Can the Geometry Prove the General Relativity Incorrect?

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Abstract

In this paper it is shown that the General Relativity Theory (GRT), which belongs to a class of metric theories of gravity, is based on a wrong assumption, contradicts the well-established laws of physics, and also its own maximum speed postulate. It is shown, based on simple arguments of geometry, that in the GRT the motion of a test body in an orbit around the centrally gravitating mass does not satisfy the conservation of angular momentum and that the equation describing the equivalence of acceleration with the force of gravity violates the contravariance principle. Finally, it is shown that the GRT also violates the Gauss law. The critique rests on a comparison of the Schwarzschild metric, which is derived from Einstein field equations, with the previously derived author's new metric from which the Schwarzschild metric follows as a first order approximation, but which is not derived from Einstein field equations.

Key words: Schwarzschild Metric; Metric Theory of Gravity; Einstein Field Equations; General Relativity Theory; Black Hole Topology; Gauss Law in GRT; Komar Mass; Ricci Tensor; Critique of General Relativity Theory; Harmonic Coordinates; Isotropic Coordinates.

1. Introduction

A new scientific truth does not triumph by convincing its opponents and making them to see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.

Max Planck

The critique of the General Relativity Theory (GRT) presented in this paper is based on a comparison of the Schwarzschild metric with the author's derived new metric published previously elsewhere [1]. To be historically accurate, it is also necessary to bring attention to the little publicized fact that the originally published Schwarzschild metric [2] is not the same as the metric that is called today the Schwarzschild metric. However, to be consistent with the volumes of published literature the Schwarzschild metric quoted in this paper will be the one that is commonly used. Both metrics, the new one and the today's Schwarzschild, describe the space-time of a non-rotating centrally gravitating body, therefore, the derivations are simple and easily understandable. The metrics also satisfy the well-known four tests of gravitation theory to a precision available in today's observations although in a recent publication [3] it was shown that in the GRT the light deflection by a gravitating body is not calculated correctly, therefore, the GRT actually does not satisfy this famous test.

The general differential metric line element corresponding to either space-time the metrics describe can be written as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\varphi\varphi}d\Omega^2 \ , \qquad (1)$$

where: $d\Omega^2 = \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$ and where the metric coefficients depend only on the radial coordinate. This form of metric assumes that according to the Riemann hypothesis the motion

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can be represented by a curved space-time in which the test bodies move in a free fall along geodesic lines not experiencing any forces in contrast to a flat space-time with the fields and forces that guide the motion. This concept forms the basis for all metric theories of gravity (MTG) and has been also adapted by Einstein in his derivation of general relativity theory. The Einstein's GRT, however, includes additional assumption related to the Ricci tensor that led to the derivation of Einstein field equations with the Schwarzschild metric as a solution. The Riemann principle is thus more general than the GRT and allows derivation of other metrics describing the space-time not only the Schwarzschild metric. For the purposes of this article the details of the metrics derivations are not very important only the derivation assumptions are and whether the metrics' descriptions of reality do not contradict the well-established laws of physics and observational facts.

The reader that is familiar only with the standard GRT and firmly believes that all the assumptions used in its derivation are true and cannot be challenged should keep in mind that in a true science the assumptions can always be questioned and must be constantly justified. It is most likely not possible to prove the GRT incorrect using only the GRT arguments, since from the same assumptions one can only come to the same conclusions. Therefore, in order to resolve the discrepancies pointed out in this paper it is necessary to scrutinize the GRT assumptions and modify them in such a way that an agreement with the available data is obtained and a theory of gravity consistent with the fundamental laws of physics is developed.

2. Definitions of metrics

The metrics and their differential metric line elements that will be studied and compared against each other and against the well-established laws of physics are described next. The first is the well-known and celebrated Schwarzschild metric:

$$ds^2 = (1 - R_s / r)(cdt)^2 - (1 - R_s / r)^{-1} dr^2 - r^2 d\Omega^2, \tag{2}$$

with its metric coefficients defined as: $g_{tt} = 1 - R_s / r$, $g_{rr} g_{tt} = 1$, and $g_{\varphi\varphi} = r^2$, where the usual parameters are defined as: $R_s = 2\kappa\, M / c^2$, M is the mass of the main body, c is the local intergalactic vacuum speed of light, and κ is the gravitational constant. The second is the new metric derived by the author elsewhere [1] with its differential metric line element as follows:

$$ds^2 = e^{-R_s/\rho}(cdt)^2 - e^{R_s/\rho} dr^2 - \rho^2 e^{-R_s/\rho} d\Omega^2, \tag{3}$$

where the function variable $\rho = \rho(r)$ is found from the differential equation:

$$d\rho = \sqrt{g_{rr}}\, dr = e^{R_s/2\rho} dr, \tag{4}$$

and where the metric coefficients are: $g_{tt} = \exp(-R_s / \rho)$, $g_{rr} g_{tt} = 1$, and $g_{\varphi\varphi} = \rho^2 g_{tt}$.

3. Meaning of coordinates and the topology of a “Black Hole”

This section may not be important for the derivations, but it may be helpful to some readers since it clearly defines the terms used in this paper and explains their meaning. The literature sometimes defines the same terms differently and this may cause confusion.

In this paper only two types of coordinates are used. The natural or standard coordinates that are for example appearing in the metric line elements given in Eq.1, Eq.2, and Eq.3, and the physical coordinates, that are defined similarly as, for example, in Eq.4. It is important to realize that the physical coordinates are not directly observable, must be computed from some suitable physics formula (that is why the name physical), are unique and not affected or changed by the gravity, by adding a massive body into the space-time. The natural coordinates, on the other hand, are observable and directly measurable by clocks and sticks. The amount of distortion of natural coordinates is given by the metric coefficients and the gravity induced distortion affects everything in the natural space-time including our bodies since we are living in this space-time. The clocks and the measuring sticks are, of course, also affected by the gravity. The recently published paper [3] describes the experimental proof that the natural coordinates correspond to reality and that the various other coordinates and coordinate transformations popular in the literature such as, for example, harmonic or isotropic coordinates are just meaningless mathematical manipulations that do not have any basis in reality. From the coordinates definitions it is obvious that the physical coordinate differentials and the natural coordinate differentials become identical far away from the gravitating bodies where the gravity has no effect. It is therefore also clear that the space-time distortion can be easily evaluated against the physical coordinates when the metric coefficients are known. An example for the Schwarzschild metric is given below, where the physical coordinate radius $\rho_s(r)$ is calculated according to Eq.4, since for this metric it can be calculated analytically:

$$\rho_s(r) = \int_{R_s}^{r} \frac{dr}{\sqrt{1-R_s/r}} = R_s \ln\left(\sqrt{\frac{r}{R_s}-1} + \sqrt{\frac{r}{R_s}}\right) + R_s\sqrt{\frac{r}{R_s}-1}\sqrt{\frac{r}{R_s}} \,. \tag{5}$$

