A Mathematical Treatise on Polychronous Wavefront Computation and its Application into Modeling Neurosensory Systems

Abstract

Polychronous Wavefront Computation is the name given to a recently proposed geometric approach on how the brain can be modeled to perform computations. It departs from the conventional use of artificial neural networks involving synapses for systems modeling.

This paper, written as a mathematical sequel to the original work on the subject, builds two closely related Neurocomputational Models that together furnishes a plausible mechanism grounded in geometry, for sensory representation of motion, shape and time in the brain.

Keywords

Polychronous Wavefront Computation; Coincidence Detectors; Peripheral Sensory Field; Central Neural Field; Inter-Sensor Stimulation Time Interval; Organizational Maps

List of Abbreviations

PWC – Polychronous Wavefront Computation
ISI – Inter-Sensor stimulation Time Interval
IPI – Inter-Pulse Interval
CNF – Central Neural Field
PNF – Peripheral Neural Field

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1. Introduction

1.1 Synopsis of Polychronous Wavefront Computation (PWC)

A basic outline of the qualitative aspects of the theory behind PWC is given here. Within this framework, pre-synaptic neurons are visualized as point sources, post-synaptic neurons as point receivers and the intricate axonal network that connect them as a medium through which impulses can propagate uniformly in all possible directions, akin to ripples of disturbance on the surface of a pond[1, 2]. The manner of arrival of impulses arising from different pre-synaptic neurons at a post-synaptic neuron, ultimately determines the strength of the latter's response. A stronger response can be expected when two or more impulses converge simultaneously than when apart.

To illustrate the PWC principle, consider two sources A and B lying in the XY-plane, stimulated in succession (see Figure 1). The circular waves emanated from these sources will intersect at different spatial points for different snapshots in time. If source A is stimulated before source B, then the points of intersection of the two wavefronts over time traces out the locus of a branch (or arm) of a hyperbola*, with its mouth open towards B. Similarly, if source B is stimulated before source A, then the points of intersection of the two wavefronts over time traces out the locus of the complementary branch of the hyperbola with its mouth open towards A.

In neuroscience, there are a postulated group of neurons called Coincidence Neuron Detectors which 'switch on' only upon the reception of two or more impulses simultaneously from two or more sources respectively. A good description of the biological utility of this special class of neurons can be found in Jeffress' Place Model of the Auditory System [3, 4]. If these coincidence neuron detectors were distributed along the trace of one of the hyperbolic branches, then they will fire away only for that particular time interval, spanning the duration of the inter-source stimulation. That is to say, the magnitude of inter-stimulation time interval (ISI) is encoded in the hyperbolic arrangement of the detectors. A natural outcome of adopting this kind of a scheme for modeling a two dimensional sheet of neurons is the existence of an organized map for a range of different ISIs.

*The original 2009 paper states that the traced locus is a parabola. The author mathematically demonstrates that it is actually a hyperbola (see §2.2.2.2 of Main Text below and also page 2 of the Supplementary Material).

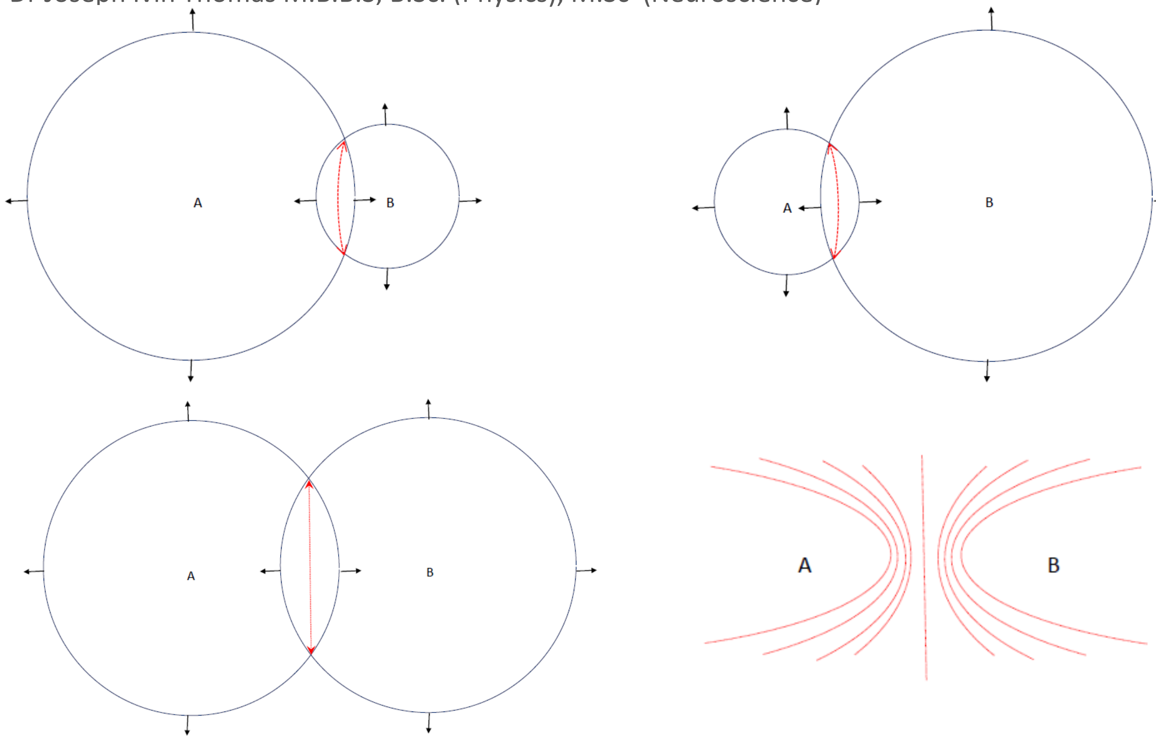


Figure 1: *Illustration of PWC Principle.*

Top: (L) When Source A is stimulated before B (i.e. $ISI = t_B - t_A > 0$*) a right hyperbolic arm is traced. (R) When Source B is stimulated before A (i.e.* $ISI = t_A - t_B > 0$*) a left hyperbolic arm is traced.*

Bottom: (L) When Sources A and B are stimulated simultaneously (i.e. $ISI = 0$*) a Straight line is traced. (R) The family of all possible hyperbolic traces for different ISIs.*

1.2 Modeling Sensory Systems using PWC

In order to model a sensory system in a generic fashion using the PWC principle, it is necessary to construct two parallel 2D cell arrays, labeling one as the Peripheral Sensory Field and the other as the Central Source (or Neural) Field (see Figure 2). The former, consists of a grid of Sensors that represents a peripheral sense organ (e.g. the skin). The latter, consists of a grid of Sources that represents the portion of the brain which receives and processes sensory input from the periphery (e.g. the somatosensory cortex). The layout of the cells in each plane is isomorphic. However, it should be noted that the uniform spacing between adjacent cells in the peripheral field is not the same as that of the central field (not shown in the illustration, for clarity sake). Both groups of cells are linked by means of one to one connections. Finally, there is a third group of cells – Coincidence Detectors – that surround each Source, like satellites. And each Source gives off a multitude of radiations to these coincidence detectors (see Figure 3).

It reasonably follows from the above arrangement, that any movement of a stimulus over the sensors in the Peripheral Field, will be accompanied by parallel activation of the sources in the Central Field, ultimately leading to the activation of the satellite Coincidence Detectors. This paper proposes that the unique geometric pattern and location of activation of the Coincidence Detectors, acts as the mechanism for encoding various aspects of motion in the brain, like speed, angle of contact, duration between successive adjacent sensor contacts and the shape of the moving stimulus.

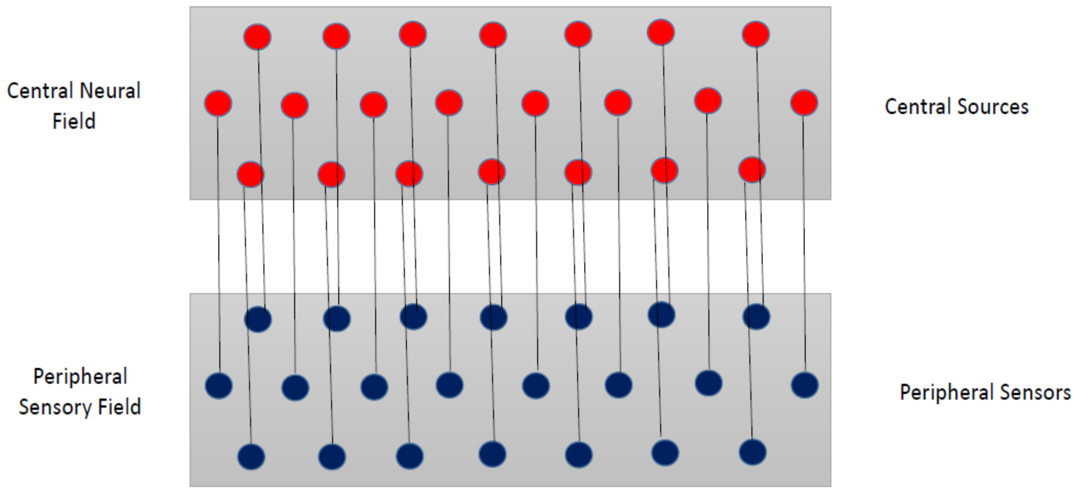
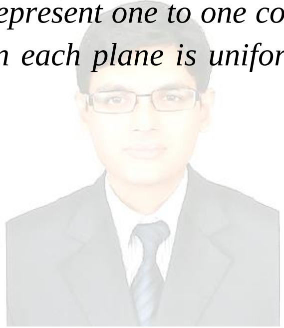


Figure 2: *Construction of a General Framework necessary for a PWC Model. Sources in the Central Field are shown as red circles. Sensors in the Peripheral Field are shown as blue circles. Black lines represent one to one connections between the cells of each plane. Spacing between cells within each plane is uniform. Also the layout of cells in both planes are isomorphic.*



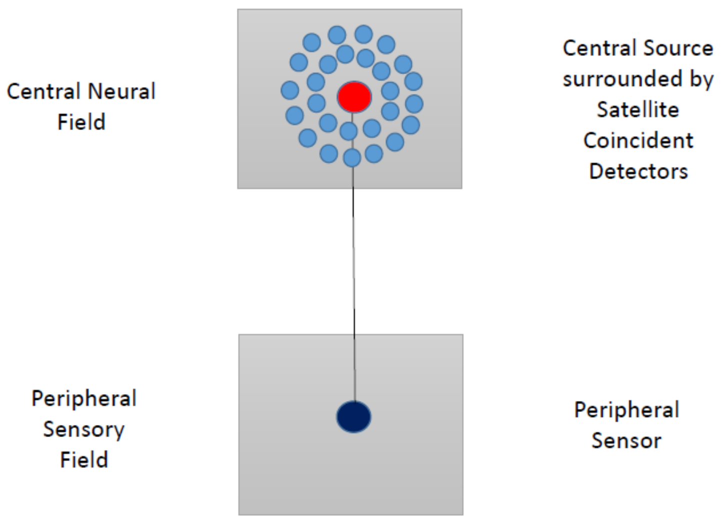


Figure 3: *An Individual Sensor-Source Unit of the PWC Model*
Coincidence Detectors clustered around a Source, receiving radiations (not shown) from it.

1.3 Aims and Objectives

1.3.1 Aims

This paper aims at laying down the quantitative foundations needed to fully account for a Two-Sensor and Three-Sensor System using a PWC approach, and then explore some interesting implications of both these models. Neural system configurations of even greater complexity can then be constructed by exploiting many of the key results obtained here. This would entail the construction of a general model for an ‘n-sensor’ system (where n = 4, 5, 6…).

1.3.2 Objectives

The construction of the Two-Sensor Model and Three-Sensor Model follow a similar format. First, the common underlying postulates are laid out and then the pictorial framework for each model is introduced. Next, the expressions for Inter-Sensor Time Interval is presented for different spatial and numerical configurations of sensors in the Peripheral Field, when a stimulus of a prescribed shape (either straight or convex or concave) is made to pass through it. These formulae, relate ISI to the speed ‘v’ with which the stimulus, inclined at a steady angle β to a reference line, sweeps through the sensor field. Then the equations of the hyperbolas formed consequent to tracing the intersection points of two (or three) advancing circular wavefronts emanated from two (or three) stimulated sources in the Central Field are presented, along with some closing remarks on the criteria on which they are formed. Lastly, a parametric analysis is carried out in the case of the Two Sensor Model to demonstrate how distinct Organizational Maps exist in the central neural field that encode such parameters as the angle of inclination (β) as the stimulus passes through the Peripheral Field, it’s speed (v) and the inter-sensor time interval (ISI) between adjacent sensor stimulations. In the case of the Three Sensor Model, Organizational Maps encoding velocity of stimulus motion and shape are generated. In principle, the existence of such Organizational Maps in the brain, provides a basis in geometry, of how a sensory apparatus can make estimations about various stimuli features in the surrounding environment.

2. Method and Materials

2.1 Postulates underlying the PWC based Neurocomputational Models

1. Senso-topy

There exists a one to one structural-functional correspondence between sensors in the Peripheral Field and sources in the Central Field. That is, the spatial layout of the peripheral sensors is faithfully reflected by the spatial layout of the central sources. Also the stimulation of a single peripheral sensor, is accompanied by a maximal response from its corresponding central source after a short connection delay.

2. Equal Sensor to Source Connection Delays a priori

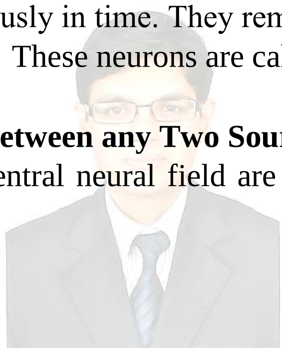
The connection delay for impulse propagation from each peripheral sensor to its corresponding central source is equal. That is, the sensor to source connection delay for each sensor-source pair is equal. Thus, the time interval between two adjacent peripheral sensor stimulations will be equal to the time interval between the corresponding two central source stimulations.

3. Coincidence Neuron Detectors

There exists a class of neurons in the central neural field that switch ‘on’ only upon reception of two or more impulses simultaneously in time. They remain switched ‘off’ for the reception of any impulse arriving in isolation. These neurons are called Coincidence Detectors.

4. Hyperbolic Distributions between any Two Sources

The coincidence detectors of the central neural field are distributed in a hyperbolic fashion between any two sources.



2.2 The Two-sensor Model (see Figure 4)

2.2.1 Framework

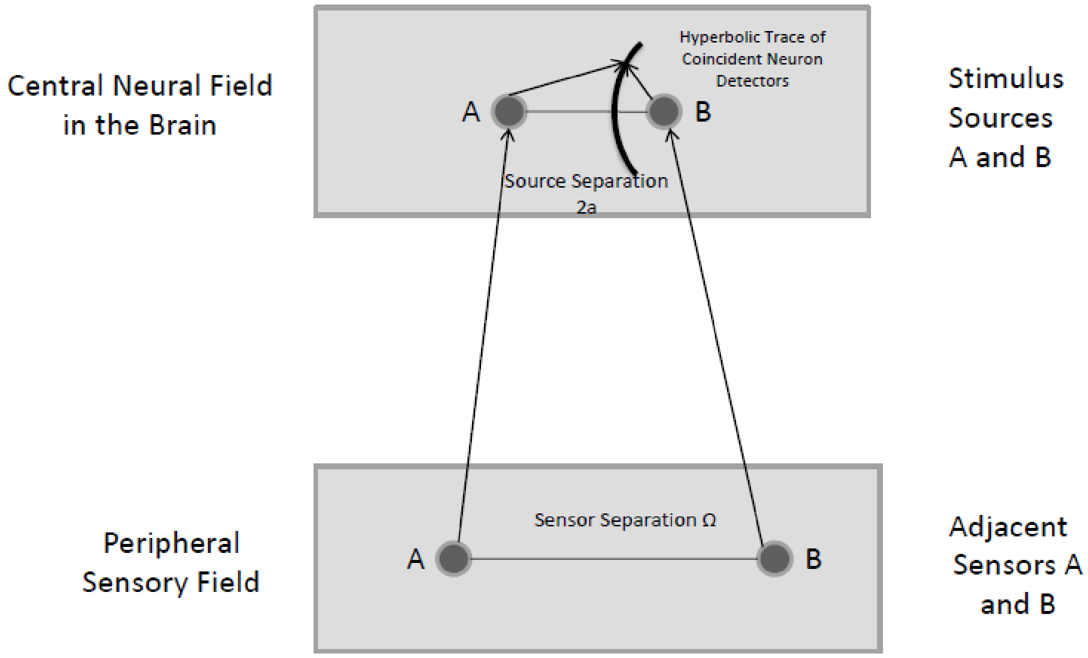


Figure 4: *Framework of the Two Sensor Model*
The peripheral sensory field and the central neural field are represented by two rectangular sheets. Two adjacent sensors labelled A and B are connected to their corresponding stimulus sources A and B respectively, via separate one to one connections. The sensors are separated by a distance denoted by Ω *and the sources are separated by a distance denoted by* $2a$*. Impulses arising from each source, converge onto coincidence detector neurons embedded in the plane of the central neural field which are distributed in a hyperbolic fashion between the two sources.*

2.2.2 Theory

2.2.2.1 Inter-Sensor Time Interval

2.2.2.1.1 When the Stimulus is an infinitely long Straight Line (see Figure 5)

Consider a peripheral sensor field consisting of just two sensors represented by the points A and B separated by a distance Ω, which are contacted successively by an infinitely long straight line stimulus denoted by 'l', at times t_A and t_B,respectively. Assume also that 'l' is inclined at an angle β with the line joining A and B, and moves with a velocity v in the direction, θ with respect to AB (N.B. $t_A < t_B$). The expression for ISI is given by (see Supplementary Material):

$$\Delta t_{AB} = \Omega.\frac{\text{Sin}\beta}{v} \qquad \ldots(1)$$

2.2.2.1.2 When the Stimulus is a Convex or Concave Semi-circle (see Figures 6&7)

For the case of a convex/concave shaped semi-circle stimulus of radius ρ, whose center O moves with a velocity v towards the midpoint O' of the line AB joining Sensors A and B, such that OO' makes an angle β with AB, the expression for ISI is given by (see Supplementary Material):

$$\Delta t_{AB} = \Omega.\frac{Cos\beta}{v} \qquad \ldots(2)$$

The condition to be fulfilled for the both sensors A and B to be successively contacted by the semicircular stimulus, is $\rho \geq \Omega/2$ when $\beta = 90^\circ$. In all the three cases (straight/convex/concave shaped stimuli), the angle of inclination has a prescribed range: $0^\circ < \beta < 90^\circ$.

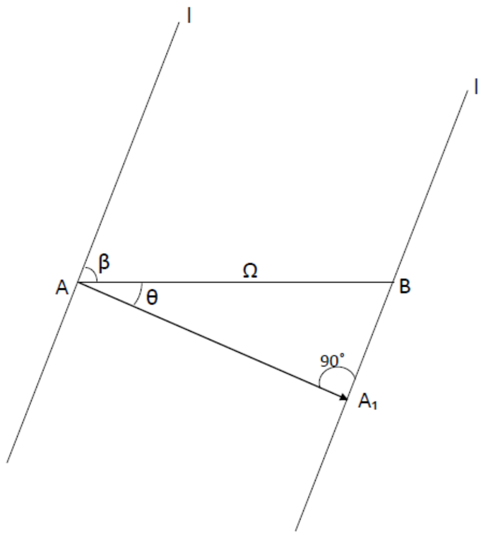


Figure 5: *Infinitely long straight stimulus 'l' contacts two Sensors A and B in succession*

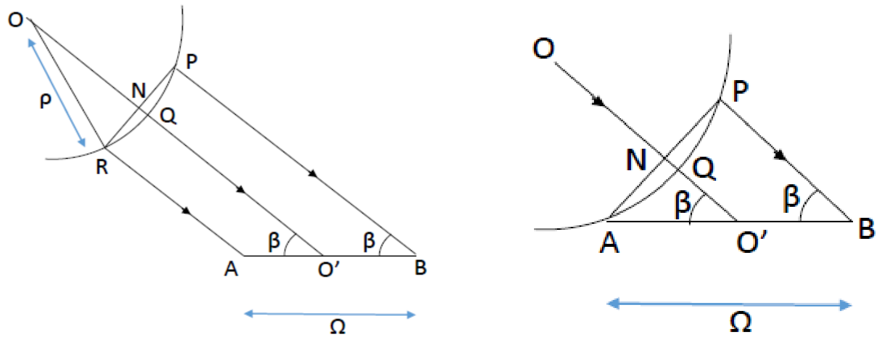


Figure 6: *Convex Semicircular stimulus of radius 'ρ' approaches and contacts two Sensors A and B in succession*

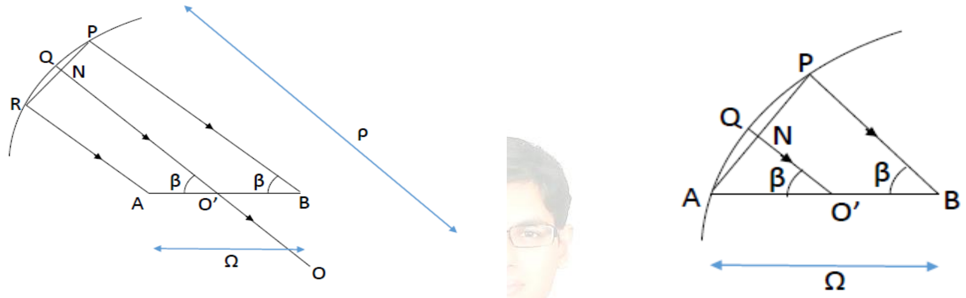


Figure 7: *Concave Semicircular stimulus of radius 'ρ' approaches and contacts two Sensors A and B in succession*

2.2.2.2 Equation of the Hyperbola forming the locus of the points of intersections of two advancing circular wavefronts over time (see Figure 8)

Consider two Sources A and B, located at positions $(-a, 0)$ and $(a, 0)$ respectively in the plane of the Central Field. When each source is stimulated successively at times t_A and t_B say, circular wavefronts emanate from them that propagate radially outwards with a constant speed u, say. If we assume A to have been stimulated before B, i.e. $t_A < t_B$ then there will be a particular instant, call it τ, when the two wavefronts travelling at equal speeds, meet at exactly one point along the line joining A and B, call it V. With the passage of time ($t > \tau$), this single point becomes two points of intersection, call them P and P' that are symmetrically placed about the line AB. The points P and P' get progressively shifted towards the right and diverge apart. On joining all the points of intersections for different snapshots in time, the trace of a branch of a hyperbola with point V as vertex is obtained. The equation of this hyperbola in terms of the Inter-pulse Interval (IPI = $\Delta t_{AB} = t_B - t_A$), the impulse propagation speed u and the source positions $(\pm a, 0)$ with respect to the origin, is given by (see Supplementary Material):

$$\frac{x^2}{\left(\frac{u\Delta t_{AB}}{2}\right)^2} - \frac{y^2}{a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2} = 1 \qquad ...(3)$$

By invoking Postulate-2, the expressions for Inter-Stimulus Interval (ISI) presented in Section 2.2.2.1 may be substituted in place of Inter-Pulse Interval (IPI) in the above equation.

An identical equation can be derived in the reversed case scenario, where the Source B is stimulated before the Source A (i.e. $t_B < t_A$), with the difference that Δt_{AB} is replaced by Δt_{BA}. Also, in order to make the equation of the hyperbola more conducive for graphical simulation, a new parameter $J_{AB} = \frac{u\Delta t_{AB}}{2}$ is introduced, which we shall from here on refer to as the J-parameter. The ensuing J-parameterized equation is:

$$x^2 = J^2 . \left(1 + \frac{y^2}{a^2 - J^2}\right) \qquad ...(4)$$

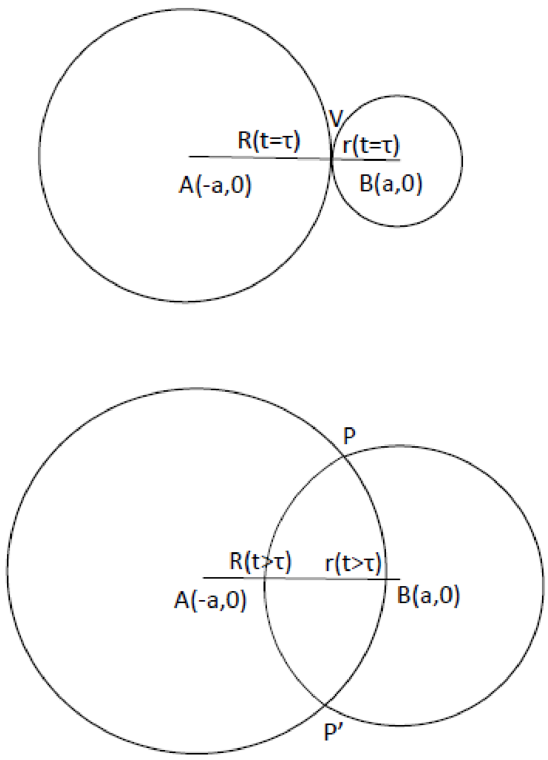


Figure 8:

Top: Two advancing circular wavefronts at the instant of single point contact at V

Bottom: Two advancing circular wavefronts at the instant of two point contact at P & P'

2.2.2.3 Remarks

A. When the time interval spanning the stimulation of sources A and B in succession of each other in any order is brought to zero, that is as $\Delta t_{AB} \to 0$ or as $\Delta t_{BA} \to 0$, both vertices approach the origin $O(0,0)$ and the branches gradually straighten out to coincide with the Y-axis, whose equation is $x = 0$.

B. It can be shown that the Principal Condition to be satisfied for the Generation of Hyperbolas is $|R(t) - r(t)| < 2a$, where $R(t)$, $r(t)$ and $2a$ are the instantaneous radii of the circular wavefronts emanated from sources A & B and the source separation length respectively.

C. Ancillary Conditions for the Generation of a Hyperbola
On examining the equation of the hyperbola, it is clear that the denominator of the y^2 term should remain a positive quantity (lest, we get an ellipse instead). That is, $a^2 - J^2 > 0 \Rightarrow J^2 < a^2 \Rightarrow -a < J < a \Rightarrow |J/a| < 1 \Rightarrow \left|\frac{u\Delta t_{AB}}{2a}\right| < 1$. But since the ISI (and IPI) depends on the shape of the stimulus, the condition to be satisfied for each alternative will be:

(i) For a Straight Line Stimulus:

$$\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\mathrm{Sin}\beta\right| < 1$$

(ii) For a Convex/Concave Semicircle Stimulus:

$$\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\mathrm{Cos}\beta\right| < 1$$

2.2.3 Results

2.2.3.1 Parametric Analysis of the Equation of the Hyperbola

2.2.3.1.1 Generating an Organizational Map for Angle of Inclination (β) in the Central Neural Field (see Figure 9)

By varying the angle of inclination β with which the stimulus (straight/convex/concave) moves through the two sensor field within the range $0^\circ \; to \; 90^\circ$, an organizational map encoding β can be generated in the plane of the central neural field using the J-parameterized form of the equation of the hyperbola (Eq.4).

For the purpose of graphical illustration, the following numerical values are adopted in the simulation: {Ω, a, u, v} = {1 mm, 0.2 mm, 0.1 mm/ms, 1 mm/ms} where 'Ω' is the sensor separation length, '2a' is the source separation length, 'u' is the propagation speed of the wavefront (or equivalently, the speed of nerve impulse conduction) and 'v' is the speed of the stimulus with respect to the stationary sensor field.

2.2.3.1.2 Generating an Organizational Map for Velocity in the Central Neural Field (see Figure 10)

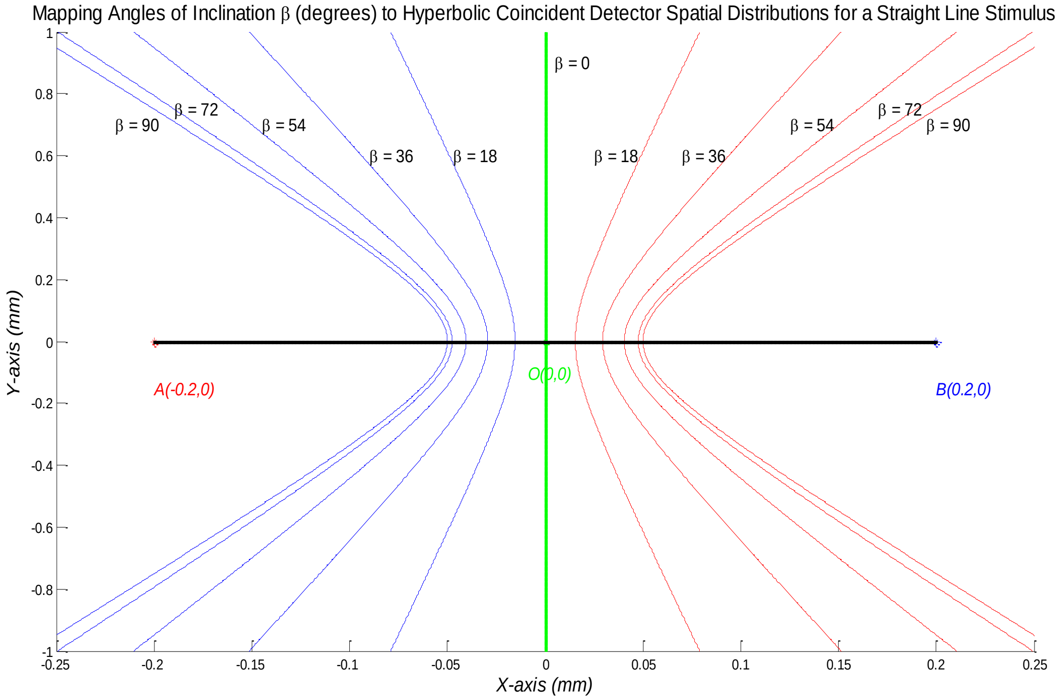
By varying the speed v with which the stimulus (straight/convex/concave) moves through the two sensor field, within a range 1 mm/ms to 3 mm/ms, an organizational map encoding v can be generated in the plane of the central neural field using the J-parameterized form of the equation of the hyperbola (Eq.4).

For the purpose of graphical illustration, the following numerical values are adopted in the simulation: {Ω, a, u, β} = {1 mm, 0.2 mm, 0.1 mm/ms, 60°} where β is the fixed inclination for the straight and convex/concave stimulus.

2.2.3.1.3 Generating an Organizational Map for Inter-Sensor Time Interval (ISI) in the Central Neural Field (see Figure 11)

By varying the magnitude of the Inter-Sensor Time Interval (ISI) between the stimulations of two adjacent sensors in the peripheral sensory field, through the range 0ms to 3ms, an organizational map encoding ISI can be generated in the plane of the central neural field using the un-parameterized equation of the hyperbola (Eq.3). (These maps are the same regardless of the shape of the stimulus).

For the purpose of graphical illustration, the following numerical values are adopted in the simulation: {a, u} = {0.2 mm, 0.1 mm/ms}.



