On the Hyper-symmetric Maxwell Equations and its Applications

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Abstract: In this paper, we propose a new space-time structure that is the three-dimensional time structure by comparison to the three-dimensional space. These two space-time structures are completely symmetrical. We improve the Maxwell equations in the new space-time structure by adding a new set of equations. The new set of equations has the space-time symmetrical structure by comparison to the old one. We can also obtain another type of wave equation by solving the hyper-symmetric Maxwell equations. The new wave equation contains both time and space coordinates. The general solution of this new wave equation can be divided into two parts. One part corresponds to the free particles, which means the localized electromagnetic wave or virtual photon. The amplitude of the localized electromagnetic wave will decrease exponentially when the distance from the mass center increasing. Another part of the general solution corresponds to the bound state of the particles. The equation of the localized electromagnetic wave in bound state is consistent with the Schrödinger's equation. So we can draw the conclusion that the Schrödinger's equation is just a special case of the localized electromagnetic wave equation. We can get the new interpretation of the wave function in quantum mechanics based on these analyses. The new interpretation shows that the essence of the wave function in quantum theory is the localized electromagnetic wave or virtual photon. So we can solve the problem of the collapse of the wave function based on the new interpretation. In order to proof the correctness of the localized electromagnetic wave or virtual photon, we apply it to solve the problem of Helium atom's ground state energy. The theoretic calculation results are very satisfactory. Our calculation shows that the theoretic value of Helium atom's ground state energy is -2.9033864868188(69)a.u., which is very close to the experiment results.

Key Words: Wave function; Maxwell equations; Helium atom; Ground state energy

1 Introduction

Maxwell equations have some extent of symmetric structure. However, that the divergence of magnetic field equals to zero decreases the symmetry of the equations. Dirac found a method to solve this problem by adding the magnetic monopole [1].

The new problems also rose up although the symmetric problem was solved by adding the monopole. The most serious problem is that there is no explicit evidences show that the monopole exists in the cosmic while the Dirac's symmetric Maxwell equations predict that it must be exist.

It means that we must do a choice: keep the symmetric structure of the Maxwell equation with the nonexistent magnetic monopole or abandon the symmetric structure of the Maxwell equations without considering the magnetic monopole.

Many authors do the second selection, but some authors do the first selection. Some authors declared that the symmetric of the Maxwell equations can be kept by introducing the

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three-dimensional time [2–7]. However, some criticisms also pointed out that the postulation of three-dimensional time lacks of the support of the evidence [8][9].

This article selects the first choice: keep the symmetric structure by introducing the eight-dimensional time-space structure. We introduce the new equations in three-dimensional time in order to keep the hyper-symmetric structure of the two set of equations.

2 Three-dimensional time

In current space-time structure, the space is three-dimensional while the time is one dimensional. Here we postulate that there is a new space-time structure, in which the time is three-dimensional while the space is one dimensional.

2.1 The differential operator

In the first, we make some conventions for the symbols used in this paper.

The space and time parameters are expressed as x_1, x_2, x_3, y in three-dimensional space structure. The low case letter x represents space and the subscripts represent the dimensions. The low case letter y represents time.

The time and space parameters are expressed as y_1, y_2, y_3, x in three-dimensional time structure.

The relationships between y and t are shown below:

$$y=ct,\ y_1=ct_1,\ y_2=ct_2,\ y_3=ct_3$$

The reason why multiply t by c is to ensure that all the coordinates must have the same magnitude in two space-time structure.

Then the Nabla operator can be expressed in different space-time structure.

In three-dimensional space structure:

$$\nabla = \hat{x}_1 \frac{\partial}{\partial x_1} + \hat{x}_2 \frac{\partial}{\partial x_2} + \hat{x}_3 \frac{\partial}{\partial x_3}$$

In three-dimensional time structure:

$$\nabla_y = \hat{y}_1 \frac{\partial}{\partial y_1} + \hat{y}_2 \frac{\partial}{\partial y_2} + \hat{y}_3 \frac{\partial}{\partial y_3}$$

For convenience, we define:

$$\nabla_t = c\nabla_y$$

Or:

$$\nabla_t = \hat{t}_1 \frac{\partial}{\partial t_1} + \hat{t}_2 \frac{\partial}{\partial t_2} + \hat{t}_3 \frac{\partial}{\partial t_3}$$

Then we can get the divergence and curl of a physics quantity in two different space-time structures.

