always have $r = 2s - 1$.

To search for another source of uniqueness, lets look the slepton sector.

For the charged leptons, we have nothing new. Every charged scalar lepton is a composite of a quark and antiquark of different type, so $rs = 2N$.

For neutral leptons, we have some options. On the Standard Model side, we could consider the known degrees of freedom of neutrinos, only as left- handed particles, or add a right-handed neutrino in each generation. On the composition side, the question is what group must be use to classify the colour neutral composites. Given that they are similar to mesons (they could even be interpreted as the mesons themselves) we will classify them with $SU(r+s)$. We could try $U(r+s)$, having an extra neutral state, but then we had a odd number of neutral states and no solution.

So we have finally two possible equations.

1) With both left and right handed neutrinos:

$$r^2 + s^2 - 1 = 4N \tag{4}$$

2) With only left handed neutrinos

$$r^2 + s^2 - 1 = 2N \tag{5}$$

And we can discard the second option: when combined with the previous equations, it does not produce an integer solution. Interestingly, the sBootstrap can be extended to the lepton sector only if we use the extra degrees of freedom that come with the addition of right-handed neutrinos.

Furthermore, the extension produces a system of equations with an unique integer solution, the one with $N = 3$.

So, even if we do not want to use QCD beta-function as an argument, we also have an unique solution if we ask the the sBootstrap to produce all the scalars in the SSM.

To conclude: we have found that the sBootstrap requisites predict that the number of fermion families must be <u>three</u>.

Looking the solution group-theoretically, we can say that the scalars are produced from 3 "down quarks" and 2 "up quarks" according the decomposition of a flavour group $SU(5)$ to $SU(3) \times SU(2)$

We get the sleptons extracted from a **24** of this flavour group

$$24 = (1,1) + (3,1) + (2,3) + (2,\bar{3}) + (1,8) \tag{6}$$

And the squarks from the two sextets that appear in the decomposition of **15**.

$$15 = (3,1) + (2,3) + (1,6) \tag{7}$$

And similarly the anti-squarks.

In the solution, the number of "light" down-type quarks is $r = 3$, equal to the number $N = 3$ of generations, but the number of "light" up-type quarks is $s = 2$.

Having a third generation discovered with one massive "up type" quark, this should be considered a striking ~~prediction~~ postdiction from the theory of dual models, or string theory, how they call it nowadays.

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