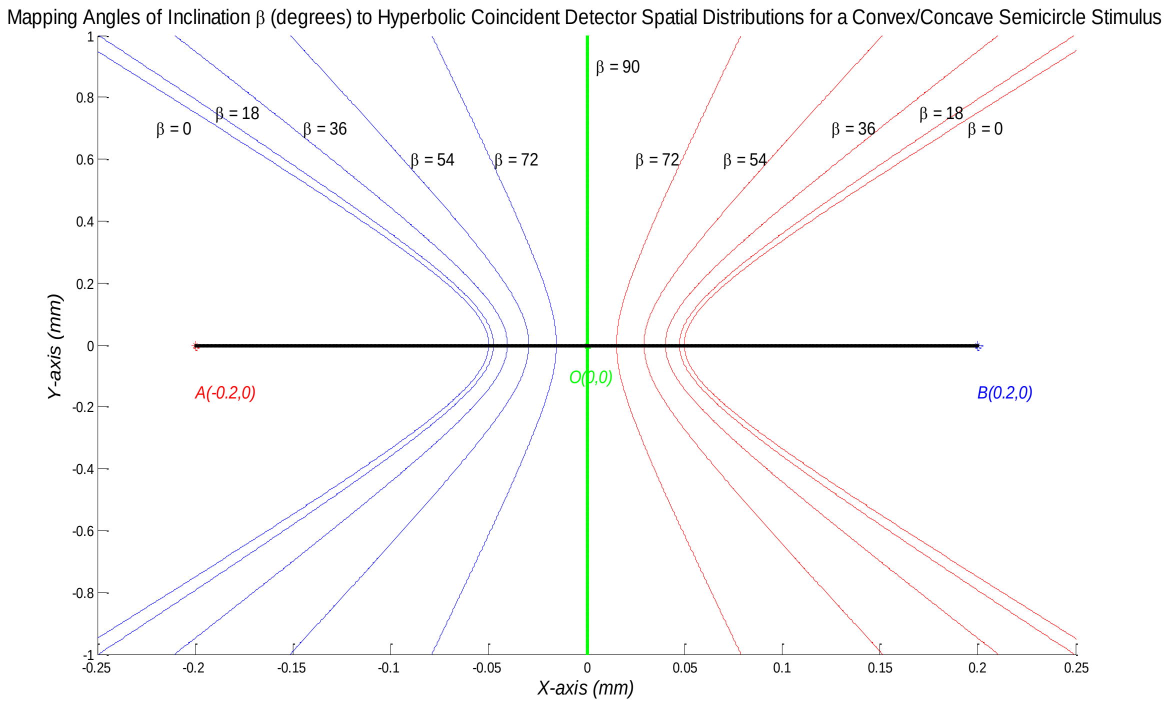
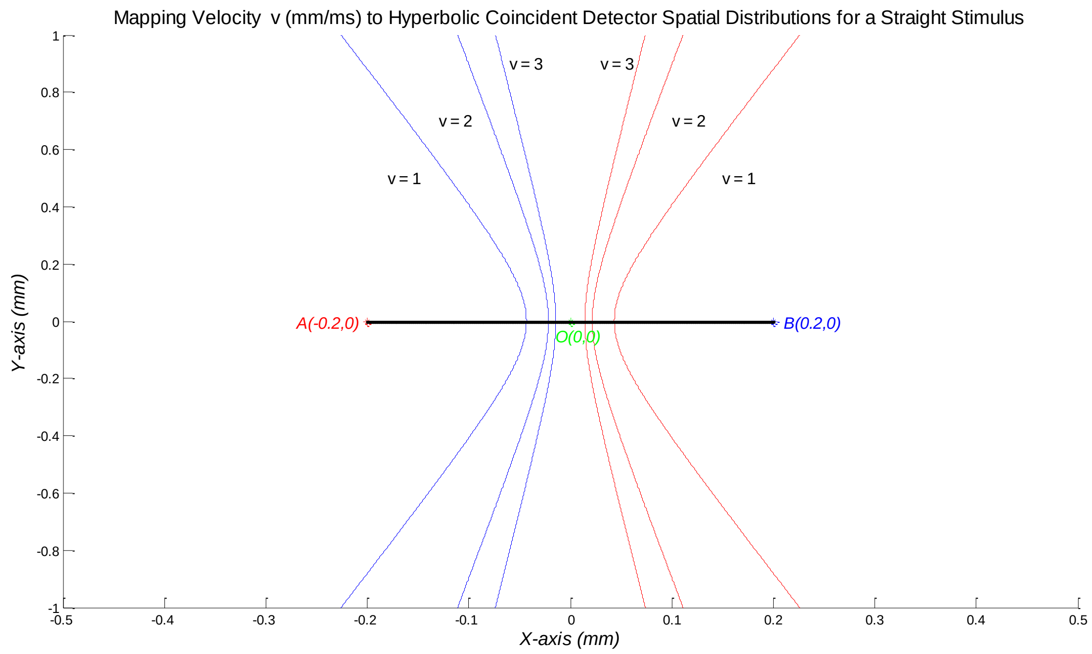


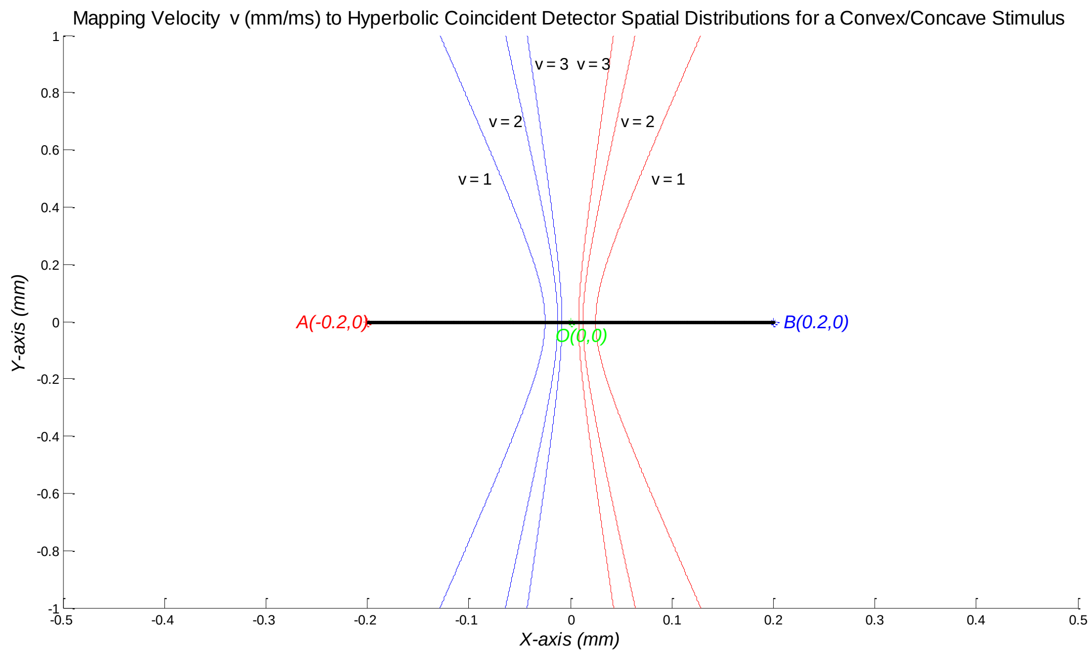
Figure 9: *Organizational Map for β in the CNF*

Top: For Straight line stimulus

Bottom: For Convex/Concave stimulus

The red hyperbolic arms correspond to the case where sensor A is stimulated before sensor B and the blue hyperbolic arms correspond to the case where sensor B is stimulated before sensor A. The green straight line in the center corresponds to the case where both the sensors A and B are stimulated simultaneously. (Angles are in degrees)





Figures 10: *Organizational Map for v in the CNF*

Top: For Straight line stimulus

Bottom: For Convex/Concave stimulus

(Velocity in mm/ms)

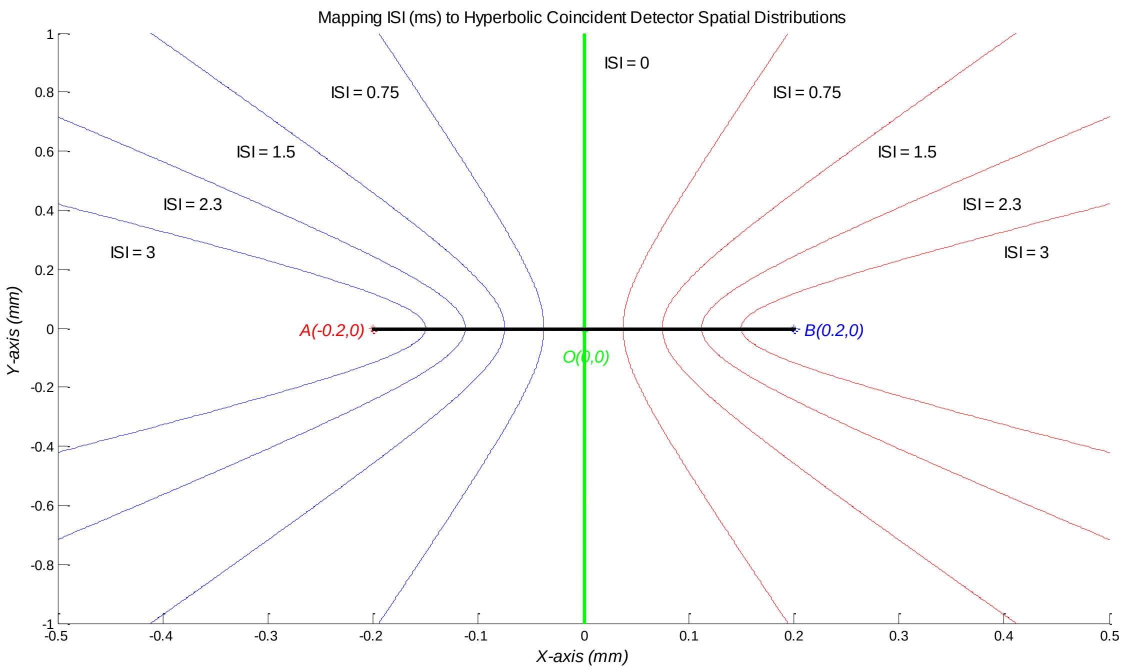


Figure 11: *Organizational Map for ISI in the CNF (Time Intervals are in ms)*

2.3 The Three Sensor Model (see Figure 12)

2.3.1 Framework

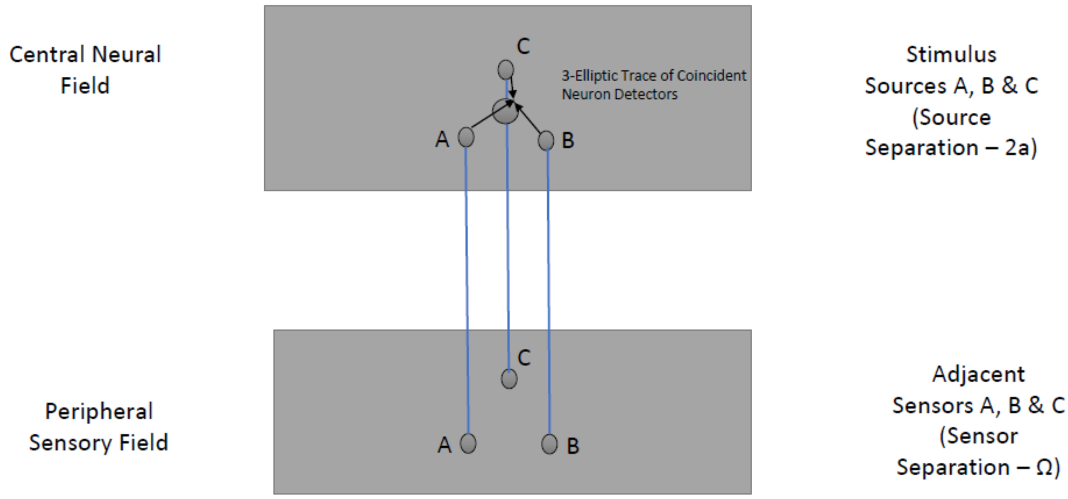


Figure 12: *Framework of the Three Sensor Model*

The peripheral sensory field and the central neural field are represented by two rectangular sheets. Three adjacent sensors labelled A, B & C located at the vertices of an equilateral triangle of side length Ω, are connected to their corresponding sources A, B & C via separate one to one connections, that are also located on the vertices of an equilateral triangle of side length 2a (refer to Postulate-1). Impulses arising from each source converge onto coincidence detector neurons that are distributed in distinct patterns depending on the stimulus shape (3-elliptic trace shown in figure corresponds to a straight line stimulus).

2.3.2 Theory

2.3.2.1 Inter-Sensor Time Interval

2.3.2.1.1 When the Stimulus is an infinitely long Straight Line (Figure 13)

Consider a peripheral sensor field consisting of three sensors represented by the points A, C and B forming the vertices of an equilateral triangle of side length Ω, which are contacted successively by an infinitely long straight line stimulus denoted by 'l', at times t_A, $t_C$and t_B,respectively. Assume also that 'l' is inclined at an angle β with the line joining A and B, and moves with a speed v in the direction θ with respect to AB (N.B. $t_A < t_C < t_B$). The expressions for ISIs are given by (see Supplementary Material):

$$\Delta t_{AC} = \Omega.\frac{\mathrm{Sin}(\beta - 60^\circ)}{v}$$
$$\Delta t_{AB} = \Omega.\frac{\mathrm{Sin}\beta}{v} \qquad \ldots(5)$$
$$\Delta t_{CB} = \Omega.\frac{\mathrm{Cos}(\beta - 30^\circ)}{v}$$

The prescribed range of angle of inclination for a specific sequence of three Sensor stimulations is $60^\circ \leq \beta \leq 120^\circ$.

2.3.2.1.2 When the Stimulus is a Convex Semicircle (Figure 14)

For a convex semicircle of radius ρ whose center O approaches the Sensor B at a steady inclination β with the line joining A and B and with a speed v, the expressions for ISIs are (see Supplementary Material):

$$\Delta t_{AC} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{v}$$
$$\Delta t_{AB} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho}{v} \qquad \ldots(6)$$
$$\Delta t_{CB} = \frac{\Omega \mathrm{Cos}(60^\circ - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)}}{v}$$

2.3.2.1.3 When the Stimulus is a Concave Semicircle (Figure 15)

For a concave semicircle of radius ρ whose center O approaches the Sensor B at a steady inclination β with the line joining A and B and with a speed v, the expressions for ISIs are (see Supplementary Material):

$$\Delta t_{AC} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{v}$$
$$\Delta t_{AB} = \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta}{v} \qquad \ldots(7)$$
$$\Delta t_{CB} = \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega \mathrm{Cos}(60^\circ - \beta)}{v}$$

The ISI expressions for both the Convex and Concave Semicircular Stimuli elucidated above are subject to the following constraints:

(i) $0^\circ \leq \beta \leq 30^\circ$ (Prescribed range for a specific sequence of three Sensor stimulations)

(ii) $\rho \geq \frac{\sqrt{3}}{2}\Omega$

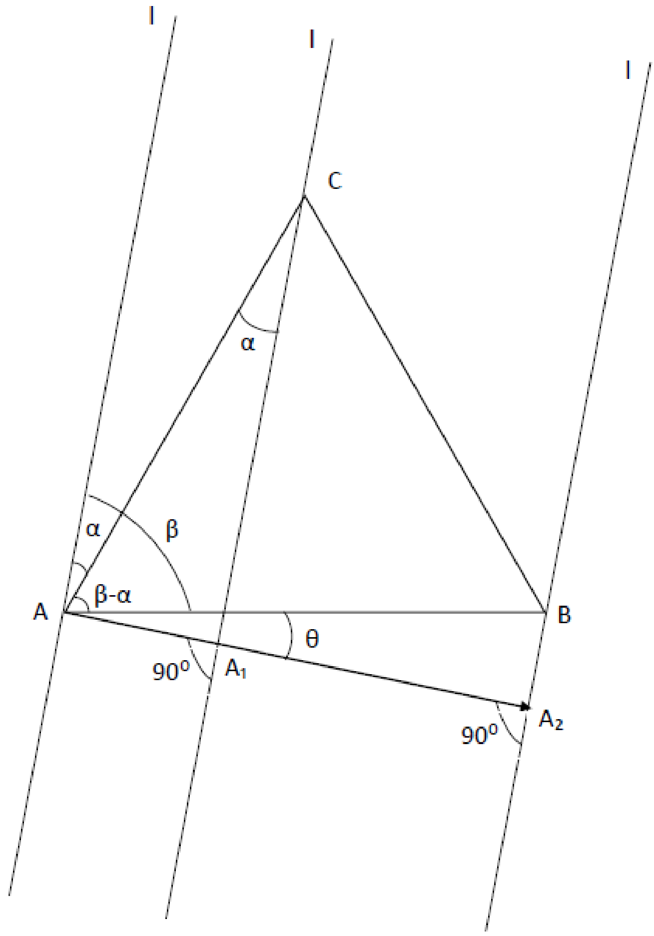


Figure 13: *Infinitely long straight stimulus 'l' contacts three Sensors A, C and B in succession*

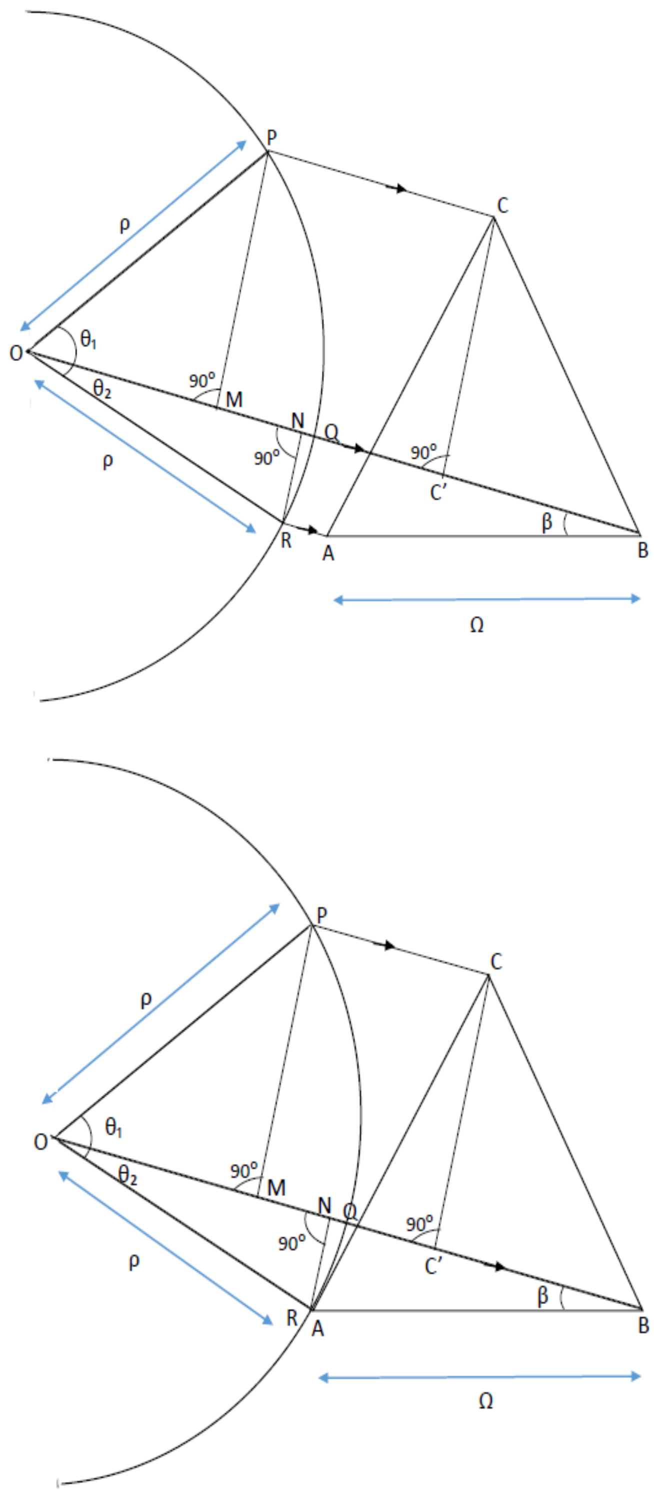
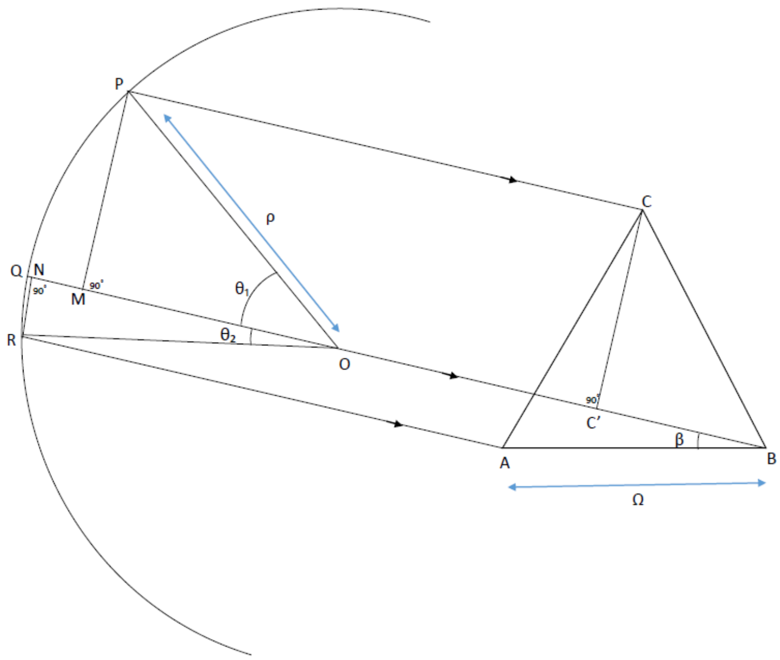


Figure 14: *Convex Semicircular stimulus of radius 'ρ' approaches three Sensors (top) and contacts A, C and B in succession (bottom)*



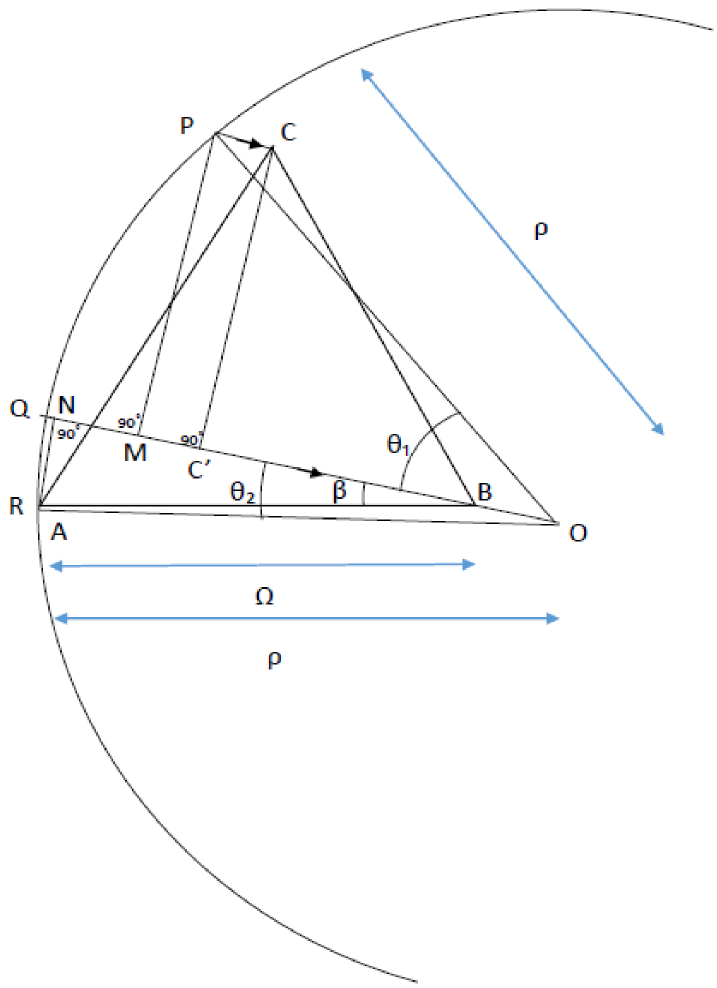


Figure 15: *Concave Semicircular stimulus of radius 'ρ' approaches three Sensors (top) and contacts A, C and B in succession (bottom)*

2.3.2.2 Equations of the Hyperbolas generated when Three Sources located at the Vertices of an Equilateral Triangle are stimulated in Temporal Succession (see Figures 16)

2.3.2.2.1 Equation of the Hyperbola generated when Sources A and B are stimulated in succession (see Figure 17)

By fixing the origin O of the XY frame to the midpoint of base AB of the equilateral ΔABC with side length 2a, the co-ordinates of the vertices A, B and C can be written as $(-a, 0)$, $(a, 0)$ and $(0, \sqrt{3}a)$, respectively. The equation of the hyperbola formed on stimulating the Sources located at vertices A and B in succession is given by Eq.(3) which is reiterated below:

$$\frac{x^2}{J_{AB}{}^2} - \frac{y^2}{a^2 - J_{AB}{}^2} = 1 \qquad \ldots(8)$$

Where $J_{AB} = \frac{u\Delta t_{AB}}{2}$

2.3.2.2.2 Equation of the Hyperbola generated when Sources A and C are stimulated in succession (see Figure 18)

Following frame rotation (60˚ clockwise) and translation operations, it may be shown that the equation of the hyperbola formed on stimulating the Sources located at the vertices A and C in succession is (see Supplementary Material):

$$y = \frac{-\sqrt{3}.a\,(ax - a^2 + 2J_{AC}{}^2) \pm 4J_{AC}\sqrt{(a^2 - J_{AC}{}^2)(x^2 + ax + a^2 - J_{AC}{}^2)}}{(3a^2 - 4J_{AC}{}^2)} \qquad \ldots(9)$$

Where $J_{AC} = \frac{u\Delta t_{AC}}{2}$

2.3.2.2.3 Equation of the Hyperbola generated when Sources C and B are stimulated in succession (see Figure 19)

Again, following frame rotation (60˚ anticlockwise) and translation operations, it may be shown that the equation of the hyperbola formed on stimulating the Sources located at the vertices C and B in succession is (see Supplementary Material):

$$y = \frac{\sqrt{3}.a(ax + a^2 - 2J_{CB}{}^2) \pm 4J_{CB}\sqrt{(a^2 - J_{CB}{}^2)(x^2 - ax + a^2 - J_{CB}{}^2)}}{(3a^2 - 4J_{CB}{}^2)} \qquad \ldots(10)$$

Where $J_{CB} = \frac{u\Delta t_{CB}}{2}$

2.3.2.2.4 Remarks

(I) The conditions for the generation of hyperbolas along the sides of ΔABC are:

A. For a Straight Line Stimulus

(i) Along AB : $\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\text{Sin}\beta\right| < 1$

(ii) Along AC : $\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\text{Sin}(\beta - 60^\circ)\right| < 1$

(iii)Along CB : $\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\text{Cos}(\beta - 30^\circ)\right| < 1$

B. For a Convex Semicircular Stimulus

(i) Along AB : $\left|u\frac{(\sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho)}{2av}\right| < 1$

(ii) Along AC : $\left|u\frac{(\sqrt{\rho^2-\Omega^2 Sin^2\beta} - \sqrt{\rho^2-\Omega^2 Sin^2(60^\circ-\beta)} + \Omega(Cos\beta - Cos(60^\circ-\beta)))}{2av}\right| < 1$

(iii)Along CB : $\left|u\frac{(\Omega Cos(60^\circ-\beta) - \rho + \sqrt{\rho^2-\Omega^2 Sin^2(60^\circ-\beta)})}{2av}\right| < 1$

C. For a Concave Semicircular Stimulus

(i) Along AB : $\left|u\frac{(\rho - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega Cos\beta)}{2av}\right| < 1$

(ii) Along AC : $\left|u\frac{(\sqrt{\rho^2-\Omega^2 Sin^2(60^\circ-\beta)} - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60^\circ-\beta)))}{2av}\right| < 1$

(iii)Along CB : $\left|u\frac{(\rho - \sqrt{\rho^2-\Omega^2 Sin^2(60^\circ-\beta)} + \Omega Cos(60^\circ-\beta))}{2av}\right| < 1$

(II) Special Cases

For certain values of angle of inclination β within the prescribed range for a particular shape of stimulus, a given pair of sources may be stimulated simultaneously resulting in the generation of a straight line that is perpendicular to the side of the triangle and passing through its midpoint, instead of the generation of a hyperbola. These angles are 60° & 120° in the case of a straight line stimulus and 30° in the case of a convex/concave stimulus. The straight line equations for each side are:

(i) Along AB : $x = 0$

(ii) Along AC : $y = \frac{-x+a}{\sqrt{3}}$

(iii)Along CB : $y = \frac{x+a}{\sqrt{3}}$

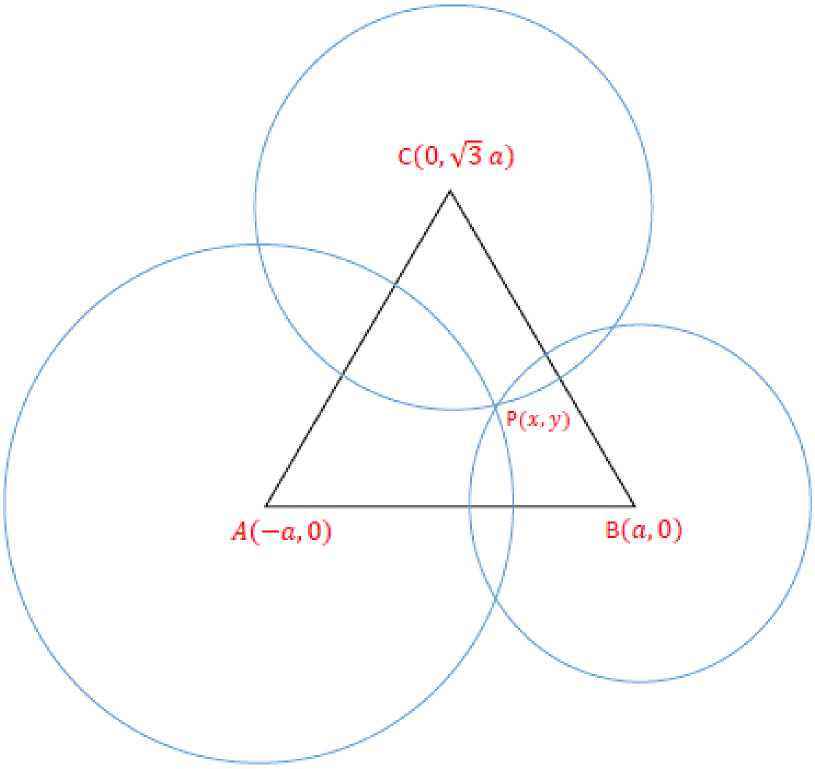


Figure 16: *Three Sources located at the vertices of an Equilateral Triangle stimulated in succession*

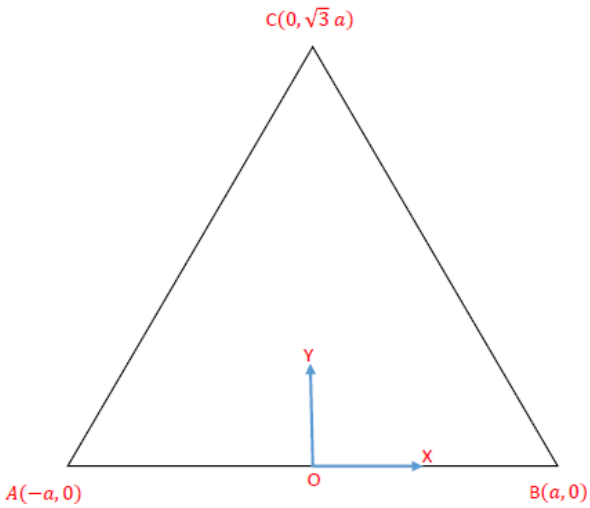


Figure 17: *XOY frame for the Hyperbola with transverse axis along the side AB*

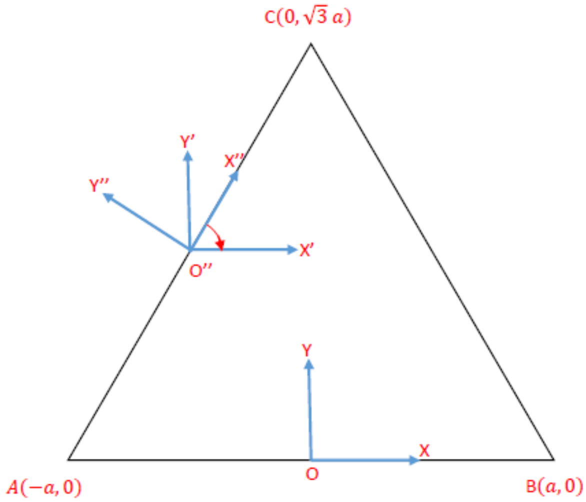


Figure 18: *X''O''Y'' frame for the Hyperbola with transverse axis along the side AC*

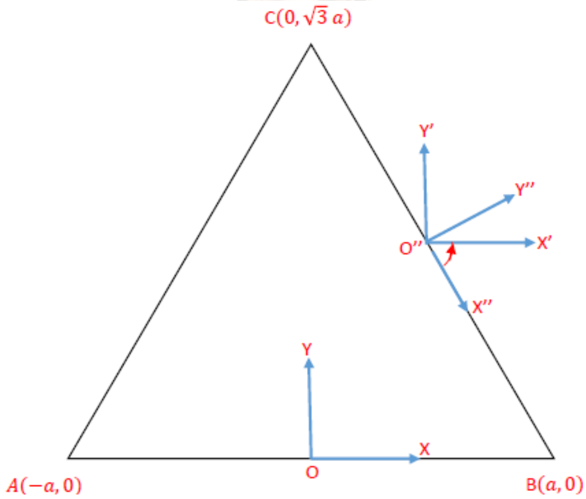


Figure 19: *X''O''Y'' frame for the Hyperbola with transverse axis along the side CB*

2.3.2.3 ISIs and J-Parameters for different Sequences of Sensor (or Source) Stimulations

When three sensors (or sources) A, B and C located on the vertices of an equilateral ΔABC are stimulated in a particular sequence by a moving stimulus that makes a steady angle β with its base, the time intervals between successive source stimulations will depend directly on β and the chosen temporal ordering of the sides (that is, on which side is taken as the first side, the base and the second side). For example, if the temporal ordering of the sides is the sequence (AC → AB → CB), it implies that the source A is stimulated before source C, which is stimulated before source B (i.e. A → C → B). By labeling the sides of the triangle as Side-1, Base and Side-2 the general expressions for time intervals and the corresponding J-parameters can be written for each specifically shaped stimuli.

2.3.2.3.1 ISIs and J-Parameters for an infinitely long Straight Stimulus

The expressions for ISIs are (see §2.3.2.1.1):

$$\begin{aligned} \Delta t_{Side-1} &= \Omega.\frac{\text{Sin}(\beta - 60^\circ)}{v} \\ \Delta t_{Base} &= \Omega.\frac{\text{Sin}\beta}{v} \\ \Delta t_{Side-2} &= \Omega.\frac{\text{Cos}(\beta - 30^\circ)}{v} \end{aligned} \qquad \ldots(11)$$

The corresponding expressions for J-parameters are:

$$\begin{aligned} J_{Side-1} &= u\Omega.\frac{\text{Sin}(\beta - 60^\circ)}{2v} \\ J_{Base} &= u\Omega.\frac{\text{Sin}\beta}{2v} \\ J_{Side-2} &= u\Omega.\frac{\text{Cos}(\beta - 30^\circ)}{2v} \end{aligned} \qquad \ldots(12)$$

2.3.2.3.2 ISIs and J-Parameters for a Convex Semicircular Stimulus

The expressions for ISIs are (see §2.3.2.1.2):

$$\begin{aligned} \Delta t_{Side-1} &= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{v} \\ \Delta t_{Base} &= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho}{v} \\ \Delta t_{Side-2} &= \frac{\Omega \text{Cos}(60^\circ - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)}}{v} \end{aligned} \qquad \ldots(13)$$

The corresponding expressions for J-parameters are:

$$\begin{aligned} J_{Side-1} &= u\frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{2v} \\ J_{Base} &= u\frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho}{2v} \\ J_{Side-2} &= u\frac{\Omega \text{Cos}(60^\circ - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)}}{2v} \end{aligned} \qquad \ldots(14)$$

2.3.2.3.3 ISIs and J-Parameters for a Concave Semicircular Stimulus

The expressions for ISIs are (see §2.3.2.1.3):

$$\begin{aligned}
\Delta t_{Sde-1} &= \frac{\sqrt{\rho^2-\Omega^2 Sin^2(60°-\beta)} - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60°-\beta))}{v} \\
\Delta t_{Base} &= \frac{\rho - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega Cos\beta}{v} \\
\Delta t_{Side-2} &= \frac{\rho - \sqrt{\rho^2-\Omega^2 Sin^2(60°-\beta)} + \Omega Cos(60°-\beta)}{v}
\end{aligned} \quad \ldots(15)$$

The corresponding expressions for J-parameters are:

$$\begin{aligned}
J_{Sde-1} &= u\frac{\sqrt{\rho^2-\Omega^2 Sin^2(60°-\beta)} - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60°-\beta))}{2v} \\
J_{Base} &= u\frac{\rho - \sqrt{\rho^2-\Omega^2 Sin^2\beta} + \Omega Cos\beta}{2v} \\
J_{Side-2} &= u\frac{\rho - \sqrt{\rho^2-\Omega^2 Sin^2(60°-\beta)} + \Omega Cos(60°-\beta)}{2v}
\end{aligned} \quad \ldots(16)$$

2.3.2.3.4 Allotment of Sides of ΔABC to categories 'Side-1', 'Base' and 'Side-2' according to the sequence of Sensor (or Source) Stimulations

Using Table 1 below as a guide, the triplet expressions for ISIs & J-parameters for each possible sequence of sensor (or source) stimulations of our analysis can be written.