The vectors in two space-time structure can be expressed as:

$$\mathbf{X} = x_1\hat{x}_1 + x_2\hat{x}_2 + x_3\hat{x}_3 = \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3$$

$$\mathbf{Y} = y_1\hat{y}_1 + y_2\hat{y}_2 + y_3\hat{y}_3 = \mathbf{y}_1 + \mathbf{y}_2 + \mathbf{y}_3$$

2.2 The expressions of the general parameters

2.2.1 The general electric and magnetic field strength

In order to make comparison between these two space-time structures, the magnitude of the electric field strength and magnetic field strength must be the same. We define the general electric field strength as

$$\mathbf{F} = \sqrt{\varepsilon}\mathbf{E}$$

The general magnetic field strength is

$$\mathbf{G} = \sqrt{\mu}\mathbf{H}$$

2.2.2 The general electric charge and general magnetic charge

Here we use e represents the electric charge, m represents the magnetic charge.
The general electric charge is β_e. The general magnetic charge is β_m
The general electric charge is:

$$\beta_e = \frac{e}{\sqrt{\varepsilon}}$$

The general magnetic charge is:

$$\beta_m = \sqrt{\mu}m$$

The electric charge density is:

$$\rho_e = \frac{e}{V}$$

The magnetic charge density is:

$$\rho_m = \frac{m}{V}$$

Then the general electric charge density is:

$$g_e = \frac{\rho_e}{\sqrt{\varepsilon}}$$

The general magnetic charge density is:

$$g_m = \sqrt{\mu}\rho_m$$

These conventions promise the magnitudes of all of the charges and densities can be the same.

2.2.3 The general electric current and magnetic current

If we assume the electric charge density flowing through unit area is constant, then the electric current can be expressed as:

$$\mathbf{I} = \frac{dq}{dt}\hat{I} = q\mathbf{v}$$

The general electric current density can be defined as:

$$\mathbf{J}_e = \sqrt{\mu}\mathbf{J} = \sqrt{\mu}\rho_e\mathbf{v} = \sqrt{\mu}\sqrt{\varepsilon}g_e\mathbf{v} = g_e\frac{\mathbf{v}}{c}$$

The relevant general magnetic current density can be defined as:

$$\mathbf{J}_m = g_m\frac{c}{v}\hat{\mathbf{J}}_m$$

3 The new Maxwell equations

By considering the symmetrical of the electric and magnetic fields, time and space, we use the transformation rules below to construct the new Maxwell equations.

$$\begin{aligned}
&\nabla \leftrightarrow \nabla_y \\
&x \leftrightarrow y \\
&g_e \leftrightarrow g_m \\
&\mathbf{J}_e \leftrightarrow \mathbf{J}_m \\
&\mathbf{G} \leftrightarrow \mathbf{F} \\
&\mathbf{F} = \sqrt{\varepsilon}\mathbf{E} \\
&\mathbf{G} = \sqrt{\mu}\mathbf{H}
\end{aligned}$$

The new Maxwell equations will be divided into two sets.

Equations (1) ~ (4) reflect the relationships between electric field and magnetic field in three-dimensional space structure, which are consistent with the old Maxwell equations. Equations (5) ~ (8) reflect the relationships in three-dimensional time structure, which are the new equations.

The Maxwell equations in three-dimensional space are

$$\begin{cases}
\nabla \cdot \mathbf{F} = g_e \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1) \\
\nabla \cdot \mathbf{G} = 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2) \\
\nabla \times \mathbf{F} = -\dfrac{\partial \mathbf{G}}{\partial y} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3) \\
\nabla \times \mathbf{G} = \dfrac{\partial \mathbf{F}}{\partial y} + \mathbf{J}_e \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)
\end{cases}$$

The Maxwell equations in three-dimensional time are

$$\begin{cases} \nabla_y \cdot \mathbf{G} = g_m \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5) \\ \nabla_y \cdot \mathbf{F} = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6) \\ \nabla_y \times \mathbf{G} = -\dfrac{\partial \mathbf{F}}{\partial x} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7) \\ \nabla_y \times \mathbf{F} = \dfrac{\partial \mathbf{G}}{\partial x} + \mathbf{J}_m \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8) \end{cases}$$

4 The calculation of the magnetic charge

We use the magnetic monopole hypothesis proposed by Dirac to calculate the magnetic charge. By using the electric charge quantization condition of Schwinger, we can get:

$$em = \frac{2h}{\mu}$$

We assume that there are the relationship between general magnetic charge and electric charge:

$$\beta_e = \alpha \beta_m$$

Then

$$\frac{e}{\sqrt{\varepsilon}} = \alpha \sqrt{\mu} m$$

Where

$$\alpha = \frac{e^2}{2\varepsilon hc}$$

It is the fine structure constant.