The physical radius is not distorted by gravity, as already mentioned, it is identical for either metric discussed in this paper and it actually expresses the distortion of the natural coordinate radius as an inverse of the function in Eq.5 that cannot be analytically calculated. The 3D plots of dependence of the physical coordinate radius plotted as a function of the natural coordinate radius are shown in Fig.1 for the Schwarzschild metric and for the new metric, both normalized to R_s. The new metric indicates that the gravitating body compresses the natural space-time as if it were pushed into the body and rolled up into a compact massive knot. In the Schwarzschild metric space-time, on the other hand, the addition of a massive body first stretches the natural space-time, breaking up its contiguity by making an empty void in it, and then after certain distance: $\rho_s = r_n = 1.467396...$, again compresses this space-time. This does not seem reasonable, since only a monotonic compressive space deformation is expected. This strange and unnatural deformation thus creates an important inaccessible region $r_n \leq 1$ called the Black Hole (BH) bounded by the so called event horizon at $r_n = 1$. As the mass of the body changes the size of the BH also changes, but the physical coordinates and the natural coordinates must match at infinity and this helps to visualize why the physical coordinates remain unchanged. There are many papers published in the literature where through the introduction of various coordinates and coordinate transformations, for example the Kruskal-Szekeres coordinates, the space-time is analytically extended into the BH. It is also claimed that inside of the BH resides the famous Schwarzschild singularity. However, this is a pure speculation not based on any evidence or any observation from reality. It is also worth noting that the original Schwarzschild metric [2] did not include this singularity. From the definition of the physical radius given above it is clear that the space-time extension into the BH does not make sense, since the physical radius already covers the entire space $0 \leq \rho \leq \infty$ and cannot be negative. The extension that might possibly be meaningful, based on the rational geometry considerations, could be made only in the positive direction of the physical coordinate radius, forming a dome shaped surface or be flat. It is also clear that such an extension cannot be analytic. The BH boundary is therefore a singular membrane in the 3D space at the Schwarzschild radius where all the mass should then reside.

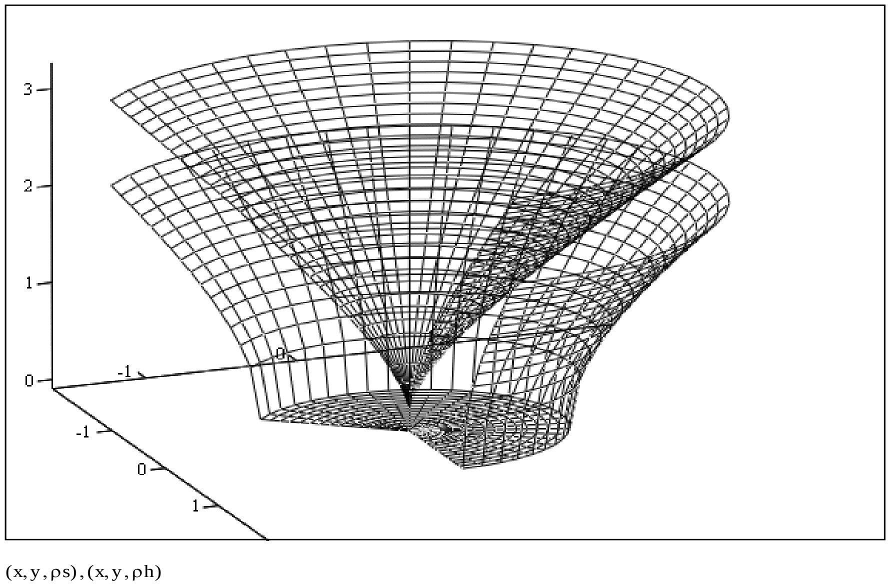

Fig.1. Using coordinates normalized to R_s the plots are showing the dependency of the physical radius ρ_s for the Schwarzschild metric and ρ_h for the new metric on the natural coordinate radius $r_n = \sqrt{x^2 + y^2}$. For the Schwarzschild metric the inaccessible Black Hole region $r_n < 1$ bounded by the event horizon at $r_n = 1$ is located at $\rho_s = 0$. There is no analytic extension shown into the interior of the BH since the physical radius ρ_s cannot have negative values. For the new metric the plot terminates at the surface of the compacted gravitating mass at $r_n \sim 0.01$ or equivalently at $\rho_h = 1/4$. There is no BH region for this metric there is always $\rho_h > 0$ for any $r_n > 0$.

Furthermore, any idea of mapping the BH interior must be rejected also on the purely logical grounds. Since it is not possible in principle to communicate with the interior and return any data from it there is no meaning in constructing any theories about it. There is no possibility of the theory falsification or verification. Any theory of the BH interior including the massive point singularity thus becomes a matter of belief, belonging to the realm of religion instead of science. The BH mass manifests its presence by the gravitational field only outside of the Schwarzschild radius, so the BH mass must be located at most at $r_n = 1$ following the simple logical reasoning. This conclusion can also be supported by the positivistic philosophical point of view, which would claim that the BH interior in a sense of any measurable space-time actually does not exist.

In order to better understand this problem and the fundamental differences between the studied metrics it is interesting to calculate the time of fall of a test body from a certain distance to the surface of the centrally gravitating body as observed by a distant observer and by an observer riding on the test body. For the Schwarzschild metric the time of fall is infinite when observed by a distant observer even if the test body is infinitesimally close to the BH's event horizon. This is a clear function discontinuity, which again does not seem reasonable and physically realistic. For the co-moving observer the fall time is always finite but there is no indication that he should be crossing the event horizon after reaching it. Joining the BH mass at the event horizon is the unceremonious end of his journey. For the new metric both times are finite, described by continuous functions, as expected. The results of the falling time calculations normalized to R_s/c are shown in Fig.2.