Sequence of Sensor (or Source) Stimulations with respect to the Vertices of ΔABC	Corresponding Sequence of Sensor (or Source) Stimulations with respect to the Sides of ΔABC		
	Side1	Base	Side2
A to C to B	AC	AB	CB
A to B to C	AB	AC	BC
B to A to C	BA	BC	AC
B to C to A	BC	BA	CA
C to A to B	CA	CB	AB
C to B to A	CB	CA	BA

Table 1

2.3.2.4 Equations of the Hyperbolas generated for Specific Sequences of Source Stimulations

In §2.3.2.2, the three analytical equations of the hyperbolas (8),(9) and(10) generated when Sources located at $A(-a,0)$, $B(a,0)$ and $C(0,\sqrt{3}a)$ are successively stimulated, with respect to the origin $O(0,0)$ (fixed to the midpoint of side AB) were presented. They are reiterated below:

$$y = \frac{-\sqrt{3}.a(ax - a^2 + 2{J_{AC}}^2) \pm 4J_{AC}\sqrt{(a^2 - {J_{AC}}^2)(x^2 + ax + a^2 - {J_{AC}}^2)}}{(3a^2 - 4{J_{AC}}^2)}$$

$$y = \pm\sqrt{(a^2 - {J_{AB}}^2)\left(\frac{x^2}{{J_{AB}}^2} - 1\right)} \qquad \ldots(17)$$

$$y = \frac{\sqrt{3}.a(ax + a^2 - 2{J_{CB}}^2) \pm 4J_{CB}\sqrt{(a^2 - {J_{CB}}^2)(x^2 - ax + a^2 - {J_{CB}}^2)}}{(3a^2 - 4{J_{CB}}^2)}$$

The above hyperbolas have their transverse axes fixed to the sides AC, AB and CB of ΔABC respectively. Depending on which source is stimulated first and the ordering of subsequent source stimulations, the J-parameters will vary (since J depends on Δt which further depends on β and v). Thus, the J-parameter in each equation ultimately determines the shape & position of the individual hyperbolas and therefore also the common point of intersection between them (see Figure 16). Using Table-1 again and Equations (17), a triplet set of hyperbolic equations for each of the possible sequences of sensor (or source) stimulations can be obtained.

2.3.3 Results

2.3.3.1 Organizational Maps for Velocity in the Central Neural Field

2.3.3.1.1 When the Stimulus is an infinitely long Straight line (see Figure 20)

On calculating the J-parameters of §2.3.2.3 with numerical values {Ω, a, u, {β}} = {1mm, 0.2mm, 0.1mm/ms, {60˚, 90˚, 120˚}} and {v} = {1mm/ms, 2mm/ms, 3mm/ms}. Substituting these into the triplet set of hyperbolic equations of §2.3.2.4, it is possible to generate velocity maps of the moving straight line stimulus in the central neural field.

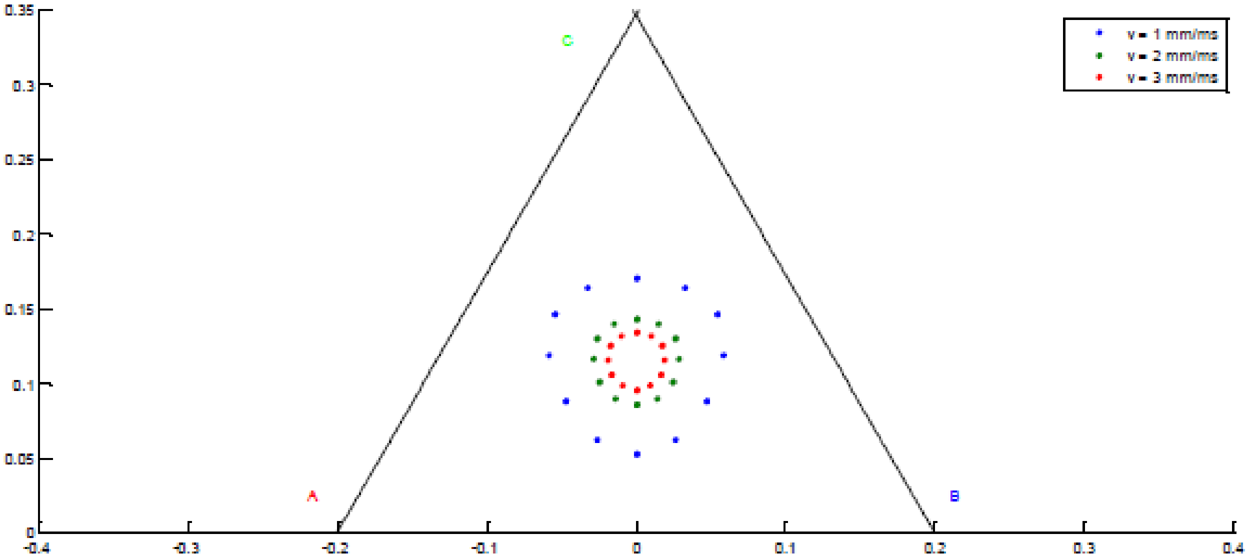


Figure 20: *Organizational Map for v in the CNF when stimulus is a Straight line*

2.3.3.1.2 When the Stimulus is a Convex Semicircle (see Figure 21)

On calculating the J-parameters of §2.3.2.3 using numerical values {ρ, Ω, a, u, {β}} = {1mm, 1mm, 0.2mm, 0.1mm/ms, {0˚, 15˚, 30˚}} and {v} = {1mm/ms, 2mm/ms, 3mm/ms}. Substituting these into the hyperbolic equations of §2.3.2.4, it is possible to generate velocity maps of the moving convex semicircular stimulus in the central neural field.

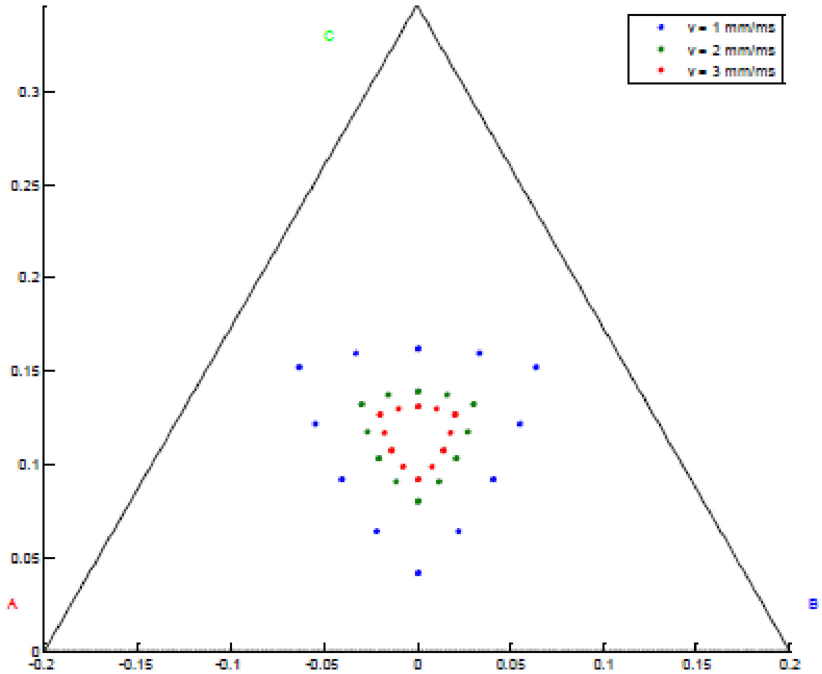


Figure 21: *Organizational Map for v in the CNF when stimulus is a Convex Semicircle*

2.3.3.1.3 When the Stimulus is a Concave Semicircle (see Figure 22)

On calculating the J-parameters of §2.3.2.3 using numerical values {ρ, Ω, a, u, {β}} = {1mm, 1mm, 0.2mm, 0.1mm/ms, {0˚, 15˚, 30˚}} and {v} = {1mm/ms, 2mm/ms, 3mm/ms}. Substituting these into the hyperbolic equations of §2.3.2.4, it is possible to generate velocity maps of the moving concave semicircular stimulus in the central neural field.

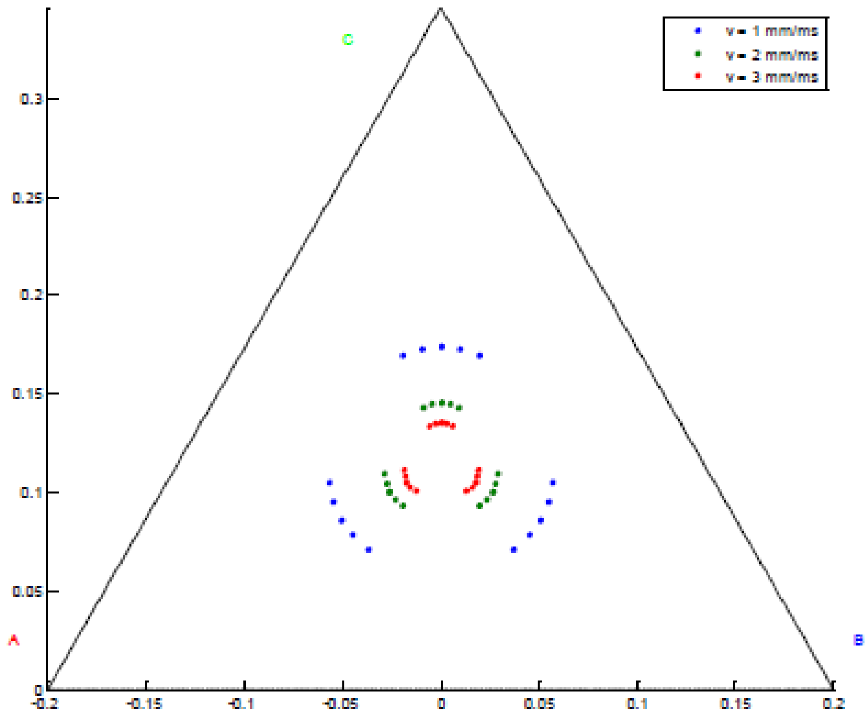


Figure 22: *Organizational Map for v in the CNF when stimulus is a Concave Semicircle*

3. Discussion

3.1 A Contrast between the Two Sensor Model and Three Sensor Model

3.1.1 Limitation of the Two Sensor Model: A Problem of Ambiguity

The Two Sensor Model predicts that each hyperbolic distribution of coincidence detectors in the Central Neural Field, encodes information regarding Inter-Sensor Time Interval, Angle of contact made with the sensor separation length Ω, the speed of the stimulus sweeping through the stationary Two Sensor Field and also the shape of the stimulus. However, the fact that only a single J-parameter determines the shape and position of the hyperbolic arm with respect to the source positions, is a cause for ambiguity in ascertaining specific features of stimulus motion such as speed v and angle of contact β (hence direction). Recall that for a straight line stimulus Eq. (1):

$$J_{AB} = \frac{u\Delta t_{AB}}{2} = \text{u } \Omega.\frac{\text{Sin}\beta}{2v} \qquad \ldots(18)$$

From the above formula, it is clear that if $\frac{\text{Sin}\beta}{v}$ were set as constant for different possible doublet values of (v, β) that satisfy the constraints described in §2.2.2.3, with all other factors (u, Ω) remaining equal, then the J-parameter will consequently remain also a constant for all the (v, β) values.

Formally stated, $J_{AB} = \{constant \ \forall \ (v, \beta) : (\frac{\text{Sin}\beta}{v} = constant) \wedge (\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2a}\right)\text{Sin}\beta\right| < 1)\}$.

For instance, let $\{\text{u}, \Omega\} = \{0.1, 1\}$, then for all $\{(v, \beta)\} = \{\left(\frac{1}{2}, 30^{\circ}\right), \left(\frac{\sqrt{3}}{2}, 60^{\circ}\right), (1{,}90^{\circ})\}$ the parameter $J_{AB} = 0.05$. This implies that the same hyperbolic spatial distribution of detectors encodes multiple stimulus velocities (speed & direction). Such ambiguity is eliminated if an extra sensor and two additional inter-sensor intervals are incorporated into this simplistic model.

3.1.2 Resolution of Ambiguity with the Three Sensor Model

The Three Sensor Model is what follows when three sensors and three inter-sensor intervals are taken into consideration. This model predicts that there is exactly one point location for a coincidence detector in the Central Neural Field (CNF) that encodes a specific stimulus shape and velocity. It is located at the point of intersection of the three hyperbolic arms generated when circular wavefronts emanated from successively stimulated Sources meet in space and in time. The shape and velocity of the stimulus determines the values taken up by the three J-parameters involved in the formalism. These parameters further dictate the whereabouts of the common point of intersection, such that any change in the value of one or more of them would result in a change in the location of that point.

Depending on the shape and velocity of the stimulus, different concentric patterns of distribution of coincidence detectors in the CNF are observed, which share a common characteristic of diminishing size with increasing velocity. These patterns are summed up below:

(i) For a Straight Line Stimulus - 3-elliptic pattern (see Figure 20)
(ii) For a Convex Semicircular Stimulus - equilateral triangloid pattern (see Figure 21)
(iii)For a Concave Semicircular Stimulus - 3-cusp pattern (see Figure 22)

3.2 An Extension: The Three Sensor Model with an Isosceles Right Triangle Configuration (see Figure 23)

The Three Sensor Model developed in the previous sections, was constructed for an Equilateral Triangle Configuration of Sensors and Sources in the Peripheral Sensory Field and Central Neural Field, respectively. A similar detailed formalism has been developed for an Isosceles Right Triangle Configuration of Sensors (and Sources). Only the final simulation result showing the velocity maps is presented here (see Figure 23).

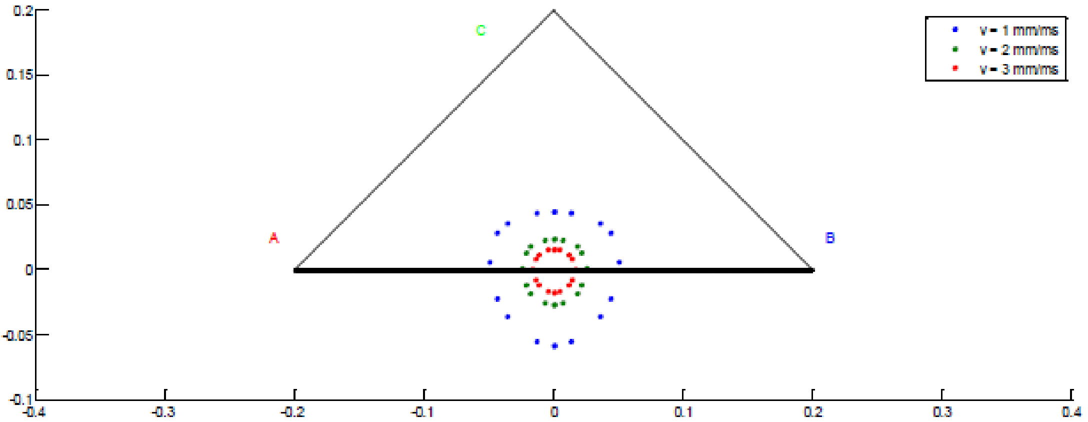


Figure 23: *Organizational Map for v in the CNF*

3.3 Outline of the Three Sensor Model Algorithm

The hallmark of the algorithm used to develop the Three Sensor Models lies in the derivations of the novel analytical equations describing the three hyperbolas (Equations 17), each having one side of the triangle as a transverse axis. They are expressed in the explicit functional form $y = f(x)$ following which plots are generated for different (v, β) values. The common points of intersection between the hyperbolas are ascertained in each case, and plotted collectively on a scatter diagram to obtain the Organizational Maps for velocity in the Central Neural Field. The accuracy of the (x, y) solutions can be increased by simply increasing the point resolution along the X-axis. The proposed algorithm could also have potential implications in the fields of Communication Engineering and the Navigational Sciences, particularly for the localization of a receiver/transmitter station [5].

3.4 Future Models: Scheme for the Construction of a General n-Sensor Model (see Figure 24)

For a Field with a hexagonal array of Sensors (or Sources), each hexagonal unit is composed of six equilateral triangles. Similarly, for a Field with a square array of Sensors (or Sources), each square unit is composed of two right isosceles triangles. It is thus conceivable that using the key results derived for both variants of the Three Sensor Model, that a generalized Sensor Model can be developed with n number of sensors (where n ≥ 4). For such an n-Sensor Model, there are an associated $n(n-1)/2$ inter-sensor time intervals involved in the formalism. It should be noted however, that a truly general model is one that can accommodate even an irregular array.

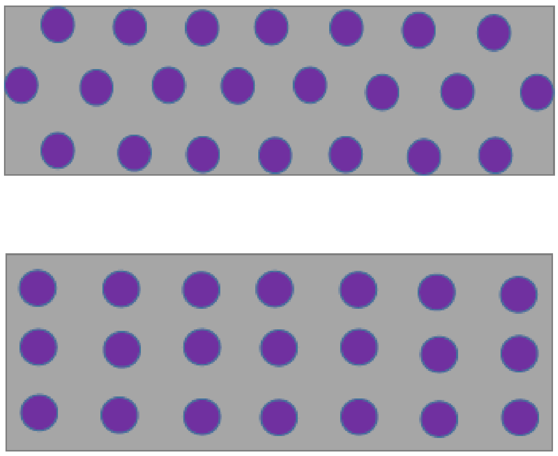


Figure 24: *Array of Sensors in the Peripheral Neural Field: Hexagonal Array (top) and Square Array (bottom)*

3.5 Some Closing Speculation

The hypothesized geometric models could potentially find implementation into any sensory system wherein the postulates of §2.1 are satisfied. For instance, retinotopy has been a long established property of the visual system [6]. So if this system were modelled along the lines of the PWC principle, then one prediction that would naturally follow is that various features of a moving stimulus (like orientation) is encoded in the unique spatial position of neurons distributed in a particular layer of the visual cortex.

This paper focuses exclusively on modeling a generic Sensory System, which is one that is characterized by flow of information from the periphery (sense organ) to the center (brain). The author forwards the hypothesis that the same paradigm, may find pertinence in the reverse case scenario as well. That is, where the flow of information is from the center to the periphery. In other words, a Motor System.

Acknowledgements

Gloria in excelsis Deo

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Supplementary Material

Derivations of Equations (1) and (2)

(1) When the Stimulus is an Infinitely long straight line (see Fig. 5 Main Text)

Consider Right ΔAA_1B,

$$AB = \Omega \quad \text{...(1a)}$$

$$\beta + \theta = 90° \quad \text{...(1b)}$$

$$\Delta t_{AB} = t_B - t_A$$

$$= \frac{AA_1}{v}$$

$$= \frac{AB.Cos\theta}{v} \quad (\text{In } \Delta AA_1B,\ Cos\theta = AA_1/AB)$$

$$= \frac{\Omega.Cos\theta}{v} \quad \text{(By (1a))}$$

$$= \frac{\Omega.Cos(90°-\beta)}{v} \quad \text{(By (1b))}$$

$$= \frac{\Omega.Sin\beta}{v}$$

$$\boxed{\Delta t_{AB} = \Omega.\frac{\mathrm{Sin}\beta}{v}}$$

(2) When the Stimulus is a Convex/Concave Semicircle (see Fig. 6&7 Main Text)

Consider Right ΔAPB,

$$AO' = O'B = \frac{AB}{2} = \frac{\Omega}{2} \quad \text{...(2a)}$$

$$\Delta t_{AB} = t_B - t_A$$

$$= \frac{PB}{v}$$

$$= \frac{AB.Cos\beta}{v} \quad (\text{In } \Delta RPB,\ Cos\beta = PB/AB)$$

$$= \frac{\Omega.Cos\beta}{v} \quad \text{(By (1a))}$$

$$\boxed{\Delta t_{AB} = \Omega.\frac{\mathrm{Cos}\beta}{v}}$$

NOTE:
In case (1), the straight line stimulus will always contact both sensors A and B will in succession, provided that it is infinitely long. However, in case (2) for the convex/concave semicircular stimulus, the sensors A and B will be touched in succession provided that $\rho_{min} \geq \frac{\Omega}{2} Sin\beta$. This is clearly evident from the ΔAPB, $Sin\beta = \frac{AP}{AB} = \frac{2\rho Sin\theta}{\Omega}$ where we put ∠AON = ∠PON = θ. Hence, we can say that the minimum limit of the diameter of the stimulus 2ρ should be greater than Ω.Sinβ, for a given inclination β and sensor separation Ω.

Derivation of Equation (3)
(see Fig. 8 Main Text)

The equation of the circular wavefront emanating from source $A(-a, 0)$ stimulated at time t_A, is given by:

$$(x + a)^2 + y^2 = R^2 \qquad \text{...(3a)}$$

The equation of the circular wavefront emanating from source $B(a, 0)$ stimulated at time t_B, is given by:

$$(x - a)^2 + y^2 = r^2 \qquad \text{...(3b)}$$

R and r are the instantaneous radii of either wavefronts, where $R > r$ for any instant, since A was stimulated before B (i.e. $t_A < t_B$).

The speed of propagation of both wavefronts is equal and uniform in all directions and is given by:

$$u = \frac{dR}{dt} = \frac{dr}{dt} \qquad \text{...(3c)}$$

Subtracting Equation (3b) from (3a) yields,

$$(x + a)^2 - (x - a)^2 = R^2 - r^2$$

On simplifying the above,

$$x = \frac{(R^2 - r^2)}{4a} \qquad \text{...(3d)}$$

Squaring Equation (3d),

$$x^2 = \frac{(R^2 - r^2)^2}{16a^2} \qquad \text{....(3e)}$$

Differentiating Equation (3e) with respect to time,

$$2x\frac{dx}{dt} = \frac{2(R^2 - r^2)(2R.\frac{dR}{dt} - 2r.\frac{dr}{dt})}{16a^2}$$

$$2x\frac{dx}{dt} = \frac{4u(R^2 - r^2)(R - r)}{16a^2} \qquad \text{(By (3c))}$$

$$2x\frac{dx}{dt} = \frac{4u(R + r)(R - r)^2}{16a^2} \qquad \text{....(3f)}$$

Substituting x from Equation (3d) into Equation (3a),

$$y^2 = R^2 - (x + a)^2$$

$$= R^2 - \left(\frac{(R^2 - r^2)}{4a} + a\right)^2$$

$$= \left(R + \left(\frac{(R^2 - r^2)}{4a} + a\right)\right)\left(R - \left(\frac{(R^2 - r^2)}{4a} + a\right)\right)$$

$$= \frac{(R^2 - r^2 + 4a^2 + 4aR).(-R^2 + r^2 - 4a^2 + 4aR)}{16a^2}$$

$$= -\frac{(R^4 + r^4 + 16a^4 - 2R^2r^2 - 8a^2R^2 - 8a^2r^2)}{16a^2}$$

$$= -\frac{\left[(R^2 + r^2 - 4a^2)^2 - 4R^2r^2\right]}{16a^2}$$

$$= -\frac{\left[((R-r)^2 + 2Rr - 4a^2)^2 - 4R^2r^2\right]}{16a^2}$$

$$= -\frac{\left[((R-r)^2 + 2Rr - 4a^2) + 2Rr\right]\left[((R-r)^2 + 2Rr - 4a^2) - 2Rr\right]}{16a^2}$$

$$= -\frac{((R-r)^2 + 4Rr - 4a^2)((R-r)^2 - 4a^2)}{16a^2}$$

$$y^2 = -\frac{((R+r)^2 - 4a^2)((R-r)^2 - 4a^2)}{16a^2} \qquad \text{...(3g)}$$

From (3g), it is clear that in order for $y \in \boldsymbol{R}$, either one of the following two conditions must hold true:

(i) $R + r > 2a$ and $R - r < 2a$, or

(ii) $R + r < 2a$ and $R - r > 2a$

In order that the two circular wavefronts intersect each other to trace out the locus of a hyperbola, with one of the vertices V lying somewhere on the line AB joining the point sources A and B, it is necessary that condition (i) holds true. Condition (ii) would geometrically imply that the circles intersect nowhere in the XY-plane and is therefore rejected. So provided condition (i) holds true, we can write:

$$y = \pm\sqrt{-\frac{((R+r)^2 - 4a^2)((R-r)^2 - 4a^2)}{16a^2}} \quad \in \boldsymbol{R} \qquad \text{...(3h)}$$

Differentiating Equation (3g) with respect to time,

$$2y.\frac{dy}{dt} = -\frac{\left[((R+r)^2 - 4a^2).2(R-r)\left(\frac{dR}{dt} - \frac{dr}{dt}\right) + ((R-r)^2 - 4a^2).2(R+r)\left(\frac{dR}{dt} + \frac{dr}{dt}\right)\right]}{16a^2}$$

$$\Rightarrow \qquad 2y.\frac{dy}{dt} = -\frac{4u(R+r)((R-r)^2 - 4a^2)}{16a^2} \qquad \text{...(3i)} \qquad \text{(By (3c))}$$

To re-iterate, if t_A and t_B be the times at which sources A and B are stimulated respectively ($t_A < t_B$), and τ is the instant at which they both meet at a point V lying on the line AB, then we can say that the wavefront arising from source A, would have grown from a radius $R = 0$ to $R = R(\tau)$ in the time interval spanning t_A to τ. Similarly, the wavefront arising from source B, would have grown from a radius $r = 0$ to $r = r(\tau)$ in the time interval spanning t_B to τ. So it should be possible to integrate equation (3c), keeping in mind that the speed of propagation of the wavefronts is equal and uniform in all directions from both the sources and that $t_A < t_B < \tau$:

$$\int_0^{R(\tau)} dR = \int_{t_A}^{\tau} u.dt \Rightarrow R(\tau) = u(\tau - t_A) \qquad \text{...(3j)}$$

$$\int_0^{r(\tau)} dr = \int_{t_B}^{\tau} u.dt \Rightarrow r(\tau) = u(\tau - t_B) \qquad \text{...(3k)}$$

At the instant, $t = \tau$, both wavefronts meet at the point V on the line $AB = 2a$,

$$R(\tau) + r(\tau) = 2a \qquad \text{...(3l)}$$

Subtracting Equation (29) from (28),

$$R(\tau) - r(\tau) = u(t_B - t_A) = u.\Delta t_{AB} \qquad \text{...(3m) (By definition of IPI)}$$

The (x, y) co-ordinates of the set of points (P&P') of intersections of the two circular wavefronts at times $t > \tau$, can be generically defined by equations (3d) and (3h):

$$\left(\frac{(R(t)^2 - r(t)^2)}{4a}, \pm\sqrt{-\frac{((R(t)+r(t))^2 - 4a^2)((R(t)-r(t))^2 - 4a^2)}{16a^2}}\right) \qquad \text{...(3n)}$$

The co-ordinate of the point V lying on AB can be found by substituting (3l) & (3m) in (3n):

$$\left(\frac{u\Delta t_{AB}}{2}, 0\right) \qquad \text{...(3o)}$$

Since the two circular wavefronts propagate outwards with the same rate of expansion u, we can expect that the instantaneous difference in their radii, $R(t) - r(t)$ to be constant with time. A formal justification of this statement can be made as follows:

$$\frac{d(R(t) - r(t))}{dt} = \frac{dR}{dt} - \frac{dr}{dt} = u - u = 0 \qquad \text{(By (3c))}$$

$$\Rightarrow R(t) - r(t) = constant$$

This would imply that Equation (3m) should hold true for all times, $t \geq \tau$. That is,

$$R(t) - r(t) = u(t_B - t_A) = u.\Delta t_{AB} \qquad \text{...(3p)}$$

This would satisfy the characteristic property of a hyperbola being the locus of the point whose difference in the distances from two fixed points (foci) is a constant. So this would mean that the locus of the point of intersections of two circular wavefronts emanating from sources A and B takes the shape of a hyperbola, since the differences in their instantaneous radii has been shown to be always constant. Therefore, $V\left(\frac{u\Delta t_{AB}}{2}, 0\right)$ will be the co-ordinate of the Vertex of one branch of a hyperbola generated when source A is stimulated before source B. The Vertex of the complementary branch of the hyperbola which is generated when source B is stimulated before source A has its vertex at the co-ordinate $V'\left(-\frac{u\Delta t_{BA}}{2}, 0\right)$, since $\Delta t_{AB} = t_B - t_A = -(t_A - t_B) = -\Delta t_{BA}$.

The general equation of a hyperbola with center at origin and transverse axis along the X-axis is:

$$\frac{x^2}{C^2} - \frac{y^2}{D^2} = 1 \qquad \text{...(3q)}$$

Where C and D are the semi-lengths of the transverse and conjugate axes respectively. The value of the constant C is already known to us from (3o) since it represents the distance of the vertex of the hyperbola from the origin. That is,

$$C = \frac{u\Delta t_{AB}}{2} \qquad \text{...(3r)}$$

However, the value of the constant D is yet to be determined. Once D is found and put into (3q), we would have arrived at the required equation of the hyperbola. (Note that the sources $A(-a,0)$ and $B(a,0)$ lie at the foci of the hyperbola).

Differentiating Equation (3q) with respect to time,

$$\frac{1}{C^2}2x\frac{dx}{dt} - \frac{1}{D^2}2y\frac{dy}{dt} = 0$$

The above equation should hold true for all times $t \geq \tau > t_B > t_A$. This would mean that for $t = \tau$,

$$\frac{1}{C^2}.2x\frac{dx}{dt}_{t=\tau} - \frac{1}{D^2}.2y\frac{dy}{dt}_{t=\tau} = 0 \qquad \text{...(3s)}$$

From Equations (3i), (3l) and (3m),

$$2x\frac{dx}{dt}_{t=\tau} = \frac{4u(R(\tau)+r(\tau))(R(\tau)-r(\tau))^2}{16a^2} = 4u.\,2a.\frac{(u\Delta t_{AB})^2}{16a^2} = \frac{u^3(\Delta t_{AB})^2}{2a} \qquad \text{...(3t)}$$

From Equations (27), (3l) and (3m),

$$2y.\frac{dy}{dt}_{t=\tau} = -\frac{4u(R(\tau)+r(\tau))\left((R(\tau)-r(\tau))^2 - 4a^2\right)}{16a^2} = -4u.\frac{2a\left((u.\Delta t_{AB})^2 - 4a^2\right)}{16a^2} = -\frac{u\left((u.\Delta t_{AB})^2 - 4a^2\right)}{2a} \qquad \text{...(3u)}$$

Substituting (3t), (3u) and (3r) in Equation (3s),

$$\frac{1}{(\frac{u\Delta t_{AB}}{2})^2}\frac{u^3(\Delta t_{AB})^2}{2a} - \frac{1}{D^2}\left(-\frac{u\left((u.\Delta t_{AB})^2 - 4a^2\right)}{2a}\right) = 0$$

On algebraic simplification of the above, we get:

$$D^2 = a^2 - \frac{u^2(\Delta t_{AB})^2}{4} = a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2 = a^2 - C^2 \qquad \text{...(3v)} \quad \text{(By(3r))}$$

Substituting (3v) and (3r) in (3q), we finally arrive at,

$$\boxed{\frac{x^2}{\left(\frac{u\Delta t_{AB}}{2}\right)^2} - \frac{y^2}{a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2} = 1}$$

This is the analytical equation of the hyperbola representing the locus of all the points of intersection between two circular wavefronts emanating from sources A and B when stimulated at times t_A and t_B respectively ($t_A < t_B$) .

It is expressed in terms of the Inter-Pulse Interval Δt_{AB}, the speed of propagation of the circular wavefront u and the position of the sources $(\pm a, 0)$ with respect to the origin, lying midway between the sources.