5 Some characteristics of the three-dimensional time

5.1 General velocity

The velocity is defined in three-dimensional space as

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \frac{cd\mathbf{x}}{dy}$$

We can define the velocity in three-dimensional time as

$$\mathbf{w} = \frac{d\mathbf{t}}{dx} = \frac{d\mathbf{y}}{cdx}$$

The two velocities have

$$\mathbf{w} = \frac{1}{v} \hat{y}$$

Here we define the general velocity in three-dimensional space as

$$\mathbf{v}_c = \frac{\mathbf{v}}{c}$$

The corresponding general velocity in three-dimensional time is

$$\mathbf{w}_c = v_c^{-1}\hat{y}$$

The general electric current and magnetic current can be expressed as:

$$\mathbf{J}_e = g_e \mathbf{v}_c$$

$$\mathbf{J}_m = g_m \mathbf{w}_c$$

5.2 The electric-magnetic wave in three-dimensional time

We can also derive the electric-magnetic wave equation by using the Maxwell equations (5) ~ (8) in three-dimensional time structure. By using equations (7) without considering the current density, we can get:

$$\nabla_y \times \nabla_y \times \mathbf{G} = -\frac{\partial \nabla_y \times \mathbf{F}}{\partial x}$$

Then

$$\nabla_y{}^2 \mathbf{G} = \frac{\partial^2 \mathbf{G}}{\partial x^2}$$

Or

$$\nabla_t{}^2 \mathbf{G} = c^2 \frac{\partial^2 \mathbf{G}}{\partial x^2} \quad \text{.......... (9)}$$

It is a wave equation that is consistent with the known electric-magnetic wave equation. However, the velocity of the wave is:

$$w = c^{-1}$$

Or the general velocity is

$$w_c = 1$$

Because the particle's velocity cannot be larger than light in three-dimensional space, we can also assume that the relativity principles still correct in three-dimensional time for the demanding of the space-time symmetrical. The maximum velocity in three-dimensional time structure of a particle w equals 1/c.

6 Another wave equation

6.1 The wave equation that across two different space-time structure

Here we do the depth analysis of the new Maxwell equations in order to understand the relationships between two different space-time structures. By using the equation (7), we can take the space curl in both sides of the equation. Then we can get

$$\nabla \times \nabla_y \times \mathbf{G} = -\frac{\partial \nabla \times \mathbf{F}}{\partial x} \quad (10)$$

Considering equation (3), we can get

$$\nabla_y (\nabla \cdot \mathbf{G}) - (\nabla \cdot \nabla_y)\mathbf{G} = \frac{\partial^2 \mathbf{G}}{\partial x \partial y} \quad (11)$$

Then we can get the wave equation

$$(\nabla \cdot \nabla_y)\mathbf{G} = -\frac{\partial^2 \mathbf{G}}{\partial x \partial y}$$

Or

$$(\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i \partial y_i} + \frac{\partial^2}{\partial x \partial y})\mathbf{G} = 0 \quad (12)$$

Similarly, we can get

$$(\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i \partial y_i} + \frac{\partial^2}{\partial x \partial y})\mathbf{F} = 0 \quad (13)$$

From equation (13), we can get

$$(\nabla \cdot \nabla_y)\mathbf{F} = -\frac{\partial y}{\partial x}\frac{\partial^2 \mathbf{F}}{\partial y \partial y} \quad (14)$$

If we assume that the electric charge and the magnetic charge are co-existence, the movement of the electric charge in three-dimensional space will cause the corresponding magnetic charge moving in three-dimensional time for the symmetric demanding. The “distance” that they moved will equal to each other.