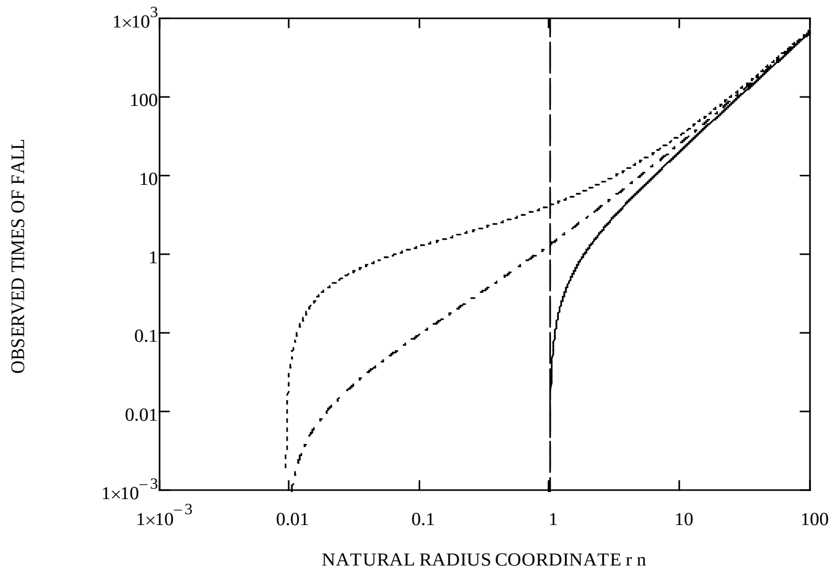

Fig.2. The graphs show the times of fall normalized to R_s/c from a given distance normalized to R_s to the surface of the gravitating body for a distant observer and for an observer co-moving with the test body. The dotted and the dot-dashed lines represent the new metric falling times for a distant and co-moving observer respectively. The solid line represents the falling time of the co-moving observer for the Schwarzschild metric. The distant observer falling time for the Schwarzschild metric is infinite, represented by a discontinuous vertical dashed line step function from zero to infinity.

Therefore, in this paper it is considered that at the Schwarzschild radius the singularity is real, not only a coordinate singularity as is commonly believed, and that there is an empty void inside of the BH that cannot be reached from the exterior region and where the space-time loses its meaning. The disadvantage of various coordinate transformations that are used for mapping the interior of BHs, particularly when the time and space coordinates are mixed together, is that their meaning and the relation to the natural and physical coordinates used in this paper is lost. When new coordinates or coordinate transformations are introduced it is necessary to always clearly identify whether they can be observed and how they are used in measurements. This is often not done and this leads to only mathematical manipulations without any physical meaning behind them or a connection to reality. This paper makes an effort to avoid this problem.

There is no impact of the modified BH model on the derivations presented in this paper, since it is clear that the interior of the BH does not have to be mapped by any coordinate system and taken into account, in particular when it is shown that other metrics exist describing the space-time of the centrally gravitating body without the event horizon and the BH singularity.

4. General MTG equations

The metrics introduced in section **2** will be tested and evaluated by studying the motion of small test bodies in the space-times that these metrics describe. In order to simplify this task only two kinds of motion will be investigated: the radial motion and the orbital motion in a circular orbit. It is therefore advantageous to first derive general equations for these motions based on Eq.1 and then apply them to the respective metrics.

First, for the circular orbital motion, using the well-known Lagrange formalism, considering for simplicity the motion only in the equatorial plane: $\vartheta = \pi / 2$, the Lagrangian describing such a motion of a small test body is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 - g_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 . \tag{6}$$

The first integral of Euler-Lagrange (EL) equation derived from the variational principle $\delta\int_\tau L d\tau = 0$ that corresponds to the time coordinate is:

$$g_{tt}\left(\frac{dt}{d\tau}\right) = k , \tag{7}$$

where k is an arbitrary constant of integration typically set to $k = 1$ when it is necessary to satisfy the initial condition at infinity. The EL equation of motion corresponding to the radial coordinate is as follows:

$$-\frac{d}{d\tau}\left(2g_{rr}\frac{dr}{d\tau}\right) = \dot{g}_{tt}\left(\frac{cdt}{d\tau}\right)^2 - \dot{g}_{rr}\left(\frac{dr}{d\tau}\right)^2 - \dot{g}_{\varphi\varphi}\left(\frac{d\varphi}{d\tau}\right)^2 , \tag{8}$$

where the dot represents the partial derivative with respect to the radial coordinate. Since for the circular orbit the radial coordinate is constant with $dr / d\tau = 0$ and $d^2 r / d\tau^2 = 0$, Eq.8 becomes:

$$\left(\frac{d\varphi}{dt}\right)^2 = c^2 \frac{\dot{g}_{tt}}{\dot{g}_{\varphi\varphi}} , \tag{9}$$

where Eq.7 was used to eliminate the variable τ. Considering now that the natural coordinate orbital time t_o, which is the observable quantity referenced to the central mass coordinate system, is found when the angle is set to: $\varphi = 2\pi$, the following equation results:

$$t_o = \frac{2\pi}{c}\sqrt{\frac{\dot{g}_{\varphi\varphi}}{\dot{g}_{tt}}} . \tag{10}$$

This is a general formula that can be used for any metric of the form given in Eq.1 describing the space-time of a non-rotating centrally gravitating body. More details about this formula and the Kepler's third law have been published elsewhere [4]. From the Lagrangian in Eq.6 also follows the general formula for the conservation of angular momentum. The first integral corresponding to the angular coordinate is:

$$g_{\varphi\varphi}\frac{d\varphi}{d\tau} = k\alpha . \tag{11}$$

Dividing Eq.11 by Eq.7 replaces the invariant $d\tau$ by the natural time coordinate differential dt, which is an observable parameter. The result is the general formula for the conservation of angular momentum, which is valid in any MTG metric that satisfies the form given in Eq.1:

$$\frac{g_{\varphi\varphi}}{g_{tt}}\frac{d\varphi}{dt} = \alpha . \tag{12}$$

In Eq.12 it might seem reasonable to expect that the ratio $g_{\varphi\varphi}/g_{tt}$ should be equal to the natural radial distance squared. However, a similar conservation of angular momentum formula: $\rho^2 d\varphi_{ph}/dt_{ph} = \alpha$ must also be valid in the flat physical space-time, therefore, it must hold that: $g_{\varphi\varphi}/g_{tt} = \rho^2$. The proof follows from Eq.12, by substituting for the natural angle and the natural time the relations: $\sqrt{g_{\varphi\varphi}}d\varphi = \rho d\varphi_{ph}$, and $\sqrt{g_{tt}}dt = dt_{ph}$. The formula: $g_{\varphi\varphi}/g_{tt} = \rho^2$ can thus be considered an equivalent expression for the conservation of angular momentum. The validity of this formula will be reconfirmed again later since it must be consistent with the Kepler's third law, the Gauss law, and the Newton gravitational law. The new metric coefficients satisfy this relation, while the Schwarzschild metric coefficients do not. The conservation of angular momentum formula in Eq.12 can also be evaluated for circular orbits by plotting the integration constant α squared as a function of the natural radial distance. For the Newton case this function is linear equal to: $\alpha_n^2 = \kappa M r$. Using Eq.9 and Eq.12 the result for the Schwarzschild metric is:

