Structural Unification of Newtonian and Rational Gravity

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Abstract If there were no electromagnetic interaction in the Solar system from the beginning then no heavy bodies like Earth or Sun would have existed and elementary particles would miss collision with each other due to the sparse population of particles. The solar system would have been a tiny "spiral galaxy" or "elliptical galaxy" because of the structuring nature of Newton's gravity.

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1 Astronomical Observation

Our global world is mighty and magnificent which, however, is a mass point as seen to the scale of the Solar system. This is because the Solar system is too vast and those massive materials are too rare to be caught by the Earth for its expansion. In the beginning, Earth might be formed by the gravitational pull of other materials. Later it reached a stable state. In 1600s, Newton summarized a formula of gravity for stable and isolated mass points. In 1910s, Einstein, based on his theory of relativity, generalized the formula whose accurate testification, however, is based on isolated mass points only.

Now we further expand our observation to the scale of galaxies, and we see that the Milky way is a disk-shaped, globally smooth stellar system which can not be considered to be a mass point. It has an internal structure that is solely resulted from gravity. All galaxies demonstrate some kind of internal structure. And those galaxies which are relatively independent, can be classified to be either three-dimensional ellipticals or disk-shaped spirals. Therefore, gravity should be considered to be a structural force. Because galaxies fall into a few types, the structuring nature of gravity can be studied. For the first time [1-3], the rational gravity (a kind of structuring force) was discovered based on the analysis of galaxy images and the concept of rational structure.

Rational Gravity: To any point on a rational structure, there correspond three Darwin surfaces which pass the point and are orthogonal to each other. To any Darwin surface there exists the corresponding component of the gravitational force at the point whose direction is normal to the surface and whose magnitude is proportional to the Gaussian curvature of the surface at the point.

The current paper is dedicated to the discussion on the possibility of the unification of Newtonian and rational gravity. Section 2 is a review of rational structure. The general formula of rational gravity is presented in Section 3. Section 4 is focused on the unification of Newtonian gravity with the rational one. The final Section is a short discussion on general relativity, current difficulties in large-scale astrophysics, and our future work.

2 Rational Structure and Line Magnitude

Let us consider the simplest case: two dimensional rational structure in the plane (x, y) which corresponds to disk-shaped galaxies. The fundamental idea of rational structure is the assumption of a net of orthogonal curves

$$\begin{cases} x = x(\lambda, \mu), \\ y = y(\lambda, \mu) \end{cases} \tag{1}$$

so that the matter density $\rho(x, y)$ on one side of each curve is in constant ratio to the one on the other side [4]. These curves are called Darwin curves [5]. Rational structure is a pattern of density distribution, and our theory on galaxy structure is essentially a pattern theory. Therefore, all our results depend solely on the derivatives to the logarithmic density

$$f(x, y) = \ln \rho(x, y) \tag{2}$$

not on the absolute strength of the density.

The basic quantities describing these curves are their line magnitudes, $P(\lambda, \mu) = \sqrt{x_\lambda'^2 + y_\lambda'^2}$, $Q(\lambda, \mu) = \sqrt{x_\mu'^2 + y_\mu'^2}$. We have proved that an orthogonal net of curves is equivalent to an analytic complex function [6]. Therefore, we can always choose x and y to be the real and imaginary parts, respectively, of an analytic complex function $x(\lambda, \mu) + \mathrm{i}\, y(\lambda, \mu)$. That is, the Cauchy-Riemann equations are always satisfied,

$$\begin{cases} x_\lambda' = y_\mu', \\ x_\mu' = -y_\lambda' \end{cases} \tag{3}$$

Accordingly, the two line magnitudes become equivalent

$$P = \sqrt{x_\lambda'^2 + x_\mu'^2} = \sqrt{y_\lambda'^2 + y_\mu'^2} = Q \tag{4}$$

The fundamental quantities describing the rational structure are the directional derivatives to the logarithmic density $f(x, y)$ along the orthogonal net of curves, $\hat{u}(\lambda, \mu), \hat{v}(\lambda, \mu)$. The parallel law is that they are the functions of single variables only, $\hat{u} = \hat{u}(\lambda),\ \hat{v} = \hat{v}(\mu)$. The skew law is that both the logarithmic density and the line magnitude are the functions of the single variable

$$\hat{C} = \hat{U}(\lambda) + \hat{V}(\mu) \tag{5}$$

where $\hat{u} = d\hat{U}(\lambda)/d\lambda, \hat{v} = d\hat{V}(\mu)/d\mu$ (see [6]). That is,

$$f(\hat{C}) = h \int P(C) dC \tag{6}$$

where we have introduced a common constant h so that $\hat{C} = hC$, $\hat{U} = hU$, etc. Note that natural density distribution usually requires $h < 0$. The mathematical relations among partial derivatives, directional derivatives and line magnitude are

$$\begin{aligned} f_\lambda' &= \hat{u} P(\lambda, \mu), \\ f_\mu' &= \hat{v} P(\lambda, \mu) \end{aligned} \tag{7}$$