Then we have

$$\frac{\partial Y}{\partial X} = 1$$

And we can get

$$(\nabla \cdot \nabla_y)\mathbf{F} = -\frac{\partial Y}{\partial X}\frac{\partial y}{\partial x}\frac{\partial^2 \mathbf{F}}{\partial y^2}$$

$$(\nabla \cdot \nabla_y)\mathbf{F} = -\frac{w_c}{v_c}\frac{\partial^2 \mathbf{F}}{\partial y^2} \quad \text{.......(15)}$$

Considering the definition of the general velocity

$$\frac{w_c}{v_c} = \frac{c^2}{vw^{-1}}$$

Then we can get the new wave equation below.

$$(\nabla \cdot \nabla_y)\mathbf{F} = -\frac{c^2}{vw^{-1}}\frac{\partial^2 \mathbf{F}}{\partial y^2} \equiv -\frac{1}{vw^{-1}}\frac{\partial^2 \mathbf{F}}{\partial t^2} \quad \text{.......(16)}$$

Considering the demanding of symmetric, we can get

$$vw^{-1} = c^2$$

It can satisfy

$$\lim_{v \to c} w = c^{-1}, \quad \lim_{v \to 0} w = 0$$

The limitation means that when velocity of the electric charge is zero in three-dimensional space, the velocity of the corresponding magnetic charge in three-dimensional time also equals to zero. If the velocity of the electric charge is c in three-dimensional space, the velocity of the corresponding magnetic charge in three-dimensional time also equals to 1/c.

Formula (16) shows us a new kind of electromagnetic wave. This wave's properties are similiar to the normal electromagnetic wave whoes speed is equal to c. However, the new electromagnetic wave's velocity is lower then the speed of light. We call this new electromagnetic wave as the slow electromagnetic wave or virtual photon.

6.2 The separation of variables

By using the method of separation of variables, we get

$$\mathbf{F} = \mathbf{F}(\mathbf{X}, \mathbf{Y})F(t)$$

Where

$$F(t) = e^{-i\omega t} \quad \text{.......(17)}$$

Substitute F(t) into equation (16), we can get

$$(\nabla \cdot \nabla_y)\mathbf{F}(\mathbf{X}, \mathbf{Y}) - k^2\mathbf{F}(\mathbf{X}, \mathbf{Y}) = 0 \quad \text{.......(18)}$$

Or

$$\sum_{i=1}^{3}\frac{\partial^2 \mathbf{F}(\mathbf{X}, \mathbf{Y})}{\partial x_i \partial y_i} - k^2\mathbf{F}(\mathbf{X}, \mathbf{Y}) = 0 \quad \text{.......(19)}$$

Where

$$k = \frac{\omega}{\sqrt{vw^{-1}}} = \frac{\omega}{c}$$

6.3 Solution of the wave equation

The equation (19) has four particular solutions:

$$\mathbf{F} = \mathbf{F}_0 e^{-\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})}$$

$$\mathbf{F} = \mathbf{F}_0 e^{\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})}$$

$$\mathbf{F} = \mathbf{F}_0 e^{-i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})}$$

$$\mathbf{F} = \mathbf{F}_0 e^{i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})}$$

Since the four particular solutions are linearly independent, we can get the general solution of the equation (19) as

$$\mathbf{F} = \mathbf{F}_0[C_1 e^{-i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})-i\omega t} + C_2 e^{i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})-i\omega t} + C_3 e^{-\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})-i\omega t} + C_4 e^{\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})-i\omega t}]$$

Or

$$\mathbf{F} = \mathbf{F}_0[C_1 e^{-i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})} + C_2 e^{i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})} + C_3 e^{-\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})} + C_4 e^{\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})}]e^{-i\omega t} \qquad \text{............(20)}$$

6.3.1 The virtual photon

Consider the boundary conditions for free particle:

$$\mathbf{F}\big|_{\mathbf{X},\mathbf{Y}\to\infty} = 0$$

$$\mathbf{F}\big|_{\mathbf{X},\mathbf{Y}=0} = \mathbf{F}_0$$

We can get

$$\mathbf{F} = \mathbf{F}_0 e^{-\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})-i\omega t}$$

Similarly, we can get

$$\mathbf{G} = \mathbf{G}_0 e^{-\mathbf{k}\cdot(\mathbf{X}+\mathbf{Y})-i\omega t}$$

It represents the oscillating electromagnetic field that propagated in a velocity smaller than the velocity of light. However, the amplitude of the oscillating electromagnetic field decreasing exponentially, which shows the localization characteristic. It just like a virtual photon held by the free particle.

