On the Infinity of Primes of the Form $2x^2$–1

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Abstract

In this paper we consider primes of the form $2x^2$–1 and discover there is a very great probability for appearing of such primes, and give an argument for the infinity of primes of the form $2x^2$–1 by the infinity of near-square primes of Mersenne primes.

Keywords: primes of the form $2x^2$–1; near-square primes of Mersenne primes.

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1. A Great probability for appearing of primes of the form $2x^2$–1

We consider primes of the form $2x^2$–1 where x is positive integer, and found 44 such primes for x less than 100 as follows

x=2: 7, x=3: 17, x=4: 31, x=6: 71, x=7: 97, x=8: 127, x=10: 199, x=11: 241, x=13: 337, x=15: 449, x=17: 577, x=18: 647, x=21: 881, x=22: 967, x=24: 1151, x=25: 1249, x=28: 1567, x=34: 2311, x=36: 2591, x=38: 2887, x=39: 3041, x=41: 3361, x=42: 3527, x=45: 4049, x=46: 4231, x=49: 4801, x=50: 4999, x=52: 5407,

x=56: 6271, x=59: 6961, x=62: 7687, x=63: 7937, x=64: 8191, x=69: 9521, x=73: 10657, x=76: 11551, x=80: 12799, x=81: 13121, x=85: 14449, x=87: 15137, x=91: 16561, x=92: 16927, x=95: 18049, x=98: 19207.

More such primes can be viewed in Encyclopedia of Integer Sequences[1]. From above results we see that among 99 x-values, there are 44 x-values to generate primes the form $2x^2–1$. It implies that there is a very great probability for appearing of primes of the form $2x^2–1$. Are these primes with such great probability infinite? However, it is very difficult to give proof for infinity of a prime number sequence with formula, for example, a theorem proved in 1997 by J. Friedlander and H. Iwaniec shows that there are infinitely many primes of the form x^2+y^4[2] but there have not been more such proofs for other prime number sequences with formula.

Considering near-square primes of Mersenne primes $W_p = 2M_p^2 –1$ to be a subset of primes of the form $2x^2–1$, we may give an argument for the infinity of primes of the form $2x^2–1$ by argument about infinity of near-square primes of Mersenne primes.

2. The infinity of near-square primes of Mersenne primes

Definition 2.1 If M_p is a Mersenne prime then $W_p = 2M_p^2 –1$ is called a near-square number of Mersenne prime.

Definition 2.2 If $W_p = 2M_p^2 –1$ is a prime number then W_p is called a near-square prime of Mersenne prime.

All near-square numbers of Mersenne primes is an infinite sequence if Mersenne primes are infinite. But it does not mean near-square primes of Mersenne primes are infinite. The argument on infinity of near-square primes of Mersenne primes should be given. The first few near-square primes of Mersenne primes have been verified as follows[3]

$W_2=2^5–2^4+1=17$,

$W_3=2^7–2^5+1=97$,

$W_7=2^{15}–2^9+1=32257$,

$W_{17}=2^{35}–2^{19}+1=34359214081$

$W_{19}=2^{39}–2^{21}+1=549753716737$

…

Definition 2.3 Exponents of all Mersenne primes M_p are called basic sequence of number of near-square primes of Mersenne primes.

Basic sequence of number of near-square primes of Mersenne primes are infinite if Mersenne primes are infinite by Definition 2.3.

Definition 2.4 If the first few continuous exponents of Mersenne primes p make $W_p = 2M_p^2 –1$ become near-square primes of Mersenne primes in basic sequence of number of near-square primes of Mersenne primes then these exponents are called original continuous prime number sequence of near-square primes of Mersenne primes.

Lemma 2.5 The original continuous prime number sequence of near-square primes of Mersenne primes is p=2,3.

Proof. Since the first two near-square numbers of Mersenne primes i. e. W_2=17 and W_3=97 are all prime but the third near-square number of Mersenne prime i. e. W_5=1921 is composite, by Definition 2.4 the original continuous prime number sequence of near-square primes of Mersenne primes is p=2,3.

Definition 2.6 Near-square primes of Mersenne primes are strongly finite if the first few continuous terms generated from the original continuous prime number sequence are prime but all larger terms are composite.

Fermat prime criterion 2.7 Near-square primes of Mersenne primes are infinite if both the sum of corresponding original continuous prime number sequence and the first such prime are Fermat primes, but such primes are strongly finite if one of them is not Fermat prime.

Corollary 2.8 If Fermat prime criterion 2.7 is true, then near-square primes of Mersenne primes are infinite.

Proof. Since the sum of original continuous prime number sequence of near-square primes of Mersenne primes i.e. 2+3=5 is a Fermat prime i.e. F_1 and the first near-square prime of Mersenne prime W_2=17 is also a Fermat prime i.e. F_2, by Fermat prime criterion 2.7 near-square primes of Mersenne primes are infinite.

Proposition 2.9 Primes of the form $2x^2–1$ are infinite.

Proof. By Corollary 2.8 near-square primes of Mersenne primes are infinite and near-square primes of Mersenne primes $W_p = 2M_p^2 –1$ are a subset of primes of the form $2x^2–1$, hence primes of the form $2x^2–1$ are infinite.

References

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