On the de Bruijn-Newman constant

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Abstract

We use the positivity axiom of inner product spaces to prove the equivalent statement of the Riemann hypothesis.

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1 Introduction

Let $H = \Phi(t)e^{\lambda t^2}$. de Bruijn [dB50] proved that H has only real zeros for $\lambda \geq 1/2$. Newman [New76] proved that there exists a constant Λ such that H has only real zeros if and only if $\lambda \geq \Lambda$. The one of lower bounds [COSV93] is $\Lambda > -5.895 \times 10^{-9}$. The better lower bound [Odl00] is $\Lambda > -2.7 \times 10^{-9}$.

The Riemann hypothesis is equivalent to $\Lambda \leq 0$. To prove the Riemann hypothesis, we must show that $\Lambda \leq 0$. In this paper, we have done this task.

2 The result

First, we use the positivity axiom of inner product spaces.

Axiom 2.1 (Positivity Axiom)**.** Let $\langle \mathbf{u}, \mathbf{v} \rangle$ be the inner product. Then $\langle \mathbf{u}, \mathbf{u} \rangle \geq 0$.

We define a number ϕ such that $\phi := \langle \mathbf{u}, \mathbf{u} \rangle$.

Theorem 2.2. $\phi \geq 0$.

Proof. Use Axiom 2.1. □

Theorem 2.3. $\Lambda \leq 0$.

Proof. By Theorem 2.2, $\phi \geq 0$. Multiplying by -1, we have $-\phi \leq 0$. Set $\Lambda = -\phi$. □

References

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