The momentum of the virtual photon is

$$\mathbf{p}_\gamma = \hbar\mathbf{k}$$

Then we have

$$\mathbf{F} = \mathbf{F}_0 e^{-\mathbf{p}_\gamma \cdot (\mathbf{X}+\mathbf{Y})/\hbar - i\omega t} \quad \text{.......(21)}$$

6.3.2 The bound state condition

Consider

$$k^2 = \left(\frac{p_\gamma}{\hbar}\right)^2$$

We have

$$E_k \approx \frac{{p_\gamma}^2}{2m}$$

Then

$$p_\gamma{}^2 \approx 2mE_k$$

$$k^2 = \frac{2mE_k}{\hbar^2}$$

The total energy of a particle in a potential field

$$E = E_k + V(\mathbf{X})$$

Then

$$k^2 = \frac{2m[E - V(\mathbf{X})]}{\hbar^2}$$

Substitute it into equation (19), we can get

$$\sum_{i=1}^{3} \frac{\partial^2 \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i \partial y_i} - \frac{2m[E - V(\mathbf{X})]}{\hbar^2} \mathbf{F}(\mathbf{X},\mathbf{Y}) = 0 \quad \text{.......(22)}$$

Since the electromagnetic field must decrease to zero in the limited area in a bound state, the particular solution $e^{\pm \mathbf{k} \cdot (\mathbf{X}+\mathbf{Y})} \neq 0$ will not satisfy the condition.

Therefore, we can get

$$C_3 = C_4 = 0$$

The general solution (20) will be changed into

$$\mathbf{F} = \mathbf{F}_0 [C_1 e^{-i\mathbf{k} \cdot (\mathbf{X}-\mathbf{Y})} + C_2 e^{i\mathbf{k} \cdot (\mathbf{X}-\mathbf{Y})}] \quad \text{.......(23)}$$

For this solution, we have

$$\frac{\partial \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial y_i} = ik\mathbf{F}_0 [C_1 e^{-i\mathbf{k} \cdot (\mathbf{X}-\mathbf{Y})} - C_2 e^{i\mathbf{k} \cdot (\mathbf{X}-\mathbf{Y})}]$$

$$\frac{\partial \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i} = -ik\mathbf{F}_0[C_1 e^{-i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})} - C_2 e^{i\mathbf{k}\cdot(\mathbf{X}-\mathbf{Y})}]$$

Therefore

$$\frac{\partial \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i} = -\frac{\partial \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial y_i}$$

We can get

$$\sum_{i=1}^{3}\frac{\partial^2 \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i \partial y_i} = -\sum_{i=1}^{3}\frac{\partial^2 \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i^{\,2}} \quad \text{......(24)}$$

Substitute it into equation (22), we have

$$\sum_{i=1}^{3}\frac{\partial^2 \mathbf{F}(\mathbf{X},\mathbf{Y})}{\partial x_i^{\,2}} - \frac{2m}{\hbar^2}[V(\mathbf{X})-E]\mathbf{F}(\mathbf{X},\mathbf{Y}) = 0 \quad \text{......(25)}$$

If we define

$$\mathbf{F}(\mathbf{X},\mathbf{Y}) = \psi(\mathbf{X},\mathbf{Y})\hat{F}$$

Then we have

$$\sum_{i=1}^{3}\frac{\partial^2 \psi(\mathbf{X},\mathbf{Y})}{\partial x_i^{\,2}} - \frac{2m}{\hbar^2}[V(\mathbf{X})-E]\psi(\mathbf{X},\mathbf{Y}) = 0 \quad \text{......(26)}$$

It is consistent with the time-independent Schrödinger equation. However, the wave function in equation (26) includes the coordinates of three-dimensional space and time.

Since the equation is derived from the slow electromagnetic wave equation (19), we can draw the conclusion that **the nature of wave function in quantum theory is the slow electromagnetic wave or virtual photon.**

6.4 The energy density of the virtual photon

For the virtual photon carried by a free particle, we can calculate the energy density

$$W = \frac{1}{2}\mathbf{F}\cdot\mathbf{F}^* + \frac{1}{2}\mathbf{G}\cdot\mathbf{G}^* = \frac{1}{2}(\mathbf{F}_0^{\,2} + \mathbf{G}_0^{\,2})e^{-2\mathbf{P}\cdot\mathbf{X}/\hbar} \equiv W_0 e^{-2\mathbf{P}\cdot\mathbf{X}/\hbar}$$

It reflects the energy distribution in the space of a virtual photon.

