The Critical Error in the Formulation of the Special Relativity

Radwan M. Kassir

radwan.elkassir@dargroup.com

Abstract

The perception of events in two inertial reference frames in relative motion is analyzed from the perspective of the special relativity (SR) postulates. Straightforward inconsistencies have been identified, and subsequent mathematical contradictions disproving the SR predictions have been determined. The approach used in the SR formulation to get around the identified contradictions is revealed.

Introduction

The SR time dilation prediction is based on the transformation of the time interval between two events occurring at the origin (or co-local events) in one reference frame to another frame in relative translational motion with respect to the first. It has been shown in earlier works[1,2] that such transformation would be invalid, as it involves coordinates having zero value. In this paper, further analysis of event perceptions relative to both frames will reconfirm the invalidity of such transformation, hence the invalidity of the SR prediction of the time dilation. Similar analysis for simultaneous events proves the invalidity of the SR length contraction prediction.

Temporal Events Analysis

Consider two inertial frames of reference, $K\left(x,y,z,t\right)$ and $K'\left(x',y',z',t'\right)$, in translational relative motion with parallel corresponding axes, and let their origins be aligned along the overlapped x- and x'- axes. Let v be the relative motion velocity in the x-x' direction. K and K' are assumed to be overlapping at the time $t = t' = 0$.

Arbitrary non-origin events

Let's suppose that at the frames overlapping instant, an event $E_1(x',0,0,0)$ $[E_1(x,0,0,0)]$ takes place at distance x' with respect to K' origin (x with respect to K origin) on the x-x' axis. According to the SR light speed postulate, this event is perceived by an observer at K' origin at the time:

$$t' = \frac{x'}{c}, \qquad (1)$$

and by an observer at K origin at the time:

$$t = \frac{x}{c}. \tag{2}$$

With respect to the K' observer, the origin of K' at this perception time is at a distance of vt' from that of K, and using the SR speed of light postulate, the same event will be perceived (with respect to the K' observer) by an observer at K origin at the time:

$$t = t' + \frac{vt'}{c}. \tag{3}$$

To account for any time scaling due to the relative motion between the inertial frames K and K', let's write equation (3) in the following form.

$$t = \gamma\left(t' + \frac{vt'}{c}\right), \tag{4}$$

where γ is a real positive factor depending on v.

In fact, this scaling factor is essential for the speed of light postulate to be retained, since using the light speed postulate, the inverse of Eq. (3) can be written as:

$$t' = t - \frac{vt}{c},$$

which, when substituted in Eq. (3) will lead to $t = t'$, and consequently to $v = 0$.

Replacing Eq. (1) into Eq. (4) yields:

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right). \tag{5}$$

Multiplying both sides of Eq. (4) by c, and using Eqs. (1) and (2) leads to:

$$x = \gamma(x' + vt'). \tag{6}$$

Using the SR first postulate and the isotropic property of space, the inverse of the transformation Eqs. (5) and (6) can be obtained by swapping the primed and unprimed coordinates, and replacing v with $-v$:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right). \tag{7}$$

$$x' = \gamma(x - vt). \tag{8}$$

Solving Eqs. (5), (6), (7), and (8) for γ results in:[1,2]

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{9}$$

Eqs. (5), (6), (7) and (8) are therefore the inverse and direct Lorentz transformation equations for the coordinates in the relative motion direction.

Co-local events at K' origin

Now, suppose an event $E_2(0,0,0,t')$ $[E_2(vt,0,0,t)]$ occurs at K' origin,

$$x' = 0, \tag{10}$$

at the time t' with respect to K' $(t$ with respect to $K)$.

Again, with respect to the K' observer, the origin of K' at the event perception time is at a distance of vt' from that of K, and using the SR speed of light postulate, the same event will be perceived (with respect to the K' observer) by an observer at K origin at the time:

$$t = t' + \frac{vt'}{c}, \tag{11}$$

or, to account for any time scaling, at the time:

$$t = \gamma\left(t' + \frac{vt'}{c}\right). \tag{12}$$

However, in this case Eq. (1) doesn't hold, and therefore Eq. (5) doesn't follow. Yet, in SR it is customary for such events (occurring at K' origin) to replace Eq. (10) $(x' = 0)$ in Lorentz transformation Eq. (5), inapplicable in this case, since it is derived for events having $x' = ct'$ invalid for co-local events having $x' = 0$ and $t' > 0$.