Remarks:

A. On the Hyperbola Equation

- The vertices of the hyperbola lie at the co-ordinate points $V\left(\frac{u\Delta t_{AB}}{2},0\right)$ and $V'\left(-\frac{u\Delta t_{BA}}{2},0\right)$
- The foci of the hyperbola lies at the co-ordinate points $A(-a,0)$ and $B(a,0)$
- The center of the hyperbola lies at the origin $O(0,0)$

B. In the reverse situation, where source B is stimulated before source A (i.e. $t_B < t_A$), an identical equation can be derived with the difference that Δt_{AB} is replaced by Δt_{BA}. Note that $\Delta t_{BA} = t_A - t_B = -(t_B - t_A) = -\Delta t_{AB}$.

$$\frac{x^2}{\left(\frac{u\Delta t_{BA}}{2}\right)^2} - \frac{y^2}{a^2 - \left(\frac{u\Delta t_{BA}}{2}\right)^2} = 1$$

C. When the time interval spanning the stimulation of either sources A and B in succession of each other in any order is brought to zero, that is as $\Delta t_{AB} \to 0$ or as $\Delta t_{BA} \to 0$, both vertices approach the origin $O(0,0)$ and the branches gradually straighten out to coincide with the Y-axis, whose equation is $x = 0$. To illustrate this, put $\Delta t_{AB} = 0$ in the hyperbola equation:

$$\frac{x^2}{0^2} - \frac{y^2}{a^2} = 1 \;\Rightarrow\; x^2 = 0^2.\left(1 + \frac{y^2}{a^2}\right) = 0 \;\Rightarrow x = 0$$

D. Eccentricity of the Hyperbola

We found that that the expressions for the squared lengths of semi-transverse axis and semi-conjugate axis to be $C^2 = \left(\frac{u\Delta t_{AB}}{2}\right)^2$ and $D^2 = a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2$ respectively. The eccentricity e of a hyperbola is related to these quantities by the expression:

$$D^2 = C^2(e^2 - 1)$$

$$\Rightarrow \qquad e = \sqrt{1 + \frac{D^2}{C^2}}$$

$$\Rightarrow \qquad e = \sqrt{1 + \frac{a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2}{\left(\frac{u\Delta t_{AB}}{2}\right)^2}}$$

$$\Rightarrow \qquad e = \frac{2a}{u\Delta t_{AB}} > 1$$

Since the eccentricity of a hyperbola is always greater than unit. This would mean that:

$$u\Delta t_{AB} < 2a$$

$$R(t) - r(t) < 2a \qquad \text{...(3w) (By (3p))}$$

Equation (45) holds true when source A is stimulated before source B. However, for the reverse scenario when source B is stimulated before source A, the following inequality should hold:

$$r(t) - R(t) < 2a \qquad \text{...(3x)}$$

From the inequalities (3w) and (3x), it may be concluded that the Principal Condition to be fulfilled for the generation of hyperbolas is:

$$|R(t) - r(t)| < 2a \qquad \text{...(3y)}$$

E. Ancillary Condition for the Generation of a Hyperbola

On inspection of the hyperbola equation, it is clear that the denominator of the y^2 term be a positive quantity (lest, we get an ellipse instead). That is,

$$a^2 - \left(\frac{u\Delta t_{AB}}{2}\right)^2 > 0 \Rightarrow \left(\frac{u\Delta t_{AB}}{2a}\right)^2 < 1 \Rightarrow \left|\frac{u\Delta t_{AB}}{2a}\right| < 1$$

We found earlier for a Straight Line/Convex/Concave Semicircular Stimulus, the expressions for Δt_{AB} are given by equations (1) and (2). On substituting these into the above inequality, we arrive at the ancillary conditions for the generation of the hyperbola. These are stated in §2.2.2.3 of the main text, and also reiterated here below:

(i) For Straight line Stimulus:

$$\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2\mathrm{a}}\right)\mathrm{Sin}\beta\right| < 1$$

(ii) For Convex/Concave Semicircular Stimulus:

$$\left|\left(\frac{u}{v}\right)\left(\frac{\Omega}{2\mathrm{a}}\right)\mathrm{Cos}\beta\right| < 1$$

Derivation of Equations (5), (6) and (7)

(1) When the Stimulus is an Infinitely long straight line (see Fig. 13 Main Text)

$$AB = BC = CA = \Omega \qquad \text{...(5a)}$$

$$\beta + \theta = 90° \qquad \text{...(5b)}$$

$$\beta - \alpha = 60° \qquad \text{...(5c)}$$

$$AA_1 = AC.Sin\alpha = \Omega.Sin\alpha = \Omega.Sin(\beta - 60°) \qquad \text{...(5d)}$$ (By (5c) and In ΔAA_1C, $Sin\alpha = \frac{AA_1}{AC}$)

$$AA_2 = AB.Cos\theta = \Omega.Cos\theta = \Omega.Cos(90° - \beta) = \Omega.Sin\beta \qquad \text{...(5e)}$$ (By (5b) and In ΔAA_2C, $Cos\theta = \frac{AA_2}{AB}$)

$$\Delta t_{AC} = t_C - t_A = \frac{AA_1}{v} = \Omega.\frac{\mathrm{Sin}(\beta - 60°)}{v} \qquad \text{...(5f)} \qquad \text{(By (5d))}$$

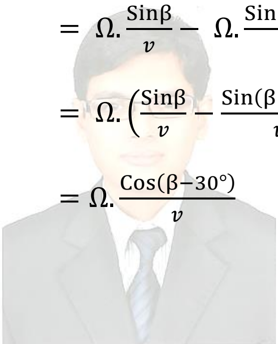
$$\Delta t_{AB} = t_B - t_A = \frac{AA_2}{v} = \Omega.\frac{\mathrm{Sin}\beta}{v} \qquad \text{...(5g)} \qquad \text{(By (5e))}$$

$$\Delta t_{CB} = t_B - t_C = \frac{A_1A_2}{v} = \frac{AA_2 - AA_1}{v}$$

$$= \Omega.\frac{\mathrm{Sin}\beta}{v} - \Omega.\frac{\mathrm{Sin}(\beta - 60°)}{v} \qquad \text{(By (5d)\&(5e))}$$

$$= \Omega.\left(\frac{\mathrm{Sin}\beta}{v} - \frac{\mathrm{Sin}(\beta - 60°)}{v}\right)$$

$$= \Omega.\frac{\mathrm{Cos}(\beta - 30°)}{v} \qquad \text{...(5h)}$$



Summarizing (5f), (5g) and (5h):

$$\Delta t_{AC} = \Omega.\frac{\mathrm{Sin}(\beta - 60°)}{v}$$

$$\Delta t_{AB} = \Omega.\frac{\mathrm{Sin}\beta}{v}$$

$$\Delta t_{CB} = \Omega.\frac{\mathrm{Cos}(\beta - 30°)}{v}$$

(2) When the Stimulus is a Convex Semicircle (see Fig. 14 Main Text)

From Right ΔCC'B,

$$CB^2 = CC'^2 + C'B^2$$

$$(\Omega)^2 = (CC')^2 + (CBCos(60^\circ - \beta))^2$$

$$\Omega^2 = (CC')^2 + \Omega^2 Cos^2(60^\circ - \beta)$$

$$CC' = \Omega.Sin(60^\circ - \beta) \qquad \text{...(6a)}$$

From Right ΔOMP,

$$PM = \rho Sin\theta_1 \qquad \text{...(6b)}$$

From the top figure it is clear that PM is equal and parallel to CC'. So it follows from (6a)&(6b),

$$\rho Sin\theta_1 = \Omega Sin(60^\circ - \beta) \qquad \text{...(6c)}$$

$$\Rightarrow \rho Cos\theta_1 = \sqrt{\rho^2 Cos^2\theta_1}$$

$$= \sqrt{\rho^2(1 - Sin^2\theta_1)}$$

$$= \sqrt{\rho^2 - \rho^2 Sin^2\theta_1}$$

$$= \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} \qquad \text{...(6d) (By (6c))}$$

From Right ΔONR,

$$ON = \rho Cos\theta_2 \qquad \text{...(6e)}$$

$$NR = \rho Sin\theta_2 \qquad \text{...(6f)}$$

From Right ΔRNB,

$$NB = \Omega Cos\beta \qquad \text{...(6g)}$$

$$NR = \Omega Sin\beta \qquad \text{...(6h)}$$

From (6f) and (6h),

$$\rho Sin\theta_2 = \Omega Sin\beta \qquad \text{...(6i)}$$

$$\Rightarrow \rho Cos\theta_2 = \sqrt{\rho^2 Cos^2\theta_2}$$

$$= \sqrt{\rho^2(1 - Sin^2\theta_2)}$$

$$= \sqrt{\rho^2 - \rho^2 Sin^2\theta_2}$$

$$= \sqrt{\rho^2 - \Omega^2 Sin^2\beta} \qquad \text{...(6j) (By (6i))}$$

From (6e) and (6g),

$$OB = ON + NB = \rho Cos\theta_2 + \Omega Cos\beta \qquad ...(6k)$$

From Right ΔPMO,

$$OM = \rho Cos\theta_1 \qquad ...(6l)$$

From Right ΔCC'B,

$$C'B = \Omega Cos(60^\circ - \beta) \qquad ...(6m)$$

Also,

$$MC' = OB - (OM + C'B)$$

$$= \rho Cos\theta_2 + \Omega Cos\beta - (\rho Cos\theta_1 + \Omega Cos(60^\circ - \beta)) \text{ (By (6k), (6l), (6m))}$$

From both figures it is clear that MC' is equal and parallel to PC. So it follows that,

$$PC = \rho Cos\theta_2 + \Omega Cos\beta - (\rho Cos\theta_1 + \Omega Cos(60^\circ - \beta)) \qquad ...(6n)$$

Distance covered between contacting Sensors A and C in succession = PC

Time Interval between contacting Sensors A and C in succession = Δt_{AC}

$$= \frac{PC}{v}$$

$$= \frac{\rho Cos\theta_2 + \Omega Cos\beta - (\rho Cos\theta_1 + \Omega Cos(60^\circ - \beta))}{v}$$

(By (6n))

$$= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - (\sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega Cos(60^\circ - \beta))}{v}$$

(By (6d),(6j))

$$= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{v}$$

Distance covered between contacting Sensors A and B in succession = QB

$$= OB - OQ$$

$$= \rho Cos\theta_2 + \Omega Cos\beta - \rho \qquad \text{(By (6k))}$$

$$= \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho \qquad ...(6o) \qquad \text{(By (6j))}$$

Time Interval between contacting Sensors A and B in succession $= \Delta t_{AB}$

$$= \frac{QB}{v}$$

$$= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho}{v}$$

(By (6o))

From the bottom figure it is clear that when the point P on the convex stimulus coincides with the sensor at point C, the point M will coincide with the point C' because PC and MC' are equal and parallel to each other. Also the point Q on the convex stimulus remains always ahead of the point M by a distance equal to MQ.

Distance covered between contacting sensors C and B in succession = Distance travelled by point Q towards Sensor B, after point P contacts sensor C $= QB$

$$= C'B - MQ$$

$$= CB.Cos(60^{\circ} - \beta) - (OQ - OM)$$

$$= \Omega Cos(60^{\circ} - \beta) - (\rho - \rho Cos\theta_1)$$

$$= \Omega Cos(60^{\circ} - \beta) - \rho + \rho Cos\theta_1$$

$$= \Omega Cos(60^{\circ} - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)}$$

...(6p) (By (6d))

Time Interval between contacting Sensors C and B in succession $= \Delta t_{CB}$

$$= \frac{QB}{v}$$

$$= \frac{C'B - MQ}{v}$$

$$= \frac{\Omega Cos(60^{\circ} - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)}}{v}$$

(By (6p))

Summarizing the ISIs below:

$$\Delta t_{AC} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)} + \Omega(Cos\beta - Cos(60^{\circ} - \beta))}{v}$$

$$\Delta t_{AB} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta - \rho}{v}$$

$$\Delta t_{CB} = \frac{\Omega Cos(60^{\circ} - \beta) - \rho + \sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)}}{v}$$

Where the condition $\rho \geq \frac{\sqrt{3}}{2}\Omega$ must be satisfied in order that the convex stimulus make successive sensors contacts, for the entire angular range, $0^{\circ} \leq \beta \leq 30^{\circ}$.

(3) When Stimulus is a Concave Semicircle (see Figures 15 Main Text)

From Right ΔCC'B,

$$CB^2 = (CC')^2 + (C'B)^2$$

$$\Rightarrow \Omega^2 = (CC')^2 + \left(\Omega Cos(60^{\circ} - \beta)\right)^2$$

$$\Rightarrow CC' = \Omega Sin(60^{\circ} - \beta) \qquad \text{...(7a)}$$

From Right ΔOMP,

$$PM = \rho Sin\theta_1 \qquad \text{...(7b)}$$

From the top figure it is clear that PM is equal and parallel to CC'. So it follows from (7a)&(7b),

$$\rho Sin\theta_1 = \Omega Sin(60^{\circ} - \beta) \qquad \text{...(7c)}$$

$$\Rightarrow \rho Cos\theta_1 = \sqrt{\rho^2 Cos^2\theta_1}$$

$$= \sqrt{\rho^2(1 - Sin^2\theta_1)}$$

$$= \sqrt{\rho^2 - \rho^2 Sin^2\theta_1}$$

$$= \sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)} \qquad \text{...(7d)} \quad \text{(By (7c))}$$

From Right ΔONR,

$$ON = \rho Cos\theta_2 \qquad \text{...(7e)}$$

$$NR = \rho Sin\theta_2 \qquad \text{...(7f)}$$

From Right ΔRNB,

$$NB = \Omega Cos\beta \qquad \text{...(7g)}$$

$$NR = \Omega Sin\beta \qquad \text{...(7h)}$$

From (7f) and (7h),

$$\rho Sin\theta_2 = \Omega Sin\beta \qquad \text{...(7i)}$$

$$\Rightarrow \rho Cos\theta_2 = \sqrt{\rho^2 Cos^2\theta_2}$$

$$= \sqrt{\rho^2(1 - Sin^2\theta_2)}$$

$$= \sqrt{\rho^2 - \rho^2 Sin^2\theta_2}$$

$$= \sqrt{\rho^2 - \Omega^2 Sin^2\beta} \qquad \text{...(7j)} \quad \text{(By (7i))}$$

It can be seen from the bottom figure that,

$$OB = ON - NB = \rho Cos\theta_2 - \Omega Cos\beta \qquad \text{...(7k)} \quad \text{(By (7e)\&(7g))}$$

From Right ΔPMO,

$$OM = \rho Cos\theta_1 \qquad \text{...(7l)}$$

From Right ΔCC'B,

$$C'B = \Omega Cos(60^{\circ} - \beta) \qquad \text{...(7m)}$$

Also,

$$\begin{aligned} MC' &= MO - C'O \\ &= MO - (C'B + BO) \\ &= \rho Cos\theta_1 - (\,\Omega Cos(60^{\circ} - \beta) + \rho Cos\theta_2 - \Omega Cos\beta) \quad \text{(By (7k), (7l), (7m))} \end{aligned}$$

From both figures it is clear that MC' is equal and parallel to PC. So it follows that,

$$PC = \rho Cos\theta_1 - (\,\Omega Cos(60^{\circ} - \beta) + \rho Cos\theta_2 - \Omega Cos\beta) \qquad \text{...(7n)}$$

Distance covered between contacting Sensors A and C in succession = PC

Time Interval between contacting Sensors A and C in succession $= \Delta t_{AC}$

$$= \frac{PC}{v}$$

$$= \frac{\rho Cos\theta_1 - (\,\Omega Cos(60^{\circ} - \beta) + \rho Cos\theta_2 - \Omega Cos\beta)}{v}$$

(By (7n))

$$= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)} - (\,\Omega Cos(60^{\circ} - \beta) + \sqrt{\rho^2 - \Omega^2 Sin^2\beta} - \Omega Cos\beta)}{v}$$

(By (7d),(7j))

$$= \frac{\sqrt{\rho^2 - \Omega^2 Sin^2(60^{\circ} - \beta)} - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60^{\circ} - \beta))}{v}$$

Distance covered between contacting Sensors A and B in succession $=$ QB

$$\begin{aligned} &= QO - BO \\ &= \rho - (\rho Cos\theta_2 - \Omega Cos\beta) \quad \text{(By (7k))} \\ &= \rho - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta \quad \text{...(7o)} \quad \text{(By (7j))} \end{aligned}$$

Time Interval between contacting Sensors A and B in succession = Δt_{AB}

$$= \frac{QB}{v}$$

$$= \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta}{v}$$

(By (7o))

From the bottom figure it is clear that when the point P on the concave stimulus coincides with the sensor at point C, the point M will coincide with the point C' because PC and MC' are equal and parallel to each other. Also the point Q on the concave stimulus remains always behind the point M by a distance equal to MQ.

Distance covered between contacting sensors C and B in succession = Distance travelled by point Q towards Sensor B, after point P contacts sensor C = QB

$$= QM + C'B$$

$$= (QO - MO) + C'B$$

$$= \rho - \rho Cos\theta_1 + \Omega Cos(60^\circ - \beta)$$

$$= \rho - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega Cos(60^\circ - \beta)$$

…(7p) (By (7d))

Time Interval between contacting Sensors C and B in succession = Δt_{CB}

$$= \frac{QB}{v}$$

$$= \frac{QM + C'B}{v}$$

$$= \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega Cos(60^\circ - \beta)}{v}$$

(By (7p))

Summarizing the ISIs below:

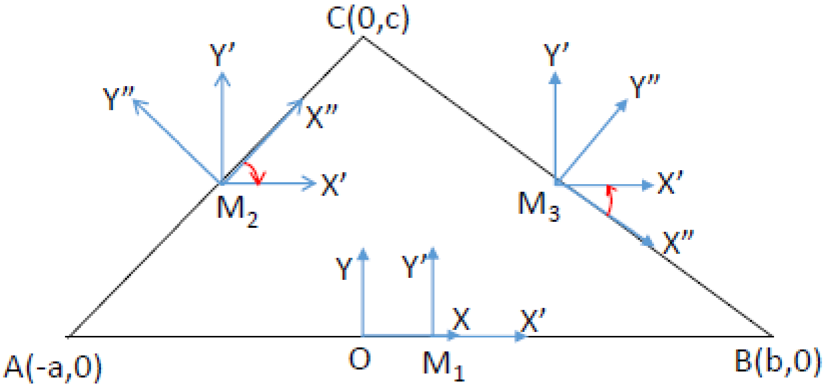
$$\Delta t_{AC} = \frac{\sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega(Cos\beta - Cos(60^\circ - \beta))}{v}$$

$$\Delta t_{AB} = \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2\beta} + \Omega Cos\beta}{v}$$

$$\Delta t_{CB} = \frac{\rho - \sqrt{\rho^2 - \Omega^2 Sin^2(60^\circ - \beta)} + \Omega Cos(60^\circ - \beta)}{v}$$

Where the condition $\rho \geq \frac{\sqrt{3}}{2}\Omega$ must be satisfied in order that the convex stimulus make successive sensors contacts, for the entire angular range, $0^\circ \leq \beta \leq 30^\circ$.

Derivation of Equations (8), (9) and (10)

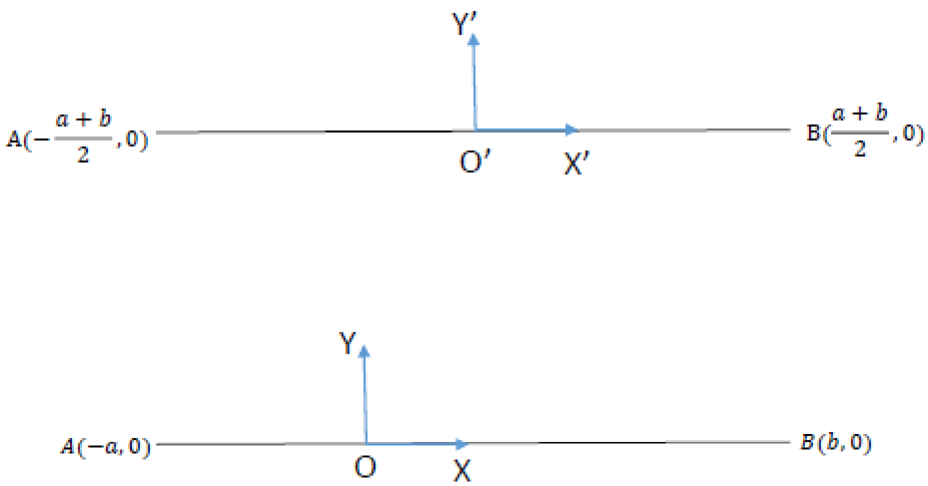


Let us consider the general most case, where three Sensors are placed at the vertices $A(-a, 0), B(b, 0)$ *and* $C(0, c)$ of a scalene triangle, with respect to the origin O of XOY Frame as shown in the figure below (N.B. $a > 0,\ b > 0, c > 0$. Also, M_1, M_2 and M_3 are the mid-points of sides AB, AC and CB respectively of the scalene ΔABC).

The stimulus is made to contact these Sensors located in the Peripheral Field, in temporal succession. Consequently, three circular wavefronts are emanated outwards from the isomorphically placed Sources in the Central Field (each Source acting as a center for a circular wavefront). The geometrical analysis is done in three parts, by taking pair wise source stimulations and finding the equations of the hyperbolas with the sides AB, AC and CB as transvers axes, with respect to the fixed origin $O(0,0)$ of the XY-frame.

(i) Successive Stimulation of Sources $A(-a, 0)$ and $B(b, 0)$

From the figure below, two frames are considered XOY and X'O'Y'. The Origin O' is located midway between the Sensors A and B.



$$|AB| = |AO| + |OB| = a + b, \qquad |AO'| = |O'B| = \frac{|AB|}{2} = \frac{a+b}{2}$$

The coordinate of O' with respect to O is $\left(\frac{b-a}{2}, 0\right)$.

With respect to the X'O'Y' Frame, the equation of the hyperbola generated on stimulating the Sources A and B successively, takes the form of Eq(3).

$$\frac{(x')^2}{(J_{AB})^2} - \frac{(y')^2}{\left(\frac{AB}{2}\right)^2 - (J_{AB})^2} = 1 \qquad \text{...(8a)}$$

Where $J_{AB} = \frac{u.\Delta t_{AB}}{2}$ and $AB = a + b$.

With respect to the XOY Frame, the coordinate transformation equations for translation operation are given by:

$$x = x' + p$$

$$y = y' + q$$

Where (p, q) represents the coordinates of the origin O' w.r.t O. That is, $\left(\frac{b-a}{2}, 0\right)$.

Therefore, we can write,

$$\left.\begin{aligned} x' &= x - p = x - \frac{b-a}{2} \\ y' &= y - q = y - 0 = y \end{aligned}\right\} \qquad \text{...(8b)}$$

Substituting (8b) in (8a),

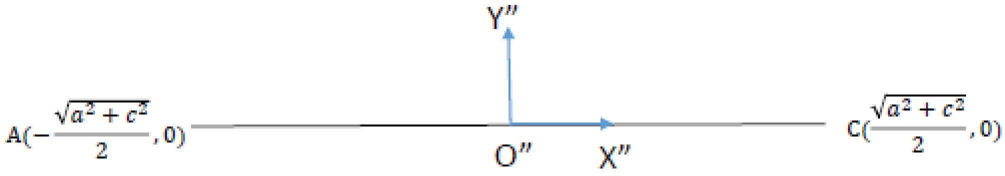
$$\frac{\left(x - \frac{b-a}{2}\right)^2}{J^2{}_{AB}} - \frac{(y)^2}{\left(\frac{a+b}{2}\right)^2 - J^2{}_{AB}} = 1$$

$$\Rightarrow y^2 = \left(\left(\frac{a+b}{2}\right)^2 - J^2{}_{AB}\right)\left(\frac{\left(x - \frac{b-a}{2}\right)^2}{J^2{}_{AB}} - 1\right)$$

$$\boxed{\Rightarrow y = \pm\sqrt{\left(\left(\frac{a+b}{2}\right)^2 - J^2{}_{AB}\right)}\sqrt{\left(\frac{\left(x - \frac{b-a}{2}\right)^2}{J^2{}_{AB}} - 1\right)}}$$

This is the required equation of the hyperbola with transverse axis along the side AB of ΔABC, generated when sources A and B are stimulated successively.

(ii) Successive Stimulation of Sources $A(-a,0)$ and $C(0,c)$



The midpoint of side AC of ΔABC is chosen as the origin O'' of the X"O"Y" Frame. From the previous figure, we can see that in Right ΔAOC, where O is the origin of the XOY Frame, the following relation holds,

$$AC^2 = AO^2 + CO^2 = a^2 + c^2$$

$$\Rightarrow AC = \sqrt{a^2 + c^2}$$

$$\Rightarrow AO" = O"C = \frac{AC}{2} = \frac{\sqrt{a^2 + c^2}}{2}$$

So we can write the respective coordinates of vertices A and C with respect to the X"O"Y" Frame to be $(-\frac{\sqrt{a^2+c^2}}{2}, 0)$ and $(\frac{\sqrt{a^2+c^2}}{2}, 0)$.

With respect to the X"O"Y" Frame, the equation of the hyperbola generated on successively stimulating the Sources A and C, takes the form of Eq(3).

$$\frac{(x'')^2}{(J_{AC})^2} - \frac{(y'')^2}{\left(\frac{AC}{2}\right)^2 - (J_{AC})^2} = 1 \quad \text{...(9a)}$$

Where $J_{AC} = \frac{u.\Delta t_{AC}}{2}$ and $AC = \frac{\sqrt{a^2+c^2}}{2}$.

Consider a clockwise rotated X'O'Y' Frame with respect to the X"O"Y" Frame, where their respective origins O' and O" are coincident and the positive X'-axis is aligned parallel to the positive X-axis of the XOY Frame.

Note that O' (and O") lie at the midpoint of the side AC of ΔABC, and can therefore be ascribed the coordinate $(-\frac{a}{2}, \frac{c}{2})$ with respect to origin O of the XOY Frame.

Then with respect to the X'O'Y' Frame, the coordinate transformation equations for rotation operation are given by:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix} \quad \text{...(9b)}$$

Where $\theta = -\propto$ for clockwise rotation. The magnitude of $\propto$ is equal to the angle ∠CAB.

Applying the Cosine Rule for ΔABC,

$$Cos(\angle CAB) = \frac{AC^2 + AB^2 - CB^2}{2.AC.AB}$$

$$\Rightarrow\ Cos \propto = \frac{(\sqrt{a^2+c^2})^2 + (a+b)^2 - (\sqrt{b^2+c^2})^2}{2.(\sqrt{a^2+c^2}).(a+b)}$$

$$\left.\begin{aligned} \Rightarrow Cos \propto &= \tfrac{a}{\sqrt{a^2+c^2}} \\ \Rightarrow Sin \propto &= \tfrac{c}{\sqrt{a^2+c^2}} \end{aligned}\right\} \quad \text{...(9c)}$$

Substituting (9c) in (9b),

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} Cos(-\propto) & Sin(-\propto) \\ -Sin(-\propto) & Cos(-\propto) \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix}$$

$$= \begin{pmatrix} Cos \propto & -Sin \propto \\ Sin \propto & Cos \propto \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a}{\sqrt{a^2+c^2}} & -\frac{c}{\sqrt{a^2+c^2}} \\ \frac{c}{\sqrt{a^2+c^2}} & \frac{a}{\sqrt{a^2+c^2}} \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix}$$

That is,

$$\left.\begin{aligned} x' &= \tfrac{a}{\sqrt{a^2+c^2}}x" - \tfrac{c}{\sqrt{a^2+c^2}}y" \\ y' &= \tfrac{c}{\sqrt{a^2+c^2}}x" + \tfrac{a}{\sqrt{a^2+c^2}}\ y" \end{aligned}\right\} \quad \text{...(9d)}$$

With respect to the XOY Frame, the coordinate transformation equations for translation operation are given by:

$$\left.\begin{aligned} x &= x' + p \\ y &= y' + q \end{aligned}\right\} \quad \text{...(9e)}$$

Where (p, q) represents the coordinates of the origin O' w.r.t O. That is, $(-\frac{a}{2}, \frac{c}{2})$.

From (9d) and (9e),

$$x = \tfrac{a}{\sqrt{a^2+c^2}}x" - \tfrac{c}{\sqrt{a^2+c^2}}y" - \tfrac{a}{2}$$

$$y = \tfrac{c}{\sqrt{a^2+c^2}}x" + \tfrac{a}{\sqrt{a^2+c^2}}\ y" + \tfrac{c}{2}$$

On rearranging the above terms, we can get,

$$x" = \frac{a}{\sqrt{a^2+c^2}}x + \frac{c}{\sqrt{a^2+c^2}}y + \frac{a^2-c^2}{2\sqrt{a^2+c^2}}$$

$$y" = -\frac{c}{\sqrt{a^2+c^2}}x + \frac{a}{\sqrt{a^2+c^2}}\ y - \frac{ac}{\sqrt{a^2+c^2}}$$

Multiplying both sides of the above two equations by $\sqrt{a^2+c^2}$ and then squaring the result,

$$\left.\begin{aligned}(a^2+c^2)(x'')^2 &= (ax+cy+\tfrac{a^2-c^2}{2})^2 \\ (a^2+c^2)(y'')^2 &= (-cx+ay-ac)^2\end{aligned}\right\} \quad \text{...(9f)}$$

Reiterating equation (9a) below,

$$\frac{(x'')^2}{(J_{AC})^2} - \frac{(y'')^2}{\left(\frac{AC}{2}\right)^2 - (J_{AC})^2} = 1$$

Multiplying both sides by (a^2+c^2) and substituting $AC = \sqrt{a^2+c^2}$,

$$\frac{(a^2+c^2)(x'')^2}{(J_{AC})^2} - \frac{(a^2+c^2)(y'')^2}{\frac{a^2+c^2}{4} - (J_{AC})^2} = (a^2+c^2)$$

Multiplying both sides by $(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$,

$$\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)(a^2+c^2)(x'')^2 - (J_{AC})^2(a^2+c^2)(y'')^2 = (a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$

...(9g)

Substituting (9f) in (9g),

$$\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)\left(ax+cy+\frac{a^2-c^2}{2}\right)^2 - (J_{AC})^2(-cx+ay-ac)^2 = (a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right) \quad \text{...(9h)}$$

After a great deal of algebraic simplification* of (9h), we arrive at the following quadratic expression in y,

$$\begin{aligned}4(c^2-4J^2)y^2 + 4[2acx + c(a^2-c^2) + 4cJ^2]y + 4(a^2-4J^2)x^2 \\ + 4[a(a^2-c^2) - 4aJ^2]x + (a^2-c^2)^2 - 8J^2(a^2+c^2-2J^2) = 0\end{aligned}$$

Which is of the form,

$$Py^2 + Qy + R = 0$$

*see page 27

Where,

$$P = 4(c^2 - 4J^2)$$

$$Q = 4[2acx + c(a^2 - c^2) + 4cJ^2]$$

$$R = 4(a^2 - 4J^2)x^2 + 4[a(a^2 - c^2) - 4aJ^2]x + (a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)$$

N.B. The subscript AC has been dropped from the J-parameter for notational convenience.

The Discriminant of the quadratic equation in y, can be shown after much lengthy algebraic simplification** to be,

$$\Delta = Q^2 - 4PR$$

$$= 64J^2(a^2 + c^2 - 4J^2)(4x^2 + 4ax + a^2 + c^2 - 4J^2)$$

Therefore, the solution to the quadratic equation in y is,

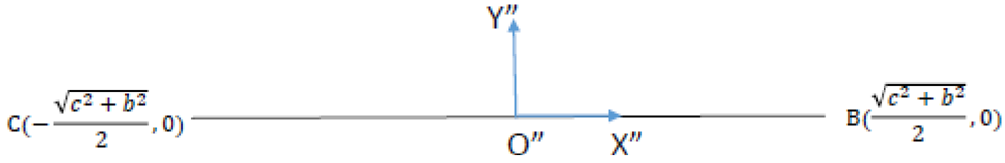
$$y = \frac{-Q \pm \sqrt{\Delta}}{2P}$$

$$y = \frac{-4\left[2acx + c\left(a^2 - c^2\right) + 4cJ^2\right] \pm \sqrt{64J^2(a^2 + c^2 - 4J^2)(4x^2 + 4ax + a^2 + c^2 - 4J^2)}}{8(c^2 - 4J^2)}$$

This is the required equation of the hyperbola with transverse axis along the side AC of ΔABC, generated when sources A and C are stimulated successively.

** see page 32

(iii) Successive Stimulation of Sources $C(0, c)$ and $B(b, 0)$



The midpoint of side CB of ΔABC is chosen as the origin O'' of the X"O"Y" Frame. From the previous figure, we can see that in Right ΔCOB, where O is the origin of the XOY Frame, the following relation holds,

$$CB^2 = CO^2 + OB^2 = c^2 + b^2$$

$$\Rightarrow CB = \sqrt{c^2 + b^2}$$

$$\Rightarrow CO" = O"B = \frac{CB}{2} = \frac{\sqrt{c^2 + b^2}}{2}$$

So we can write the respective coordinates of vertices C and B with respect to the X"O"Y" Frame to be $(-\frac{\sqrt{c^2+b^2}}{2}, 0)$ and $(\frac{\sqrt{c^2+b^2}}{2}, 0)$.

With respect to the X"O"Y" Frame, the equation of the hyperbola generated on successively stimulating the Sources C and B, takes the form of Eq(3).

$$\frac{(x'')^2}{(J_{CB})^2} - \frac{(y'')^2}{\left(\frac{CB}{2}\right)^2 - (J_{CB})^2} = 1 \qquad \text{...(10a)}$$

Where $J_{CB} = \frac{u.\Delta t_{CB}}{2}$ and $CB = \frac{\sqrt{c^2+b^2}}{2}$.

Consider an anti-clockwise rotated X'O'Y' Frame with respect to the X"O"Y" Frame, where their respective origins O' and O" are coincident and the positive X'-axis is aligned parallel to the positive X-axis of the XOY Frame.

Note that O' (and O") lie at the midpoint of the side CB of ΔABC, and can therefore be ascribed the coordinate $(\frac{b}{2}, \frac{c}{2})$ with respect to origin O of the XOY Frame.

Then with respect to the X'O'Y' Frame, the coordinate transformation equations for rotation operation are given by:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix} \qquad \text{...(10b)}$$

Where $\theta = +\beta$ for anti-clockwise rotation. The magnitude of β is equal to the angle ∠CBA.

Applying the Cosine Rule for ΔABC,

$$Cos(\angle \text{CBA}) = \frac{CB^2 + AB^2 - CA^2}{2.CB.AB}$$

$$\Rightarrow\ Cos\beta\ = \frac{(\sqrt{c^2+b^2})^2 + (a+b)^2 - (\sqrt{a^2+c^2})^2}{2.(\sqrt{c^2+b^2}).(a+b)}$$

$$\left.\begin{aligned} \Rightarrow Cos\beta\ &= \frac{b}{\sqrt{c^2+b^2}} \\ \Rightarrow Sin\beta\ &= \frac{c}{\sqrt{c^2+b^2}} \end{aligned}\right\} \quad \text{...(10c)}$$

Substituting (10c) in (10b),

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} Cos\beta & Sin\beta \\ -Sin\beta & Cos\beta \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix}$$

$$= \begin{pmatrix} \frac{b}{\sqrt{c^2+b^2}} & \frac{c}{\sqrt{c^2+b^2}} \\ \frac{-c}{\sqrt{c^2+b^2}} & \frac{b}{\sqrt{c^2+b^2}} \end{pmatrix} \begin{pmatrix} x" \\ y" \end{pmatrix}$$

That is,

$$\left.\begin{aligned} x' &= \frac{b}{\sqrt{c^2+b^2}} x" + \frac{c}{\sqrt{c^2+b^2}} y" \\ y' &= \frac{-c}{\sqrt{c^2+b^2}} x" + \frac{b}{\sqrt{c^2+b^2}} y" \end{aligned}\right\} \quad \text{...(10d)}$$

With respect to the XOY Frame, the coordinate transformation equations for translation operation are given by:

$$\left.\begin{aligned} x &= x' + p \\ y &= y' + q \end{aligned}\right\} \quad \text{...(10e)}$$

Where (p, q) represents the coordinates of the origin O' w.r.t O. That is, $(\frac{b}{2}, \frac{c}{2})$.