Notice:

$$\mathbf{F}^* = \mathbf{F}_0 e^{-\mathbf{P}\cdot(\mathbf{X}-\mathbf{Y})/\hbar + i\omega t}$$

Then

$$\mathbf{F}\cdot\mathbf{F}^* = \mathbf{F}_0^{\,2} e^{-2\mathbf{P}\cdot\mathbf{X}/\hbar} \quad \text{......(27)}$$

The energy density of virtual photon is equivalent to the probability density of the wave function in quantum theory.

For free particles, the wave packet solution of Schrodinger's equation is:

$$\psi(\mathbf{X},t)=\frac{1}{(2\pi)^{3/2}\hbar}\int C(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{X}/\hbar-i\omega t}d\mathbf{p} \quad (28)$$

C(**p**) is an undetermined coefficients.

From formula (28), we can find that there are chromatic dispersion phenomena in Schrodinger's free particle solution when the particle moving. By contrast, there is no chromatic dispersion in localized electromagnetic wave solution (27), which is consistent with the experiment results. It can be an important support to the hyper-symmetric Maxwell equation.

By using the electromagnetic wave function of virtual photon to replace the probability wave function, we can also solve the problems of the collapse of the wave function. The virtual photons will transmit to other particles after interaction. So the slow electromagnetic wave will always exist in the particles instead of collapsing.

6.5 An object of mass m held an virtual photon

Considering an object of mass m that held an virtual photon $h\nu$, the total energy of the object will be:

$$E^2=m^2c^4+h^2\nu^2$$

Or:

$$E^2=m^2c^4+p^2c^2$$

It is consistent with the relativity energy formula. We shall paid attention to the orthogonality of the virtual photon's energy and static mass.

So the kinetic energy in low velocities will be:

$$E_k=\frac{h^2\nu^2}{2mc^2}=\frac{h^2}{8\pi^2mx^2} \quad (29)$$

In which the wave length of the virtual photon is :

$$\lambda=2\pi x$$

7 An Accurate calculation of Helium atom's ground state energy

7.1 Essential model

There are at least three particles in helium atom, so we cannot use Schrodinger's equation to solve these problems accurately. However, we can use some approximation methods, including perturbation method or variational method, to get some approximation solutions. Here, we try to use the localization electromagnetic wave solutions to get more accurate results.

The two electrons will hold two virtual photons due to the interactions between electrons and proton. Or they will fall into the proton. For the symmetric consideration, the two virtual photons shall have same energy.

The relationship between electrons and proton can be described by figure 1. The distance between electron and proton is x. The distance between two electrons is 2x.

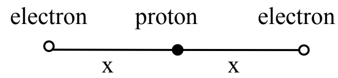

Fig.1 The structure of the Helium

First we consider the interaction between electrons and proton. For the consideration of symmetry requirement, the virtual photon's wave length shall equal to the perimeter of electrons moving around proton. Then the momentum of each virtual photon will be：

$$p_\gamma = \frac{\hbar}{x}$$

We can get the total kinetic energy of two electrons according to formula (29):

$$E_{ep} = \frac{2h^2}{8\pi^2 m x^2}$$

Secondary, we shall also consider the interaction between two electrons. So the two electrons shall also hold a virtual photon. The wave length of this virtual photon is equal to: λ=2πr=4πx. Then we can calculate the virtual photon's momentum of this two electrons system is:

$$p_{ee} = \frac{\hbar}{r} = \frac{\hbar}{2x}$$

Or:

$$x p_{ee} = \frac{\hbar}{2}$$

We presume that this two electrons system is equal to a mass 2m object. So the kinetic energy of this two electrons system is:

$$E_{ee} = \frac{h^2}{64\pi^2 m x^2}$$

In the end, we can get the formula that can be used to calculate the Helium atom's ground state energy:

$$E = \frac{2h^2}{8\pi^2 m x^2} + \frac{h^2}{64\pi^2 m x^2} - \frac{4e^2}{4\pi\varepsilon x} + \frac{e^2}{8\pi\varepsilon x} \cdots\cdots \quad (30)$$

Since:

$$\frac{\partial E}{\partial x} = -\frac{17h^2}{32\pi^2 m x} + \frac{7e^2}{8\pi\varepsilon} = 0$$

So:

$$x = \frac{17}{28} a_0$$

$$E_{\min} = -\frac{49}{17}\frac{e^2}{a_0}$$

$$E_{\min} = -2.88\frac{e^2}{a_0}(eV)$$

Since the two electrons system is unstable, the error of this result is relatively large by comparing with the experiment value.

