Conjectured Primality and Compositeness Tests for Numbers of Special Forms

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Abstract: Conjectured polynomial time primality and compositeness tests for numbers of special forms are introduced .

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1 Introduction

In number theory the Riesel primality test [1] , is the fastest deterministic primality test for numbers of the form $k \cdot 2^n - 1$ with k odd and $k < 2^n$. The test was developed by Hans Riesel and it is based on Lucas-Lehmer test [2] . In 1960 Kusta Inkeri provided unconditional , deterministic , lucasian type primality test for Fermat numbers [3] . In 2008 Ray Melham provided unconditional , probabilistic , lucasian type primality test for generalized Mersenne numbers [4] . In 2010 Pedro Berrizbeitia ,Florian Luca and Ray Melham provided polynomial time compositeness test for numbers of the form $(2^p + 1)/3$, see Theorem 2 in [5] . In this note I present lucasian type primality and compositeness tests for numbers of special forms .

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$, where m and x are positive integers .

Conjecture 2.1. *Let* $N = k \cdot 2^n - 1$ *such that* $n > 2$, $3 \mid k$, $k < 2^n$ *and*

$$\begin{cases} k \equiv 1 \pmod{10} \ with \ n \equiv 2,3 \pmod 4 \\ k \equiv 3 \pmod{10} \ with \ n \equiv 0,3 \pmod 4 \\ k \equiv 7 \pmod{10} \ with \ n \equiv 1,2 \pmod 4 \\ k \equiv 9 \pmod{10} \ with \ n \equiv 0,1 \pmod 4 \end{cases}$$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(3) \textit{, thus}$$
$$N \textit{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.2. *Let* $N = k \cdot 2^n - 1$ *such that* $n > 2$, $3 \mid k$, $k < 2^n$ *and*

$$\begin{cases} k \equiv 3 \pmod{42} \textit{ with } n \equiv 0,2 \pmod{3} \\ k \equiv 9 \pmod{42} \textit{ with } n \equiv 0 \pmod{3} \\ k \equiv 15 \pmod{42} \textit{ with } n \equiv 1 \pmod{3} \\ k \equiv 27 \pmod{42} \textit{ with } n \equiv 1,2 \pmod{3} \\ k \equiv 33 \pmod{42} \textit{ with } n \equiv 0,1 \pmod{3} \\ k \equiv 39 \pmod{42} \textit{ with } n \equiv 2 \pmod{3} \end{cases}$$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(5) \textit{, thus}$$
$$N \textit{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.3. *Let* $N = k \cdot 2^n + 1$ *such that* $n > 2$, $k < 2^n$ *and*

$$\begin{cases} k \equiv 5,19 \pmod{42} \textit{ with } n \equiv 0 \pmod{3} \\ k \equiv 13,41 \pmod{42} \textit{ with } n \equiv 1 \pmod{3} \\ k \equiv 17,31 \pmod{42} \textit{ with } n \equiv 2 \pmod{3} \\ k \equiv 23,37 \pmod{42} \textit{ with } n \equiv 0,1 \pmod{3} \\ k \equiv 11,25 \pmod{42} \textit{ with } n \equiv 0,2 \pmod{3} \\ k \equiv 1,29 \pmod{42} \textit{ with } n \equiv 1,2 \pmod{3} \end{cases}$$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(5) \textit{, thus}$$
$$N \textit{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.4. *Let* $N = k \cdot 2^n + 1$ *such that* $n > 2$, $k < 2^n$ *and*

$$\begin{cases} k \equiv 1 \pmod{6} \textit{ and } k \equiv 1,7 \pmod{10} \textit{ with } n \equiv 0 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 1,3 \pmod{10} \textit{ with } n \equiv 1 \pmod{4} \\ k \equiv 1 \pmod{6} \textit{ and } k \equiv 3,9 \pmod{10} \textit{ with } n \equiv 2 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 7,9 \pmod{10} \textit{ with } n \equiv 3 \pmod{4} \end{cases}$$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(8) \textit{, thus}$$
$$N \textit{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.5. *Let* $N = 3 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 1,2 \pmod{4}$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with}$$
$$S_0 = \begin{cases} P_3(32), & \textit{if } n \equiv 1 \pmod{4} \\ P_3(28), & \textit{if } n \equiv 2 \pmod{4} \end{cases}$$
$$\textit{thus}$$
$$N \textit{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.6. *Let* $N = 5 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 1, 3 \pmod 4$

$$\text{Let } S_i = P_2(S_{i-1}) \text{ with}$$
$$S_0 = \begin{cases} P_5(28), & \text{if } n \equiv 1 \pmod 4 \\ P_5(32), & \text{if } n \equiv 3 \pmod 4 \end{cases}$$
$$\text{thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod N$$

Conjecture 2.7. *Let* $N = 7 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 0, 2 \pmod 4$

$$\text{Let } S_i = P_2(S_{i-1}) \text{ with}$$
$$S_0 = \begin{cases} P_7(8), & \text{if } n \equiv 0 \pmod 4 \\ P_7(32), & \text{if } n \equiv 2 \pmod 4 \end{cases}$$
$$\text{thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod N$$