Now we see that the formula (6) does lead to the above relations. The above formulas (6) and (7) are very important. If the line magnitude $P(C)$ does determine an orthogonal net of curves which satisfies the Cauchy-Riemann equations (3), then the resulting structure (6) must be rational. This is because the directional derivatives $\hat{u}$ and $\hat{v}$ given by the formulas (7) are single-variable functions themselves.

The paper [6] showed that the line magnitudes given by the Riccati equation with constant coefficients do determine orthogonal nets of curves which satisfy the Cauchy-Riemann equations. In fact, the Riccati equation is the necessary and sufficient condition for rational structure. Therefore, there exist only a few solutions to rational structure. Coincidentally, astronomical observation indicates that, ignoring those strongly interacted ones, there exist only a few types of galaxies.

3 Rational Gravity and Line Magnitude

Now we calculate the rational gravity in two-dimensional rational structure. That is, we need calculate the curvature vectors of the orthogonal Darwin curves (1). A circle has everywhere its curvature equal to the reciprocal of its radius. A smaller circle bends sharply and has larger curvature. The curvature of a straight line is zero. The curvature of a smooth curve is defined as the curvature of its osculating circle at each point. A curvature vector takes into account of its direction of the bend as well as its sharpness. At any point in a rational structure, there cross two Darwin curves at a right angle. Hence there exist a pair of curvature vectors at the point, and the curvature vector of one curve is tangent to the other curve. The sum of the two vectors is the rational gravity according to the definition given in Section 1.

The components of the tangent vector to the Darwin curve in parameter λ are [6],

$$\begin{cases} x'_\lambda = P\sin\alpha, \\ y'_\lambda = -x'_\mu = -P\cos\alpha \end{cases} \tag{8}$$

The squared modulus of the vector is

$$x'^2_\lambda + y'^2_\lambda = P^2 \tag{9}$$

The direction of the vector is $(\sin\alpha, -\cos\alpha)$. Therefore, the tangent direction of the μ curve is $(\cos\alpha, \sin\alpha)$. The second derivatives to the λ curve are

$$\begin{aligned} x''_{\lambda\lambda} &= P'_\lambda \sin\alpha + P\cos\alpha\ \alpha'_\lambda = P'_\lambda \sin\alpha + P\cos\alpha\,(-(\ln P)'_\mu) \\ &= P'_\lambda \sin\alpha - P'_\mu \cos\alpha, \\ y''_{\lambda\lambda} &= -P'_\lambda \cos\alpha + P\sin\alpha\ \alpha'_\lambda = -P'_\lambda \cos\alpha + P\sin\alpha\,(-(\ln P)'_\mu) \\ &= -P'_\lambda \cos\alpha - P'_\mu \sin\alpha \end{aligned} \tag{10}$$

To obtain the curvature k_1 of the λ-curve, we calculate

$$\begin{aligned} &|x'_\lambda y''_{\lambda\lambda} - y'_\lambda x''_{\lambda\lambda}| \\ &= P\,|-P'_\lambda \sin\alpha\cos\alpha - P'_\mu \sin^2\alpha + P'_\lambda \sin\alpha\cos\alpha - P'_\mu \cos^2\alpha| \\ &= P\,|P'_\mu| \end{aligned} \tag{11}$$

Finally we have the curvature

$$k_1 = \frac{1}{R_1} = \frac{|x'_\lambda y''_{\lambda\lambda} - y'_\lambda x''_{\lambda\lambda}|}{(x'^2_\lambda + y'^2_\lambda)^{3/2}} = \frac{|P'_\mu|}{P^2} \tag{12}$$

In vectorial form, it is

$$\mathbf{k}_1 = \frac{|P'_\mu|}{P^2}(\cos\alpha, \sin\alpha) \tag{13}$$

Similarly the curvature vector for the μ curve is

$$\mathbf{k}_2 = \frac{|P'_\lambda|}{P^2}(\sin\alpha, -\cos\alpha) \tag{14}$$

Finally the rational gravity is

$$\mathbf{F}(x, y) \propto \mathbf{k}_1 + \mathbf{k}_2 = \frac{1}{P^2}(|P'_\mu|\cos\alpha + |P'_\lambda|\sin\alpha, |P'_\mu|\sin\alpha - |P'_\lambda|\cos\alpha) \tag{15}$$