$$\alpha_s^2 = \frac{\alpha_n^2}{(1 - R_s/r)^2}, \tag{13}$$

and for the new metric:

$$\alpha_h^2 = \alpha_n^2 \frac{\rho(r)/r}{1 + R_s/2\rho(r)}. \tag{14}$$

For a better clarity the graphs of these dependencies normalized to α_n^2 are shown in Fig.3.

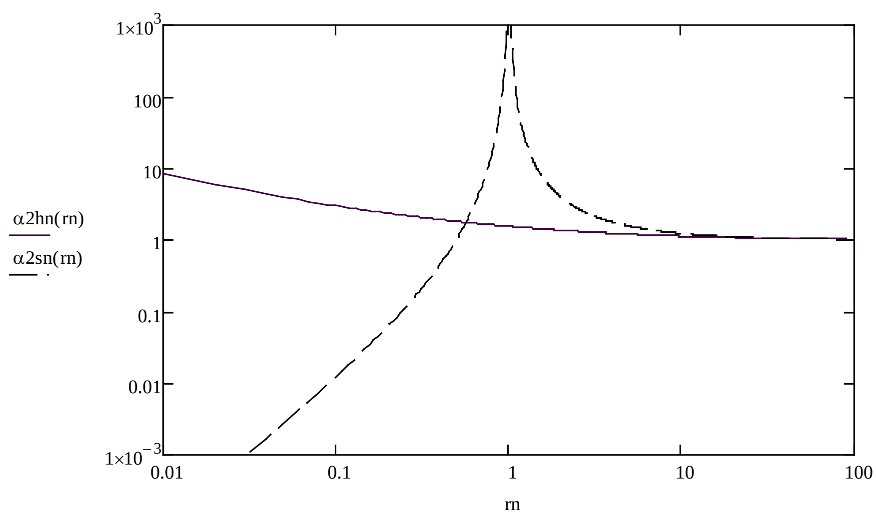

Fig.3. The graphs show the angular momentum integration constants for the circular orbits squared and normalized to α_n^2 that are plotted as functions of the natural radial distance normalized to R_s. For the Schwarzschild metric (dashed line) the constant becomes infinite at the Schwarzschild radius and negative in the interior of BH (the negative value is not shown in the graph since only the absolute values were plotted). For the new metric the normalized constant α_h^2/α_n^2 is always finite and significantly departs from unity only for the orbits that are close to the surface of the ultimately compacted body, which is located at the $r_n \sim 0.01$.

The infinite value of the angular momentum integration constant α_s for the Schwarzschild metric at the Schwarzschild radius is not reasonable and cannot correspond to reality.

In the second step, turning the attention to tests where the test body moves only in the radial direction, the Lagrangian describing such a motion is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 . \qquad (15)$$

Since the Lagrangian itself is also the first integral equal to $L = c^2$ it is simple to derive the following relations:

$$g_{tt}\left(\frac{dt}{d\tau}\right) = k , \qquad (16)$$

$$g_{rr} g_{tt}\left(\frac{dr}{d\tau}\right)^2 = c^2 k^2 - c^2 g_{tt} . \qquad (17)$$

After some rearrangement and differentiation of Eq.17 with respect to τ, assuming that the metric coefficients are functions of the gravitational potential, the following equation is obtained:

$$\frac{d^2 r}{d\tau^2} = -\frac{c^2}{2}\frac{\partial}{\partial\varphi_n}\left(\frac{1}{g_{rr}} - \frac{k^2}{g_{rr} g_{tt}}\right)\frac{\partial\varphi_n}{\partial r} . \qquad (18)$$

In order to maintain the same contravariance character on both sides of Eq.18 it is easily seen that the partial derivative of the terms in parenthesis must be a component of a contravariant metric tensor. This is well known from the tensor calculus since the metric tensor is used to raise or lower indices of covariant or contravariant geometric objects. There are only two possibilities how to satisfy this requirement since there are only two components of the metric tensor available in the radial motion case; the parenthesis terms derivative equal either to g^{rr} or to g^{tt}. The solution for the spherical coordinates for an arbitrary k is: $g_{rr} g_{tt} = 1$, or alternately written as: $g_{tt} = 1/g_{rr} = g^{rr}$, knowing that the equatorial plane coordinate system is orthogonal. This makes the parameterized acceleration (the natural radius differentiated twice with respect to the invariant $d\tau$) independent of the parameterized velocity and simplifies Eq.18 into:

$$\frac{d^2 r}{d\tau^2} = -\left(\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\varphi_n}\right) g^{rr}\frac{\partial\varphi_n}{\partial r} . \qquad (19)$$

Furthermore, the parenthesis term in Eq.19 must be equal to unity, therefore it must hold that:

$$\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\varphi_n} = 1, \qquad (20)$$

in order to obtain the Newton-like equation with the gravitating force being equal to acceleration according to the Einstein equivalence principle:

$$\frac{d^2 r}{d\tau^2} = -g^{rr} \frac{\partial \varphi_n}{\partial r} . \tag{21}$$

The relation $g_{rr} g_{tt} = 1$ is well known from the Schwarzschild metric and can also be derived for the new metric from the condition that the natural space deformation by the gravity is locally isotropic. The second solution of Eq.18 is: $g_{rr} = 1$ corresponding to cylindrical coordinates that are not discussed in this paper. The solution of Eq.20 that is valid for either coordinate system is easily found, assuming the zero potential boundary condition at infinity, as follows:

$$g_{tt} = e^{2\varphi_n / c^2} . \tag{22}$$

This equation thus allows calculating the gravitational potential and the gravitational field intensity from the metric coefficient standing by the time coordinate. It is interesting to note that Eq.22 and the condition $g_{rr} g_{tt} = 1$ follow from Eq.1 once it is assumed that the gravitational field has a potential and that the metric coefficients depend on it.