Therefore, for an event occurring at K' origin $(x'=0)$ at time t', the SR predicted time t with respect to K is concluded from the invalid (for this case) Eq. (5) as:

$$t = \gamma t'. \tag{13}$$

Whereas, Eq. (12) predicts this time to be:

$$t = \gamma t'\left(1+\frac{v}{c}\right). \tag{14}$$

Comparing Eqs. (13) and (14) results in the contradiction:

$$v = 0. \tag{15}$$

It follows that the SR conversion $x'=0;\ t=\gamma t'$, predicting time dilation, is invalid.

The same analysis of the above two events can be performed from the perspective of an observer at K origin, with a similar contradiction being obtained.

Simultaneous events

Similarly, Lorentz transformation Eq. (6) is not applicable for events having $t'=0$ and $x'\neq 0$, as it is derived under Eqs. (1) and (2), requiring $x'=0$ for $t'=0$. However, in SR interpretation of Lorentz transformation Eq.(6), length contraction from the perspective of K' is predicted by setting $t'=0$ (for simultaneous events duration) to get the relation $x=\gamma x'$, ignoring the restriction imposed by the basic speed of light constancy Eqs. (1) and (2). Hence follows the invalidity of the SR length contraction prediction. The same reasoning is applicable to Eq. (8) to show the invalidity of the SR time contraction prediction from the perspective of K.

The Special Relativity approach

It is ascertained in the previous sections that the Lorentz transformation time equations:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right),$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right),$$

are principally derived on the basis of events having $x = ct;\ x' = ct'$, invalid for co-local events having $x = 0$ and $t > 0$ $(x' = 0$ and $t' > 0)$. These restrictions are obviously fatal for the SR formulation requiring such co-local events—separated by a time interval—for the interpretation of the Lorentz transformation. In order to overcome this obstacle, the equations:

$$x = ct, \qquad (16)$$

and

$$x' = ct', \qquad (17)$$

expressing the basic speed of light constancy principle, were manipulated and combined into the equation:

$$x^2 - c^2t^2 = x'^2 - c^2t'^2, \qquad (18)$$

set as the principle equation representing the SR speed of light postulate.[3] Setting $x = 0$ with $t > 0$ (or $x' = 0$ with $t' > 0$); or $t = 0$ with $x \neq 0$ (or $t' = 0$ with $x' \neq 0$), is made now possible with the constructed Eq.(18), while the conditions $x = 0;\ t = 0$ $(x' = 0;\ t' = 0)$, imposed by the original light speed constancy Eqs.(16) and (17), are ignored!

It should be noted that Eq.(18) can also be obtained from the light sphere equations, namely:

$$x^2 + y^2 + z^2 = c^2t^2, \qquad (19)$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2, \qquad (20)$$

representing the light speed constancy principle in the three-dimensional space, by subtracting the two equations from each other, and using the invariance of the y and z coordinates (i.e., $y = y',\ z = z'$). However, Eqs.(19) and (20) also require that at the instant of time $t = t' = 0$—the moment when the spherical light wave front is emitted from the coinciding frame origins—the spatial coordinates must be zero as well, i.e. $x = x' = 0$, $y = y' = 0$, and $z = z' = 0$; these initial conditions are not attributed to the resulting Eq.(18) in the SR formulation.

Eq.(18) forms the basis of the Lorentz transformation derivation in the SR formulation.[3] The Lorentz transformation equations are indeed derivable, yet more tediously, from Eq.(18) being mathematically equivalent to the deriving Eqs. (16) and (17)—except with no consideration given to the coordinate values obtained from these equations at the space and time origins (i.e., ignoring the initial conditions required by equations (16) and (17)). Such a critical violation undermines the validity of the SR predictions, in agreement with the findings of earlier studies.[1,2] In fact, these studies demonstrate that the Lorentz transformation equations result in mathematical contradictions when applied for co-local or simultaneous events.