Conjecture 2.8. *Let* $N = 9 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 2, 3 \pmod 4$

$$\text{Let } S_i = P_2(S_{i-1}) \text{ with}$$
$$S_0 = \begin{cases} P_9(28), & \text{if } n \equiv 2 \pmod 4 \\ P_9(32), & \text{if } n \equiv 3 \pmod 4 \end{cases}$$
$$\text{thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod N$$

Conjecture 2.9. *Let* $N = 11 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 1, 3 \pmod 4$

$$\text{Let } S_i = P_2(S_{i-1}) \text{ with}$$
$$S_0 = \begin{cases} P_{11}(8), & \text{if } n \equiv 1 \pmod 4 \\ P_{11}(28), & \text{if } n \equiv 3 \pmod 4 \end{cases}$$
$$\text{thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod N$$

Conjecture 2.10. *Let* $N = 13 \cdot 2^n + 1$ *such that* $n > 2$ *and* $n \equiv 0, 2 \pmod 4$

$$\text{Let } S_i = P_2(S_{i-1}) \text{ with}$$
$$S_0 = \begin{cases} P_{13}(32), & \text{if } n \equiv 0 \pmod 4 \\ P_{13}(8), & \text{if } n \equiv 2 \pmod 4 \end{cases}$$
$$\text{thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod N$$

Conjecture 2.11. *Let* $F = 2^{2^n} + 1$ *such that* $n \geq 2$. *Let* $S_i = P_4(S_{i-1})$ *with* $S_0 = 8$, *thus*

$$F \text{ is prime iff } S_{2^{n-1}-1} \equiv 0 \pmod F$$

Conjecture 2.12. *Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$, $k > 0$, $k \equiv 3, 9 \pmod{10}$ *and* $k < 6^n$

$$\text{Let } S_i = P_6(S_{i-1}) \text{ with } S_0 = P_{3k}(P_3(3)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.13. *Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$, $k > 0$, $k \equiv 5 \pmod{42}$ *and* $k < 6^n$

$$\text{Let } S_i = P_6(S_{i-1}) \text{ with } S_0 = P_{3k}(P_3(5)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.14. *Let* $N = k \cdot b^n - 1$ *such that* $n > 2$, k *is odd* , $3 \nmid k$, b *is even* , $3 \nmid b$, $5 \nmid b$, $k < b^n$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{bk/2}(P_{b/2}(4)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.15. *Let* $N = k \cdot b^n - 1$ *such that* $n > 2$, $k < b^n$ *and*

$$\begin{cases} k \equiv 3 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \end{cases}$$

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{bk/2}(P_{b/2}(5778)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.16. *Let* $N = k \cdot b^n - 1$ *such that* $n > 2$, $k < b^n$ *and*

$$\begin{cases} k \equiv 9 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \end{cases}$$

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{bk/2}(P_{b/2}(5778)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.17. *Let* $N = k \cdot b^n - 1$ *such that* $n > 2$, $k < b^n$ *and*

$$\begin{cases} k \equiv 21 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 2, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 1, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 1, 2 \pmod{4} \end{cases}$$

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{bk/2}(P_{b/2}(3)) \text{ , thus}$$
$$N \text{ is prime iff } S_{n-2} \equiv 0 \pmod{N}$$

Conjecture 2.18. *Let* $F_n(b) = b^{2^n} + 1$ *such that* $n > 1$, b *is even* , $3 \nmid b$ *and* $5 \nmid b$.

$$\textit{Let } S_i = P_b(S_{i-1}) \textit{ with } S_0 = P_{b/2}(P_{b/2}(8)) \textit{, thus}$$
$$F_n(b) \textit{ is prime iff } S_{2^n-2} \equiv 0 \pmod{F_n(b)}$$

Conjecture 2.19. *Let* $N = k \cdot 3^n - 2$ *such that* $n \equiv 0 \pmod 2$, $n > 2$, $k \equiv 1 \pmod 4$ *and* $k < 3^n$.

$$\textit{Let } S_i = P_3(S_{i-1}) \textit{ with } S_0 = P_{3k}(4) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_1(4) \pmod N$$

Conjecture 2.20. *Let* $N = k \cdot 3^n - 2$ *such that* $n \equiv 1 \pmod 2$, $n > 2$, $k \equiv 1 \pmod 4$ *and* $k < 3^n$.

$$\textit{Let } S_i = P_3(S_{i-1}) \textit{ with } S_0 = P_{3k}(4) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_3(4) \pmod N$$

Conjecture 2.21. *Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$, $k \equiv 1, 3 \pmod 8$ *and* $k < 3^n$.

$$\textit{Let } S_i = P_3(S_{i-1}) \textit{ with } S_0 = P_{3k}(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_3(6) \pmod N$$