7.2 Considering the exchange of virtual photon between particles

In addition to the bound state of two electrons, which is resulted from the attractive force between electrons and proton, the repulsive force between two electrons will cause them to jump out of the bound state. It will cause the system deviating from stable state, radiating and absorbing virtual photons.

Since the interactions among electrons and proton are electromagnetic interactions, the interaction coupling constant is the fine structure constant α.

7.2.1 The probability that a ground state electron radiates a virtual photon

The probability that a ground state electron radiates a virtual photon is:

$$j_{e(1)} = \alpha$$

So the probability that a ground state electron does not radiate a virtual photon is:

$$j_{c(1)} = 1 - \alpha$$

7.2.2 The probability that a ground state electron radiates a virtual photon and being absorbed by a proton

Considering that the probability that a proton absorbs a virtual photon can also be α, then if the virtual photon radiated by a ground state electron is absorbed by the proton, the total probability of this process will be:

$$j_{\gamma(1)} = \alpha^2$$

This is the first order process. For the second order process, in the condition of the virtual photon radiated by ground state electron being absorbed by the proton, the probability that a proton radiates a virtual photon again is α, while the probability of the virtual photon radiated by the proton being absorbed by the electron is α^2. Then we have:

$$j_{e(2)} = \alpha^2 \cdot \alpha$$

$$j_{\gamma(2)} = \alpha^2 \cdot \alpha^2$$

And so on, we can calculate the probabilities of three or more orders process.

7.2.3 The probabilities of interactions among ground state electrons and proton

By merge together all of the processes, we can get the total probability of interaction between electron and proton, or between electron and electron.

$$p = 1 - j_{e(1)} + j_{\gamma(1)} - j_{e(2)} + j_{\gamma(2)} + \ldots$$

Therefore:

$$p = 1 - \alpha + \alpha^2 - \alpha^3 + \alpha^4 + ...$$

$$p = \frac{1}{1+\alpha} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (31)$$

7.3 An accurate calculation formula of Helium atom's ground state energy

If we insert formula (31) into formula (30), we can get：

$$E = \left(\frac{2h^2}{8\pi^2 mx^2} + \frac{h^2}{64\pi^2 mx^2}\right)\frac{1}{1+\alpha} - \frac{4e^2}{4\pi\varepsilon x} + \frac{e^2}{8\pi\varepsilon x} \cdots\cdots\cdots \quad (32)$$

Then we can get:

$$E_{\min} = -\frac{49(1+\alpha)me^4}{68\varepsilon^2 h^2}$$

$$E_{\min} = -\frac{49\,(1+\alpha)}{17}\frac{e^2}{a_0} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (33)$$

7.4 Data Analysis

We can calculate the ground state energy of Helium atom by using formula (33):

$$E_{\min} = -\frac{49\,(1 + 7.2973525698\,(24) \times 10^{-3})}{17}\frac{e^2}{a_0} = -2.9033864868188\,(69)\,\frac{e^2}{a_0}\,(eV)$$

Tab.1 The compare of the calculation and experiment value of Helium atom's ground state energy

1st Ionization Energy(eV)	2nd Ionization Energy(eV)	Experiment value E_e(a.u.)	Theoretical value E (a.u.)	Error Δ**
54.41776217(2)*	24.587387512(25)	-2.90338583(13)	-2.9033864868188(69)	2.26e-7

*1st Ionization Energy in table 1 is the theoretical value.

**Error calculation formula: $\Delta = \frac{|E| - |E_e|}{|E_e|}$

Hartree energy in eV:

1a.u.= 27.21138505(60) eV.

Data derived from NIST: http://physics.nist.gov/cgi-bin/cuu/Value?threv (Retrieved May 17, 2014)

The value of Helium atom's first and second Ionization energy retrieved from NIST.