Finally, once the gravitational field intensity is known it should also satisfy the Gauss law as any other standard field intensity. The Gauss law relates the field intensity integrated over an arbitrary but enclosed surface to the amount of the enclosed rest mass that generates the field, similarly as in the Maxwell's EM field theory where the integrated electrical field intensity is related to the enclosed charge. In order to avoid the possibility of a mistake it is best to first write the Gauss law formula in the flat physical space-time notation as follows:

$$\oint_S \frac{\partial \varphi_n}{\partial \rho} \rho^2 dS = 4\pi\rho^2 \frac{\partial \varphi_n}{\partial \rho} = 4\pi\kappa M . \tag{23}$$

Since the integrating surface S can be spherical the integration was easily carried out. The formula can be then converted to natural space-time variables by substituting for the potential from Eq.22 into Eq.23 and the Gauss law written in terms of the metric coefficients as:

$$\frac{g_{\varphi\varphi}}{g_{tt}\sqrt{g_{tt}}} \frac{\partial g_{tt}}{\partial r} = R_s . \tag{24}$$

More details of derivation are given in the Appendix. Any metric of MTG describing the non-rotating centrally gravitating body with mass M should thus also satisfy the condition in Eq.24. The validity of the Gauss law suggests that the field energy, which is negative or removed from the space around the gravitating body, was converted by some sort of condensation process or the space-time phase change into the tangible gravitating mass of the body. This also implies that the total energy W of the field plus the energy equivalent of the condensed mass of the body can be zero and that the gravitation field around the central body does not have any equivalent tangible gravitating rest mass. The total energy can then be calculated as follows:

$$W = Mc^2 + \int_0^M \varphi_n dM . \tag{25}$$

This becomes for the new metric at the physical mass equivalent radius $\rho_e = R_s / 4$ or equivalently at the natural mass equivalent radius $r_e = 0.009384... \cdot R_s$ equal to:

$$W = Mc^2 - \frac{\kappa M^2}{2\rho_e} = 0. \tag{26}$$

The mass equivalent radius or diameter is the ultimate size and the final stage of the gravitational collapse of a very massive gravitating star. For the Schwarzschild metric, however, the result is:

$$W = \frac{1}{2}Mc^2 - \frac{1}{2}Mc^2\left(\frac{r_{es}}{R_s} - 1\right)\ln\left(1 - \frac{R_s}{r_{es}}\right) > 0. \tag{27}$$

There is no natural mass equivalent radius r_{es} here that would make the total energy zero for this space-time and thus stop the gravitational collapse. The collapse continues for eternity [5]. The energy to create this space-time with masses and fields in them therefore has to be supplied from somewhere else and from an unknown source. To correct this problem in the classical BH theory, it should be considered that one half of the BH mass is trapped in the void in form of a negative energy [5], which again cannot be in principle falsified or verified.

To summarize the results obtained in this section, it is convenient to arrange all the conditions that were found and that any MTG must satisfy into a table as shown below in **Table I**. These are the general formulas independent of the GRT assumption about the Ricci tensor or equivalently the mass-energy tensor T_{jk}, or any other assumptions that could be construed as a field theory, in addition to the metric such as for example in Ref. [6], that the metrics must satisfy.

Table I: Conditions that any metric of the form given in Eq.1 must satisfy.

1	Time coordinate metric coefficient	$g_{tt} = e^{2\varphi_n/c^2}$
2	Gravitational potential from g_{tt}	$\varphi_n = c^2 \ln\sqrt{g_{tt}}$
3	Radius and time metric coefficients relation	$g_{rr}g_{tt} = 1$
4	Conservation of angular momentum	$\frac{g_{\varphi\varphi}}{g_{tt}}\frac{d\varphi}{dt} = \alpha$
5	Orbital time, the Kepler's 3rd law for circular orbits	$t_o = \frac{2\pi}{c}\sqrt{\frac{\dot{g}_{\varphi\varphi}}{\dot{g}_{tt}}}$
6	Gauss law formula for the enclosed rest mass	$\frac{g_{\varphi\varphi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}} = R_s$
7	Acceleration-g force equivalence formula	$\frac{d^2r}{d\tau^2} = -g^{rr}\frac{\partial\varphi_n}{\partial r}$
8	Total energy; field plus mass equivalent energy	$Mc^2 + \int_0^M \varphi_n dM = 0$

There are four conditions in **Table I** defining the metric coefficients (rows 1,3,4,6) and only three metric coefficients in the metric line elements. It is therefore obvious that there must be some interdependency between the coefficients. By inspecting the formulas it is easy to conclude that all the parameters are defined, except for the constant R_s, when it is considered that the angular momentum of an orbiting test body is conserved and the relation: $g_{\varphi\varphi}/g_{tt}=\rho^2$ holds. Also, as will become apparent later from **Table II**, the gravitational potential at large distances approaches the classical Newton gravitational potential as it should in order to obtain an agreement with observations. This is reasonable, since it is well known that the Newton gravitational potential unlike the Schwarzschild gravitational potential satisfies the condition of minimum field energy: $\delta\int_\rho(\partial\varphi_n/\partial\rho)^2\rho^2 d\rho=0$ as shown in more detail in the Appendix. The above reasoning can be reversed and restated that from the Newton gravitational potential: $\varphi_n=-\kappa\, M/\rho(r)$ follows the equivalent formula for the conservation of angular momentum: $g_{\varphi\varphi}/g_{tt}=\rho^2$ when the Gauss law is satisfied. Finally, it is possible to conclude that if the above conservation of angular momentum formula and the Newton gravitational potential formula hold, then the Gauss law in Eq.24 must be satisfied and the gravitational field cannot have any tangible gravitating rest mass. If one metric coefficient is specified incorrectly, as is for example in the Schwarzschild metric where: $g_{\varphi\varphi}=r^2$, this violates observations[3], the mathematical consistency of the metric, or some well-known law of physics as is discussed in more detail in section 5.

Adding the condition of Ricci tensor, or equivalently the mass-energy tensor T_{jk}, equal to zero in deriving Einstein field equations for the centrally gravitating body and from that the Schwarzschild metric is thus not permissible and is over specifying the problem. The Ricci tensor and the mass-energy tensor are the derived geometrical objects that must be computed from the particular metric. The metric cannot be calculated by arbitrarily specifying the mass-energy tensor first as is postulated in the GRT.