We see that rational gravity is determined by the orthogonal Darwin curves, not by the density distribution. However, logarithmic density and line magnitude determine each other uniquely (see (6)). Accordingly we can derive an expression of the rational gravity with the logarithmic density f instead of the line magnitude P. Making use of the formula (6), we have

$$\begin{aligned} P'_\lambda &= P'_C\,\hat{u}/h = f''_{\hat{C}}\,\hat{u}, \\ P'_\mu &= P'_C\,\hat{v}/h = f''_{\hat{C}}\,\hat{v} \end{aligned} \tag{16}$$

Therefore,

$$\mathbf{F}(x, y) \propto \mathbf{k}_1 + \mathbf{k}_2 = \frac{|f''_{\hat{C}}|}{f'^2_{\hat{C}}}\,\nabla f \tag{17}$$

where we see that the constant h is completely canceled out (see the formula (6)). The new expression turns out to be very simple. Its deep meaning should be explored in the future. Note that this formula is applicable to two-dimensional rational structure only. In the case of three-dimensional rational structure, the Gaussian curvature vectors for Darwin surfaces must be employed.

Now we turn to the important example of ordinary spiral galaxies which are simply an exponential disk. The disk has infinite nets of orthogonal Darwin curves. Surprisingly, different nets share the same rational gravity [3]. That is, application of the formula (17) to exponential disk results in

$$F(x, y) \propto |\mathbf{k}_1 + \mathbf{k}_2| = \frac{1}{r} \tag{18}$$

The gravitational field is inversely proportional to the distance from the disk center. This result explains the phenomenon of constant rotational curves elegantly [3].

4 Unification of Newtonian and Rational Gravity

The formula (18) reveals a way in which Newtonian and rational Gravity may be unified. This formula misses a proportion constant which should be determined by galaxy observation. Here, however, a choice of the proportion constant is attempted so that the formula (18) reduces to Newtonian gravity in some process. The choice is

$$F(x, y) = G_2 M \frac{1}{r} \tag{19}$$

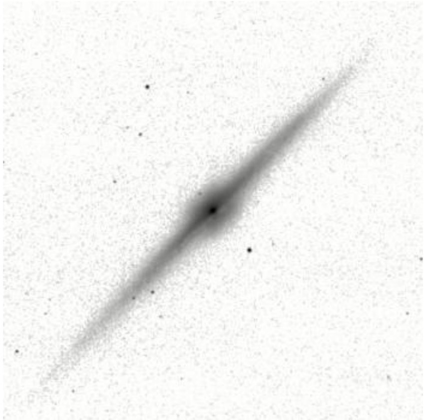

Figure 1: A longer-wavelength image of edge-on spiral galaxy NGC 4565 (Courtesy of Jarrett *et al* [8]). The central bulge is easily recognized.

where G_2 is the gravitational constant in two-dimensional gravity and M is the total mass of the disk galaxy. A spiral galaxy is a thin disk (see Figure 1 and [8]), and we humans' first impression is that the gravitational force field should be two-dimensional (i. e., cylindrical) so that the disk would not collapse [1]. Hence, the vectors of the force field in different layers of the disk are parallel to the disk. Note that the above formula is applicable to ordinary spiral galaxies only and the rational gravity is inversely proportional to the distance from the vertical axis at the galaxy center. One way to generalize the two-dimensional rational structure to three-dimensional is to rotate spatially the net of orthogonal Darwin curves (1) about its symmetric axis [7]. In the example of ordinary spiral galaxies, a spherically symmetric distribution of stars is expected whose rational gravity is similar to Newton's one

$$F(x, y) = GM\frac{1}{r^2} \tag{20}$$

where G is the usual gravitational constant. However, the difference between Newton's gravity and rational one must be emphasized. Newton's gravity is between two stable and isolated mass points whereas the rational one is the self-gravity inside a rational structure

$$\rho(x, y, z) = \rho_0 e^{f(x,y,z)} \tag{21}$$

In the classical Newtonian theory of smooth mass distribution, people employ Poisson's equation which, however, is a differential expression of the two-body interaction and the law of action at a distance. This theory of galactic dynamics has been used to predict kinematical phenomena of galaxies [9]. These predictions do not match galaxy observation generally. The first example was presented by Zwichy [10]. Another well-known example is the problem of constant rotation curves. To maintain the status of Newton's theory, people introduced the concept of dark matter which, however, has never been observed directly.