From (10d) and (10e),

$$x\ = \frac{b}{\sqrt{c^2+b^2}} x" + \frac{c}{\sqrt{c^2+b^2}} y" + \frac{b}{2}$$

$$y\ = \frac{-c}{\sqrt{c^2+b^2}} x" + \frac{b}{\sqrt{c^2+b^2}} y" + \frac{c}{2}$$

On rearranging the above terms, we can get,

$$x"\ = \frac{b}{\sqrt{c^2+b^2}} x - \frac{c}{\sqrt{c^2+b^2}} y + \frac{c^2-b^2}{2\sqrt{c^2+b^2}}$$

$$y"\ = \frac{c}{\sqrt{c^2+b^2}} x + \frac{b}{\sqrt{c^2+b^2}} y - \frac{cb}{\sqrt{c^2+b^2}}$$

Multiplying both sides of the above two equations by $\sqrt{c^2+b^2}$ and then squaring the result,

$$\left.\begin{aligned}(c^2+b^2)(x")^2 &= (bx - cy + \tfrac{c^2-b^2}{2})^2\\ (c^2+b^2)(y")^2 &= (cx + by - cb)^2\end{aligned}\right\} \quad \text{...(10f)}$$

Reiterating equation (10a) below,

$$\frac{(x'')^2}{(J_{CB})^2} - \frac{(y'')^2}{\left(\frac{CB}{2}\right)^2 - (J_{CB})^2} = 1$$

Multiplying both sides by (c^2+b^2) and substituting $CB = \sqrt{c^2+b^2}$,

$$\frac{(c^2+b^2)(x'')^2}{(J_{CB})^2} - \frac{(c^2+b^2)(y'')^2}{\frac{c^2+b^2}{4} - (J_{CB})^2} = (c^2+b^2)$$

Multiplying both sides by $(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$,

$$\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)(c^2+b^2)(x'')^2 - (J_{CB})^2(c^2+b^2)(y'')^2 = (c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

...(10g)

Substituting (10f) in (10g),

$$\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)\left(bx - cy + \frac{c^2-b^2}{2}\right)^2 - (J_{CB})^2(cx+by-cb)^2 = (c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right) \quad \text{...(10h)}$$

After a great deal of algebraic simplification† of (10h), we arrive at the following quadratic expression in y,

$$\begin{aligned}4(c^2-4J^2)y^2 + 4[-2bcx + c(b^2-c^2) + 4cJ^2]y + 4(b^2-4J^2)x^2\\ + 4[-b(b^2-c^2) + 4bJ^2]x + (b^2-c^2)^2 - 8J^2(b^2+c^2-2J^2) = 0\end{aligned}$$

Which is of the form,

$$Py^2 + Qy + R = 0$$

Where,

$$P = 4(c^2-4J^2)$$

$$Q = 4[-2bcx + c(b^2-c^2) + 4cJ^2]$$

$$R = 4(b^2-4J^2)x^2 + 4[-b(b^2-c^2) + 4bJ^2]x + (b^2-c^2)^2 - 8J^2(b^2+c^2-2J^2)$$

N.B. The subscript CB has been dropped from the J-parameter for notational convenience.

†see page 34

The Discriminant of the quadratic equation in y, can be shown after much lengthy algebraic simplification†† to be,

$$\Delta = Q^2 - 4PR$$

$$= 64J^2(b^2 + c^2 - 4J^2)(4x^2 - 4bx + b^2 + c^2 - 4J^2)$$

Therefore, the solution to the quadratic equation in y is,

$$y = \frac{-Q \pm \sqrt{\Delta}}{2P}$$

$$y = \frac{-4[-2bcx + c(b^2 - c^2) + 4cJ^2] \pm \sqrt{64J^2(b^2 + c^2 - 4J^2)(4x^2 - 4bx + b^2 + c^2 - 4J^2)}}{8(c^2 - 4J^2)}$$

This is the required equation of the hyperbola with transverse axis along the side CB of ΔABC, generated when sources C and B are stimulated successively.

Summarizing Equations (8), (9) and (10), **for a Scalene Triangle**

(i) Equation of Hyperbola generated when Sources A and B are stimulated successively

$$y = \pm\sqrt{\left(\left(\frac{a+b}{2}\right)^2 - J^2{}_{AB}\right)}\sqrt{\left(\frac{\left(x - \frac{b-a}{2}\right)^2}{J^2{}_{AB}} - 1\right)}$$

(ii) Equation of Hyperbola generated when Sources A and C are stimulated successively

$$y = \frac{-4[2acx + c(a^2 - c^2) + 4c{J_{AC}}^2] \pm \sqrt{64{J_{AC}}^2(a^2 + c^2 - 4{J_{AC}}^2)(4x^2 + 4ax + a^2 + c^2 - 4{J_{AC}}^2)}}{8(c^2 - 4{J_{AC}}^2)}$$

(iii) Equation of Hyperbola generated when Sources C and B are stimulated successively

$$y = \frac{-4[-2bcx + c(b^2 - c^2) + 4c{J_{CB}}^2] \pm \sqrt{64{J_{CB}}^2(b^2 + c^2 - 4{J_{CB}}^2)(4x^2 - 4bx + b^2 + c^2 - 4{J_{CB}}^2)}}{8(c^2 - 4{J_{CB}}^2)}$$

††see page 39

For an Equilateral Triangle

The co-ordinates of the vertices of an equilateral ΔABC of side length $2a$, are obtained by placing $b = a$ and $c = \sqrt{3}a$. Hence, the vertex co-ordinates become $A(-a, 0)$, $B(a, 0)$ and $C(0, \sqrt{3}a)$. Substituting these in the equations (8), (9) and (10) for a scalar triangle.

(i) Equation of Hyperbola generated when Sources A and B are stimulated successively

$$y = \pm\sqrt{(a^2 - J^2{}_{AB})}\sqrt{\left(\frac{x^2}{J^2{}_{AB}} - 1\right)}$$

(ii) Equation of Hyperbola generated when Sources A and C are stimulated successively

$$y = \frac{-\sqrt{3}.a(ax - a^2 + 2J_{AC}{}^2) \pm 4J_{AC}\sqrt{(a^2 - J_{AC}{}^2)(x^2 + ax + a^2 - J_{AC}{}^2)}}{(3a^2 - 4J_{AC}{}^2)}$$

(iii) Equation of Hyperbola generated when Sources C and B are stimulated successively

$$y = \frac{\sqrt{3}.a(ax + a^2 - 2J_{CB}{}^2) \pm 4J_{CB}\sqrt{(a^2 - J_{CB}{}^2)(x^2 - ax + a^2 - J_{CB}{}^2)}}{(3a^2 - 4J_{CB}{}^2)}$$

For an Isosceles Right Triangle

The co-ordinates of the vertices of an equilateral ΔABC with hypotenuse $2a$ and adjacent side lengths $\sqrt{2}a$, are obtained by placing $b = a$ and $c = a$. Hence, the vertex co-ordinates become $A(-a,0),\ B(a,0)\ and\ C(0,a)$. Substituting these in the equations (8), (9) and (10) for a scalar triangle.

(i) Equation of Hyperbola generated when Sources A and B are stimulated successively

$$y = \pm\sqrt{(a^2 - J^2{}_{AB})}\sqrt{\left(\frac{x^2}{J^2{}_{AB}} - 1\right)}$$

(ii) Equation of Hyperbola generated when Sources A and C are stimulated successively

$$y = \frac{-a(ax + 2{J_{AC}}^2) \pm \sqrt{4{J_{AC}}^2(a^2 - 2{J_{AC}}^2)(2x^2 + 2ax + a^2 - 2{J_{AC}}^2)}}{(a^2 - 4{J_{AC}}^2)}$$

(iii) Equation of Hyperbola generated when Sources C and B are stimulated successively

$$y = \frac{a(ax - 2{J_{CB}}^2) \pm \sqrt{4\,{J_{CB}}^2(a^2 - 2{J_{CB}}^2)(2x^2 - 2ax + a^2 - 2{J_{CB}}^2)}}{(a^2 - 4{J_{CB}}^2)}$$

Algebraic Simplification of (9h)

Reiterating (9h) below,

$$\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)\left(ax+cy+\frac{a^2-c^2}{2}\right)^2-(J_{AC})^2(-cx+ay-ac)^2=(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)$$

Multiplying both sides by 4,

$$(a^2+c^2-4(J_{AC})^2)\left(ax+cy+\frac{a^2-c^2}{2}\right)^2-4(J_{AC})^2(-cx+ay-ac)^2=4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)$$

...(9h1)

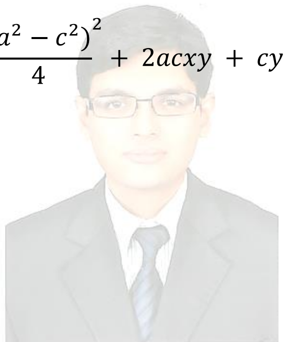
Expanding the first term of LHS,

$$(a^2+c^2-4(J_{AC})^2)\left(ax+cy+\frac{a^2-c^2}{2}\right)^2$$

$$=(a^2+c^2-4(J_{AC})^2)\left(a^2x^2+c^2y^2+\left(\frac{a^2-c^2}{2}\right)^2+2acxy+2cy.\frac{a^2-c^2}{2}+2.\frac{a^2-c^2}{2}.ax\right)$$

$$=(a^2+c^2-4(J_{AC})^2)\left(a^2x^2+c^2y^2+\frac{(a^2-c^2)^2}{4}+2acxy+cy(a^2-c^2)+ax(a^2-c^2)\right)$$

...(9h2)



Expanding the second term of LHS,

$$4(J_{AC})^2(-cx+ay-ac)^2$$

$$=4(J_{AC})^2(c^2x^2+a^2y^2+a^2c^2-2acxy-2a^2cy+2ac^2x)$$

...(9h3)

Subtracting (9h3) from (9h2), we get LHS of (9h1),

$$LHS=(a^2+c^2-4(J_{AC})^2)\left(ax+cy+\frac{a^2-c^2}{2}\right)^2-4(J_{AC})^2(-cx+ay-ac)^2$$

$$=\left[a^2\left(a^2+c^2-4(J_{AC})^2\right)-4c^2(J_{AC})^2\right]x^2+\left[c^2\left(a^2+c^2-4(J_{AC})^2\right)-4a^2(J_{AC})^2\right]y^2+$$
$$\left[2ac\left(a^2+c^2-4(J_{AC})^2\right)+8ac(J_{AC})^2\right]xy+\left[\left(a^2+c^2-4(J_{AC})^2\right)a(a^2-c^2)-8ac^2(J_{AC})^2\right]x+$$
$$\left[\left(a^2+c^2-4(J_{AC})^2\right)c(a^2-c^2)+8a^2c(J_{AC})^2\right]y+\left(a^2+c^2-4(J_{AC})^2\right)\frac{(a^2-c^2)^2}{4}-4a^2c^2(J_{AC})^2$$

The RHS of (9h1) contains only a constant term.

$$RHS=4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)$$

Therefore, after transposing and clubbing the constant term of the RHS with the constant term of the LHS, we may write equation (9h1) as follows,

$$\left[a^2\left(a^2+c^2-4(J_{AC})^2\right)-4c^2(J_{AC})^2\right]x^2+\left[c^2\left(a^2+c^2-4(J_{AC})^2\right)-4a^2(J_{AC})^2\right]y^2+\left[2ac\left(a^2+c^2-4(J_{AC})^2\right)+8ac(J_{AC})^2\right]xy+\left[\left(a^2+c^2-4(J_{AC})^2\right)a(a^2-c^2)-8ac^2(J_{AC})^2\right]x+\left[\left(a^2+c^2-4(J_{AC})^2\right)c(a^2-c^2)+8a^2c(J_{AC})^2\right]y+\left(a^2+c^2-4(J_{AC})^2\right)\frac{(a^2-c^2)^2}{4}-4a^2c^2(J_{AC})^2-4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)=0$$

Which is of the form,

$$Ax^2+By^2+Cxy+Dx+Ey+F=0 \qquad \text{...(9h4)}$$

Where,

$$A=a^2(a^2+c^2-4(J_{AC})^2)-4c^2(J_{AC})^2$$

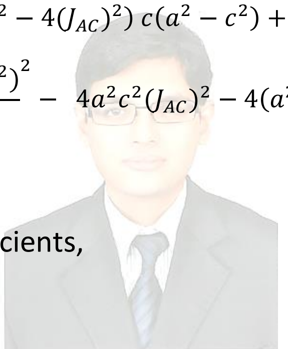
$$B=c^2(a^2+c^2-4(J_{AC})^2)-4a^2(J_{AC})^2$$

$$C=2ac(a^2+c^2-4(J_{AC})^2)+8ac(J_{AC})^2$$

$$D=(a^2+c^2-4(J_{AC})^2)a(a^2-c^2)-8ac^2(J_{AC})^2$$

$$E=(a^2+c^2-4(J_{AC})^2)\,c(a^2-c^2)+8a^2c(J_{AC})^2$$

$$F=(a^2+c^2-4(J_{AC})^2)\frac{(a^2-c^2)^2}{4}-4a^2c^2(J_{AC})^2-4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4}-(J_{AC})^2\right)$$



Simplifying each of the above coefficients,

$$\begin{aligned}A&=a^2(a^2+c^2-4(J_{AC})^2)-4c^2(J_{AC})^2\\&=a^2(a^2+c^2)-4a^2(J_{AC})^2-4c^2(J_{AC})^2\\&=a^2(a^2+c^2)-4(J_{AC})^2(a^2+c^2)\\&=(a^2+c^2)(a^2-4(J_{AC})^2)\end{aligned}$$

$$\begin{aligned}B&=c^2(a^2+c^2-4(J_{AC})^2)-4a^2(J_{AC})^2\\&=c^2(a^2+c^2)-4c^2(J_{AC})^2-4a^2(J_{AC})^2\\&=c^2(a^2+c^2)-4(J_{AC})^2(c^2+a^2)\\&=(a^2+c^2)(c^2-4(J_{AC})^2)\end{aligned}$$

$$\begin{aligned}C&=2ac(a^2+c^2-4(J_{AC})^2)+8ac(J_{AC})^2\\&=2ac(a^2+c^2)\end{aligned}$$

$$\begin{aligned}D&=(a^2+c^2-4(J_{AC})^2)a(a^2-c^2)-8ac^2(J_{AC})^2\\&=(a^2+c^2)a(a^2-c^2)-4(J_{AC})^2a(a^2-c^2)-8ac^2(J_{AC})^2\end{aligned}$$

$$= (a^2+c^2)a(a^2-c^2) - 4a(J_{AC})^2(a^2-c^2+\ 2c^2)$$
$$= (a^2+c^2)a(a^2-c^2) - 4a(J_{AC})^2(a^2+c^2)$$
$$= (a^2+c^2)(a(a^2-c^2) - 4a(J_{AC})^2)$$

$$E = (a^2+c^2-4(J_{AC})^2)\,c(a^2-c^2) + 8a^2c(J_{AC})^2$$
$$= (a^2+c^2)c(a^2-c^2) - 4(J_{AC})^2c(a^2-c^2) + 8a^2c(J_{AC})^2$$
$$= (a^2+c^2)c(a^2-c^2) - 4c(J_{AC})^2(a^2-c^2-\ 2a^2)$$
$$= (a^2+c^2)c(a^2-c^2) + 4c(J_{AC})^2(a^2+c^2)$$
$$= (a^2+c^2)(c(a^2-c^2) + 4c(J_{AC})^2)$$

$$F = (a^2+c^2-4(J_{AC})^2)\frac{(a^2-c^2)^2}{4} - 4a^2c^2(J_{AC})^2 - 4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$
$$= (a^2+c^2)\frac{(a^2-c^2)^2}{4} - 4(J_{AC})^2\frac{(a^2-c^2)^2}{4} - 4a^2c^2(J_{AC})^2 - 4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$
$$= (a^2+c^2)\frac{(a^2-c^2)^2}{4} - (J_{AC})^2\ (a^2-c^2)^2 - 4a^2c^2(J_{AC})^2 - 4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$
$$= (a^2+c^2)\frac{(a^2-c^2)^2}{4} - (J_{AC})^2[(a^2-c^2)^2 + 4a^2c^2] - 4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$
$$= (a^2+c^2)\frac{(a^2-c^2)^2}{4} - (J_{AC})^2(a^2+c^2)^2 - 4(a^2+c^2)(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)$$
$$= (a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4} - (J_{AC})^2(a^2+c^2) - 4(J_{AC})^2\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)\right\}$$
$$= (a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4} - (J_{AC})^2\left[(a^2+c^2) + 4\left(\frac{a^2+c^2}{4} - (J_{AC})^2\right)\right]\right\}$$
$$= (a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4} - (J_{AC})^2[2(a^2+c^2) - 4(J_{AC})^2]\right\}$$
$$= (a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4} - 2(J_{AC})^2(a^2+c^2-2(J_{AC})^2)\right\}$$

Summary of Coefficients,

$$A = (a^2+c^2)(a^2-4(J_{AC})^2)$$
$$B = (a^2+c^2)(c^2-4(J_{AC})^2)$$
$$C = 2ac(a^2+c^2)$$
$$D = (a^2+c^2)(a(a^2-c^2) - 4a(J_{AC})^2)$$
$$E = (a^2+c^2)(c(a^2-c^2) + 4c(J_{AC})^2)$$
$$F = (a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4} - 2(J_{AC})^2(a^2+c^2-2(J_{AC})^2)\right\}$$

Substituting above coefficients in (9h4),

$(a^2+c^2)(a^2-4(J_{AC})^2)x^2+(a^2+c^2)(c^2-4(J_{AC})^2)y^2+\ 2ac(a^2+c^2)xy+(a^2+c^2)(a(a^2-c^2)-4a(J_{AC})^2)x+(a^2+c^2)(c(a^2-c^2)+4c(J_{AC})^2)y+(a^2+c^2)\left\{\frac{(a^2-c^2)^2}{4}-2(J_{AC})^2(a^2+c^2-2(J_{AC})^2)\right\}=\ 0$

Dividing both sides by (a^2+c^2),

$(a^2-4(J_{AC})^2)x^2+(c^2-4(J_{AC})^2)y^2+\ 2acxy+(a(a^2-c^2)-4a(J_{AC})^2)x+(c(a^2-c^2)+4c(J_{AC})^2)y+\left\{\frac{(a^2-c^2)^2}{4}-2(J_{AC})^2(a^2+c^2-2(J_{AC})^2)\right\}=\ 0$

Multiplying both sides by 4,

$4(a^2-4(J_{AC})^2)x^2+4(c^2-4(J_{AC})^2)y^2+\ 8acxy+4(a(a^2-c^2)-4a(J_{AC})^2)x+4(c(a^2-c^2)+4c(J_{AC})^2)y+(a^2-c^2)^2-8(J_{AC})^2(a^2+c^2-2(J_{AC})^2)=\ 0$

Rearranging terms to form a quadratic expression in y,

$$4\left(c^2-4{J_{AC}}^2\right)y^2+4\left[2acx+c(a^2-c^2)+4c{J_{AC}}^2\right]y+4\left(a^2-4{J_{AC}}^2\right)x^2+4\left[a(a^2-c^2)-4a{J_{AC}}^2\right]x+(a^2-c^2)^2-8{J_{AC}}^2\left(a^2+c^2-2{J_{AC}}^2\right)=0$$

…(9h)

This is the required quadratic equation (9h) in the main text, which is of the form,

$$Py^2+Qy+R=0$$

Where,

$$P=4(c^2-4J^2)$$

$$Q=4[2acx+c(a^2-c^2)+4cJ^2]$$

$$R=4(a^2-4J^2)x^2+4[a(a^2-c^2)-4aJ^2]x+(a^2-c^2)^2-8J^2(a^2+c^2-2J^2)$$

Its discriminant is given by,

$$\Delta=\ Q^2-4PR$$

Evaluating Δ term-wise,

$Q^2=16(2acx+c(a^2-c^2)+4cJ^2)^2$

$=16(4a^2c^2x^2+c^2(a^2-c^2)^2+16c^2J^4+4ac^2x(a^2-c^2)+8c^2J^2(a^2-c^2)+16ac^2xJ^2)$

$= 64a^2c^2x^2 + 16c^2(a^2 - c^2)^2 + 256c^2J^4 + 64ac^2x(a^2 - c^2) + 128c^2J^2(a^2 - c^2) + 256ac^2xJ^2$

$= 64a^2c^2x^2 + 64ac^2x(a^2 - c^2) + 256ac^2xJ^2 + 16c^2(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2)$

$= 64a^2c^2x^2 + [64ac^2(a^2 - c^2) + 256ac^2J^2]x + 16c^2(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2)$

$P.R = \{4(c^2 - 4J^2)\}\{4(a^2 - 4J^2)x^2 + 4[a(a^2 - c^2) - 4aJ^2]x + (a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)\}$

$= 16(c^2 - 4J^2)(a^2 - 4J^2)x^2 + 16(c^2 - 4J^2)[a(a^2 - c^2) - 4aJ^2]x + 4(c^2 - 4J^2)\{(a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)\}$

Multiplying both sides by 4,

$4P.R = 64(c^2 - 4J^2)(a^2 - 4J^2)x^2 + 64(c^2 - 4J^2)[a(a^2 - c^2) - 4aJ^2]x + 16(c^2 - 4J^2)\{(a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)\}$

Substituting Q^2 and $4PR$ into the expression for Δ,

$\Delta = Q^2 - 4PR$

$= \{64a^2c^2 - 64(c^2 - 4J^2)(a^2 - 4J^2)\}x^2 + \{64ac^2(a^2 - c^2) + 256ac^2J^2 - 64(c^2 - 4J^2)[a(a^2 - c^2) - 4aJ^2]\}x + 16c^2(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2) - 16(c^2 - 4J^2)\{(a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)\}$

Coefficient of x^2 in Δ,

$64a^2c^2 - 64(c^2 - 4J^2)(a^2 - 4J^2)$
$= 64a^2c^2 - \{64a^2c^2 - 256c^2J^2 - 256a^2J^2 + 1024J^4\}$

$= 256c^2J^2 + 256a^2J^2 - 1024J^4$

$= 256J^2(c^2 + a^2 - 4J^2)$

Coefficient of x in Δ,
$64ac^2(a^2 - c^2) + 256ac^2J^2 - 64(c^2 - 4J^2)[a(a^2 - c^2) - 4aJ^2]$

$= 64ac^2(a^2 - c^2) + 256ac^2J^2 - 64a(c^2 - 4J^2)(a^2 - c^2) + 256aJ^2(c^2 - 4J^2)$

$= 64ac^2(a^2 - c^2) + 256ac^2J^2 + 256aJ^2(c^2 - 4J^2) - 64a(c^2 - 4J^2)(a^2 - c^2)$

$= 64ac^2(a^2 - c^2) + 256aJ^2(c^2 + c^2 - 4J^2) - 64a(c^2 - 4J^2)(a^2 - c^2)$

$= 64ac^2(a^2 - c^2) + 512aJ^2(c^2 - 2J^2) - 64a(c^2 - 4J^2)(a^2 - c^2)$

$= 64ac^2(a^2 - c^2) - 64a(c^2 - 4J^2)(a^2 - c^2) + 512aJ^2(c^2 - 2J^2)$

$= 64a(a^2 - c^2)(c^2 - c^2 + 4J^2) + 512aJ^2(c^2 - 2J^2)$

$= 256aJ^2(a^2 - c^2) + 512aJ^2(c^2 - 2J^2)$

$= 256aJ^2(a^2 - c^2 + 2(c^2 - 2J^2))$
$= 256aJ^2(a^2 + c^2 - 4J^2)$

Coefficient of x^0 in Δ,

$16c^2(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2) - 16(c^2 - 4J^2)\{(a^2 - c^2)^2 - 8J^2(a^2 + c^2 - 2J^2)\}$

$= 16c^2(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2) - 16(c^2 - 4J^2)(a^2 - c^2)^2 + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 16c^2(a^2 - c^2)^2 - 16(c^2 - 4J^2)(a^2 - c^2)^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2) + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 16(a^2 - c^2)^2(c^2 - c^2 + 4J^2) + 256c^2J^4 + 128c^2J^2(a^2 - c^2) + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 64(a^2 - c^2)^2J^2 + 256c^2J^4 + 128c^2J^2(a^2 - c^2) + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 64(a^2 - c^2)^2J^2 + 128c^2J^2(a^2 - c^2) + 256c^2J^4 + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 64J^2(a^2 - c^2)(a^2 - c^2 + 2c^2) + 256c^2J^4 + 128J^2(c^2 - 4J^2)(a^2 + c^2 - 2J^2)$

$= 64J^2(a^2 - c^2)(a^2 + c^2) + 256c^2J^4 + 128J^2(c^2 - 4J^2)(a^2 + c^2) - 256J^4(c^2 - 4J^2)$

$= 64J^2(a^2 - c^2)(a^2 + c^2) + 128J^2(c^2 - 4J^2)(a^2 + c^2) + 256c^2J^4 - 256J^4(c^2 - 4J^2)$

$= 64J^2(a^2 + c^2)\left(a^2 - c^2 + 2(c^2 - 4J^2)\right) + 256J^4\left(c^2 - (c^2 - 4J^2)\right)$

$= 64J^2(a^2 + c^2)(a^2 + c^2 - 8J^2) + 1024J^6$

$= 64J^2\{(a^2 + c^2)(a^2 + c^2 - 8J^2) + 16J^4\}$

$= 64J^2(a^4 + a^2c^2 - 8a^2J^2 + c^2a^2 + c^4 - 8c^2J^2 + 16J^4)$

$= 64J^2(a^4 + c^4 + 16J^4 + 2a^2c^2 - 8c^2J^2 - 8a^2J^2)$
$= 64J^2(a^2 + c^2 - 4J^2)^2$

Therefore, the discriminant Δ becomes,

$\Delta = 256J^2(c^2 + a^2 - 4J^2)x^2 + 256aJ^2(a^2 + c^2 - 4J^2)x + 64J^2(a^2 + c^2 - 4J^2)^2$

$= 64J^2(c^2 + a^2 - 4J^2)(4x^2 + 4ax + a^2 + c^2 - 4J^2)$

The solution of the quadratic equation (9h) in y is,

$$y = \frac{-Q \pm \sqrt{\Delta}}{2P}$$

That is,

$$y = \frac{-4[2acx + c(a^2 - c^2) + 4cJ^2] \pm \sqrt{64J^2(c^2 + a^2 - 4J^2)(4x^2 + 4ax + a^2 + c^2 - 4J^2)}}{8(c^2 - 4J^2)}$$

This is one of the required equations in the box on page 24.

Algebraic Simplification of (10h)

Reiterating (10h) below,

$$\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)\left(bx-cy+\frac{c^2-b^2}{2}\right)^2-(J_{CB})^2(cx+by-cb)^2=(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)$$

Multiplying both sides by 4,

$$(c^2+b^2-4(J_{CB})^2)\left(bx-cy+\frac{c^2-b^2}{2}\right)^2-4(J_{CB})^2(cx+by-cb)^2=4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)$$

…(10h1)

Expanding the first term of LHS of (10h1),

$$(c^2+b^2-4(J_{CB})^2)\left(bx-cy+\frac{c^2-b^2}{2}\right)^2$$

$$=(c^2+b^2-4(J_{CB})^2)\left(b^2x^2+c^2y^2+\left(\frac{c^2-b^2}{2}\right)^2-2bcxy-2cy.\frac{c^2-b^2}{2}+2.\frac{c^2-b^2}{2}.bx\right)$$

$$=(c^2+b^2-4(J_{CB})^2)\left(b^2x^2+c^2y^2+\frac{(c^2-b^2)^2}{4}-2bcxy-cy(c^2-b^2)+bx(c^2-b^2)\right)$$

…(10h2)

Expanding the second term of LHS,

$$4(J_{CB})^2(cx+by-cb)^2$$

$$=4(J_{CB})^2(c^2x^2+b^2y^2+c^2b^2+2cbxy-2b^2cy-2bc^2x)$$

…(10h3)

Subtracting (10h3) from (10h2), we get LHS of (10h1),

$$LHS=(c^2+b^2-4(J_{CB})^2)\left(bx-cy+\frac{c^2-b^2}{2}\right)^2-4(J_{CB})^2(cx+by-cb)^2$$

$$=\left[b^2\left(c^2+b^2-4(J_{CB})^2\right)-4c^2(J_{CB})^2\right]x^2+\left[c^2\left(c^2+b^2-4(J_{CB})^2\right)-4b^2(J_{CB})^2\right]y^2+\left[-2bc\left(c^2+b^2-4(J_{CB})^2\right)-8cb(J_{CB})^2\right]xy+\left[\left(c^2+b^2-4(J_{CB})^2\right)b(c^2-b^2)+8bc^2(J_{CB})^2\right]x+\left[-\left(c^2+b^2-4(J_{CB})^2\right)c(c^2-b^2)+8b^2c(J_{CB})^2\right]y+\left(c^2+b^2-4(J_{CB})^2\right)\frac{(c^2-b^2)^2}{4}-4c^2b^2(J_{CB})^2$$

The RHS of (10h1) contains only a constant term.

$$RHS=4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)$$

Therefore, after transposing and clubbing the constant term of the RHS with the constant term of the LHS, we may write equation (10h1) as follows,

$$\left[b^2\left(c^2+b^2-4(J_{CB})^2\right)-4c^2(J_{CB})^2\right]x^2+\left[c^2\left(c^2+b^2-4(J_{CB})^2\right)-4b^2(J_{CB})^2\right]y^2+\left[-2bc\left(c^2+b^2-4(J_{CB})^2\right)-8cb(J_{CB})^2\right]xy+\left[\left(c^2+b^2-4(J_{CB})^2\right)b(c^2-b^2)+8bc^2(J_{CB})^2\right]x+\left[-\left(c^2+b^2-4(J_{CB})^2\right)c(c^2-b^2)+8b^2c(J_{CB})^2\right]y+\left(c^2+b^2-4(J_{CB})^2\right)\frac{(c^2-b^2)^2}{4}-4c^2b^2(J_{CB})^2-4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)=0$$

Which is of the form,

$$Ax^2+By^2+Cxy+Dx+Ey+F=0 \qquad \text{...(10h4)}$$

Where,

$$A=b^2(c^2+b^2-4(J_{CB})^2)-4c^2(J_{CB})^2$$

$$B=c^2(c^2+b^2-4(J_{CB})^2)-4b^2(J_{CB})^2$$

$$C=-2bc(c^2+b^2-4(J_{CB})^2)-8cb(J_{CB})^2$$

$$D=(c^2+b^2-4(J_{CB})^2)b(c^2-b^2)+8bc^2(J_{CB})^2$$

$$E=-(c^2+b^2-4(J_{CB})^2)\,c(c^2-b^2)+8b^2c(J_{CB})^2$$

$$F=(c^2+b^2-4(J_{CB})^2)\frac{(c^2-b^2)^2}{4}-4c^2b^2(J_{CB})^2-4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4}-(J_{CB})^2\right)$$

Simplifying each of the above coefficients,

$$\begin{aligned}A&=b^2(c^2+b^2-4(J_{CB})^2)-4c^2(J_{CB})^2\\&=b^2(c^2+b^2)-4b^2(J_{CB})^2-4c^2(J_{CB})^2\\&=b^2(c^2+b^2)-4(J_{CB})^2(b^2+c^2)\\&=(c^2+b^2)(b^2-4(J_{CB})^2)\end{aligned}$$

$$\begin{aligned}B&=c^2(c^2+b^2-4(J_{CB})^2)-4b^2(J_{CB})^2\\&=c^2(c^2+b^2)-4c^2(J_{CB})^2-4b^2(J_{CB})^2\\&=c^2(c^2+b^2)-4(J_{CB})^2(c^2+b^2)\\&=(c^2+b^2)(c^2-4(J_{CB})^2)\end{aligned}$$

$$\begin{aligned}C&=-2bc(c^2+b^2-4(J_{CB})^2)-8cb(J_{CB})^2\\&=-2bc(c^2+b^2)\end{aligned}$$

$$\begin{aligned}D&=(c^2+b^2-4(J_{CB})^2)b(c^2-b^2)+8bc^2(J_{CB})^2\\&=(c^2+b^2)b(c^2-b^2)-4(J_{CB})^2b(c^2-b^2)+8bc^2(J_{CB})^2\\&=(c^2+b^2)b(c^2-b^2)-4b(J_{CB})^2(c^2-b^2-2c^2)\\&=(c^2+b^2)b(c^2-b^2)+4b(J_{CB})^2(c^2+b^2)\\&=(c^2+b^2)(b(c^2-b^2)+4b(J_{CB})^2)\end{aligned}$$

$$\begin{aligned} E &= -(c^2+b^2-4(J_{CB})^2)\,c(c^2-b^2) + 8b^2c(J_{CB})^2 \\ &= -(c^2+b^2)c(c^2-b^2) + 4(J_{CB})^2c(c^2-b^2) + 8b^2c(J_{CB})^2 \\ &= -(c^2+b^2)c(c^2-b^2) + 4c(J_{CB})^2(c^2-b^2+2b^2) \\ &= -(c^2+b^2)c(c^2-b^2) + 4c(J_{CB})^2(c^2+b^2) \\ &= -(c^2+b^2)(c(c^2-b^2) - 4c(J_{CB})^2) \end{aligned}$$

$$F = (c^2+b^2-4(J_{CB})^2)\frac{(c^2-b^2)^2}{4} - 4c^2b^2(J_{CB})^2 - 4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

$$= (c^2+b^2)\frac{(c^2-b^2)^2}{4} - 4(J_{CB})^2\frac{(c^2-b^2)^2}{4} - 4c^2b^2(J_{CB})^2 - 4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

$$= (c^2+b^2)\frac{(c^2-b^2)^2}{4} - (J_{CB})^2\ (c^2-b^2)^2 - 4c^2b^2(J_{CB})^2 - 4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

$$= (c^2+b^2)\frac{(c^2-b^2)^2}{4} - (J_{CB})^2[(c^2-b^2)^2 + 4c^2b^2] - 4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

$$= (c^2+b^2)\frac{(c^2-b^2)^2}{4} - (J_{CB})^2(c^2+b^2)^2 - 4(c^2+b^2)(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)$$

$$= (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - (J_{CB})^2(c^2+b^2) - 4(J_{CB})^2\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)\right\}$$

$$= (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - (J_{CB})^2\left[(c^2+b^2) + 4\left(\frac{c^2+b^2}{4} - (J_{CB})^2\right)\right]\right\}$$

$$= (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - (J_{CB})^2[2(c^2+b^2) - 4(J_{CB})^2]\right\}$$

$$= (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - 2(J_{CB})^2(c^2+b^2-2(J_{CB})^2)\right\}$$

Summary of Coefficients,

$$A = (c^2+b^2)(b^2-4(J_{CB})^2)$$

$$B = (c^2+b^2)(c^2-4(J_{CB})^2)$$

$$C = -2bc(c^2+b^2)$$

$$D = (c^2+b^2)(b(c^2-b^2) + 4b(J_{CB})^2)$$

$$E = -(c^2+b^2)(c(c^2-b^2) - 4c(J_{CB})^2)$$

$$F = (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - 2(J_{CB})^2(c^2+b^2-2(J_{CB})^2)\right\}$$