5. Metrics evaluation

The two metrics introduced in the metric definition section **2** will now be investigated and compared according to the criteria listed in **Table I**. Since the formulas are simple it is easy to substitute the corresponding metric coefficients into them and construct another table, **Table II**, for the metrics of interest. By inspecting the formulas in **Table II** row by row it becomes easy to compare the metrics and arrive at the following conclusions:

1. The Schwarzschild metric has a problem at the Schwarzschild radius, called the event horizon. There is no problem for the new metric at that radius; no Black Holes exist where a mass would be hidden behind the event horizon. The metric coefficient standing by the time coordinate must be an exponential function of the gravitational potential, the Schwarzschild metric does not satisfy this requirement.
2. The Schwarzschild gravitational potential when calculated from the metric coefficient is infinitely negative at the event horizon. No radiation or BH evaporation is thus possible. There is no problem for the new metric at the event horizon. The potential for the new metric resembles the Newton gravitational potential that satisfies the minimum field energy condition. The only difference is that the natural coordinate radius is replaced by the physical

coordinate radius following Eq.4. The minimum of this gravitational potential at the surface of the body that has collapsed to its ultimate minimum size is equal to: $\varphi_{n\min} = -2c^2$.

3. The relations between the time and the radius metric coefficients are the same for both metrics. This is the only condition that the Schwarzschild metric satisfies correctly.
4. The Schwarzschild metric does not seem to conserve the angular momentum of orbiting test bodies. There is zero angular momentum at the Schwarzschild radius if the classical formula is used. The BHs should not rotate [5] contrary to empirical evidence of high speed rotation of compact astronomical objects. On the other hand, if the rotation is accepted and the orbit time of test bodies follows the Kepler's third law as discussed below, then the angular momentum integration constant α_s must be infinity at the Schwarzschild radius and reverse its sign inside of the BH. This is again a strange behavior and an inconsistency that cannot be reflecting the real world. The new metric keeps the finite α_h and therefore preserves the conservation of angular momentum even if the test bodies orbit close to the surface of the ultimately compacted main body. The conservation of angular momentum is a well know fundamental law of physics. It is thus necessary that for the angular metric coefficient the formula: $g_{\varphi\varphi} = \rho^2 g_{tt}$ is always satisfied. This is violated by the Schwarzschild metric.
5. For the Schwarzschild metric the orbital time (the Kepler's third law) approaches zero when the mass of the centrally gravitating body tends to very large values. This implies an infinite orbital speed. For example: at the radius $r = R_s / 2$ the test bodies would whiz around with the vacuum speed of light inside of the BHs, and for smaller radii even faster, assuming that BHs have any interior regions according to the classical view. The event horizon does not seem to pose a problem here. It is also strange that this formula is identical with the formula derived classically from the Newton inertial and gravitational laws that are valid only in a flat space-time. There are no effects from the curvature of space-time included in the classical formula. For the new metric, however, there is a limit equal to the physical orbital length divided by the vacuum speed of light and an effect from the space-time curvature. This is interesting and expected from any reasonable theory of gravity, but fails in the GRT. The detail discussion of this problem has been published elsewhere [4].
6. An important finding that is disturbing is that the gravitational field of the Schwarzschild metric does not satisfy the typical Gauss law. The rest mass, when calculated from the field, should not depend on the radius of the integrating spherical surface once all the rest mass generating the field has been fully enclosed. The fact that the rest mass depends on r is a fatal flaw of GRT particularly when the new metric has no problem with it. It seems that in the GRT the field calculated rest mass is infinite at the Schwarzschild radius and then steadily diminishes approaching M as if the field had a negative compensating tangible rest mass. This is not reasonable, no negative tangible field rest mass with a required density has ever been found. The new metric result that the gravitational field has no tangible gravitating rest mass is interesting and in some sense similar to claims used in the GRT that in an empty space around the gravitating body the mass energy tensor T_{jk} is zero. This finding places a restriction on the validity of the famous Einstein energy-mass equivalence formula: $E = m_i c^2$. This formula holds true for the inertial mass in both directions, but for the gravitational mass this is not always true. For example, photons do not have a gravitating mass [1,7]: $m_{gph} \neq E / c^2$.

7. It is surprising and strange that for the Schwarzschild metric the inertial and gravitational forces follow the classical Newton laws formulas, except for the natural time being replaced by the metric invariant τ, without any effect from the curvature of space-time. The new metric formula includes these effects. It is also necessary that the contravariant character of geometric objects from the natural space-time is equal on both sides of the equation. The Schwarzschild metric formula does not seem to satisfy this elementary requirement of tensor calculus and geometry.
8. Finally, the field energy plus the mass equivalent energy of the centrally gravitating body being zero for the new metric is an appealing property also from the philosophical point of view, since it balances the energy of the entire visible mass of the Universe to zero. The current Schwarzschild metric model obviously does not offer this possibility.

Table II: Metrics evaluation according to criteria given in **Table I**.

	Evaluation Parameter	**Schwarzschild Metric of GRT**	**Author's New Metric**
1	Time coordinate metric coefficient	$g_{tt}=1-\frac{R_s}{r}$	$g_{tt}=e^{\frac{-R_s}{\rho}}$, $d\rho=e^{\frac{R_s}{2\rho}}dr$
2	Gravitational potential from g_{tt}	$\varphi_s=\frac{c^2}{2}\ln\left(1-\frac{R_s}{r}\right)$	$\varphi_n=-\frac{\kappa M}{\rho(r)}$
3	Radius and time coefficients relation	$g_{rr}g_{tt}=1$	$g_{rr}g_{tt}=1$
4	Conservation of angular momentum	$r^2\frac{d\varphi}{dt}=\alpha\left(1-\frac{R_s}{r}\right)$	$\rho^2\frac{d\varphi}{dt}=\alpha$, $\frac{g_{\varphi\varphi}}{g_{tt}}=\rho^2$
5	Orbital time, the Kepler's 3rd law for circular orbits	$t_{os}=2\pi\sqrt{\frac{r^3}{\kappa M}}$	$t_{oh}=2\pi\sqrt{\frac{\rho^3}{\kappa M}+\frac{\rho^2}{c^2}}$
6	Gauss law formula for the enclosed rest mass	$\frac{g_{\varphi\varphi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}}=R_s\left(1-\frac{R_s}{r}\right)^{\frac{-3}{2}}$	$\frac{g_{\varphi\varphi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}}=R_s$
7	Acceleration-g force equivalence formula	$\frac{d^2r}{d\tau^2}=-\frac{\kappa M}{r^2}$	$\frac{d^2r}{d\tau^2}=-\frac{\kappa M}{\rho^2}\sqrt{g^{rr}}$
8	Total energy; field plus mass equivalent energy	$Mc^2+\int_0^M\varphi_s dM>0$	$Mc^2-\frac{\kappa M^2}{2\rho_e}=0,\ \rho_e=\frac{R_s}{4}$

