An answer to Beal's Conjecture

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BEAL'S CONJECTURE: If $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor.

This document explains the simple validity of Beal's conjecture.

Solution:

Given that x, y & z are positive integers and greater than 2.
Assume that A1, B1, & C1 are positive integers which are co-prime.
We know that for any two positive integers

- ⇨ $A1^x + B1^x = C1$ ----------------I
- ⇨ $A1^x - B1^x = C1$ if A1>B1
  By rearranging $A1^x = B1^x + C1$ ----------------II
- ⇨ $A1^x - B1^x = -C1$ if A1<B1
  By rearranging $A1^x + C1 = B1^x$ ----------------III

All the three equations are in the form of A^x (+/-) B^x = +/- C

Step1: Multiply equation I with a common factor $C1^x$ or its multiples[#]

- ⇨ $C1^x * A1^x + C1^x * B1^x = C1^{(x+1)}$
- ⇨ $(C1*A1)^x + (C1*B1)^x = C1^{(x+1)}$ --------------IV

Let A=C1*A1, B=C1*B1, C=C1 & z=x+1
Since A1, B1, C1 are positive integers, A, B, C are also positive integers.
Now equation IV can be rewritten as

$A^x + B^x = C^z$

If the numbers A, B & C can be factored further then value of x, z will change further to x, y & z and the equation can be rewritten as $\mathbf{A^x + B^y = C^z}$

Here it is proved that if $\mathbf{A^x + B^y = C^z}$ where A, B, C, x, y, z are positive integers x, y, z are greater than 2 then there is a common factor C1 between the three numbers.

Now if we apply the same from step1 to equations II & III we can conclude that $\mathbf{A^x + B^y = C^z}$ where C1 is the common factor.

Hence from the above we can say that the numbers which are following Beal's Conjecture can be rewritten in the form of $A^x + B^x = C$ or $A^x = B^x + C$ or $A^x + C = B^x$

Example1:

Let us assume any 2 numbers say A=7, B=3 & x=4

Now we know that $7^4 + 3^4 = 2482$
Let us assume a multiplication factor 2482^4. (We can take multiples of 2482 also. Since there is no other common factor between the three numbers, any other factor may not result into Beal conjecture).
Now $7^4 * 2482^4 + 3^4 * 2482^4 = 2482 * 2482^4$

- ⇨ $\mathbf{17374^4 + 7746^4 = 2482^5}$
  which is a Beal's conjecture with a common factor of 2482^4.

Instead of addition if we assume $7^4-3^4=2320$
By rearranging we can say that $3^4+2320=7^4$
Here we can assume a multiplication factor of 2320^4 or its multiples.
Now $3^4*2320^4+2320*2320^4=7^4*2320^4$

⇨ **$6960^4+2320^5=16240^4$**
which is a Beal's conjecture with a common factor of 2320^4.

Example2:

Let us assume 2 smaller numbers say A=3, B=1 & x=3

Now we know that $3^3+1^3=28$
Let us assume a multiplication factor 2^3*224^3.
Now $3^3*2^3*224^3+1^3*2^3*224^3=28*2^3*224^3$

⇨ **$1344^3+448^3=224^4$**
which is a Beal's conjecture with a common factor of 448^3.

Instead of addition if we assume $3^3-1^3=26$
By rearranging we can say that $1^3+26=3^3$
Here we can assume a multiplication factor of $2^3 208^3$ or its multiples.
Now $1^3*2^3*208^3+26*2^3*208^3=3^3*2^3*208^3$

⇨ **$416^3+208^4=1248^3$**
which is a Beal's conjecture with a common factor of 416^3.

Table-A below shows some examples of conjecture which are satisfying Beal's criteria (in the form of $A^x+B^y=C^z$) and can be rewritten in the form of $A1^x$ (+/-) $B1^x$=C1 after division by the HCF. We can note that A1, B1 & C1 are co-prime.

Table - A

Sl No	$A^x+B^y=C^z$ form	HCF	$A1^x(+/-) B1^x= +/-C1$ form
1	$27^4+162^3=9^7$	9^6	$1^3+2^3=9$
2	$3^3+6^3=3^5$	3^3	$1^3+2^3=9$
3	$144^3+288^3=72^4$	$72^3 2^1$	$1^3+2^3=9$
4	$117^4+234^3=585^3$	117^3	$117+2^3=5^3$
5	$294^3+98^4=490^3$	98^3	$3^3+98=5^3$
6	$54^5+54^5=972^3$	54^5	$1^5+1^5=2$
7	$91^3+13^5=104^3$	13^3	$7^3+169=8^3$
8	$961^3+31^5=62^5$	31^5	$31+1^5=2^5$
9	$61^4+244^3=305^3$	61^3	$61+4^3=5^3$
10	$91^4+455^3=546^3$	91^3	$91+5^3=6^3$
11	$211^6+422^5=633^5$	211^5	$211+2^5=3^5$
12	$2^3+2^3=2^4$	2^3	$1^3+1^3=2$
13	$32^3+8^5=4^8$	8^5	$1^5+1^5=2$
14	$512^3+512^3=128^4$	$128^3 4^3$	$1^3+1^3=2$
15	$32^4+32^4=8^7$	$8^4 4^4$	$1^4+1^4=2$
16	$152^4+608^3=912^3$	$152^3 2^3$	$19+2^3=3^3$
17	$242^5+242^6=726^5$	242^5	$1^5+242=3^5$
18	$63^3+63^4=252^3$	63^3	$1^3+63=4^3$
19	$273^3+364^3=91^4$	91^3	$3^3+4^3=91$
20	$65^3+260^3=65^4$	65^3	$1^3+4^3=65$

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21	$455^3+1001^3=182^4$	91^3	$5^3+11^3=1456$ $(91 *2^4)$
22	$35^5+310^3=435^3$	5^3	$420175+62^3=87^3$
23	$37^4+111^3=148^3$	37^3	$37+3^3=4^3$
24	$127^4+762^3=889^3$	127^3	$127+6^3=7^3$
25	$88^7+176^5=528^5$	176^5	$242+1^5=3^5$

Conclusion:

BEAL'S conjecture $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, has been derived from the basic expression $A^x +/- B^x=C$ where A, B, C & x are positive integers & x>2, by multiplication with a suitable common factor. We know that the above expression $A^x +/- B^x=C$ is true for any positive integers & ***therefore Beal's conjecture is also true for the condition specified.***

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Note: #- Any multiplication factor will not change the results of the equation. But we will not be able to form the Beal's conjecture. Only a set of multiplication factor will leads to the formation of Beal's conjecture.

A, B, C, x, y, z are general terms & hence need not be the same between the two different expressions.

A, B, C, x, y, z are assumed to be finite positive integers.

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