Substituting above coefficients in (10h4),

$$(c^2+b^2)(b^2-4(J_{CB})^2)x^2 + (c^2+b^2)(c^2-4(J_{CB})^2)y^2 - 2bc(c^2+b^2)xy + (c^2+b^2)(b(c^2-b^2) + 4b(J_{CB})^2)x - (c^2+b^2)(c(c^2-b^2) - 4c(J_{CB})^2)y + (c^2+b^2)\left\{\frac{(c^2-b^2)^2}{4} - 2(J_{CB})^2(c^2+b^2-2(J_{CB})^2)\right\} = 0$$

Dividing both sides by (c^2+b^2),

$$(b^2-4(J_{CB})^2)x^2+(c^2-4(J_{CB})^2)y^2-2bcxy+(b(c^2-b^2)+4b(J_{CB})^2)x-(c(c^2-b^2)-4c(J_{CB})^2)y+\left\{\frac{(c^2-b^2)^2}{4}-2(J_{CB})^2(c^2+b^2-2(J_{CB})^2)\right\}=0$$

Multiplying both sides by 4,

$$4(b^2-4(J_{CB})^2)x^2+4(c^2-4(J_{CB})^2)y^2-8bcxy+4(b(c^2-b^2)+4b(J_{CB})^2)x-4(c(c^2-b^2)-4c(J_{CB})^2)y+(c^2-b^2)^2-8(J_{CB})^2(c^2+b^2-2(J_{CB})^2)=0$$

Rearranging terms to form a quadratic expression in y,

$$4(c^2-4(J_{CB})^2)y^2+4[-2bcx-c(c^2-b^2)+4c(J_{CB})^2]y+4\left(b^2-4(J_{CB})^2\right)x^2+4\left(b\left(c^2-b^2\right)+4b(J_{CB})^2\right)x+\left(c^2-b^2\right)^2-8(J_{CB})^2\left(c^2+b^2-2(J_{CB})^2\right)=0$$

…(10h)

This is the required quadratic equation (10h) in the main text, which is of the form,

$$Py^2+Qy+R=0$$

Where,

$$P=4(c^2-4(J_{CB})^2)$$

$$Q=4[-2bcx-c(c^2-b^2)+4c(J_{CB})^2]$$

$$R=4(b^2-4(J_{CB})^2)x^2+4(b(c^2-b^2)+4b(J_{CB})^2)x+(c^2-b^2)^2-8(J_{CB})^2(c^2+b^2-2(J_{CB})^2)$$

Its discriminant is given by,

$$\Delta=Q^2-4PR$$

Evaluating Δ term-wise,

$$Q^2=16\left(-2bcx-c(c^2-b^2)+4c(J_{CB})^2\right)^2$$

$$=16(4b^2c^2x^2+c^2(c^2-b^2)^2+16c^2J^4+4bc^2x(c^2-b^2)-8c^2J^2(c^2-b^2)-16bc^2xJ^2)$$

$$=64b^2c^2x^2+16c^2(c^2-b^2)^2+256c^2J^4+64bc^2x(c^2-b^2)-128c^2J^2(c^2-b^2)-256bc^2xJ^2$$

$$=64b^2c^2x^2+64bc^2x(c^2-b^2)-256bc^2xJ^2+16c^2(c^2-b^2)^2+256c^2J^4-128c^2J^2(c^2-b^2)$$

$$=64b^2c^2x^2+[64bc^2(c^2-b^2)-256bc^2J^2]x+16c^2(c^2-b^2)^2+256c^2J^4-128c^2J^2(c^2-b^2)$$

$$P.R = \{4(c^2 - 4(J_{CB})^2)\}\{4(b^2 - 4(J_{CB})^2)x^2 + 4(b(c^2 - b^2) + 4b(J_{CB})^2)x + (c^2 - b^2)^2 - 8(J_{CB})^2(c^2 + b^2 - 2(J_{CB})^2)\}$$

$$= 16(c^2 - 4J^2)(b^2 - 4J^2)x^2 + 16(c^2 - 4J^2)[b(c^2 - b^2) + 4bJ^2]x + 4(c^2 - 4J^2)\{(c^2 - b^2)^2 - 8J^2(c^2 + b^2 - 2J^2)\}$$

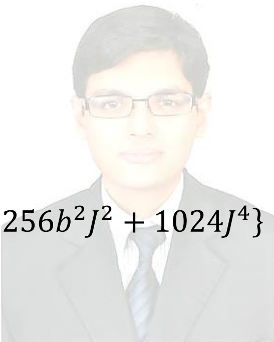
Multiplying both sides by 4,

$$4P.R = 64(c^2 - 4J^2)(b^2 - 4J^2)x^2 + 64(c^2 - 4J^2)[b(c^2 - b^2) + 4bJ^2]x + 16(c^2 - 4J^2)\{(c^2 - b^2)^2 - 8J^2(c^2 + b^2 - 2J^2)\}$$

Substituting Q^2 and $4PR$ into the expression for Δ,

$$\Delta = Q^2 - 4PR$$

$$= \{64b^2c^2 - 64(c^2 - 4J^2)(b^2 - 4J^2)\}x^2 + \{64bc^2(c^2 - b^2) - 256bc^2J^2 - 64(c^2 - 4J^2)[b(c^2 - b^2) + 4bJ^2]\}x + 16c^2(c^2 - b^2)^2 + 256c^2J^4 - 128c^2J^2(c^2 - b^2) - 16(c^2 - 4J^2)\{(c^2 - b^2)^2 - 8J^2(c^2 + b^2 - 2J^2)\}$$



Coefficient of x^2 in Δ,

$$64b^2c^2 - 64(c^2 - 4J^2)(b^2 - 4J^2)$$

$$= 64b^2c^2 - \{64b^2c^2 - 256c^2J^2 - 256b^2J^2 + 1024J^4\}$$

$$= 256c^2J^2 + 256b^2J^2 - 1024J^4$$

$$= 256J^2(c^2 + b^2 - 4J^2)$$

Coefficient of x in Δ,

$$64bc^2(c^2 - b^2) - 256bc^2J^2 - 64(c^2 - 4J^2)[b(c^2 - b^2) + 4bJ^2]$$

$$= 64bc^2(c^2 - b^2) - 256bc^2J^2 - 64b(c^2 - 4J^2)(c^2 - b^2) - 256bJ^2(c^2 - 4J^2)$$

$$= 64bc^2(c^2 - b^2) - 256bc^2J^2 - 256bJ^2(c^2 - 4J^2) - 64b(c^2 - 4J^2)(c^2 - b^2)$$

$$= 64bc^2(c^2 - b^2) - 256bJ^2(c^2 + c^2 - 4J^2) - 64b(c^2 - 4J^2)(c^2 - b^2)$$

$$= 64bc^2(c^2 - b^2) - 512bJ^2(c^2 - 2J^2) - 64b(c^2 - 4J^2)(c^2 - b^2)$$

$$= 64bc^2(c^2 - b^2) - 64b(c^2 - 4J^2)(c^2 - b^2) - 512bJ^2(c^2 - 2J^2)$$

$$= 64b(c^2 - b^2)(c^2 - c^2 + 4J^2) - 512bJ^2(c^2 - 2J^2)$$

$$= 256bJ^2(c^2 - b^2) - 512bJ^2(c^2 - 2J^2)$$

$$= 256bJ^2(c^2 - b^2 - 2(c^2 - 2J^2))$$

$$= -256bJ^2(c^2 + b^2 - 4J^2)$$

Coefficient of x^0 in Δ,

$16c^2(c^2-b^2)^2+256c^2J^4-128c^2J^2(c^2-b^2)-16(c^2-4J^2)\{(c^2-b^2)^2-8J^2(c^2+b^2-2J^2)\}$

$=16c^2(c^2-b^2)^2+256c^2J^4-128c^2J^2(c^2-b^2)-16(c^2-4J^2)(c^2-b^2)^2+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=16c^2(c^2-b^2)^2-16(c^2-4J^2)(c^2-b^2)^2+256c^2J^4-128c^2J^2(c^2-b^2)+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=16(c^2-b^2)^2(c^2-c^2+4J^2)+256c^2J^4-128c^2J^2(c^2-b^2)+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=64(c^2-b^2)^2J^2+256c^2J^4-128c^2J^2(c^2-b^2)+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=64(c^2-b^2)^2J^2-128c^2J^2(c^2-b^2)+256c^2J^4+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=64J^2(c^2-b^2)(c^2-b^2-2c^2)+256c^2J^4+128J^2(c^2-4J^2)(c^2+b^2-2J^2)$

$=-64J^2(c^2-b^2)(c^2+b^2)+256c^2J^4+128J^2(c^2-4J^2)(c^2+b^2)-256J^4(c^2-4J^2)$

$=-64J^2(c^2-b^2)(c^2+b^2)+128J^2(c^2-4J^2)(c^2+b^2)+256c^2J^4-256J^4(c^2-4J^2)$

$=-64J^2(c^2+b^2)\big(c^2-b^2-2(c^2-4J^2)\big)+256J^4\big(c^2-(c^2-4J^2)\big)$

$=64J^2(c^2+b^2)(c^2+b^2-8J^2)+1024J^6$

$=64J^2\{(c^2+b^2)(c^2+b^2-8J^2)+16J^4\}$

$=64J^2(c^4+c^2b^2-8c^2J^2+b^2c^2+b^4-8b^2J^2+16J^4)$

$=64J^2(c^4+b^4+16J^4+2c^2b^2-8c^2J^2-8b^2J^2)$

$=64J^2(c^2+b^2-4J^2)^2$

Therefore, the discriminant Δ becomes,

$\Delta=\ 256J^2(c^2+b^2-4J^2)x^2-256bJ^2(c^2+b^2-4J^2)x+64J^2(c^2+b^2-4J^2)^2$

$\quad=64J^2(c^2+b^2-4J^2)(4x^2-4bx+c^2+b^2-4J^2)$

The solution of the quadratic equation (10h) in y is,

$$y=\frac{-Q\pm\sqrt{\Delta}}{2P}$$

That is,

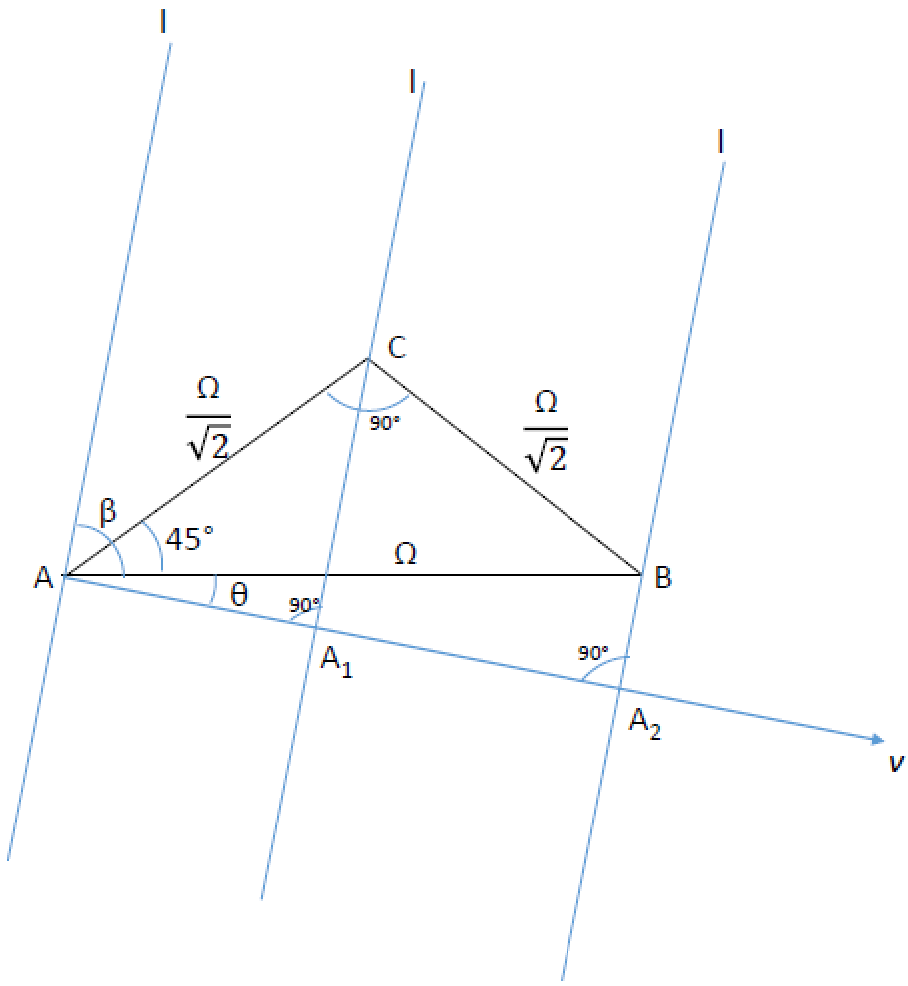
$$y=\frac{-4[-2bcx-c(c^2-b^2)+4cJ^2]\pm\sqrt{64J^2(c^2+b^2-4J^2)(4x^2-4bx+c^2+b^2-4J^2))}}{8(c^2-4J^2)}$$

This is the other required equation in the box on page 24.

Expressions for Inter-Sensor Stimulation Time Interval and J-parameters in the case of a Right Isosceles Triangle Distribution of Sensors

N.B. The Right Isosceles ΔABC is right angled at C and AC = CB

(i) For the Sequence of Sensor Stimulation $A \to C \to B$ and $B \to C \to A$



$$\beta + \theta = 90° \qquad \text{(where } 45° \leq \beta \leq 135°\text{)}$$

$$AB = \Omega$$

$$AC = BC = \frac{\Omega}{\sqrt{2}}$$

From Right ΔAA_1C,

$$AA_1 = AC.Cos(45° + \theta)$$

$$= \frac{\Omega}{\sqrt{2}} Cos(45° + 90° - \beta)$$

$$= \frac{\Omega}{\sqrt{2}} Cos\big(90° - (\beta - 45°)\big)$$

$$= \frac{\Omega}{\sqrt{2}} Sin(\beta - 45°)$$

$$= \Omega . Sin45° . Sin(\beta - 45°)$$

$$= \Omega \left\{ -\frac{1}{2} \left(Cos(45° + \beta - 45°) - Cos(45° - \beta + 45°) \right) \right\}$$

$$= \Omega \left\{ -\frac{1}{2} \left(Cos\beta - Cos(90° - \beta) \right) \right\}$$

$$= \Omega \left\{ -\frac{1}{2} (Cos\beta - Sin\beta) \right\}$$

$$= \frac{\Omega}{2} (Sin\beta - Cos\beta)$$

From Right ΔAA_2B,

$$AA_2 = AB . Cos\theta$$

$$= \Omega Cos(90° - \beta)$$

$$= \Omega Sin\beta$$

$$A_1A_2 = AA_2 - AA_1$$

$$= \Omega Sin\beta - \left\{ \frac{\Omega}{2} (Sin\beta - Cos\beta) \right\}$$

$$= \frac{\Omega}{2} (Sin\beta + Cos\beta)$$

Expressions for ISIs are,

$$\Delta t_{AC} = \frac{AA_1}{v} = \frac{\Omega}{2v} (Sin\beta - Cos\beta)$$

$$\Delta t_{AB} = \frac{AA_2}{v} = \frac{\Omega Sin\beta}{v}$$

$$\Delta t_{CB} = \frac{A_1A_2}{v} = \frac{\Omega (Sin\beta + Cos\beta)}{2v}$$

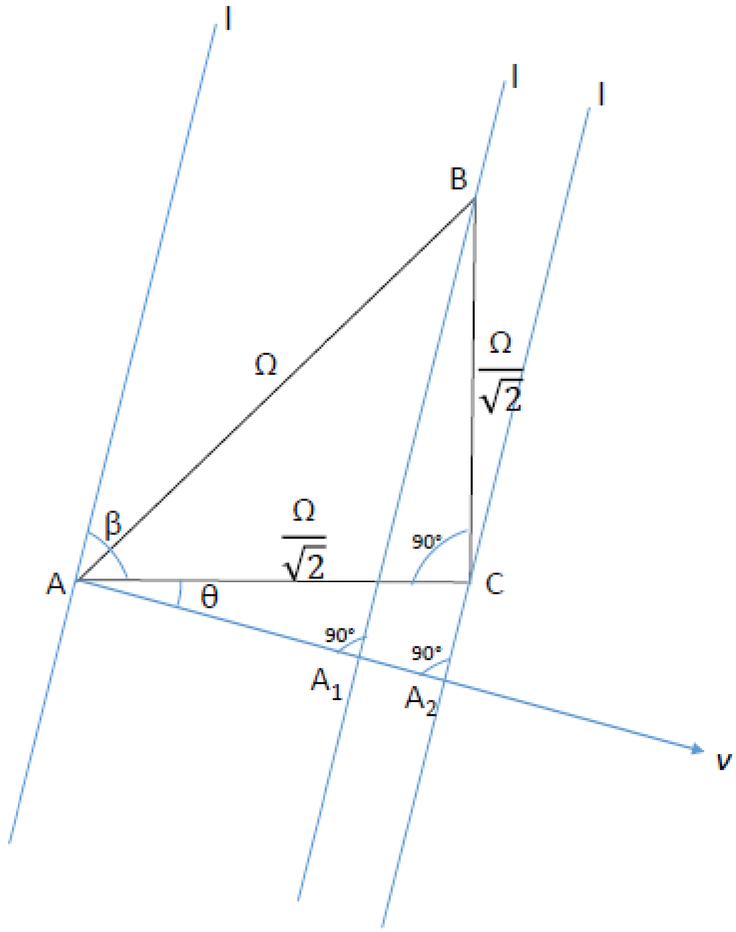
Expressions for J-parameters are,

$$J_{AC} = \frac{u}{2} \Delta t_{AC} = \frac{u\Omega}{4v} (Sin\beta - Cos\beta)$$

$$J_{AB} = \frac{u}{2} \Delta t_{AB} = \frac{u\Omega Sin\beta}{2v}$$

$$J_{CB} = \frac{u}{2} \Delta t_{CB} = \frac{u\Omega (Sin\beta + Cos\beta)}{4v}$$

(ii) For the Sequence of Sensor Stimulation $A \rightarrow B \rightarrow C \ and \ B \rightarrow A \rightarrow C$



$$\beta + \theta = 90° \qquad \text{(where } 45° \leq \beta \leq 90°\text{)}$$

$$AB = \Omega$$

$$AC = BC = \frac{\Omega}{\sqrt{2}}$$

From Right ΔAA₁B,

$$AA_1 = AB.Cos(45° + \theta)$$

$$= \Omega.Cos(45° + 90° - \beta)$$

$$= \Omega.Cos\big(90° - (\beta - 45°)\big)$$

$$= \Omega.Sin(\beta - 45°)$$

From Right ΔAA₂C,

$$AA_2 = AC.Cos\theta$$

$$= \frac{\Omega}{\sqrt{2}}Cos(90° - \beta)$$

$$= \frac{\Omega}{\sqrt{2}} Sin\beta$$

$$= \Omega. Sin45°. Sin\beta$$

$$= \Omega. Sin\beta. Sin45°$$

$$= -\frac{\Omega}{2}\big(Cos(\beta + 45°) - Cos(\beta - 45°)\big)$$

$$= -\frac{\Omega}{2}\big(Cos(90° + \beta - 45°) - Cos(\beta - 45°)\big)$$

$$= -\frac{\Omega}{2}\big(-Sin(\beta - 45°) - Cos(\beta - 45°)\big)$$

$$= \frac{\Omega}{2}\big(Sin(\beta - 45°) + Cos(\beta - 45°)\big)$$

$$A_1A_2 = AA_2 - AA_1$$

$$= \frac{\Omega}{2}\big(Sin(\beta - 45°) + Cos(\beta - 45°)\big) - \Omega. Sin(\beta - 45°)$$

$$= \frac{\Omega}{2}\big(Cos(\beta - 45°) - Sin(\beta - 45°)\big)$$

Expressions for ISIs are,

$$\Delta t_{AB} = \frac{AA_1}{v} = \frac{\Omega. Sin(\beta - 45°)}{v}$$

$$\Delta t_{AC} = \frac{AA_2}{v} = \frac{\Omega\big(Sin(\beta - 45°) + Cos(\beta - 45°)\big)}{2v}$$

$$\Delta t_{BC} = \frac{A_1A_2}{v} = \frac{\Omega\big(Cos(\beta - 45°) - Sin(\beta - 45°)\big)}{2v}$$

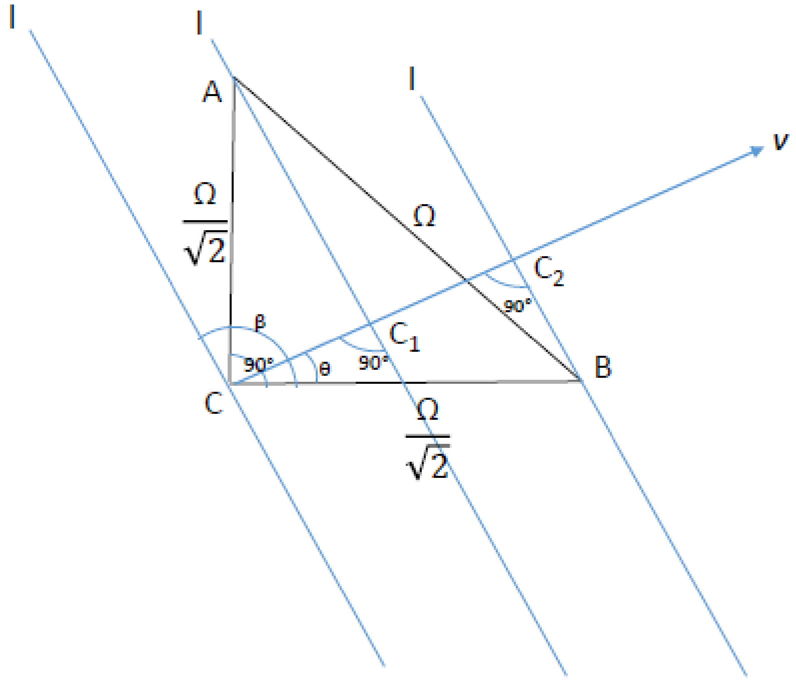
Expressions for J-parameters are,

$$J_{AB} = \frac{u}{2}\Delta t_{AB} = \frac{u\Omega. Sin(\beta - 45°)}{2v}$$

$$J_{AC} = \frac{u}{2}\Delta t_{AC} = \frac{u\Omega\big(Sin(\beta - 45°) + Cos(\beta - 45°)\big)}{4v}$$

$$J_{BC} = \frac{u}{2}\Delta t_{BC} = \frac{u\Omega\big(Cos(\beta - 45°) - Sin(\beta - 45°)\big)}{4v}$$

(iii) For the Sequence of Sensor Stimulation $C \to A \to B \text{ and } C \to B \to A$



$$\beta - \theta = 90° \qquad \text{(where } 90° \leq \beta \leq 135°\text{)}$$

$$AB = \Omega$$

$$AC = BC = \frac{\Omega}{\sqrt{2}}$$

From Right ΔAC_1C,

$$CC_1 = AC.Cos(90° - \theta)$$

$$= \frac{\Omega}{\sqrt{2}}.Sin\theta$$

$$= \frac{\Omega}{\sqrt{2}}.Sin(\beta - 90°)$$

From Right ΔCC_2B,

$$CC_2 = CB.Cos\theta$$

$$= \frac{\Omega}{\sqrt{2}}Cos(\beta - 90°)$$

$$C_1C_2 = CC_2 - CC_1$$

$$= \frac{\Omega}{\sqrt{2}}Cos(\beta - 90°) - \frac{\Omega}{\sqrt{2}}.Sin(\beta - 90°)$$

$$= \frac{\Omega}{\sqrt{2}}\left(Cos(\beta - 90°) - Sin(\beta - 90°)\right)$$

Expressions for ISIs are,

$$\Delta t_{CA} = \frac{CC_1}{v} = \frac{\Omega}{v\sqrt{2}}.Sin(\beta - 90°)$$

$$\Delta t_{CB} = \frac{CC_2}{v} = \frac{\Omega}{v\sqrt{2}}Cos(\beta - 90°)$$

$$\Delta t_{AB} = \frac{C_1C_2}{v} = \frac{\Omega}{v\sqrt{2}}\left(Cos(\beta - 90°) - Sin(\beta - 90°)\right)$$

Expressions for J-parameters are,

$$J_{CA} = \frac{u}{2}\Delta t_{CA} = \frac{u\Omega}{2v\sqrt{2}}.Sin(\beta - 90°)$$

$$J_{CB} = \frac{u}{2}\Delta t_{CB} = \frac{u\Omega}{2v\sqrt{2}}Cos(\beta - 90°)$$

$$J_{AB} = \frac{u}{2}\Delta t_{AB} = \frac{u\Omega}{2v\sqrt{2}}\left(Cos(\beta - 90°) - Sin(\beta - 90°)\right)$$

MATLAB Coding

A. Two Sensor Model

A.1. Mapping Angles of Inclination

A.1.1. For Straight Line Stimulus (see Figure 9 – top, in Main Text)

```
omeg = 1 ;                           % millimeter
a = 0.2 ;                            % millimeter
u = 0.1  ;                           % millimeter/millisec
v = 1 ;                              % millimeter/millisec

beta = linspace(0,pi/2,6)       ;    % radians
J = (u.*omeg.*sin(beta))./(2.*v)   ;

y = linspace(-1,1,1000) ;


hold on
for i = 1:6 ;     % corresponds to beta equal to {0, 18, 36, 54, 72, 90}
degrees
    plot((J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'r');
    plot(-(J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'b');
    disp([J(i),beta(i),(180/pi).*beta(i)])
end

axis([-0.25 0.25 -1 1])

plot(-0.2,0,'r*', 0.2,0,'b*', 0,0,'g*')          % marking out the positions
of the sources A and B

plot(0,linspace(-1,1,1000),'g')                  % corresponds to beta = 0
plot(linspace(-0.2,0.2,1000),0,'k')              % corresponds to line AB

text(-0.2,-0.15,'\itA(-0.2,0)','color','r','fontsize',12) % labeling source
A
text(0.2,-0.15,'\itB(0.2,0)','color','b','fontsize',12)   % labeling source
B
text(-0.009, -0.1,'\itO(0,0)','color','g','fontsize',12 ) % labeling
midpoint O of line AB

% labeling the beta angles corresponding to the hyperbolas
text(0.005,0.9,'\beta = 0','color','k','fontsize',12)
text(0.025,0.6,'\beta = 18','color','k','fontsize',12)
text(0.07,0.6,'\beta = 36','color','k','fontsize',12)
text(0.125,0.7,'\beta = 54','color','k','fontsize',12)
text(0.17,0.75,'\beta = 72','color','k','fontsize',12)
text(0.195,0.7,'\beta = 90','color','k','fontsize',12)
text(-0.047,0.6,'\beta = 18','color','k','fontsize',12)
text(-0.09,0.6,'\beta = 36','color','k','fontsize',12)
text(-0.145,0.7,'\beta = 54','color','k','fontsize',12)
text(-0.19,0.75,'\beta = 72','color','k','fontsize',12)
text(-0.22,0.7,'\beta = 90','color','k','fontsize',12)

% labeling axes
xlabel('\itX-axis (mm)','color','k','fontsize',14)
ylabel('\itY-axis (mm)','color','k','fontsize',14)
title('Mapping Angles of Inclination \beta (degrees) to Hyperbolic
Coincident Detector Spatial Distributions for a Straight Line
Stimulus','color','k','fontsize',14)
```

A.1.2. For Convex/Concave Semicircular Stimulus (see Figure 9 – bottom, in Main Text)

```
omega = 1 ;                             % millimeter
a = 0.2 ;                               % millimeter
u = 0.1  ;                              % millimeter/millisec
v = 1 ;                                 % millimeter/millisec
ro = 2 ;                                % millimeter

beta = linspace(0,pi/2,6)      ;        % radians
J = (u.*omega.*cos(beta))./(2.*v)  ;

y = linspace(-1,1,1000) ;


hold on
for i = 1:6 ;     % corresponds to beta equal to {0, 18, 36, 54, 72, 90}
degrees
    plot((J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'r');
    plot(-(J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'b');
    disp([J(i),beta(i),(180/pi).*beta(i)])
end

axis([-0.25 0.25 -1 1])

plot(-0.2,0,'r*', 0.2,0,'b*', 0,0,'g*')          % marking out the positions
of the sources A & B and origin O

plot(0,linspace(-1,1,1000),'g')                  % corresponds to beta = 90
plot(linspace(-0.2,0.2,1000),0,'k')              % corresponds to line AB

text(-0.2,-0.15,'\itA(-0.2,0)','color','r','fontsize',12) % labeling source
A
text(0.2,-0.15,'\itB(0.2,0)','color','b','fontsize',12)   % labeling source
B
text(-0.009, -0.1,'\itO(0,0)','color','g','fontsize',12 ) % labeling
midpoint O of line AB

% labeling the beta angles corresponding to the hyperbolas
text(0.005,0.9,'\beta = 90','color','k','fontsize',12)
text(0.025,0.6,'\beta = 72','color','k','fontsize',12)
text(0.07,0.6,'\beta = 54','color','k','fontsize',12)
text(0.125,0.7,'\beta = 36','color','k','fontsize',12)
text(0.17,0.75,'\beta = 18','color','k','fontsize',12)
text(0.195,0.7,'\beta = 0','color','k','fontsize',12)
text(-0.047,0.6,'\beta = 72','color','k','fontsize',12)
text(-0.09,0.6,'\beta = 54','color','k','fontsize',12)
text(-0.145,0.7,'\beta = 36','color','k','fontsize',12)
text(-0.19,0.75,'\beta = 18','color','k','fontsize',12)
text(-0.22,0.7,'\beta = 0','color','k','fontsize',12)


% labeling axes
xlabel('\itX-axis (mm)','color','k','fontsize',14)
ylabel('\itY-axis (mm)','color','k','fontsize',14)
title('Mapping Angles of Inclination \beta (degrees) to Hyperbolic
Coincident Detector Spatial Distributions for a Convex/Concave Semicircle
Stimulus','color','k','fontsize',14)
```

A.2. Mapping Velocity

A.2.1. For Straight Line Stimulus (see Figure 10 – top, in Main Text)

```
omega = 1 ;                                  % millimeter
a = 0.2 ;                                    % millimeter
u = 0.1 ;                                    % millimeter/sec
beta = pi/3 ;                                % radians

v = linspace(1, 3, 3)      ;                 % millimeters/sec
J = (u.*omega.*sin(beta))./(2.*v)   ;

y = linspace(-1,1,1000) ;

hold on
for i = 1:3 ;
    plot((J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'r');
    plot(-(J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'b');
    disp(v(i))
end

axis([-0.5 0.5 -1 1])

plot(-0.2,0,'r*', 0.2,0,'b*', 0,0,'g*')         % marking out the positions
of the sources A & B and origin O

plot(linspace(-0.2,0.2,1000),0,'k')             % corresponds to line AB

text(-0.27,0,'\itA(-0.2,0)','color','r','fontsize',12)  % labeling source A
text(0.21,0,'\itB(0.2,0)','color','b','fontsize',12)    % labeling source B
text(-0.015, -0.05,'\itO(0,0)','color','g','fontsize',12 ) % labeling
midpoint O of line AB

% labeling the velocities corresponding to the hyperbolas
text(0.03,0.9,'v = 3','color','k','fontsize',12)
text(-0.13,0.7,'v = 2','color','k','fontsize',12)
text(0.15,0.5,'v = 1','color','k','fontsize',12)
text(-0.06,0.9,'v = 3','color','k','fontsize',12)
text(0.1,0.7,'v = 2','color','k','fontsize',12)
text(-0.18,0.5,'v = 1','color','k','fontsize',12)


% labeling axes
xlabel('\itX-axis (mm)','color','k','fontsize',14)
ylabel('\itY-axis (mm)','color','k','fontsize',14)
title('Mapping Velocity  v (mm/ms) to Hyperbolic Coincident Detector
Spatial Distributions for a Straight Stimulus','color','k','fontsize',14)
```

A.2.2. For Convex/Concave Semicircular Stimulus (see Figure 10 – bottom, in Main Text)

```
omega = 1 ;                             % millimeter
a = 0.2 ;                               % millimeter
u = 0.1 ;                               % millimeter/sec
beta = pi/3 ;                           % radians

v = linspace(1, 3, 3)       ;           % millimeters/sec
J = (u.*omega.*cos(beta))./(2.*v)   ;

y = linspace(-1,1,1000) ;

hold on
for i = 1:3 ;
    plot((J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'r');
    plot(-(J(i)).*sqrt(1 + (y.^2)./(a.^2 - (J(i)).^2)), y, 'b');
    disp(v(i))
end

axis([-0.5 0.5 -1 1])

plot(-0.2,0,'r*', 0.2,0,'b*', 0,0,'g*')         % marking out the positions
of the sources A & B and origin O

plot(linspace(-0.2,0.2,1000),0,'k')             % corresponds to line AB

text(-0.27,0,'\itA(-0.2,0)','color','r','fontsize',14)  % labeling source A
text(0.21,0,'\itB(0.2,0)','color','b','fontsize',14)    % labeling source B
text(-0.015, -0.05,'\itO(0,0)','color','g','fontsize',14 ) % labeling
midpoint O of line AB

% labeling the velocities corresponding to the hyperbolas
text(0.007,0.9,'v = 3','color','k','fontsize',12)
text(-0.08,0.7,'v = 2','color','k','fontsize',12)
text(0.08,0.5,'v = 1','color','k','fontsize',12)
text(-0.035,0.9,'v = 3','color','k','fontsize',12)
text(0.05,0.7,'v = 2','color','k','fontsize',12)
text(-0.11,0.5,'v = 1','color','k','fontsize',12)


% labeling axes
xlabel('\itX-axis (mm)','color','k','fontsize',14)
ylabel('\itY-axis (mm)','color','k','fontsize',14)
title('Mapping Velocity  v (mm/ms) to Hyperbolic Coincident Detector
Spatial Distributions for a Convex/Concave
Stimulus','color','k','fontsize',14)
```