From this summary, and also from the recent publication [3] where it is shown that the traditional calculation of the light deflection by a gravitating body in the GRT is not correct, it is clear that the Schwarzschild metric does not correspond to reality. From this conclusion, therefore, also follows that the GRT is not the correct theory of gravity and should be abandoned. The assumption about the Ricci tensor (or equivalently the mass energy tensor T_{jk}) being zero used in the derivation of Einstein field equations is not permissible. Einstein field equations and the Schwarzschild metric should not be used in a search for the correct model of gravity, in a search for the Gravity Waves, for modeling of the Universe, or in any other advanced theory such as the various string theories. The Schwarzschild metric is only a first order approximation for the weak gravitational field as is shown in the next section.

6. Weak field approximation

The first order approximation for the new metric time coordinate coefficient g_{tt} is found by integrating Eq.4 as follows:

$$r = \int_0^{\rho} e^{\frac{-R_s}{2\rho}} d\rho = \frac{R_s}{2} \int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x^2} dx = \rho e^{\frac{-R_s}{2\rho}} - \frac{R_s}{2} \int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x} dx = \rho e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2} Ei\left(-\frac{R_s}{2\rho}\right), \quad (28)$$

where $Ei(x)$ is the Euler exponential integral function and where $x = R_s / 2\rho$. Unfortunately there is no analytic expression for ρ as function of r, so the approximation needs to be found iteratively. For large distances ($0 < x << 1$), the Euler exponential integral is approximated as:

$$Ei(-x) = \ln(\gamma' x) + \ldots, \quad (29)$$

where $\gamma' = 1.781072...$ and $\ln(\gamma') = \gamma$ is the famous Euler constant $\gamma = 0.577215...$. It is therefore possible to write the following:

$$\rho e^{\frac{-R_s}{2\rho}} = r - \frac{R_s}{2} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + \ldots. \quad (30)$$

Rearranging this result as follows:

$$\frac{1}{\rho} = \frac{1}{r} e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2r\rho} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + \ldots = \frac{1}{r} - \frac{R_s}{2r\rho} + \frac{R_s}{2r\rho} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + \ldots, \quad (31)$$

and substituting for $1/\rho$ from the left hand side of Eq.31 to the right hand side, the iterative expression for $1/\rho$ valid for large distances becomes:

$$\frac{1}{\rho} = \frac{1}{r} - \frac{1}{2} \frac{R_s}{r^2} \left[1 + \ln\left(\frac{2r}{\gamma' R_s}\right)\right] + \ldots. \quad (32)$$

From this formula then follows the approximation for g_{tt}:

$$g_{tt} = e^{\frac{-R_s}{\rho}} = 1 - \frac{R_s}{r} + \frac{R_s^2}{r^2} \left(1 + \ln\sqrt{\frac{2r}{\gamma' R_s}}\right) + \ldots. \quad (33)$$

In the next step the second order term in Eq.33 can be neglected since the increase of the logarithmic function of $\sqrt{r}$ is slower than the first power of r and R_s is always very much smaller than r. This leads to the familiar formula for the metric coefficient g_{tt}:

$$g_{tt} = 1 - \frac{R_s}{r} + \ldots. \quad (34)$$

Similarly, the first order approximation for the new metric angular coefficient $g_{\varphi\varphi}$ is found from Eq.30, which can be rearranged and approximated when *r* tends to large values as follows:

$$\rho e^{\frac{-R_s}{2\rho}} = r - \frac{R_s}{2}\ln\left(\frac{\gamma' R_s}{2\rho}\right) + \ldots = r\left[1 + \frac{R_s}{r}\ln\sqrt{\frac{2r}{\gamma' R_s}}\right] + \ldots \underset{r\to\infty}{=} r . \tag{35}$$

Substituting these approximations into Eq.3 the formula for the differential metric line element then becomes the celebrated Schwarzschild metric differential metric line element:

$$ds^2 = \left(1 - \frac{R_s}{r}\right)(cdt)^2 - \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 . \tag{36}$$

However, the convergence of the square bracket in Eq.35 to unity is slow, which eventually leads to a failure of the Schwarzschild metric to satisfy the conservation of angular momentum of orbiting test bodies and the failure to agree with the light bending experiments, as described in more detail elsewhere [3], when the Schwarzschild limit value is used for this metric coefficient.

The derived approximation also allows finding a condition that needs to be satisfied to obtain a reasonable description of the space-time geometry by this metric for large *r*, ($r > R_s$). The condition is:

$$\frac{r}{R_s} >> 1 + \ln\sqrt{\frac{r}{R_s}} . \tag{37}$$

From the above derivations there is no doubt that the Schwarzschild metric is only the first order approximation of the metric introduced in Eq.3 and thus only the first order approximation of reality. The reality is being defined here as a space-time described by a metric whose time metric coefficient is an exponential function of gravitational potential, the metric that does not permit larger than the speed of light velocities, and where the test body trajectories conserve the angular momentum. Of course it is also necessary that the four tests of GRT are satisfied: the Mercury perihelion advance, the light bending by the Sun, the Shapiro delay, and the gravitational red shift, which the new metric satisfies [11,3]. Since the Schwarzschild metric is the correct and unique solution of Einstein field equations for the spherical case, according to the well-known Birkhoff theorem [8], therefore, there can be only one inescapable conclusion that Einstein field equations yield only the first order approximations of the correct metrics when the energy-momentum tensor T_{jk} and thus the Ricci tensor are set to zero resulting in all the problems and discrepancies described in this paper. While the study of Einstein field equations and various Einstein Spaces described by these equations can be an interesting and intellectually rewarding mathematical exercise with a large amount of work already devoted to this topic [9], it is clear that very little of this work can actually be applied to reality.