Mathematical contradictions

Indeed, substituting Eq. (7) into Eq. (5), returns:

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right),$$

which can be simplified to:

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right). \qquad (21)$$

Since, as shown earlier, Eqs. (7) and (5) are derived under the condition of $x = ct;\ x' = ct'$, then Eq. (21) can be written as:

$$t\left(\gamma^2 - 1\right) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \qquad (22)$$

If Eqs. (7), (5) and (22) were generalized (i.e., applied to conversions other than $x = ct;\ x' = ct'$) and particularly applied to an event with the time $t' = 0$, then according to Eq. (7), the transformed t- coordinate with respect to K would be $t = vx / c^2$. Consequently, for $t \neq 0$, Eq. (22) would reduce to:

$$t\left(\gamma^2 - 1\right) = t\gamma^2, \qquad (23)$$

yielding the contradiction:

$$\gamma^2 - 1 = \gamma^2, \quad \text{or} \quad 0 = 1.$$

It follows that the conversion of the time coordinate $t' = 0$ to $t = vx / c^2$, for $x \neq 0$, by Lorentz transformation Eq. (7), is proved to be invalid, since it leads to a contradiction when used in Eq. (22), resulting from the Lorentz transformation equations for $t \neq 0$ (i.e., beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$)—Letting $t = 0$ would satisfy Eq. (23), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $t' = 0$ would be $t = vx / c^2 = 0$, yielding $v = 0$, as we're addressing the conversion of $t' = 0$ to $t = vx / c^2$ for $x \neq 0$.

A similar contradiction is obtained by substituting Eq. (5) into Eq. (7), and applying Eq.(5) for the conversion $t=0;\ t'=-vx'/c^2$ of the time coordinate $t=0$.

Furthermore, substituting Eq. (8) into Eq. (6), yields:

$$x=\gamma\left(\gamma\left(x-vt\right)+vt'\right);$$

$$x\left(\gamma^2-1\right)=\gamma v\left(\gamma t-t'\right);$$

$$x\left(\gamma^2-1\right)=\gamma vt\left(\gamma-\frac{t'}{t}\right). \qquad (24)$$

Since Eqs. (8) and (6), along with Eqs. (7) and (5), are derived under the condition of $x=ct;\ x'=ct'$, Eq. (24) can be written as:

$$x\left(\gamma^2-1\right)=\gamma vt\left(\gamma-\frac{x'}{x}\right). \qquad (25)$$

If Eqs. (8), (6) and (25) were generalized (i.e., applied to conversions other than $x=ct;\ x'=ct')$, and particularly applied to an event with the coordinate $x'=0$, then according to Eq. (8), the transformed x-coordinate with respect to K would be $x=vt$. Consequently, for $x\neq 0$, Eq. (25) would reduce to:

$$x\left(\gamma^2-1\right)=x\gamma^2, \qquad (26)$$

$\gamma^2 - 1 = \gamma^2$, or $0 = 1$.

It follows that the conversion of the space coordinate $x' = 0$ of K' origin to $x = vt$, at time $t > 0$, with respect to K by Lorentz transformation Eq. (8), is invalid, since it leads to a contradiction when used in Eq. (25), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e., beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$) —Letting $x = 0$ would satisfy Eq. (26), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $x' = 0$ would be $x = vt = 0$, yielding $v = 0$, as we're addressing the conversion of $x' = 0$ to $x = vt$ for $t > 0$.

A similar contradiction would follow upon substituting Eq. (6) into Eq. (8), and applying Eq. (6) for the conversion $x = 0$; $x' = -vt'$ of the space coordinate $x = 0$.

Conclusion

The Lorentz transformation equations are shown to be merely applicable for events satisfying the basic light speed constancy equations $x = ct$ and $x' = ct'$. The erroneous application of the Lorentz transformation on co-local events ($x' = 0$; $t' > 0$, in K', or $x = 0$; $t > 0$, in K), or simultaneous events ($t' = 0$; $x' \neq 0$, in K', or $t = 0$; $x \neq 0$, in K), is shown to result in mathematical contradictions and invalid predictions of time dilation, or length contraction, respectively.

1 Kassir, R. M. On Lorentz Transformation and Special Relativity: Critical Mathematical Analyses and Findings. *Physics Essays* **27**, 16 (2014).

2 Kassir, R. M. On Special Relativity: Root cause of the problems with Lorentz transformation. *Physics Essays* **27**, 198-203 (2014).

3 Einstein, A. Einstein's comprehensive 1907 essay on relativity, part I. *English translations in Am. Jour. Phys. 45 (1977), Jahrbuch der Radioaktivitat und Elektronik* **4** (1907).