Why is our rational gravity the generalization to Newton's one? We know from the formulas (13) and (14) that rational gravity is independent of the constant $h(< 0)$. If we choose an infinite value of h then $f(x, y)$ goes to $-\infty$ and the resulting rational structure becomes an isolated mass point. The total mass M is kept constant in the process and Newton's gravity finally comes to the stage. This is the unification of Newton's theory with rational gravity.

In short, Newton's gravity is proposed to be a structural force. If there were no electromagnetic interaction in the Solar system from the beginning then no heavy bodies like

Earth or Sun would have existed and elementary particles would miss collision with each other due to the sparse population of particles. The solar system would have been a tiny "spiral galaxy" or "elliptical galaxy" because of the structuring nature of Newton's gravity.

5 Discussion

Our proposition is not contradictory with general relativity (GR) which is applicable to strong gravity only. Galaxy structure, however, is under the category of weak gravity. Nevertheless, GR and our proposition share the same feature of geometrization of gravity. According to the theory of relativity, light speed c is assumed to be the maximum one. Therefore, any quantity with a factor c is considered to be a large quantity. In GR, ct is such a large quantity which serves the forth dimension of spacetime manifold that has an indefinite metric. The inverse of another large quantity $cr/2GM$ is used for the approximation of GR. The requirement is that the zeroth-order approximation reduces to Newton's gravity. GR is testified precisely only in the first-order approximation. However, the testification is more consistent if we assume that spacetime is always locally curved from a larger flat background spacetime [11]. The coordinates in the flat background can be used to describe the curved spacetime. This new explanation of GR does not destroy (instead keeps) the basic principles of old GR (e. g., equivalence principle, curved manifold, and general covariance). We know that there exist only a few solutions to rational structure. Therefore, the unification of GR with rational gravity should be no problem as we did for newtonian gravity in the last Section.

Certainly GR is not directly applicable to galaxies. For example, the weak field approximation of GR is called gravitoelectromagnetism (GEM, see [12]) which maintains the Poisson's equation. For a mass point, the Poisson's equation reduces to Newton's formula of universal gravity. Because the left side is linear with the gravitational potential and the right side is linear with the mass density, the Poisson's equation in its general form is nothing but the linear summation of Newton's universal gravity between mass points. That is, Poisson's equation is a differential expression of the two-body interaction and the law of action at a distance. It is hard to imagine that the stars far away from the galaxy center suffer instant forces from the stars near the galaxy center. If gravity was consistent with a theory resembling Maxwell's electromagnetism as suggested by GR, then gravitational waves would be ubiquitous because of omnipresent black holes. However, the waves have not been detected by now. Currently astrophysicists use Newton's universal gravity and dark matter to explain the kinematic phenomena of galaxies. However, the purpose of the addition of dark matter is to make Poisson's equation hold. This theory needs to be changed even though we assume dark matter would exist.

Furthermore, there exists the stability issue with the galactic application of GR and Newtonian gravity. Spiral galaxies which are always thin disks (see Figure 1), may require a two-dimensional gravitational field parallel to the disks so that the thin disks are not only possible but also stable. Surprisingly, rational gravity which is derived directly from rational structure, provides such a gravitational field. However, GR and Newtonian gravity without their generalization can not provide in their first principles such a cylindrical gravitational field. Their galactic application encounters many instability issues (see bar instability as an example [13]). It was also pointed out that a massive dark halo would be needed in stabilizing galactic disks if Newtonian gravity continued to be true [14]. Rational gravity, however, is a “stabilizing force” itself. Image fitting indicates that galaxies are rational structure and their arms follow Darwin curves [4, 5]. Therefore, galaxy arms themselves are the result of the perturbation to rational structure. The perturbation still meets the principle of constant ratios: To achieve a minimum perturbation to rational structure, galaxy arms try to be developed along the curves of constant ratios, that is, along the Darwin curves [4, 5]. In other words, “constant ratios” are directly the structuring force, i. e., the rational gravity. These results need further quantitative investigation.

In short, difficulties in large scale astrophysics may have a solution if we assume that gravitational field should have a local origin. In this regard, our rational gravity is promising. However, our study on rational structure and rational gravity are based on relatively isolated galaxy structure, that is, based on spiral and elliptical galaxies. Irregular galaxies which are strongly interacted systems with their environment, suffer external gravity in addition to their self-gravity. Generalization of our discussion to strongly interacted systems is of great challenge. This is left for the future work.

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