Conjecture 2.22. *Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$, $k \equiv 5, 7 \pmod 8$ *and* $k < 3^n$.

$$\textit{Let } S_i = P_3(S_{i-1}) \textit{ with } S_0 = P_{3k}(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_1(6) \pmod N$$

Conjecture 2.23. *Let* $N = k \cdot 2^n - c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 3, 5 \pmod 8$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv -P_{\lfloor c/2 \rfloor}(6) \pmod N$$

Conjecture 2.24. *Let* $N = k \cdot 2^n + c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 3, 5 \pmod 8$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv -P_{\lceil c/2 \rceil}(6) \pmod N$$

Conjecture 2.25. *Let* $N = k \cdot 2^n - c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 1, 7 \pmod 8$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_{\lceil c/2 \rceil}(6) \pmod N$$

Conjecture 2.26. *Let* $N = k \cdot 2^n + c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 1, 7 \pmod 8$

$$\textit{Let } S_i = P_2(S_{i-1}) \textit{ with } S_0 = P_k(6) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_{\lfloor c/2 \rfloor}(6) \pmod N$$

Conjecture 2.27. *Let* $N = k \cdot 10^n - c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 3, 5 \pmod 8$

$$\textit{Let } S_i = P_{10}(S_{i-1}) \textit{ with } S_0 = P_{5k}(P_5(6)) \textit{, thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv -P_{5\lfloor c/2 \rfloor}(6) \pmod N$$

Conjecture 2.28. *Let* $N = k \cdot 10^n + c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 3, 5 \pmod{8}$

$$\text{Let } S_i = P_{10}(S_{i-1}) \text{ with } S_0 = P_{5k}(P_5(6)) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv -P_{5\lceil c/2 \rceil}(6) \pmod{N}$$

Conjecture 2.29. *Let* $N = k \cdot 10^n - c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 1, 7 \pmod{8}$

$$\text{Let } S_i = P_{10}(S_{i-1}) \text{ with } S_0 = P_{5k}(P_5(6)) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{5\lceil c/2 \rceil}(6) \pmod{N}$$

Conjecture 2.30. *Let* $N = k \cdot 10^n + c$ *such that* $n > 2c$, $k > 0$, $c > 0$ *and* $c \equiv 1, 7 \pmod{8}$

$$\text{Let } S_i = P_{10}(S_{i-1}) \text{ with } S_0 = P_{5k}(P_5(6)) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{5\lfloor c/2 \rfloor}(6) \pmod{N}$$

Conjecture 2.31. *Let* $R = (3^p - 1)/2$ *such that* $p > 3$ *and* p *is an odd prime .*

$$\text{Let } S_i = P_3(S_{i-1}) \text{ with } S_0 = P_3(4) \text{ , thus}$$
$$\text{If } R \text{ is prime then } S_{p-1} \equiv P_3(4) \pmod{R}$$

Conjecture 2.32. *Let* $R = (10^p - 1)/9$ *such that* p *is an odd prime .*

$$\text{Let } S_i = P_{10}(S_{i-1}) \text{ with } S_0 = P_5(6) \text{ , thus}$$
$$\text{If } R \text{ is prime then } S_{p-1} \equiv P_5(6) \pmod{R}$$

Conjecture 2.33. *Let* $N = b^n - b - 1$ *such that* $n > 2$, $b \equiv 0, 6 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$$

Conjecture 2.34. *Let* $N = b^n - b - 1$ *such that* $n > 2$, $b \equiv 2, 4 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$$

Conjecture 2.35. *Let* $N = b^n + b + 1$ *such that* $n > 2$, $b \equiv 0, 6 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{b/2}(6) \pmod{N}$$

Conjecture 2.36. *Let* $N = b^n + b + 1$ *such that* $n > 2$, $b \equiv 2, 4 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$$

Conjecture 2.37. *Let* $N = b^n - b + 1$ *such that* $n > 3$, $b \equiv 0, 2 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{b/2}(6) \pmod{N}$$

Conjecture 2.38. *Let* $N = b^n - b + 1$ *such that* $n > 3$, $b \equiv 4, 6 \pmod{8}$.

$$\textit{Let } S_i = P_b(S_{i-1}) \textit{ with } S_0 = P_{b/2}(6) \textit{ , thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv -P_{(b-2)/2}(6) \pmod{N}$$

Conjecture 2.39. *Let* $N = b^n + b - 1$ *such that* $n > 3$, $b \equiv 0, 2 \pmod{8}$.

$$\textit{Let } S_i = P_b(S_{i-1}) \textit{ with } S_0 = P_{b/2}(6) \textit{ , thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv P_{(b-2)/2}(6) \pmod{N}$$

Conjecture 2.40. *Let* $N = b^n + b - 1$ *such that* $n > 3$, $b \equiv 4, 6 \pmod{8}$.

$$\textit{Let } S_i = P_b(S_{i-1}) \textit{ with } S_0 = P_{b/2}(6) \textit{ , thus}$$
$$\textit{If } N \textit{ is prime then } S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$$

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