A.3. Mapping ISI (see Figure 11, in Main Text)

```
delt = linspace(0,3,5) ;                % milliseconds
a = 0.2 ;                               % millimeter
u = 0.1 ;                               % millimeter/millisecond
K = (u.*delt)./2 ;

y = linspace(-1,1,1000) ;

hold on
for i = 1:5 ;
    plot(K(i).*sqrt(1 + (y.^2)./(a.^2 - (K(i)).^2)), y, 'r');
    plot(-K(i).*sqrt(1 + (y.^2)./(a.^2 - (K(i)).^2)), y, 'b');
    disp(delt(i))
end

axis([-0.5 0.5 -1 1])

plot(-0.2,0,'r*', 0.2,0,'b*', 0,0,'g*')            % marking out the positions
of the sources A and B

plot(0,linspace(-1,1,1000),'g')                    % corresponds to delt = 0
plot(linspace(-0.2,0.2,1000),0,'k')                % corresponds to line AB

text(-0.27,0,'\itA(-0.2,0)','color','r','fontsize',12)  % labeling source A
text(0.21,0,'\itB(0.2,0)','color','b','fontsize',12)    % labeling source B
text(-0.02,-0.09,'\itO(0,0)','color','g','fontsize',12) % labeling origin O

% labeling the IWIs corresponding to the hyperbolas
text(0.02,0.9,'ISI = 0','color','k','fontsize',12)
text(0.18,0.8,'ISI = 0.75','color','k','fontsize',12)
text(0.28,0.6,'ISI = 1.5','color','k','fontsize',12)
text(0.36,0.42,'ISI = 2.3','color','k','fontsize',12)
text(0.4,0.26,'ISI = 3','color','k','fontsize',12)
text(-0.24,0.8,'ISI = 0.75','color','k','fontsize',12)
text(-0.33,0.6,'ISI = 1.5','color','k','fontsize',12)
text(-0.40,0.42,'ISI = 2.3','color','k','fontsize',12)
text(-0.45,0.26,'ISI = 3','color','k','fontsize',12)

% labeling axes
xlabel('\itX-axis (mm)','color','k','fontsize',12)
ylabel('\itY-axis (mm)','color','k','fontsize',12)
title('Mapping ISI (ms) to Hyperbolic Coincident Detector Spatial
Distributions','color','k','fontsize',12)
```

B. Three Sensor Model – Equilateral Triangle Array of Sensors (or Sources)

B.1. For Straight Line Stimulus

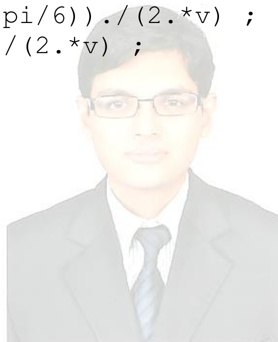
B.1.1. Calculation of J-parameters for different Sequences of Stimulation

```
format long

% Sequence A to C to B

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_AC = (u.*omega.*sin(beta - pi/3))./(2.*v) ;
J_AB = (u.*omega.*sin(beta))./(2.*v) ;
J_CB = (u.*omega.*cos(beta - pi/6))./(2.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_CB)
```

```
% Sequence A to B to C

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_AC = (u.*omega.*sin(beta))./(2.*v) ;
J_AB = (u.*omega.*sin(beta - pi/3))./(2.*v) ;
J_BC = (u.*omega.*cos(beta - pi/6))./(2.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_BC)



% Sequence B to A to C

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_AC = (u.*omega.*cos(beta - pi/6))./(2.*v) ;
J_BA = (u.*omega.*sin(beta - pi/3))./(2.*v) ;
J_BC = (u.*omega.*sin(beta))./(2.*v) ;

disp(J_AC)
disp(J_BA)
disp(J_BC)
```

```
% Sequence B to C to A

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_CA = (u.*omega.*cos(beta - pi/6))./(2.*v) ;
J_BA = (u.*omega.*sin(beta))./(2.*v) ;
J_BC = (u.*omega.*sin(beta - pi/3))./(2.*v) ;

disp(J_CA)
disp(J_BA)
disp(J_BC)
```

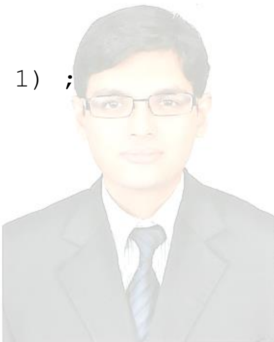
```
% Sequence C to A to B

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_CA = (u.*omega.*sin(beta - pi/3))./(2.*v) ;
J_AB = (u.*omega.*cos(beta - pi/6))./(2.*v) ;
J_CB = (u.*omega.*sin(beta))./(2.*v) ;

disp(J_CA)
disp(J_AB)
disp(J_CB)
```



```
% Sequence C to B to A

omega = 1;
a = 0.2;
u = 0.1;
v = 1;
beta = [pi/3 pi/2 2*pi/3];

J_CA = (u.*omega.*sin(beta))./(2.*v) ;
J_BA = (u.*omega.*cos(beta - pi/6))./(2.*v) ;
J_CB = (u.*omega.*sin(beta - pi/3))./(2.*v) ;

disp(J_CA)
disp(J_BA)
disp(J_CB)
```

B.1.2. Hyperbolas generated for Sequences of Stimulation A to C to B

```
% beta = 60 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =      ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 90 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 = ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

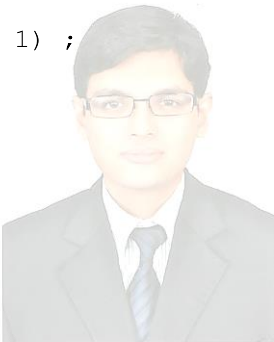
```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 120 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

B.1.3. Hyperbolas generated for Sequences of Stimulation A to B to C

```
% beta = 60 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

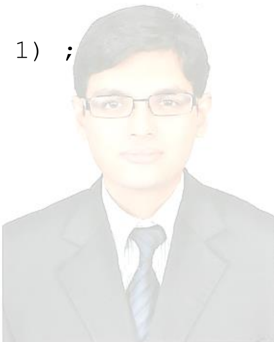


```
% beta = 90 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 120 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

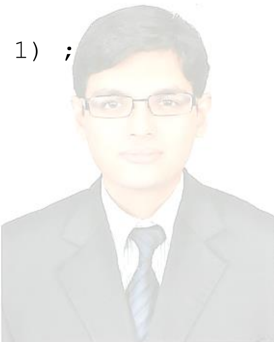
B.1.4. Hyperbolas generated for Sequences of Stimulation B to A to C

```
% beta = 60 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =    ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 90 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

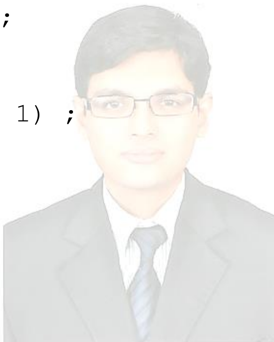


```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 120 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

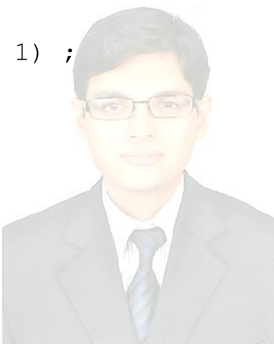
B.1.5. Hyperbolas generated for Sequences of Stimulation B to C to A

```
% beta = 60 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

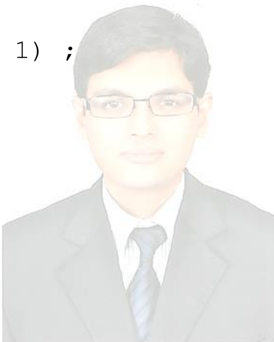


```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 90 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



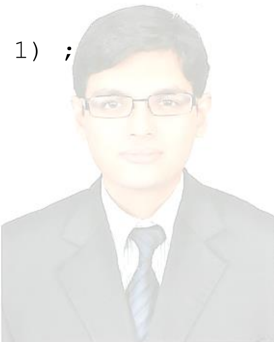
```
% beta = 120 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



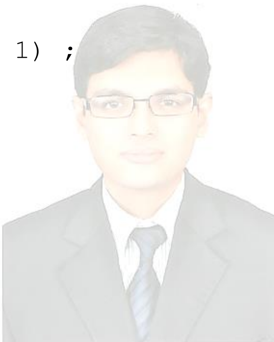
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

B.1.6. Hyperbolas generated for Sequences of Stimulation C to A to B

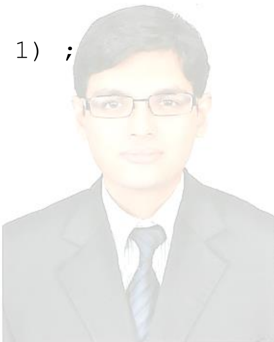
```
% beta = 60 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



```
% beta = 90 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



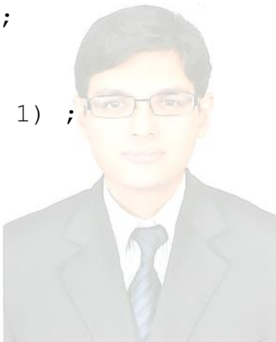
```
% beta = 120 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

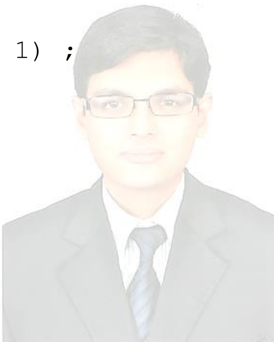
B.1.7. Hyperbolas generated for Sequences of Stimulation C to B to A

```
% beta = 60 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

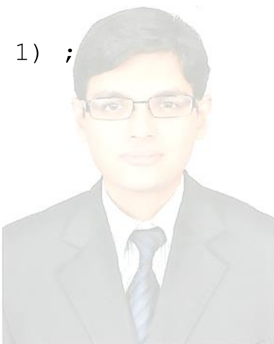


```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 90 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



```
% beta = 120 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



B.2. For Convex Semicircular Stimulus

B.2.1. Calculation of J-parameters for different Sequences of Stimulation

```
format long

% A to C to B

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega (condition ro > (sqrt(3)/2)*omega =
0.8660*omega)

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_AC = u.*(a - b + c)./(2.*v) ;
J_AB = u.*(a + d - ro)./(2.*v) ;
J_CB = u.*(e - ro + b)./(2.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_CB)
```

```
% A to B to C

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_AB = u.*(a - b + c)./(2.*v) ;
J_AC = u.*(a + d - ro)./(2.*v) ;
J_BC = u.*(e - ro + b)./(2.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_BC)
```

```
% B to A to C

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_BA = u.*(a - b + c)./(2.*v) ;
J_BC = u.*(a + d - ro)./(2.*v) ;
J_AC = u.*(e - ro + b)./(2.*v) ;

disp(J_AC)
disp(J_BA)
disp(J_BC)
```

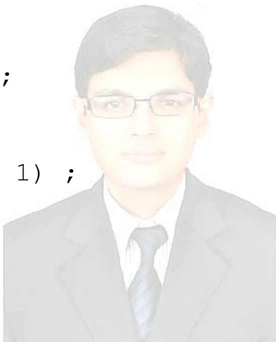
```
% B to C to A

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_BC = u.*(a - b + c)./(2.*v) ;
J_BA = u.*(a + d - ro)./(2.*v) ;
J_CA = u.*(e - ro + b)./(2.*v) ;

disp(J_CA)
disp(J_BA)
disp(J_BC)
```

```
% C to A to B

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_CA = u.*(a - b + c)./(2.*v) ;
J_CB = u.*(a + d - ro)./(2.*v) ;
J_AB = u.*(e - ro + b)./(2.*v) ;

disp(J_CA)
disp(J_AB)
disp(J_CB)
```

```
% C to B to A

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 1 ; %taken to be equal to omega

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_CB = u.*(a - b + c)./(2.*v) ;
J_CA = u.*(a + d - ro)./(2.*v) ;
J_BA = u.*(e - ro + b)./(2.*v) ;

disp(J_CA)
disp(J_BA)
disp(J_CB)
```

B.2.2. Hyperbolas generated for Sequences of Stimulation A to C to B

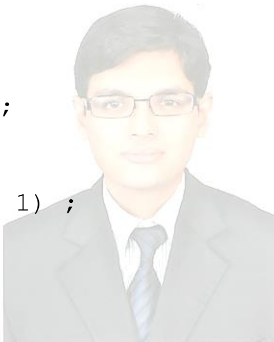
```
% beta = 0 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =     ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

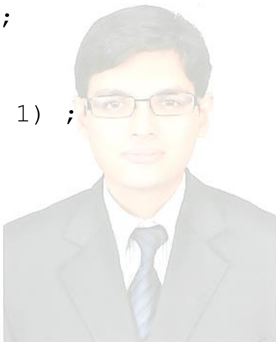
B.2.3. Hyperbolas generated for Sequences of Stimulation A to B to C

```
% beta = 0 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



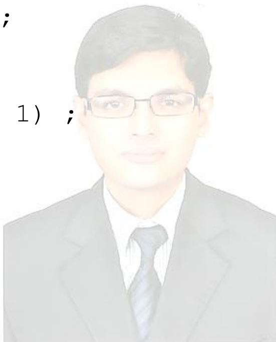
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

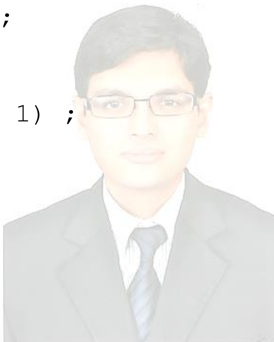
```
% beta = 30 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



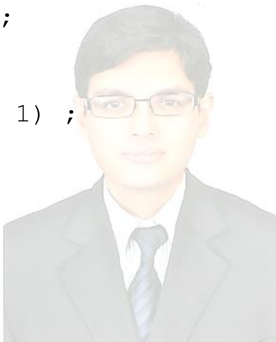
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

B.2.4. Hyperbolas generated for Sequences of Stimulation B to A to C

```
% beta = 0 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

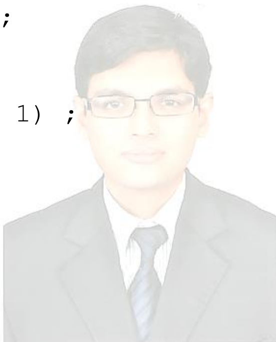


```
% beta = 15 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

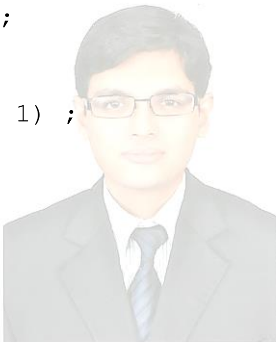
```
% beta = 30 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



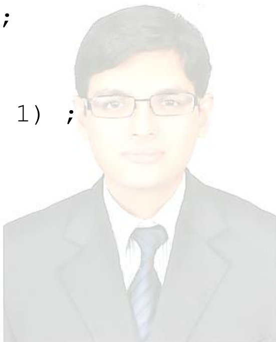
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

B.2.5. Hyperbolas generated for Sequences of Stimulation B to C to A

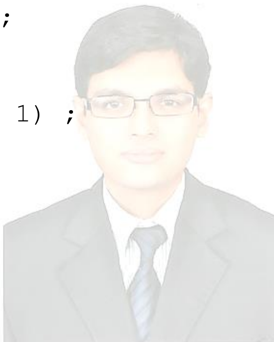
```
% beta = 0 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



```
% beta = 15 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



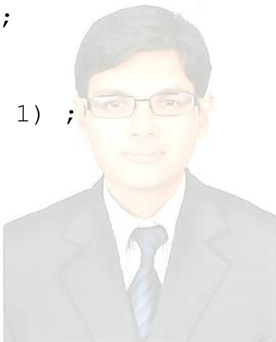
```
% beta = 30 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

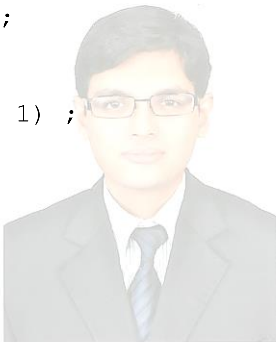
B.2.6. Hyperbolas generated for Sequences of Stimulation C to A to B

```
% beta = 0 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



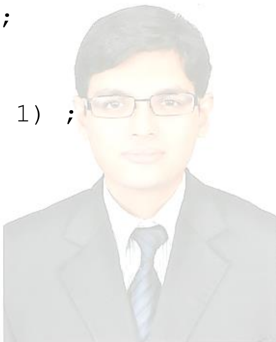
```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



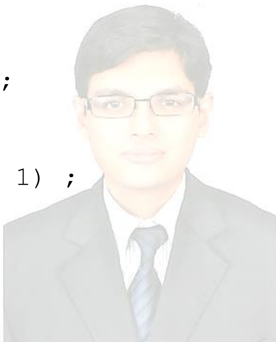
```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 30 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



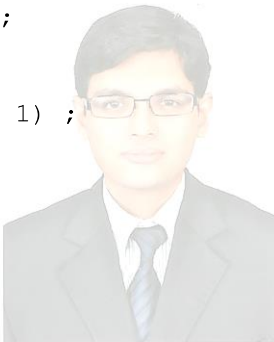
B.2.7. Hyperbolas generated for Sequences of Stimulation C to B to A

```
% beta = 0 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

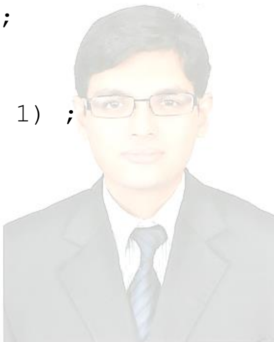


```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



```
% beta = 30 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 1000000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 1000000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 1000000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

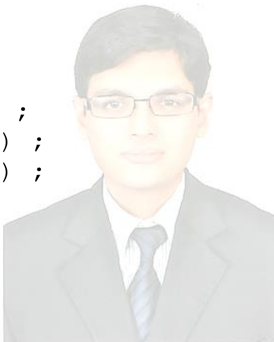
B.3. For Concave Semicircular Stimulus

B.3.1. Calculation of J-parameters for different Sequences of Stimulation

```
format long

% A to C to B

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_AC = u.*(b - a + c)./(2.*v) ;
J_AB = u.*(ro - a + d)./(2.*v) ;
J_CB = u.*(ro - b + e)./(2.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_CB)
```

```
% A to B to C

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_AB = u.*(b - a + c)./(2.*v) ;
J_AC = u.*(ro - a + d)./(2.*v) ;
J_BC = u.*(ro - b + e)./(2.*v) ;

disp(J_AB)
disp(J_AC)
disp(J_BC)



% B to A to C

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied
```

```
a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_BA = u.*(b - a + c)./(2.*v) ;
J_BC = u.*(ro - a + d)./(2.*v) ;
J_AC = u.*(ro - b + e)./(2.*v) ;

disp(J_BA)
disp(J_BC)
disp(J_AC)



% B to C to A

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_BC = u.*(b - a + c)./(2.*v) ;
J_BA = u.*(ro - a + d)./(2.*v) ;
J_CA = u.*(ro - b + e)./(2.*v) ;

disp(J_BC)
disp(J_BA)
disp(J_CA)
```

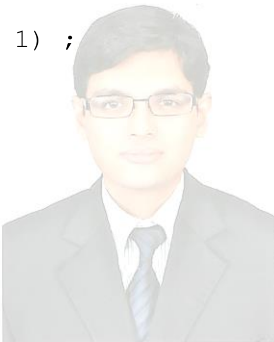


```
% C to A to B

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_CA = u.*(b - a + c)./(2.*v) ;
J_CB = u.*(ro - a + d)./(2.*v) ;
J_AB = u.*(ro - b + e)./(2.*v) ;

disp(J_CA)
disp(J_CB)
disp(J_AB)
```

```
% C to B to A

omega = 1;
u = 0.1;
v = 1;
beta = [0 pi/12 pi/6];
ro = 2 ; % condition ro > omega should be satisfied

a = sqrt(((ro).^2) - ((omega).^2).*((sin(beta)).^2)) ;
b = sqrt(((ro).^2) - ((omega).^2).*((sin(pi/3 - beta)).^2)) ;
c = omega.*(cos(beta) - cos(pi/3 - beta)) ;
d = omega.*cos(beta) ;
e = omega.*cos(pi/3 - beta) ;

J_CB = u.*(b - a + c)./(2.*v) ;
J_CA = u.*(ro - a + d)./(2.*v) ;
J_BA = u.*(ro - b + e)./(2.*v) ;

disp(J_CB)
disp(J_CA)
disp(J_BA)
```

B.3.2. Hyperbolas generated for Sequences of Stimulation A to C to B

```
% beta = 0 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =     ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

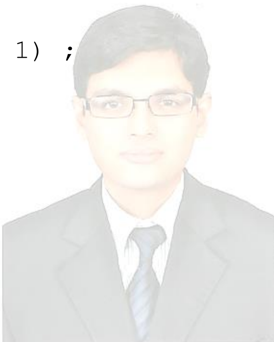
```
% beta = 15 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 = ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 30 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =   ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;



% hyperbolas with axis CB

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

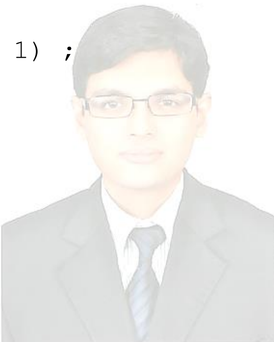
B.3.3. Hyperbolas generated for Sequences of Stimulation A to B to C

```
% beta = 0 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

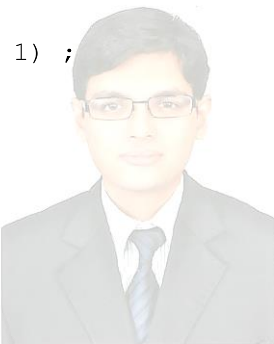


```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

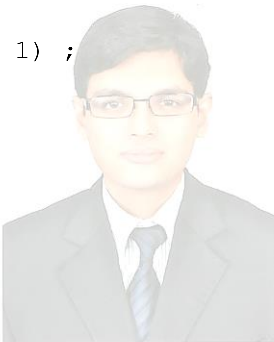


```
% beta = 30 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



B.3.4. Hyperbolas generated for Sequences of Stimulation B to A to C

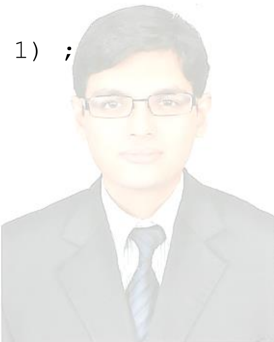
```
% beta = 0 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

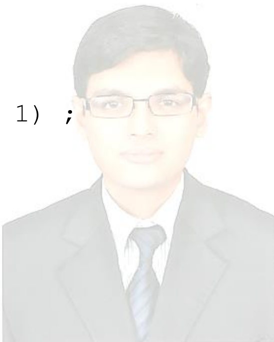
```
% beta = 15 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 30 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



B.3.5. Hyperbolas generated for Sequences of Stimulation B to C to A

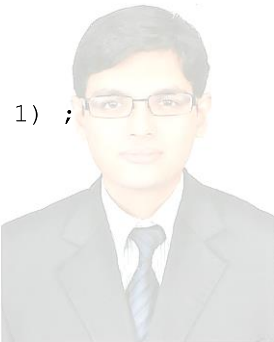
```
% beta = 0 degrees w.r.t BA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t BA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

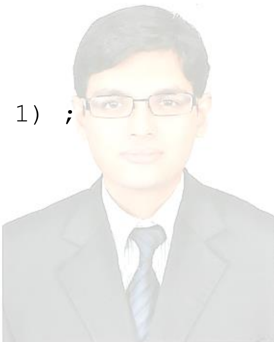
```
% beta = 30 degrees w.r.t BA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =     ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

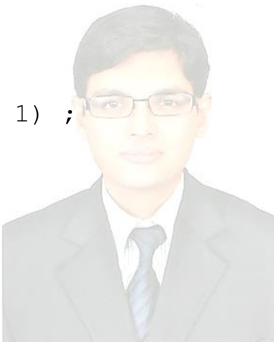
B.3.6. Hyperbolas generated for Sequences of Stimulation C to A to B

```
% beta = 0 degrees w.r.t CB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



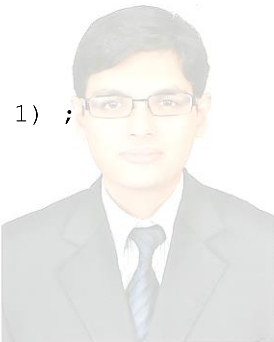
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t CB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



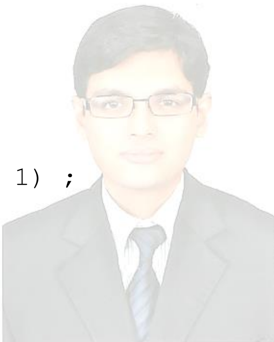
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 30 degrees w.r.t CB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 = ;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



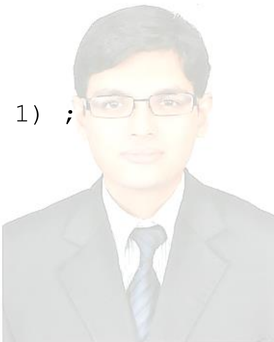
B.3.7. Hyperbolas generated for Sequences of Stimulation C to B to A

```
% beta = 0 degrees w.r.t CA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



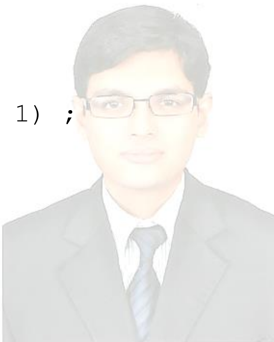
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 15 degrees w.r.t CA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```

```
% beta = 30 degrees w.r.t CA as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -sqrt(3).*a.*(a.*x - a.^2 + 2.*(J_1).^2) ;
M_1 = 4.*(J_1).*sqrt((a.^2 - (J_1).^2).*(x.^2 + a.*x + a.^2 - (J_1).^2)) ;
N_1 = 3.*a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;


% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = sqrt(3).*a.*(a.*x + a.^2 - 2.*(J_3).^2) ;
M_3 = 4.*(J_3).*sqrt((a.^2 - (J_3).^2).*(x.^2 - a.*x + a.^2 - (J_3).^2)) ;
N_3 = 3.*a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 0 0.4])
hold off
```



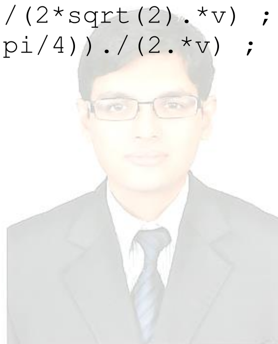
C. Three Sensor Model with Right Isosceles Triangle Array of Sensors (or Sources) and Straight Line Stimulus

C.1. Calculation of J-parameters for different Sequences of Stimulations

```
format long


% A to C to B

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/3 pi/2 2*pi/3];

J_AC = (u.*omega.*(sin(beta) - cos(beta)))./(4.*v) ;
J_AB = (u.*omega.*sin(beta))./(2.*v) ;
J_CB = (u.*omega.*(sin(beta) + cos(beta)))./(4.*v) ;

disp(J_AC)
disp(J_AB)
disp(J_CB)
```

```
% A to B to C

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/4 pi/3 pi/2];

J_AB = (u.*omega.*(sin(beta - pi/4)))./(2.*v) ;
J_AC = (u.*omega.*sin(beta))./(2*sqrt(2).*v) ;
J_BC = (u.*omega.*cos(beta))./(2*sqrt(2).*v) ;

disp(J_AB)
disp(J_AC)
disp(J_BC)



% B to A to C

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/4 pi/3 pi/2];

J_BA = (u.*omega.*(sin(beta - pi/4)))./(2.*v) ;
J_BC = (u.*omega.*sin(beta))./(2*sqrt(2).*v) ;
J_AC = (u.*omega.*cos(beta))./(2*sqrt(2).*v) ;

disp(J_BA)
disp(J_BC)
disp(J_AC)
```

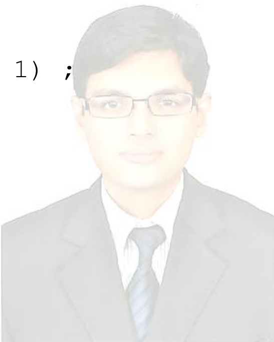
```
% B to C to A

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/3 pi/2 2*pi/3];

J_BC = (u.*omega.*(sin(beta) - cos(beta)))./(4.*v) ;
J_BA = (u.*omega.*sin(beta))./(2.*v) ;
J_CA = (u.*omega.*(sin(beta) + cos(beta)))./(4.*v) ;

disp(J_BC)
disp(J_BA)
disp(J_CA)



% C to A to B

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/2 2*pi/3 3*pi/4];

J_CA = (u.*omega.*sin(beta - pi/2))./(2*sqrt(2).*v) ;
J_CB = (u.*omega.*sin(beta))./(2*sqrt(2).*v) ;
J_AB = (u.*omega.*cos(beta - pi/4))./(2.*v) ;

disp(J_CA)
disp(J_CB)
disp(J_AB)
```



```
% C to B to A

omega = 1;
a = 0.2;
u = 0.1;
v = 3;
beta = [pi/2 2*pi/3 3*pi/4];

J_CB = (u.*omega.*sin(beta - pi/2))./(2*sqrt(2).*v) ;
J_CA = (u.*omega.*sin(beta))./(2*sqrt(2).*v) ;
J_BA = (u.*omega.*cos(beta - pi/4))./(2.*v) ;

disp(J_CB)
disp(J_CA)
disp(J_BA)
```

C.2. Hyperbolas generated for Sequence of Stimulation A to C to B

```
% beta = 60 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 90 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

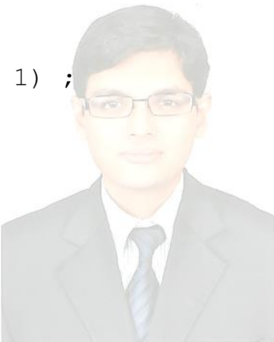
```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 120 degrees w.r.t AB as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

C.3. Hyperbolas generated for Sequence of Stimulation A to B to C

```
% beta = 45 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 60 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

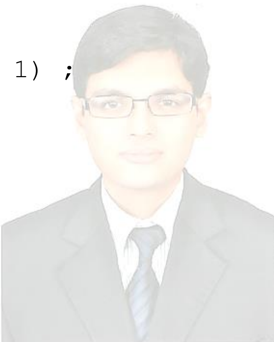
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =  ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 90 degrees w.r.t AC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =  ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

C.4. Hyperbolas generated for Sequence of Stimulation B to A to C

```
% beta = 45 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

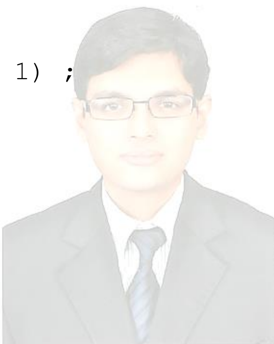
```
% beta = 60 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 90 degrees w.r.t BC as base
% hyperbolas with axis AC

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;



% hyperbolas with axis BC

a = 0.2 ;
J_3 =     ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

C.5. Hyperbolas generated for Sequence of Stimulation B to C to A

```
% beta = 60 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =     ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 90 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

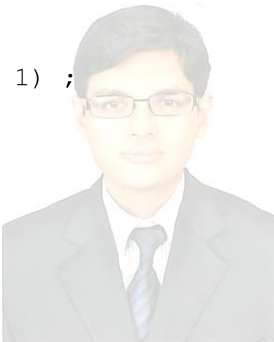
```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 120 degrees w.r.t BA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis BC

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

C.6. Hyperbolas generated for Sequence of Stimulation C to A to B

```
% beta = 90 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 120 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

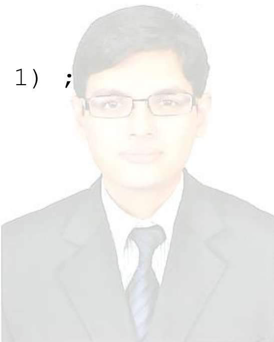
```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =   ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 135 degrees w.r.t CB as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis AB

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

C.6. Hyperbolas generated for Sequence of Stimulation C to B to A

```
% beta = 90 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =   ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```



```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 120 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =  ;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =    ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

```
% beta = 135 degrees w.r.t CA as base
% hyperbolas with axis CA

a = 0.2 ;
J_1 =;

x = linspace(-1, 1, 100000) ;

L_1 = -((2.*a.*(J_1).^2) + (a.^2).*x) ;
M_1 = sqrt((4.*(J_1).^2).*((a.^2 - 2.*(J_1).^2).*(2.*(x.^2) + 2.*a.*x +
a.^2 - 2.*(J_1).^2))) ;
N_1 = a.^2 - 4.*(J_1).^2 ;

y_11 = (L_1 + M_1)./N_1 ;
y_21 = (L_1 - M_1)./N_1 ;



% hyperbolas with axis BA

a = 0.2 ;
J_2 =     ;

x = linspace(-1, 1, 100000) ;

L_2 = sqrt(a.^2 - (J_2).^2) ;
M_2 = sqrt((x.^2./(J_2).^2) - 1) ;

y_12 = (L_2).*(M_2) ;
y_22 = -(L_2).*(M_2) ;
```

```
% hyperbolas with axis CB

a = 0.2 ;
J_3 =    ;

x = linspace(-1, 1, 100000) ;

L_3 = -((2.*a.*(J_3).^2) - (a.^2).*x) ;
M_3 = sqrt((4.*(J_3).^2).*((a.^2 - 2.*(J_3).^2).*(2.*(x.^2) - 2.*a.*x +
a.^2 - 2.*(J_3).^2))) ;
N_3 = a.^2 - 4.*(J_3).^2 ;

y_13 = (L_3 + M_3)./N_3 ;
y_23 = (L_3 - M_3)./N_3 ;



hold on
plot(x,y_11,'r',x,y_21,'r')
plot(x,y_12,'g',x,y_22,'g')
plot(x,y_13,'b',x,y_23,'b')
axis equal
axis([-0.2 0.2 -0.4 0.4])
hold off
```