7. Conclusions

In this paper it was clearly shown, based on the simple arguments of geometry and on the well-established fundamental principles of physics, that the GRT is a wrong theory of gravity and that the Schwarzschild metric, which is the unique solution of Einstein field equations, is only a first order approximation of reality. It is thus concluded that Einstein field equations and their various derivatives are not correct and should be abandoned. Therefore, using Einstein field equations in search for the metric to model the strong gravitational field, to study the propagation and detection of gravitational waves, or on a larger scale to model the Universe, or using them in various string theories, leads to wrong results that do not correspond to the real world. The assumption about the Ricci tensor, or equivalently the mass-energy tensor, being zero in an empty space around the gravitating body is not correct, the zero is not a permissible value for these tensors when the centrally gravitating mass generating the field is not zero. If a correct mass-energy tensor corresponding to reality were used in Einstein field equations the equations would become identities without yielding any useful metric solutions. The basic concept of Einstein field equations is thus fundamentally ill conceived.

Appendix

The details of the derivation of Eq.24 using the new metric and starting from Eq.23, which is expanded and written below, again for a convenience, are as follows:

$$\oint_S \frac{\partial \varphi_n}{\partial \rho}\rho^2 dS = 4\pi\rho^2 \frac{\partial \varphi_n}{\partial \rho} = 4\pi\left(g^{rr}\frac{\partial \varphi_n}{\partial r}\right)\rho^2\sqrt{g_{rr}} = 4\pi\kappa M \ . \qquad \text{(A1)}$$

The term in the parenthesis can be identified with the contravariant radial component of the gravitational field intensity vector $E^r_{(g)}$ and the square root term associated with the covariant component of the surface area vector $n_r = \rho^2\sqrt{g_{rr}}$, which is perpendicular in this case to the unity radius integrating spherical surface: $\oint_S dS = 4\pi$. Eq.A1 can thus be rewritten in the familiar integral formula for the Gauss law as is also well known in the Maxwell EM field theory:

$$\oint_S \vec{E}_{(g)} \cdot \vec{n}\ dS = 4\pi\kappa M \ . \qquad \text{(A2)}$$

The Newton gravitational potential φ_n used in Eq.A1 is derived from the minimum field energy condition:

$$\delta\int_\rho \left(\frac{\partial \varphi_n}{\partial \rho}\right)^2 \rho^2 d\rho = 0 \ , \qquad \text{(A3)}$$

written here again for simplicity in the flat physical space-time notation. The first integral of EL equation corresponding to this variational problem is:

$$\frac{\partial \varphi_n}{\partial \rho} = \frac{\kappa\, M}{\rho^2} \ , \qquad \text{(A4)}$$

where the constant of integration was set equal to: κM in order to obtain an agreement with observations and measurements. After integrating this result once more, setting the potential at the infinity to zero, the familiar Newton gravitational potential formula is obtained:

$$\varphi_n = -\frac{\kappa\, M}{\rho(r)} \ , \qquad \text{(A5)}$$

but with the natural radius replaced by the physical radius. This is a reasonable result, following the discussion in section **3**, where it was explained that the gravitation field distorts the natural space-time thus affecting the natural radius, but leaving the physical radius unchanged. The physical radius and consequently the gravitational potential are the spatial invariants independent of a particular choice of the spatial coordinate system thus satisfying the Einstein general covariance principle in a static field. This covariance is violated by the Schwarzschild gravitational potential, which demonstrates once more that the Schwarzschild metric does not describe the reality correctly.

It is also convenient to rearrange Eq.A1 and rewrite the parenthesis term as follows:

$$\left(g^{rr}\frac{\partial\varphi_n}{\partial r}\right)\rho^2\sqrt{g_{rr}} = \rho^2 g_{tt}\frac{\partial\varphi_n}{\partial r}\sqrt{g_{rr}} = g_{\varphi\varphi}\frac{\partial\varphi_n}{\partial r}\sqrt{g_{rr}} = \kappa M\ , \qquad \text{(A6)}$$

where the relation $g_{\varphi\varphi} = \rho^2 g_{tt}$ is identified with the equivalent expression for the conservation of angular momentum. In these derivations it is, of course, considered that the equatorial plane coordinate system is orthogonal, because: $\vartheta = \pi/2$, and that the metric coefficients further satisfy the relation: $g_{tt}g_{rr} = 1$ also written as: $g_{tt} = g^{rr}$. Substituting into Eq.A6 for the potential from Eq.22 then leads to the result:

$$g_{\varphi\varphi}\sqrt{g_{rr}}\frac{\partial\varphi_n}{\partial r} = \frac{g_{\varphi\varphi}}{g_{tt}\sqrt{g_{tt}}}\frac{c^2}{2}\frac{\partial g_{tt}}{\partial r} = \kappa M\ . \qquad \text{(A7)}$$

Finally, multiplying Eq.A7 by $2/c^2$ results in the general formula introduced in Eq.24 that any metric of the form given in Eq.1 must satisfy:

$$\frac{g_{\varphi\varphi}}{g_{tt}\sqrt{g_{tt}}}\frac{\partial g_{tt}}{\partial r} = R_s\ . \qquad \text{(A8)}$$

Eq.A8 is the Gauss law relating the metric coefficients to the amount of the fully enclosed centrally gravitating rest mass by the integrating spherical surface. The formula is similar to the definition of Komar mass [10] used in the GRT, but with some small differences due to the MTG approach used in this derivation that are not to be discussed here any further since they are outside of the scope of this paper.

From the above derivations and the derivations presented in the main body of the paper it is therefore clear that the metric coefficient g_{tt} follows uniquely from Eq.22 and the modified Newton gravitational potential formula as derived in Eq.A5. The metric coefficient g_{rr} follows from the condition: $g_{tt}g_{rr} = 1$, and the metric coefficient $g_{\varphi\varphi}$ follows from Eq.A8, or more directly from the equivalent formula for the conservation of angular momentum $g_{\varphi\varphi} = \rho^2 g_{tt}$.

Einstein field equations are thus not necessary for finding the correct metric for the space-time of the non-rotating centrally gravitating body, which corresponds to reality, does not lead to absurdities such as Black Holes and Event Horizons, and what is most important, does not lead to contradictions with the well-established laws of physics.

It is thus left up to the reader to decide whether the arguments and derivations presented in this paper are sufficient to prove the GRT incorrect. The main stream science unfortunately will never admit that their more than 100 years long belief in the GRT dogma could possibly be wrong. Hundreds of papers and publications are printed every year to perpetuate this belief and worship it whenever possible. The GRT is a religion, not a science, and thus cannot correct itself. For additional critique of the GRT and the related topics, such as the non-existence of the Gravitomagnetic field, the reader is referred to Ref. [11] and Ref. [12].

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