Website: http://physics.nist.gov/PhysRefData/ASD/ionEnergy.html (Retrieved May 17, 2014)

It seems that the theoretical value calculated from formula (33) is in good agreement with the experiment value. However, we can find that there is slight difference between theoretical value and experiment value from table 1. The reason why there is difference mainly due to the first ionization energy data that NIST gives is the theoretical value, which may have some slight differences from the real experiment value. We had found another experiment value that given by article [10] in the references of this paper is: [10]：

$$E_e = -2.90338648 \text{ a.u.}$$

It is in good agreement with the theoretical value calculated by formula (33).

It seems that we need to do more accurate experiments to test formula (33).

7.5 Relativity energy correction

Because formula (30) uses the classical mechanics' kinetic energy express, which is only the first order relativity energy correction, we should do second order or more correction to obtain more accurate theoretical value.

Considering the relativity energy formula: $E = mc^2(1+\frac{h^2\nu^2}{m^2c^4})^{\frac{1}{2}}$

Where $h\nu$ is the energy of virtual photon. After we do the second order correction, we can get:

$$E \approx mc^2 + \frac{h^2\nu^2}{2mc^2} - \frac{h^4\nu^4}{8m^3c^6}$$

So the second correction is:

$$\Delta E^{II} = \frac{2h^4}{128\pi^4 m^3c^2x^4} + \frac{h^4}{16384\pi^4 m^3c^2x^4} \quad \cdots\cdots\cdots\cdots (34)$$

We can also use:

$$x = \frac{17}{28}a_0 = \frac{17}{28}\frac{\varepsilon h^2}{\pi me^2}$$

Substituting the x into formula (34), we can get:

$$\Delta E^{II} = \frac{257h^4}{16384\pi^4 m^3c^2}\left(\frac{28}{17}\frac{\pi me^2}{\varepsilon h^2}\right)^4$$

We can get:

$$\Delta E^{II} = \frac{257e^4}{16c^2\varepsilon^2}\left(\frac{7}{17}\right)^4\frac{me^4}{4\varepsilon^2h^2} \sim 4.3\times10^{-73}(a.u.)$$

From the result above, we can find that the high order relativity energy corrections have very small impact on calculation results. So the high order relativity energy corrections can be ignored.

8 Conclusions

We improve Maxwell equation by assuming the existence of the three-dimensional time structure corresponding to the three-dimensional space structure. The improved Maxwell equations have better symmetry characteristics.

We can get some useful properties of the magnetic charge by analyzing the hyper-symmetric Maxwell equations. We can get the relationship between the magnetic charge in three-dimensional time structure and electric charge in three-dimensional space structure by using the Schwinger electric charge quantization condition. The relationship between magnetic charge and electric charge is related to the fine structure constant.

First, we get the electromagnetic wave equation in three-dimensional time structure that is consistent with the equation in three-dimensional space structure by solving Maxwell equations in three-dimensional time structure.

Second, we get the localized electromagnetic wave equation that contains the coordinates of two different space-time structures based on solving the hyper-symmetric Maxwell equations. Then we get the general solution of this equation. The general solution includes two parts. The first part includes the particular solutions that the wave's amplitude decreased exponentially from the mass center. These particular solutions correspond to the wave function of the free particle. We call it the slow electromagnetic wave. Since the electromagnetic wave obtained from the slow electromagnetic wave solution is localized and the velocity of the localized electromagnetic wave is smaller than the velocity of light, we can also call it the virtual photon corresponding to Feynman's virtual photon hypothesis. The second part of the general solution includes the particular solutions that the wave's amplitude oscillates both in time and space. We find that these particular solutions are consistent with the solutions of time-independent Schrödinger's equation. It reflects the movement of particles in the bound state. We draw the conclusion that the essence of wave function in quantum theory is the slow electromagnetic wave or virtual photon. The probability density of the wave function is corresponding to the energy density of the slow electromagnetic wave.

In order to verify the correctness of hyper symmetric Maxwell equations, we try to use the slow electromagnetic wave solutions to solve the problem of Helium atom's ground state energy. Since there are two electrons in Helium atom, there are two bound states and one unbound state exist in Helium atom. The two bound states reflect the interactions between electrons and proton. Unbound state reflects the interactions between two electrons. Schrodinger's equation can only solve problems of bound states. That is the reason why we cannot get the accurate solution of Helium atom's ground state energy in the past. Our method can obtain the formula (33), and the calculation result -2.9033864868188(69)a.u. is very close to the experiment results.

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