D. Velocity Maps

D.1. Straight Line Stimulus to an Equilateral Array of Sensors (or Sources)

```
% Combined Velocity Maps v = {1,2,3} (mm/ms}

a_1 = [0.0475 0.0587 0.0542 0.0000 0.0325 0.0542 0.0000 -0.0325 -0.0542 -
0.0475 -0.0587 -0.0542 0.0475 0.0264 0.0000 -0.0475 -0.0264 0.0000] ;
b_1 = [0.0881 0.1191 0.1467 0.1703 0.1645 0.1467 0.1703  0.1645  0.1467
0.0881  0.1191  0.1467 0.0881 0.0629 0.0529  0.0881  0.0629 0.0529] ;

a_2 = [0.0244 0.0290 0.0259 0.0000 0.0153 0.0259 0.0000 -0.0153 -0.0259 -
0.0244 -0.0290 -0.0259 0.0244 0.0138  0.0000 -0.0244 -0.0138 0.0000] ;
b_2 = [0.1014 0.1164 0.1304 0.1435 0.1401 0.1304 0.1435 0.1401 0.1304
0.1014 0.1164 0.1304 0.1014 0.0899 0.0856 0.1014 0.0899 0.0856] ;

a_3 = [0.0164 0.0193 0.0170 0.0000 0.0101 0.0170 0.0000 -0.0101 -0.0170 -
0.0164 -0.0193 -0.0170 0.0164 0.0093 0.0000 -0.0164 -0.0093 0.0000] ;
b_3 = [0.1060 0.1159 0.1253 0.1343 0.1320 0.1253 0.1343 0.1320 0.1253
0.1060 0.1159 0.1253 0.1060 0.0986 0.0958 0.1060 0.0986 0.0958] ;


hold on
scatter(a_1,b_1,'.')
scatter(a_2,b_2,'.')
scatter(a_3,b_3,'.')

legend('v = 1 mm/ms','v = 2 mm/ms','v = 3 mm/ms')

x = linspace(-1,1,1000) ;
y_1 = sqrt(3).*(x+0.2) ; %side AC of triangle ABC
y_2 = -sqrt(3).*(x-0.2) ; %side CB of triangle ABC

plot(x,y_1,'k',x,y_2,'k',x,0,'k')
axis equal
axis([-0.2 0.2 0 0.3464])
text(-0.22, 0.025, 'A', 'color', 'r')
text(0.21, 0.025, 'B', 'color', 'b')
text(-0.05, 0.33, 'C', 'color', 'g')
hold off
```

D.2. Convex Semicircular Stimulus to an Equilateral Array of Sensors (or Sources)

```
% Combined Velocity Maps v = {1,2,3} (mm/ms}

a_1 = [0.0636 0.0550 0.0404 0.0636 0.0333 0.0000 -0.0636 -0.0333 0.0000 -
0.0636 -0.0550 -0.0404 0.0000 0.0218 0.0404 0.0000 -0.0218 -0.0404] ;
b_1 = [0.1522 0.1221 0.0921 0.1522 0.1598 0.1621  0.1522  0.1598 0.1621
0.1522  0.1221  0.0921 0.0420 0.0645 0.0921 0.0420  0.0645  0.0921] ;

a_2 = [0.0301 0.0271 0.0206 0.0301 0.0157 0 -0.0301 -0.0157 0 -0.0301 -
0.0271 -0.0206 0 0.0114 0.0206 0 -0.0114 -0.0206] ;
b_2 = [0.1328 0.1179 0.1036 0.1328 0.1377 0.1393 0.1328 0.1377 0.1393
0.1328 0.1179 0.1035 0.0807 0.0908 0.1036 0.0807 0.0908 0.1036] ;

a_3 = [0.0202 0.018 0.0139 0.0202 0.0103 0 -0.0202 -0.0103 0 -0.0202 -0.018
-0.0139 0 0.00767 0.0139 0 -0.00766 -0.0139] ;
b_3 = [0.1271 0.117 0.1075 0.1271 0.1303 0.1315 0.1271 0.1303 0.1315 0.1271
0.117 0.1075 0.0922 0.0991 0.1075 0.0922 0.0991 0.1075] ;


hold on
scatter(a_1,b_1,'.')
scatter(a_2,b_2,'.')
scatter(a_3,b_3,'.')

legend('v = 1 mm/ms','v = 2 mm/ms','v = 3 mm/ms')

x = linspace(-1,1,1000) ;
y_1 = sqrt(3).*(x+0.2) ; %side AC of triangle ABC
y_2 = -sqrt(3).*(x-0.2) ; %side CB of triangle ABC

plot(x,y_1,'k',x,y_2,'k',x,0,'k')
axis equal
axis([-0.2 0.2 0 0.3464])
text(-0.22, 0.025, 'A', 'color', 'r')
text(0.21, 0.025, 'B', 'color', 'b')
text(-0.05, 0.33, 'C', 'color', 'g')
hold off
```

D.3. Concave Semicircular Stimulus to an Equilateral Array of Sensors (or Sources)

```
% Combined Velocity Maps v = {1,2,3} (mm/ms}

a_1 = [0.0569 0.0547 0.0509 0.0198 0.00965 0 -0.0198 -0.00965 0 -0.0569 -
0.0547 -0.0509 0.0371 0.045 0.0509 -0.0371 -0.045 -0.0509] ;
b_1 = [0.1055 0.0950 0.0861 0.1698 0.173 0.1742 0.1698 0.173 0.1742 0.1055
0.095 0.0861 0.0711 0.0783 0.0861 0.0711 0.0783 0.0861] ;

a_2 = [0.0286 0.0278 0.0261 0.009285 0.0045077 0 -0.009285 -0.0045077 0 -
0.0286 -0.0278 -0.0261 0.0193 0.0232 0.0261 -0.0193 -0.0232 -0.0261] ;
b_2 = [0.1097 0.1047 0.1004 0.1431 0.1449 0.1456 0.1431 0.1449 0.1456
0.1097 0.1047 0.1004 0.0936 0.0968 0.1004 0.0936 0.0968 0.1004] ;

a_3 = [0.0191 0.0186 0.0175 0.006055 0.002934 0 -0.006055 -0.002934 0 -
0.0191 -0.0186 -0.0175 0.013 0.0157 0.0175 -0.013 -0.0157 -0.0175] ;
b_3 = [0.1114 0.1081 0.1053 0.134 0.1353 0.1357 0.134 0.1353 0.1357 0.1114
0.1081 0.1053 0.101 0.103 0.1053 0.101 0.103 0.1053] ;


hold on
scatter(a_1,b_1,'.')
scatter(a_2,b_2,'.')
scatter(a_3,b_3,'.')

legend('v = 1 mm/ms','v = 2 mm/ms','v = 3 mm/ms')

x = linspace(-1,1,1000) ;
y_1 = sqrt(3).*(x+0.2) ; %side AC of triangle ABC
y_2 = -sqrt(3).*(x-0.2) ; %side CB of triangle ABC

plot(x,y_1,'k',x,y_2,'k',x,0,'k')
axis equal
axis([-0.2 0.2 0 0.3464])
text(-0.22, 0.025, 'A', 'color', 'r')
text(0.21, 0.025, 'B', 'color', 'b')
text(-0.05, 0.33, 'C', 'color', 'g')
hold off
```

D.4. Straight Line Stimulus to a Right Isosceles Array of Sensors (or Sources)

```
% Combined Velocity Maps v = {1,2,3} (mm/ms}

a_1 = [0.0436 0.050 0.0438 0 0.0133 0.0359 0 -0.0133 -0.0359 -0.0436 -0.05
-0.0438 0.0359 0.0134 0 -0.0359 -0.0134 0] ;
b_1 = [-0.022 0.00625 0.0284 0.045 0.044 0.0359 0.045 0.044 0.0359 -0.022
0.00625 0.0284 -0.0359 -0.0555 -0.0583 -0.0359 -0.0555 -0.0583] ;

a_2 = [0.0217 0.0250 0.0217 0 0.006543 0.0178 0 -0.006543 -0.0178 -0.0217 -
0.025 -0.0217 0.0178 0.0065535 0 -0.0178 -0.0065535 0] ;
b_2 = [-0.0117 0.00156 0.0133 0.0236 0.0229 0.0178 0.0236 0.0229 0.0178 -
0.0117 0.001562 0.0133 -0.0178 -0.0256 -0.0268 -0.0178 -0.0256 -0.0268] ;

a_3 = [0.0144 0.0167 0.0144 0 0.0043135 0.0118 0 -0.0043136 -0.0118 -0.0145
-0.0167 -0.0144 0.0118 0.004316 0 -0.0118 -0.004316 0] ;
b_3 = [-0.00795 0.0007 0.00864 0.016 0.0155 0.0118 0.016 0.0155 0.0118 -
0.00796 0.0007 0.00865 -0.0118 -0.0168 -0.0174 -0.0118 -0.0168 -0.0174] ;


hold on
scatter(a_1,b_1,'.')
scatter(a_2,b_2,'.')
scatter(a_3,b_3,'.')

x = linspace(-0.2,0.2,10000) ;
y_1 = 0.2 + x ; %side CA of triangle ABC
y_2 = 0.2 - x ; %side CB of triangle ABC

legend('v = 1 mm/ms', 'v = 2 mm/ms', 'v = 3 mm/ms')
plot(x,y_1,'k',x,y_2,'k',x,0,'k')
axis equal
axis([-0.4 0.4 -0.1 0.2])
text(-0.22, 0.025, 'A', 'color', 'r')
text(0.21, 0.025, 'B', 'color', 'b')
text(-0.06, 0.185, 'C', 'color', 'g')
hold off
```

E. Simulation Results on MATLAB

E.1. Straight Line Stimulus to an Equilateral Triangular Array

J parameters for v = 1 mm/ms (Speed of the Stimulus)

SEQUENCE A TO C TO B (Base AB)	60°	90°	120°
J_{AC}	0	0.025	0.043301270189222
J_{AB}	0.043301270189222	0.05	0.043301270189222
J_{CB}	0.043301270189222	0.025	0

SEQUENCE A TO B TO C (Base AC)	60°	90°	120°
J_{AC}	0.043301270189222	0.05	0.043301270189222
J_{AB}	0	0.025	0.043301270189222
J_{BC}	0.043301270189222	0.025	0

SEQUENCE B TO A TO C (Base BC)	60°	90°	120°
J_{AC}	0.043301270189222	0.025	0
J_{BA}	0	0.025	0.043301270189222
J_{BC}	0.043301270189222	0.05	0.043301270189222

SEQUENCE B TO C TO A (Base BA)	60°	90°	120°
J_{CA}	0.043301270189222	0.025	0
J_{BA}	0.043301270189222	0.05	0.043301270189222
J_{BC}	0	0.025	0.043301270189222

SEQUENCE C TO A TO B (Base CB)	60°	90°	120°
J_{CA}	0	0.025	0.043301270189222
J_{AB}	0.043301270189222	0.025	0
J_{CB}	0.043301270189222	0.05	0.043301270189222

SEQUENCE C TO B TO A (Base CA)	60°	90°	120°
J_{CA}	0.043301270189222	0.05	0.043301270189222
J_{BA}	0.043301270189222	0.025	0
J_{CB}	0	0.025	0.043301270189222

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
60°	0.0475	0.0881
90°	0.0587	0.1191
120°	0.0542	0.1467

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
60°	0	0.1703
90°	0.0325	0.1645
120°	0.0542	0.1467

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
60°	0	0.1703
90°	-0.0325	0.1645
120°	-0.0542	0.1467

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
60°	-0.0475	0.0881
90°	-0.0587	0.1191
120°	-0.0542	0.1467

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
60°	0.0475	0.0881
90°	0.0264	0.0629
120°	0	0.0529

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
60°	-0.0475	0.0881
90°	-0.0264	0.0629
120°	0	0.0529

J parameters for v = 2 mm/ms

SEQUENCE A TO C TO B (Base AB)	60°	90°	120°
J_{AC}	0	0.0125	0.021650635094611
J_{AB}	0.021650635094611	0.025	0.021650635094611
J_{CB}	0.021650635094611	0.0125	0

SEQUENCE A TO B TO C (Base AC)	60°	90°	120°
J_{AC}	0.021650635094611	0.025	0.021650635094611
J_{AB}	0	0.0125	0.021650635094611
J_{BC}	0.021650635094611	0.0125	0

SEQUENCE B TO A TO C (Base BC)	60°	90°	120°
J_{AC}	0.021650635094611	0.0125	0
J_{BA}	0	0.0125	0.021650635094611
J_{BC}	0.021650635094611	0.025	0.021650635094611

SEQUENCE B TO C TO A (Base BA)	60°	90°	120°
J_{CA}	0.021650635094611	0.0125	0
J_{BA}	0.021650635094611	0.025	0.021650635094611
J_{BC}	0	0.0125	0.021650635094611

SEQUENCE C TO A TO B (Base CB)	60°	90°	120°
J_{CA}	0	0.0125	0.021650635094611
J_{AB}	0.021650635094611	0.0125	0
J_{CB}	0.021650635094611	0.025	0.021650635094611

SEQUENCE C TO B TO A (Base CA)	60°	90°	120°
J_{CA}	0.021650635094611	0.025	0.021650635094611
J_{BA}	0.021650635094611	0.0125	0
J_{CB}	0	0.0125	0.021650635094611

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
60°	0.0244	0.1014
90°	0.029	0.1164
120°	0.0259	0.1304

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
60°	0	0.1435
90°	0.0153	0.1401
120°	0.0259	0.1304

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
60°	0	0.1435
90°	-0.0153	0.1401
120°	-0.0259	0.1304

SEQUENCE B TO C TO A (Angle w.r.t BA)	x	Y
60°	-0.0244	0.1014
90°	-0.029	0.1164
120°	-0.0259	0.1304

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
60°	0.0244	0.1014
90°	0.0138	0.0899
120°	0	0.0856

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
60°	-0.0244	0.1014
90°	-0.0138	0.0899
120°	0	0.0856

J parameters for v = 3 mm/ms

SEQUENCE A TO C TO B (Base AB)	$60°$	$90°$	$120°$
J_{AC}	0	0.008333333333333	0.014433756729741
J_{AB}	0.014433756729741	0.016666666666667	0.014433756729741
J_{CB}	0.014433756729741	0.008333333333333	0

SEQUENCE A TO B TO C (Base AC)	$60°$	$90°$	$120°$
J_{AC}	0.014433756729741	0.016666666666667	0.014433756729741
J_{AB}	0	0.008333333333333	0.014433756729741
J_{BC}	0.014433756729741	0.008333333333333	0

SEQUENCE B TO A TO C (Base BC)	$60°$	$90°$	$120°$
J_{AC}	0.014433756729741	0.008333333333333	0
J_{BA}	0	0.008333333333333	0.014433756729741
J_{BC}	0.014433756729741	0.016666666666667	0.014433756729741

SEQUENCE B TO C TO A (Base BA)	$60°$	$90°$	$120°$
J_{CA}	0.014433756729741	0.008333333333333	0
J_{BA}	0.014433756729741	0.016666666666667	0.014433756729741
J_{BC}	0	0.008333333333333	0.014433756729741

SEQUENCE C TO A TO B (Base CB)	$60°$	$90°$	$120°$
J_{CA}	0	0.008333333333333	0.014433756729741
J_{AB}	0.014433756729741	0.008333333333333	0
J_{CB}	0.014433756729741	0.016666666666667	0.014433756729741

SEQUENCE C TO B TO A (Base CA)	$60°$	$90°$	$120°$
J_{CA}	0.014433756729741	0.016666666666667	0.014433756729741
J_{BA}	0.014433756729741	0.008333333333333	0
J_{CB}	0	0.008333333333333	0.014433756729741

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	x	Y
60°	0.0164	0.1060
90°	0.0193	0.1159
120°	0.0170	0.1253

SEQUENCE A TO B TO C (Angle w.r.t AC)	x	Y
60°	0	0.1343
90°	0.0101	0.1320
120°	0.0170	0.1253

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
60°	0	0.1343
90°	-0.0101	0.1320
120°	-0.0170	0.1253

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
60°	-0.0164	0.1060
90°	-0.0193	0.1159
120°	-0.0170	0.1253

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
60°	0.0164	0.1060
90°	0.0093	0.0986
120°	0	0.0958

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
60°	-0.0164	0.1060
90°	-0.0093	0.0986
120°	0	0.0958

E.2. Convex Semicircular Stimulus to an Equilateral Triangular Array

J parameters for v = 1 mm/ms (Speed of the Stimulus)

SEQUENCE A TO C TO B (Base AB)	0°	15°	30°
J_{AC}	0.05	0.0259	0
J_{AB}	0.05	0.0466	0.0366
J_{CB}	0	0.0207	0.0366

SEQUENCE A TO B TO C (Base AC)	0°	15°	30°
J_{AC}	0.05	0.0466	0.0366
J_{AB}	0.05	0.0259	0
J_{BC}	0	0.0207	0.0366

SEQUENCE B TO A TO C (Base BC)	0°	15°	30°
J_{AC}	0	0.0207	0.0366
J_{BA}	0.05	0.0259	0
J_{BC}	0.05	0.0466	0.0366

SEQUENCE B TO C TO A (Base BA)	0°	15°	30°
J_{CA}	0	0.0207	0.0366
J_{BA}	0.05	0.0466	0.0366
J_{BC}	0.05	0.0259	0

SEQUENCE C TO A TO B (Base CB)	0°	15°	30°
J_{CA}	0.05	0.0259	0
J_{AB}	0	0.0207	0.0366
J_{CB}	0.05	0.0466	0.0366

SEQUENCE C TO B TO A (Base CA)	0°	15°	30°
J_{CA}	0.05	0.0466	0.0366
J_{BA}	0	0.0207	0.0366
J_{CB}	0.05	0.0259	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
0°	0.0636	0.1522
15°	0.055	0.1221
30°	0.0404	0.0921

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.0636	0.1522
15°	0.0333	0.1598
30°	0	0.1621

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.0636	0.1522
15°	-0.0333	0.1598
30°	0	0.1621

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
0°	-0.0636	0.1522
15°	-0.055	0.1221
30°	-0.0404	0.0921

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0	0.042
15°	0.0218	0.0645
30°	0.0404	0.0921

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	0	0.042
15°	-0.0218	0.0645
30°	-0.0404	0.0921

J parameters for v = 2 mm/ms

SEQUENCE A TO C TO B (Base AB)	0°	15°	30°
J_{AC}	0.025	0.0129	0
J_{AB}	0.025	0.0233	0.0183
J_{CB}	0	0.0104	0.0183

SEQUENCE A TO B TO C (Base AC)	0°	15°	30°
J_{AC}	0.025	0.0233	0.0183
J_{AB}	0.025	0.0129	0
J_{BC}	0	0.0104	0.0183

SEQUENCE B TO A TO C (Base BC)	0°	15°	30°
J_{AC}	0	0.0104	0.0183
J_{BA}	0.025	0.0129	0
J_{BC}	0.025	0.0233	0.0183

SEQUENCE B TO C TO A (Base BA)	0°	15°	30°
J_{CA}	0	0.0104	0.0183
J_{BA}	0.025	0.0233	0.0183
J_{BC}	0.025	0.0129	0

SEQUENCE C TO A TO B (Base CB)	0°	15°	30°
J_{CA}	0.025	0.0129	0
J_{AB}	0	0.0104	0.0183
J_{CB}	0.025	0.0233	0.0183

SEQUENCE C TO B TO A (Base CA)	0°	15°	30°
J_{CA}	0.025	0.0233	0.0183
J_{BA}	0	0.0104	0.0183
J_{CB}	0.025	0.0129	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
0°	0.0301	0.1328
15°	0.0271	0.1179
30°	0.0206	0.1036

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.0301	0.1328
15°	0.0157	0.1377
30°	0	0.1393

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.0301	0.1328
15°	-0.0157	0.1377
30°	0	0.1393

SEQUENCE B TO C TO A (Angle w.r.t BA)	x	Y
0°	-0.0301	0.1328
15°	-0.0271	0.1179
30°	-0.0206	0.1035

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0	0.0807
15°	0.0114	0.0908
30°	0.0206	0.1036

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	0	0.0807
15°	-0.0114	0.0908
30°	-0.0206	0.1036

J parameters for v = 3 mm/ms

SEQUENCE A TO C TO B (Base AB)	0°	15°	30°
J_{AC}	0.017	0.0087	0
J_{AB}	0.017	0.0155	0.0122
J_{CB}	0	0.0069	0.0122

SEQUENCE A TO B TO C (Base AC)	0°	15°	30°
J_{AC}	0.017	0.0155	0.0122
J_{AB}	0.017	0.0087	0
J_{BC}	0	0.0069	0.0122

SEQUENCE B TO A TO C (Base BC)	0°	15°	30°
J_{AC}	0	0.0069	0.0122
J_{BA}	0.017	0.0087	0
J_{BC}	0.017	0.0155	0.0122

SEQUENCE B TO C TO A (Base BA)	0°	15°	30°
J_{CA}	0	0.0069	0.0122
J_{BA}	0.017	0.0155	0.0122
J_{BC}	0.017	0.0087	0

SEQUENCE C TO A TO B (Base CB)	0°	15°	30°
J_{CA}	0.017	0.0087	0
J_{AB}	0	0.0069	0.0122
J_{CB}	0.017	0.0155	0.0122

SEQUENCE C TO B TO A (Base CA)	0°	15°	30°
J_{CA}	0.017	0.0155	0.0122
J_{BA}	0	0.0069	0.0122
J_{CB}	0.017	0.0087	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	x	Y
0°	0.0202	0.1271
15°	0.018	0.117
30°	0.0139	0.1075

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.0202	0.1271
15°	0.0103	0.1303
30°	0	0.1315

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.0202	0.1271
15°	-0.0103	0.1303
30°	0	0.1315

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
0°	-0.0202	0.1271
15°	-0.018	0.117
30°	-0.0139	0.1075

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0	0.0922
15°	0.00767	0.0991
30°	0.0139	0.1075

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	0	0.0922
15°	-0.00766	0.0991
30°	-0.0139	0.1075

E.3. Concave Semicircular Stimulus to an Equilateral Triangular Array

J parameters for v = 1 mm/ms (Speed of the Stimulus)

SEQUENCE A TO C TO B (Base AB)	$0°$	$15°$	$30°$
J_{AC}	0.0151	0.0073	0
J_{AB}	0.05	0.0491	0.0465
J_{CB}	0.0349	0.0418	0.0465

SEQUENCE A TO B TO C (Base AC)	$0°$	$15°$	$30°$
J_{AC}	0.05	0.0491	0.0465
J_{AB}	0.0151	0.0073	0
J_{BC}	0.0349	0.0418	0.0465

SEQUENCE B TO A TO C (Base BC)	$0°$	$15°$	$30°$
J_{AC}	0.0349	0.0418	0.0465
J_{BA}	0.0151	0.0073	0
J_{BC}	0.05	0.0491	0.0465

SEQUENCE B TO C TO A (Base BA)	$0°$	$15°$	$30°$
J_{CA}	0.0349	0.0418	0.0465
J_{BA}	0.05	0.0491	0.0465
J_{BC}	0.0151	0.0073	0

SEQUENCE C TO A TO B (Base CB)	$0°$	$15°$	$30°$
J_{CA}	0.0151	0.0073	0
J_{AB}	0.0349	0.0418	0.0465
J_{CB}	0.05	0.0491	0.0465

SEQUENCE C TO B TO A (Base CA)	$0°$	$15°$	$30°$
J_{CA}	0.05	0.0491	0.0465
J_{BA}	0.0349	0.0418	0.0465
J_{CB}	0.0151	0.0073	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
0°	0.0569	0.1055
15°	0.0547	0.095
30°	0.0509	0.0861

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.0198	0.1698
15°	0.0096	0.173
30°	0	0.1742

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.0198	0.1698
15°	-0.0096	0.173
30°	0	0.1742

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
0°	-0.0569	0.1055
15°	-0.0547	0.095
30°	-0.0509	0.0861

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0.0371	0.0711
15°	0.045	0.0783
30°	0.0509	0.0861

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	-0.0371	0.0711
15°	-0.045	0.0783
30°	-0.0509	0.0861

J parameters for v = 2 mm/ms

SEQUENCE A TO C TO B (Base AB)	0°	15°	30°
J_{AC}	0.00755	0.00365	0
J_{AB}	0.025	0.02455	0.02325
J_{CB}	0.01745	0.0209	0.02325

SEQUENCE A TO B TO C (Base AC)	0°	15°	30°
J_{AC}	0.025	0.02455	0.02325
J_{AB}	0.00755	0.00365	0
J_{BC}	0.01745	0.0209	0.02325

SEQUENCE B TO A TO C (Base BC)	0°	15°	30°
J_{AC}	0.01745	0.0209	0.02325
J_{BA}	0.00755	0.00365	0
J_{BC}	0.025	0.02455	0.02325

SEQUENCE B TO C TO A (Base BA)	0°	15°	30°
J_{CA}	0.01745	0.0209	0.02325
J_{BA}	0.025	0.02455	0.02325
J_{BC}	0.00755	0.00365	0

SEQUENCE C TO A TO B (Base CB)	0°	15°	30°
J_{CA}	0.00755	0.00365	0
J_{AB}	0.01745	0.0209	0.02325
J_{CB}	0.025	0.02455	0.02325

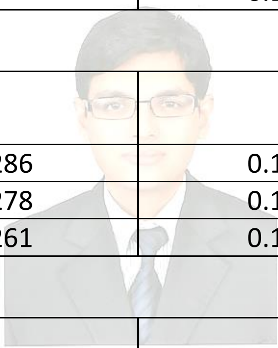
SEQUENCE C TO B TO A (Base CA)	0°	15°	30°
J_{CA}	0.025	0.02455	0.02325
J_{BA}	0.01745	0.0209	0.02325
J_{CB}	0.00755	0.00365	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
0°	0.0286	0.1097
15°	0.0278	0.1047
30°	0.0261	0.1004

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.009285	0.1431
15°	0.0045077	0.1449
30°	0	0.1456

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.009285	0.1431
15°	-0.0045077	0.1449
30°	0	0.1456



SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
0°	-0.0286	0.1097
15°	-0.0278	0.1047
30°	-0.0261	0.1004

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0.0193	0.0936
15°	0.0232	0.0968
30°	0.0261	0.1004

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	-0.0193	0.0936
15°	-0.0232	0.0968
30°	-0.0261	0.1004

J parameters for v = 3 mm/ms

SEQUENCE A TO C TO B (Base AB)	0°	15°	30°
J_{AC}	0.00503	0.00243	0
J_{AB}	0.0166	0.01636	0.0155
J_{CB}	0.01163	0.01393	0.0155

SEQUENCE A TO B TO C (Base AC)	0°	15°	30°
J_{AC}	0.0166	0.01636	0.0155
J_{AB}	0.00503	0.00243	0
J_{BC}	0.01163	0.01393	0.0155

SEQUENCE B TO A TO C (Base BC)	0°	15°	30°
J_{AC}	0.01163	0.01393	0.0155
J_{BA}	0.00503	0.00243	0
J_{BC}	0.0166	0.01636	0.0155

SEQUENCE B TO C TO A (Base BA)	0°	15°	30°
J_{CA}	0.01163	0.01393	0.0155
J_{BA}	0.0166	0.01636	0.0155
J_{BC}	0.00503	0.00243	0

SEQUENCE C TO A TO B (Base CB)	0°	15°	30°
J_{CA}	0.00503	0.00243	0
J_{AB}	0.01163	0.01393	0.0155
J_{CB}	0.0166	0.01636	0.0155

SEQUENCE C TO B TO A (Base CA)	0°	15°	30°
J_{CA}	0.0166	0.01636	0.0155
J_{BA}	0.01163	0.01393	0.0155
J_{CB}	0.00503	0.00243	0

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	x	Y
0°	0.0191	0.1114
15°	0.0186	0.1081
30°	0.0175	0.1053

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
0°	0.006055	0.134
15°	0.002934	0.1353
30°	0	0.1357

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
0°	-0.006055	0.134
15°	-0.002934	0.1353
30°	0	0.1357

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
0°	-0.0191	0.1114
15°	-0.0186	0.1081
30°	-0.0175	0.1053

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
0°	0.013	0.101
15°	0.0157	0.103
30°	0.0175	0.1053

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
0°	-0.013	0.101
15°	-0.0157	0.103
30°	-0.0175	0.1053

E.4. Straight Line Stimulus to a Right Isosceles Triangular Array

J parameters for v = 1 mm/ms (Speed of the Stimulus)

SEQUENCE A TO C TO B (Base AB)	60°	90°	120°
J_{AC}	0.0091506	0.025	0.0341506
J_{AB}	0.0433012	0.05	0.0433012
J_{CB}	0.0341506	0.025	0.0091506

SEQUENCE A TO B TO C (Base AC)	45°	60°	90°
J_{AC}	0.025	0.0306186	0.035355
J_{AB}	0	0.0129409	0.035355
J_{BC}	0.025	0.0176776	0

SEQUENCE B TO A TO C (Base BC)	45°	60°	90°
J_{AC}	0.025	0.0176776	0
J_{BA}	0	0.0129409	0.035355
J_{BC}	0.025	0.0306186	0.035355

SEQUENCE B TO C TO A (Base BA)	60°	90°	120°
J_{CA}	0.0341506	0.025	0.0091506
J_{BA}	0.0433012	0.05	0.0433012
J_{BC}	0.0091506	0.025	0.0341506

SEQUENCE C TO A TO B (Base CB)	90°	120°	135°
J_{CA}	0	0.0176776	0.025
J_{AB}	0.035355	0.0129409	0
J_{CB}	0.035355	0.0306186	0.025

SEQUENCE C TO B TO A (Base CA)	90°	120°	135°
J_{CA}	0.035355	0.0306186	0.025
J_{BA}	0.035355	0.0129409	0
J_{CB}	0	0.0176776	0.025

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
$60°$	0.0436	-0.0220
$90°$	0.050	0.00625
$120°$	0.0438	0.0284

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
$45°$	0	0.045
$60°$	0.0133	0.044
$90°$	0.0359	0.0359

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
$45°$	0	0.045
$60°$	-0.0133	0.044
$90°$	-0.0359	0.0359

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
$60°$	-0.0436	-0.022
$90°$	-0.05	0.00625
$120°$	-0.0438	0.0284

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
$90°$	0.0359	-0.0359
$120°$	0.0134	-0.0555
$135°$	0	-0.0583

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
$90°$	-0.0359	-0.0359
$120°$	-0.0134	-0.0555
$135°$	0	-0.0583

J parameters for v = 2 mm/ms

SEQUENCE A TO C TO B (Base AB)	60°	90°	120°
J_{AC}	0.0046	0.0125	0.0171
J_{AB}	0.0217	0.025	0.0217
J_{CB}	0.0171	0.0125	0.0046

SEQUENCE A TO B TO C (Base AC)	45°	60°	90°
J_{AC}	0.0125	0.0153	0.0177
J_{AB}	0	0.0065	0.0177
J_{BC}	0.0125	0.0088	0

SEQUENCE B TO A TO C (Base BC)	45°	60°	90°
J_{AC}	0.0125	0.0088	0
J_{BA}	0	0.0065	0.0177
J_{BC}	0.0125	0.0153	0.0177

SEQUENCE B TO C TO A (Base BA)	60°	90°	120°
J_{CA}	0.0171	0.0125	0.0046
J_{BA}	0.0217	0.025	0.0217
J_{BC}	0.0046	0.0125	0.0171

SEQUENCE C TO A TO B (Base CB)	90°	120°	135°
J_{CA}	0	0.0088	0.0125
J_{AB}	0.0177	0.0065	0
J_{CB}	0.0177	0.0153	0.0125

SEQUENCE C TO B TO A (Base CA)	90°	120°	135°
J_{CA}	0.0177	0.0153	0.0125
J_{BA}	0.0177	0.0065	0
J_{CB}	0	0.0088	0.0125

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	X	Y
$60°$	0.0217	-0.0117
$90°$	0.0250	0.00156
$120°$	0.0217	0.0133

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
$45°$	0	0.0236
$60°$	0.006543	0.0229
$90°$	0.0178	0.0178

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
$45°$	0	0.0236
$60°$	-0.006543	0.0229
$90°$	-0.0178	0.0178

SEQUENCE B TO C TO A (Angle w.r.t BA)	x	Y
$60°$	-0.0217	-0.0117
$90°$	-0.025	0.00156
$120°$	-0.0217	0.0133

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
$90°$	0.0178	-0.0178
$120°$	0.0065	-0.0256
$135°$	0	-0.0268

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
$90°$	-0.0178	-0.0178
$120°$	-0.0065	-0.0256
$135°$	0	-0.0268

J parameters for v = 3 mm/ms

SEQUENCE A TO C TO B (Base AB)	60°	90°	120°
J_{AC}	0.0031	0.00833	0.0114
J_{AB}	0.0144	0.01667	0.0144
J_{CB}	0.0114	0.00833	0.0031

SEQUENCE A TO B TO C (Base AC)	45°	60°	90°
J_{AC}	0.0083	0.0102	0.0118
J_{AB}	0	0.0043	0.0118
J_{BC}	0.0083	0.0059	0

SEQUENCE B TO A TO C (Base BC)	45°	60°	90°
J_{AC}	0.0083	0.0059	0
J_{BA}	0	0.0043	0.0118
J_{BC}	0.0083	0.0102	0.0118

SEQUENCE B TO C TO A (Base BA)	60°	90°	120°
J_{CA}	0.0114	0.0083	0.0031
J_{BA}	0.0144	0.0167	0.0144
J_{BC}	0.0031	0.0083	0.0114

SEQUENCE C TO A TO B (Base CB)	90°	120°	135°
J_{CA}	0	0.0059	0.0083
J_{AB}	0.0118	0.0043	0
J_{CB}	0.0118	0.0102	0.0083

SEQUENCE C TO B TO A (Base CA)	90°	120°	135°
J_{CA}	0.0118	0.0102	0.0083
J_{BA}	0.0118	0.00433	0
J_{CB}	0	0.0059	0.0083

POINTS OF INTERSECTION

SEQUENCE A TO C TO B (Angle w.r.t AB)	x	Y
60°	0.0144	-0.0079
90°	0.0167	0.0007
120°	0.0144	0.0086

SEQUENCE A TO B TO C (Angle w.r.t AC)	X	Y
45°	0	0.016
60°	0.0043	0.0155
90°	0.0118	0.0118

SEQUENCE B TO A TO C (Angle w.r.t BC)	X	Y
45°	0	0.016
60°	-0.0043	0.0155
90°	-0.0118	0.0118

SEQUENCE B TO C TO A (Angle w.r.t BA)	X	Y
60°	-0.0145	-0.00796
90°	-0.0167	0.0007
120°	-0.0144	0.00865

SEQUENCE C TO A TO B (Angle w.r.t CB)	X	Y
90°	0.0118	-0.0118
120°	0.0043	-0.0168
135°	0	-0.0174

SEQUENCE C TO B TO A (Angle w.r.t CA)	X	Y
90°	-0.0118	-0.0118
120°	-0.0043	-0.0168
135°	0	-0.0174